# "Statistical Mechanics, Integrability and Combinatorics"
# Schedule for Week 1, May 11-15

**<u>Monday May 11, Room B, 14.30</u>**

**Weighted Hurwitz numbers and hypergeometric $\tau$-functions**
John Harnad *(CRM and Concordia University, Montreal)*

Parametric families in the center $Z(C[S_n])$ of the group algebra of the symmetric group $S_n$ are constructed by identifying one set of indeterminates in the Cauchy-Littlewood formula as commuting Jucys-Murphy elements, and the other as weighting parameter values. Their eigenvalues provide coefficients in the double Schur function expansion of 2D Toda $\tau$-functions of hypergeometric type. Expressing these in the basis of products of power sum symmetric functions, the coefficients are interpreted geometrically as parametric families of weighted Hurwitz numbers, enumerating weighted branched coverings of the Riemann sphere. Combinatorially, they may be interpreted as weighted sums over paths in the Cayley graph of $S_n$ generated by transpositions. Dual pairs of bases for the algebra of symmetric functions provide both the geometrical and combinatorial significance of these weighted enumerative invariants. If time permits, a quantum deformation of these results, based on the generating function for Macdonald polynomials, will be presented.

**<u>Monday May 11, Cloister, 17.30</u>**

Welcome drink

**<u>Wednesday May 13, Room B, 11.30</u>**

**Discrete parafermions and quantum-group symmetries**
Yacine Ikhlef *(CNRS-UPMC Jussieu, Paris)*

In this talk, I will give an overview of discrete parafermions in integrable lattice models, based on the Bernard-Felder construction of non-local quantum group currents, and focusing on the example of the six-vertex model and the related loop model on the square lattice.

**Thursday May 14, Room B,**
**Mini-workshop on "Integrability and Combinatorics"**

**10.00-10.45**
**Limit shapes in the Schur process**
Dan Betea *(CNRS-UPMC-Jussieu, Paris)*

We will talk about the asymptotics of large pyramid partitions, q^Volume weighted, and non-uniformly weighted Aztec diamonds from the perspective of Schur processes. Joint work with Cedric Boutillier and Mirjana Vuletić.

**11.00-11.45**
**Computing the inverse Kasteleyn matrix for domino tilings of Aztec diamonds**
Sunil Chhita *(Bonn University)*

Simulations of domino tilings of large Aztec diamonds give striking pictures due to the emergence of macroscopic regions. These regions are often referred to as solid, liquid and gas. Limiting curves separate these regions and interesting probabilistic behaviors occur around these curves, which are related to random matrix theory. One approach to analyze these behaviors is through entries of the inverse Kasteleyn matrix which give joint probabilities of dominoes occuring in a random tiling. In this talk, we present an elementary combinatorial method, via certain recurrence relations, which computes the generating function of the inverse Kasteleyn matrix for uniform domino tilings of the Aztec diamond, which contains two macroscopic regions. This method also extends to give a derivation of the generating function of the two-periodic Aztec diamond, which contains all three macroscopic phases. The talk is based on joint work with Benjamin Young (Oregon).

**11.45-12.30**
**Macdonald superpolynomials and the Ruijsenaars-Schneider model**
Olivier Blondeau-Fournier *(King's College, London)*

Macdonald polynomials were discovered around 1988 and provide a remarkable (and rich) generalization of many other symmetric polynomials, such as the Jack, Hall-Littlewood, Schur, zonal, etc. Macdonald polynomials are also related to the solutions of a certain quantum (mechanic) integrable model, the so called Ruijsenaars-Schneider (RS) model. Recently, supersymmetric extension of integrable models lead to the discovery of a new family of polynomials, called superpolynomials, that now depend on anti-commuting variables (in addition to the usual commuting variables). Quite surprising, the subset of symmetric superpolynomials, which are invariant under a diagonal action of the symmetric group, can be defined combinatorially provided one is ready to introduce new objects that generalize standard partitions: the superpartitions. In this talk I will give an introduction to superpolynomials and present a generalization of Macdonald polynomials. Depending of the time, here is an outline: 1) Classical basis. 2) Duality and orthogonality. 3) Macdonald superpolynomials. 4) Properties and conjectures. 5) Cherednik operators. 6) Supersymmetric version of the RS model.

**14.30-15.15**

**Thermodynamic limit of the six-vertex model with reflecting boundary**

Giuliano Pavan Ribeiro *(Universidade Federal de São Carlos, São Paulo)*

We study the thermodynamic limit of the six-vertex model with domain wall boundary and reflecting end. We evaluated the partition function explicitly in special cases. We calculated the homogeneous limit of the Tsuchiya determinant formula for the partition function. We evaluated the thermodynamic limit and obtain the free energy of the six-vertex model with reflecting end. We determined the free energy in the disordered regime.

**15.30-16.15**

**On Random matrices and spin chains**

Miguel Tierz *(Universidad Complutense de Madrid)*

We will introduce a one-dimensional spin chain model and review how its associated thermal correlation functions admit a random matrix description. We will show how to generalize the construction to the case of interactions beyond the nearest neighbour setting and discuss physical implications of the random matrix formulation.

**Friday May 15, Room B, 11.30**

**A special case of the XYZ model with boundaries**

Vladimir Mangazeev *(Australian National University, Canberra)*

It is known that on the combinatorial line $\eta = \pi/3$ the ground state energy of the XYZ periodic spin chain is proportional to the size of the system for even sizes only. In this paper we generalize this result to open boundaries. We consider the XYZ model at $\eta = \pi/3$ with non-diagonal boundary terms. Then we find a one-parametric family of boundary parameters when the ground state energy is proportional to the size of the system for both even and odd system sizes. In the trigonometric limit it reproduces the $U_q(sl(2))$-invariant XXZ spin Hamiltonian.

# “Statistical Mechanics, Integrability and Combinatorics” Schedule for Week 3, May 25-29

**Monday May 25, Room A, 14.30**

**From conormal bundles of Schubert varieties to loop models**
Paul Zinn-Justin *(CNRS & UMPC-Jussieu, Paris)*

In this work in collaboration with A. Knutson, we investigate the correspondence between algebraic geometry and quantum integrable systems (which has been recently popularized by the work of Maulik and Okounkov, among others) from the point of view of Grobner degenerations. The latter is very combinatorial in nature and works equally well for cohomology and K-theory. Following Knutson and Miller, I shall recall the simplest framework in which one can develop this approach, namely (matrix) Schubert varieties and Schubert and Grothendieck polynomials. After that, I shall formulate a broad extension of these results which will naturally lead us to loop models on general lattices: first noncrossing loops (Temperley-Lieb model), then, if time allows, crossing loops (Brauer model).

**Tuesday May 26, Room A, 14.30**

**Airy diffusions and $N^{1/3}$ fluctuations in the 2D and 3D Ising models**
Senya Shlosman *(Aix-Marseille Université)*

Consider the 3D Ising model at a low temperature. We will look at the level lines of the Ising droplet near its edge. I will explain that their fluctuations are of the order of $N^{1/3}$. When scaled by $N^{1/3}$ , their limiting behavior for large $N$ is given by the Airy diffusion process. This diffusion has appeared earlier in a paper by Ferrari and Spohn, where the Brownian motion above the parabolic barrier is considered.
Work in progress with D. Ioffe and Y. Velenik.

**Wednesday May 27, Room A, 11.30**

**Elliptic cohomology and real life**
Andrei Okounkov *(Columbia University, New York)*

In recent year, there has been a surge of interests in relating quantum integrable systems to questions of enumerative geometry. I will review some aspects of such connections, which prove to be very fruitful for both fields. A natural challenge is push this connection to the extreme and understand elliptic R-matrices and related integrable system from geometric perspective. This will be the topic of my talk, based on joint work with Mina Aganagic.

**Thursday May 28, Room A**
**Mini-workshop**

**Morning Session**

**9.30-10.10**
**Counting perfect matching in graphs with application in monomer-dimer models**
Afshin Behmaram *(University of Tabriz)*

In this talk, we introduce the Pfaffian method for counting perfect matching (monomer-dimer) in graphs. Using this methods we calculate the number of perfect matching in some class of lattice models. Also, we give upper and lower bound for the number of perfect matching in some class of graphs.

**10.10-10.50**
**Quasi-invariants of 2-knots and integrable models**
Dmitry Talalaev *(Moscow State University, and ITEP, Moscow)*

I'll talk about the tetrahedron equation, some natural problems associated with it, the problem of describing the invariants of 2-knots using the theory quandles cohomology and some new construction of quasi-invariants based on the 3-dimensional lattice statistical models with special properties. A few words I would say about a purely combinatorial question of describing the so-called n-simplicial complexes. I will also emphasize the relation of this structure with the theory of quantum integrable systems on two-dimensional lattices.

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**11.10-11.50**
**The Scalar product of XXZ spin chain. Application to the ground state at $\Delta = -1/2$**
Alexander Garbali *(CNRS & UPMC Jussieu, Paris)*

The computation of the correlation functions of the integrable XXZ spin chain can be done using the form factor approach. This approach relies on the scalar products of Bethe states. We study the ground state scalar product $S_n$ of the inhomogeneous XXZ $s=1/2$ spin chain of length $2n$ at its combinatorial point $\Delta = -1/2$ with twisted periodic boundary conditions. At this point the ground state eigenvalue $\tau_n$ of the transfer matrix is known and has a simple form that does not contain the Bethe roots. We use the knowledge of $\tau_n$ and the Slavnov determinant written in a suitable form to obtain a closed expression for the scalar product $S_n$ and the norm of the ground state.

**11.50-12.30**
**Sum rule for a mixed boundary qKZ equation**
Caley Finn *(The University of Melbourne)*

I will describe a graphical construction giving a generalised sum rule for components of the solution of the Temperley-Lieb qKZ equation with mixed boundaries. Our construction uses the fact that the solutions of the qKZ equation can be written as factorised products of Baxterized Hecke generators. I will also discuss the connection to the Kazhdan-Lusztig basis and other bases of the Hecke algebra of type $B_N$.

**Afternoon Session**

**14.30-15.10**

**Boundary algebras and Kac modules for logarithmic minimal models**

Alexi Morin-Duchesne *(Université Catholique de Louvain)*

Virasoro Kac modules were initially introduced indirectly as representations whose characters arise in the continuum scaling limits of certain transfer matrices in logarithmic minimal models, described using Temperley-Lieb algebras. However, the structure of the Virasoro Kac modules have remained largely unidentified. Here, we introduce the appropriate algebraic framework for the lattice analysis as a quotient of the one-boundary Temperley-Lieb algebra. The corresponding lattice modules are introduced and examined using invariant bilinear forms. The structures of the Virasoro Kac modules are inferred from these results and are found to be given by finitely generated submodules of Feigin-Fuchs modules. Additional evidence for this identification is obtained by comparing the formalism of lattice fusion with the fusion rules of the Virasoro Kac modules.

**15.10-15.50**

**Non-equilibrium dynamics of the XXZ model**

Jacopo de Nardis *(University of Amsterdam)*

We study quantum quenches in integrable spin-1/2 chains in which we unitary evolve the ground state of the antiferromagnetic Ising model with the XXZ Hamiltonian with Delta >= 1. For this non-equilibrium situation, an application of the first-principles-based quench action method allows us to give an exact description of the post-quench late time steady state in the thermodynamic limit. We show that a generalized Gibbs ensemble (GGE), implemented using all known local conserved charges, fails to reproduce the exact quench action steady state and to correctly predict post-quench equilibrium expectation values of physical (local) observables [Wouters et al., *Phys. Rev. Lett.* 113, 117202 (2014)]. This shows that the set of local conserved charges obtained from the expansion of the logarithm of the transfer matrix is not complete and more unknown conserved quantities intervene in the non-equilibrium dynamics of the XXZ spin chain in the Gapped phase and in the isotropic point Delta=1.

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**16.10-16.50**

**The Fibonacci family of dynamical universality classes**

Gunter M Schütz *(Institute for Complex Systems II, Jülich)*

We use the universal nonlinear fluctuating hydrodynamics approach to study anomalous one-dimensional transport far from thermal equilibrium in terms of the dynamical structure function. Generically for more than one conservation law mode coupling theory is shown to predict a discrete family of dynamical universality classes with dynamical exponents which are consecutive ratios of neighboring Fibonacci numbers, starting with z = 2 (corresponding to a diffusive mode) or z = 3/2 (Kardar-Parisi-Zhang (KPZ) mode). If neither a diffusive nor a KPZ mode are present, all Fibonacci modes have as dynamical exponent the golden mean $z=(1+\sqrt{5})/2$. The scaling functions of the Fibonacci modes are asymmetric Lévy distributions which are completely fixed by the macroscopic current-density relation and compressibility matrix of the system. The theoretical predictions are confirmed by Monte-Carlo simulations of a three-lane asymmetric simple exclusion process.

**16.50-17.30**

**Matrix Product Ansatz for nonequilibrium steady states of driven quantum systems: XXZ spin chain, Hubbard model and others**

Vladislav Popkov *(University of Cologne)*

We review exact results from last 3 years, concerning one-dimensional open quantum systems, connected at the ends to a dissipative baths, which sustain global gradients of magnetization, energy, etc. across the system. Recent progress has allowed to push the concepts of integrability of quantum systems beyond the thermal equilibrium: exact nonequilibrium steady states (NESS) for several celebrated 1D quantum many-body systems models (XXZ spin chain, Hubbard model) were calculated analytically. The non-equilibrium integrability results from an additional degree of freedom, hidden in the Yang-Baxter structure for its `equilibrium' counterpart. This hidden degree of freedom (related to the amplitude of the dissipative term), gives rise to non-unitary representations of quantum symmetries of the models and allows to formulate a Matrix Product Ansatz for the NESS. A transfer matrix of the problem has an underlying Yang-Baxter structure, which hints at possible integrability of the full Liouvillean dynamics.

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**Friday May 29, Room A, 11.30**

**Littlewood-Richardson coefficients and integrable tilings**

Michael Wheeler *(The University of Melbourne)*

Littlewood-Richardson coefficients are the expansion coefficients in the product of two Schur functions. The subject of this talk will be the Knutson-Tao puzzles, which are certain tilings of the triangular lattice which enumerate Littlewood-Richardson coefficients. I will describe how Knutson-Tao puzzles can be obtained using the framework of quantum integrability, and why this framework is natural for the study of symmetric functions. I will discuss various new generalizations of Knutson-Tao puzzles to other families of symmetric functions, including Grothendieck and Hall-Littlewood polynomials.

# “Statistical Mechanics, Integrability and Combinatorics”
# Schedule for Week 4, June 1-5

**Monday June 1, Room B, 11.30**

**Exact enumeration of alternating sign matrices**
Roger Behrend *(Cardiff University)*

I will review a range of classical and recent results for the exact enumeration of alternating sign matrices with prescribed values of certain statistics (in particular, the numbers of inversions and -1's, and the positions of the 1's in the first and last rows and columns), and in some cases with certain symmetry conditions imposed. The results can typically be obtained by using connections with the statistical mechanical six-vertex model with domain-wall (or related) boundary conditions.

**Monday June 1, Cloister, 17.30**

Wine and Cheese “garden party”
Accompanying persons are welcome.

**Tuesday June 2**

June 2 is National holiday in Italy (Festa della Repubblica).

GGI is closed but you are anyway authorized to come to the Institute.
Note that lunch is not available.

You can access GGI area through the small gate
in Via G. Righini 2, using the pink key.

**Wednesday June 3, Room B, 11.30**

**Combinatorial aspects of correlation functions of integrable models**
Nikolay Bogoliubov (*St.-Petersburg Department of V. A. Steklov Mathematical Institute and ITMO University)*

We discuss the connection between quantum integrable and some aspects of enumerative combinatorics and the theory of partitions. As a basic example, we consider the spin XXZ Heisenberg chain in the limiting cases of zero and infinite anisotropy. The representation of the Bethe wave functions via the Schur functions allows to apply the well-developed theory of the symmetric functions to the calculation of the thermal correlation functions as well as of the form-factors. The determinantal expressions of the form-factors and of the thermal correlation functions are obtained. We provide a combinatorial interpretation of the formula for the correlation functions in terms of nests of the self-avoiding lattice paths. The interpretation proposed is in turn related to enumeration of the boxed plane partitions. The asymptotical behavior of the thermal correlation functions is studied in the limit of small temperature provided that the characteristic parameters of the system are large enough. The leading asymptotics of the correlation functions are found to be proportional to the squared numbers of boxed plane partitions.

**Thursday June 4, Room B, 11.30**

**Non-linear integral equation approach to** sl(2|1) **integrable network models**
Andreas Klümper *(University of Wuppertal)*

An integrable sl(2|1) invariant network model with alternating 3 and $\bar 3$ representations on vertical and horizontal lines is considered. This system can be formulated equivalently as a superspin chain. The model is `solvable' by nested Bethe ansatz which yields two sets of coupled equations for the Bethe roots.
The model was introduced by Gade (1998) and was extensively investigated by Essler, Frahm, Saleur (2005) most notably by numerical techniques. There are different motivations for the study of this kind of models. One reason is due to the appearance of a non-compact degree of freedom in the continuum limit of lattice models with staggering and (by definition) compact local quantum space.
I will focus on the possibility of derivation and solution of well posed non-linear integral equations (NLIE) for suitable, finitely many auxiliary functions. This is indeed possible by an ansatz that has been developed for the thermodynamics of the supersymmetric *tJ* model. For this model a set of three auxiliary functions were shown to satisfy a closed set of NLIE. For the network model two copies of these functions satisfy a closed set of six NLIE. These NLIE can be written as NLIE for two `weakly coupled' su(2) spin-1 chains of Takhtajan-Babujian type. With some tweaks the equations can be solved numerically. This is work in progress.

**Friday June 5, Room B, 11.30**

**Application of the hidden fermionic structure to the CFT**
Hermann Boos *(University of Wuppertal)*

We discuss the scaling limit of the fermionic operators that were originally constructed for the lattice six vertex model. We relate these operators to the usual Virasoro generators modulo the integrals of motion. Also the OPE in the fermionic basis and the recursion relations for the conformal blocks discovered by Al. Zamolodchikov in eighties as well as some other aspects are discussed.

# “Statistical Mechanics, Integrability and Combinatorics”
# Schedule for Week 5, June 8-12

**<u>Monday June 8, Room B, 14.30</u>**

**Spin chains with generic boundaries**
Nicolai Kitanine (Université de Bourgogne)

In this seminar I'll consider the XXZ spin chain with the most general boundary term. It will be shown that the separation of variables method permits to construct the complete set of eigenstates and to characterize the spectrum in terms of the solutions of the inhomogeneous version of the Baxter T-Q equation. In the constrained case this equation reduces to the usual homogeneous T-Q equation. Finally I’ll discuss the ways to compute the correlation functions and the form factors for the models solvable by the separation of variables technique.

**<u>Tuesday June 9, Room B, 11.30</u>**

**Correlations and inhomogeneous field theory inside the arctic circle**
Jean-Marie Stéphan *(Max Planck Institute for the Physics of Complex Systems, Dresden)*

A one-dimensional toy-model of fermionic particles evolving in imaginary time from a domain-wall initial state is introduced, and solved. The main interest of this toy-model is that it exhibits the “arctic-circle phenomenon” originally discovered in dimer models on the Aztec diamond, namely a spatial phase separation between a critically fluctuating region and a frozen region.
The purpose of the talk is to study the critical region from a field-theoretical perspective. Large-scale correlations inside the disk are expressed in terms of correlators in a (euclidean) massless Dirac theory. It is observed that this theory is inhomogeneous: contrary to better understood models or geometries the metric is position-dependent, so it is in fact a Dirac theory in curved two-dimensional space. The technique used to solve the toy-model can be extended to deal with the transfer matrices of other models: dimers on the honeycomb lattice, on the square lattice, and the six-vertex model at the free fermion point ($\Delta = 0$). In all cases, the underlying action is Dirac in curved space.

**<u>Wednesday June 10, Room B, 11.30</u>**

**Yang-Baxter Maps, Discrete Integrable Equations and Quantum Groups**
Vladimir Bazhanov *(Australian National University, Canberra)*

For every quantized Lie algebra there exists a map from the tensor square of the algebra to itself, which by construction satisfies the set-theoretic Yang-Baxter equation. This map allows one to define an integrable discrete quantum evolution system on quadrilateral lattices, where local degrees of freedom (dynamical variables) take values in a tensor power of the quantized Lie algebra. The corresponding equations of motion admit the zero curvature representation. The commuting Integrals of Motion are defined in the standard way via the Quantum Inverse Problem Method, utilizing Baxter’s famous commuting transfer matrix approach. All elements of the above construction have a meaningful quasi-classical limit. As a result one obtains an integrable discrete Hamiltonian evolution system, where the local equation of motion are determined by a classical Yang-Baxter map and the action functional is determined by the quasi-classical asymptotics of the universal R-matrix of the underlying quantum algebra. In this paper we present detailed considerations of the above scheme on the example of the algebra $U_q(sl_2)$ leading to discrete Liouville equations, however the approach is rather general and can be applied to any quantized Lie algebra.

**Thursday June 11, Room B**
**Mini-Workshop**

**10.00-10.45**

**The entropy of six-vertex model with variety of different boundary conditions**
Thiago Silva Tavares *(Universidade Federal de São Carlos, São Paolo)*

In this work we study the dependence of the six-vertex model intensive properties on boundary conditions. In the first part I shall talk mainly on toroidal boundary conditions which mixed periodic and anti-periodic closings. We argue that these kinds of boundary conditions cannot produce different results from completely periodic boundary. In second part we address the fixed boundaries types and show some examples of boundaries agreeing or disagreeing with periodic's value. In particular we introduce the Néel boundary condition whose number of configurations we believe to be maximal among all fixed boundaries. By means of boundary merge we show that the residual entropy may take any value between zero and domain-wall's value. Then we provide some numerical results supporting the idea that the entropy may take any value between zero and periodic's value.

**10.45-11.30**

**Bruhat and Tamari orders in integrable systems**
Folkert Müller-Huissen *(Max-Planck-Institute for Dynamics and Self-Organization, Göttingen)*

A (weak) Bruhat order is a natural partial order on a symmetric group. It appears, e.g., in the scattering of KdV solitons in a "tropical limit". Tamari orders are partially ordered sets (actually lattices) based on the associativity law. They are physically realized in terms of (at fixed time tree-shaped) soliton solutions of the famous KP equation (Dimakis and M-H 2010). Addressing the combinatorics underlying simplex equations, which generalize the Yang-Baxter equation, Manin and Schechtman introduced in 1986 higher Bruhat orders. Corresponding higher (Stasheff-) Tamari orders can be obtained from the higher Bruhat orders via a kind of projection. In the same way as higher Bruhat orders encode the structure of simplex equations, higher Tamari orders determine generalizations ("polygon equations") of the pentagon equation (Dimakis and M-H, arXiv:1409.7855, to appear in SIGMA). In this talk we will try to explain all this in a fairly elementary way.

**11.45-12.30**

**Spontaneous Breaking of *U(N)* symmetry in invariant Matrix Models**
Fabio Franchini *(INFN, Sezione di Firenze)*

Matrix Models successfully capture the behavior of many strongly interacting systems in a variety of contexts, while being a wonderful playground of integrability and analytical methods. Traditionally, the requirement of base invariance has lead to the conclusion that these models describe only extended systems.

We show that deviations of the eigenvalue statistics from the Wigner-Dyson universality reflects itself on the eigenvector distribution and that gaps in the eigenvalue density break U(N) symmetry to a smaller one. This spontaneous symmetry breaking means that egeinvectors become localized to a sub-manifold of the Hilbert space (and, physically, that one describes a system which lacks ergodicity). This realization means that random matrix techniques can be lent to the study of new observables, which have not been examined before.
We also consider models with log-normal weight, such as those emerging in Chern-Simons and ABJM theories. They can be solved through q-deformed orthogonal polynomials and their eigenvalue distribution is intermediate between Wigner-Dyson and Poissonian, which candidates these models for describing a phase intermediate between the extended and the localized ones. We show that they have a much richer energy landscape than expected, with their partition functions decomposable in a large number of equilibrium configurations. We argue that this structure is a reflection of the non-trivial (multi-fractal) eigenvector statistics and comment on the implications of these results.

- F. Franchini; "On the Spontaneous Breaking of U(N) symmetry in invariant Matrix Models"; arXiv:1412.6523.
- F. Franchini; "Toward an invariant matrix model for the Anderson Transition"; arXiv:1503.03341.

**<u>Friday June 12, Room B, 11.30</u>**

**Observables at combinatorial points of solvable models.**
Bernard Nienhuis *(University of Amsterdam)*

Since the discovery of Razumov and Stroganov of remarkable properties of the ground states of the XXZ model at Delta=-1/2, I have been interested in obtaining observables of this kind of solvable models for finite size geometries. By 'this kind of models' I mean 1D quantum chains or 2D statistical models of which the ground state energy or free energy respectively has no finite size corrections. It has been useful to represent these models as loop models: sums of configurations of paths (or a single path) a 2D lattice, typically on a L x infinite geometry. In the talk I will discuss attempts (succesful or not) to obtain closed forms of observables or the relation of such quantities to objects in enumerative combinatorics or other seemingly unrelated fields.

# “Statistical Mechanics, Integrability and Combinatorics”
# Schedule for Week 6, June 15-19

**Monday June 15, Room B, 14.30**

**Plane partitions with two-periodic weights**
Sevak Mkrtchyan *(University of Rochester)*

We will discuss scaling limits of skew plane partitions with two-periodic weights under several boundary conditions. We will discuss the frozen boundary and the correlation kernel of the limiting point processes. Of particular interest is the process at the turning points. The turning points that appear in the homogeneous case split in the two-periodic case into pairs of turning points macroscopically separated by a “semi-frozen” region. As a result the point process at a turning point is not the GUE minor process, but rather a pair of GUE minor processes, non-trivially correlated. We will also discuss an intermediate regime when the weights are periodic but all converge to 1. In this regime the limit shape and correlations in the bulk are the same as in the case of homogeneous weights and periodicity is not visible in the bulk. However, the process at turning points is still not the GUE minor process.

**Monday June 15, Cloister, 17.30**

Wine and Cheese “garden party”
Accompanying persons are welcome

**Tuesday June 16, Room B, 11.30**

**Topological vertices and integrable models**
Omar Foda *(The University of Melbourne)*

A topological vertex is a combinatorial object that one associates to a plane partition. It is also a building block of instanton partition functions which, via the AGT correspondence, are expectation values of vertex operators in integrable models.
I wish to explain, with emphasis on computational details, how one can start from a set of topological vertices, glue them to construct a topological partition function, and choose the parameters of the latter to obtain conformal blocks of Virasoro minimal models.

**Wednesday June 17, Room B, 11.30**

**Generalized Smoluchowski Equations and Scalar Conservation Laws**
Fraydoun Rezakhanlou *(University of California, Berkeley)*

By a classical result of Bertoin, if initially a solution to Burgers' equation is a Levy process without positive jumps, then this property persists at later times. According to a theorem of Groeneboom, a white noise initial data also leads to a Levy process at positive times. Menon and Srinivasan observed that in the both aforementioned results the evolving Levy measure satisfies a Smoluchowski–type equation. They also conjectured that a similar phenomenon would occur if instead of Burgers' equation, we solve a general scalar conservation law with a convex flux function. Though a Levy process may evolve to a Markov process that in most cases is not Levy. The corresponding jump kernel would satisfy a generalized Smoluchowski equation. Along with Dave Kaspar, we show that of this conjecture is true for monotone solutions to scalar conservation laws. I also formulate some open questions concerning the analogous questions for Hamilton-Jacobi PDEs in higher dimensions.

## Thursday June 18, Room B

**11.00-11.45**

**Dimers on rail yard graphs**

Sanjay Ramassamy *(Brown University, Providence)*

The dimer model is a statistical mechanics model corresponding to perfect matchings on graphs. We introduce a general model of dimer coverings of certain planar bipartite graphs, which we call rail yard graphs (RYG). Using transfer matrices and dimer-localizing operators, we give explicit expressions for the partition function and for the inverse Kasteleyn matrix, which yields all dimer correlation functions. Plane partitions, domino tilings of the Aztec diamond and pyramid partitions arise as particular cases of the RYG dimer model.
This is joint work with Cédric Boutillier, Jérémie Bouttier, Guillaume Chapuy and Sylvie Corteel.

**11.45-12.30**

**Integrability of limit shape phenomenon in the six-vertex Model**

Aneth Sridhar *(University of California, Berkeley)*

The six vertex model on a planar region can be reformulated as a theory random stepped surfaces called height functions. In certain circumstances, the six vertex model exhibits the limit shape phenomenon: in the thermodynamic limit, the average height function is deterministic and conjecturally can be found by solving a certain variational problem. In this talk, we discuss the implications of the discrete integrability of the six vertex model (in the sense of commuting transfer matrices) on the integrability of the limit shape phenomenon (in the sense of commuting Hamiltonians for the PDE arising from the variational problem).

## Friday June 19, Room B, 11.30

**Appearance of determinants for stochastic growth models**

Tomohiro Sasamoto *(Tokyo Institute of Technology)*

For the Kardar-Parisi-Zhang (KPZ) equation and related discrete growth models, certain quantities can be written as Fredholm determinants. From this follows for example that the fluctuations of the models in the large time limit is described by the Tracy-Widom distributions from random matrix theory. An interesting point here is that many models in question are apparently not free-fermionic (not determinantal) but still admit the Fredholm determinant formulas for certain quantities. In this talk I will present and explain about a few such examples (TASEP, KPZ equation, O'Connell-Yor polymer etc).

# “Statistical Mechanics, Integrability and Combinatorics”
# Schedule for Week 8, June 29-July 3

**Monday June 29, Room B, 14.30**

**The Robbins number triangle and some of its symmetries**
Dan Romik *(University of California, Davis)*

The Robbins triangle is a triangle of positive integers A_{n,k} that famously appear in connection with the refined enumeration of alternating sign matrices (ASMs), as well as in several other enumeration problems. The numbers A_{n,k} are known to be given by an explicit formula involving a product of factorials. While the fascinating combinatorial properties of ASMs and their connection to square ice and other statistical physics models have justifiably gotten a lot of attention, the numbers A_{n,k} themselves possess some very intriguing "hidden" symmetries that seem worth exploring for their own sake, and this will be the focus of this talk. One of the symmetries I will discuss mysteriously appears in connection with the study of the so-called "Witten zeta function" associated with the group SU(3), and has no known relation to the combinatorics of ASMs or other related objects. The talk will be elementary and no knowledge will be assumed.

**Monday June 29, Cloister, 17.30**

Wine and Cheese “garden party”
Accompanying persons are welcome

**Tuesday June 30, Room B, 14.30**

**Towards a non-equilibrium Bethe ansatz for the Kondo model**
Eldad Bettleheim *(The Hebrew University of Jerusalem)*

An application of Slavnov's formula is presented which gives access to non-equilibrium properties in the Kondo model through non-linear integral equations extending such equations encountered in the equilibrium thermodynamic Bethe ansatz.

**Wednesday July 1, Room B, 11.30**

**The crossing probability for directed polymers in random media**
Andrea De Luca *(Univeristé Paris-Sud, Orsay)*

We study the probability that two directed polymers in the same random potential do not intersect. We use the replica method to map the problem onto the attractive Lieb-Liniger model with generalized statistics between particles. We obtain analytical expressions for the first few moments of this probability, and compare them to a numerical simulation of a discrete model at high-temperature. From these observations, several large time properties of the non-crossing probabilities are conjectured. Extensions of our formalism to more general observables are discussed.

**<u>Thursday July 2, Room B,</u>**

**<u>11.00-11.45</u>**

**The dimer model: monomers, Arctic Circle and CFT**

Nicolas Allegra *(Université de Lorraine, Nancy)*

In the first part of this presentation, some classical results of the pfaffian theory of the dimer model are introduced in a fermionic framework. The complete and detailed fermionic solution of the dimer model on the square lattice with an arbitrary number of monomers is presented [1] and some important applications will be detailed and compared to CFT results via the so-called height mapping [2]. In a second part, the arctic circle phenomenon will be introduced in a field theory point a view and some properties will be discussed.

[1] N.A, JY.Fortin, Phys. Rev. E 89, 062107

[2] N.A, Nuclear Physics B 894 (2015) 685–732

[3] N.A, J.Dubail, M.Haque, J-M Stephan and J.Viti, (in preparation).

**<u>11.45-12.30</u>**

**Off-critical interfaces in two dimensions. Exact results from field theory**

Alessio Squarcini *(SISSA, Trieste)*

We consider phase separation and interfaces of systems of classical statistical mechanics in two dimensions below criticality. While interfaces in 2D at criticality are described by SLE, exact results for the scaling limit of the order parameter profile in the off-critical regime have been obtained only by means of exact lattice computations for the 2D Ising model; a circumstance that raises the question about the role of Ising solvability. We will show how low-energy properties of two-dimensional field theory yield exact results for order parameter profiles, passage probabilities and interface structure in presence of single and double interfaces for different universality classes in 2D. Results available from the lattice solution of the Ising model in the plane and in the half-plane are recovered as a particular case.

[1] G. Delfino and A.S., Interfaces and wetting transition on the half plane. Exact results from field theory, Journal of Statistical Mechanics P05010 (2013)

[2] G. Delfino and A.S., Exact theory of intermediate phases in two dimensions, Annals of Physics 342, 171 (2014)

[3] G. Delfino and A.S., Phase separation in a wedge. Exact results, Physical Review Letters 113 (2014) 066101