Hydrodynamization and non-equilibrium attractors in the high-temperature QCD plasma

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Galileo Galilei Institute
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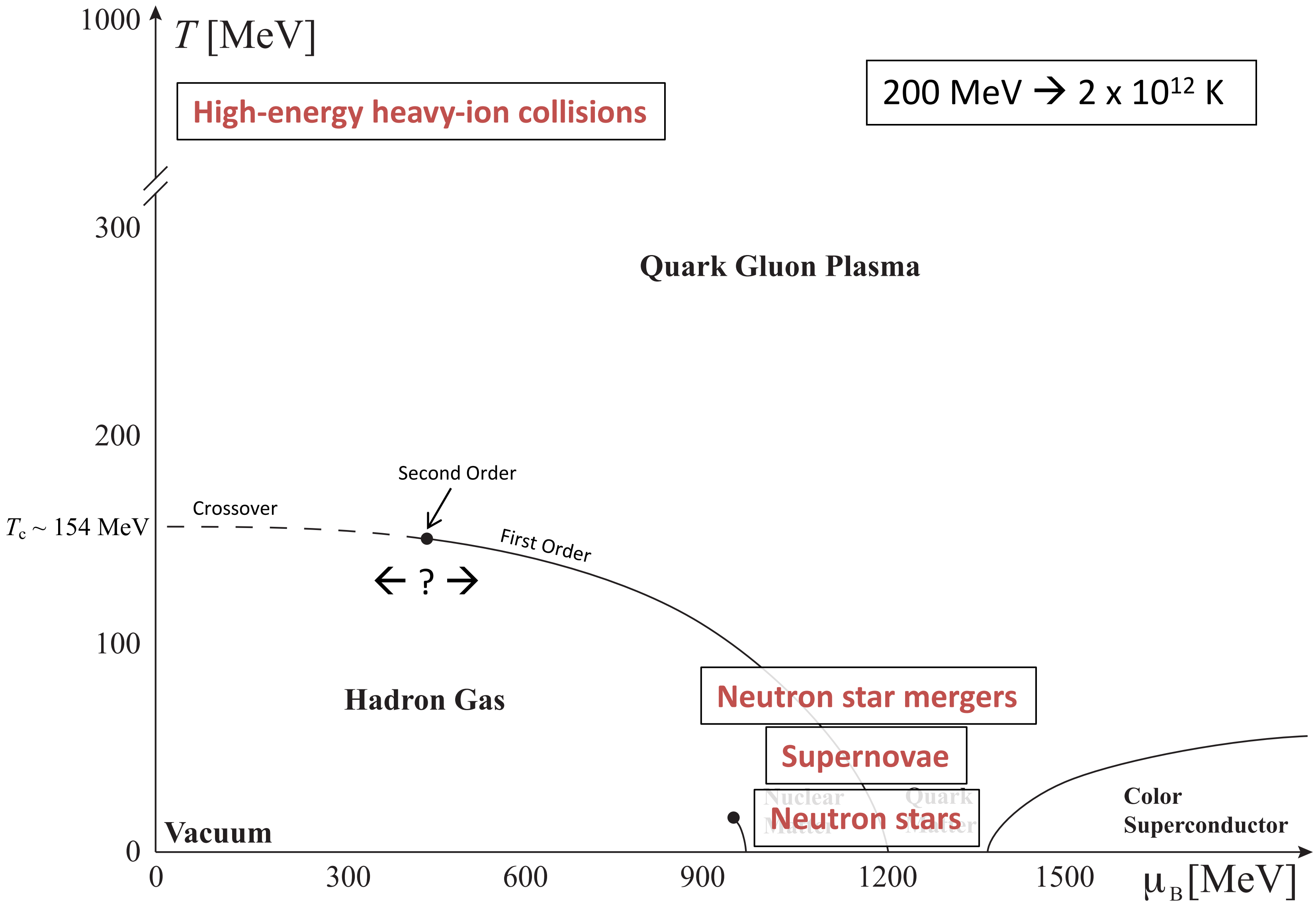
KENT STATE UNIVERSITY

U.S. DEPARTMENT OF ENERGY

Online slides and notes

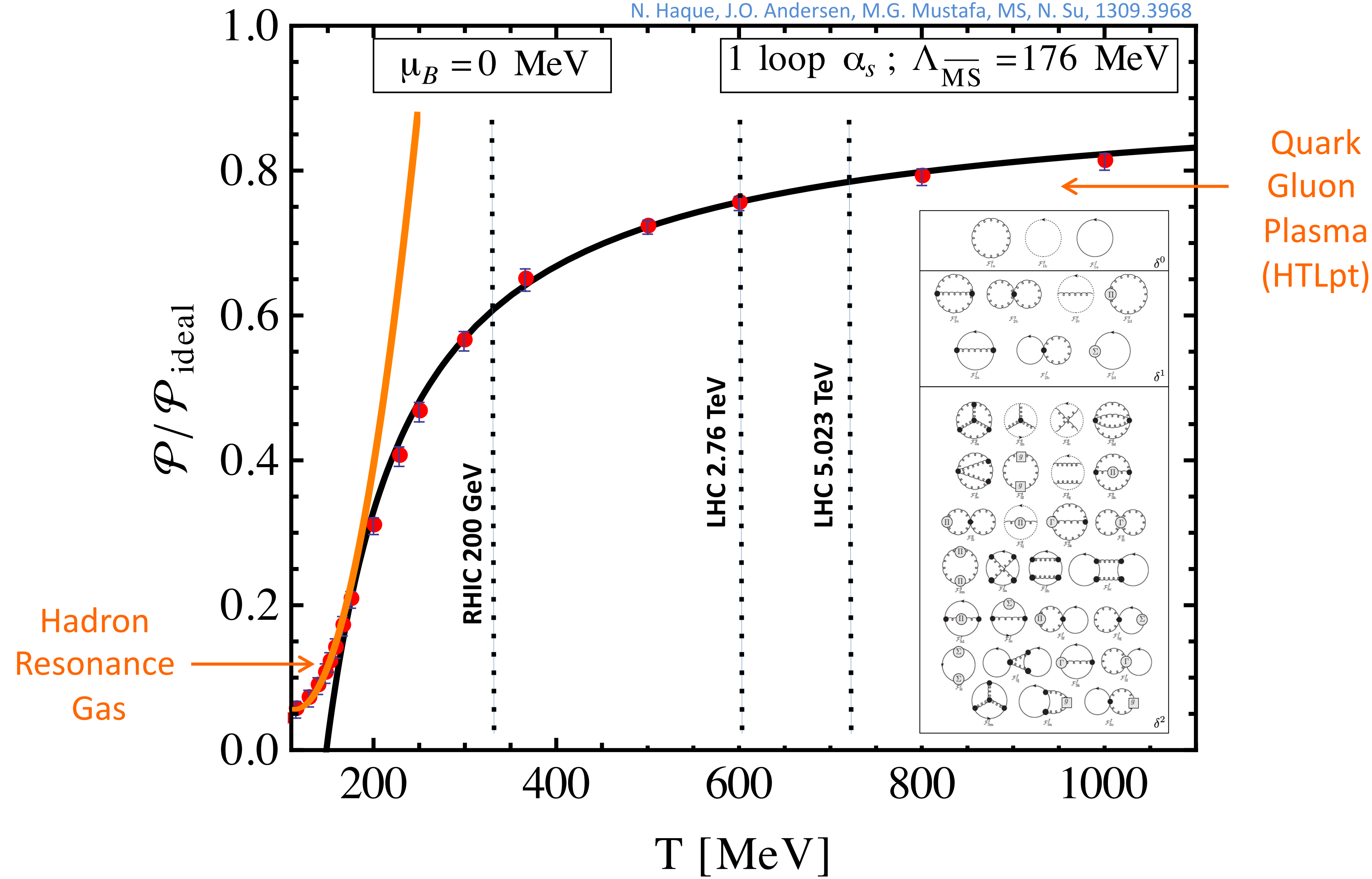
http://personal.kent.edu/~mstrick6/ggi

QCD Phase Diagram



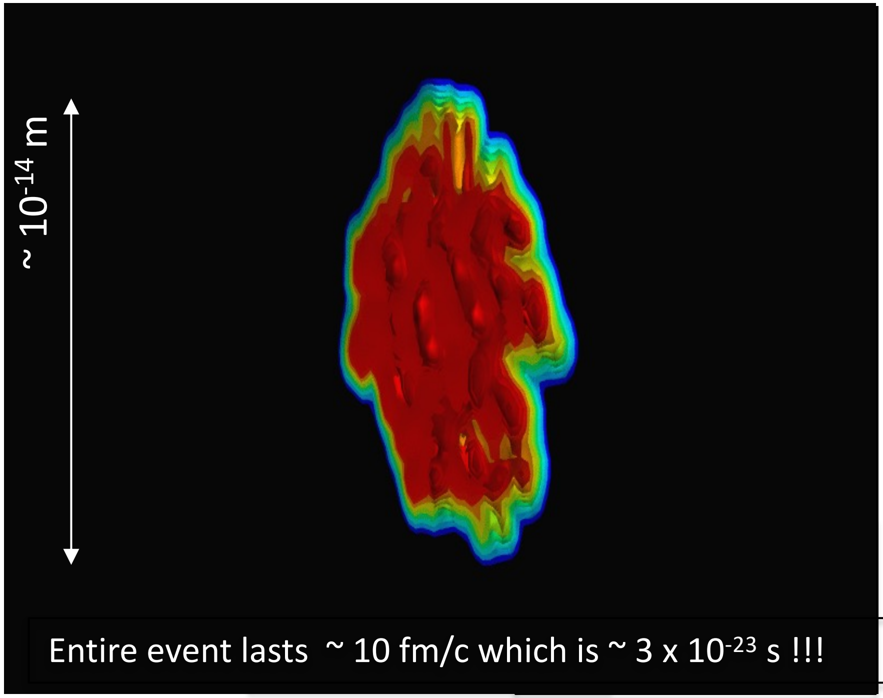
Pressure vs Temperature – μ_B = 0 MeV

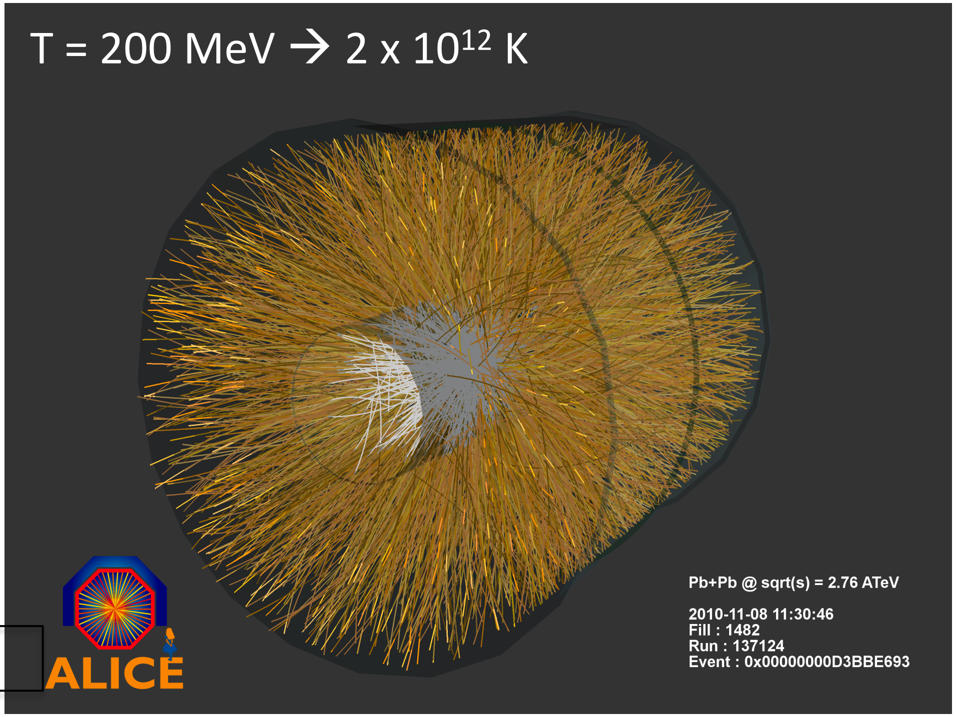
Andersen, Leganger, Su, and MS 1009.4644, 1103.2528
N. Haque, J.O. Andersen, M.G. Mustafa, MS, N. Su, 1309.3968



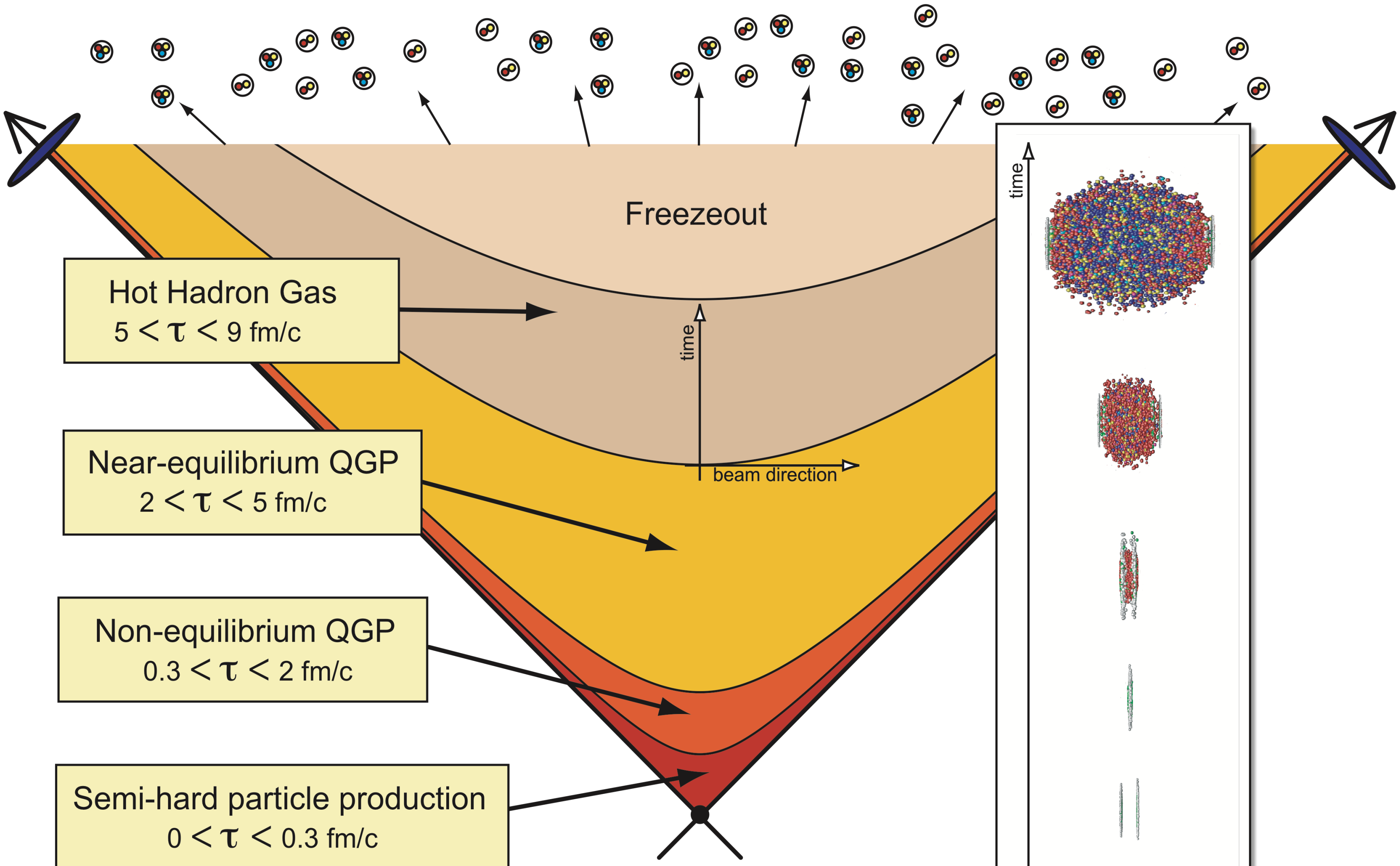
Ultrarelativistic heavy-ion collisions

- **RHIC**, BNL – Au-Au @ 200 GeV/nucleon (highest energy) → $T_0 \sim$ 400 MeV
- **LHC**, CERN – Pb-Pb @ 2.76 TeV → $T_0 \sim$ 600 MeV
- **LHC**, CERN – Pb-Pb @ 5.02 TeV → $T_0 \sim$ 700 MeV
- **RHIC**, BNL **BES** – Au-Au @ 7.7 - 39 GeV → $T_0 \sim$ 30-100 MeV [+finite density]
- **FAiR** (GSI), **NICA** (Dubna) – U-U @ 35 GeV → $T_0 \sim$ 100 MeV [+finite density]



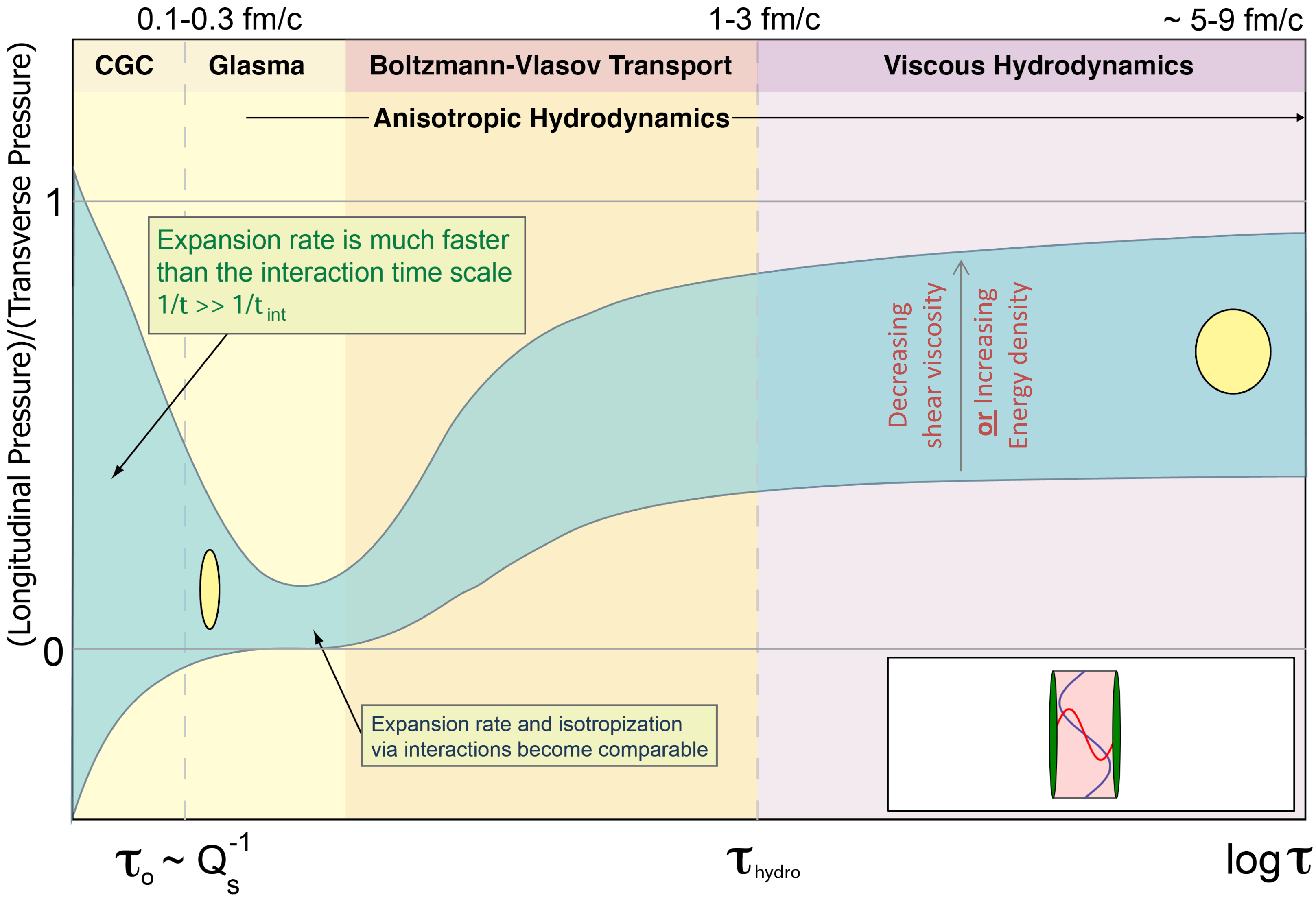


Heavy Ion Collision Timescales



1 fm/c ≅ 3 x 10^{-24} s

QGP momentum anisotropy cartoon



Non-equilibrium hydrodynamics and dynamical attractors

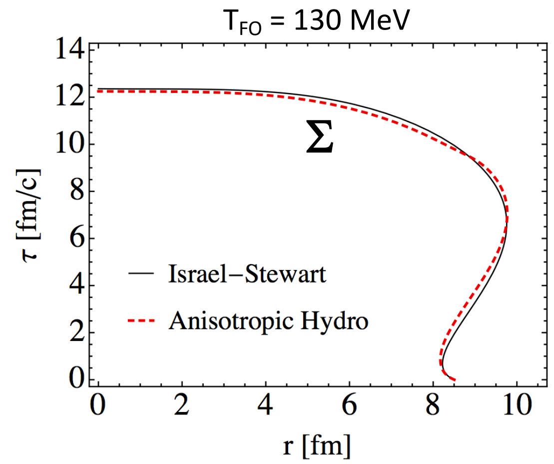
Why is this topic relevant?

- The QGP is highly **momentum-space anisotropic in the local rest frame** (All approaches agree on this, AdS-CFT based models, kinetic theory with/without plasma instabilities, 3d CGC, …).
- Despite these large momentum-space anisotropies, **dissipative hydrodynamics has proven to be quite successful** in describing experimental results (see the slides coming next).
- Why does it work so well and how can we use our understanding of "**hydrodynamization**" to improve existing hydrodynamics theories (e.g. anisotropic hydrodynamics, transseries, resurgence theory, etc.)?
- In my lectures, I will discuss the unreasonable effectiveness of dissipative hydrodynamics using the concept of **non-equilibrium dynamical attractors.** [see e.g. Heller and Spalinski, Phys. Rev. Lett. 115 (7), 072501 (2015); 1503.07514]
- I will also discuss going beyond hydro-based treatments of attractors using **non-equilibrium kinetic theory**, with both exact and numerical solutions of the Boltzmann equation.

How do hydro models work in practice?

- First, we must specify **initial conditions** for the energy density profile, viscous corrections ($\Pi^{\mu\nu}$), and fluid flow velocity vector in the full 3d volume.

- One then **numerically solves the viscous hydro differential equations** numerically using an advanced PDE solver and input from external (lattice QCD) calculations of the **equation of state**. 14 DOF are canonically: n, $T^{\mu\nu}$, u^i



- From the full 4d profile, one then extracts a 3-surface called the freeze-out hypersurface **Σ**

$$\left(p^0 \frac{dN}{d^3p}\right)_i = \frac{\mathcal{N}_i}{(2\pi)^3} \int f_i(x,p)\, p^\mu d^3\Sigma_\mu$$

  aka "**Cooper-Frye freeze-out**". Canonically, one takes constant energy density surface.
  Importantly, we need a form for f_i on **Σ.**

- This gives the "**primordial spectra**" which are then fed into a separate hadronic code which takes care of **resonance feed-down, hadronic re-scattering**, etc.

How do hydro models work in practice?

- First, we must specify **initial conditions** for the energy density profile, viscous corrections ($\Pi^{\mu\nu}$), and fluid flow velocity vector in the full 3d volume.

$$(\mathcal{E}+\mathcal{P}+\Pi)D_u u^\tau = -(u^r)^2\Big[\partial_\tau(\mathcal{P}+\Pi) - d_\nu \pi^\nu_\tau\Big] - u^\tau u^r\Big[\partial_r(\mathcal{P}+\Pi) - d_\nu \pi^\nu_r\Big],$$

$$(\mathcal{E}+\mathcal{P}+\Pi)D_u u^r = -u^\tau u^r\Big[\partial_\tau(\mathcal{P}+\Pi) - d_\nu \pi^\nu_\tau\Big] - (u^\tau)^2\Big[\partial_r(\mathcal{P}+\Pi) - d_\nu \pi^\nu_r\Big],$$

$$D_u\mathcal{E} = -(\mathcal{E}+\mathcal{P}+\Pi)\theta_u - \pi^r_r(1-v^2)^2\nabla^{\langle r}u^{r\rangle} - r^2\,\pi^\phi_\phi\nabla^{\langle\phi}u^{\phi\rangle} - \tau^2\,\pi^\varsigma_\varsigma\nabla^{\langle\varsigma}u^{\varsigma\rangle},$$

and

$$\tau_\Pi D_u\Pi + \Pi = -\zeta\theta_u - \delta_{\Pi\Pi}\Pi\theta_u - \lambda_{\Pi\pi}\Big[2r^2\pi^\phi_\phi\nabla^{\langle\phi}u^{\phi\rangle} + 2\tau^2\pi^\varsigma_\varsigma\nabla^{\langle\varsigma}u^{\varsigma\rangle} + r^2\pi^\varsigma_\varsigma\nabla^{\langle\phi}u^{\phi\rangle} + \tau^2\pi^\phi_\phi\nabla^{\langle\varsigma}u^{\varsigma\rangle}\Big],$$

$$\tau_\pi D_u\pi^\phi_\phi + \pi^\phi_\phi = -2r^2\eta\nabla^{\langle\phi}u^{\phi\rangle} - \delta_{\pi\pi}\pi^\phi_\phi\theta_u - \frac{\tau_{\pi\pi}}{3}\Big[-r^2\pi^\phi_\phi\nabla^{\langle\phi}u^{\phi\rangle} + 2\tau^2\pi^\varsigma_\varsigma\nabla^{\langle\varsigma}u^{\varsigma\rangle} + r^2\pi^\varsigma_\varsigma\nabla^{\langle\phi}u^{\phi\rangle} + \tau^2\pi^\phi_\phi\nabla^{\langle\varsigma}u^{\varsigma\rangle}\Big] - r^2\lambda_{\pi\Pi}\Pi\nabla^{\langle\phi}u^{\phi\rangle},$$

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where

$$-d_\nu\pi^\nu_\tau = v^2\partial_\tau\pi^r_r + v\partial_r\pi^r_r + \pi^r_r\Big[\partial_\tau v^2 + \partial_r v + \frac{v^2}{\tau} + \frac{v}{r}\Big] + \frac{1}{\tau}\pi^\varsigma_\varsigma,$$

$$d_\nu\pi^\nu_r = v\,\partial_\tau\pi^r_r + \partial_r\pi^r_r + \pi^r_r\Big[\partial_\tau v + \frac{v}{\tau} + \frac{2-v^2}{r}\Big] + \frac{1}{r}\pi^\varsigma_\varsigma.$$

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  aka take Imp

- This ... then fed into a separate hadronic code which takes care of **resonance feed-down, hadronic re-scattering**, etc.

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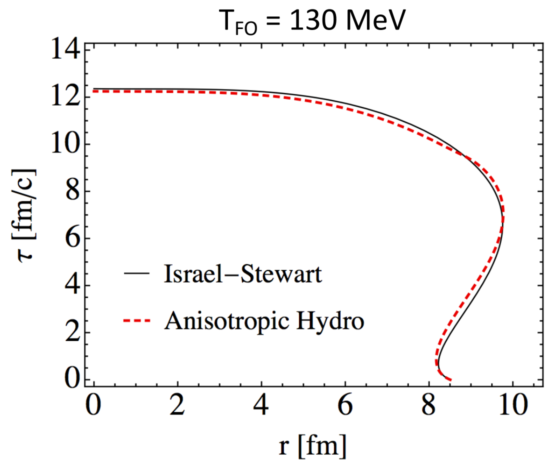

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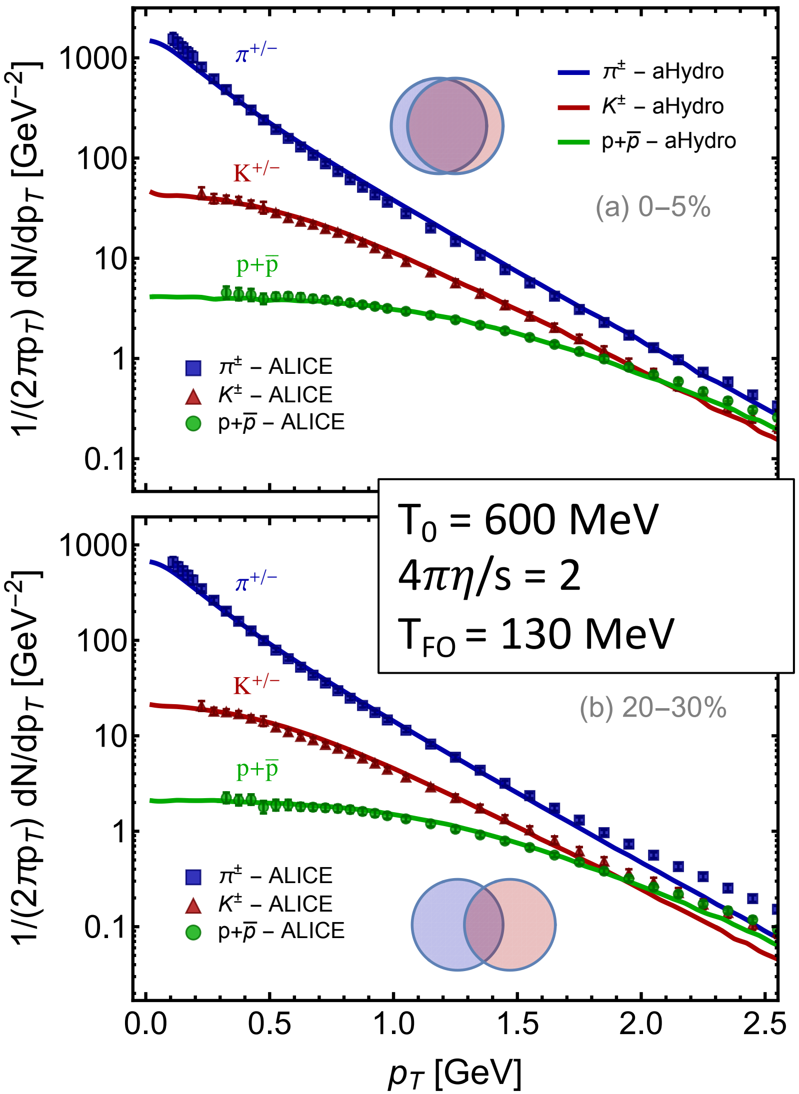
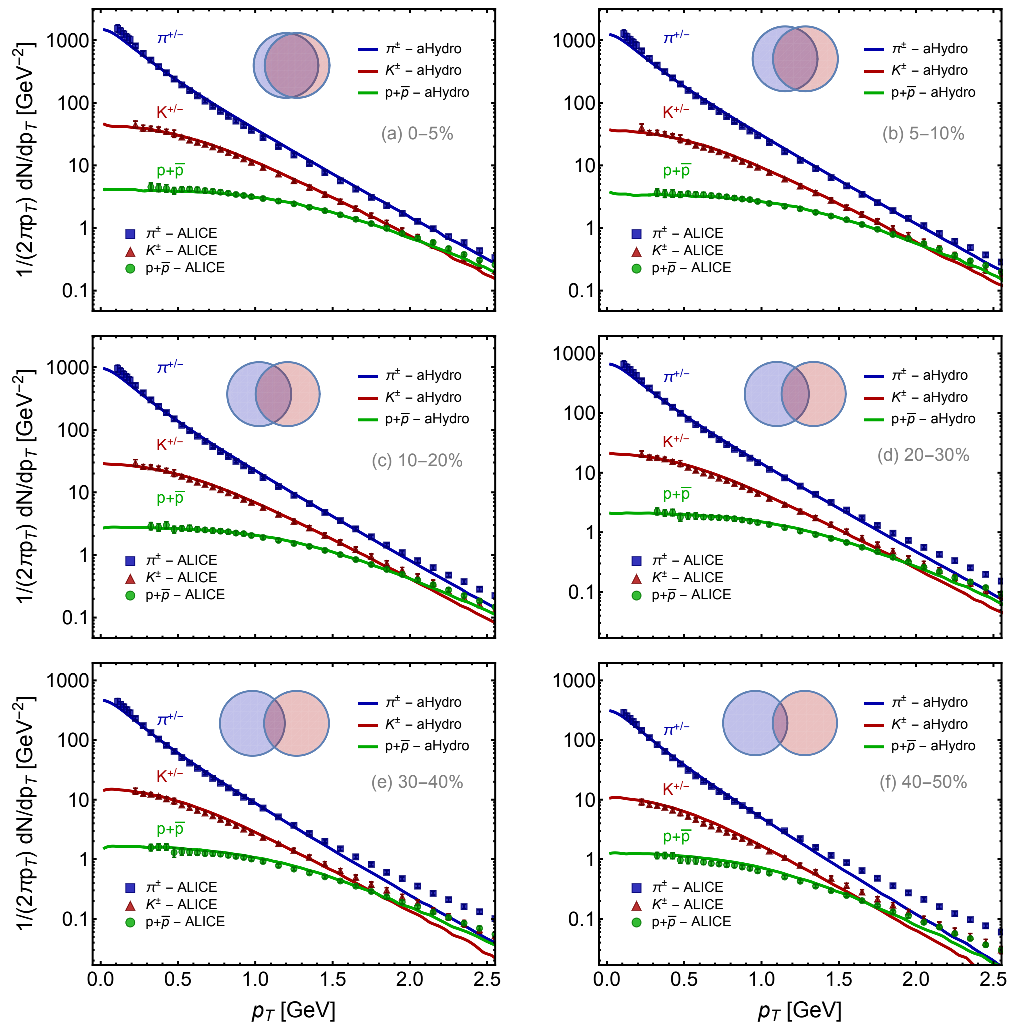
Identified particle spectra

Alqahtani, Nopoush, Ryblewski, MS, 1703.05808 (PRL); 1705.10191



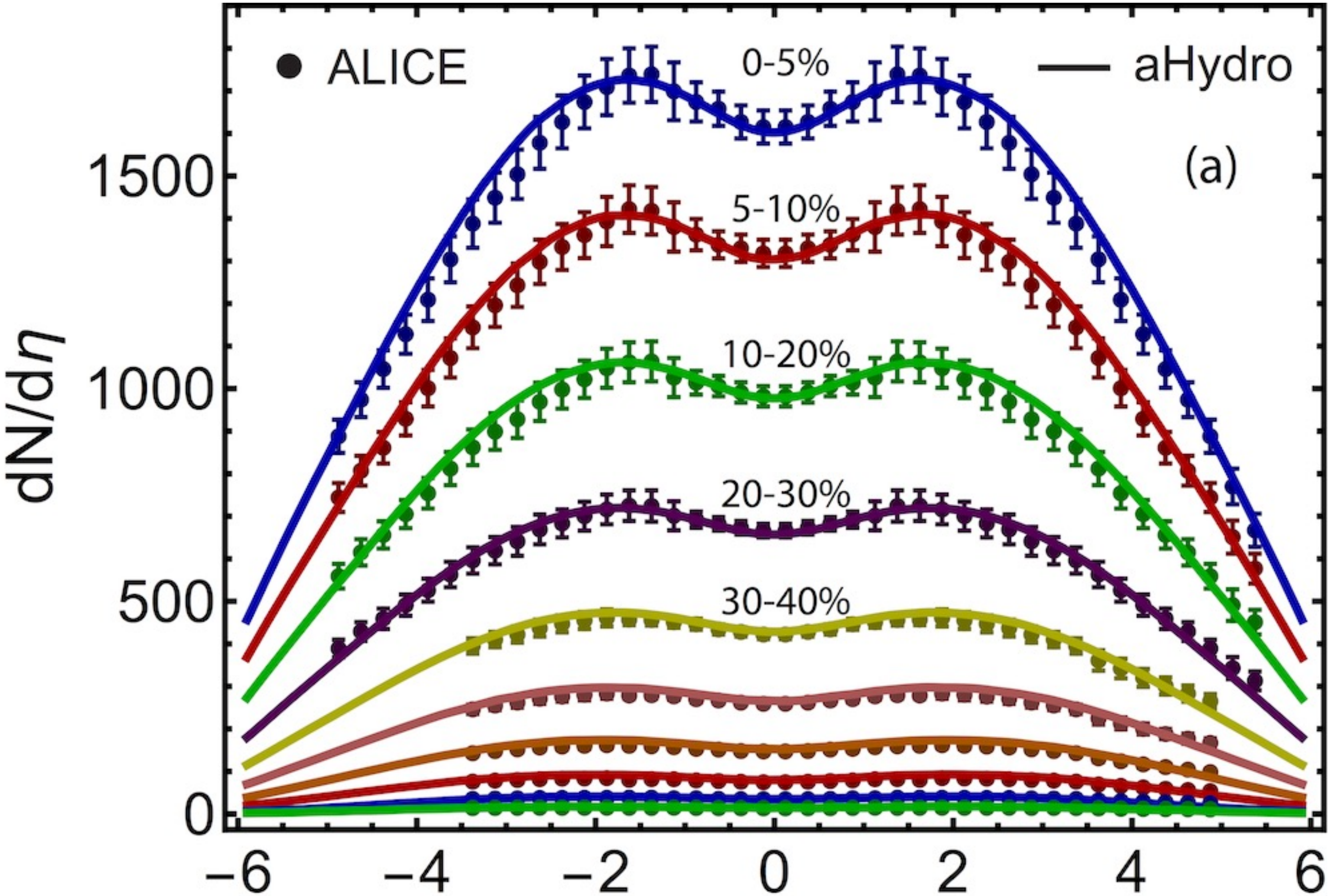
T_0 = 600 MeV
$4\pi\eta/s$ = 2
T_{FO} = 130 MeV



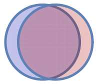
Data are from the ALICE collaboration data for **Pb-Pb collisions @ 2.76 TeV/nucleon**

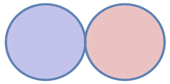
Pb-Pb vs Hydro: Charged particle multiplicities

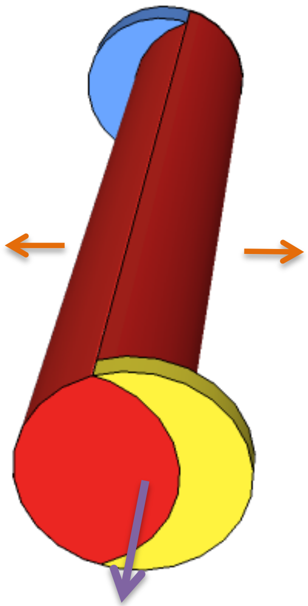
Alqahtani, Nopoush, Ryblewski, MS, 1703.05808; 1705.10191



$\eta = \text{arctanh}\left(\frac{p_L}{|\mathbf{p}|}\right)$







Collective Flow

- During non-central collisions overlap region breaks azimuthal symmetry

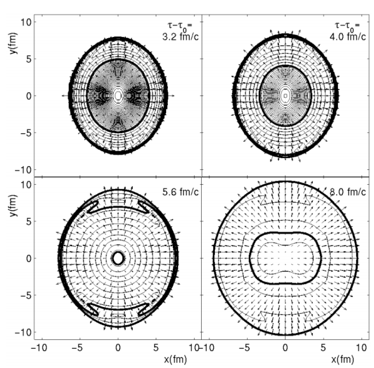
$$\frac{1}{N}\frac{dN}{d\phi} = \frac{1}{2\pi}\left(1 + 2v_1\cos\phi + 2v_2\cos 2\phi + 2v_3\cos 3\phi + \dots\right)$$

Directed Flow

Elliptic Flow

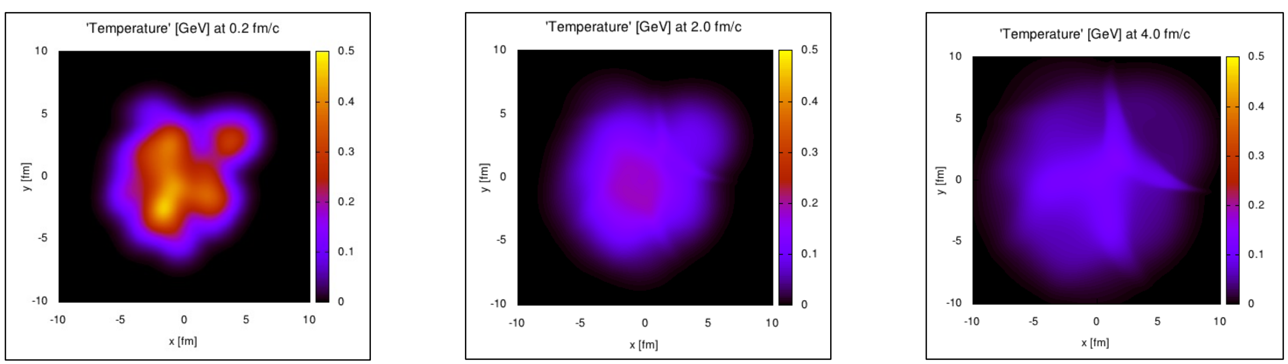
Triangular Flow

Smooth profile



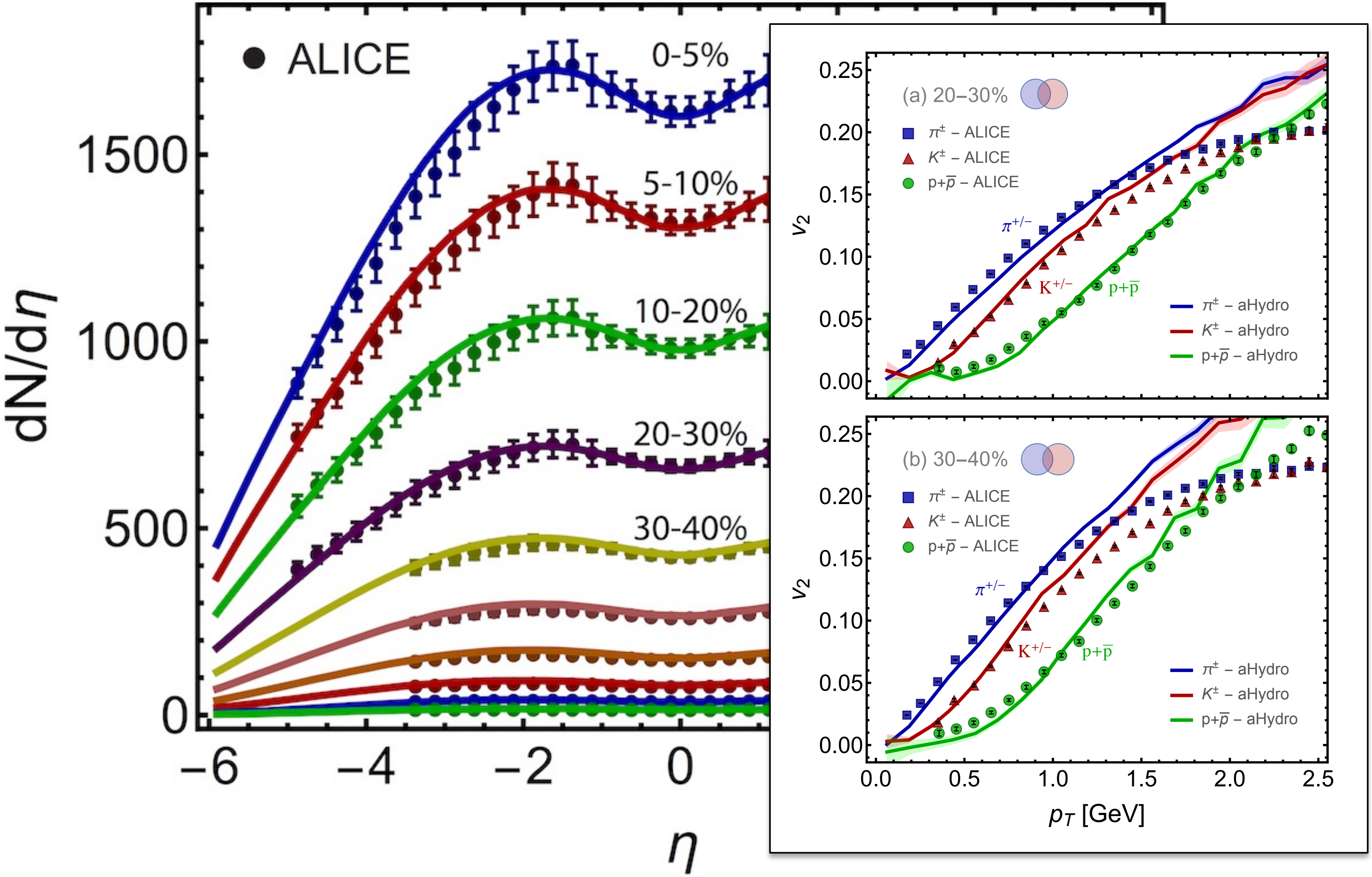
Kolb, Sollfrank, Heinz, Phys. Rev. C 62, 054909 (2000).

Event-by-Event fluctuations



Pb-Pb vs Hydro: Charged particle multiplicities

Alqahtani, Nopoush, Ryblewski, MS, 1703.05808; 1705.10191



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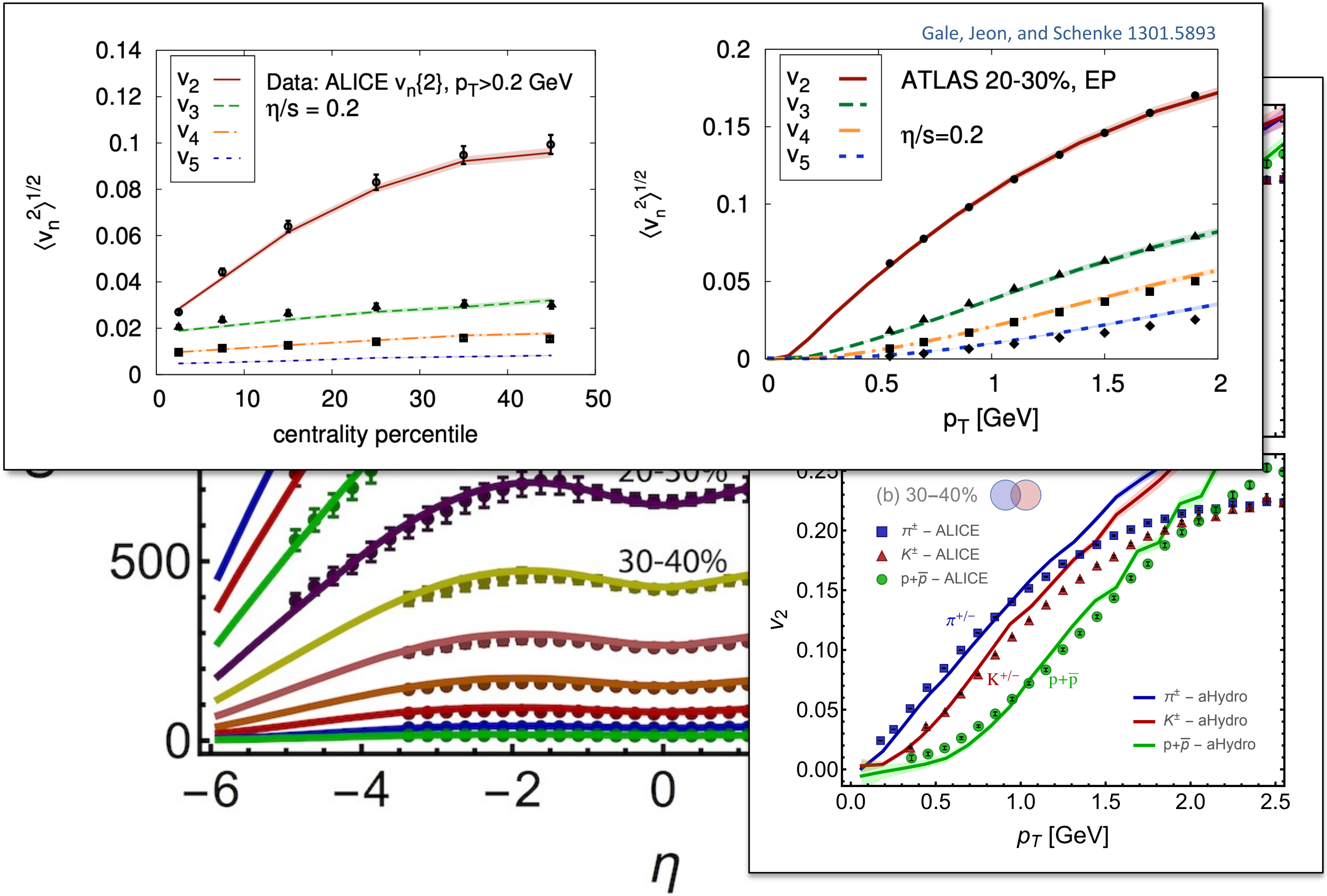
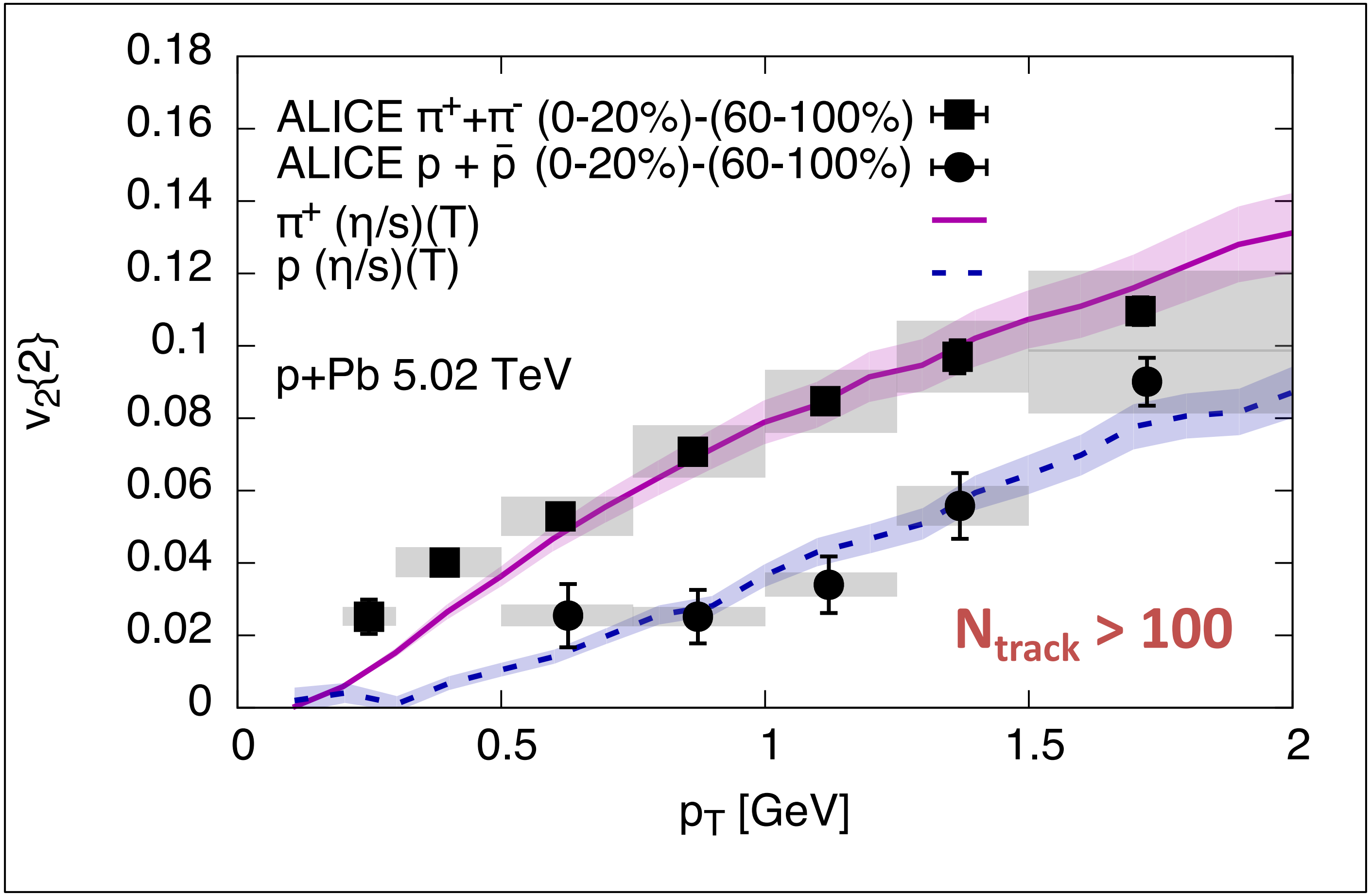
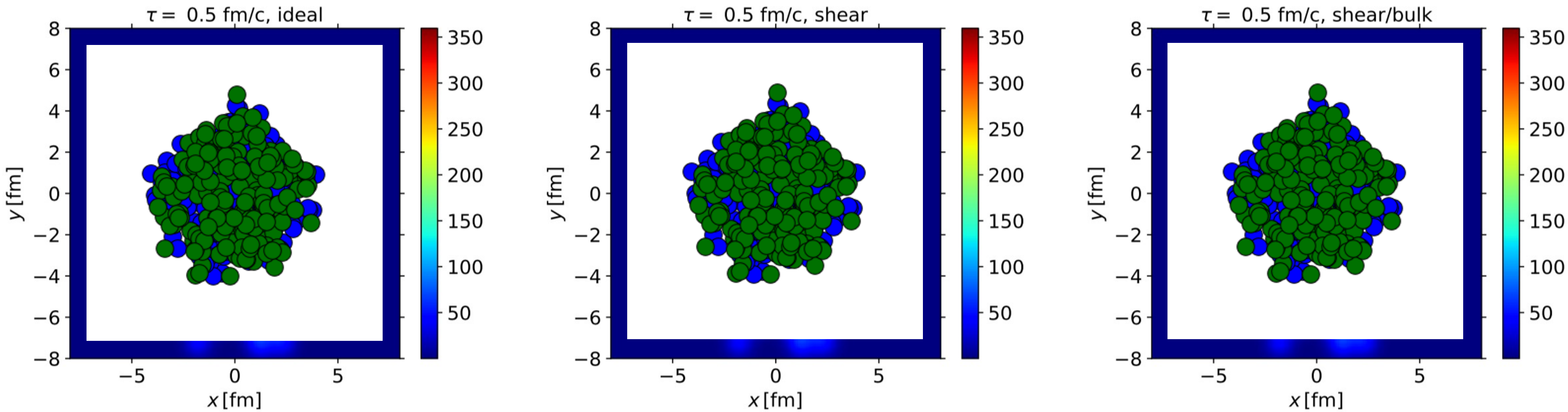


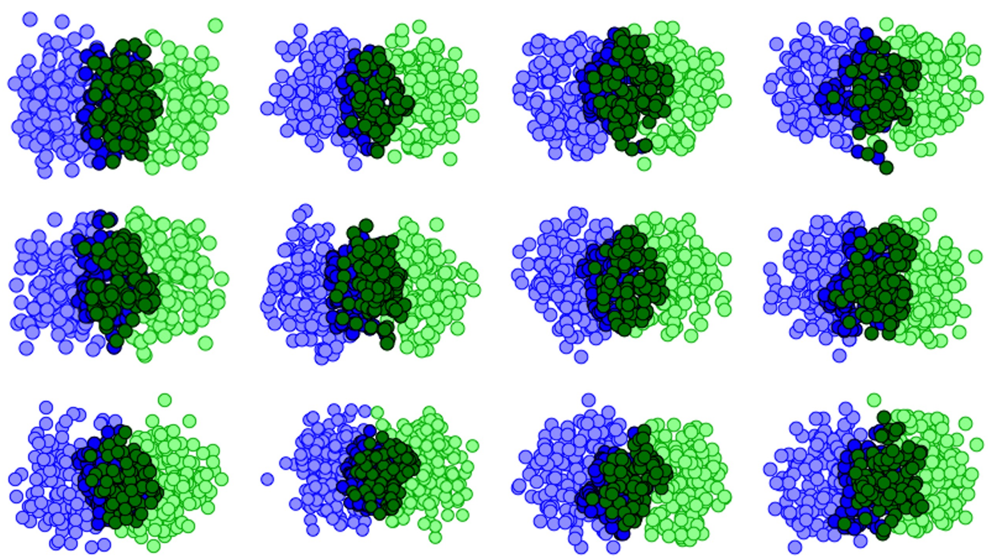
p-Pb vs Hydro: High-multiplicity events

H. Mäntysaari, B. Schenke, C.Shen, P. Tribedy, PLB 772, 681 (2017)



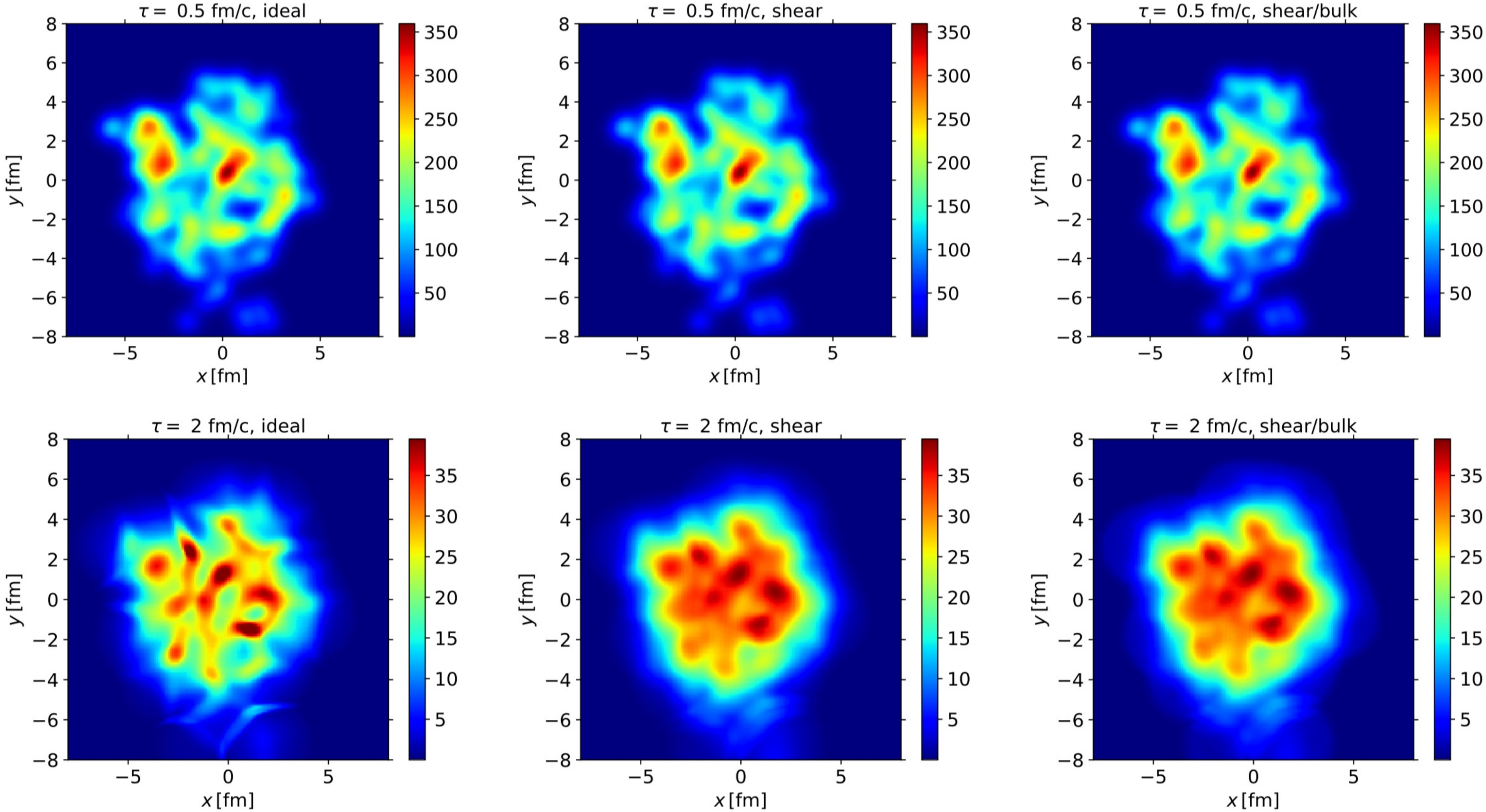
Ideal Hydro | **Shear only (η/s = 0.2)** | **Shear + Bulk**





- Variety of initial conditions (ICs) on the market. For simplicity, here I only show Monte-Carlo Glauber IC.
- Top shows animation of sampling # of participants in a central collision (b = 0 fm).
- Left shows a set of fluctuating initial conditions for b = 8 fm.

Ideal Hydro | **Shear only ($\eta/s = 0.2$)** | **Shear + Bulk**



D. Bazow, U. Heinz, and MS, 1608.06577

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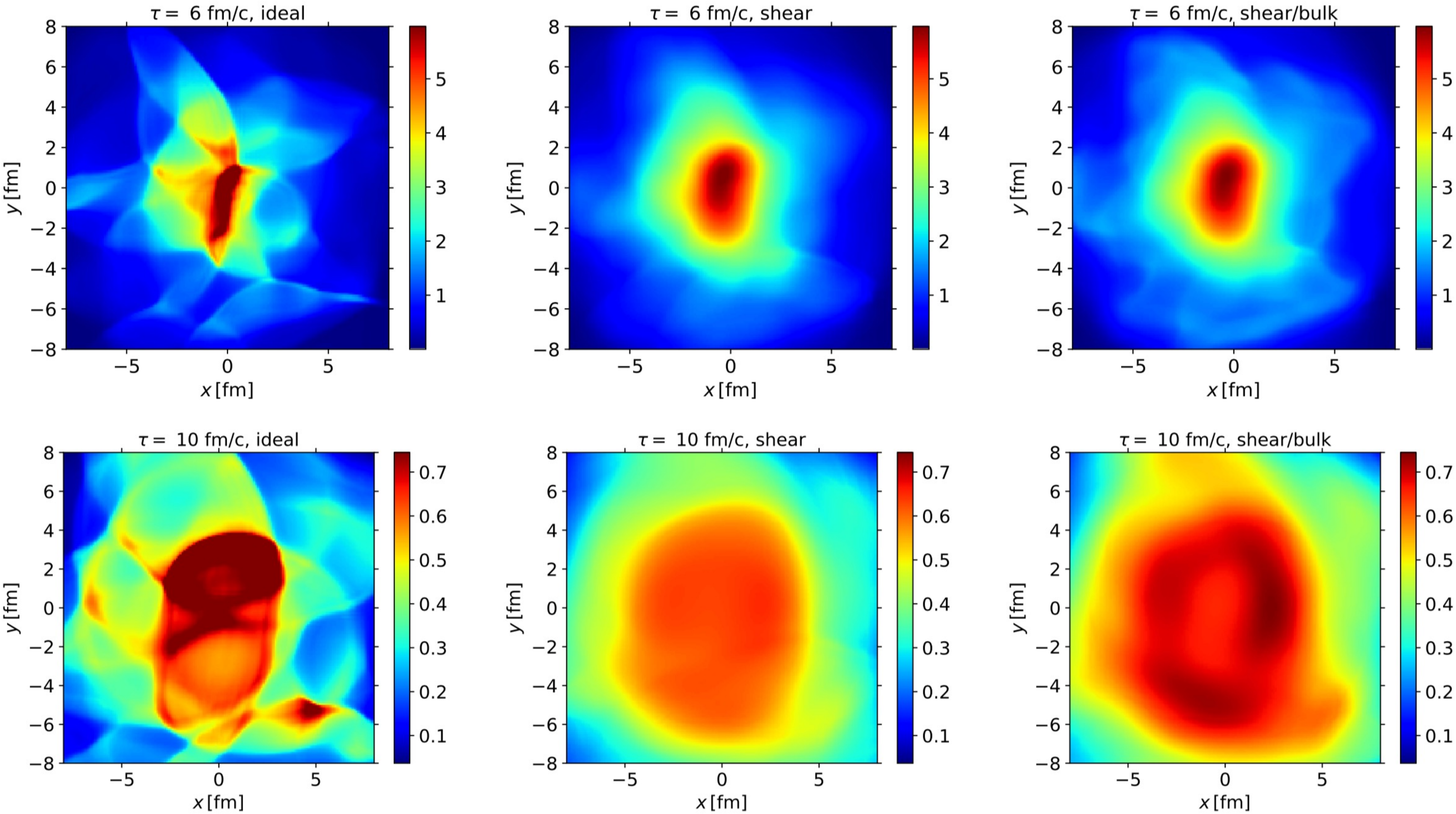


D. Bazow, U. Heinz, and MS, 1608.06577

Controlling/monitoring the evolution

The Knudsen number

Ratio of microscopic relaxation time over the macroscopic time scale which drives the system away from equilibrium

$$\mathrm{Kn} \equiv \frac{\tau_{\mathrm{micro}}}{\tau_{\mathrm{macro}}} = \tau_\pi |\theta|$$

In the environment created in heavy ion collisions one has approximately

$$\mathrm{Kn}_\pi \sim \frac{\tau_\pi}{\tau} \qquad \mathrm{Kn}_\Pi \sim \frac{\tau_\Pi}{\tau}$$

The (Inverse) Reynolds number

Ratio of the magnitude of dissipative corrections to the

$$\mathrm{R}_\pi^{-1} \equiv \frac{\sqrt{\pi^{\mu\nu}\pi_{\mu\nu}}}{\mathscr{P}_0}, \quad \mathrm{R}_\Pi^{-1} \equiv \frac{|\Pi|}{\mathscr{P}_0}.$$

Knudsen

Reynolds

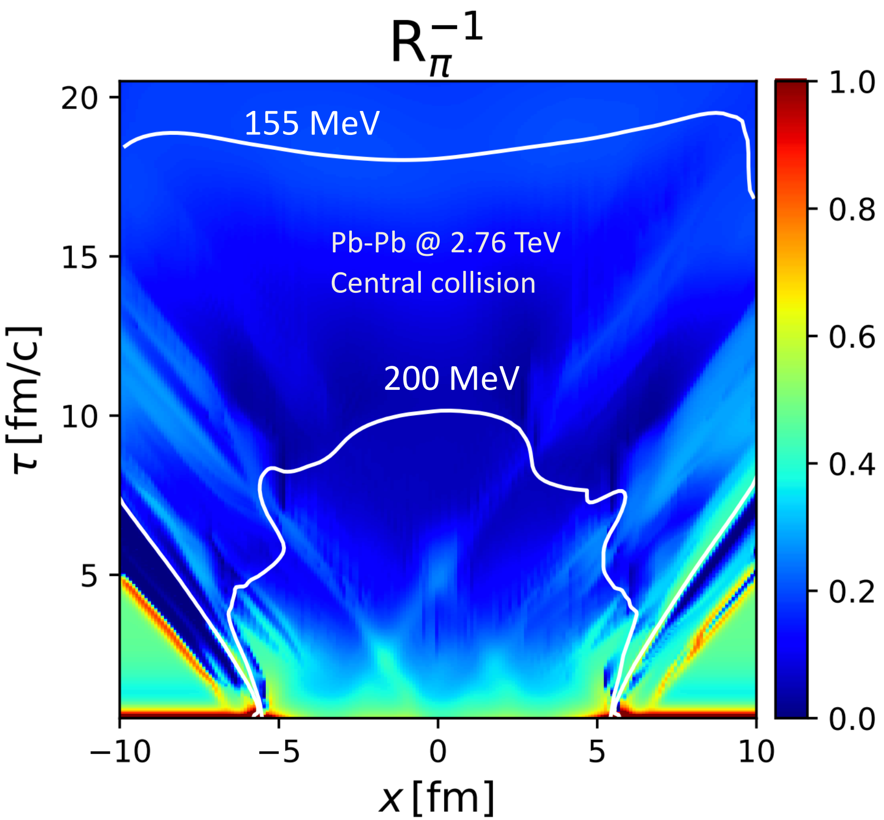
Pb-Pb @ 2.76 TeV - Don't worry, be happy?

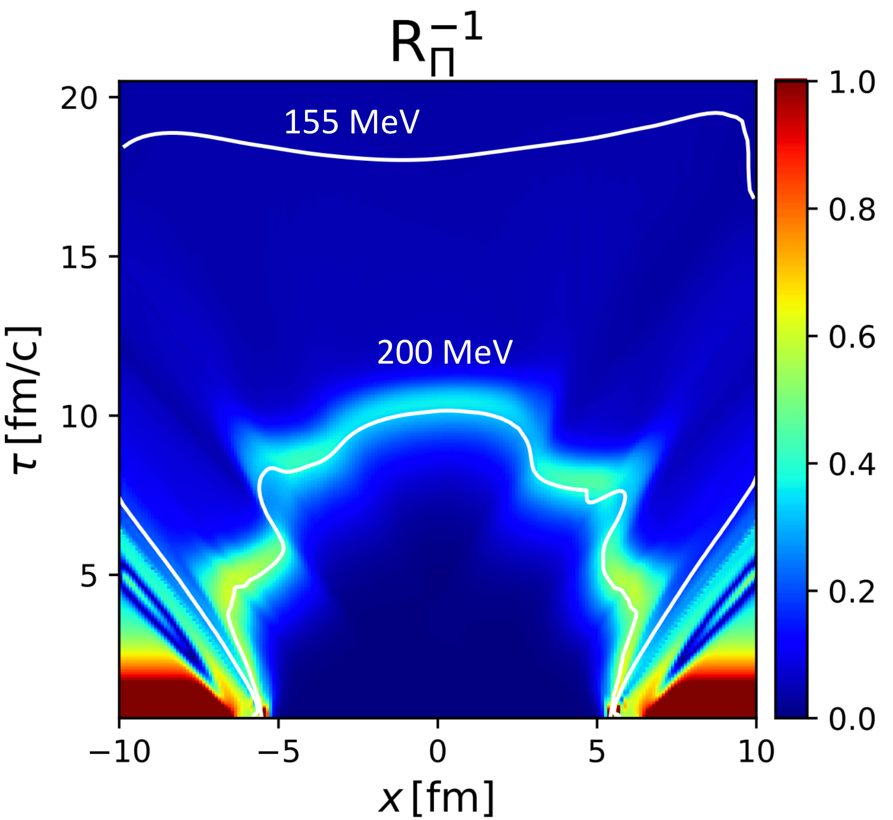
$$T^{\mu\nu} = T^{\mu\nu}_{\rm ideal} + \Pi^{\mu\nu}$$

viscous stress tensor

Shear and bulk Inverse Reynolds Numbers

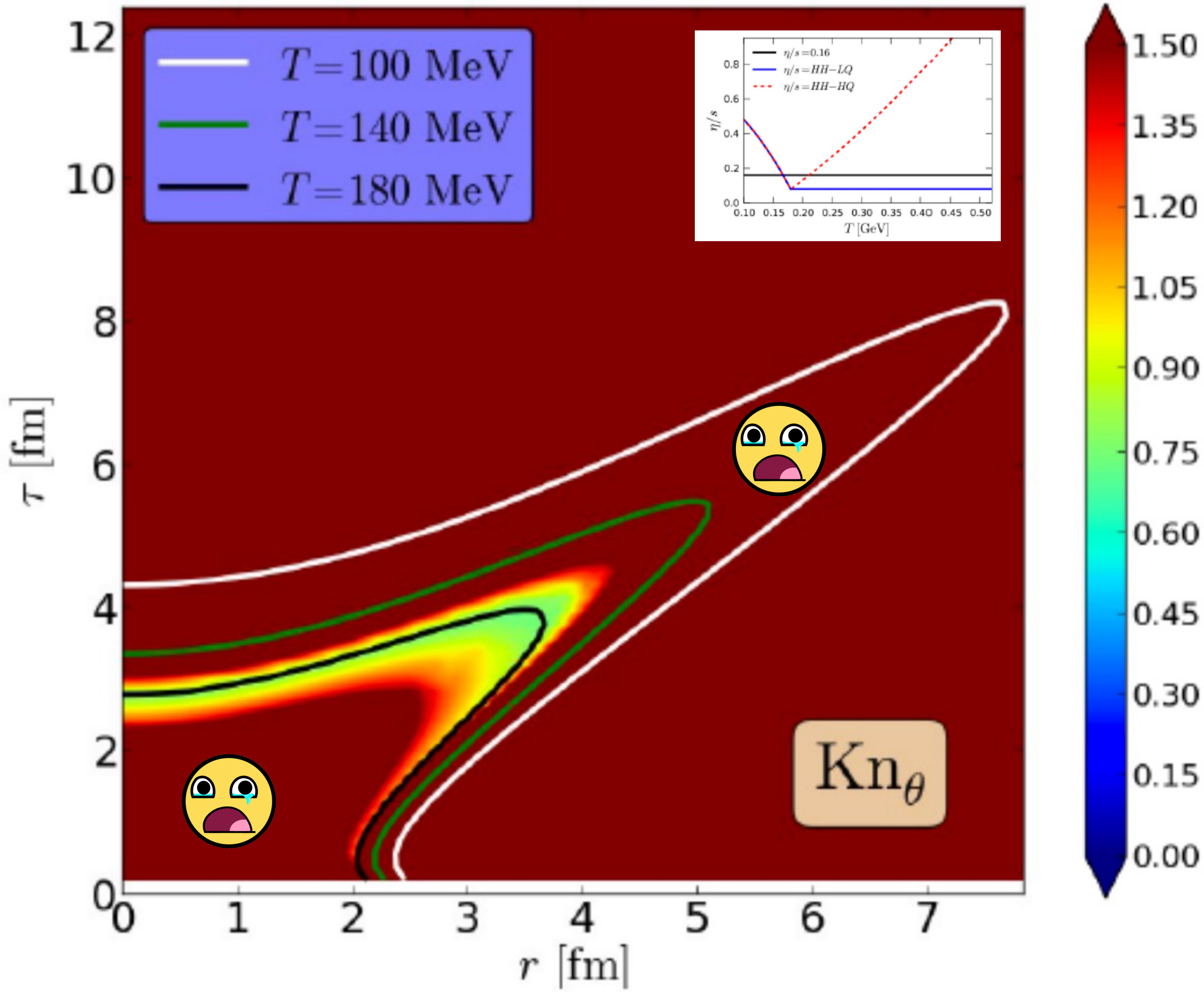
$$\mathrm{R}_\pi^{-1} \equiv \frac{\sqrt{\pi^{\mu\nu}\pi_{\mu\nu}}}{\mathscr{P}_0}, \quad \mathrm{R}_\Pi^{-1} \equiv \frac{|\Pi|}{\mathscr{P}_0}.$$



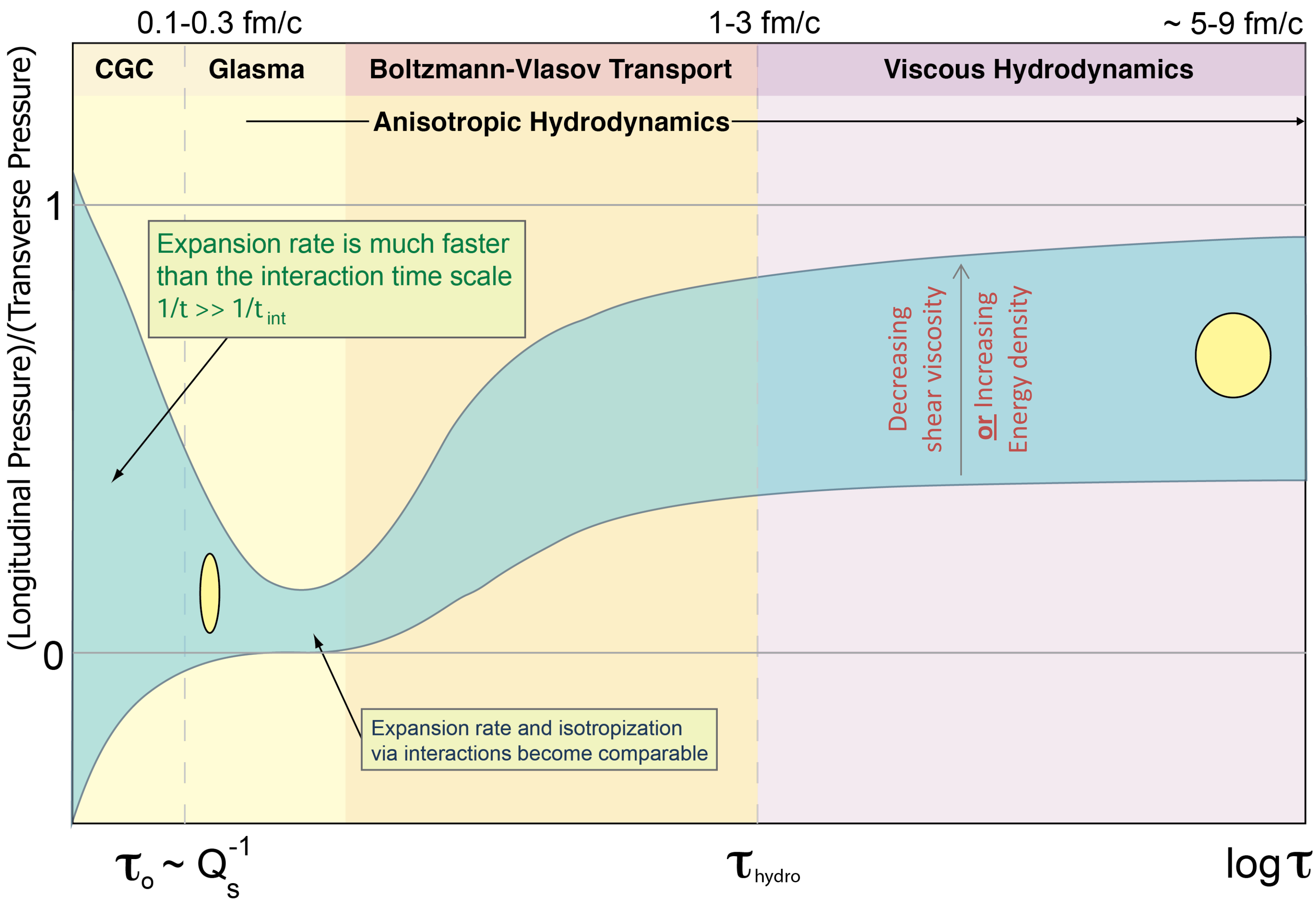


p-A @ 2.76 TeV - Don’t be happy, worry!

Figure (sans emoticons): H. Niemi and G. Denicol, 1404.7327



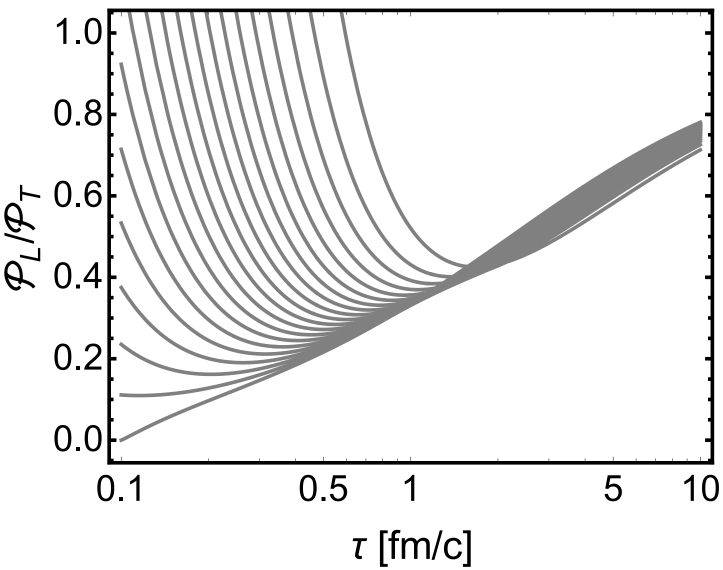
QGP momentum anisotropy cartoon



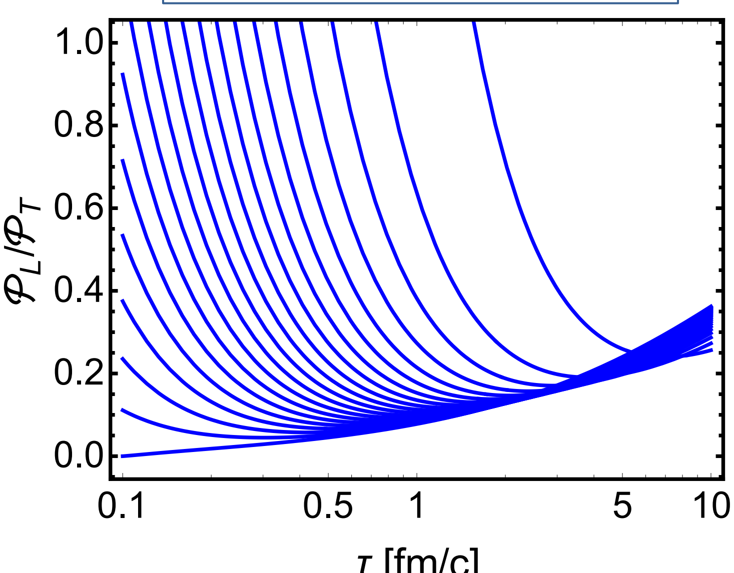
Questions

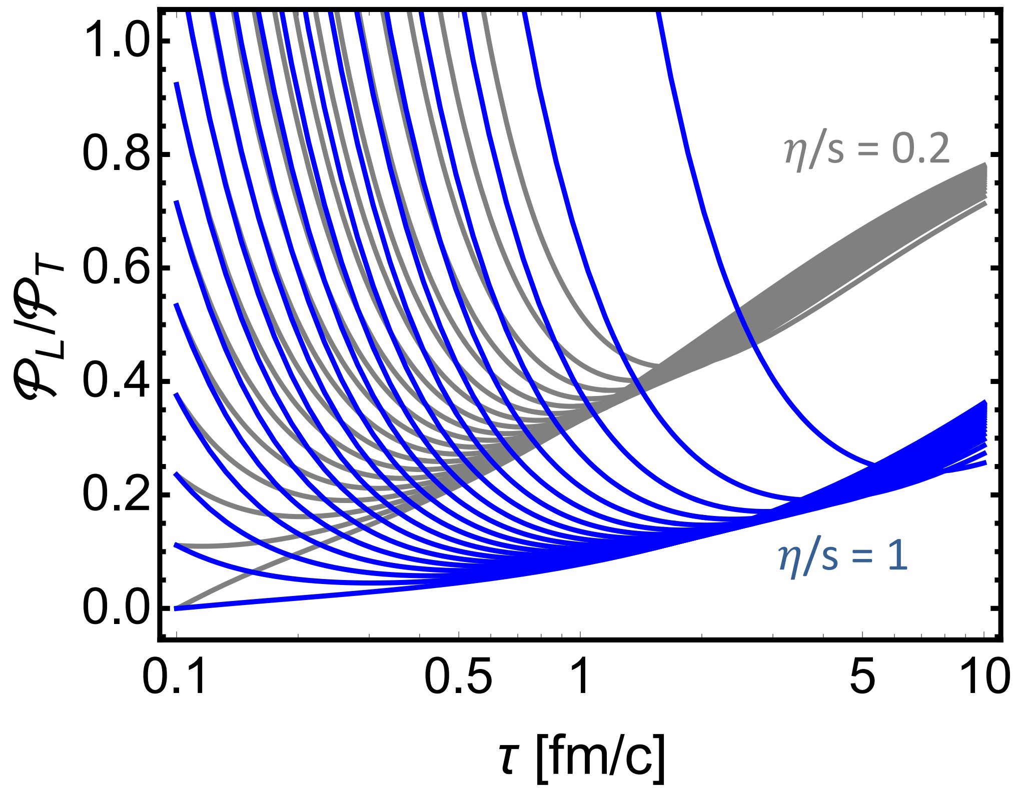
- Can standard viscous hydrodynamics treatments be able to describe the very early time dynamics reliably?
- If there are large non-eq corrections at early times, will the final results be sensitive to these?
- If I make a mistake at early time does it propagate throughout the simulation?

Assuming η/s = 0.2 and T = 500 MeV



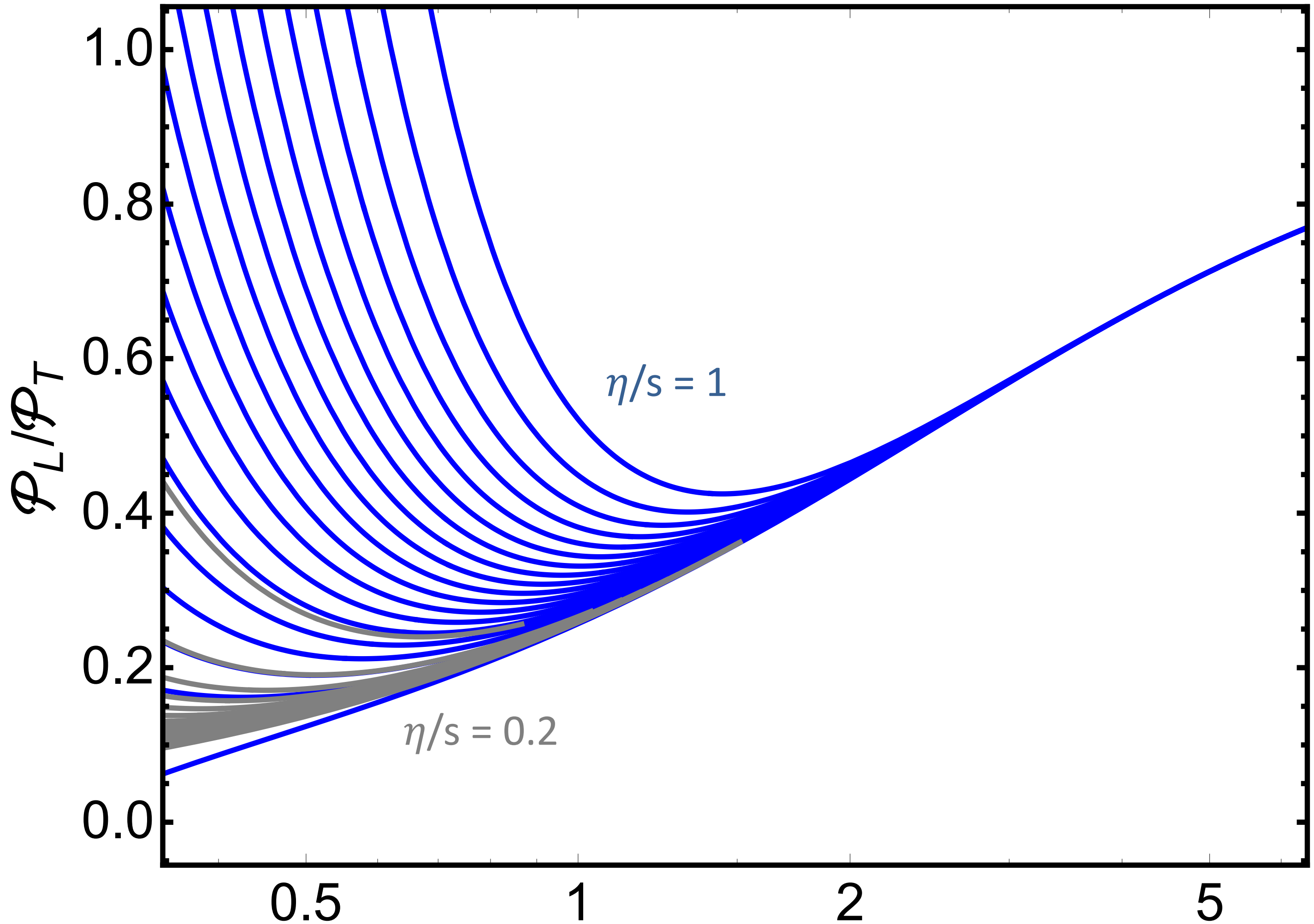
Assuming η/s = 1 and T = 500 MeV





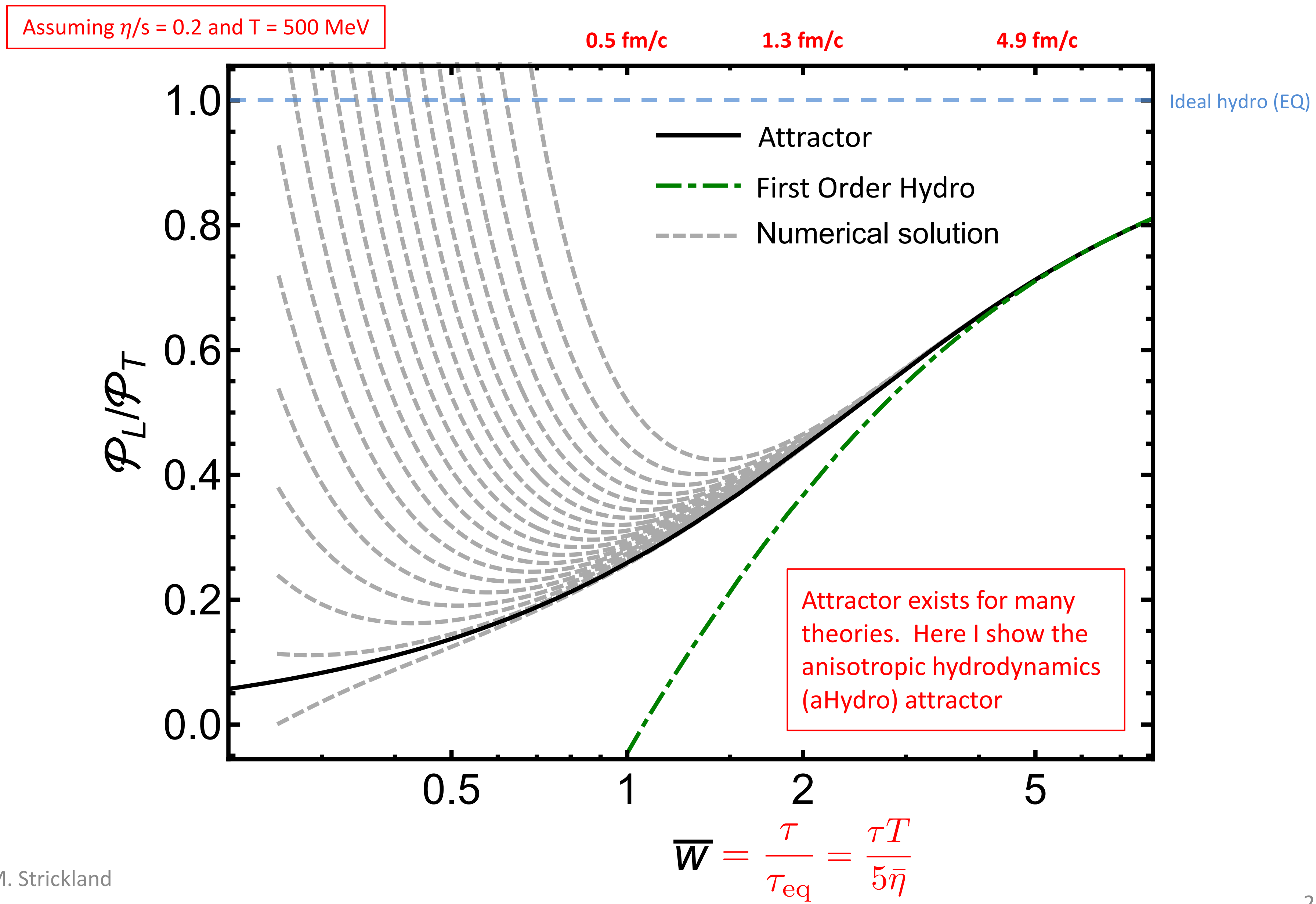
- Solve equations for different initial conditions and different values of the shear viscosity (gray vs blue)
- Hints of universal behavior at late times visible (similar levels of momentum anisotropy)

Evidence for an attractor

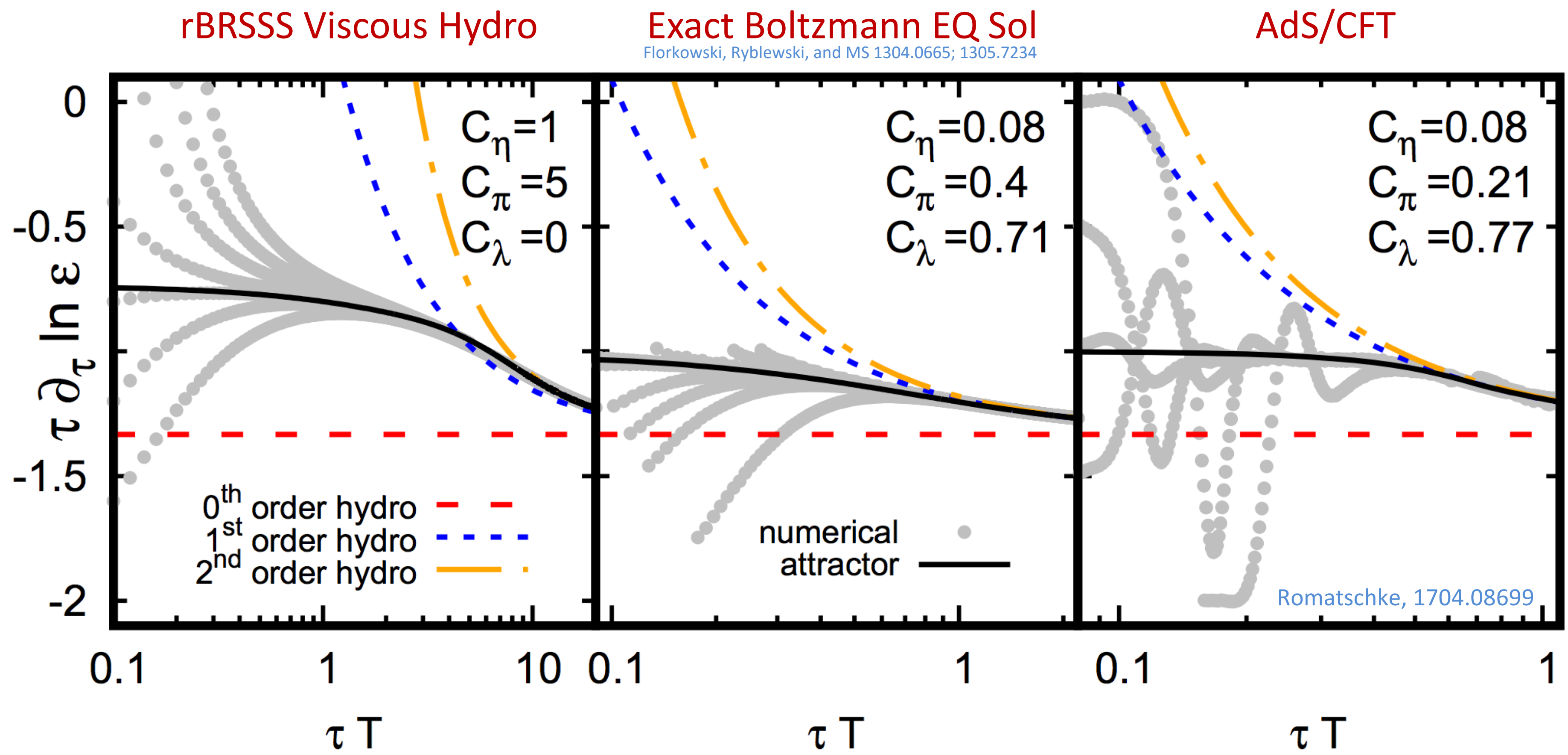


$$\overline{w} = \frac{\tau}{\tau_{\rm eq}(\tau)} = \frac{\tau T(\tau)}{5\bar{\eta}(\tau)}$$

The attractor concept

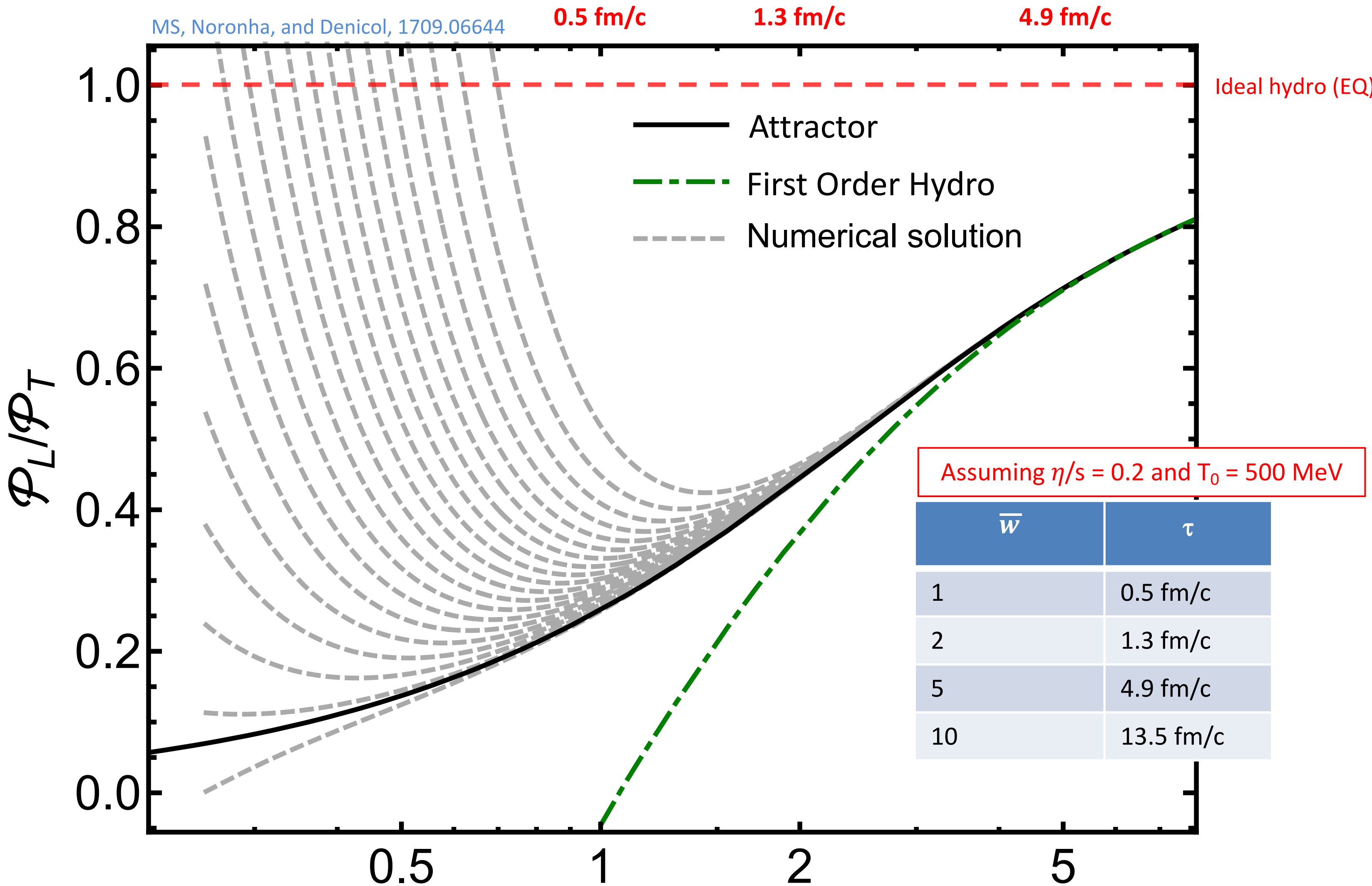


Attractor exists in many theories



Romatschke, 1704.08699; see also Keegan et al, 1512.05347

The attractor concept



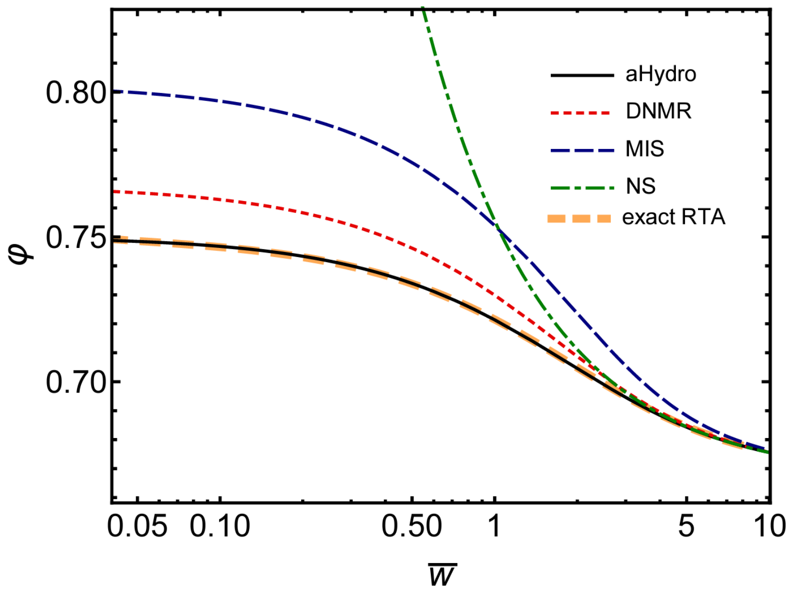
Assuming η/s = 0.2 and T_0 = 500 MeV

$\overline{w}$	τ
1	0.5 fm/c
2	1.3 fm/c
5	4.9 fm/c
10	13.5 fm/c

$$\overline{w} = \frac{\tau}{\tau_{\rm eq}} = \frac{\tau T}{5\bar{\eta}} = \text{Inverse Knudsen \#}$$

Comparisons to exact solutions in RTA

MS, G. Denicol, and J. Noronha, 1709.06644



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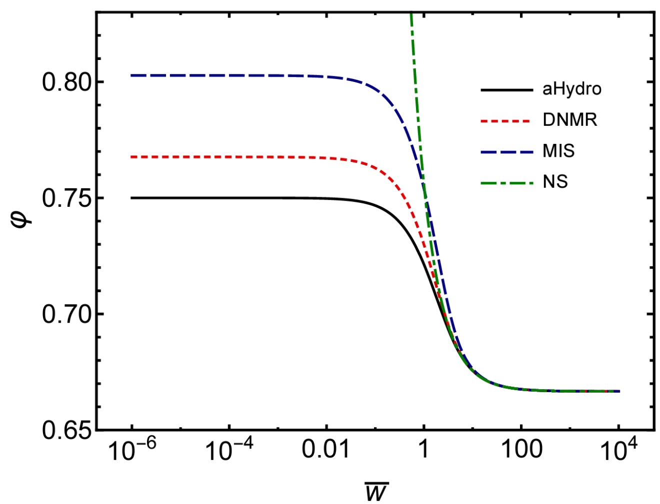

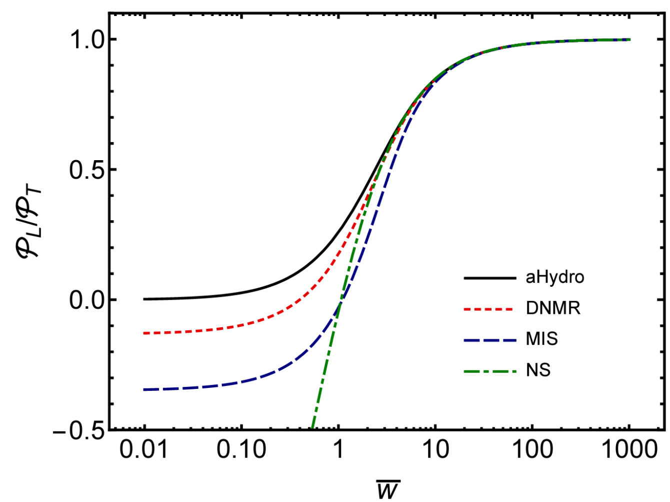

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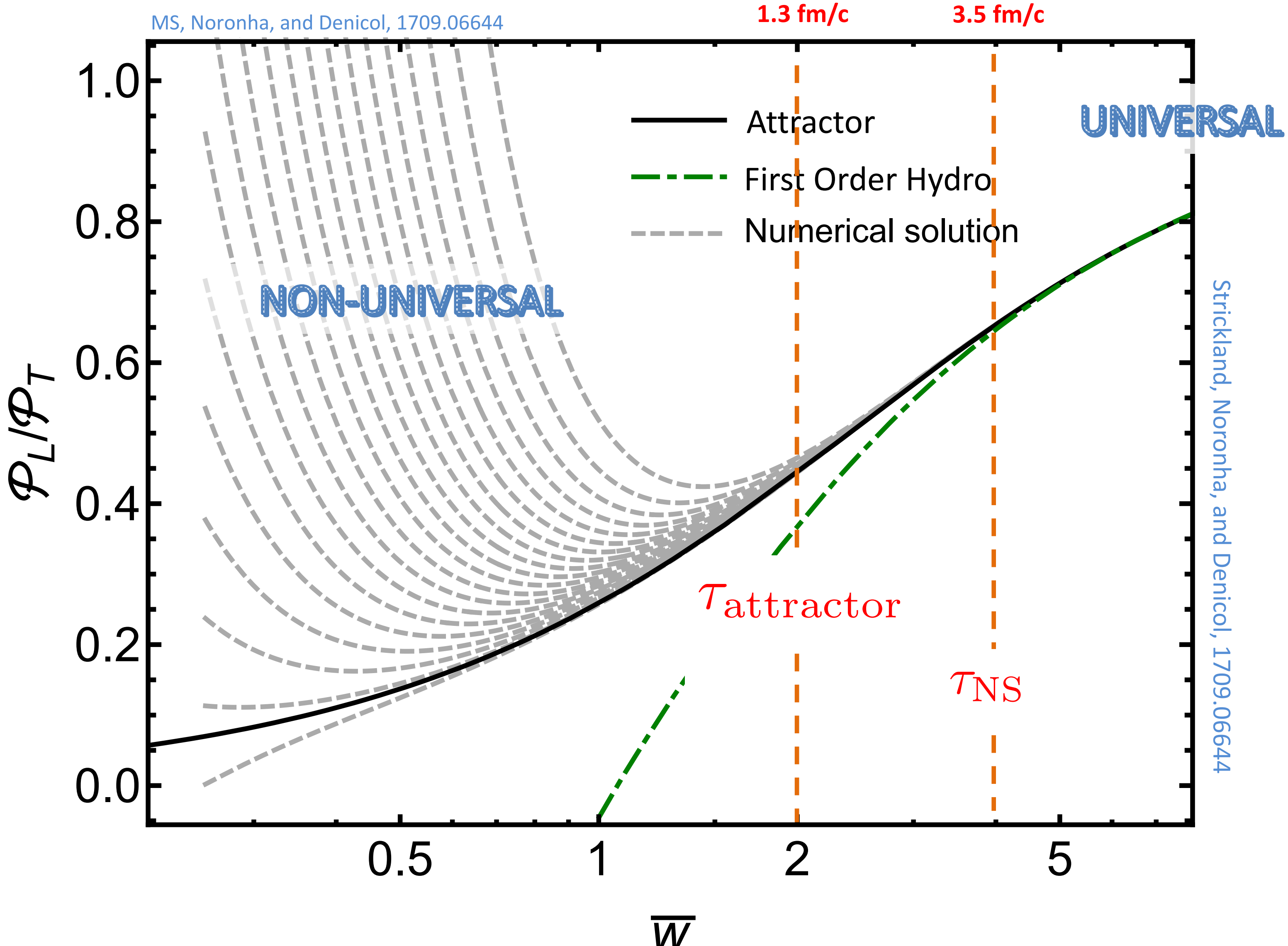


Beyond hydrodynamics?

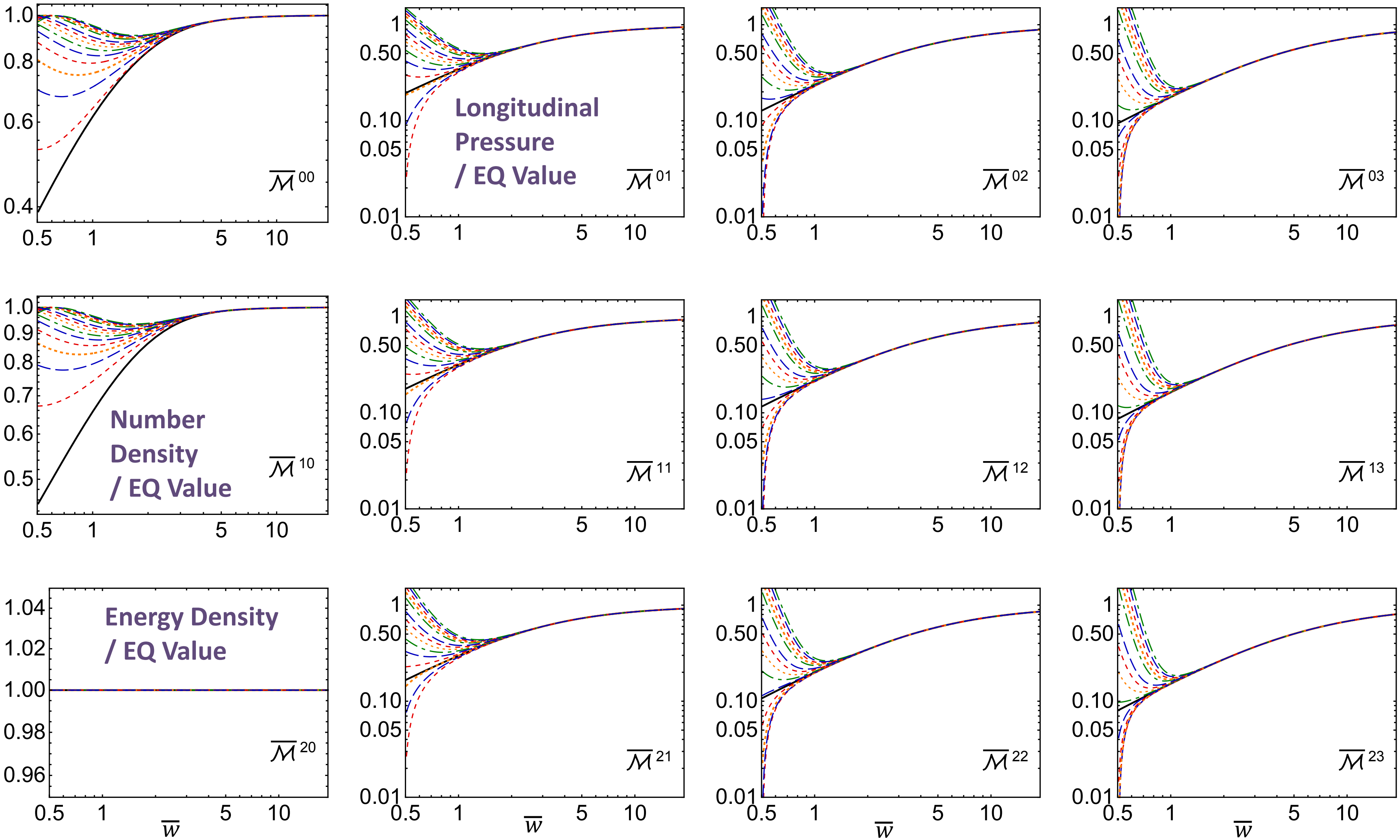
- Can the concept of a non-equilibrium attractor be extended beyond the 14 degrees of freedom described using the energy-momentum tensor, number density, and diffusion current?
- In kinetic theory we describe things in terms of a one-particle distribution function **f(x,p)** and the energy-momentum tensor is obtained from low-order moments:

$$T^{\mu\nu} = \int dP \, p^\mu p^\nu f(x,p) \qquad\qquad \int dP \equiv \int \frac{d^3p}{(2\pi)^2 E}$$

- What about more general moments of f? Particularly ones that are sensitive to higher momenta?

Behavior of higher order moments in RTA

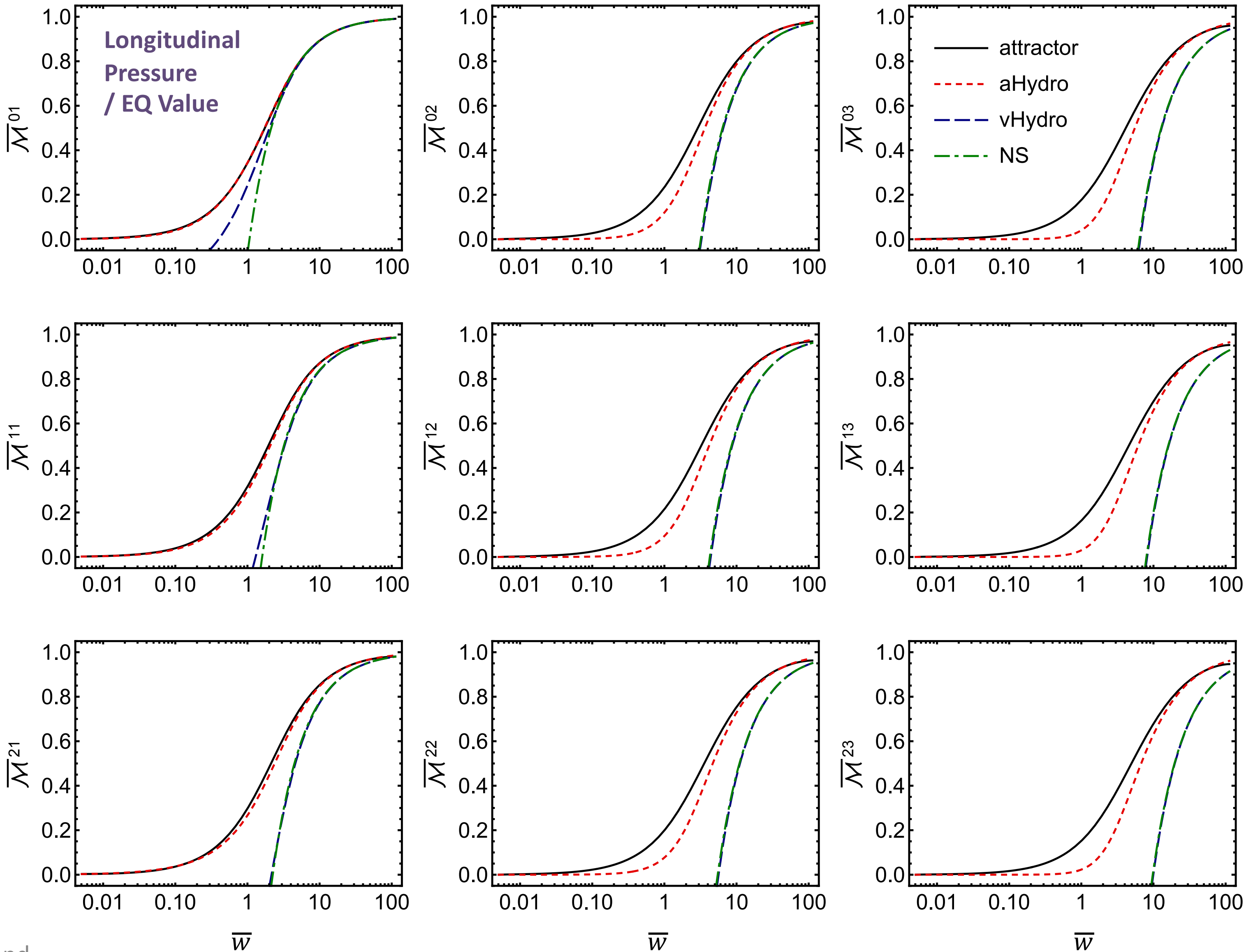
MS, Journal High Energy Physics 2018, 128 (2018).



Black Line = Attractor Solution **Dashed colored lines = scan of initial conditions**

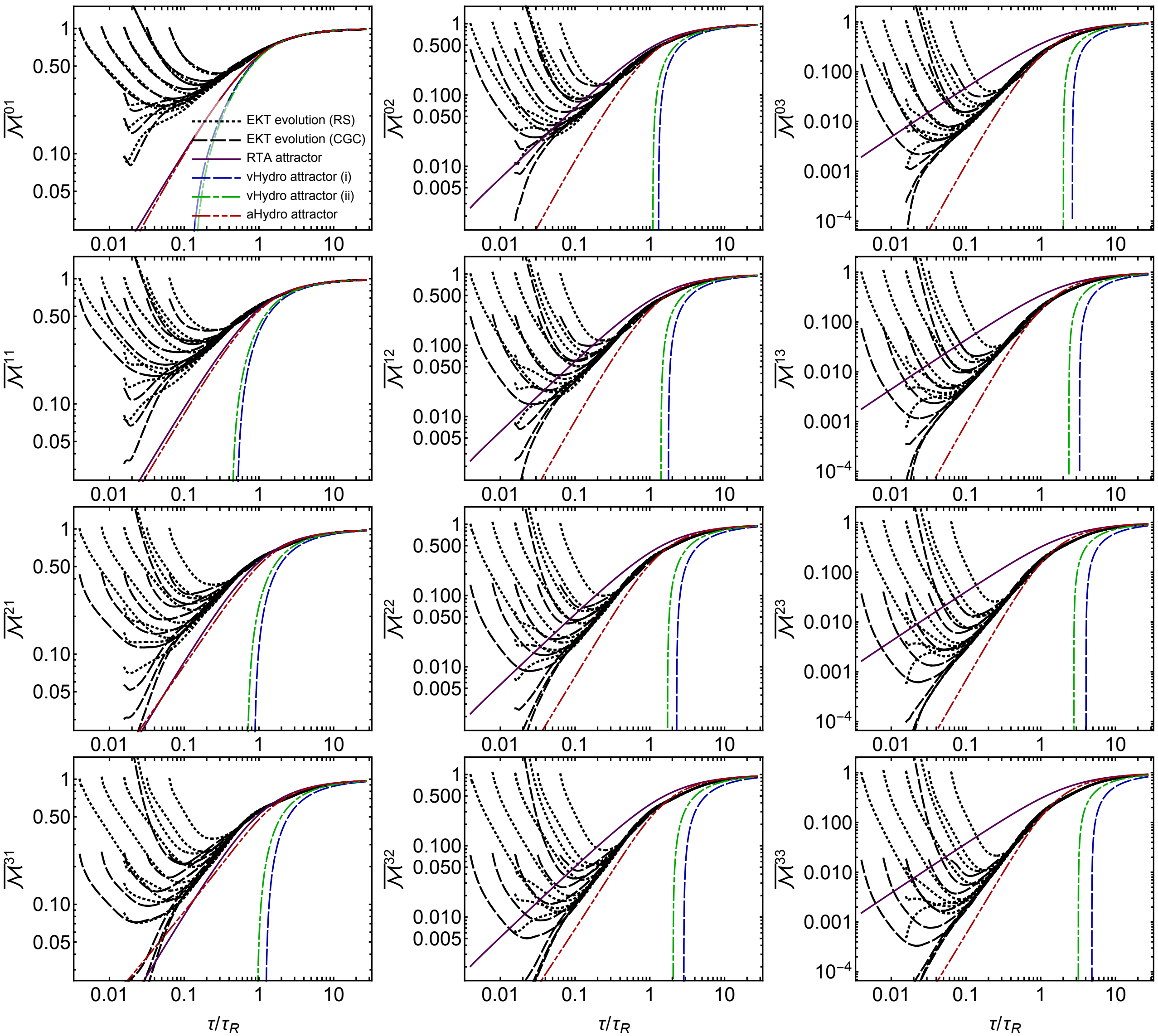
Hydrodynamic comparisons

MS, Journal High Energy Physics 2018, 128 (2018).



Similar thing happens in full QCD!

D. Almaalol, A. Kurkela, and MS, PRL 125, 122302(2020)



- Attractor seen in all moments and is the same for both types of initial conditions.
- For low order moments EKT QCD is closer to EQ than RTA and hydro predictions.
- For high order moments the opposite is true.
- **Hydrodynamization is only the tip of the iceberg → Pseudothermalization**

Similar thing happens in full QCD!

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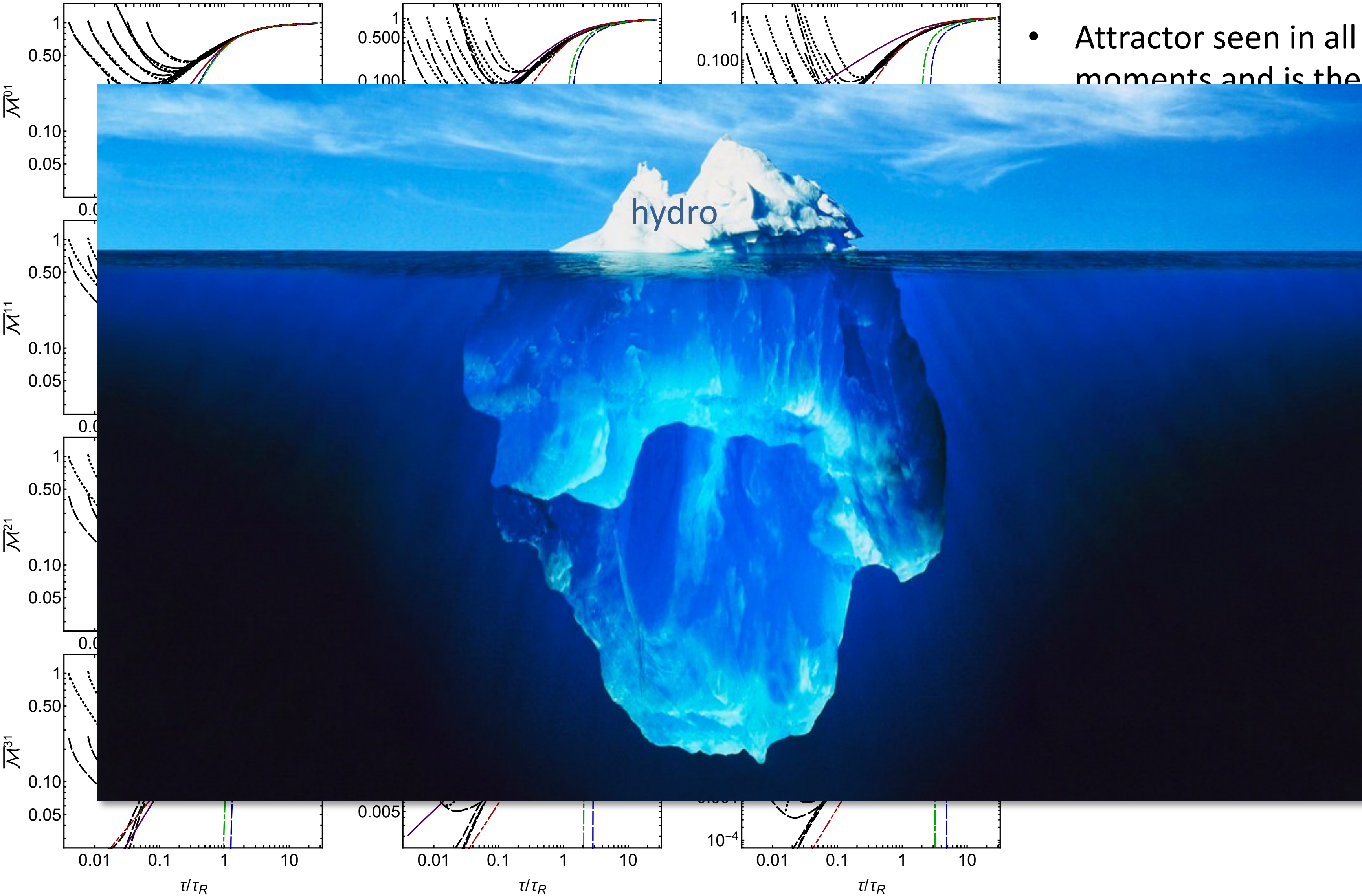

- Attractor seen in all moments and is the

Summary

- Attractors tell us that there is rapid "hydrodynamization" of the system and loss of information (e.g. initial pressure anisotropy) about the system.
- In cases where exact solutions exist, they can be used to test different approximations used in obtained hydrodynamical equations of motion.
- Attractors exist beyond hydrodynamics and extend to early times in the QGP → "pseudo-thermalization".