Small x-physics and Glasma dynamics in ultra-relativistic collisions

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1. Calculate (to lowest order, one tree diagram) the differential QED cross section
$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \frac{|\mathcal{M}|^2}{16\pi s^2} \tag{1}$$
for $e^-\mu^- \to e^-\mu^-$ scattering neglecting all the masses. Express the result in terms of the Mandelstam invariants s, t and u. What is the high energy limit of the cross section, i.e. the limit t fixed, $s \sim -u \to \infty$?

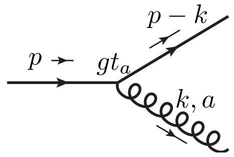
2. In the previous problem, what happens if you replace the photon by a massless scalar particle? I.e. replace $g_{\mu\nu} \to 1$ in the photon propagator and $\gamma^\mu \to 1$ in the vertex. What is the high energy limit now? (This calculation is shorter than the previous one).

3. (a) Show (this is easy) that if
$$\frac{\mathrm{d}\sigma_{\text{el.}}}{\mathrm{d}^2\mathbf{q}_T} = \left| \frac{i}{2\pi} \int \mathrm{d}^2\mathbf{b}_T e^{-i\mathbf{q}_T\cdot\mathbf{b}_T} \Gamma(\mathbf{b}_T) \right|^2$$
then
$$\sigma_{\text{el}} = \int \mathrm{d}^2\mathbf{b}_T |\Gamma(\mathbf{b}_T)|^2$$
(b) The total cross section is
$$\sigma_{\text{tot}} = 2\int \mathrm{d}^2\mathbf{b}_T \mathrm{Re}[\Gamma(\mathbf{b}_T)],$$
and the partial wave unitarity bound is $|\Gamma(\mathbf{b}_T)|^2 \leq 2\mathrm{Re}[\Gamma(\mathbf{b}_T)]$, which leads to $\sigma_{\text{el}} \leq \sigma_{\text{tot}}$ (a pretty natural requirement). Where in the complex plane can $\Gamma(\mathbf{b}_T)$ be to satisfy this?

   (c) Show that for the eikonal interaction in the Abelian theory, where we obtained
$$\Gamma(\mathbf{b}_T) = 1 - e^{i\chi(\mathbf{b}_T)}, \tag{2}$$
with a real χ, the cross section is purely elastic.

   In the CGC, we have in stead
$$\frac{\mathrm{d}\sigma_{q+A\to q+X}}{\mathrm{d}^2\mathbf{q}_T} = \left| \frac{i}{2\pi} \int \mathrm{d}^2\mathbf{b}_T e^{-i\mathbf{q}_T\cdot\mathbf{b}_T} \Gamma_{i\to j}(\mathbf{b}_T) \right|^2$$
$$\Gamma_{i\to j}(\mathbf{b}_T) = \delta_{ij} - V_{ji}(\mathbf{b}_T), \tag{3}$$
with a Wilson line $V_{ji}(\mathbf{b}_T)$ and color indices $i, j \in 1, \ldots, N_\mathrm{c}$, and the cross sections need to be averaged ($\langle\rangle$) over color configurations of the target, i.e. different $V_{ji}(\mathbf{b}_T)$.

4. (Kovchegov & Levin, exercise 5.1 a,b) The gluon field radiated from a fast-moving quark can be written using usual covariant theory Feynman rules as



$$A^a_\mu(k) = -igt^a \frac{-ig_{\mu\nu}}{k^2 + i\varepsilon} \bar{u}_\sigma(p-k)\gamma^\nu u_{\sigma'}(p)(2\pi)\delta((p-k)^2) \tag{4}$$

The incoming quark is on shell, with $p^\mu = (p^+, 0, \mathbf{0}_T)$.

(a) Simplify the Dirac structure using the eikonal kinematics $p^+ \approx (p-k)^+ \gg k^+$

(b) Then Fourier-transform

$$A^a_\mu(x) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} e^{-ik\cdot x} A^a_\mu(k) \tag{5}$$

to get the field in coordinate space

$$A^{+a}_{\text{cov}} = -\frac{g}{\pi} t^a \delta(x^-) \ln |\mathbf{x}_T|\Lambda \tag{6}$$

5. Consider two (independent of each other) transverse ($i, j \in \{1, 2\}$) pure gauge fields that depend only on transverse coordinates $A_i^{(1,2)} = A_{i,a}^{(1,2)} t^a = \frac{i}{g} V(\mathbf{x}_T)\partial_i V^\dagger(\mathbf{x}_T)$. Recall the expression for the field strength tensor $F_{\mu\nu}$ and show that these pure gauges have no longitudinal magnetic field $F_{ij}^{(1,2)} = 0$. Then consider a field that is the sum of the two: $A_i = A_i^{(1)} + A_i^{(2)}$: what is its longitudinal magnetic field F_{ij}?