The partonic structure of protons and nuclei: from current facilities to the EIC

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HAMPTON UNIVERSITY THE STANDARD OF EXCELLENCE

Jefferson Lab

Plan of the lectures

- **PART 1: QCD factorization and global PDF fitting**
  - Lecture 1 – Hadrons, partons and Deep Inelastic Scattering
  - Lecture 2 – Parton model
  - Lecture 3 – The QCD factorization theorem
  - Lecture 4 – Global PDF fits
- **PART 2: Parton distributions from nucleons to nuclei**
  - Lecture 5 / 6
- **PART 3: The next QCD frontier – The Electron-Ion collider**
  - Lextures 7 / 8

Lecture 2 - Parton Model

- **Parton model**
  - Heuristic derivation

- **DIS revisited**
  - More kinematics
  - Collinear factorization, definition of PDF
  - Parton model for DIS and its limitations

Parton Model

Parton model (see [Feynman])

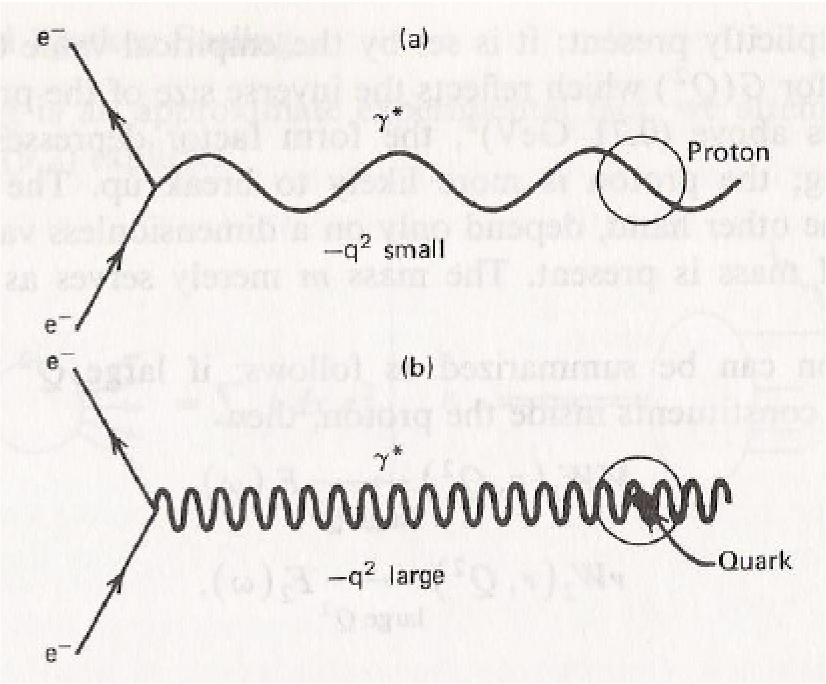
- We have evidence that a proton is a composite object made of spin ½ particles

- At high-energy, expect a “probe” to interact with these point-like objects

  – In DIS , the photon wave-length in rest frame, neglecting masses:

$$\lambda = \frac{1}{|\vec{q}|} = \frac{1}{\sqrt{\nu^2 + Q^2}} \approx \frac{1}{\nu} = \frac{2Mx}{Q^2}$$

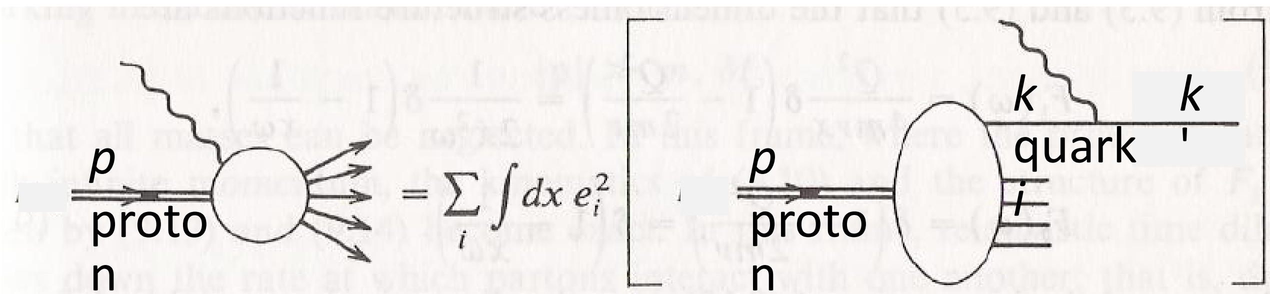
  – *E.g.*, for x=0.1, Q^2=4 GeV^2
    (and putting back *c* and *hbar*),
    $\lambda = 10^{-17}$ m = 10^{-2} fm
    to be compared with
    $R_p \approx 1$ fm



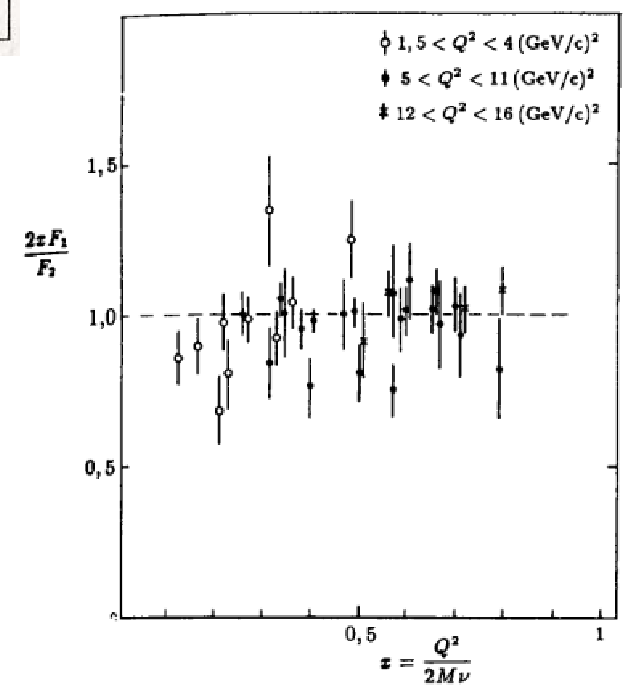
Parton model (see [Feynman])

- We have evidence that a proton is a composite object made of spin ½ particles (and should also expect some radiated gluons)

- At high-energy, expect a "probe" to interact with these point-like objects



- In DIS, the photon scatters on quasi-free quarks
  - Empirical evidence: $F_2 = 2x_B F_1$



- Seen by a high-energy probe,

> **the nucleon seems a box of practically free "partons" sharing the proton's momentum**

Parton model (see [Feynman])

- So, the nucleon is a box of practically free "partons" sharing momentum

- How can we understand this (respecting relativity, quantum mech., unitarity, etc...) ?

  - In field theory, proton wave function is specified by amplitude to find any number of partons moving with various momenta

  - This picture is however frame dependent – needs a good choice:
    - **Rest frame**: assume finite energy of interactions, *i.e.*, finite interaction times
    - **Infinite momentum frame** along z direction: times are dilated, interaction is slower and slower, until as $p_z \to \infty$ they appear to not interact at all – this is the right frame for our intuitive picture (and for precise realization in field theory)

Parton model (see [Feynman])

- What is a good variable to describe the partons?
  - A momentum fraction, say $x = k_z/p_z$, is invariant under boost along z (and phenomenologically successful in many processes)
  - Partons have "intrinsic" transverse momentum $k_T^2 \approx 0.4\ \text{GeV}^2$ (small compared to Q^2, neglect in first instance)

- In field theory, amplitude for a state of energy E to be made of n particles of total energy $E_n = E_1 + E_2 + \ldots + E_n$ is dominated in perturbation theory by

$$\mathcal{A}_n \propto \frac{C}{E - E_n} = \frac{C}{E - E_1 - \cdots - E_n}$$

  - Using $k_{z|i} = x_i p_z, \ \sum_i x_i = 1, \ E = \sqrt{p_z^2 + M^2} \approx p_z + M^2/(2p_z)$ the amplitude for 2 partons is

$$\mathcal{A}_2 \propto \frac{C'}{\frac{M^2}{2} - \frac{k_{T1}^2}{2x_1} - \frac{k_{T2}^2}{2x_2}}$$

Parton model (see [Feynman])

- Important consequences:

  - The **proton wave function depends on x_i**, and $k_{T|i}$ (but transverse momenta are small compared to Q^2 : they are negligible in first instance – but not uninteresting, ask Alexei)

  - **Partons cannot have negative x_i** (unless this is very small, see Feynman)
    - Imagine a 2 parton state, with $x_1 < 0$, then the denominator

$$E - E_{n=2} \approx (1 - |x_1| - x_2)$$

    is much larger than for 2 positive x partons, for which $E_2 \approx 0$, and

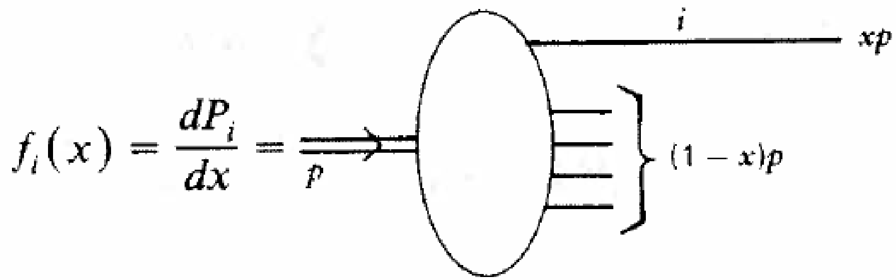
$$\mathcal{A}_2(x_1 < 0) \ll \mathcal{A}_2$$

    - States with parton of negative fractional momentum are very much suppressed compared to all $x_i>0$

Parton model (see [Feynman])

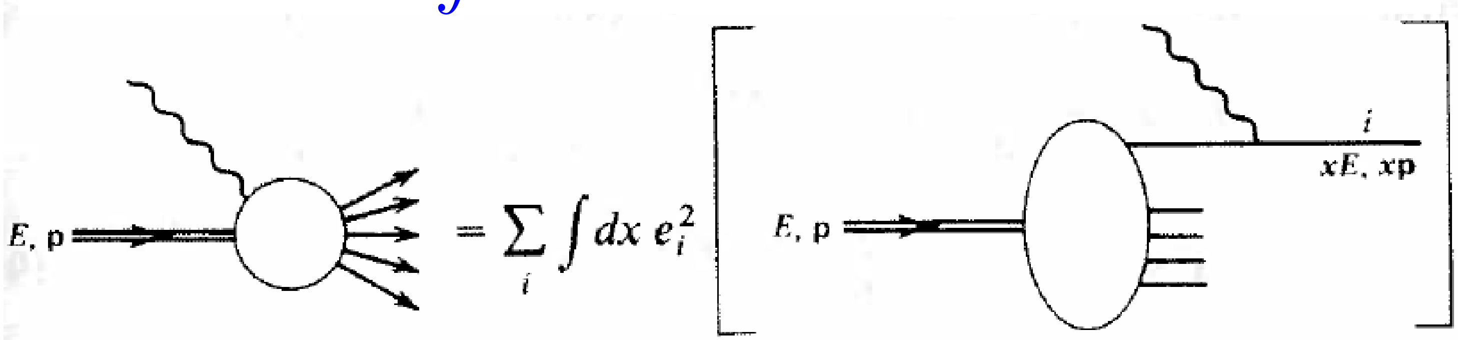
- Define a parton distribution

$$f_i(x)dx =$$ Probability of finding a parton *i* with momentum Fraction between *x* and *x+dx*



- Hard processes (e.g. a DIS cross section) should be **"factorized"**

$$\sigma(p,q) = \sum_i \int dx\, f_i(x)\hat{\sigma}^i(xp,q) = \sum_i f_i \otimes \hat{\sigma}^i$$



Sum rules

- What can we expect in general?
  - There are gluons, expect also “sea” quark anti-quark pairs
  - **Proton charge: +1**, but *u*: +2/3, *d*: -1/3, *s*: -1/3, *g*: 0

$$1 = \frac{2}{3}\int_0^1 dx\left[u(x) - \bar{u}(x)\right] - \frac{1}{3}\int_0^1 dx\left[d(x) - \bar{d}(x)\right] - \frac{1}{3}\int_0^1 dx\left[s(x) - \bar{s}(x)\right]$$

  - **Proton isospin: 1/2**, but *u*: +1/2, *d*: -1/2, *s*: 0, *g*: 0

$$\frac{1}{2} = \frac{1}{2}\int_0^1 dx\left[u(x) - \bar{u}(x)\right] - \frac{1}{2}\int_0^1 dx\left[d(x) - \bar{d}(x)\right]$$

  - **Proton strangeness: 0**, but *u*: 0, *d*: 0, *s*: 1, *g*: 0

$$0 = \int_0^1 dx\left[s(x) - \bar{s}(x)\right]$$

Sum rules

- **Parton sum rules** (for the proton)
  - Charge conservation

$$2 = \int_0^1 dx \underbrace{\left[u(x) - \bar{u}(x)\right]}_{= u_V(x)}$$

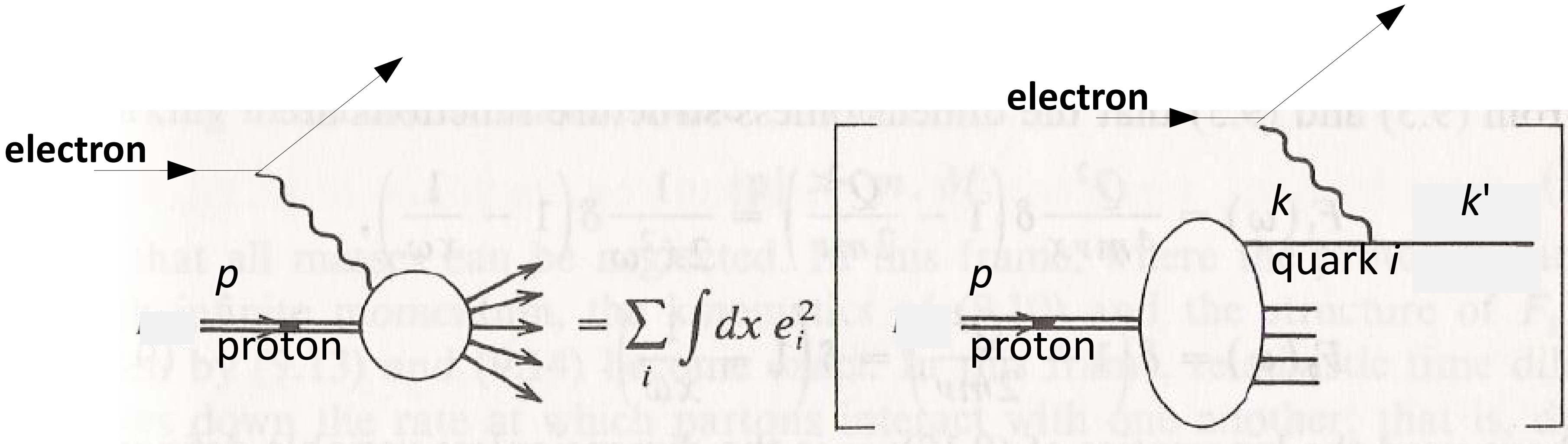
$$1 = \int_0^1 dx \underbrace{\left[d(x) - \bar{d}(x)\right]}_{= d_V(x)}$$

$$0 = \int_0^1 dx \left[s(x) - \bar{s}(x)\right]$$

  - Momentum conservation

$$1 = \sum_{i=q,g} \int_0^1 dx\, x\, f_i(x)$$

How to probe these “partons”?



- DIS ≈ photon-quark *elastic* scattering
- Interpretation of x_B
  - Parton carries fraction *x* of proton's momentum: $k^\mu = x\, p^\mu$
  - 4-momentum conservation: $k' = k+q$
  - Partons have zero mass: $k^2 = k'^2 = 0$

$$x = \frac{Q^2}{2p \cdot q} = x_B$$

- **The virtual photon probes quarks with $x = x_B$**

How to probe these "partons"?

- **Beware:** There is an inconsistency in the derivation:
  - From $k = xp$ follows that quarks are massive, $M_q = xM$!!

- A heuristic way out is to work in the "infinite momentum frame", where $|\vec{p}| \to \infty$ so that one can neglect the proton's mass:

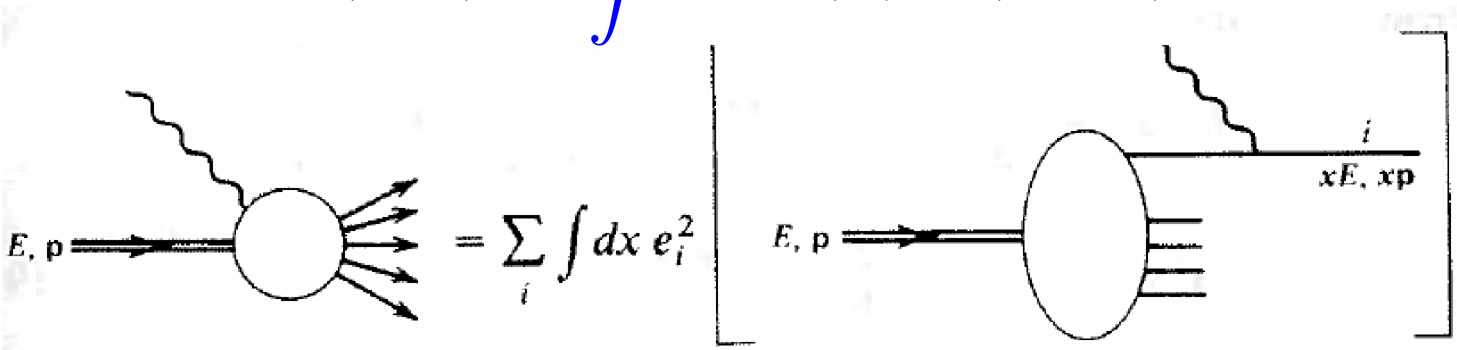
$$E_p = \sqrt{\vec{p}^2 + M^2} \to |\vec{p}|$$

  - This frame is also important to better justify the parton model
  - But quarks should be massless in any frame
    - The problem lies in the definition of x
    - We'll see a better solution in tomorrow's lecture
  - Similarly, the vector 3-momentum is not a Lorentz-invariant scale
    - In fact, M can be neglected compared to Q , not $|\vec{p}|$, as we shall see

How to probe these "partons"?

- **Caveats:**
  - **Tacit probabilistic assumption:** we are multiplying probabilities rather than amplitudes

$$\sigma(p,q) = \int dx\, f_i(x)\hat{\sigma}_i(xp,q)$$



    - justified by time dilation / smallness of photon wavelength arguments
    - Can be broken by soft (long wavelength) initial state interactions between the proton and quark lines
  - Likewise, we are assuming the same parton distribution applies to other process: **"universality"**
  - The non-trivial proof in QCD is called **"QCD factorization theorem"**

DIS revisited

More kinematics

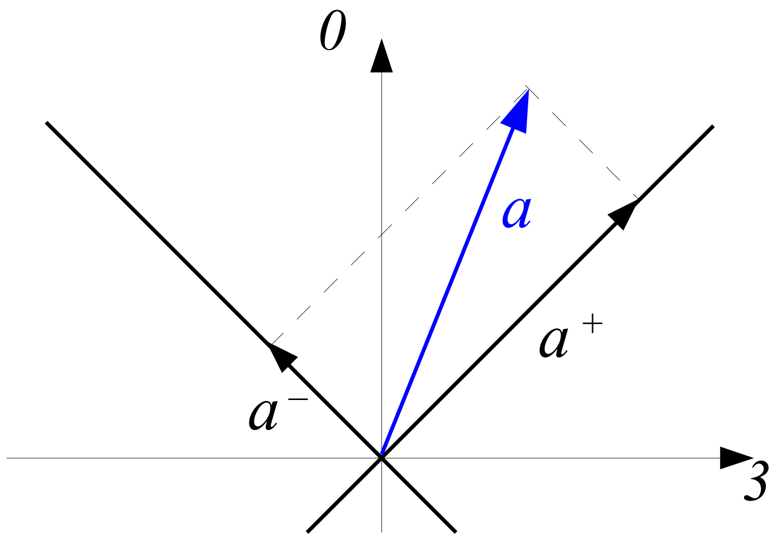
- **Light-cone coordinates**:
  a natural coord. system for processes dominated by large momentum

$$a = (a_0, \vec{a}_T, a_3)$$ Cartesian

$$a = (a^+, a^-, \vec{a}_T)$$ light-cone

$$a^\pm \equiv (a_0 \pm a_3)/\sqrt{2}$$

$$\begin{cases} a_0 = (a^+ + a^-)/\sqrt{2} \\ a_3 = (a^+ - a^-)/\sqrt{2} \end{cases}$$



$$a \cdot b = a^+ b^- + a^- b^+ - \vec{a}_T \cdot \vec{b}_T$$
$$a^2 = 2a^+ a^- - \vec{a}_T \cdot \vec{a}_T$$

More kinematics

- Boosts of velocity β in the 3-direction

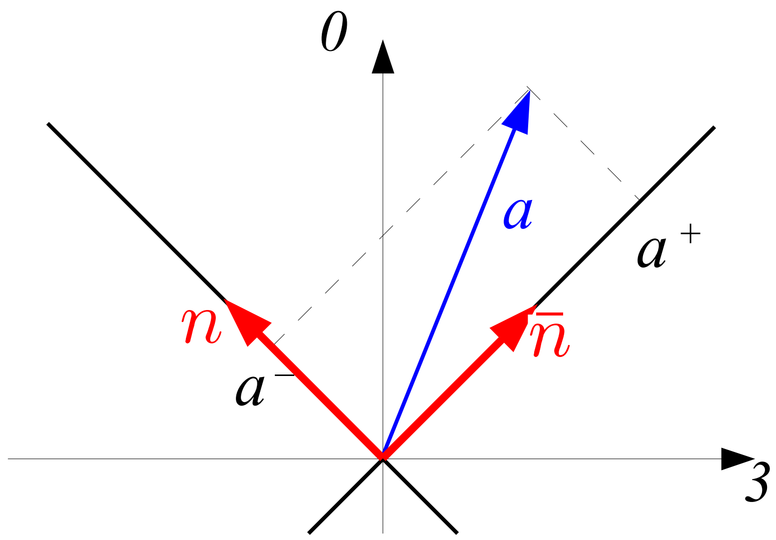
$$\begin{cases} a^{+} \Lambda a^{+} \\ a^{-} (1/\Lambda) a^{-} \end{cases} \qquad \Lambda = \frac{1-\beta}{\sqrt{1-\beta^2}}$$

  – Boost-invariant quantities:

$$a^{+}/b^{+} \; ; \quad a^{-}/b^{-}$$

$$\vec{a}_T$$

$$a^{+} b^{-}$$



- Light-cone (Sudakov) vectors:

$$\bar{\boldsymbol{n}} = (1/\sqrt{2}, \vec{0}_{\perp}, 1/\sqrt{2})$$

$$n = (1/\sqrt{2}, \vec{0}_{\perp}, -1/\sqrt{2}) \qquad a^{+} = a \cdot \bar{n} \; ; \quad a^{-} = a \cdot n$$

$$\bar{n}^2 = n^2 = 0 \quad \bar{n} \cdot n = 1 \qquad a^{\mu} = a^{+} \bar{n}^{\mu} + a^{+} n^{\mu} + a_T^{\mu}$$

More kinematics

- Boosts of velocity β in the 3-direction

$$\begin{cases} a^+ \Lambda a^+ \\ a^- (1/\Lambda) a^- \end{cases} \qquad \Lambda = \frac{1-\beta}{\sqrt{1-\beta^2}}$$

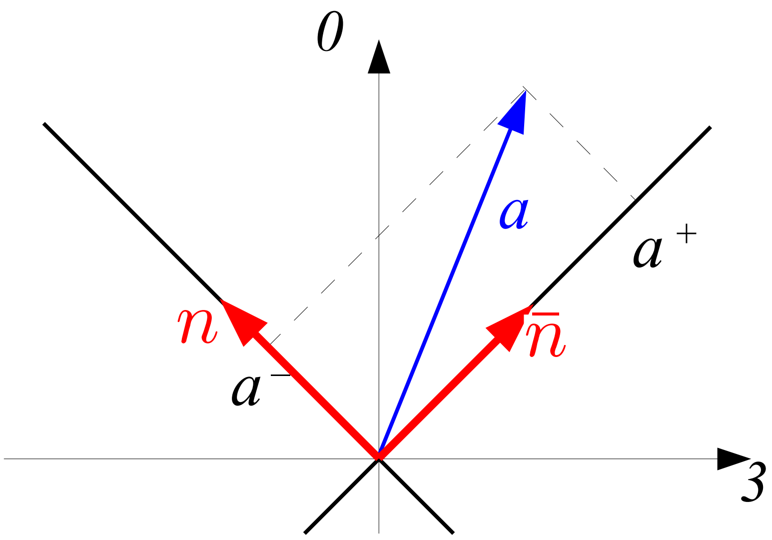
  – Boost-invariant quantities:

$$a^+/b^+ \; ; \quad a^-/b^-$$

$$\vec{a}_T$$

$$a^+ b^-$$

"fractional momenta"



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$$\bar{n} = (1/\sqrt{2}, \vec{0}_\perp, 1/\sqrt{2})$$

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$$\bar{n}^2 = n^2 = 0 \qquad \bar{n} \cdot n = 1$$

$$a^+ = a \cdot \bar{n} \; ; \quad a^- = a \cdot n$$

$$a^\mu = a^+ \bar{n}^\mu + a^+ n^\mu + a_T^\mu$$

More kinematics

- **Collinear frames**:
  - a set of frames such that p, q lie in the (+,-) plane

$$\begin{cases} p^\mu = p^+ \bar{n}^\mu + \dfrac{M^2}{2p^+} n^\mu \\[2ex] q^\mu = -\xi p^+ \bar{n}^\mu + \dfrac{Q^2}{2\xi p^+} n^\mu \end{cases}$$

with

$$\xi = \frac{2x_B}{1+\sqrt{1+4x_B^2 M^2/Q^2}} = -\frac{q^+}{p^+}$$

"Nachtmann variable"

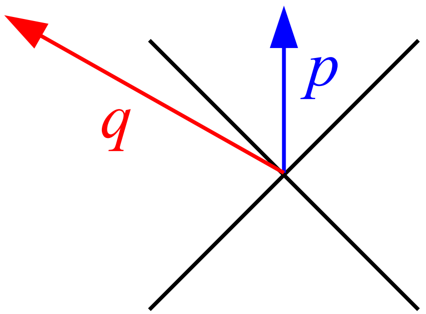
- Parameter p^+ controls boost in 3-direction
- **"massless limit"**: as $Q^2 \to \infty$, $\xi \to x_B$
- **Bjorken x_B intepreted as *fractional momentum of the photon***

  - **Ex.1** (med): derive this imposing $M^2=p^2$, $Q^2=-q^2$, $x_B=Q^2/(2p\cdot q)$;
    try first by setting M=0.

More kinematics

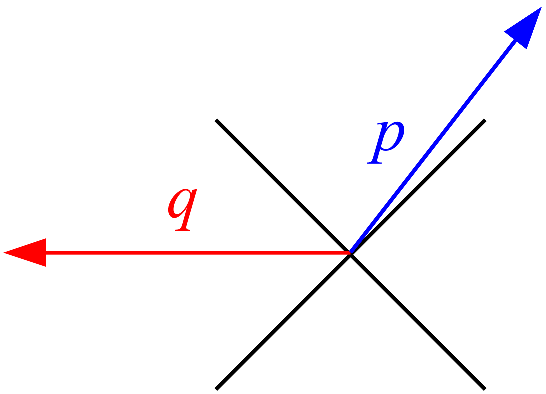
- Special cases:
  - **Proton rest frame:** $|\vec{p}| = 0 \quad (E_p = M_p)$

$$\begin{cases} p^\mu = \dfrac{M}{\sqrt{2}}\bar{n}^\mu + \dfrac{M}{\sqrt{2}}n^\mu \\ q^\mu = -\xi\dfrac{M}{\sqrt{2}}\bar{n}^\mu + \dfrac{Q^2}{\sqrt{2}\xi M}n^\mu \end{cases}$$



  - **Breit frame:** $E_{\gamma^*} = q_0 = 0$

$$\begin{cases} p^\mu = \dfrac{Q}{\sqrt{2}\xi}\bar{n}^\mu + \dfrac{M^2\xi}{\sqrt{2}Q}n^\mu \\ q^\mu = -\dfrac{Q}{\sqrt{2}}\bar{n}^\mu + \dfrac{Q}{\sqrt{2}}n^\mu \end{cases}$$



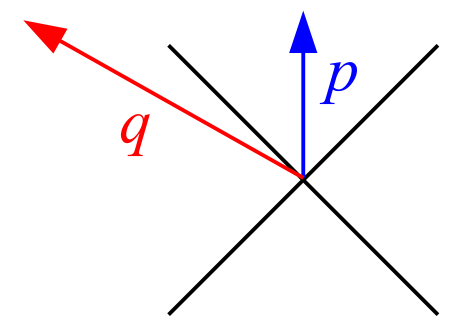
this is an (important) example of an "infinite momentum frame"

  - **Ex.2** (easy): derive these formulae

More kinematics

- Special cases:
  - **Proton rest frame:** $|\vec{p}| = 0 \quad (E_p = M_p)$

$$\begin{cases} p^\mu = \dfrac{M}{\sqrt{2}}\bar{n}^\mu + \dfrac{M}{\sqrt{2}}n^\mu \\ q^\mu = -\xi\dfrac{M}{\sqrt{2}}\bar{n}^\mu + \dfrac{Q^2}{\sqrt{2}\xi M}n^\mu \end{cases}$$



negligible only if $M^2 \ll Q^2$

  - **Breit frame:** $E_{\gamma}* = q_0 = 0$

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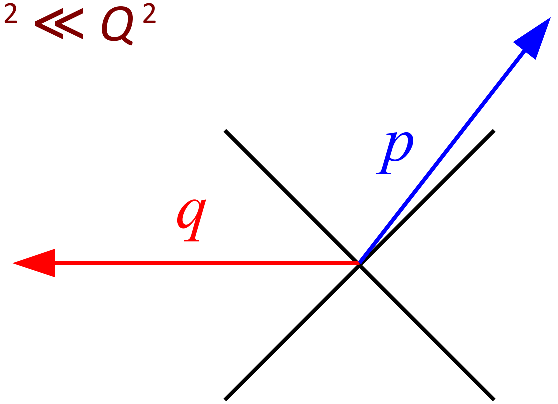

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  - **Ex.2** (easy): derive these formulae

More kinematics

- Special cases:

  – **Proton rest frame:** $|\vec{p}| = 0 \quad (E_p = M_p)$

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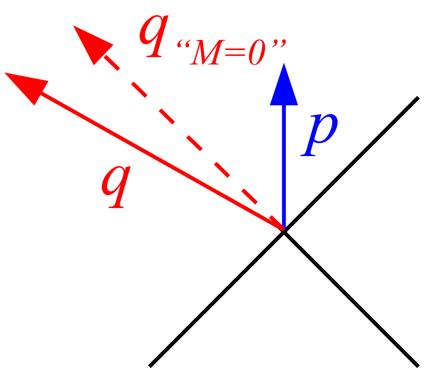

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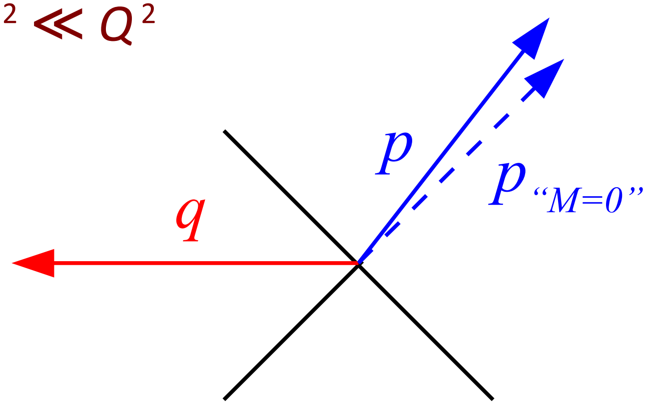


this is an (important) example of an "infinite momentum frame"

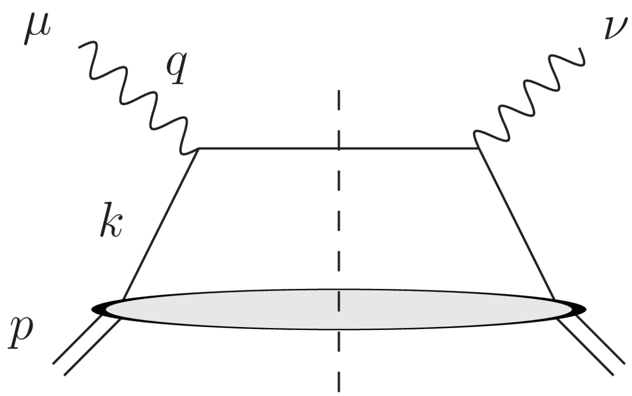
  – **Ex.2** (easy): derive these formulae

Collinear factorization in DIS at LO

Collinear factorization*

see, Accardi, Qiu, JHEP 2008 (simple)
[Collins] (full proof)

- Start from the handbag diagram



Parton fractional momentum:

$$x \equiv k^+/p^+$$

Parton's Bjorken *x*:

$$x_q \equiv -q^2/(2k \cdot q)$$

$$k^\mu = xp^+\bar{n}^\mu + \frac{k^2 + k_\perp^2}{2xp^+} n^\mu + \vec{k}_\perp$$

– Expand around on-shell ($k^2 = m_q^{\ 2} = 0$) and collinear ($k_\perp$=0) momentum

$$k^\mu = xp^+\bar{n}^\mu + O(k - \hat{k})$$

Call this $\hat{k}^\mu$

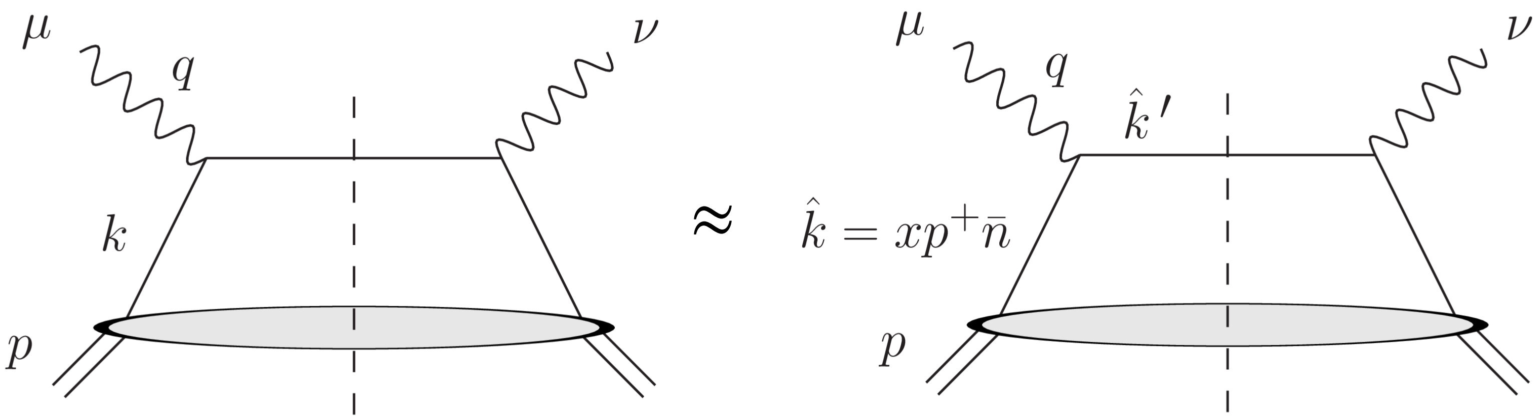
lead to $O(\Lambda^2/Q^2)$ correction in σ_{DIS}

$$\hat{x}_q \equiv -q^2/(2\hat{k} \cdot q) = x_B/x$$

*Note: will consider $M^2/Q^2 \ll 1$ for simplicity (but check the exercises)

Collinear factorization*

- Consequences:



– Now, $\hat{k}^2 = 0 \;\; p^2 = M^2$ and the quark is massless in any frame!

– Let's impose also that the final state quark $\hat{k}' = q + \hat{k}$ is on shell:

$$0 = \hat{k}'^2 = q^2 + 2\hat{k}\cdot q = (x/x_B - 1)Q^2 \quad \Rightarrow \quad \boxed{x = x_B}$$

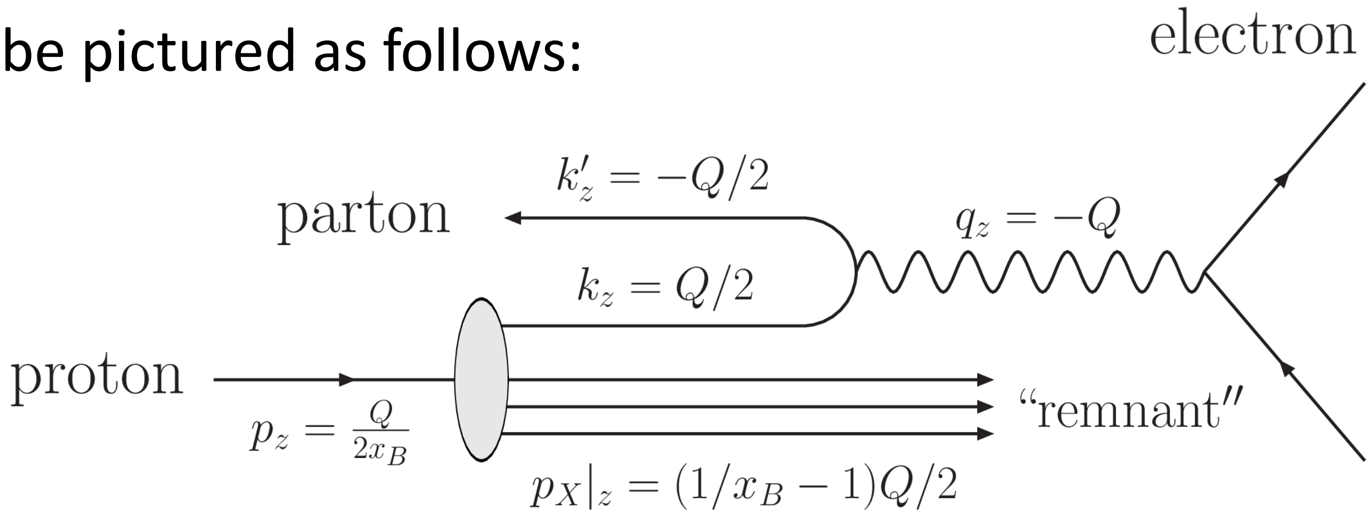
in any collinear reference frame!

– **Ex.3** (easy): show that in general, $x=\xi$

*Note: will consider $M^2/Q^2 \ll 1$ for simplicity (but check the exercises)

Back to the Breit frame for a moment

- **Ex.4** (easy): show that, in the Breit frame and for $M^2/Q^2 \ll 1$, DIS can be pictured as follows:

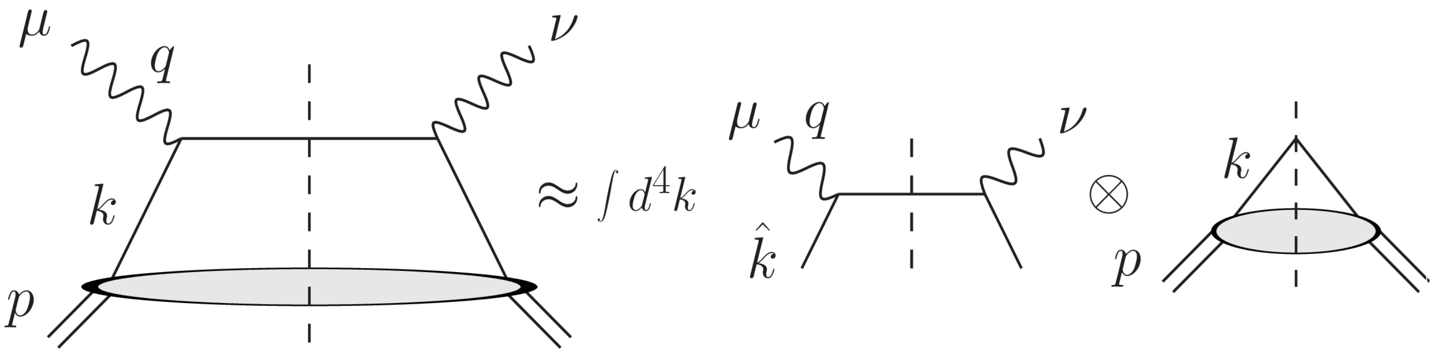


  – The scattered parton is well separated from the proton's remnant;
  – The separation in momentum increases with increasing Q^2 and decreasing x.
  – Hadrons are formed all along the intermediate momenta because of the color flux between scattered parton and remnant

- **Ex.5** (med): prove in general that $k'_z = -k_z\sqrt{1 + 4x_B^2 M^2/Q^2}$

Collinear factorization

- The diagram factorizes (need to decouple Dirac, color indexes; use "Fierz identities"):



$$W^{\mu\nu}(p,q) = \int \frac{dx}{x} \widehat{W}_q^{\mu\nu}(\hat{k},q)\, q(x)$$

so that: $q(x) = \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p|\overline{\psi}(z^- n)\frac{\gamma^+}{2}\psi(0)|p\rangle$

$$F_1(x_B,Q^2) = \sum_q \int_{x_B}^1 \frac{dx}{x} \hat{F}_1^q\left(\frac{x_B}{x},Q^2\right) q(x)$$

$$F_2(x_B,Q^2) = \sum_q \int_{x_B}^1 dx\, \hat{F}_2^q\left(\frac{x_B}{x},Q^2\right) q(x)$$

$\frac{x_B}{x} = \hat{x}_q$

Parton model result

- By explicit perturbative calculation (see Sterman "an intro to QFT"):

$$\hat{F}_2^q(\hat{x}_q) = 2\hat{F}_1^q(\hat{x}_q) = e_q \delta(1 - \hat{x}_q)$$

  with 2 consequences:

  – **Callan-Gross relation**

$$F_2(x_B, Q^2) = 2x_B F_1(x_B, Q^2)$$

  consequence of quark's spin ½ (*e.g.*, for spin 0, F_1=0)

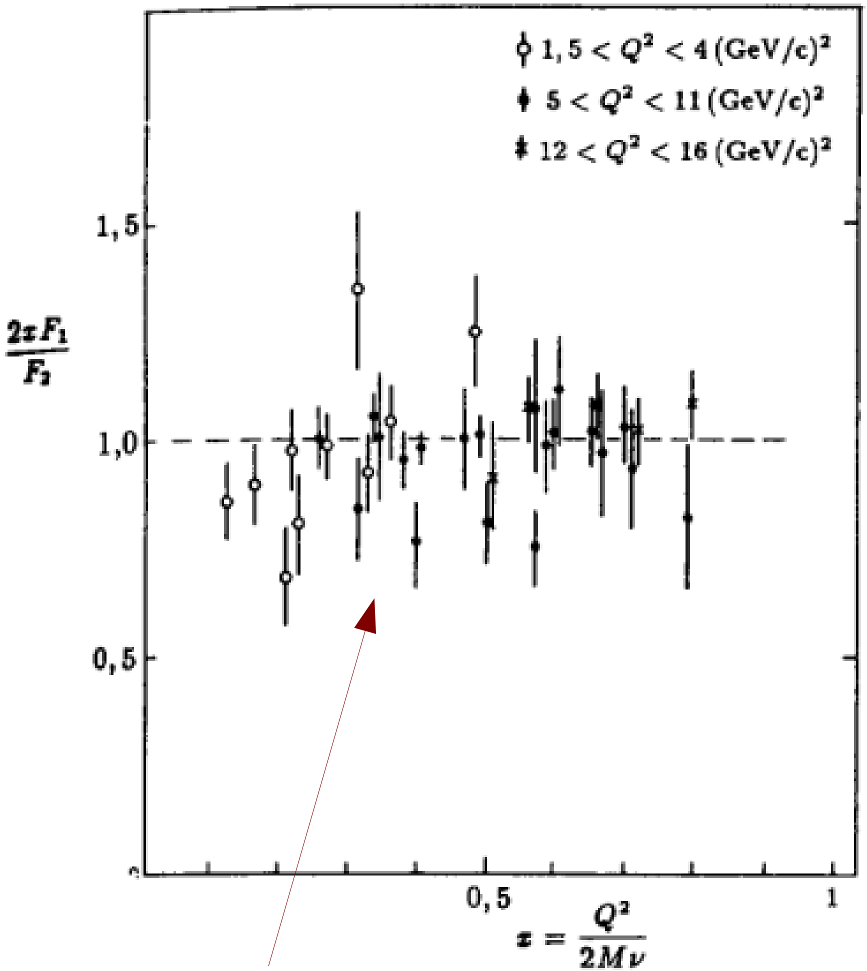
  – **Bjorken scaling**

$$F_2(x_B, Q^2) = x_B \sum_q e_q^2 q(x_B)$$

  the structure functions do not depend on Q^2

- NOTE: we have worked at LO in α_s – expect violations at NLO

Experimental data



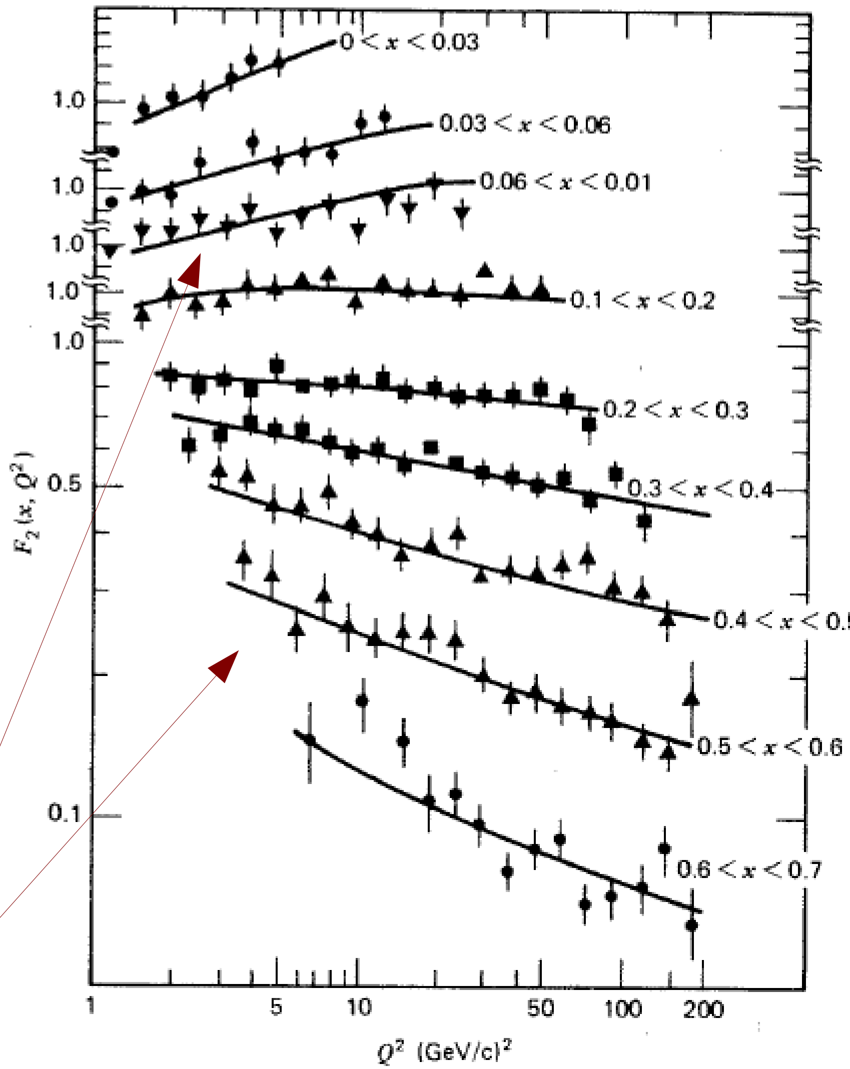


Fig. 10.10 Deviations from scaling. With increasing Q^2, the structure function $F_2(x, Q^2)$ increases at small x and decreases at large x. The data are from the CDHS counter experiment at CERN.

Charged partons have spn ½ (quarks!)

log(Q^2) scaling violations

The proton's momentum

- Partons distributions are interpreted as the probability distribution of finding a parton of momentum *x* inside the proton
  - Expect **momentum sum rule**

$$\sum_i \int_0^1 dx\, x\, q_i(x) = 1$$

- How to measure it
  - Proton is (u_V u_V d_V) – note the "Valence" subscript

$$F_2^p(x_B) = \frac{4}{9} x_B u_V(x_B) + \frac{1}{9} x_B d_V(x_B)$$

  - Neutron is (d_V d_V u_V)

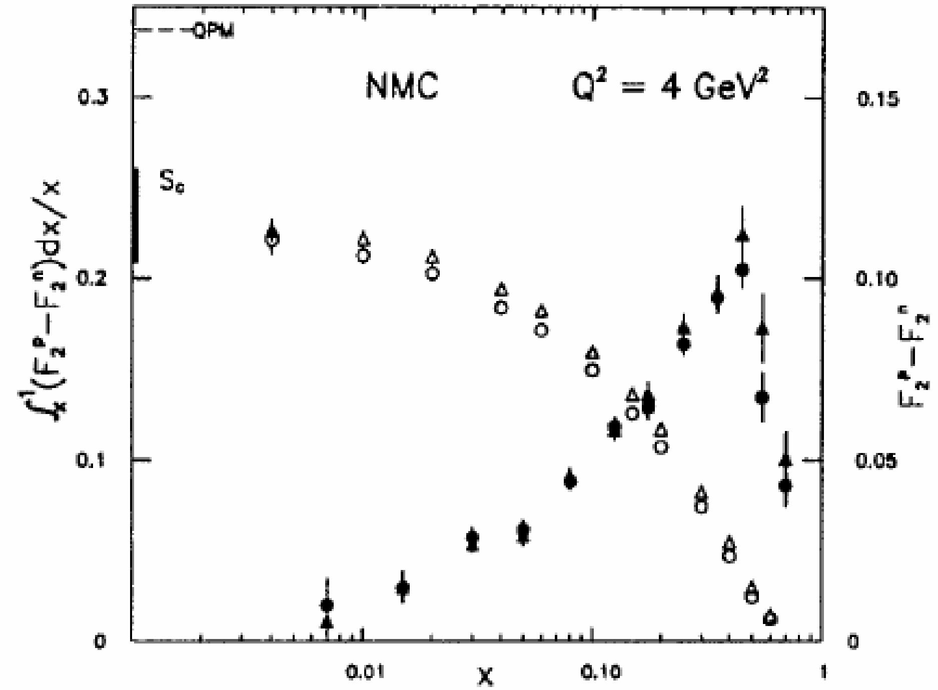
$$F_2^n(x_B) = \frac{4}{9} x_B d_V(x_B) + \frac{1}{9} x_B u_V(x_B)$$

  - Hence, expect ("Gottfried sum rule")

$$\int_0^1 dx \left(F_2^p(x) - F_2^n(x) \right) = \frac{1}{3}$$

The proton's momentum

- But data don't bear this out:

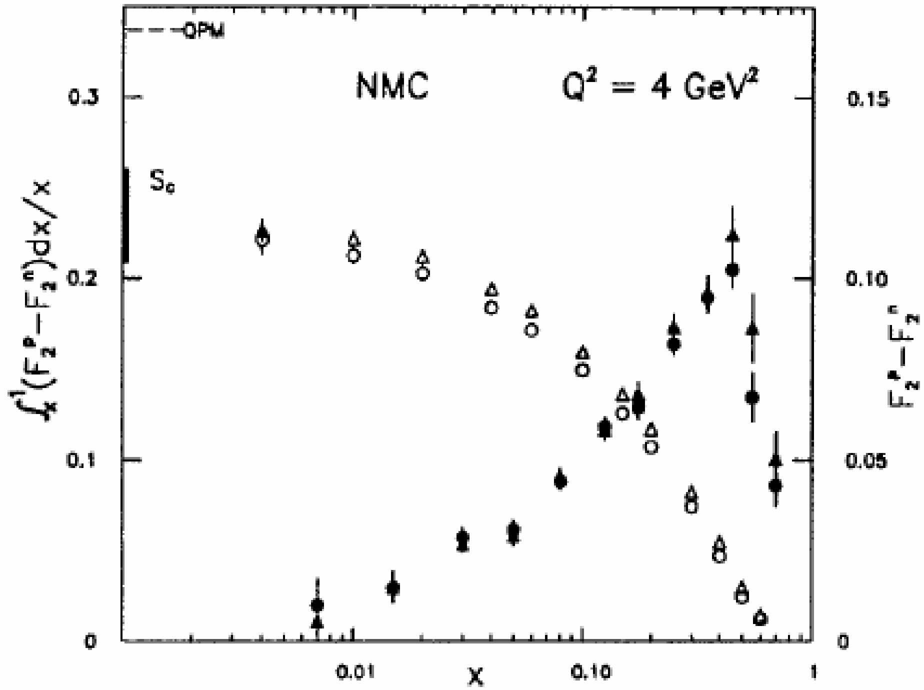


  – We are missing something!

$$\int_0^1 dx \left(F_2^p(x) - F_2^n(x) \right) = 0.235 \pm 0.026$$

The proton's momentum

- But data don't bear this out:



  – We are missing something!

$$\int_0^1 dx \left(F_2^p(x) - F_2^n(x) \right) = 0.235 \pm 0.026$$

- **Attention!** You should be jumping on your chair: there is no free neutron target! This data is from Deuterium targets, D="p+n", without any nuclear correction for binding, Fermi motion, … We will come back to this in Lecture 4 or 5.

Lecture 2 - recap

- Seen by a high-energy probe, the proton is a bag of quasi-free partons (quarks and gluons) sharing its momentum

- The simplest process probing these partons is Deep Inelastic Scattering:
  - The virtual γ interacts with partons of fractional momentum $x = x_B$

- QCD factorization at Leading Order in α_s :
  - This intuitive picture can be realized in QCD, at LO by <u>expanding</u> the parton's momentum in the interaction part of a diagram, and retaining only it's "collinear" components
  - The parton's transverse momentum appears in "higher-twist" terms, and restores gauge invariance in parton rescattering diagrams

- **Next lecture:**
  - Going NLO and the role of gluons; "improved" parton model
  - Basics of global QCD fits of parton distributions