QCD FACTORIZATION AT LO

REFS: Accardi, Qiu JHEP 2008

Bacchetta, lectures

Collins, "Fundamentals of pQCD", Oxford U. Press

Qiu, PRD 1990

Ellis-Furmanski-Petronzio, NPB...

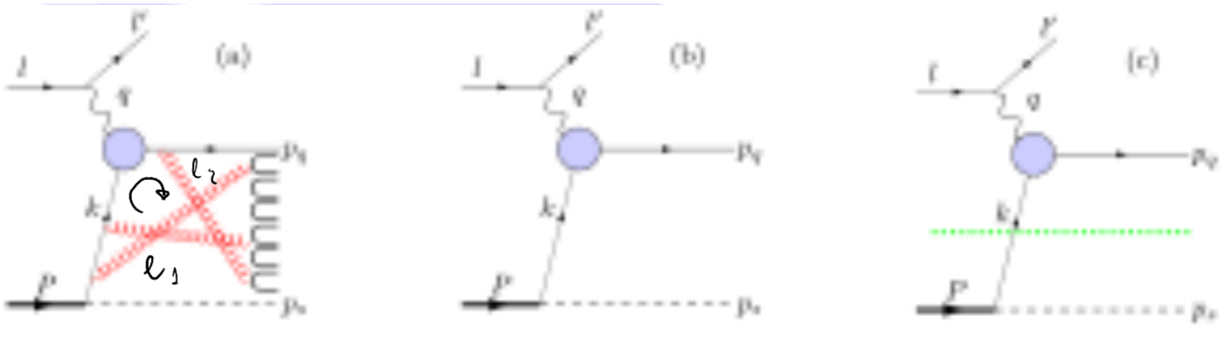
GOAL: • derive collinear factorization formulae in DIS at LO

• show how parton's k_T ends up in HT terms

• maybe, extend to DY,

↳ check universality of HT

(I) DEEP INELASTIC SCATTERING



QCD event — leading region — factorization

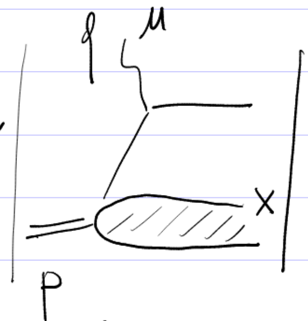
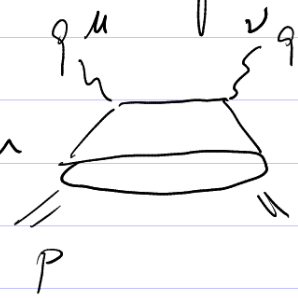
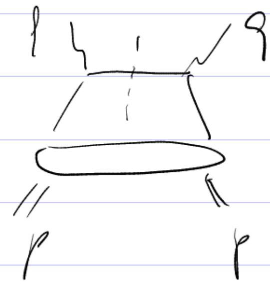
$$\sum_N \int dk \prod_{i=1}^{N} dl_i \; \alpha_s f(k, l_1 \dots l_N)$$

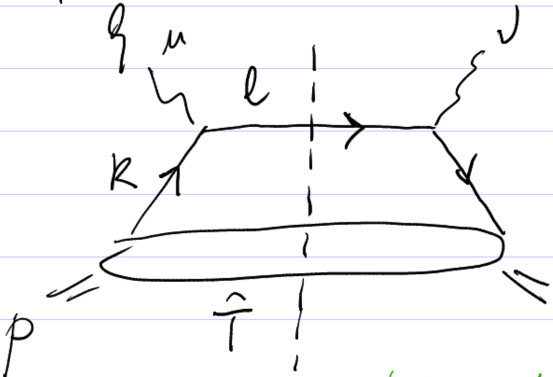
- LEADING REGION = dominant contribution in kinematics of interest:

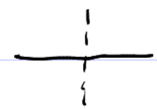
$$Q^2, \nu \to \infty \;, \quad x_B = \frac{Q^2}{2M\nu} \text{ fixed}$$

- The "leading region diagram" can be further approximated
  ↳ the approximation can be done in a way to reproduce the intuitive parton model picture of this process.

* Let's start from the l.o. diagram:

$\sigma \propto$  $= i\,\mathrm{Im}$  $=$ 

$W^{\mu\nu} \equiv$ 

 $= 2\pi i\,(\not\ell + m_q)$

"target function"

· Consider 1 parton flavor for simplicity, $m_q = 0$, 1 color

$$W^{\mu\nu}(p,q) = \frac{e^2}{8\pi}\int \frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\left[\hat{T}(k,p)\,\gamma^\nu \not\ell\, \gamma^\mu\right] 2\pi i\,\delta^{(4)}(q-k-\ell)$$

over

where  $= \sum_X \Big|$  $\Big|^2$

$$[\hat{T}(k)]_{ij} = \sum_X \delta^{(4)}\Big(p - k - \sum_{i\in X} p_i\Big)\,|\langle p|k,X\rangle|^2$$

$$= \int d^4z\, e^{iz\cdot k}\,\langle p|\bar\psi_j(z)\,\psi_i(0)|p\rangle$$

FACTORIZATION PROCEDURE

(A) Expand $\hat{T}$ in Dirac structures; only k dependence

$$\hat{T}(k) = \tau_1(k)\,\hat{\mathbb{1}} + \tau_2(k)\,\not{k} + \cancel{\tau_3(k)\,\hat{\gamma}_5} + \cancel{\tau_4(k)\,\gamma_5 \not{k}}$$

full formula in [Bacchetta]

- By parity conservation, τ_3 and τ_4 do not enter σ
- For massless quarks, the τ_1 piece also does not couple to the top part of the diagram

(1) $$\Longrightarrow \hat{T}(k) = \tau_2(k)\,\not{k}$$

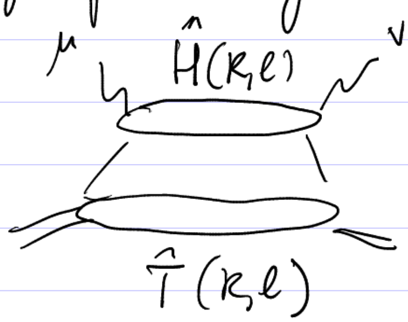
Finally,

$$W^{\mu\nu} = \int \frac{d^4k}{(2\pi)^4}\, \tau_2(k)\, \underbrace{\frac{e_j^2}{8\pi}\, \mathrm{Tr}\left[\not{k}\,\gamma^\nu\,\not{\ell}\,\gamma^\mu\right] \delta^{(4)}(q-k-\ell)}_{H^{\mu\nu}(k)}$$

"HARD SCAT. FUNCTION"

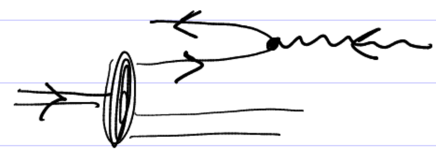
NOTE: • we called the Trace and kinematic constraints $H^{\mu\nu}$ for generality, as this reasoning can be also extended to NLO

- Graphically:



(B) Look at Breit frame, identify dominant components:

$$\begin{cases} p = p^+ \bar{n} + \frac{M^2}{2p^+} n \\ q = -\xi p^+ \bar{n} + \frac{Q^2}{2\xi p^+} n \\ k = x p^+ \bar{n} + \frac{k^2 + k_T^2}{2xp^+} n + \vec{k}_T \end{cases}$$



$q^0 = 0$

Breit frame: $q^0 = 0 \xRightarrow{E} p^+ = \frac{Q}{\sqrt{2}}$

$$\Longrightarrow k = x \frac{Q}{\sqrt{2}} \bar{n} + \frac{k^2 + k_T^2}{\sqrt{2} x Q} + \vec{k}_T$$

$k^+ = O(Q)$ $\quad$ $k^- = O(1/Q)$ $\quad$ $k_T = O(1)$

$$\Longrightarrow k^\mu = \hat{k}^\mu + \delta^\mu$$

NOTE: $\hat{k}^2 = 0$ $\Rightarrow$ massless (my definition)

where $\hat{k} = (k^+, 0, 0_\perp)$ is collinear to the proton's p

The scattered quark momentum is dominated by the k^+ component

(this holds true in any "infinite momentum" frame)

- The dominance of $k^+ \bar{n}$ also suggests to define:

$$\tau_2(k) = \frac{1}{4k^+} \mathrm{Tr}[\not{n} \hat{T}(k)]$$

(3) $$= \frac{1}{4k^+} \int d^4z \, e^{iz \cdot k} \langle p | \bar{\psi}(z) \gamma^+ \psi(0) | p \rangle$$

(at this point the definition may seem arbitrary, but when a generic scattering process ~~is considered~~, and a full expansion of $\hat{T}$ are considered, the definition (3) makes more sense
↳ see [Bacchetta]

(C) EXPAND THE HARD PART AROUND $\hat{k}$

This is the heart of collinear factorization:

$$H^{\mu\nu}(k, q) = H^{\mu\nu}(\hat{k}) + \left.\frac{\partial H}{\partial k^\alpha}\right|_{k=\hat{k}} (k^\alpha - \hat{k}^\alpha) + \ldots$$

|| C.F. IS NOT AN APPROXIMATION, RATHER AN EXPANSION AROUND COLLIN PARTON MOMENTA

↳ leading terms are retained at "leading-twist"

↳ higher order terms will become part of so called "higher-twist terms"

D) REARRANGE THE INTEGRATIONS:

$\int dk^+$ remains outside, $\int dk_T dk^-$ hit Φ_2

use:

$$d^4k = dk^+ dk^- d^2k_T$$

Find:

$$W^{\mu\nu} = \int \frac{dk^+}{k^+} \underbrace{\left[\int dk^- dk_\perp \Phi_2(k)\right]}_{\equiv f_q \text{ "QUARK DISTRIBUTION"}} H^{\mu\nu}(\hat{k})$$

$k^+ = x p^+$

Using $\int dk^- d^2k_T \, e^{iz^+k^-} e^{-i\vec{z}_T\cdot\vec{k}_T} e^{iz^-k^+} = (2\pi)^3 \delta(z^+)\delta^{(2)}(\vec{z}_T)$

$$(4) \quad \boxed{f_q = f_q(x) = \int \frac{dz^-}{2\pi} e^{-ixp^+z^-} \langle p|\bar{\psi}(z^-n)\frac{\gamma^+}{2}\psi(0)|p\rangle}$$

QUARK DISTRIBUTION FUNCTION

NOTE: <u>DELICATE STEP:</u>

- we also approximated the partonic kinematics
- $\int dk^- d^2k_T$ have become unbounded (they were <u>NOT</u> ~~bounded~~ in the full formula see MOFFAT et al 2017)

(5) OBTAIN FACTORIZED FORMULA:

$$W^{\mu\nu}(P,q) = \int \frac{dx}{x}\, H^{\mu\nu}(\tilde{k}, q)\, f_q(x)$$

in terms of Lorentz invariants,

$$H^{\mu\nu}(\tilde{k}, q) \equiv H^{\mu\nu}(\hat{x}, Q^2)$$

where: $\hat{x} \equiv \dfrac{Q^2}{2\hat{k}\cdot q} \underset{(E)}{=} \dfrac{x_B}{x}$ = PARTON-LEVEL BJORKEN INVARIANT

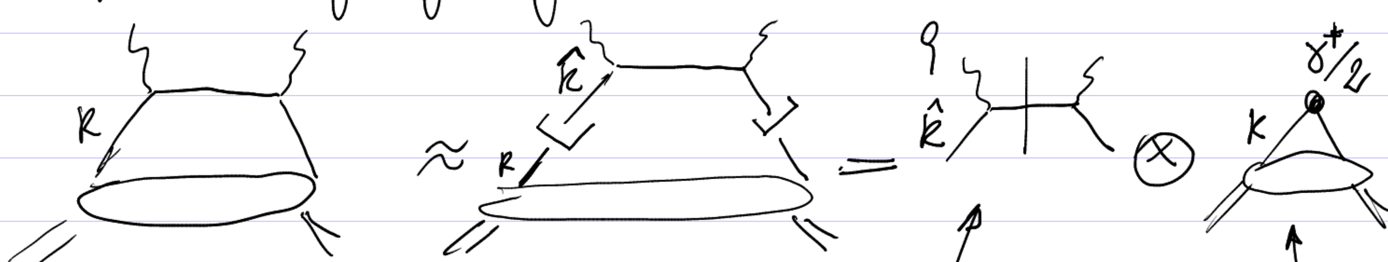
• And we obtain:

$$(S) \quad \boxed{W^{\mu\nu} = \int_{x_B}^{1} \frac{dx}{x}\, h^{\mu\nu}\left(\frac{x_B}{x}, Q^2\right) f_q(x)}$$

FACTORIZATION FORMULA FOR DIS

where the kinematic limits will be proven in a sec.

• Graphically, going back to LO:



hard scattering on a free, collinear quark

quark distrib. fn.

REMARKS:

- Equ(5) only proves that the hadronic tensor can be written as a convolution at LO

  ↳ going to NLO requires cancelling divergences, proving that the "renormalized" PDFs are universal (e.g. are the same in DY)

Only then, can we talk of FACTORIZATION THEOREM

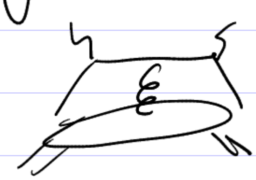
⇝ [STERMAN] for a full discussion

- What happens to the terms we neglected in the expansion of $H^{\mu\nu}$?

$$H^{\mu\nu} = H^{\mu\nu}\Big|_{\hat{k}} + \frac{\partial H^{\mu\nu}}{\partial k^\alpha}\left(k^\alpha - \hat{k}^\alpha\right) + \dots$$

→ CONTAIN k_T!

⇝ They will combine with "HIGHER TWIST DIAGRAMS"

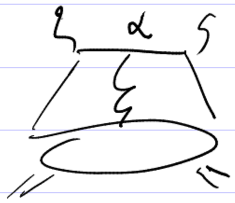


$$+ \int T \left.\frac{\partial H}{\partial k^\alpha}\right|_{\hat{k}} \left(k^\alpha - \hat{k}^\alpha\right)$$

To make these gauge invariant.

⇝ [Qiu]

Let's sketch the argument; see [Qiu] for full proof:



$$\propto \text{F.T.}\left[ig \langle p|\bar{\psi}(0) A_\alpha(y)\, \psi(z)|p\rangle \right]$$

$$k^\alpha T(k) = \int d^4z\, \frac{\partial}{\partial z^\alpha} e^{iz\cdot k} \langle p|\bar{\psi}(0)\, \psi(z)|p\rangle$$

$$= \text{F.T.}\left[\langle p|\bar{\psi}(0)\, \partial_\alpha \psi(z)|p\rangle \right]$$

Combine the two, obtain:

$$\langle p|\, \bar{\psi}\, D_\alpha \psi\, |p\rangle$$

COVARIANT DERIVATIVE

This is an example of TWIST-3 OPERATOR (has mass dimension +1 relative to TWIST-2 $\langle p|\bar{\psi}\psi|p\rangle$