GGI

Giuseppe Marchesini Memorial Conference

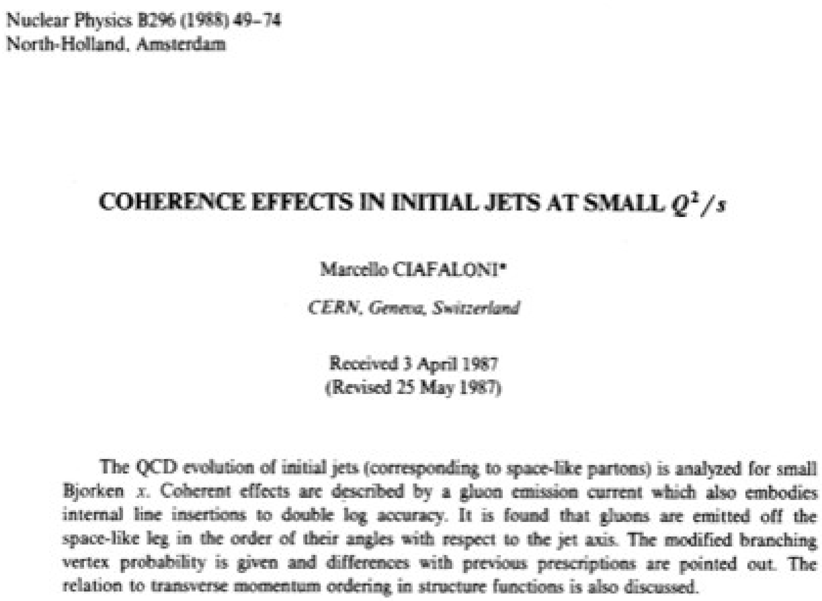
Florence, May 2017

F Hautmann

Pino and high-energy QCD:

the CCFM equation

The CCFM papers



Nuclear Physics B296 (1988) 49–74
North-Holland, Amsterdam

COHERENCE EFFECTS IN INITIAL JETS AT SMALL Q^2/s

Marcello CIAFALONI*

CERN, Geneva, Switzerland

Received 3 April 1987
(Revised 25 May 1987)

The QCD evolution of initial jets (corresponding to space-like partons) is analyzed for small Bjorken x. Coherent effects are described by a gluon emission current which also embodies internal line insertions to double log accuracy. It is found that gluons are emitted off the space-like leg in the order of their angles with respect to the jet axis. The modified branching vertex probability is given and differences with previous prescriptions are pointed out. The relation to transverse momentum ordering in structure functions is also discussed.

Nuclear Physics B336 (1990) 18–85
North-Holland

SMALL-x BEHAVIOUR OF INITIAL STATE RADIATION IN PERTURBATIVE QCD*

S. CATANI

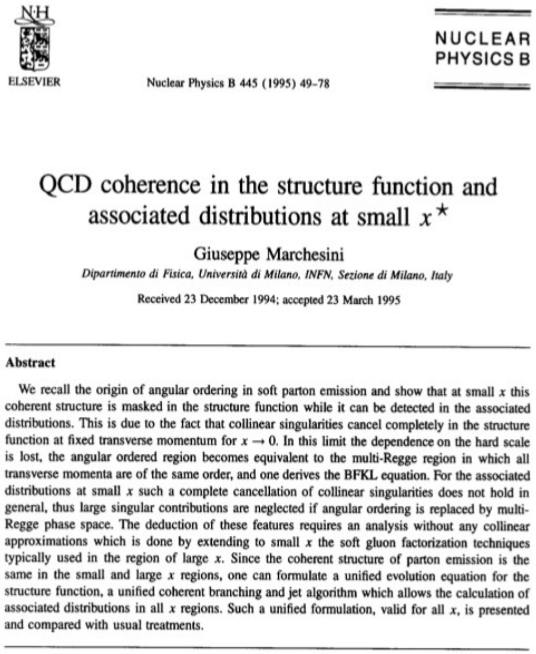
Dipartimento di Fisica, Università di Firenze, INFN, Sezione di Firenze, Italy

F. FIORANI and G. MARCHESINI

Dipartimento di Fisica, Università di Parma, INFN, Gruppo Collegato di Parma, Italy

Received 17 July 1989

We analyze in perturbative QCD the asymptotic behaviour of deep inelastic processes in the semi-hard region $x \to 0$. The study is done by extending the soft gluon insertion techniques. We confirm and extend the analysis recently performed by Ciafaloni. The main results are the following: (i) Soft gluon emission from the incoming parton takes place in a region where the angles between incoming and outgoing partons are ordered. This is due to coherent effects similar to the ones in the $x \to 1$ region. (ii) Virtual corrections involving an internal line with energy fraction x give rise, for $x \to 0$, to a new form factor of non-Sudakov type. This regularizes collinear singularities when an emitted gluon is parallel to the incoming parton. (iii) At the complete inclusive level, the new form factor plays the same role as the virtual corrections in the Lipatov equation for the Regge regime. We show that, in the semi-hard regime, the gluon anomalous dimension coincides with the Lipatov ansatz. (iv) We identify the branching structure of initial-state radiation including the semi-hard regime. The branching is formulated as a probability process given in terms of Sudakov and non-Sudakov form factors. This process, in principle, can be used to extend the existing simulations of QCD cascades to the semi-hard regime.



ELSEVIER

Nuclear Physics B 445 (1995) 49–78

NUCLEAR PHYSICS B

QCD coherence in the structure function and associated distributions at small $x^\star$

Giuseppe Marchesini

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Received 23 December 1994; accepted 23 March 1995

Abstract

We recall the origin of angular ordering in soft parton emission and show that at small x this coherent structure is masked in the structure function while it can be detected in the associated distributions. This is due to the fact that collinear singularities cancel completely in the structure function at fixed transverse momentum for $x \to 0$. In this limit the dependence on the hard scale is lost, the angular ordered region becomes equivalent to the multi-Regge region in which all transverse momenta are of the same order, and one derives the BFKL equation. For the associated distributions at small x such a complete cancellation of collinear singularities does not hold in general, thus large singular contributions are neglected if angular ordering is replaced by multi-Regge phase space. The deduction of these features requires an analysis without any collinear approximations which is done by extending to small x the soft gluon factorization techniques typically used in the region of large x. Since the coherent structure of parton emission is the same in the small and large x regions, one can formulate a unified evolution equation for the structure function, a unified coherent branching and jet algorithm which allows the calculation of associated distributions in all x regions. Such a unified formulation, valid for all x, is presented and compared with usual treatments.

- Theoretical introduction - this talk
- Comparisons with experiment – Hannes Jung's talk

CCFM and the high-energy QCD landscape – circa 1990

- Confidence in theoretical QCD predictions for hard processes not only due to asymptotic freedom but also due to methods to compute systematically large radiative contributions:

  – collinear parton radiation and renormalization group evolution

  Dokshitzer; Altarelli-Parisi; Gribov-Lipatov

  – soft gluon radiation and the role of infrared singularities

  Dokshitzer-Diakonov-Troyan; Parisi-Petronzio;
  Mueller; Bassetto-Ciafaloni-Marchesini

- What becomes of this in the high-energy regime? (relevant to SSC, LHC, HERA colliders . . .)

  Note: something must change . . . - fixed t, high s asymptotics!

  Balitski-Fadin-Kuraev-Lipatov

Soft-gluon coherence

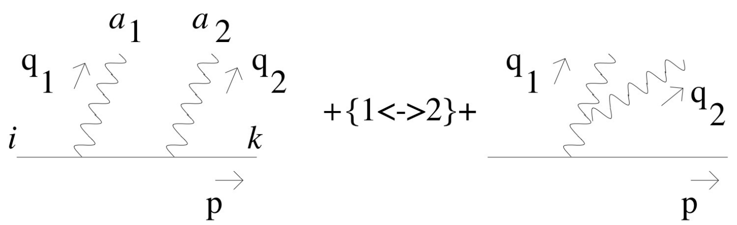
The CCFM approach: what does soft-gluon coherence look like in the high-energy (multi-Regge) kinematics

- Recall eikonal insertion currents:

$$|M_{n+1}^{a_1\ldots a_n a}(p_1,p_n,q)\rangle = \mathbf{J}^a|M_n^{a_1\ldots a_n}(p_1,p_n)\rangle \ , \quad \mathbf{J}^{a\mu} = \sum_i \mathbf{Q}_i^a \frac{p_i^\mu}{p_i\cdot q}$$

- Color coherence effect:

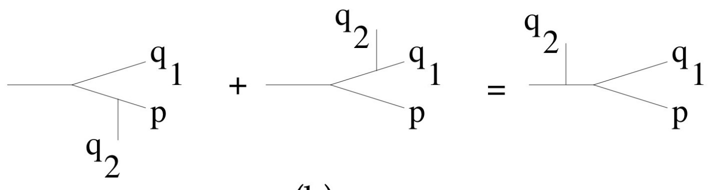
Bassetto-Ciafaloni-Marchesini



(a)

$q_2^0 \ll q_1^0$

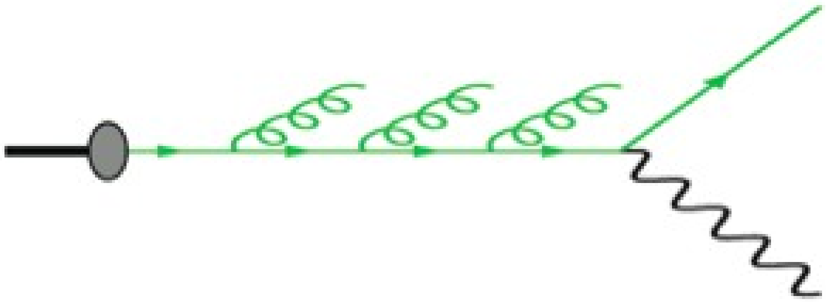
$$\begin{aligned}\mathcal{M}_{ki}^{a_1a_2} &= g_s^2\,\langle a_1\, k|\,\mathbf{J}_2\cdot\varepsilon_2\,|a'\, i'\rangle\,\langle\, i'\,|\mathbf{J}_1\cdot\varepsilon_1|\, i\,\rangle \\ &= g_s^2\,\frac{p\cdot\varepsilon_1}{p\cdot q_1}\left(\frac{p\cdot\varepsilon_2}{p\cdot q_2}\,t^{a_2}t^{a_1} + \frac{q_1\cdot\varepsilon_2}{q_1\cdot q_2}\,[t^{a_1},t^{a_2}]\right)_{ki}\end{aligned}$$



(b)

- small angle: bremsstrahlung cones
- large angle ($\theta_{pq_2} \gg \theta_{pq_1}$): sees total charge $\mathbf{Q}_p + \mathbf{Q}_{q_1}$

CCFM: coherence in the high-energy limit



- Longitudinal momenta exchanged along the initial-state decay chain become small: x << 1
- Internal emissions non-negligible
- Yet, current is factorizable at high energy!

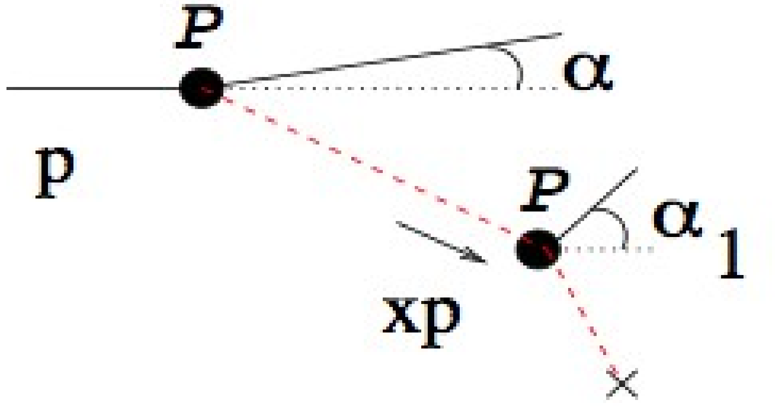
$$|M^{(n+1)}(k,p)|^2 = \{[M^{(n)}(k+q,p)]^\dagger\ [\mathbf{J}^{(R)}]^2\ M^{(n)}(k+q,p) - [M^{(n)}(k,p)]^\dagger\ [\mathbf{J}^{(V)}]^2\ M^{(n)}(k,p)\}\ .\quad \text{BUT...} \triangleright$$

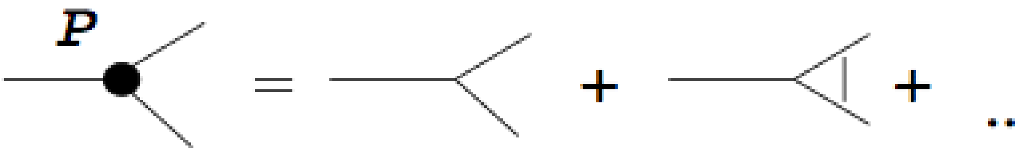
> J depends on transverse momentum transmitted down the chain
→ TMD (or unintegrated) matrix elements and
parton density functions
(→ see present-day “TMDs”)

> virtual corrections imply, besides Sudakov form factors,
modified branching probabilities
(→ “parton-shower” MC)

Exclusive evolution equation

$$\mathcal{G}(x, k_T, \mu) = \mathcal{G}_0(x, k_T, \mu) + \int \frac{dz}{z} \int \frac{dq^2}{q^2}\, \Theta(\mu - zq)$$
$$\times \underbrace{\Delta(\mu, zq)}_{\text{Sudakov}} \quad \underbrace{\mathcal{P}(z, q, k_T)}_{\text{unintegr. splitting}} \quad \mathcal{G}\left(\frac{x}{z}, k_T + (1-z)q, q\right)$$





Ciafaloni
Catani-Fiorani-Marchesini

- Returns the Balitski-Fadin-Kuraev-Lipatov (BFKL) anomalous dimension in the inclusive limit for x → 0
- Incorporates angular ordering and soft-gluon radiation with leading logarithmic accuracy to all orders in the strong coupling for x → 1

Space-like Anomalous Dimensions at N ⟶ 0

$$\gamma_{ab,N}(\alpha_S) = \sum_{k=1}^{\infty} \left[\left(\frac{\alpha_S}{N}\right)^k A_{ab}^{(k)} + \alpha_S \left(\frac{\alpha_S}{N}\right)^k B_{ab}^{(k)} + \dots \right]$$

N - moment conjugate to ln s

- The CCFM equation contains the dominant (BFKL) gluon anomalous dimensions

$$\gamma_{gg,N} = \gamma_N(\alpha_S) + \mathcal{O}(\alpha_S(\alpha_S/N)^k)$$

given by

$$1 = \frac{\bar{\alpha}_S}{N} \chi(\gamma_N(\alpha_S)) \; , \quad \chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma) = \frac{1}{\gamma}\left[1 + \sum_{k=1}^{\infty} 2\,\zeta(2k+1)\,\gamma^{2k+1}\right]$$

Power series expansion:

$$\gamma_N(\alpha_S) = \sum_{n=1}^{\infty} g_n \left(\frac{\bar{\alpha}_S}{N}\right)^n = \frac{\bar{\alpha}_S}{N} + 2\zeta(3)\left(\frac{\bar{\alpha}_S}{N}\right)^4 + 2\zeta(5)\left(\frac{\bar{\alpha}_S}{N}\right)^6 + \mathcal{O}\left(\left(\frac{\bar{\alpha}_S}{N}\right)^7\right)$$

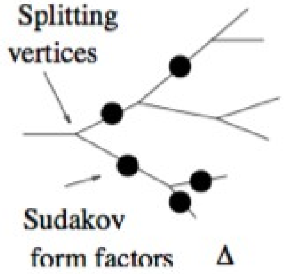
- Next-to-leading corrections at N ⟶ 0 computed in the subsequent decade.

Camici-Ciafaloni; Fadin-Lipatov; Catani-H

Application to Shower Monte Carlo Event Generators

- Factorizability of QCD x-sections ⟶ probabilistic branching picture

  ◇ QCD evolution by "parton showering" methods:

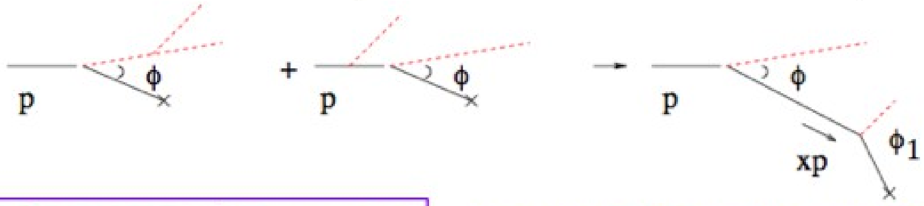


$$d\mathcal{P} = \int \frac{dq^2}{q^2} \int dz \, \alpha_S(q^2) \, P(z) \, \Delta(q^2, q_0^2)$$

$\hookrightarrow \boxed{\text{collinear, incoherent emission}}$

◇ Soft emission ⟶ interferences ⟶ ordering in decay angles:

$\hookrightarrow \boxed{\text{gluon coherence for } x \sim 1}$



◇ $\boxed{\text{Gluon coherence for } x \ll 1}$ ⇒ corrections to angular ordering:

↪ $k_\perp$-dependent parton showers

CCFM equation is TMD branching equation which contains both Sudakov physics and BFKL physics

CCFM-based Monte Carlo Event Generators

- SMALLX (Marchesini and Webber 1991, 1992)
- CASCADE (Jung and Salam 2001; Jung et al. 2010)
- LDCMC (Gustafson, Lonnblad and Miu, 2002; Lonnblad and Sjodahl, 2005)
- UPDFevolv (Jung, Taheri-Monfared and H, 2014)
- DIPSY (Gustafson, Lonnblad et al. 2011, 2015)

OUTLOOK

CCFM contributions affect both

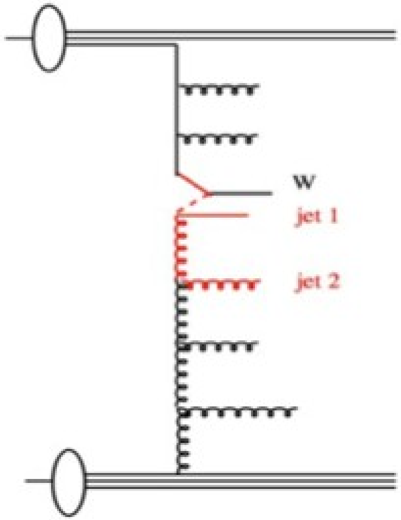
- scaling violation in inclusive structure functions

- recent determinations of TMD parton density functions from DIS data

- structure of the associated final states: jet multiplicities and angular correlations

- predictions of final-states observables at the LHC and future colliders

Applying CCFM to LHC physics: vector boson + jets final states



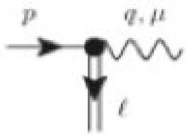
Dooling, Jung & H, Phys. Lett. B736 (2014) 293

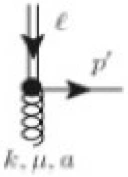
→ Use exclusive CCFM evolution

→ Determine TMD pdf from high-precision DIS data

Obtain predictions for final states associated with Drell-Yan using high-energy off-shell matrix elements

- High-energy effective theory → effective vertices

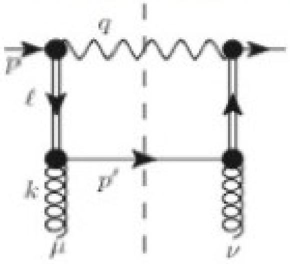




[Bogdan & Fadin, NPB740 (2006) 36]

[Lipatov & Vyazovsky, NPB597 (2001) 399]

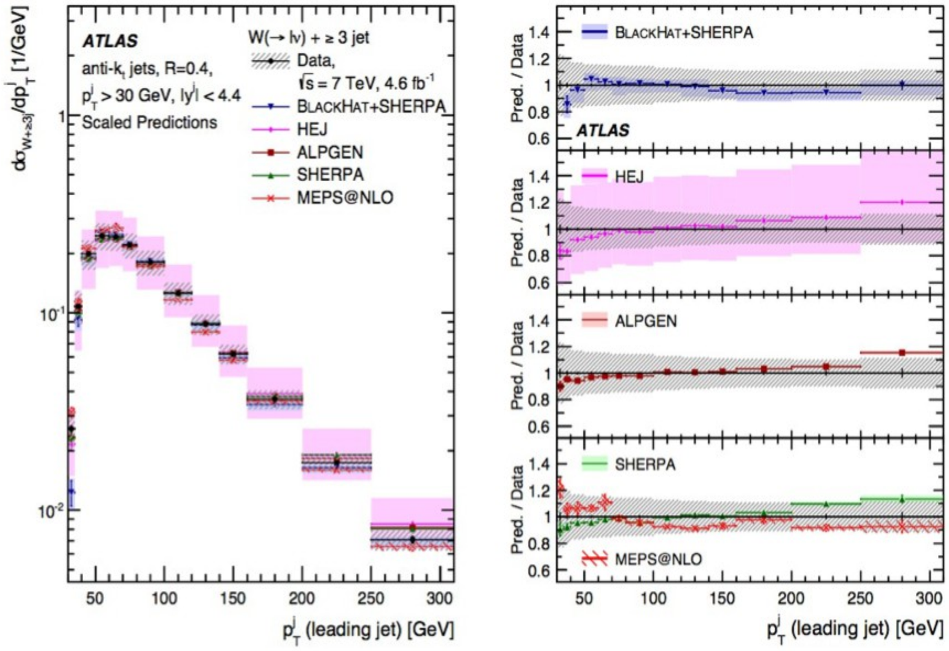
- Parton matrix elements (gauge-invariant, despite off-shell parton)



[Ball & Marzani, NPB814 (2009) 246]

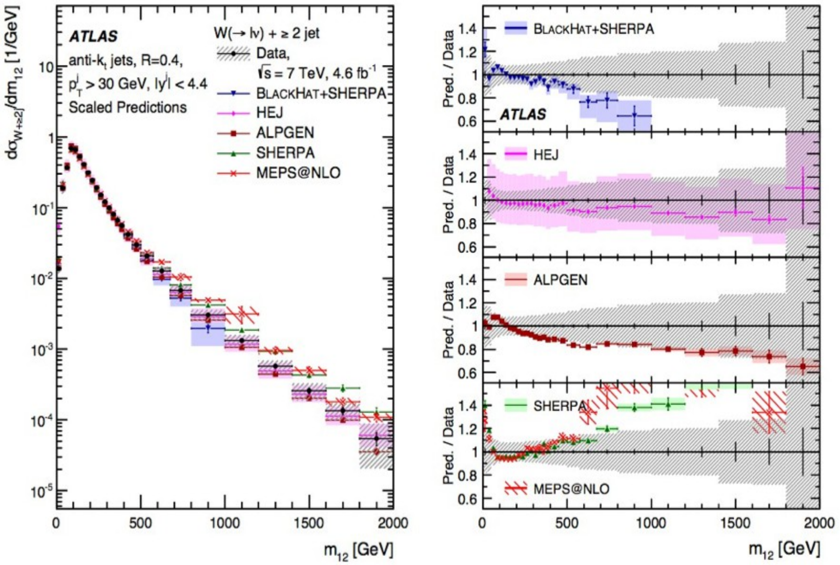
[Hentschinski, Jung & H, NPB865 (2012) 54]

ATLAS, EPJC 75 (2015) 82



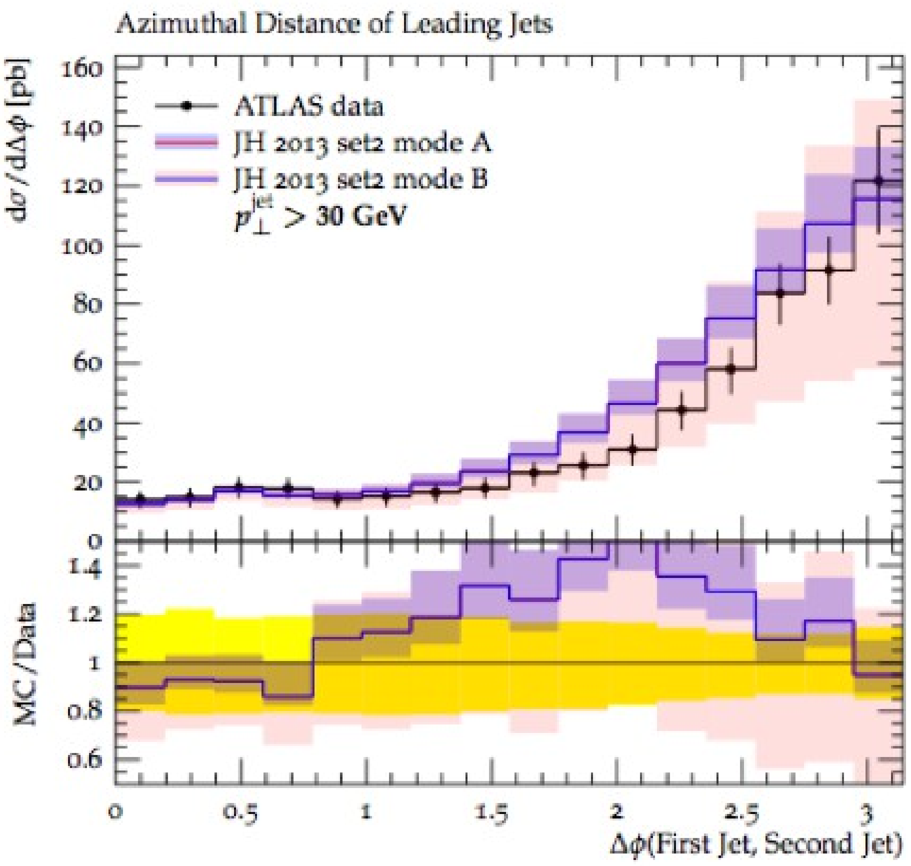
Good agreement between all predictions and data for inclusive observables

ATLAS, EPJC 75 (2015) 82



Large spread in predictions for invariant mass spectrum

Applying CCFM to LHC physics: vector boson + jets final states

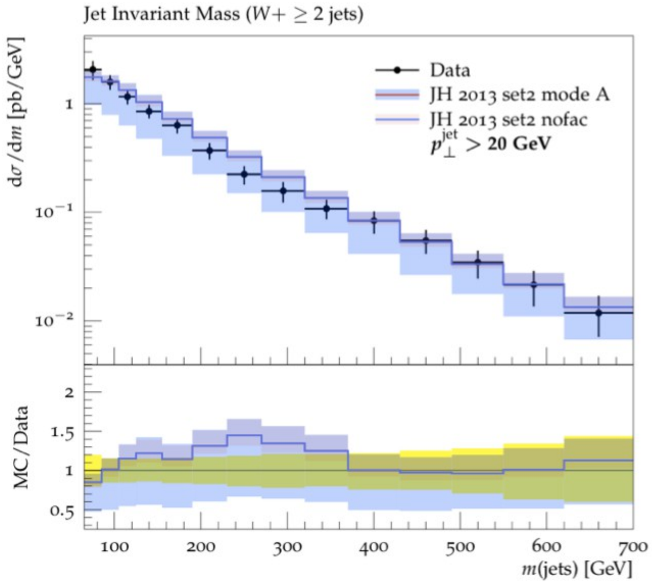


Dooling, Jung & H, Phys. Lett. B736 (2014) 293

W + n jets: dijet mass spectra from TMD-resummed approach

- not only collinear-ordered emissions but also non-ordered region which opens up at high s / pt^2 (and large pt).
- Finite angle multi-gluon radiation.

R. Angeles-Martinez et al., arXiv:1507.05267



As a conclusion . . .

- An early CCFM Monte Carlo simulation

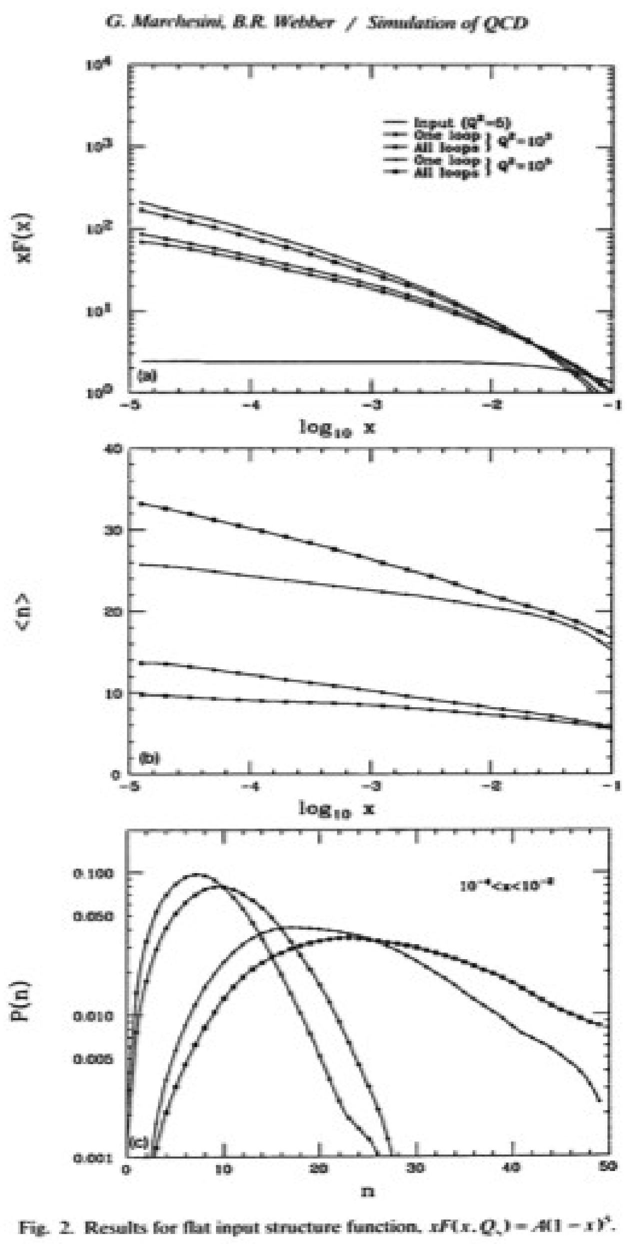
  Marchesini and Webber,

  Nucl. Phys. B349 (1991) 617

- discussed with Pino at

  Cortona Theory Meeting and

  Gran Sasso Summer Institute, 1994



G. Marchesini, B.R. Webber / Simulation of QCD

Fig. 2. Results for flat input structure function, $xF(x,Q_0) = A(1-x)^5$.

. . . a study of soft-gluon resolution scale in QCD evolution equations

Jung, Lelek, Radescu, Zlebcik & H, arXiv:1704.01757

- highlights the role of angular ordering in predicting the transverse momentum from the initial-state shower

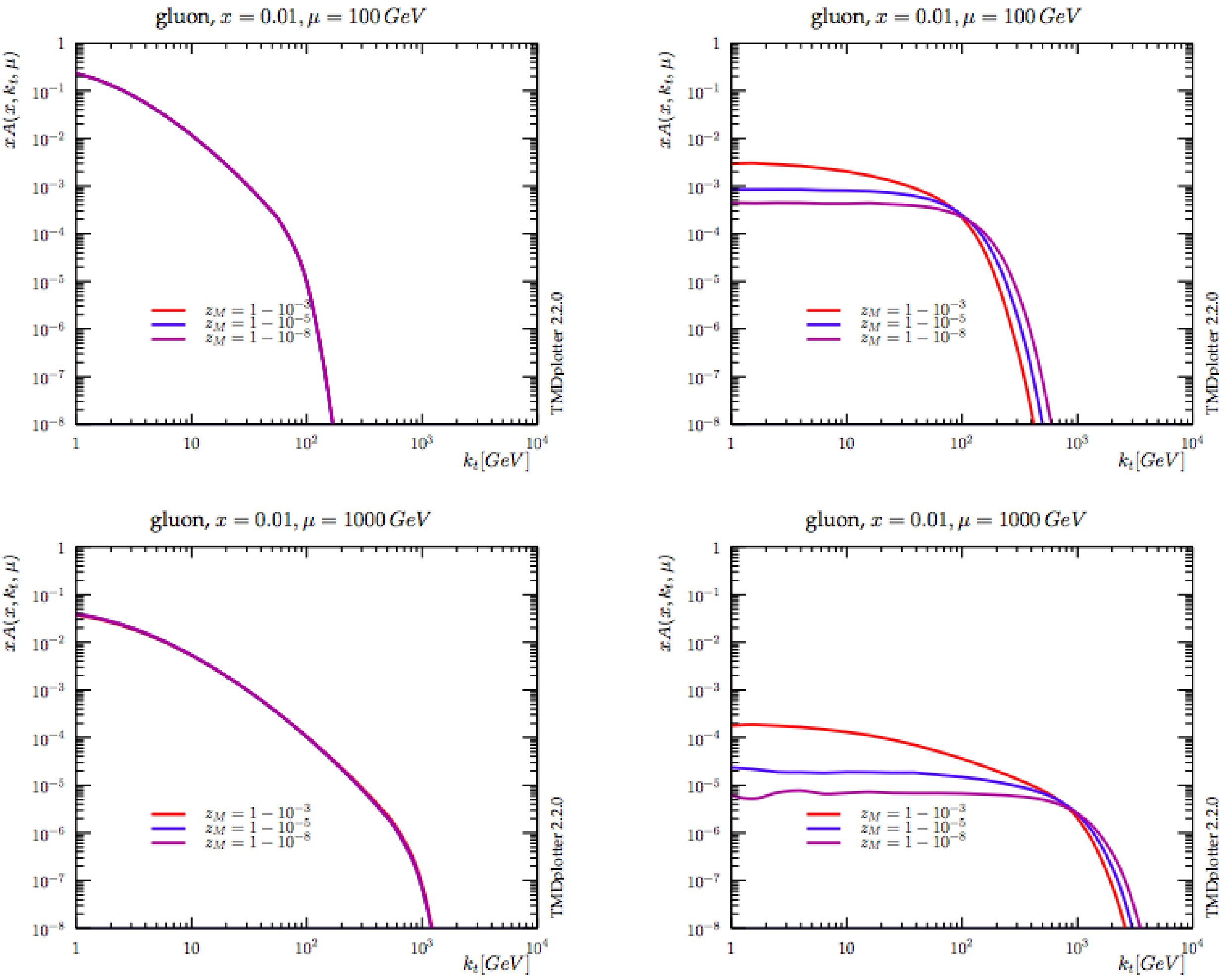


Figure 2: *Transverse momentum gluon distribution at $x = 10^{-2}$ and $\mu = 100$ GeV (upper row), $\mu = 1000$ GeV (lower row) for different values of the resolution scale parameter $1 - z_M = 10^{-3}, 10^{-5}, 10^{-8}$: (left) angular ordering; (right) transverse momentum ordering.*

CCFM evolution from the 90's to now: experiments

The challenges at HERA and LHC

- the small x issues
- how to obtain CCFM parton distribution
- Monte Carlo implementations
- final state predictions for HERA and LHC

CCFM goes public

PROCEEDINGS
SUPPLEMENTS
NUCLEAR PHYSICS B

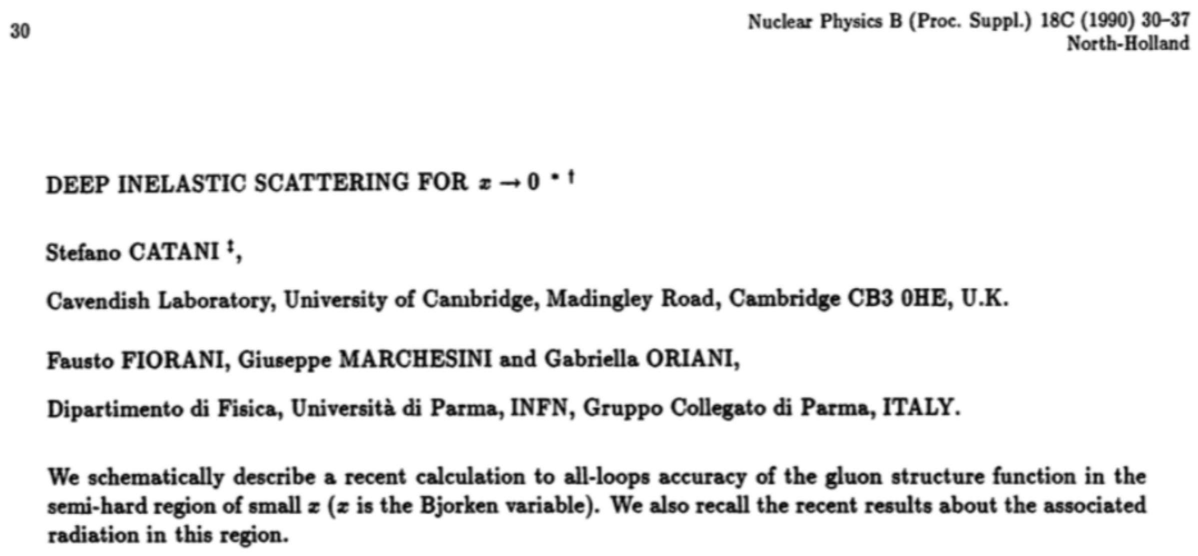
Nuclear Physics B (Proc. Suppl.) 18C (1990) FEBRUARY 1991

SMALL-X BEHAVIOR OF
DEEP INELASTIC STRUCTURE
FUNCTIONS IN QCD

Proceedings of the
DESY Topical Meeting on the
Small-x Behavior of
Deep Inelastic Structure
Functions in QCD
DESY, Hamburg, Germany
14–16 May 1990

Edited by
A. ALI and J. BARTELS

NORTH-HOLLAND



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Nuclear Physics B (Proc. Suppl.) 18C (1990) 30–37
North-Holland

DEEP INELASTIC SCATTERING FOR $x \to 0$ [*] [†]

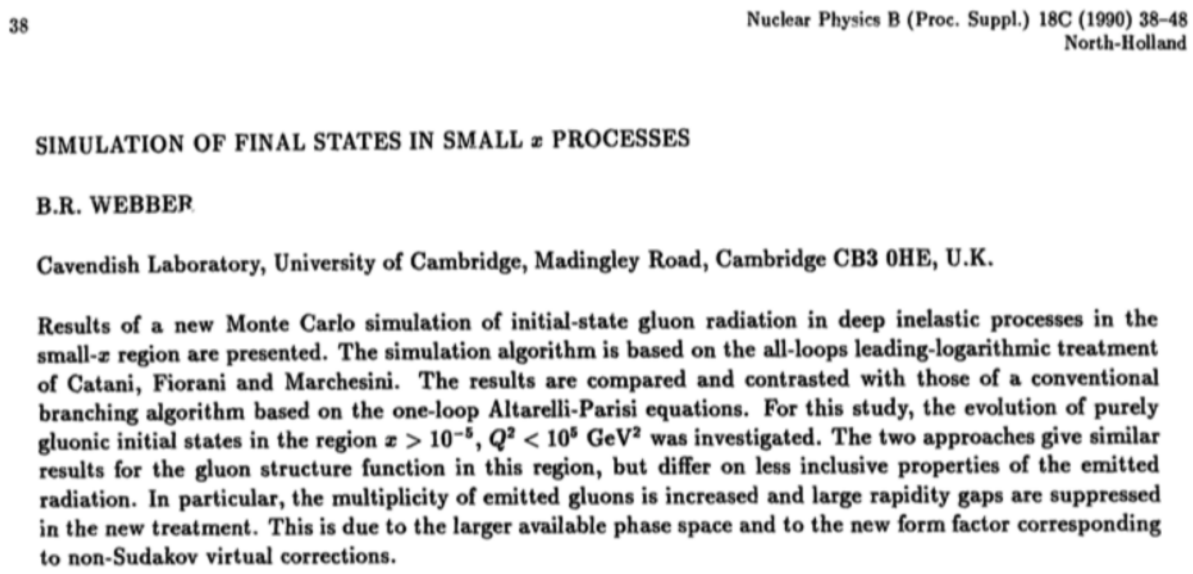
Stefano CATANI [‡],

Cavendish Laboratory, University of Cambridge, Madingley Road, Cambridge CB3 0HE, U.K.

Fausto FIORANI, Giuseppe MARCHESINI and Gabriella ORIANI,

Dipartimento di Fisica, Università di Parma, INFN, Gruppo Collegato di Parma, ITALY.

We schematically describe a recent calculation to all-loops accuracy of the gluon structure function in the semi-hard region of small x (x is the Bjorken variable). We also recall the recent results about the associated radiation in this region.



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Nuclear Physics B (Proc. Suppl.) 18C (1990) 38–48
North-Holland

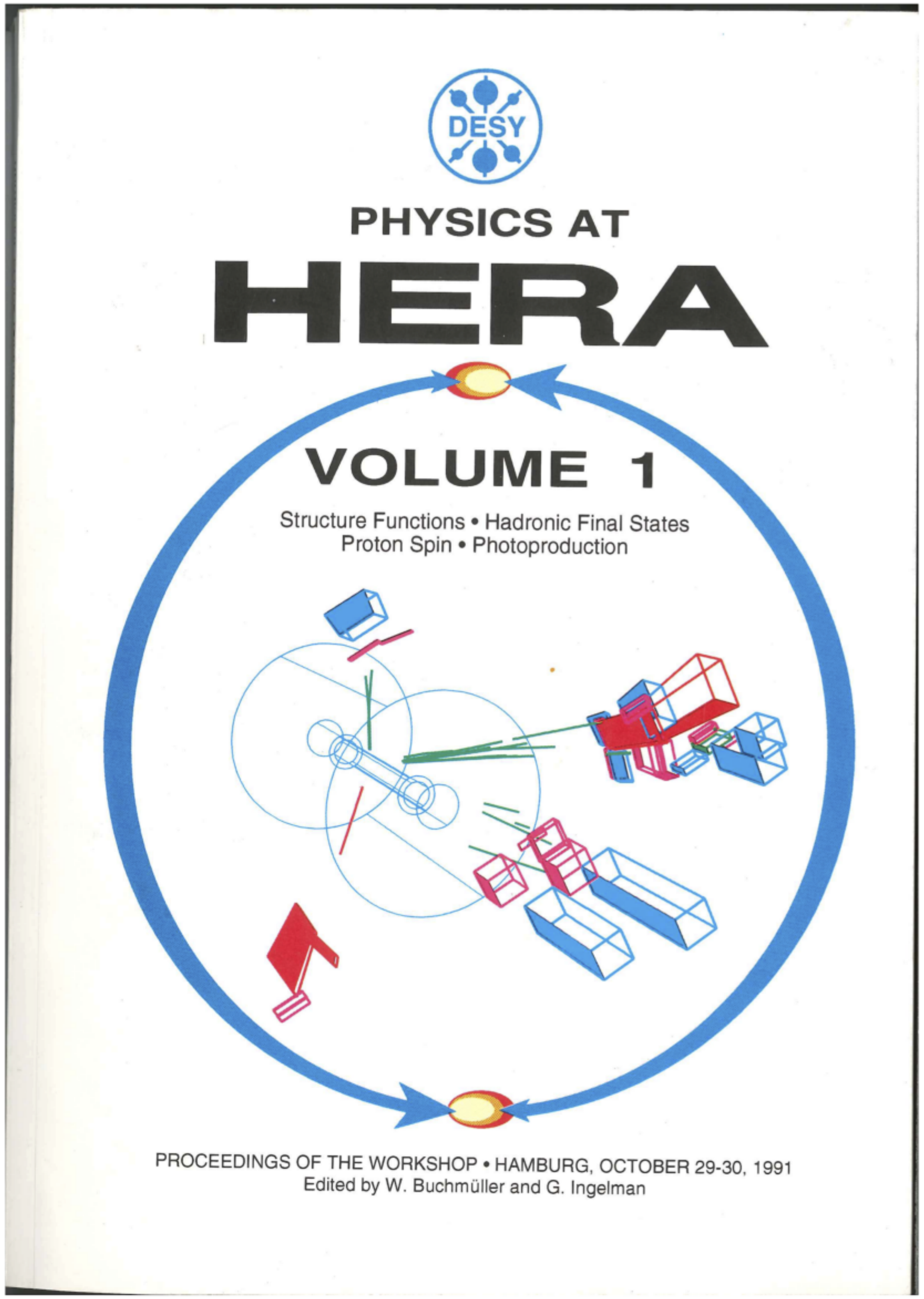
SIMULATION OF FINAL STATES IN SMALL x PROCESSES

B.R. WEBBER

Cavendish Laboratory, University of Cambridge, Madingley Road, Cambridge CB3 0HE, U.K.

Results of a new Monte Carlo simulation of initial-state gluon radiation in deep inelastic processes in the small-x region are presented. The simulation algorithm is based on the all-loops leading-logarithmic treatment of Catani, Fiorani and Marchesini. The results are compared and contrasted with those of a conventional branching algorithm based on the one-loop Altarelli-Parisi equations. For this study, the evolution of purely gluonic initial states in the region $x > 10^{-5}$, $Q^2 < 10^5$ GeV^2 was investigated. The two approaches give similar results for the gluon structure function in this region, but differ on less inclusive properties of the emitted radiation. In particular, the multiplicity of emitted gluons is increased and large rapidity gaps are suppressed in the new treatment. This is due to the larger available phase space and to the new form factor corresponding to non-Sudakov virtual corrections.

Physics at HERA workshop 1991: CCFM



DESY

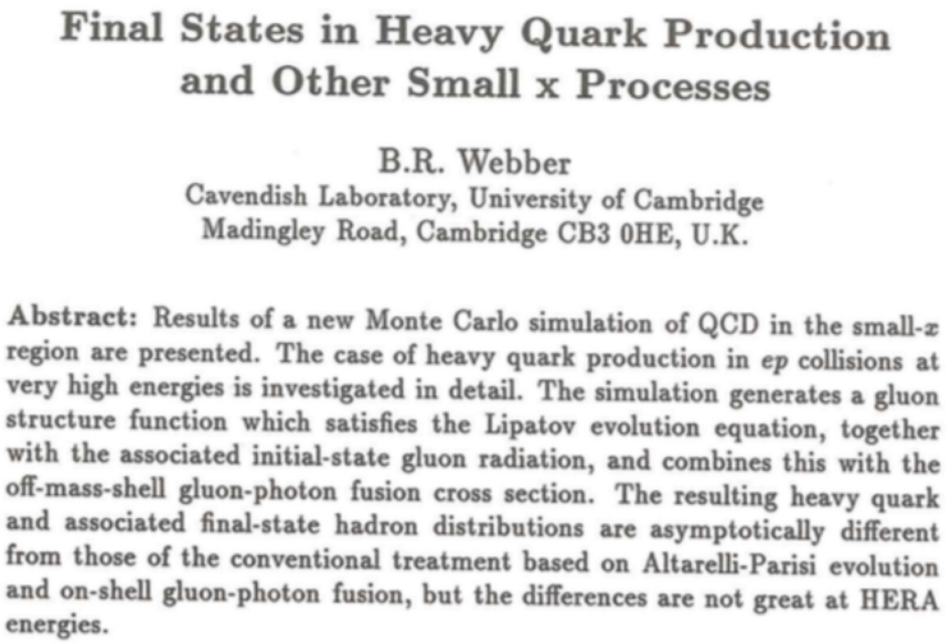
PHYSICS AT

HERA

VOLUME 1

Structure Functions • Hadronic Final States
Proton Spin • Photoproduction

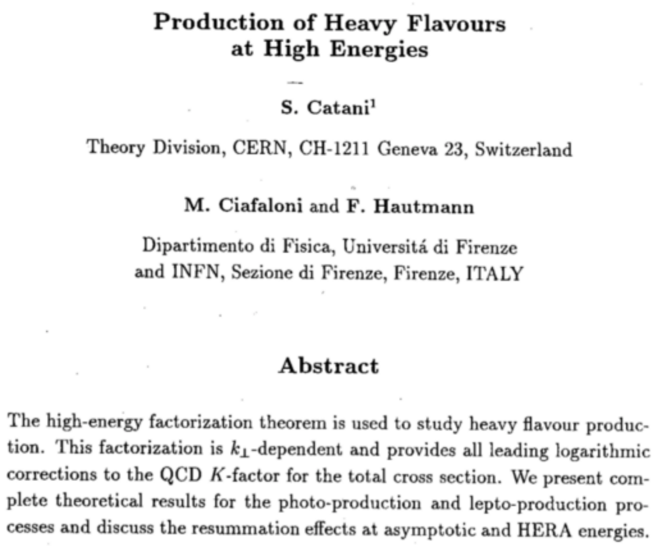
PROCEEDINGS OF THE WORKSHOP • HAMBURG, OCTOBER 29-30, 1991
Edited by W. Buchmüller and G. Ingelman



Final States in Heavy Quark Production and Other Small x Processes

B.R. Webber
Cavendish Laboratory, University of Cambridge
Madingley Road, Cambridge CB3 0HE, U.K.

Abstract: Results of a new Monte Carlo simulation of QCD in the small-x region are presented. The case of heavy quark production in *ep* collisions at very high energies is investigated in detail. The simulation generates a gluon structure function which satisfies the Lipatov evolution equation, together with the associated initial-state gluon radiation, and combines this with the off-mass-shell gluon-photon fusion cross section. The resulting heavy quark and associated final-state hadron distributions are asymptotically different from those of the conventional treatment based on Altarelli-Parisi evolution and on-shell gluon-photon fusion, but the differences are not great at HERA energies.



Production of Heavy Flavours at High Energies

S. Catani[1]

Theory Division, CERN, CH-1211 Geneva 23, Switzerland

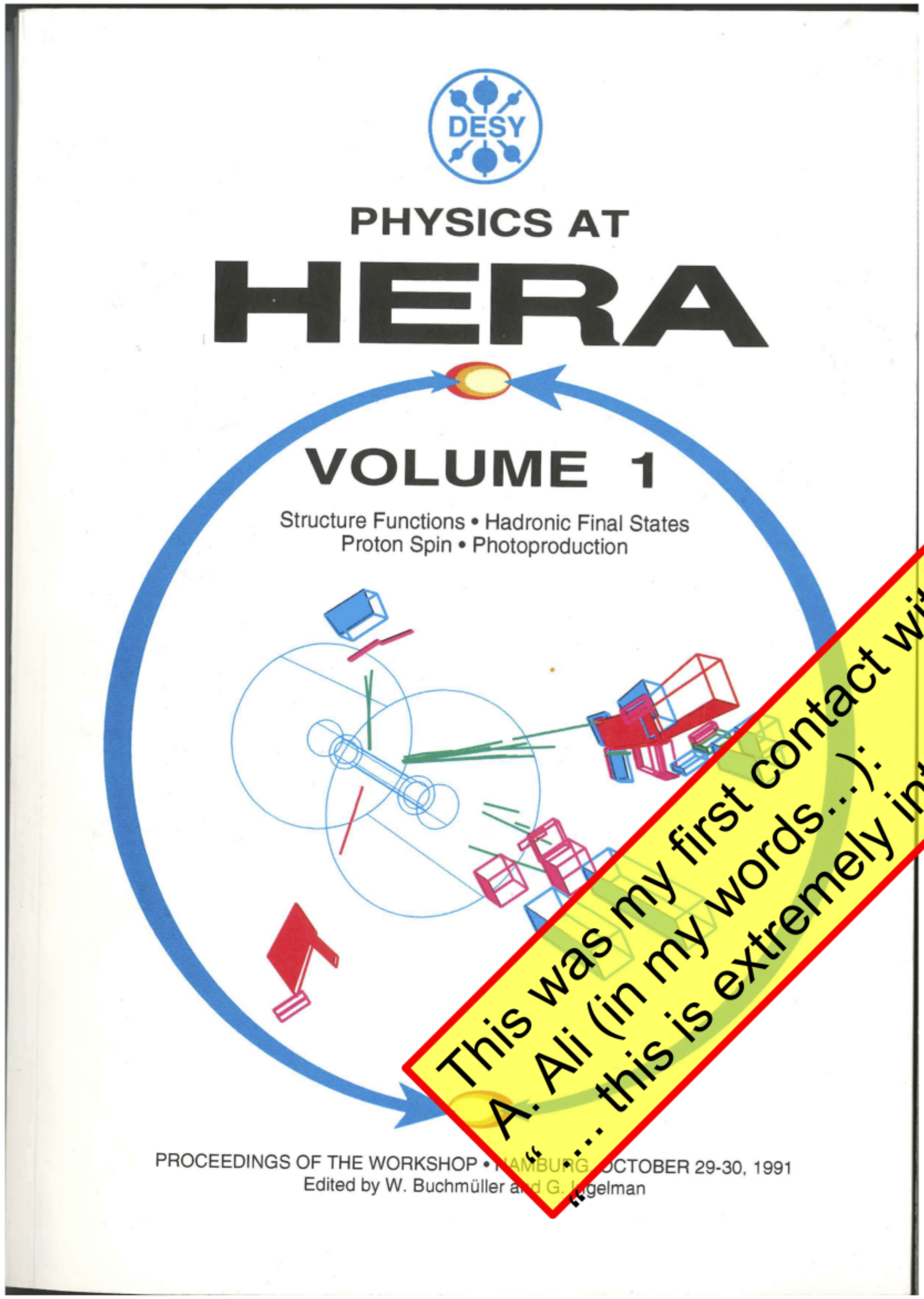
M. Ciafaloni and F. Hautmann

Dipartimento di Fisica, Universitá di Firenze
and INFN, Sezione di Firenze, Firenze, ITALY

Abstract

The high-energy factorization theorem is used to study heavy flavour production. This factorization is $k_\perp$-dependent and provides all leading logarithmic corrections to the QCD K-factor for the total cross section. We present complete theoretical results for the photo-production and lepto-production processes and discuss the resummation effects at asymptotic and HERA energies.

Physics at HERA workshop 1991: CCFM



DESY

PHYSICS AT
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Structure Functions • Hadronic Final States
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PROCEEDINGS OF THE WORKSHOP • HAMBURG • OCTOBER 29-30, 1991
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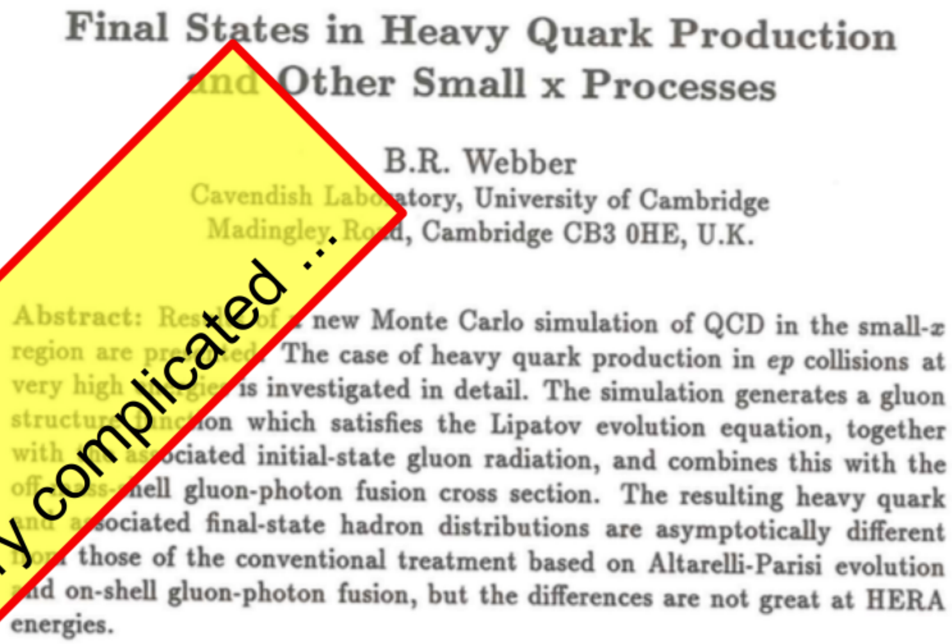
This was my first contact with CCFM
A. Ali (in my words…):
"… this is extremely interesting but very complicated …"



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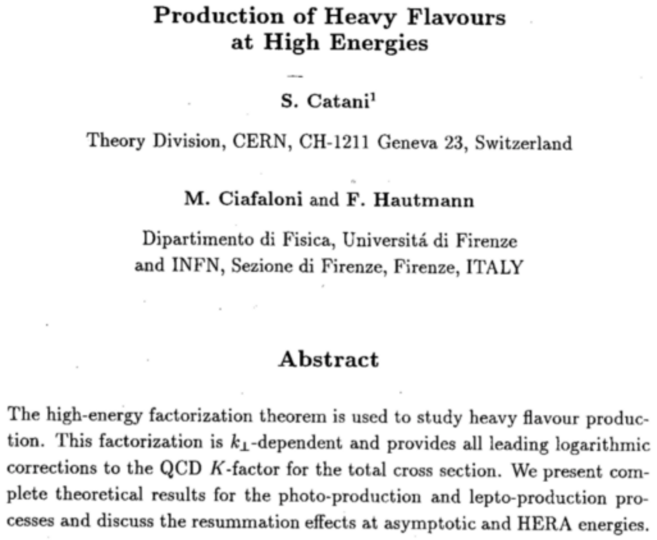

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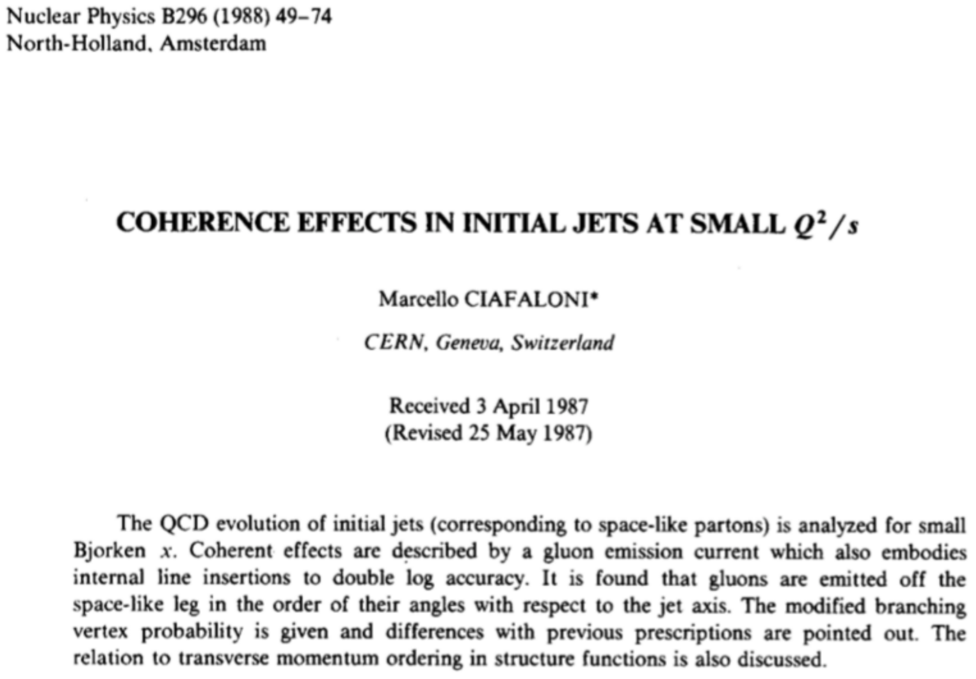
CCFM – the references



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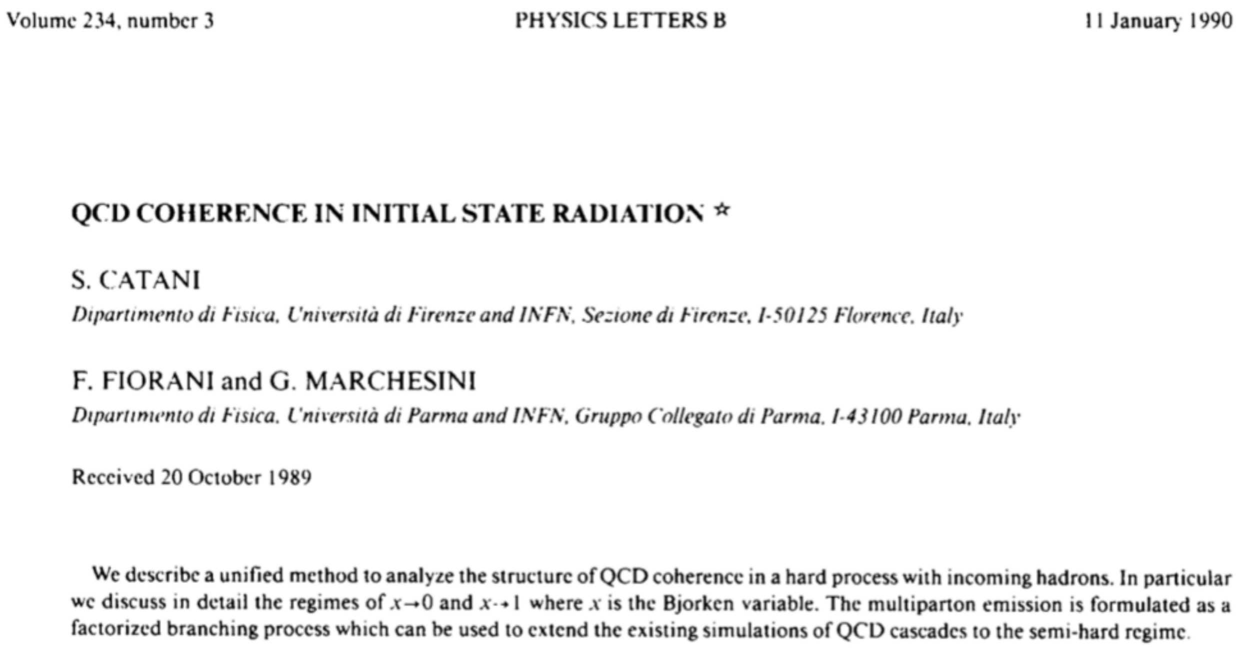
COHERENCE EFFECTS IN INITIAL JETS AT SMALL Q^2/s

Marcello CIAFALONI*
CERN, Geneva, Switzerland

Received 3 April 1987
(Revised 25 May 1987)

The QCD evolution of initial jets (corresponding to space-like partons) is analyzed for small Bjorken x. Coherent effects are described by a gluon emission current which also embodies internal line insertions to double log accuracy. It is found that gluons are emitted off the space-like leg in the order of their angles with respect to the jet axis. The modified branching vertex probability is given and differences with previous prescriptions are pointed out. The relation to transverse momentum ordering in structure functions is also discussed.



Volume 234, number 3 PHYSICS LETTERS B 11 January 1990

QCD COHERENCE IN INITIAL STATE RADIATION ☆

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Received 20 October 1989

We describe a unified method to analyze the structure of QCD coherence in a hard process with incoming hadrons. In particular we discuss in detail the regimes of $x \to 0$ and $x \to 1$ where x is the Bjorken variable. The multiparton emission is formulated as a factorized branching process which can be used to extend the existing simulations of QCD cascades to the semi-hard regime.

Nuclear Physics B336 (1990) 18–85
North-Holland

SMALL-x BEHAVIOUR OF INITIAL STATE RADIATION IN PERTURBATIVE QCD*

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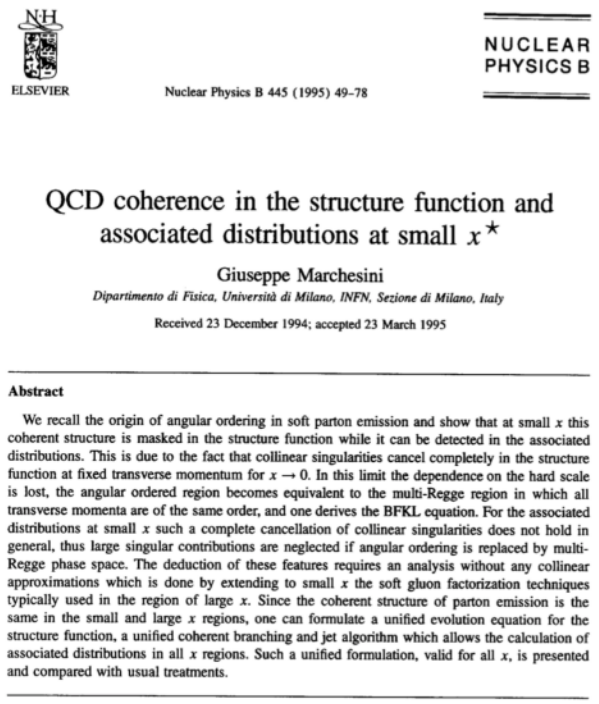
Received 17 July 1989

We analyze in perturbative QCD the asymptotic behaviour of deep inelastic processes in the semi-hard region $x \to 0$. The study is done by extending the soft gluon insertion techniques. We confirm and extend the analysis recently performed by Ciafaloni. The main results are the following: (i) Soft gluon emission from the incoming parton takes place in a region where the angles between incoming and outgoing partons are ordered. This is due to coherent effects similar to the ones in the $x \to 1$ region. (ii) Virtual corrections involving an internal line with energy fraction x give rise, for $x \to 0$, to a new form factor of non-Sudakov type. This regularizes collinear singularities when an emitted gluon is parallel to the incoming parton. (iii) At the complete inclusive level, the new form factor plays the same role as the virtual corrections in the Lipatov equation for the Regge regime. We show that, in the semi-hard regime, the gluon anomalous dimension coincides with the Lipatov ansatz. (iv) We identify the branching structure of initial-state radiation including the semi-hard regime. The branching is formulated as a probability process given in terms of Sudakov and non-Sudakov form factors. This process, in principle, can be used to extend the existing simulations of QCD cascades to the semi-hard regime.



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ELSEVIER
Nuclear Physics B 445 (1995) 49–78
NUCLEAR PHYSICS B

QCD coherence in the structure function and associated distributions at small x★

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Received 23 December 1994; accepted 23 March 1995

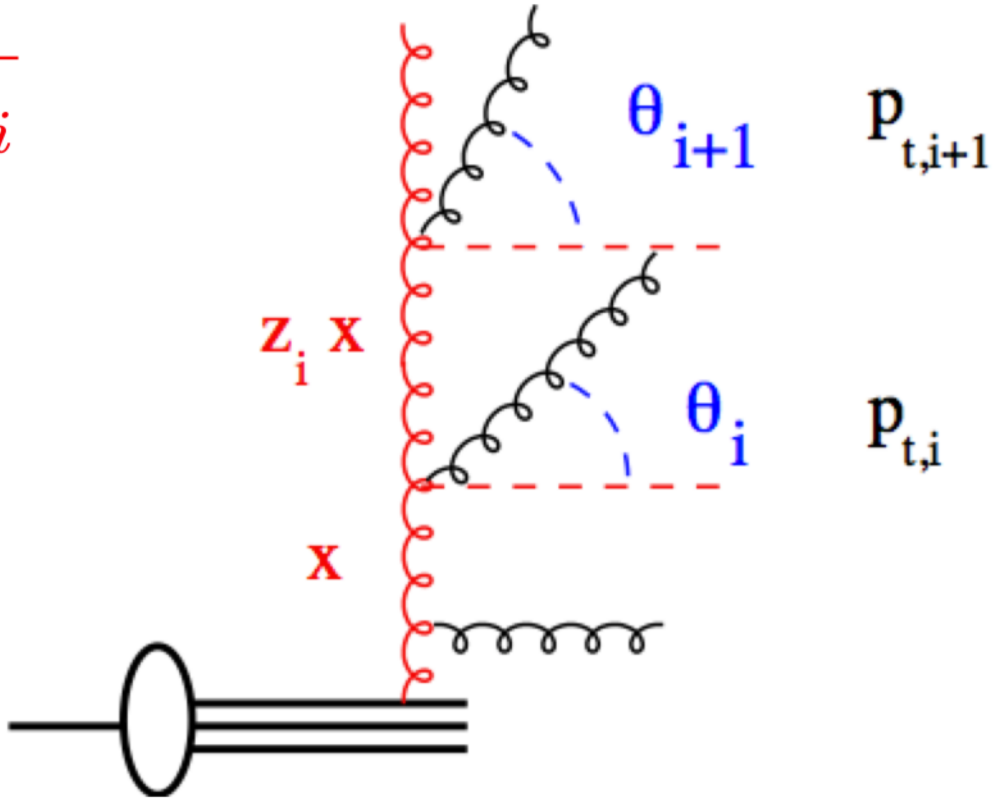
Abstract

We recall the origin of angular ordering in soft parton emission and show that at small x this coherent structure is masked in the structure function while it can be detected in the associated distributions. This is due to the fact that collinear singularities cancel completely in the structure function at fixed transverse momentum for $x \to 0$. In this limit the dependence on the hard scale is lost, the angular ordered region becomes equivalent to the multi-Regge region in which all transverse momenta are of the same order, and one derives the BFKL equation. For the associated distributions at small x such a complete cancellation of collinear singularities does not hold in general, thus large singular contributions are neglected if angular ordering is replaced by multi-Regge phase space. The deduction of these features requires an analysis without any collinear approximations which is done by extending to small x the soft gluon factorization techniques typically used in the region of large x. Since the coherent structure of parton emission is the same in the small and large x regions, one can formulate a unified evolution equation for the structure function, a unified coherent branching and jet algorithm which allows the calculation of associated distributions in all x regions. Such a unified formulation, valid for all x, is presented and compared with usual treatments.

CCFM evolution in my own words – part 1

- Color coherence requires angular ordering instead of p_t ordering ...

$$q_i > z_{i-1} q_{i-1} \qquad \text{with} \quad q_i = \frac{p_{ti}}{1 - z_i}$$



➔recover DGLAP with *q* ordering
at medium and large *x*

➔at small x, no restriction on *q*
➔ p_{ti} can perform a random walk

➔CataniCiafaloniFioraniMarchesini evolution forms a bridge between DGLAP and BFKL evolution

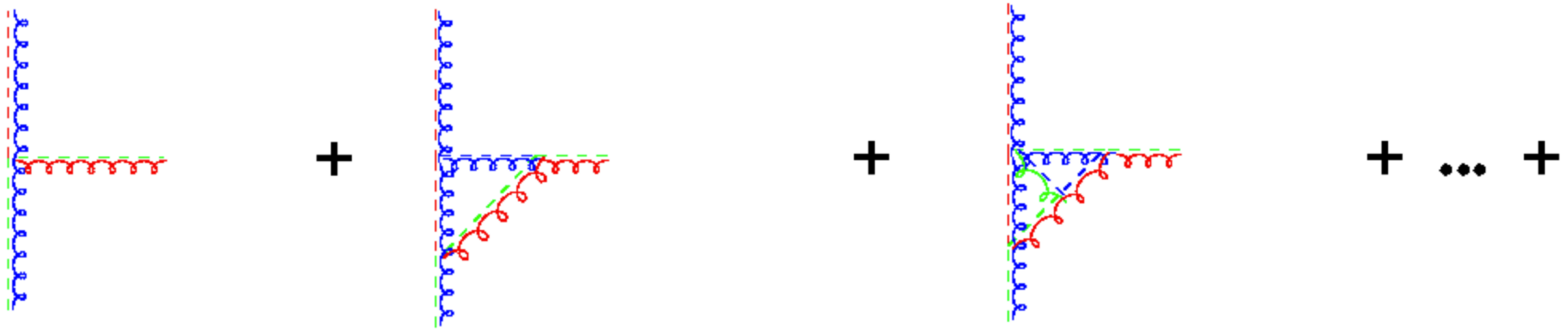
CataniCiafaloniFioraniMarchesini in my own words – part 2

$$g \to gg \quad \text{Splitting Fct} \quad \tilde{P}(z,q,k_\perp) = \frac{\bar{\alpha}_s(q(1-z))}{1-z} + \frac{\bar{\alpha}_s(k_t)}{z}\Delta_{\text{ns}}(z,q,k_\perp)$$

- Non - Sudakov form factor .... all loop resummation

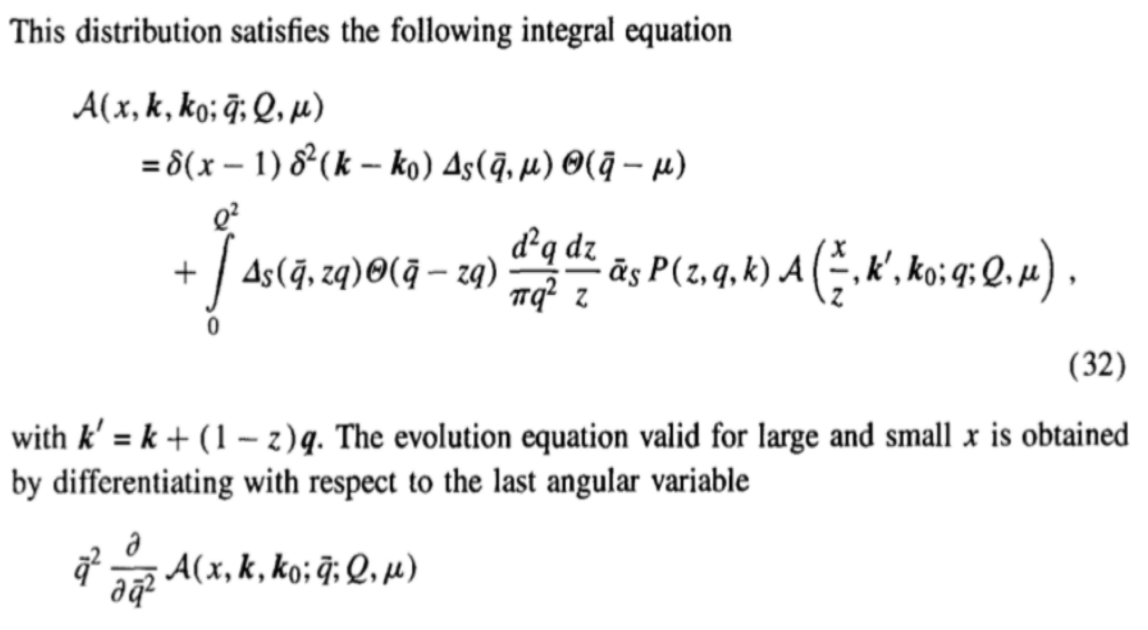
$$\Delta_{\text{ns}} = \exp\left[-\bar{\alpha}_s(k_t^2)\int_0^1 \frac{dz'}{z'}\int\frac{dq^2}{q^2}\Theta(k_t-q)\Theta(q-z'q_t)\right]$$

$$\Delta_{\text{ns}} = 1 + \left(-\bar{\alpha}_s(k_t^2)\int\frac{dz'}{z'}\int\frac{dq^2}{q^2}\right)^1 + \frac{1}{2!}\left(-\bar{\alpha}_s(k_t^2)\int\frac{dz'}{z'}\int\frac{dq^2}{q^2}\right)^2 \dots$$



$$\bar{\alpha}_s(k_t)\frac{1}{z}\left[+\quad \bar{\alpha}_s\log\left(\frac{z}{z_0}\right)\log\left(\frac{k_t^2}{z_0 z q^2}\right) + \frac{1}{2!}\left(\bar{\alpha}_s\log\left(\frac{z}{z_0}\right)\log\left(\frac{k_t^2}{z_0 z q^2}\right)\right)^2 + \cdots\right]$$

CCFM evolution equation

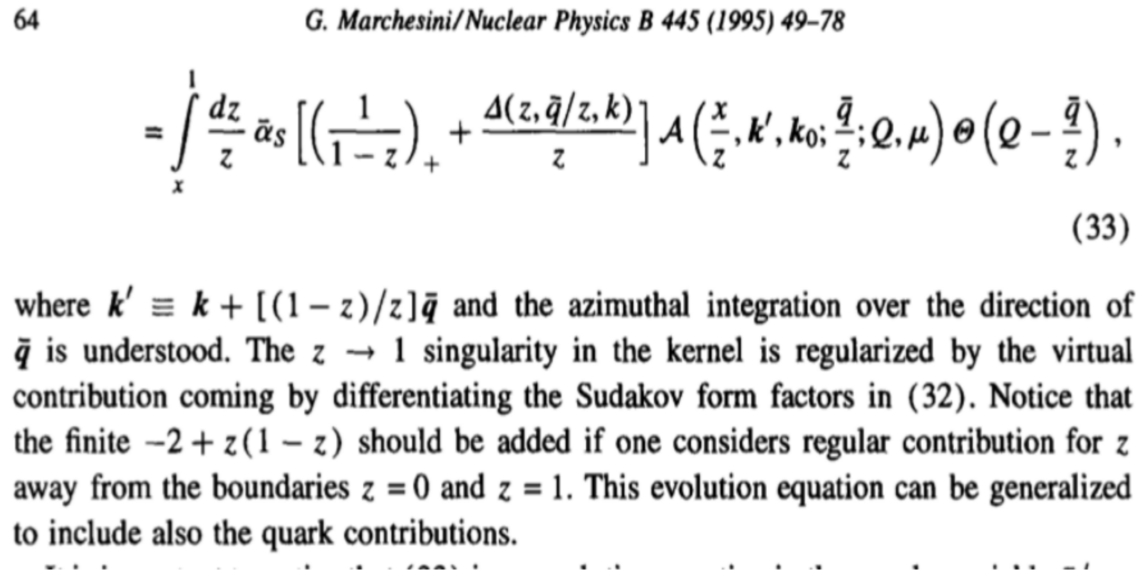


This distribution satisfies the following integral equation

$$\begin{aligned}\mathcal{A}(x,\boldsymbol{k},\boldsymbol{k}_0;\bar q;Q,\mu) &= \delta(x-1)\,\delta^2(\boldsymbol{k}-\boldsymbol{k}_0)\,\Delta_S(\bar q,\mu)\,\Theta(\bar q-\mu)\\ &\quad+\int_0^{Q^2}\Delta_S(\bar q,zq)\Theta(\bar q-zq)\,\frac{d^2q}{\pi q^2}\,\frac{dz}{z}\,\bar\alpha_S\,P(z,q,k)\,\mathcal{A}\left(\frac{x}{z},\boldsymbol{k}',\boldsymbol{k}_0;q;Q,\mu\right),\end{aligned} \tag{32}$$

with $\boldsymbol{k}'=\boldsymbol{k}+(1-z)\boldsymbol{q}$. The evolution equation valid for large and small x is obtained by differentiating with respect to the last angular variable

$$\bar q^2\,\frac{\partial}{\partial\bar q^2}\,\mathcal{A}(x,\boldsymbol{k},\boldsymbol{k}_0;\bar q;Q,\mu)$$



64 G. Marchesini/Nuclear Physics B 445 (1995) 49–78

$$=\int_x^1\frac{dz}{z}\,\bar\alpha_S\left[\left(\frac{1}{1-z}\right)_+ +\frac{\Delta(z,\bar q/z,k)}{z}\right]\mathcal{A}\left(\frac{x}{z},\boldsymbol{k}',\boldsymbol{k}_0;\frac{\bar q}{z};Q,\mu\right)\Theta\left(Q-\frac{\bar q}{z}\right), \tag{33}$$

where $\boldsymbol{k}'\equiv\boldsymbol{k}+[(1-z)/z]\bar{\boldsymbol{q}}$ and the azimuthal integration over the direction of $\bar{\boldsymbol{q}}$ is understood. The $z\to1$ singularity in the kernel is regularized by the virtual contribution coming by differentiating the Sudakov form factors in (32). Notice that the finite $-2+z(1-z)$ should be added if one considers regular contribution for z away from the boundaries $z=0$ and $z=1$. This evolution equation can be generalized to include also the quark contributions.

G. Marchesini.
QCD coherence in the structure function and
associated distributions at small x.
Nucl. Phys., B445:49–80, 1995.

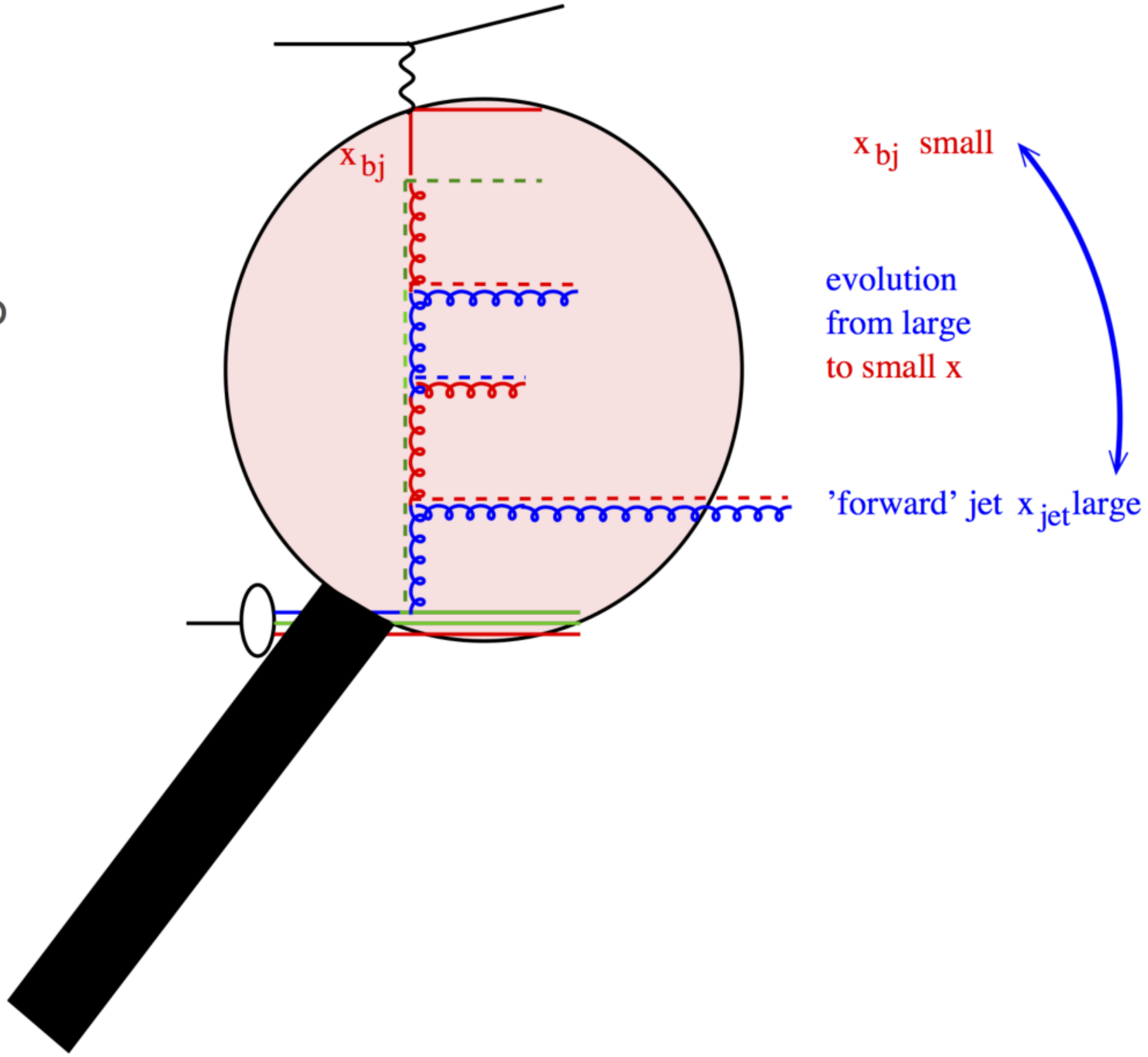
- This was one of my greatest aha !
  - ➔ CCFM written in form of an evolution equation with a structure similar to DGLAP !

CCFM evolution and experimental measurements

Comparison with inclusive measurements in ep ?

- only possible at very small x since only gluons are treated
- but F_2 and $F_2{}^c$ are also reasonably well described by standard DGLAP approaches if higher orders are included

- Need to go to comparison of final states
  - heavy flavor production
  - jet production
  - and THE BFKL signal
    - forward jet at HERA



Event simulation and CCFM evolution

- Monte Carlo program SMALLX
  (G. Marchesini & B Webber)
  - THE first small x simulation program
  - solves CCFM in a "forward evolution" producing partonic final states
  - uses $\gamma^{(*)} g^* \to Q\bar{Q}$
  - weighted events (large fluctuation)
  - further developed into updfevolv
    - for determination of gluon uPDF

- Monte Carlo generator CASCADE
  (H. Jung, G. Salam, F. Hautmann et al)
  - using uPDFs + off-shell MEs
  - backward evolution for parton shower (as std MC) using of uPDFs and Pino's evolution equation
  - hadronization

- Monte Carlo generator LDCMC
  (B. Andersson, G. Gustafson, L. Lonnblad et al)
  - re-grouping initial and final state showers in Linked Dipole Chain Model
  - evolution produces uPDFs
  - hadronization
  - further developed to DIPSY

Determination of gluon uPDF

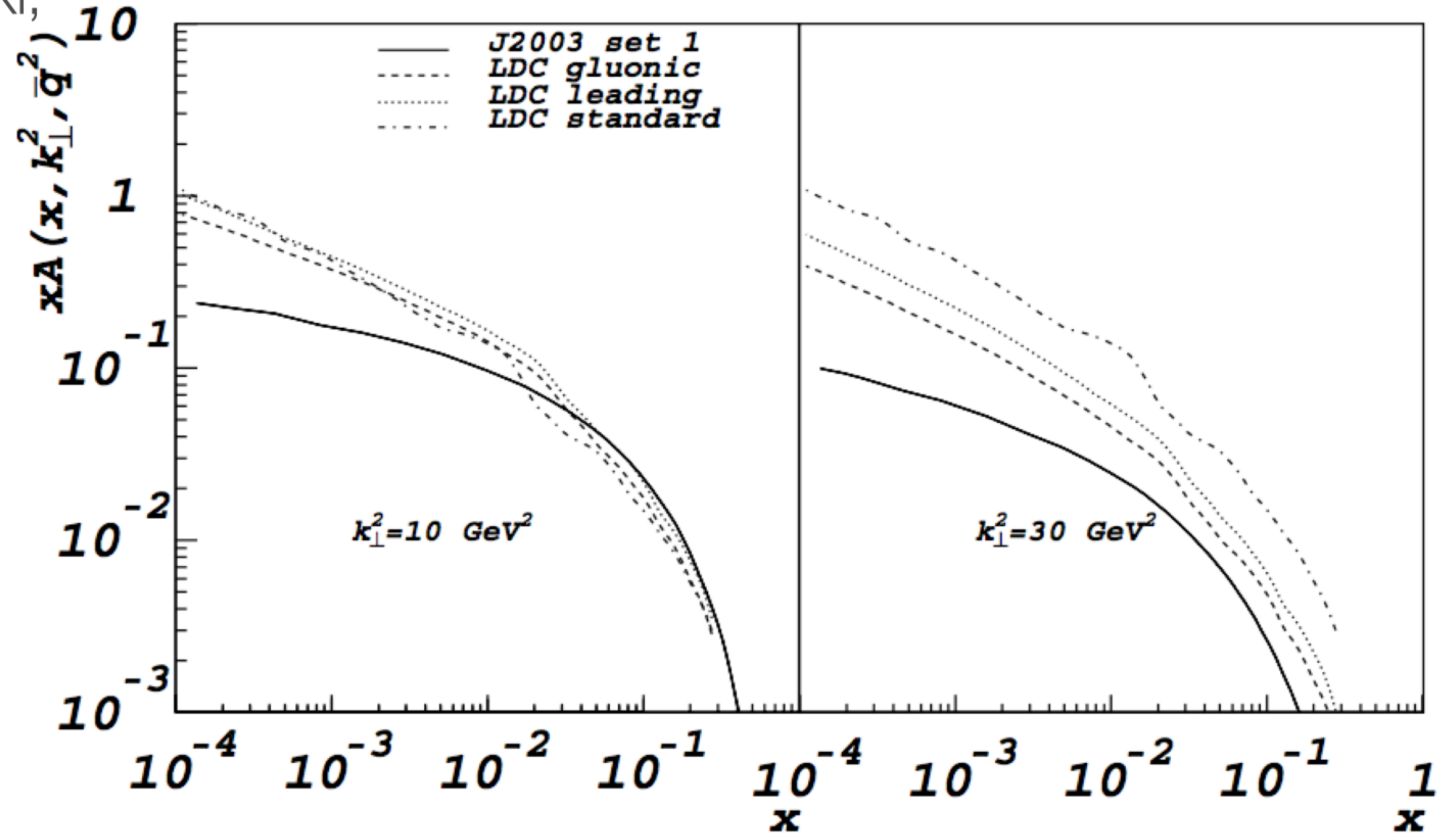
B. Andersson et al.
Small x phenomenology: Summary and status.
Eur. Phys. J., C25:77–101, 2002.

- Determination of gluon uPDF with HERA F_2 data
  - first attempts by A. Martin, J. Kwicienski, P. Sutton, A. Stasto, M. Ryskin starting in 1995

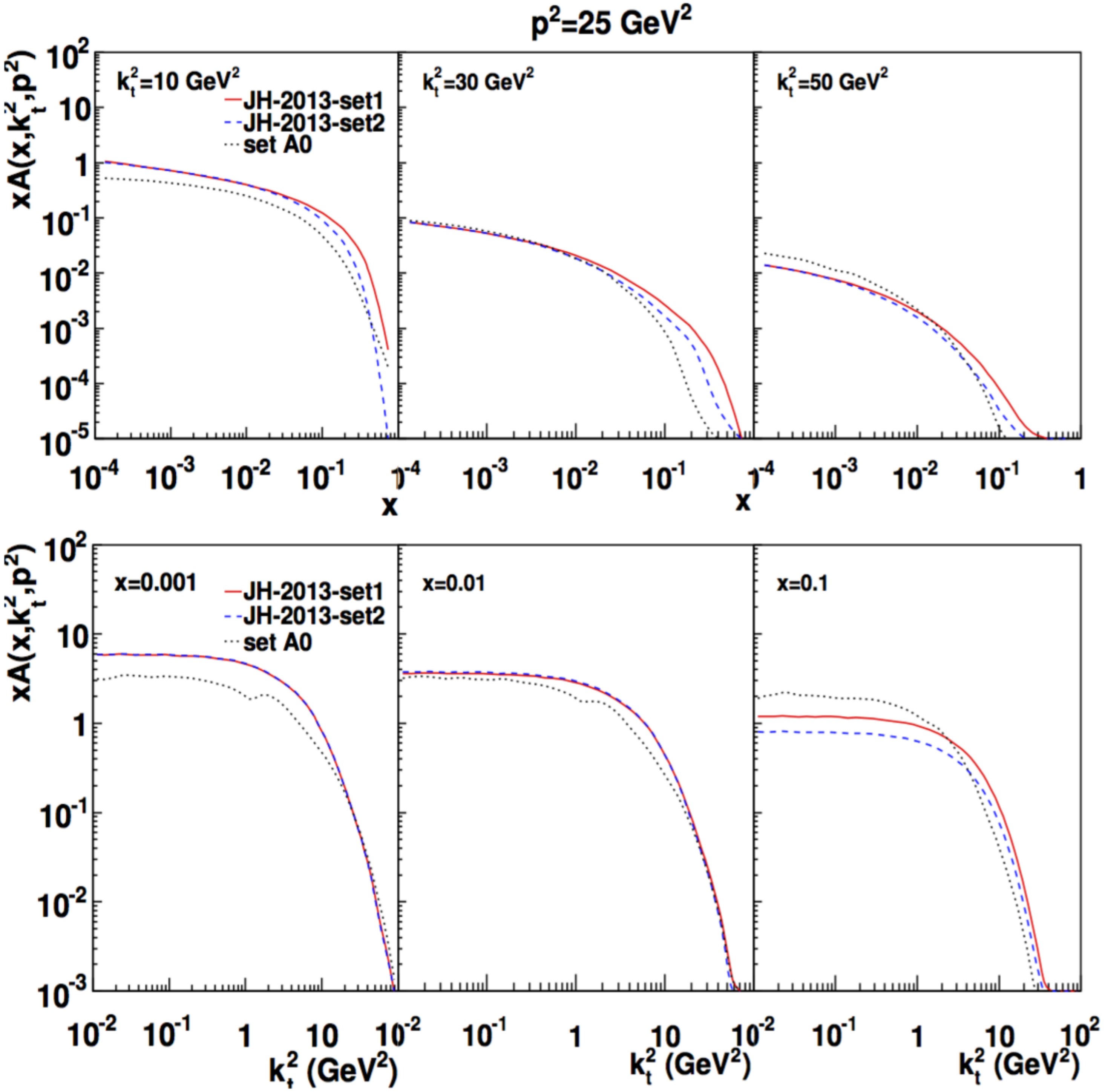
➔ comparison of CCFM gluon with LDC gluon

➔ different behavior at small and large k_t

➔ need for better constraints of k_t distribution



Determination of gluon uPDF from precision data



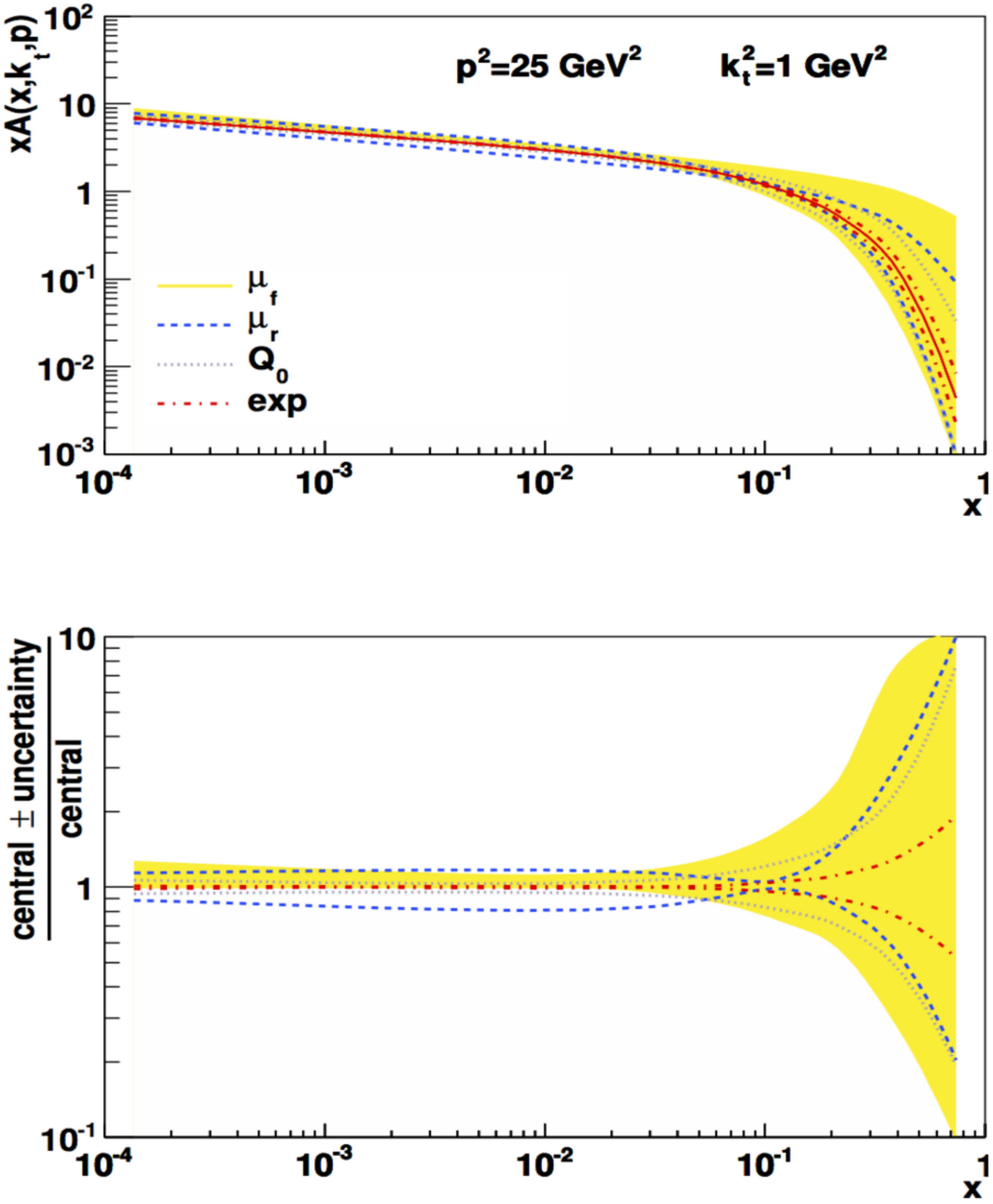
F. Hautmann and H. Jung.
Nuclear Physics B, 883:1, 2014.

- Fit function:

$$\begin{aligned}\mathcal{A}_0(x) &= N_g x^{-B_g}(1-x)^{C_g} \\ &\quad \times (1 - D_g x \\ &\quad + E_g\sqrt{x} + F_g x^2)\end{aligned}$$

  - only 3 params used in fit: no significant change for more params
- 2-loop α_S
- gluon splitting function with non-singular terms
- fits:
  - set 1: F_2 : Q^2 > 5 GeV, $x \le 0.005$
  - set 2: F_2 & $F^c{}_2$: Q^2 > 2.5 GeV
- new fit gives χ^2/ndf ~ 1.2
- details are different from previous uPDF set A0

Determination of gluon uPDF: uncertainties



F. Hautmann and H. Jung.
Nuclear Physics B, 883:1, 2014.

small k_t, small p^2

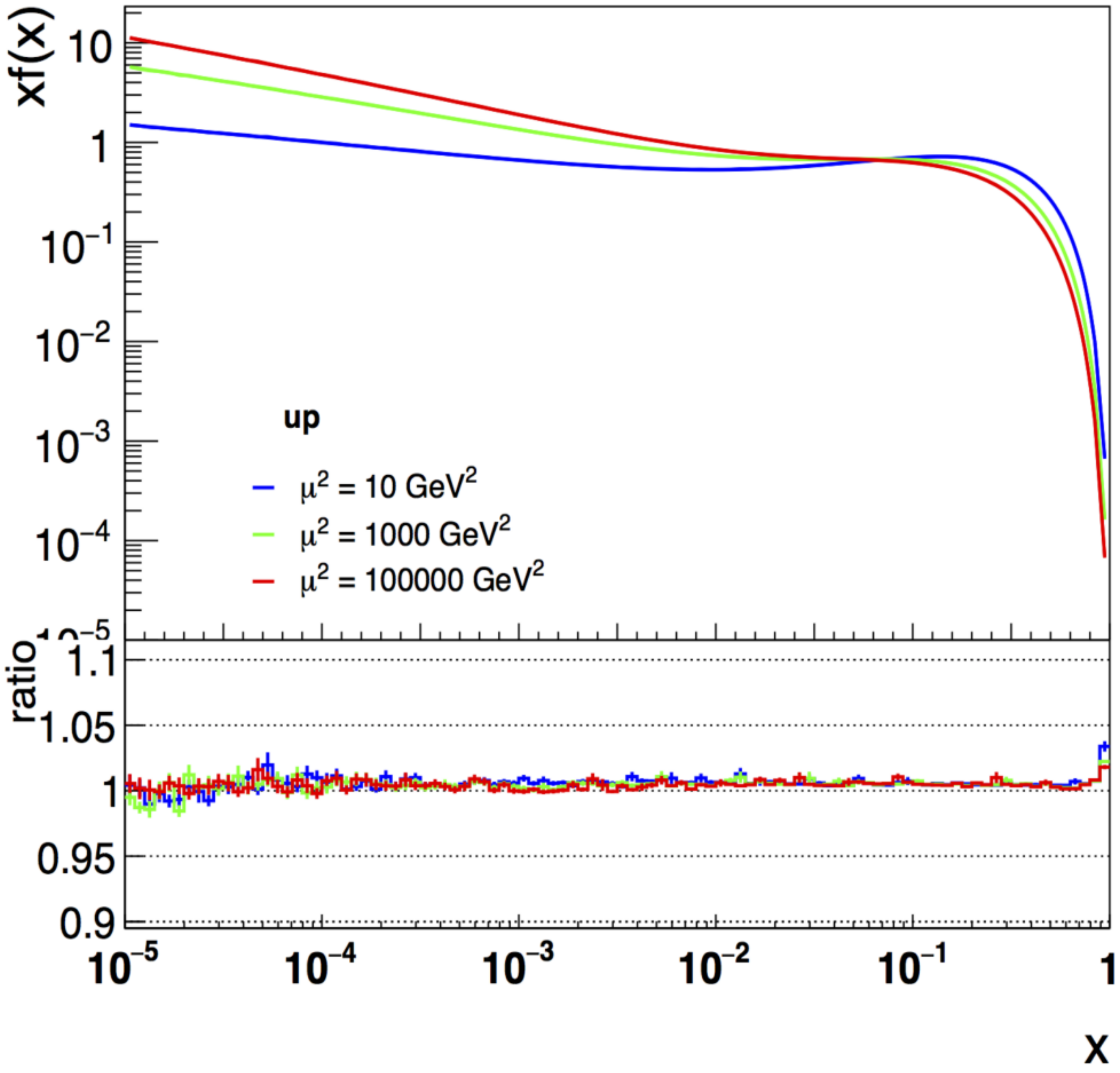
- experimental uncertainties result in 10-20 % for gluon uncertainty at medium and large x
- small uncertainties at small x
- factorization and renormalisation scale uncertainties
  - fit with shifted scales
  - large at large x, since no constrain from data: $x<0.005$, $Q^2 > 5$ GeV^2
  - ➔ dominant uncertainties

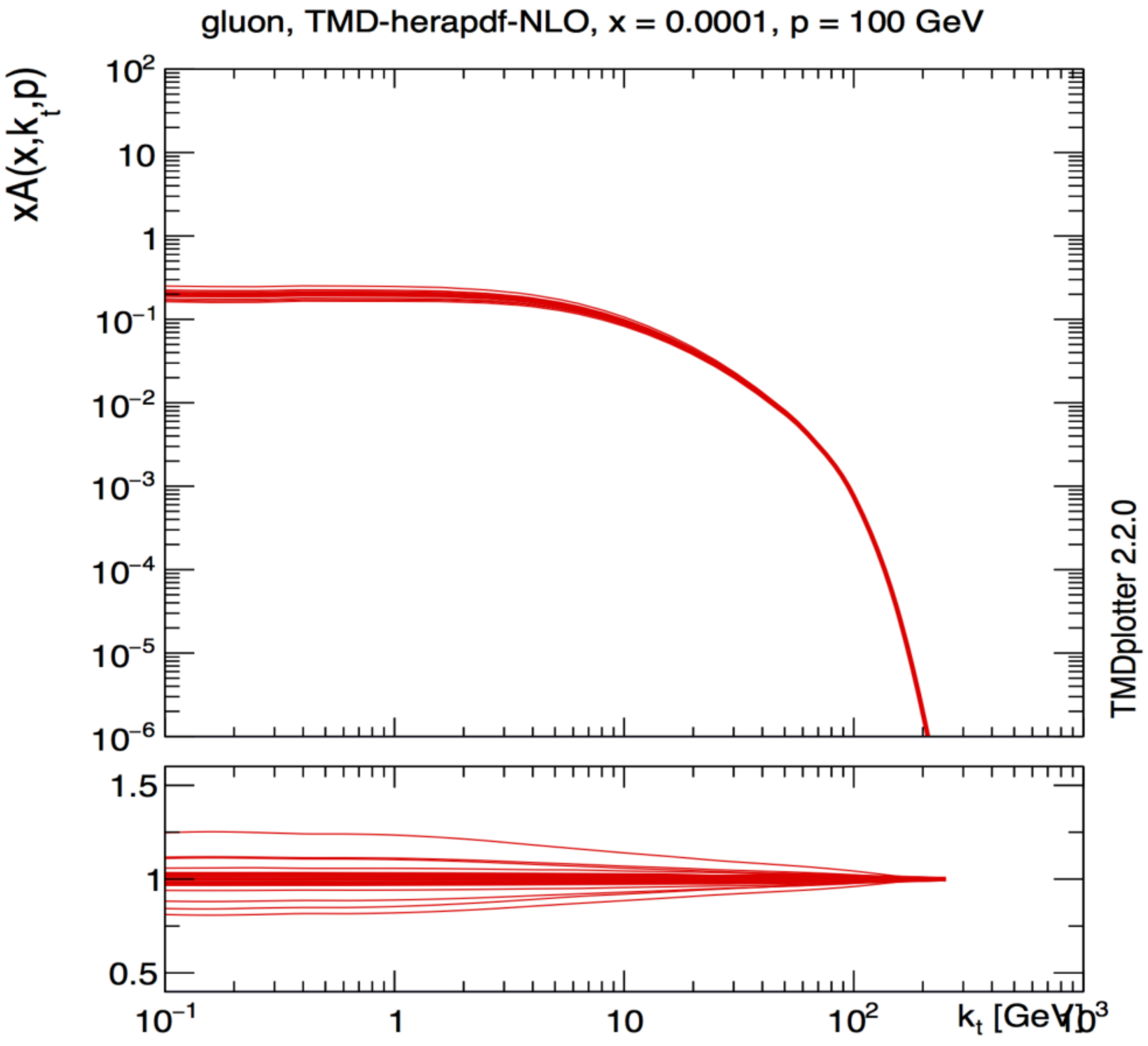
Extension to all flavor NLO uPDFs (TMDs)

- extend parton branching evolution from CCFM to all flavor (in 1-loop mode)

F. Hautmann et al arXiv:1704.01757
H. Jung, Talk at xFitter meeting 2017

- Perform fit to HERA precision data





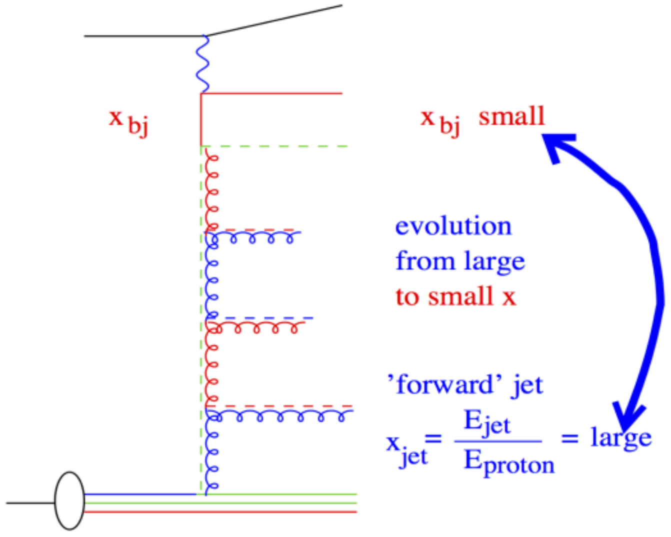
- comparison to semi-analytic calc (QCDnum) shows perfect agreement

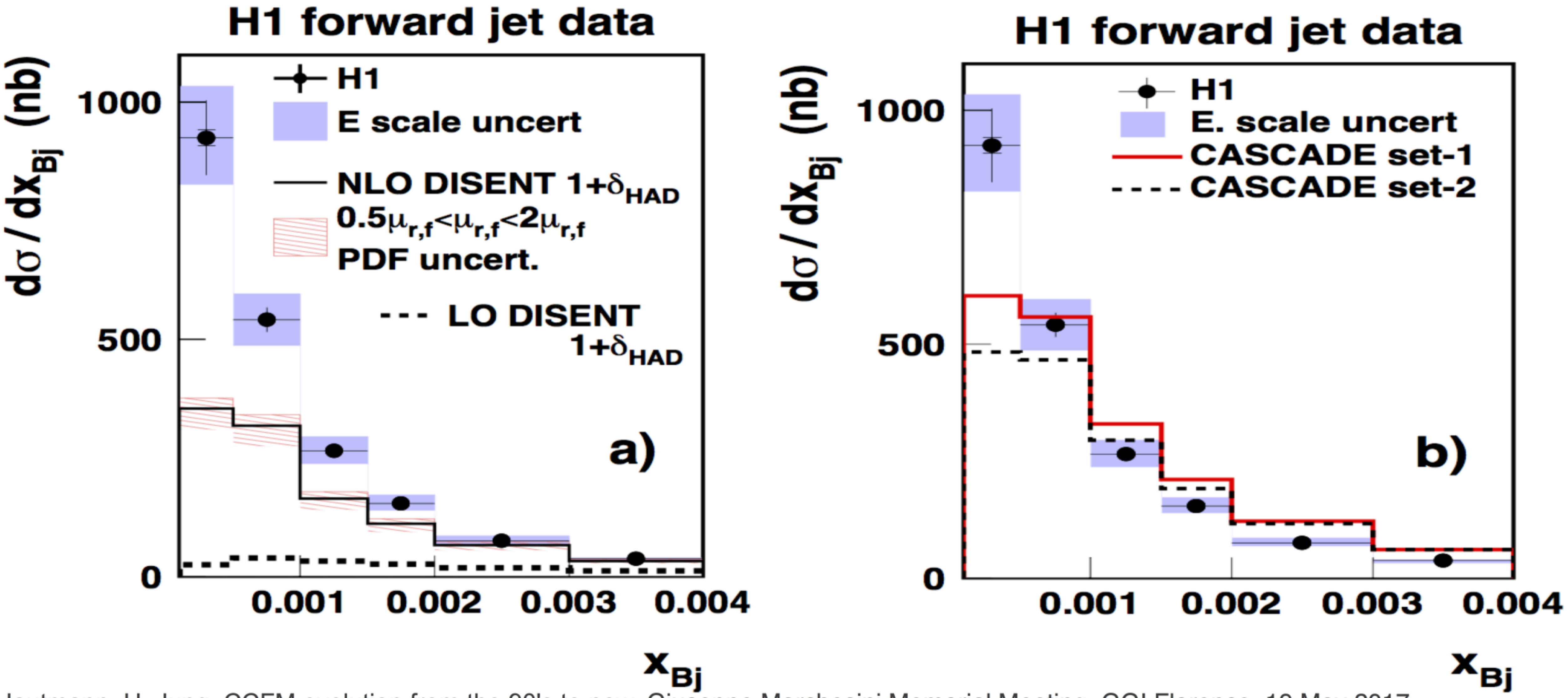
- uPDF (TMD) with full experimental uncertainties at NLO

Final state predictions: highlights from HERA

- Forward jets
  - ➔ thought to be THE signal for processes beyond DGLAP

H1 Coll
Eur.Phys.J. C46 (2006) 27-42

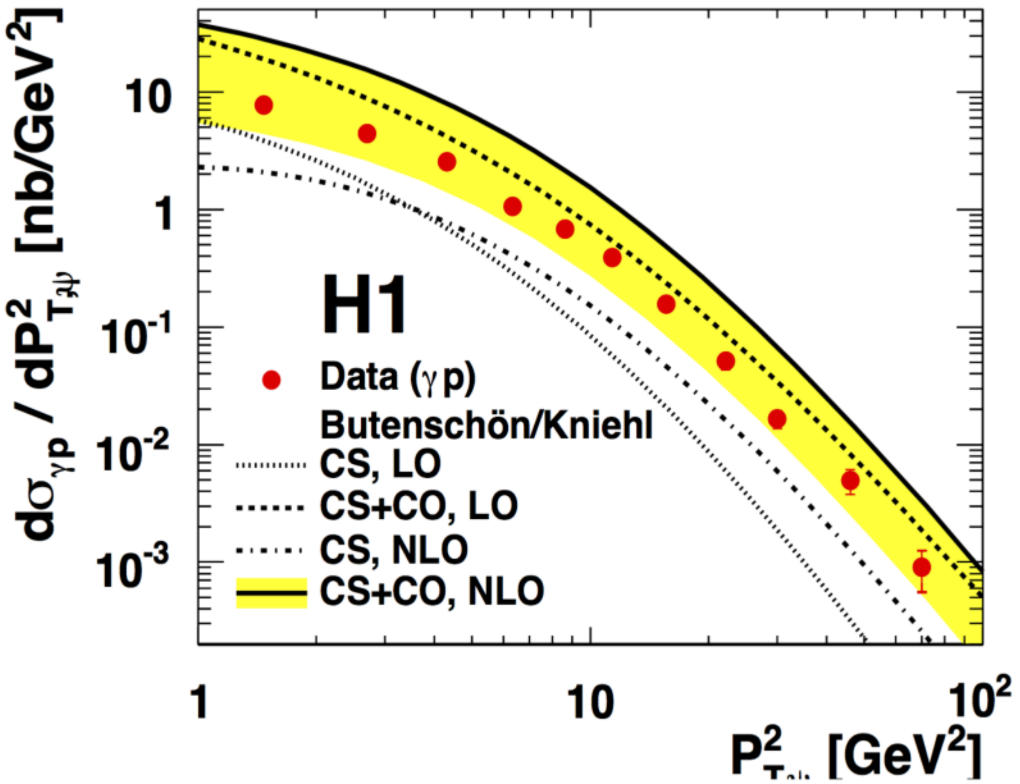


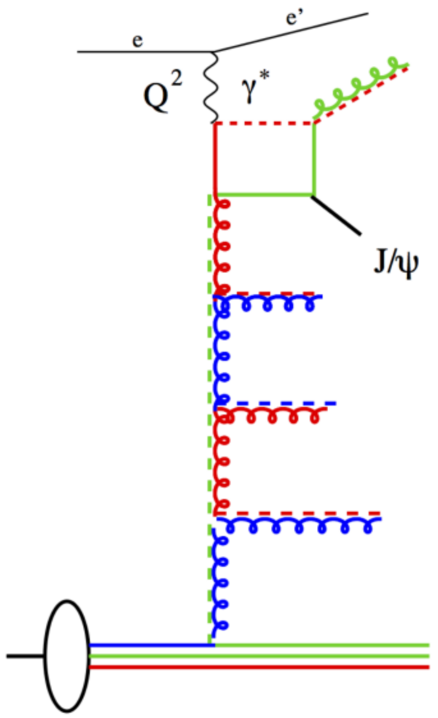


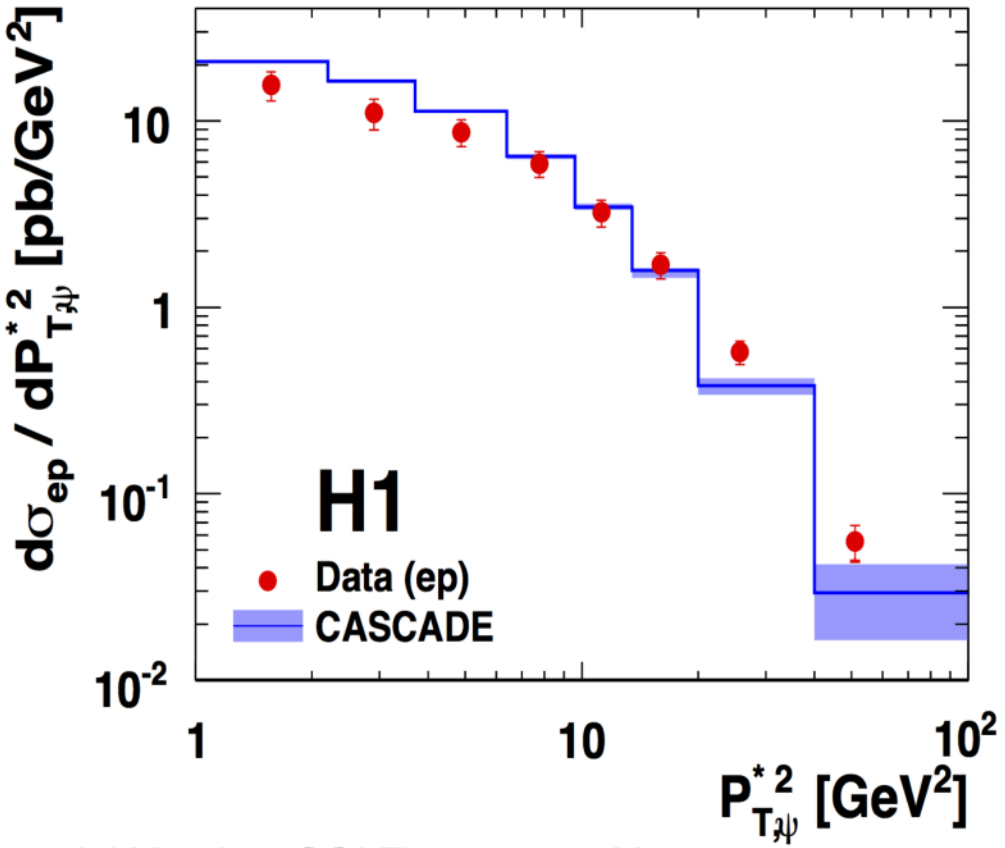
Final state predictions: highlights from HERA

- Inelastic J/ψ production
  - ➔ very sensitive to gluon density
  - ➔ calculation in color-singlet-model with off-shell gluon
  - ➔ very good description with CCFM gluon
- Note: usual collinear (NLO) calculation needs significant contribution from color-octed states

Hi Coll Eur.Phys.J.C68 (2010) 401



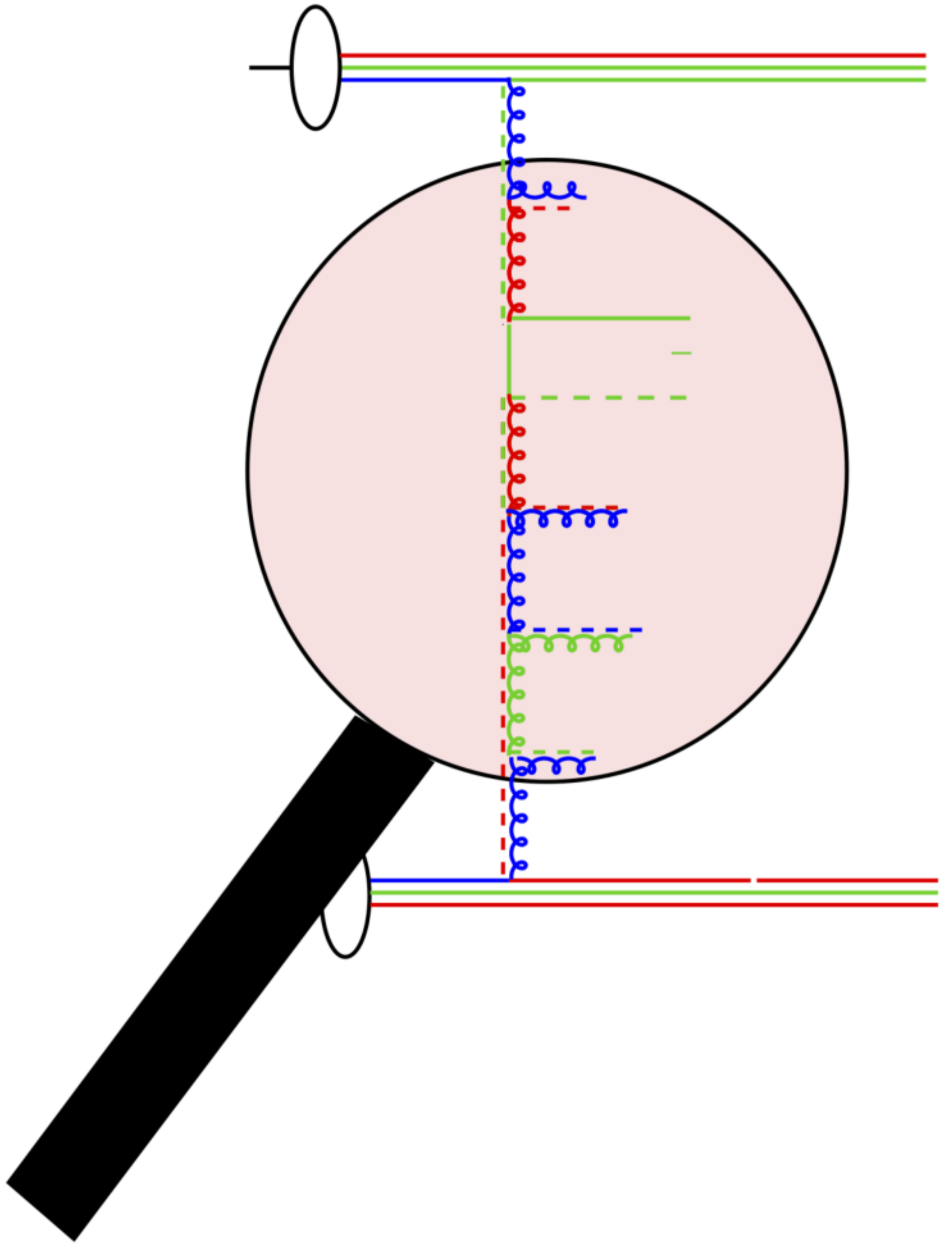




CCFM evolution and experimental measurements

Comparison with measurements in pp ?

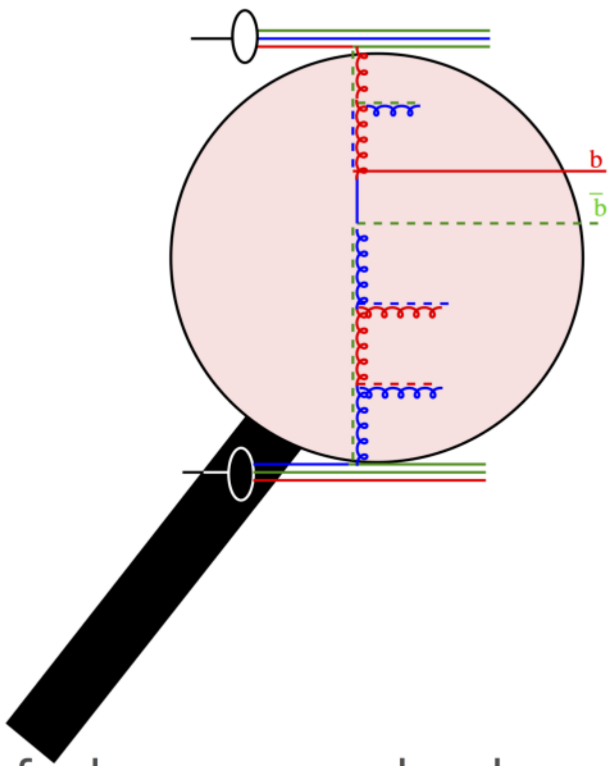
- only possible at small x since only gluons are treated
- identify processes which are gluon dominated
  - heavy flavor production
  - forward jet production
  - high pt multijets ?

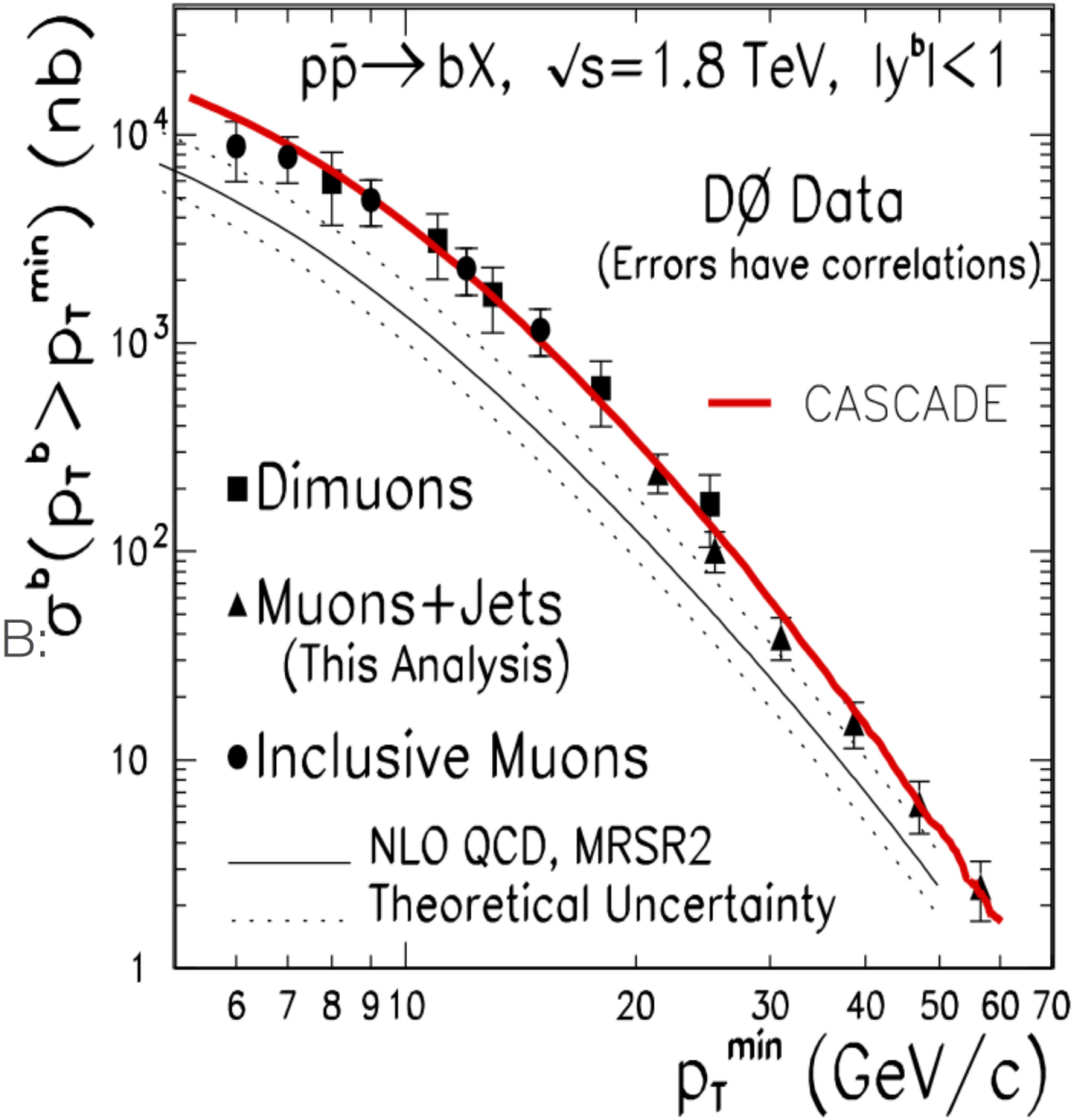


Final state predictions: highlights from TeVatron

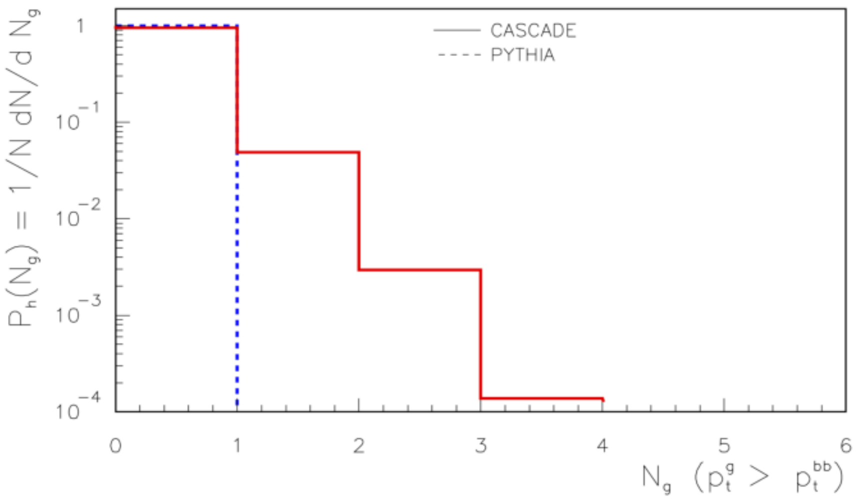
- Solution to the BBar problem at the TeVatron in 2001

H. Jung. Phys. Rev., D65:034015, 2002.
H. Jung, M. Kraemer, A. V. Lipatov, and N. P. Zotov. JHEP, 01:085, 2011.





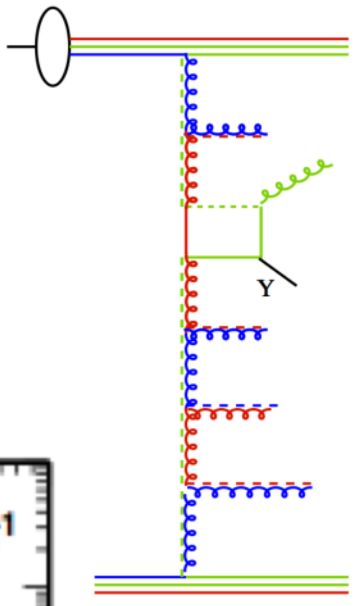
- k_t of gluons can be larger than p_t of B:



Final state predictions: highlights from LHC

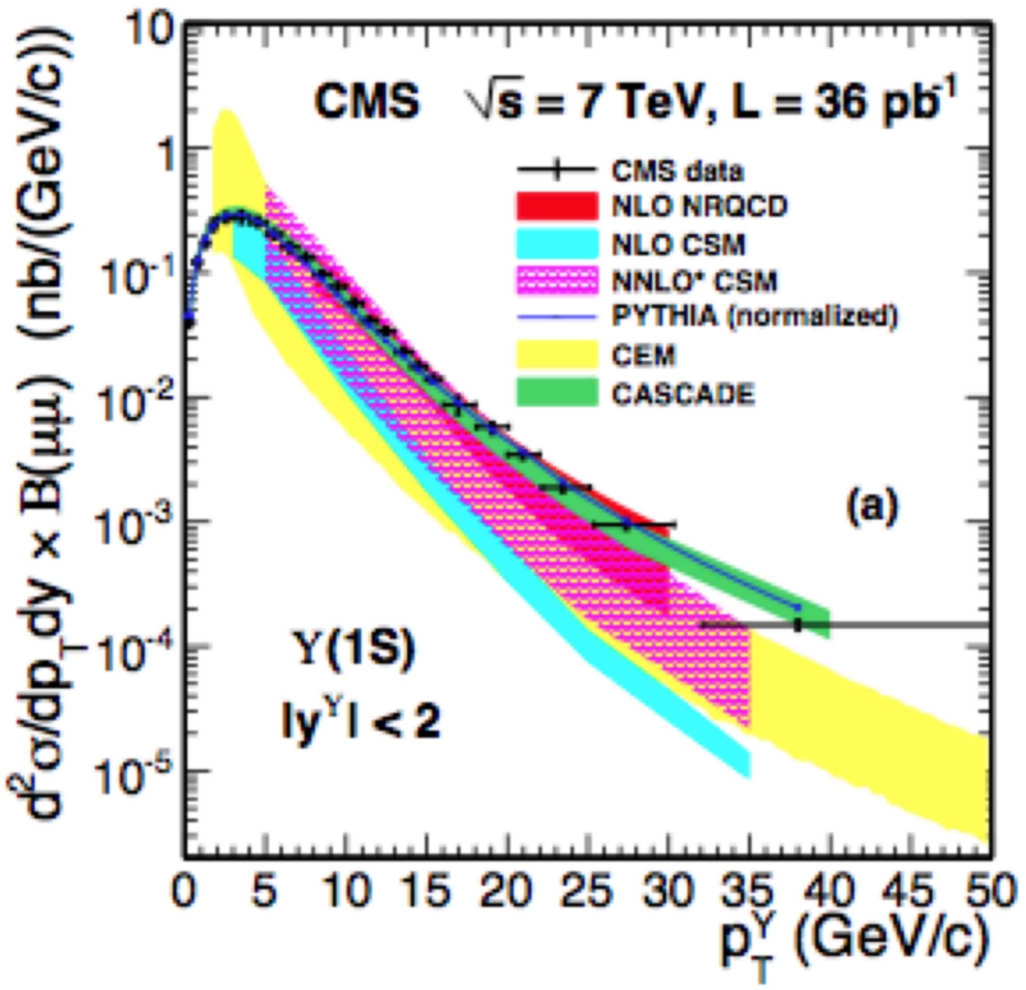
- J/psi and Upsilon production

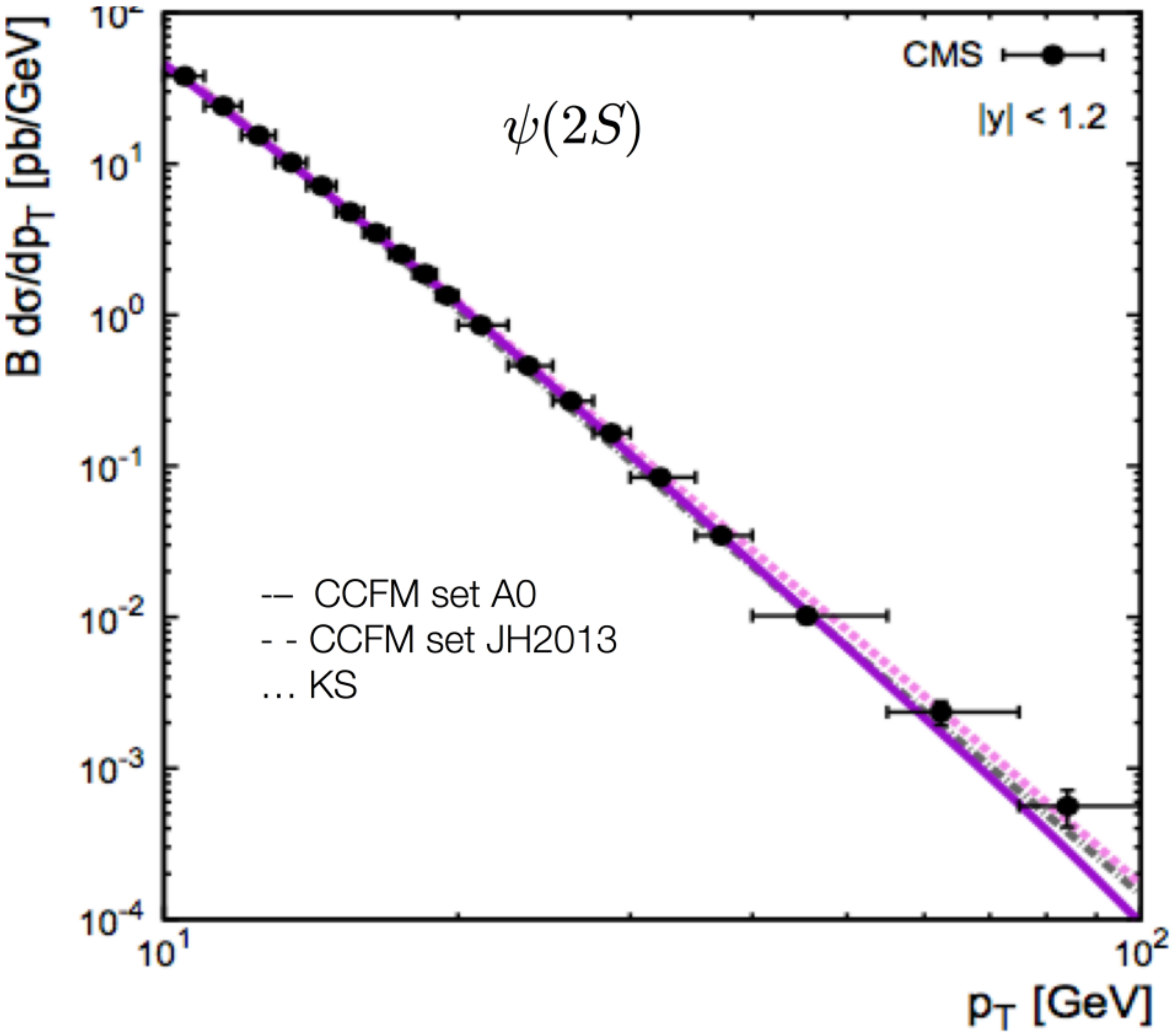
$$g^*g^* \to \Upsilon g,\ g^*g^* \to \chi_b \to \Upsilon + X$$



Prompt charmonia production and polarization at LHC in the NRQCD with kT-factorization. Part I: ψ(2S)meson
Baranov, Lipatov, Zotov Eur. Phys. J. C (2015) 75: 455.

CMS Phys.Lett. B727 (2013)101, 1303.5900
Measurement of the Y(1S), Y(2S), and Y(3S) cross sections in pp collisions at s√ = 7 TeV



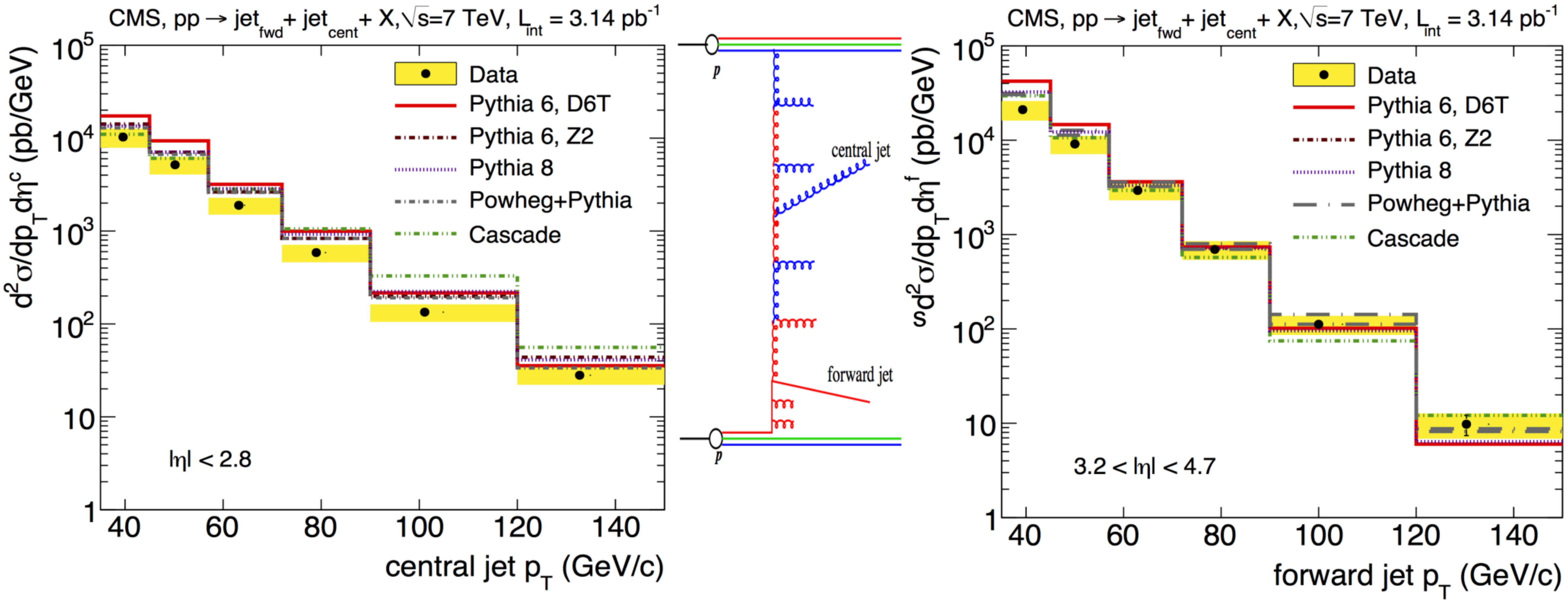


- Very good description on measurements using CCFM gluons !

Final state predictions: highlights from LHC

- central - forward jet production

CMS coll, JHEP 1206 (2012) 036

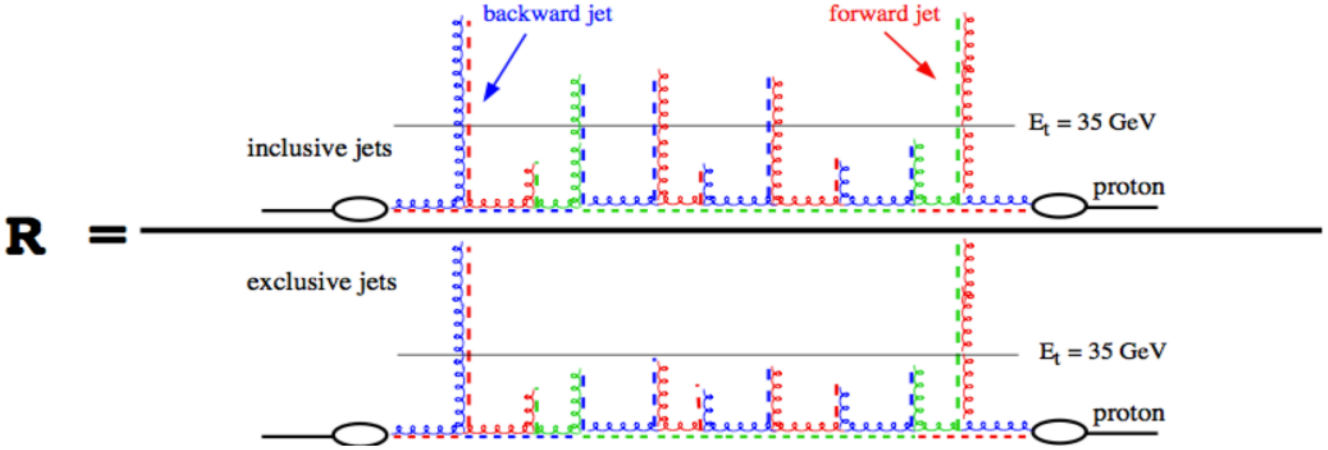


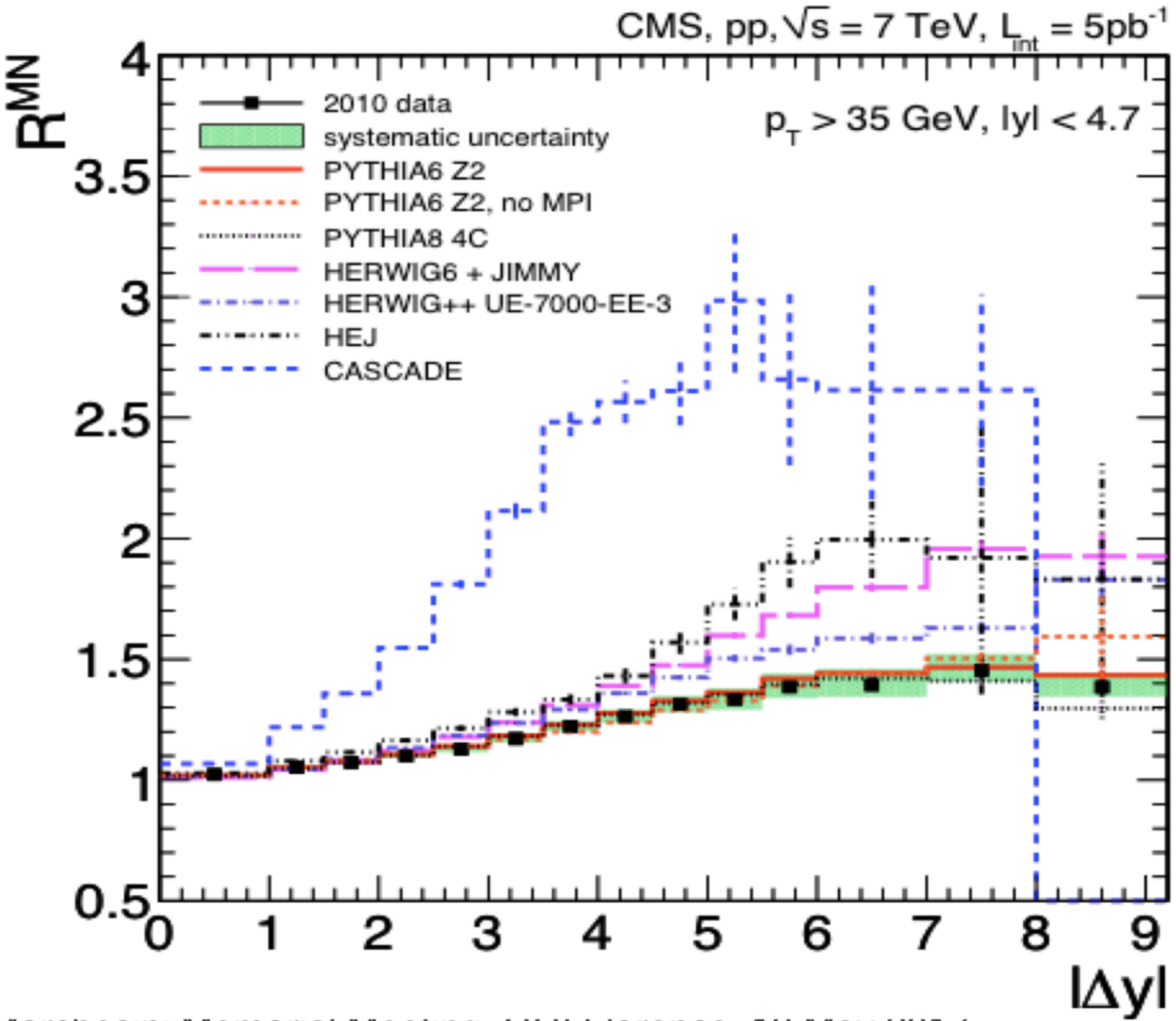
- forward – central jet production reasonably well described with CCFM gluon and valence quarks
  - ➔ but differences become visible → missing sea quark contributions
    - ➔ need for all flavor uPDFs in CCFM !

Final state predictions: highlights from LHC

CMS Coll, Eur.Phys.J. C72 (2012) 2216

- select (anti-kt) dijets with $p_{t\,min} = 35 \text{ GeV}, |y| < 4.7$
- as function of rapidity separation Δy between jets
- plot ratio of exclusive/inclusive xsection (many systematic cancel)
- for large Δy expect rising xsection due to increased phase space (BFKL effects)
- BUT …

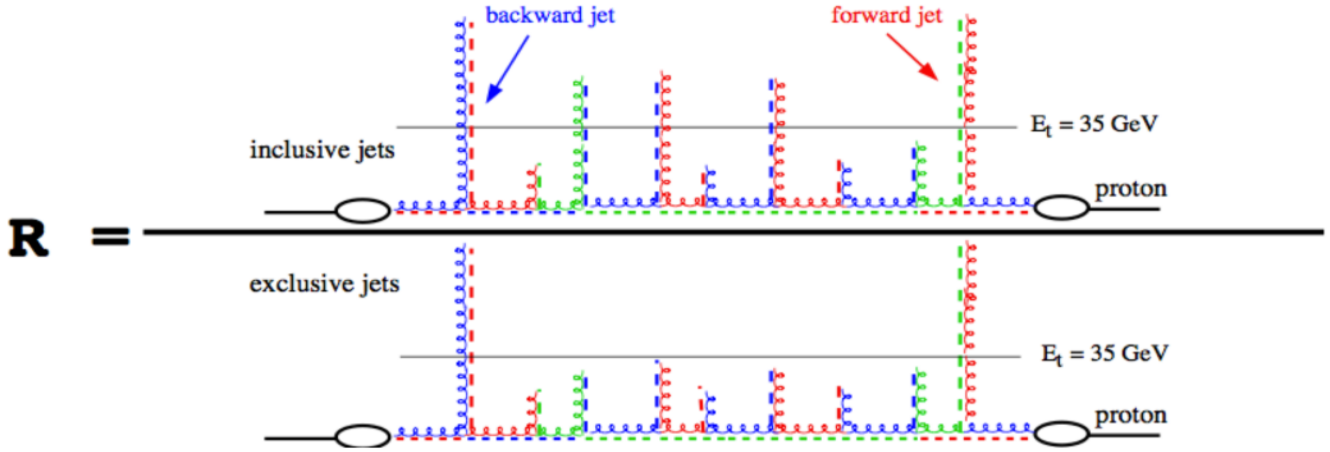


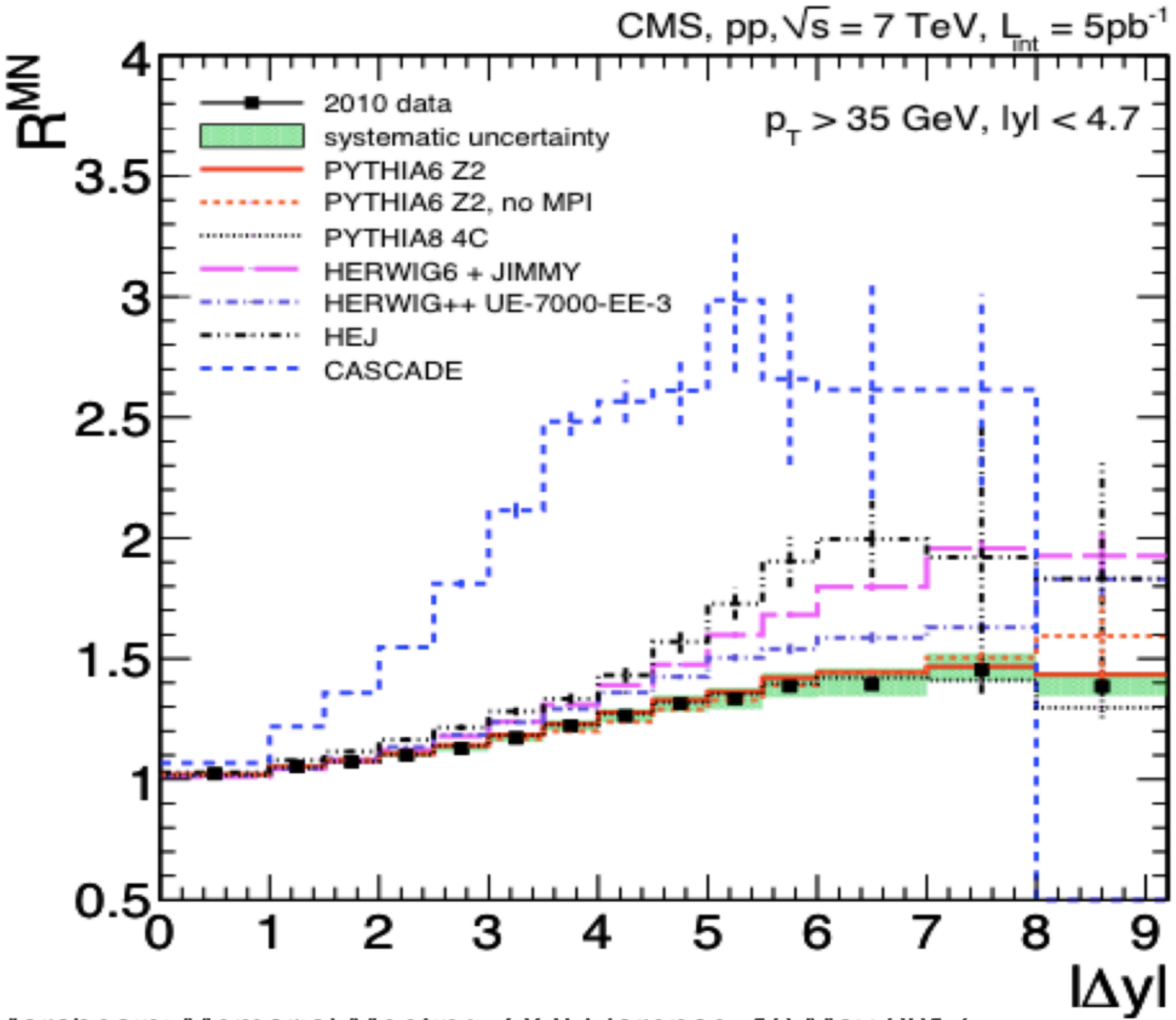


Final state predictions: highlights from LHC

- select (anti-kt) dijets with $p_{t\,min} = 35\ \mathrm{GeV},\ |y| < 4.7$
- as function of rapidity separation Δy between jets
- plot ratio of exclusive/inclusive xsection (many systematic cancel)
- for large Δy expect rising xsection due to increased phase space (BFKL effects)
- BUT
  - ➔ again this comes from gluon only uPDFs (same effect is seen in PYTHIA for gluon only)
  - ➔ need for full flavor CCFM uPDF

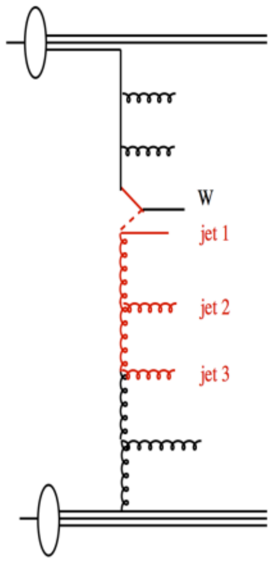
CMS Coll, Eur.Phys.J. C72 (2012) 2216

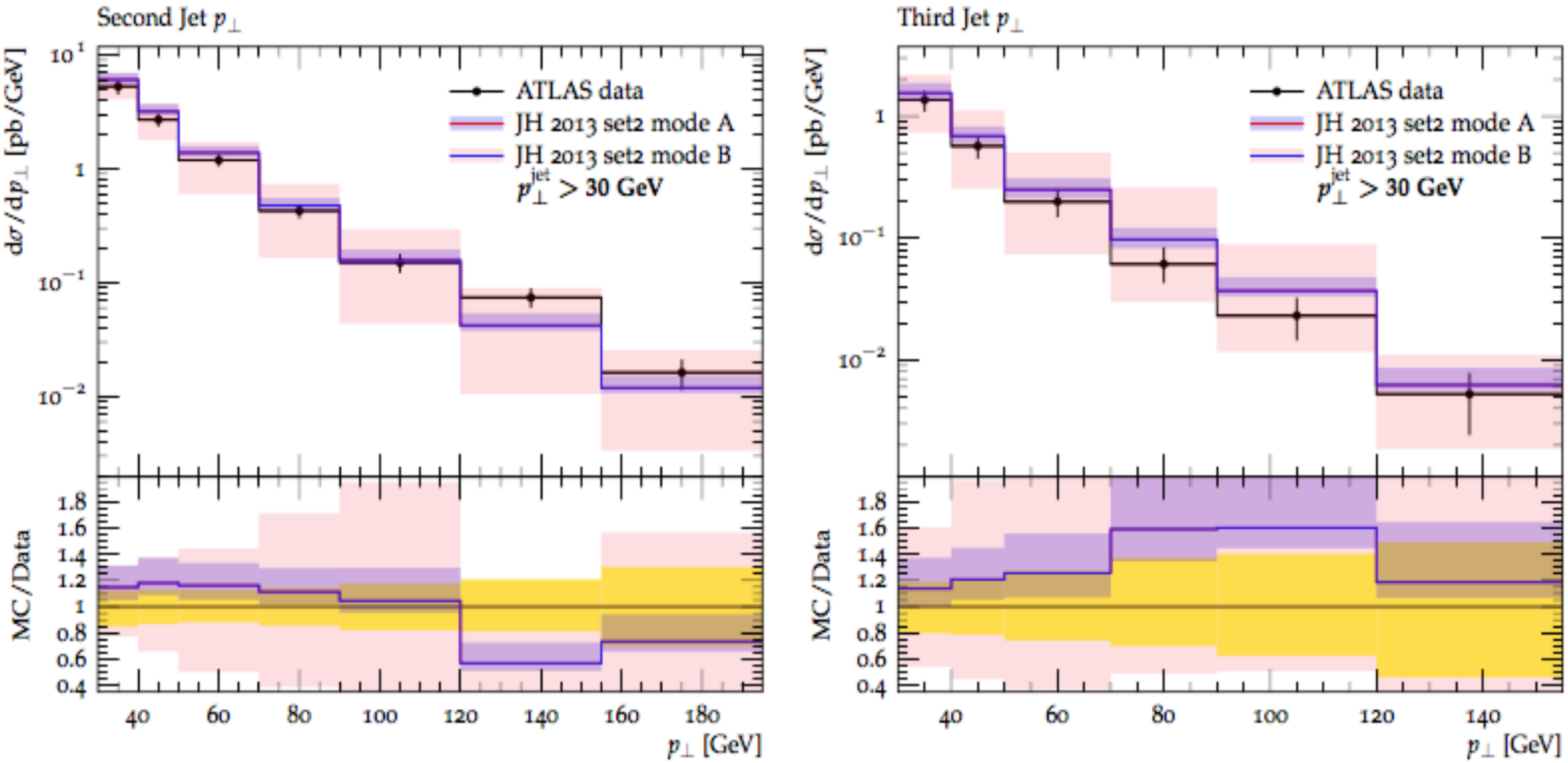




Final state predictions: highlights from LHC

S. Dooling, F. Hautmann, and H. Jung.
Phys. Lett., B736:293, 2014.





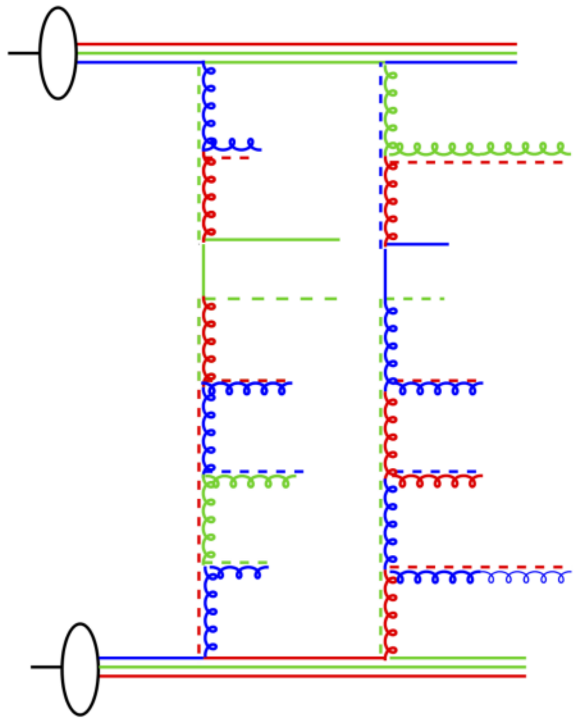
off-shell ME + CCFM k_t - shower can predict multi-jet production

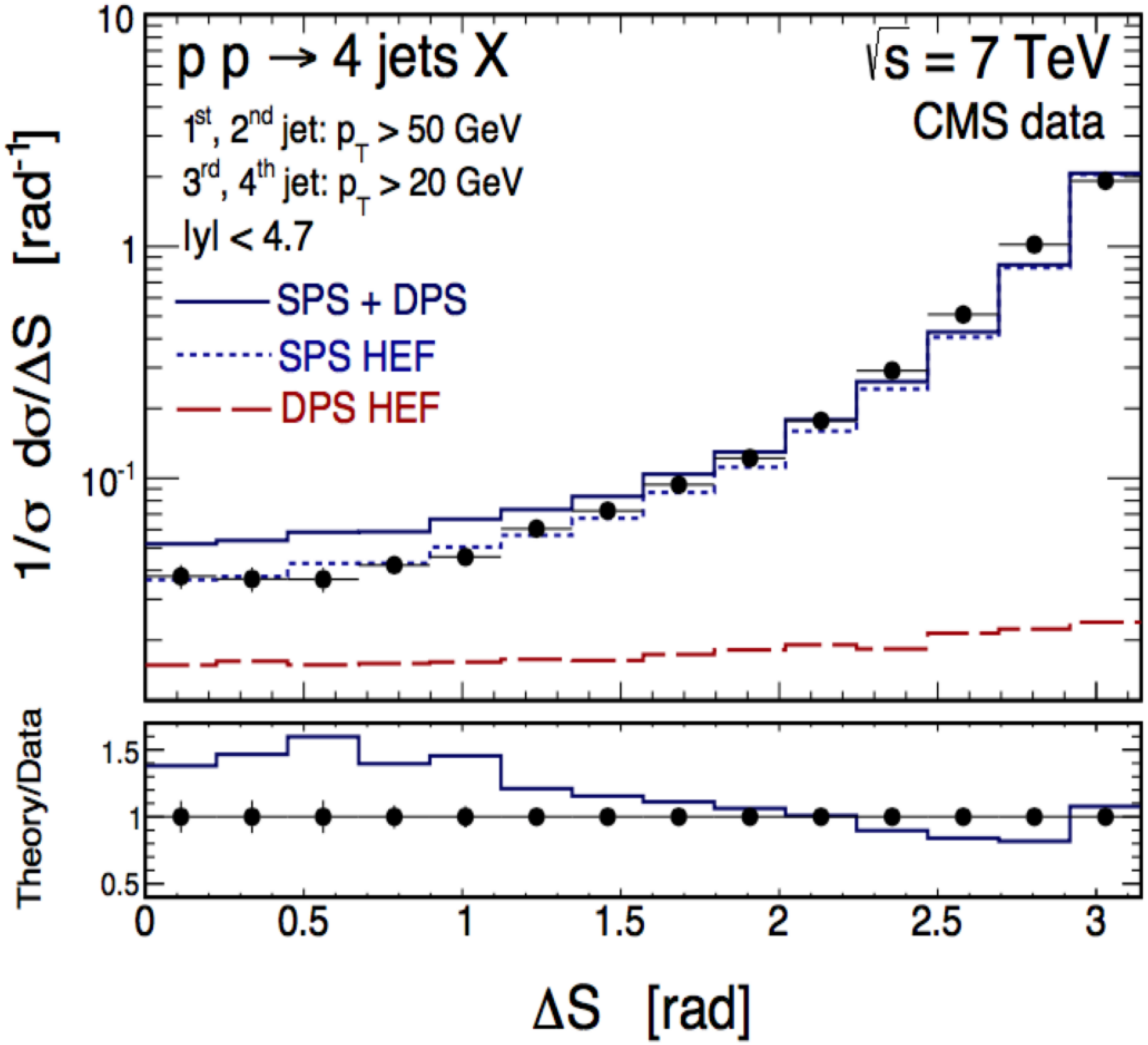
- jet p_t spectra reasonable
- multi jets come from CCFM k_t - shower
- uncertainty from pdf (including scale variation) increases with jet multiplicity
  - dominant uncertainty is scale variation

Final state predictions: highlights from LHC

- 4-jet production
  - automated off-shell ME calculation
  - 2 → 4 partons (A. van Hameren arXiv:1611.006

K. Kutak, R. Maciula, M. Serino, A. Szczurek, and A. van Hameren. Four-jet production in single- and double-parton scattering within high-energy factorization. JHEP, 04:175, 2016.





- Does k_t factorization contain DPS ?
  - what is the role of recoils ?

Instead of a summary

- CCFM description of parton branching is great !
  - consistent description of final states – emissions treated explicitly
  - angular ordering allows smooth transition from BFKL to DGLAP regions

- Bottlenecks until recently:
  - only gluons treated in fit to precision HERA F_2 data

- New developments:
  - full all flavor treatment of uPDF in 1-loop CCFM with fits to HERA precision data
  - extension to NLO and NNLO

- Next advantages
  - extension to all loop Splitting functions including k_t dependence

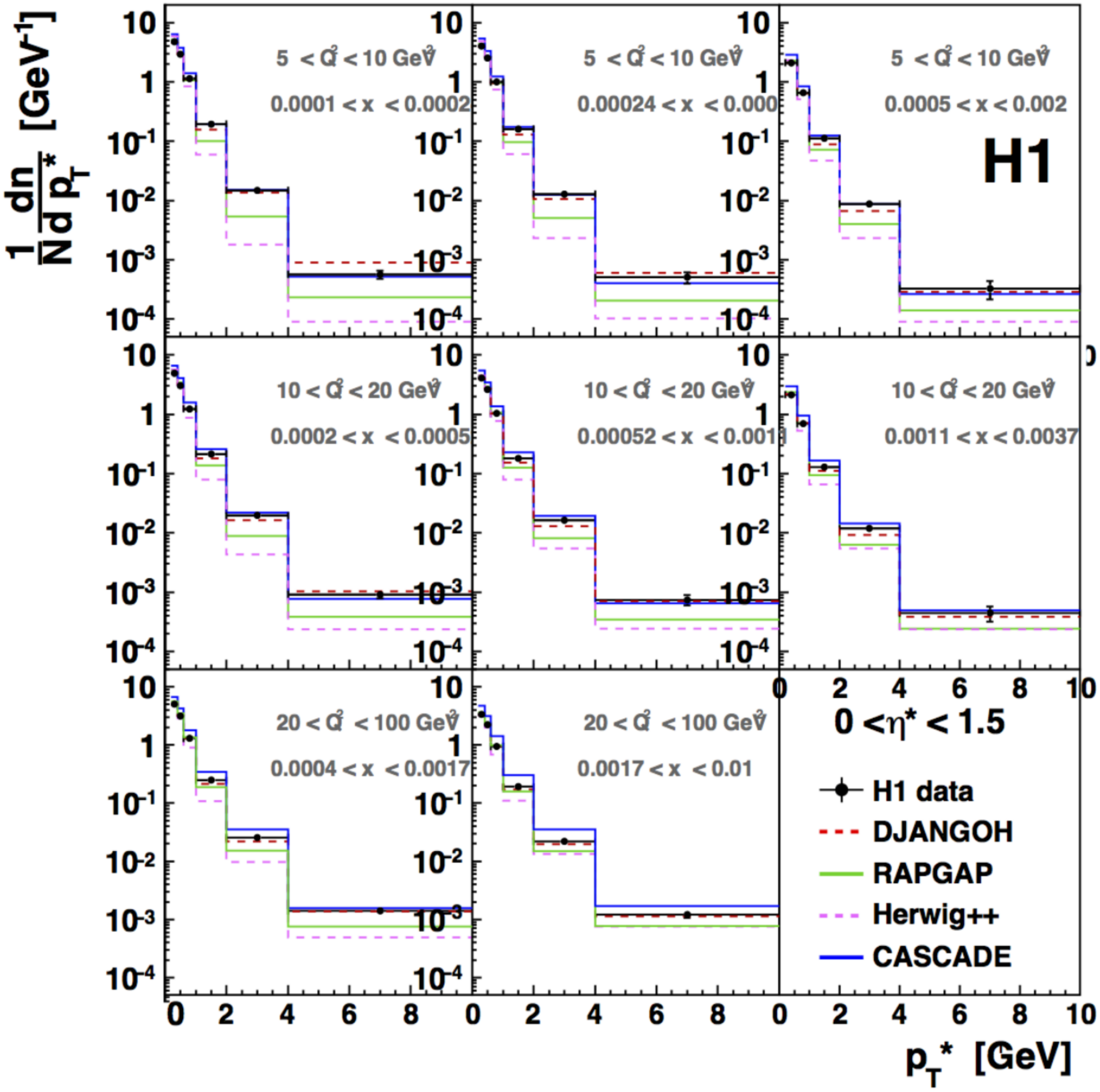
- Explore LHC phenomenology on high p_t processes

A personal remark

- I am very grateful that i had the privilege to know Pino
  - I am glad that we have invited him a few time to DESY for very interesting discussions
  - I am glad that he also came to the Lund Small x workshops to Lund

  - I learned a lot from him, scientifically and personally !

Charged particle spectra as fct of $p^*{}_t$ in DIS



H1 Coll. EPJC 73 (2013) 2406

- particle spectra as fct of $p^*{}_t$ give constraints on hardness of partons in parton shower
- collinear shower models (RAPGAP) generate too soft spectra compared to measurement
- small x improved (CCFM) shower (CASCADE) and CDM (DJANGOH) generate harder spectrum → closer to measurement at large $p^*{}_t$

Determination of gluon uPDF

- Determination of gluon uPDF with HERA F_2 data
  - ➔ first attempts by A. Martin, J. Kwicienski, P. Sutton, A. Stasto, M. Ryskin starting in 1995
  - ➔ differences are in details:
    - splitting functions
    - full evolution or last emission
    - off-shell ME
    - inclusion of DGLAP terms

B. Andersson et al.
Small x phenomenology: Summary and status.
Eur. Phys. J., C25:77–101, 2002.