Addressing hard questions to soft gluons

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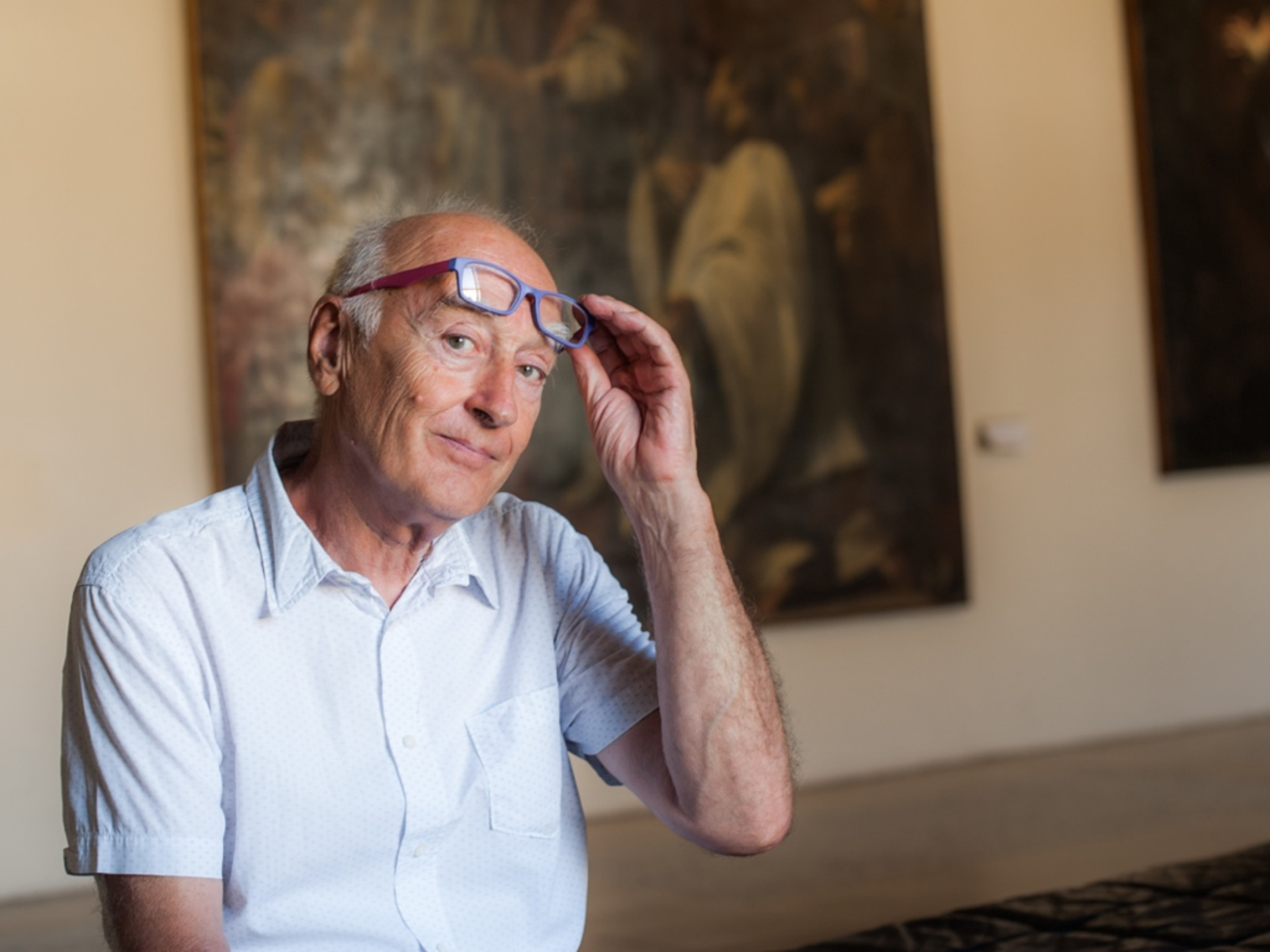


Working on a project with Pino
is like falling in love:
exciting, never boring
and with unpredictable results

20 papers
1600 citations

The main theme - physics of soft gluon radiation

Virtual convergence

Hard Processes in Quantum Chromodynamics
Yuri L. Dokshitzer, Dmitri Diakonov, S.I. Troian (St. Petersburg, INP).
Phys.Rept. 58 (1980) 269-395
Cited by 811 records

indirect multiple soft gluon radiation effects

Jet Structure and Infrared Sensitive Quantities in Perturbative QCD
A. Bassetto (Trento), M. Ciafaloni (Florence & Pisa, Scuola Normale Superiore), G. Marchesini (Parma).
Phys.Rept. 100 (1983) 201-272
Cited by 649 records

direct manifestations in the final state structure

Real interaction

Measuring color flows in hard processes via hadronic correlations **1990**

Dispersive approach to power behaved contributions in QCD hard processes **1995**

Nonperturbative effects in the energy-energy correlation **1999**

with **Bryan**. Then, a series of studies with/by **Gavin**, **Gulia** & **Andrea**

On large angle multiple gluon radiation **2003**

Hadron collisions and the fifth form-factor **2005**

Soft gluons at large angles in hadron collisions

Revisiting parton evolution and the large-x limit **2005**

with **Gavin**

N=4 SUSY Yang-Mills: three loops made simple(r) **2006**

Wise Dispersive Method

CERN 1995-96

a “WOW”

PUZZLE

Hidden message from ***QCD Radiophysics***

Quarks and gluons - QCD partons - involved in a hard interactions act as elements of a color antenna - a composite source of additional gluon radiation.

Soft gluon effects (accompanying radiation plus virtual corrections)
to **2 -> 2** hard parton scattering is rather involved
as it does not reduce to combination of “color charges” of the participating partons

Especially true for **gluon–gluon** scattering.

Here one encounters 6 (5 for SU(3)) color channels that mix with each other under soft gluon radiation ...

A difficult quest of sorting out large angle gluon radiation in all orders in $(\alpha_s \log Q)^n$ was set up and solved by George Sterman and collaborators.

Our revision of this problem has revealed a strange (if not ***mysterious***) feature ...

Puzzle of large angle Soft Gluon radiation

Soft anomalous dimension for gluon-gluon scattering

$$\frac{\partial}{\partial \ln Q} M \propto \left\{ -N_c \ln\left(\frac{t\,u}{s^2}\right) \cdot \hat{\Gamma} \right\} \cdot M$$

6=3+3. Three eigenvalues are "simple".

Three "***ain't-so-simple*** " ones were found to satisfy the cubic equation

$$\left[E_i - \frac{4}{3}\right]^3 - \frac{(1+3b^2)(1+3x^2)}{3}\left[E_i - \frac{4}{3}\right] - \frac{2(1-9b^2)(1-9x^2)}{27} = 0,$$

$$x = \frac{1}{N}, \qquad b \equiv \frac{\ln(t/s) - \ln(u/s)}{\ln(t/s) + \ln(u/s)}$$

Mark the **mysterious symmetry** w.r.t. to x -> b

*interchanging internal (**group rank**) and external (**scattering angle**) variables . . .*

An open quest for "theoretical theory"

Our experience *may* help to resolve a still hanging
"***superlogs***" **mystery**
(Mike Seymour, Jeff Forshaw & Co)

Super-leading logarithms in non-global observables in QCD (2006)
Breakdown of QCD coherence?

On the Breaking of Collinear Factorization in QCD (2012)

the origin of the trouble : in high pQCD orders,
a Coulomb exchange btw incoming color charges
starts messing with scattering dynamics

G.M. & Y.D;
thought about, but not through;
unfinished and unpublished

The main hint:

Better be damn careful when trying to solve

1. an unphysical problem by means of
2. the standard scattering theory

(representing <in| and |out> states as plane waves, in the momentum space)

N=4 SUSY :
a
CLASSICAL QFT ?

an inviting heresy

Let us see what sort of functions the N=4 parton Hamiltonian is made of

In spite of having many states ($s = 0, \frac{1}{2}, 1$), the SYM-4 parton dynamics is built of a single "universal" anomalous dimension:

$$\gamma_+(N+2) = \tilde{\gamma}_+(N+1) = \gamma_0(N) = \tilde{\gamma}_-(N-1) = \gamma_-(N-2) \equiv \gamma_{\rm uni}(N)$$

$$\gamma_{\rm uni}^{(1)}(N) = -S_1(N) = -\int_0^1 \frac{dx}{x}\left(x^N - 1\right)\cdot \frac{x}{x-1} \equiv \mathbf{M}\left[\frac{x}{(1-x)_+}\right]$$

This is nothing but (**the Mellin image of**) *the classical* (**LBK**) *gluon radiation spectrum !*

$$S_1(N) = \sum_{k=1}^N \frac{1}{k} = \psi(N+1) - \psi(1)$$

Euler Harmonic Sum

Beyond the 1st loop the answer is more complex.

New interesting functions show up

- Zagier

Euler -

harmonic sums

Euler-Zagier harmonic sums

- In higher orders enter $m > 1$

$$S_m(N) = \sum_{k=1}^{N} \frac{1}{k^m} = \frac{(-1)^m}{\Gamma(m)} \int_0^1 dx\, x^N \frac{\ln^{m-1} x}{1-x} + \zeta(m)$$

QCD and the "Basel problem"

The **problem** posed in 1644 by Pietro Mengoli, solved by 28 year old Leonhard Euler in 1735.

Euler : look upon $\frac{\sin x}{x}$ as an infinite degree polynomial **= an infinite product of roots**

"... the other results of this chapter will be of similar nature: that is, correct but unproven"

$$\zeta_2 = \sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6} \equiv S_2(\infty) \qquad \zeta_{2n} \equiv S_{2n}(\infty) \propto \pi^{2n}$$

more and more transcendental ...

Euler-Zagier harmonic sums

- In higher orders enter $m > 1$

$$S_m(N) = \sum_{k=1}^{N} \frac{1}{k^m} = \frac{(-1)^m}{\Gamma(m)} \int_0^1 dx\, x^N \frac{\ln^{m-1} x}{1-x} + \zeta(m)$$

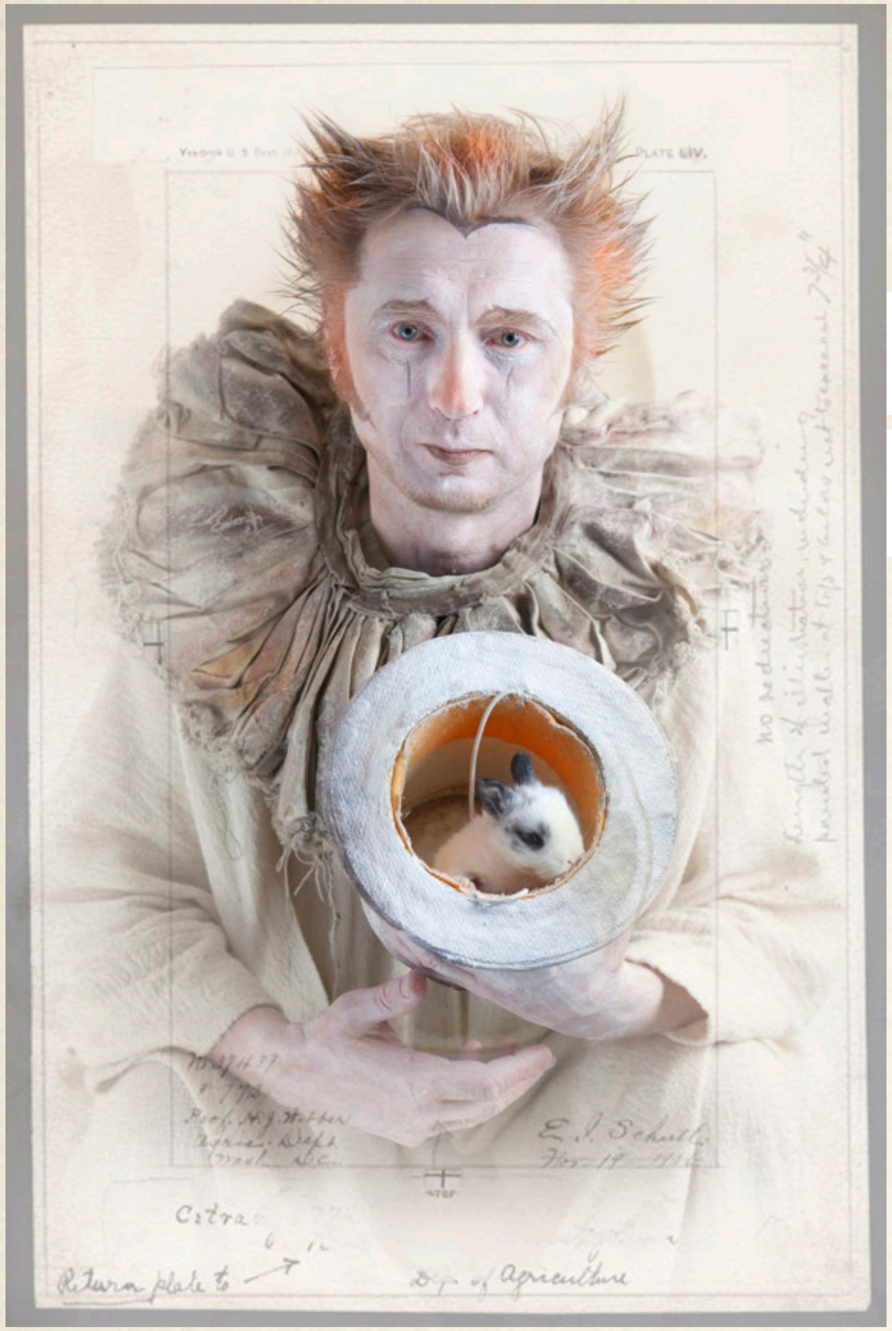
- Starting from the 2nd loop,
one encounters also *negative indices*

$$S_{-m}(N) = \sum_{k=1}^{N} \frac{(-1)^k}{k^m}$$

- multiple indices — *nested sums*

$$S_{m,\vec{\rho}}(N) = \sum_{k=1}^{N} \frac{S_{\vec{\rho}}(k)}{k^m} \qquad (\vec{\rho} = (m_1, m_2, \ldots, m_i))$$

"transcedentality" of a Harmonic Sum
= the sum of its indices



twist-2 anomalous dimension for N=4 SYM

Anatoly Kotikov - Lev Lipatov (2000)

$$\gamma_1 = -S_1$$

$$\gamma_2 = \frac{1}{2}S_3 + S_1 S_2 + \left(\frac{1}{2}S_{-3} + S_1 S_{-2} - S_{-2,1}\right)$$

→ *Principle of*
Maximal Transcedentality

hypothesis: sum of indices
= **2L-1**

Compare parton Hamiltonians

N=4 SYM		QCD	
1st loop :	1 symbol	1st loop :	1 line
2nd loop :	1 line	2nd loop :	1 page
3rd loop :	1/2 page	3rd loop :	200 pages

Exploring another hidden symmetry – "Gribov-Lipatov reciprocity"

1 line

4 loops

5 loops

... ALL loops ?

D-r & Marchesini (2006)

Beccaria & Fiorini (2009)

Romuald Janik & Co (2010+)

someone (one day)

why care ?

QCD and SUSY-QCD share the gluon sector !

Importantly, the maximal transcedentality (*clagon*) structures constitute the bulk of the QCD anomalous dimensions.

Employ $\mathcal{N}=4$ SYM to simplify the essential part of the QCD dynamics

N=4 SYM dynamics is ***classical***, in (un)certain sense

No truly quantum effects are being seen

(look at the β-function and/or the anomalous dimension)

If this is true, the goal would be

to derive a one-line-all-orders expression for γ from $\gamma^{(1)}$ in $\mathcal{N}=4$ SYM
and then to export it into QCD,
to cover "90%" of the small-distance parton dynamics