MiNLO as a merging tool: a path to NNLOPS

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Relies on work with William Astill, Wojtek Bizon, Keith Hamilton, Paolo Nason, Carlo Oleari, Emanuele Re

Precision QCD

At colliders QCD interactions are weak. Precision through higher-order expansion in the small coupling α

1. fixed order

$$\frac{\sigma}{\sigma_0} = 1 \quad \text{LO}$$
$$+ c_1\alpha \quad \text{NLO}$$
$$+ c_2\alpha^2 \quad \text{NNLO}$$
$$+ \ldots$$

2. all order (L = large logarithm)

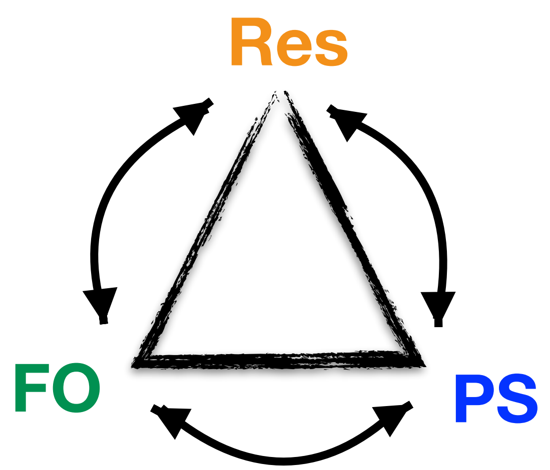
$$\ln\frac{\sigma}{\sigma_0} = \alpha^n L^{n+1} \quad \text{LL}$$
$$+ \alpha^n L^n \quad \text{NLL}$$
$$+ \alpha^n L^{n-1} \quad \text{NNLL}$$
$$+ \ldots$$

3. parton shower event generators: go from perturbative picture (quark/gluons) to realistic final state (pions, mesons etc.) [Herwig, Pythia, Sherpa …]

Interplay between 3 approaches

The three approaches not considered independently:

- insight from analytic resummation crucial to improve/develop parton showers
- analytic resummed calculations matched to fixed-order, i.e. when a calculation is expanded to fixed-order, the last order known should be reproduced exactly



- parton-showers matched to fixed-order NLO results (only in few cases to NNLO, see later)

While matching of analytic resummations to FO is trivial, as one has full analytic control, the matching of parton showers to FO is far from trivial (main issue: avoid double-counting)

PhD with Pino

My PhD with Pino focused on analytic resummations

- In 2000 the perturbative description of two jet events was already very refined (NLO+NLL accuracy). Many people worked on it.
- Pino suggested to extend this accuracy to multijet events
- Challenging because it does not admit a classical probabilistic interpretation. NLL accuracy can be reached by
  - ✓ taking into account soft inter-jet gluon radiation
  - ✓ account for hard intra-jet parton decays
  - ✓ take into account kinematical recoil effects
  - ✓ prove soft gluon exponentiation and the prescription for the running coupling

It was an ambitious program and it showed an excellent vision of where the attention would move to in the following years

PhD with Pino

- After finishing my PhD I did not work with Pino anymore
- He encouraged me to move away from Italy (ideally to the US)
- I did my first post-doc in Durham and Pino urged me to work with local people there (although he did not seem enthusiastic about the “fixed-order activity” done there)
- But this is how Pino was, always looking for something new and exciting, pushing for new experiences and adventures …

Pioneering Monte Carlos

Pino's heart was never really with fixed-oder calculations, he found it much more interesting to understand "all-order" effects and was a pioneer of Monte Carlo methods

Cavendish–HEP–87/9
December, 1987

HERWIG

A new Monte Carlo event generator for simulating Hadron Emission Reactions With Interfering Gluons [1]

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Abstract

HERWIG is a general-purpose particle physics event generator with the following novel features:

- Simulation of hard lepton-lepton, lepton-hadron and hadron-hadron scattering and soft hadron-hadron collisions in one package
- Colour coherence of partons (initial and final) in hard subprocesses
- QCD jet evolution with soft gluon interference via angular ordering
- Backward evolution of initial-state partons including interference
- Azimuthal correlations within and between jets due to interference
- Azimuthal correlations within jets due to gluon polarization
- Cluster hadronization of jets via non-perturbative gluon splitting
- Same cluster hadronization scheme for jets, soft hadronic collisions and underlying events in hard collisions.

A lot of what Pino did is used today in modern Monte Carlos

NLO + parton showers

NLO + parton shower (NLO+PS) matching achieved about 15 years ago in seminal papers

- POWHEG

Nason ’04

- MC@NLO

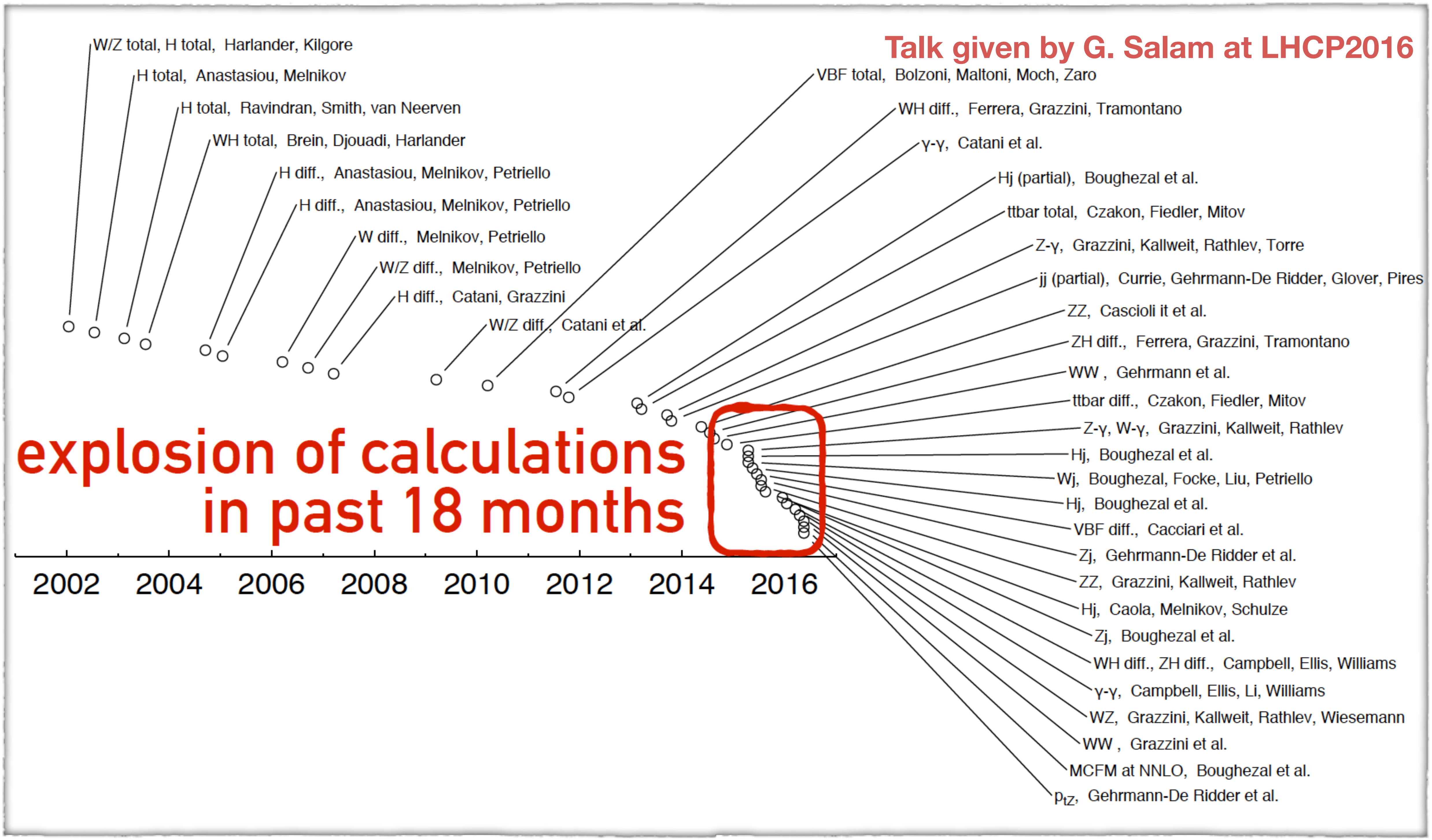
Frixione & Webber ’02

Guarantees good perturbative accuracy for inclusive cross-sections, together with a realistic description of the final state (with hadronization, multi-parton interaction, all order effects…)

Today NLO+PS codes used in all advanced LHC analysis.

But the new frontier of fixed-order calculations is now NNLO …

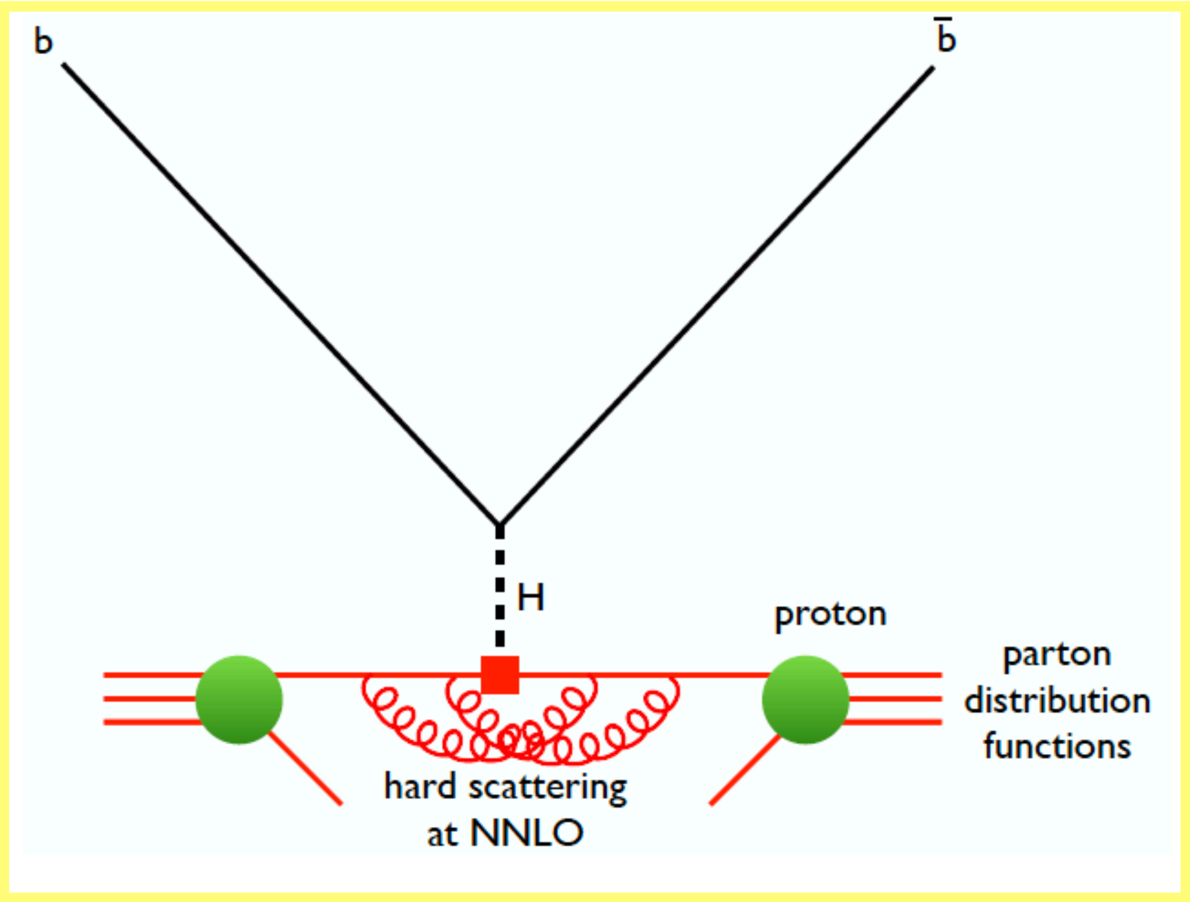
The NNLO revolution



NNLO or parton shower ?

NNLO:

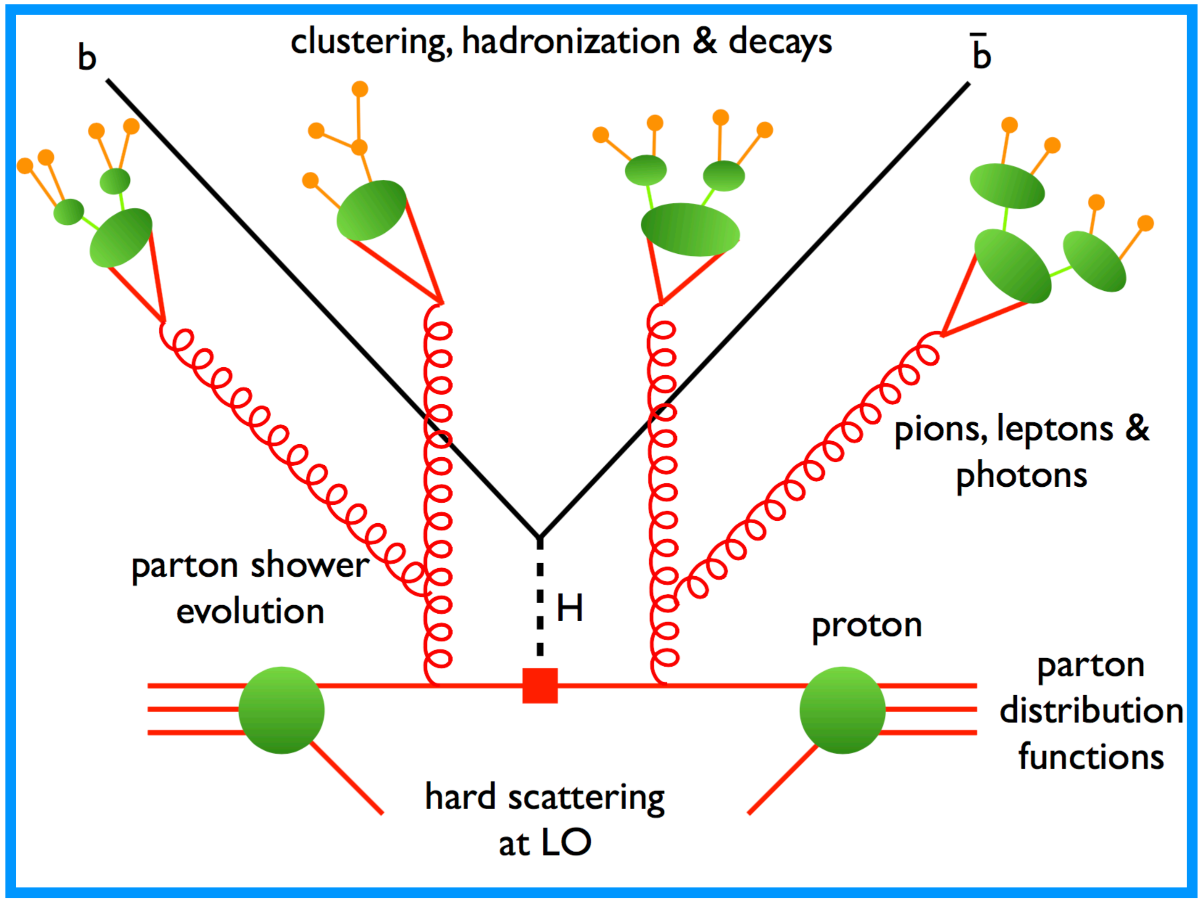
good perturbative accuracy, accurate inclusive cross-sections, but limited to low multiplicity and parton level only



?

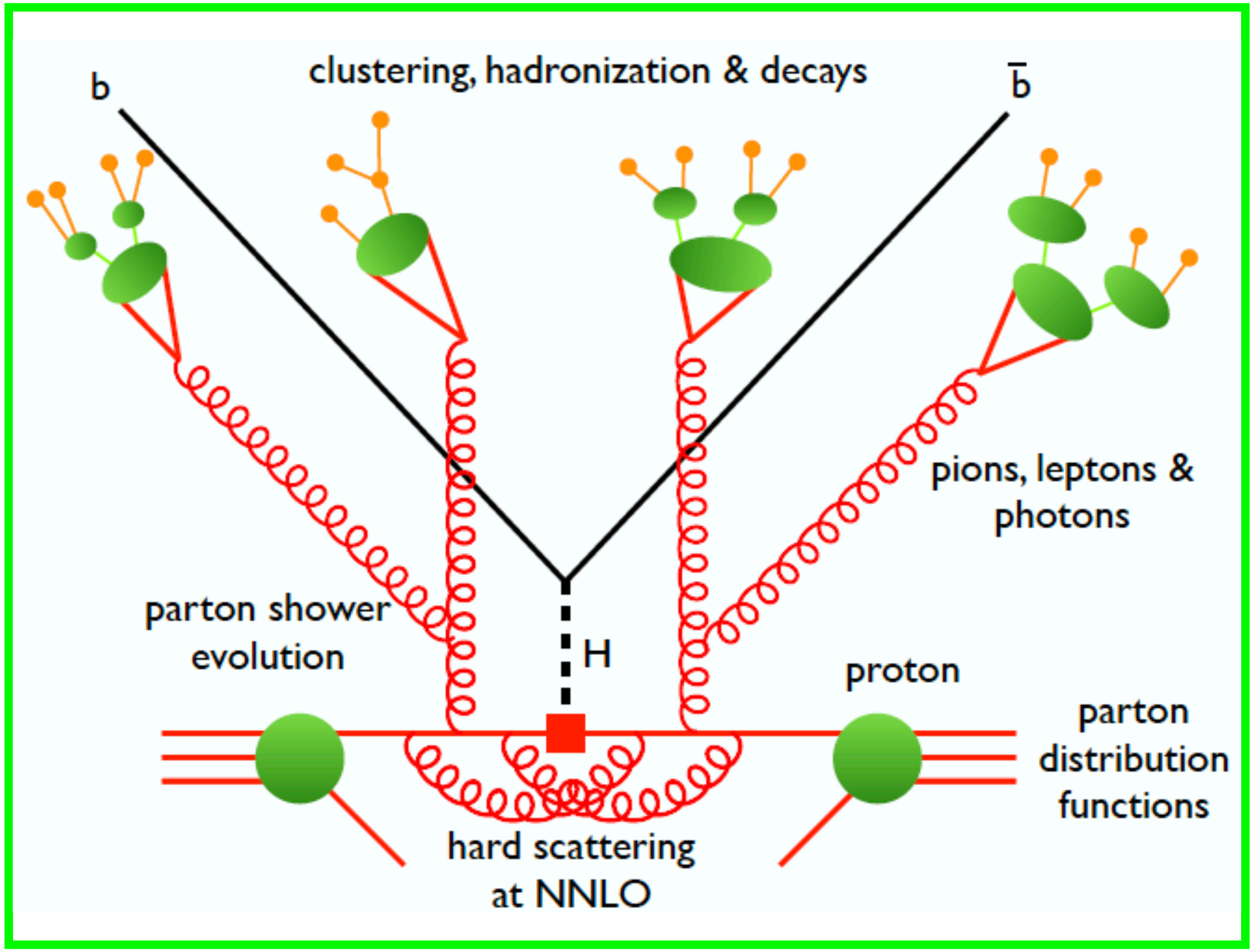
Parton shower:

less accurate, but realistic description, including multi-parton interactions, resummation, hadronization effects



NNLO + PS

Merging NNLO and parton shower (NNLOPS) is a must to have the best perturbative accuracy with a realistic description of final state



NNLOPS with MiNLO

One approach to NNLOPS is based on MiNLO

What is MiNLO (Multi Scale Improved NLO)?*

- MiNLO born as a procedure to set renormalization and factorisation scales dynamically à-la CKKW and include Sudakov form factors in NLO calculations

Why was MiNLO developed?

- Scale dependence is reduced at NLO compared to LO, but even at NLO the scale choice is an issue. Different choices can lead to a different picture/contrasting conclusions.

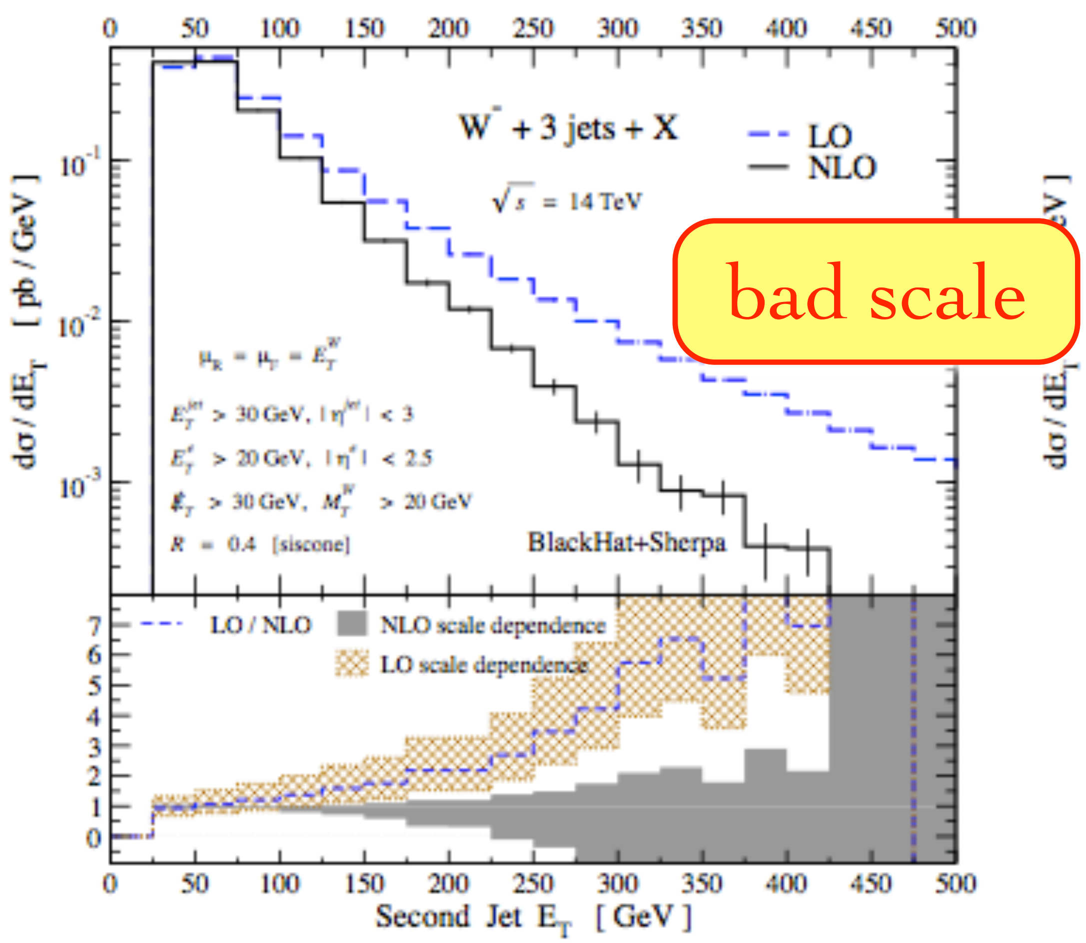
(*) The original name was SiNLO, for 'Sudakov Improved NLO', but somebody said that SiNLO reminded him of sins and wrong-doings …

Let's take a step back and discuss renormalization and factorisation scale choice at NLO

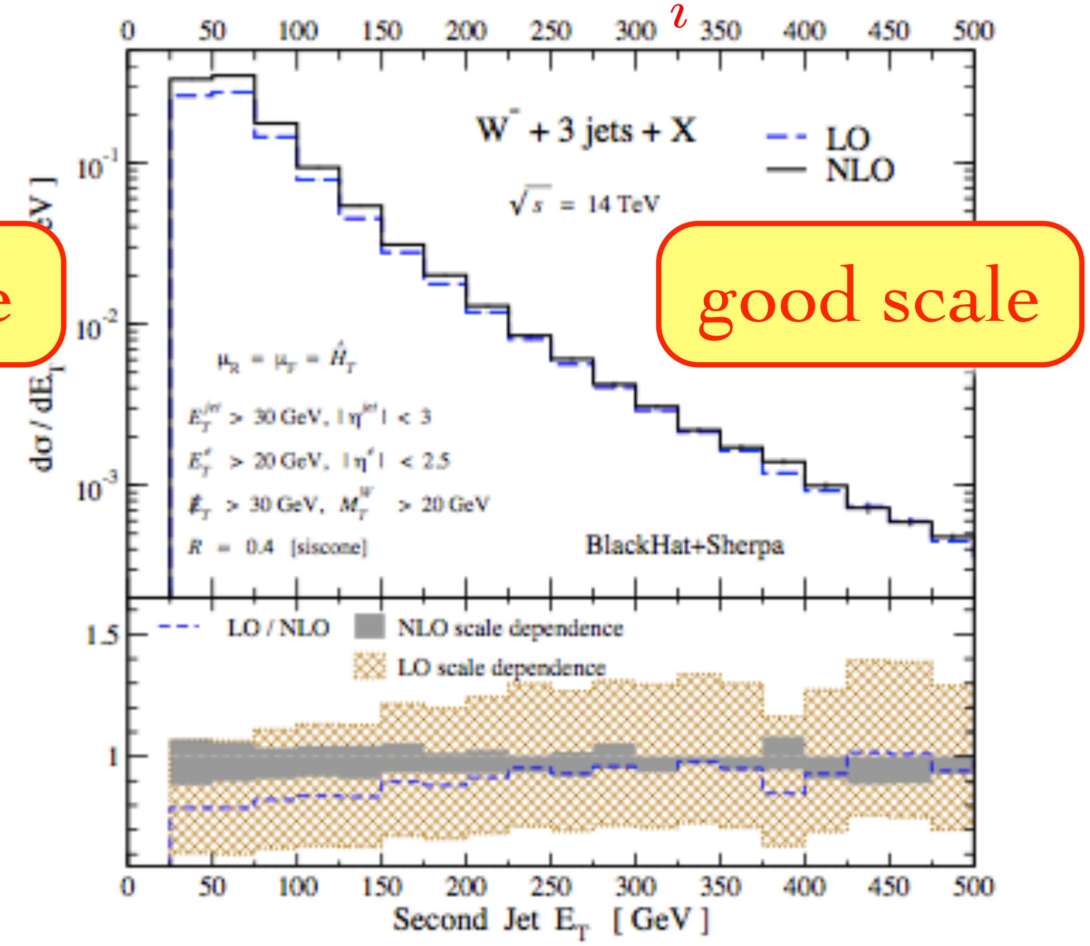
Scale choice at NLO

An example where a scale choice leads to a different picture: transverse momentum of second jet in W + 3 jet events

$$\mu_R = \mu_F = E_T^W$$



$$\mu_R = \mu_F = \sum_i p_{t,i}$$



W+ multi-jet processes are important backgrounds to SUSY searches at high transverse energies

Scale choice at NLO

Often a “good scale” is determined a posteriori, either by requiring NLO corrections to be small, or by looking where the sensitivity to the scale is minimized

Reason: bad scale ➠ large logs ➠ large NLO, large scale dependence

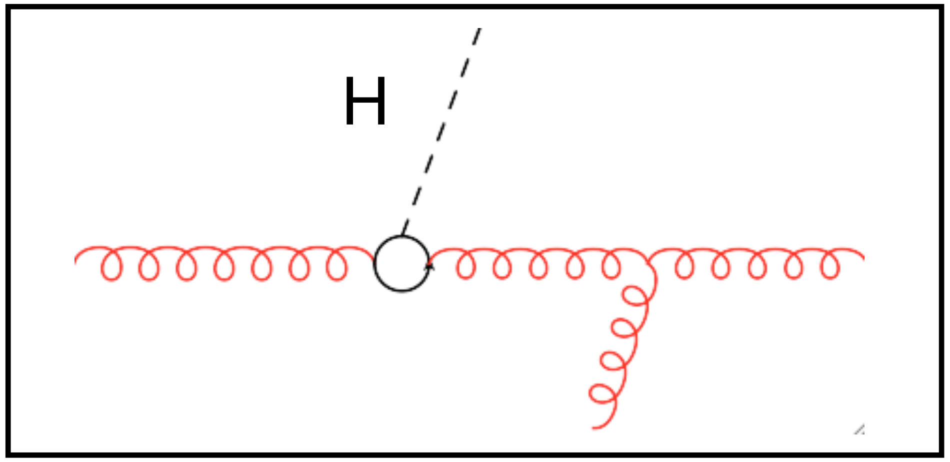
But we also know that large NLO ~~➠~~ bad scale choice, since large NLO corrections can have a “genuine” physical origin
(new channels opening up, Sudakov logarithms, color factors, large gluon flux ...)

Ambiguity in scale choice: H+1 jet

Even in simple processes the "best scale choice" is not clear

Example: H+1 jet production
The jet has most likely a small transverse momentum, so has the Higgs. Therefore two different scales are present m_H and $p_{t,H} \sim p_{t,j}$

One can argue in favour of a scale choice of the type $\alpha_s(M_H)^2\, \alpha_s(p_{T,H})$ or of the type $\alpha_s(p_{T,H})^3$



These scales lead to incompatible results at pure fixed order
(but there is no incompatibility once Sudakov form factors are properly taken into account, see later…)

Scale choice at leading order

Leading order (LO) calculations in matrix elements generators that follow the CKKW procedure are quite sophisticated in the scale choice: they use optimized/local scales at each vertex and Sudakov form factors at internal/external lines

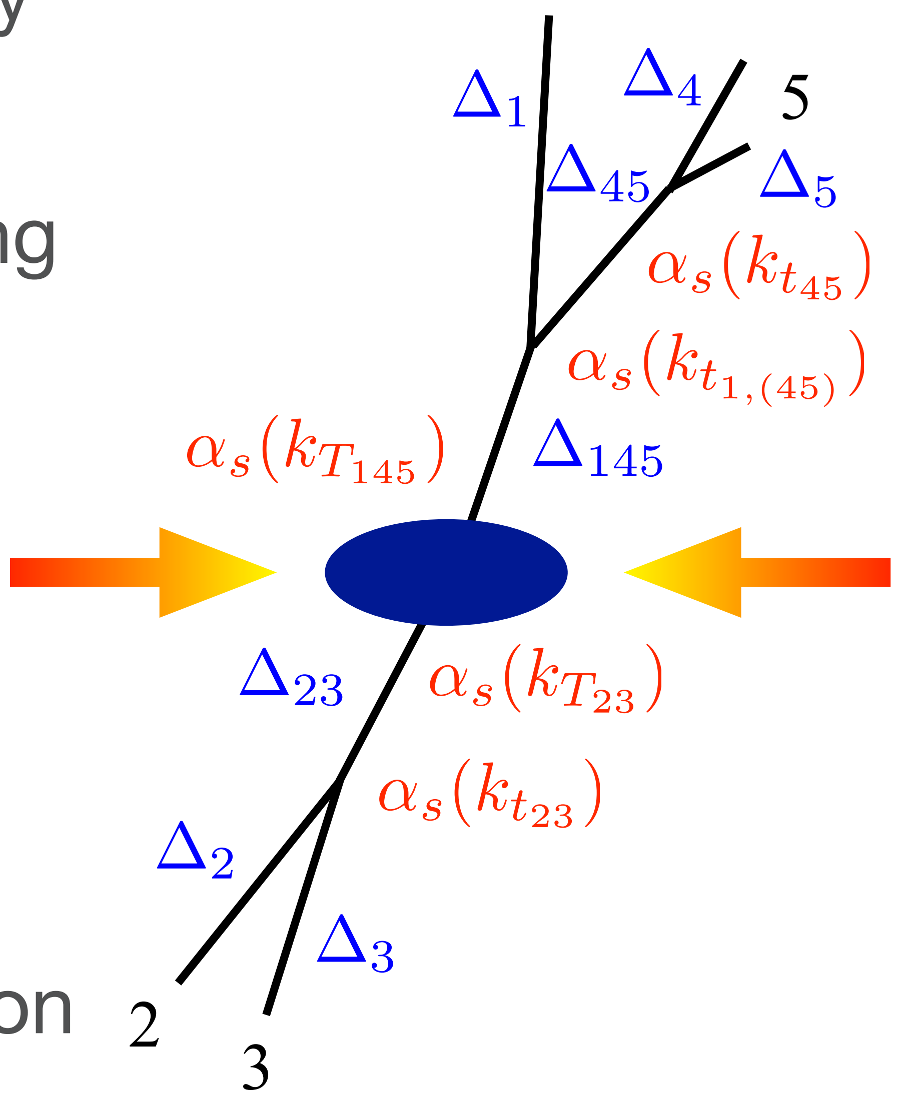
Catani, Krauss, Kuehn, Webber ’01

Reminder:

a Sudakov form factor encodes the probability of evolving from one scale Q_1 to the next Q_2 without branching above a resolution scale Q_0

Recap at CKKW procedure

- reconstruct the most likely branching history with k_t algorithm
- evaluate each α_s at the local p_t of the splitting
- for internals line between nodes at scale Q_i and Q_j include a Sudakov form factor $\Delta_{ij}=D(Q_0,Q_i)/D(Q_0,Q_j)$ that encodes the probability of evolving without emitting. For external lines include the Sudakov factor $\Delta_i=D(Q_0,Q_i)$
- match to a parton shower to include radiation below Q_0



Scale choice intertwined with inclusion of Sudakov form factors

NLO: two observations

1. A generic NLO cross-section has the form

$$\alpha_S^n(\mu_R)\,B + \alpha_S^{n+1}(\mu_R)\left(V(Q) + n b_0 \log\frac{\mu_R^2}{Q^2} B(Q)\right) + \alpha_S^{n+1}(\mu_R)\,R$$

   Adopting CKKW scales at LO, this becomes naturally

$$\alpha_s(\mu_1)\dots\alpha_s(\mu_n)B + \alpha_s^{n+1}(\mu_R')\left(V(Q) + b_0\log\frac{\mu_1^2\dots\mu_n^2}{Q^{2n}}B\right) + \alpha_s^{n+1}(\mu_R'')R$$

   and the scale choices μ_R’ and μ_R” are irrelevant for the scale compensation

2. Sudakov corrections included at LO via the CKKW procedure lead to NLO corrections that need to be subtracted to preserve NLO accuracy

The MiNLO procedure

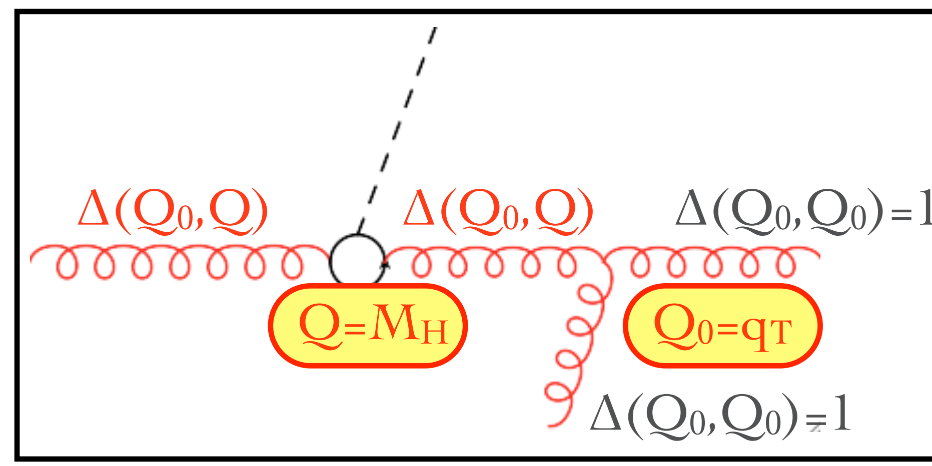
1. Find the CKKW n clustering scales $Q_1 < \ldots < Q_n$. Fix the hard scale of the process Q to the system invariant mass after clustering. Set Q_0 to Q_1 (inclusive on radiation below Q_1)
2. Evaluate the n coupling constants at the scales Q_i
3. Set μ_R in the virtual to the geometric average of these scales
4. Include Sudakov form factors for Born and virtual terms, and for the real term after the first branching
5. Subtract the NLO bit present in the CKKW Sudakov of the Born
6. Give a prescription for the $(n+1)^{th}$ power of α_s in the real and virtual terms and for the factorisation scale μ_F

MiNLO on Higgs plus one jet

Example: take e.g. H + one jet

In POWHEG it is customary to discuss the $\bar{B}$ function, which for H + 1 jet is defined as

$$\bar{B} = \alpha_s^3(\mu_R)\left[B + V(\mu_R) + \int d\Phi_{\mathrm{rad}}\, R\right]$$



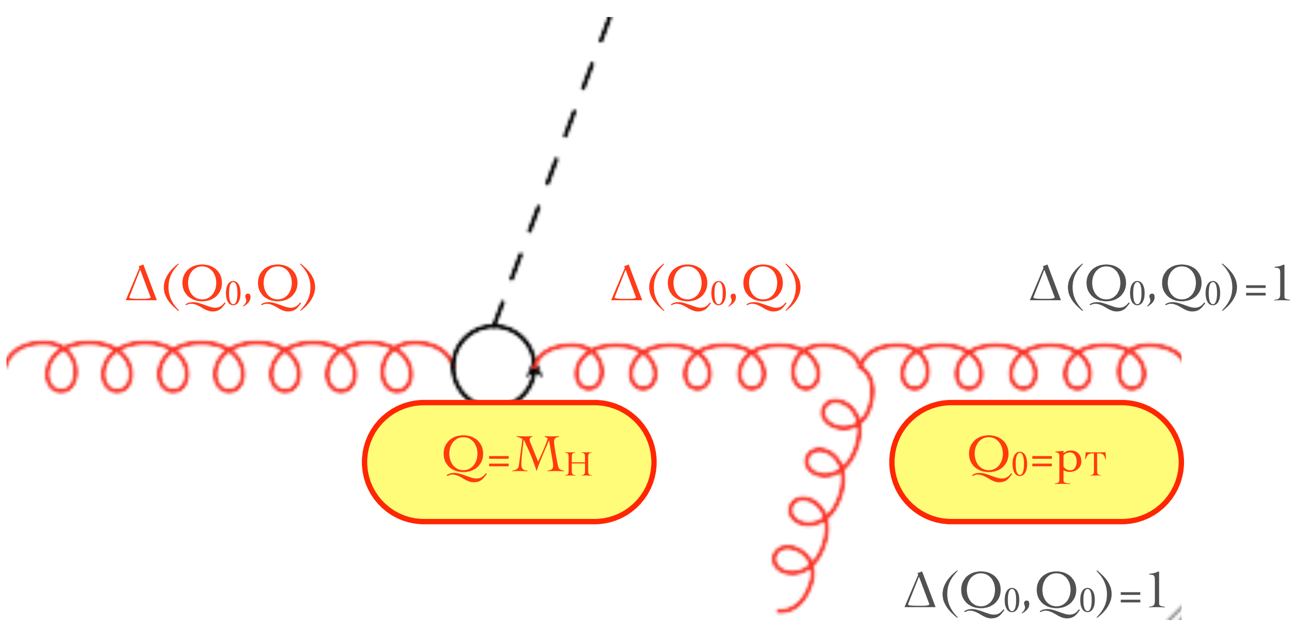
With MiNLO this function becomes

$$\bar{B} = \alpha_s^2\left(M_H^2\right)\alpha_s\left(q_T^2\right)\Delta_g^2\left(M_H, q_T\right)\left[B\left(1 - 2\Delta_g^{(1)}\left(M_H, q_T\right)\right) + V(\mu') + \int d\Phi_{\mathrm{rad}}\, R\right]$$

NB: the Sudakov form factors make the Higgs plus one jet calculation finite even without any cut on the jet transverse p_t

Clarifying the scale ambiguity in H+1j

How is the mismatch between using $\alpha_s(M_H)^2\,\alpha_s(p_{T,H})$ or $\alpha_s(p_{T,H})^3$ addressed with Sudakov form factors?



Clarifying the scale ambiguity in H+1j

The MiNLO procedure suggests to use $\alpha_s(M_H)^2\,\alpha_s(p_{T,H})$ supplemented by Sudakov form factors. There are two Sudakov factors, giving

$$F = \alpha_s^2(M_H)\alpha_s(p_T)\left\{\exp\left[-\frac{C_A}{\pi b_0}\left\{\log\frac{\log\frac{Q^2}{\Lambda^2}}{\log\frac{Q_0^2}{\Lambda^2}}\left(\frac{1}{2}\log\frac{Q^2}{\Lambda^2}-\frac{\pi b_0}{C_A}\right)-\frac{1}{2}\log\frac{Q^2}{Q_0^2}\right\}\right]\right\}^2$$ NLL Sudakov

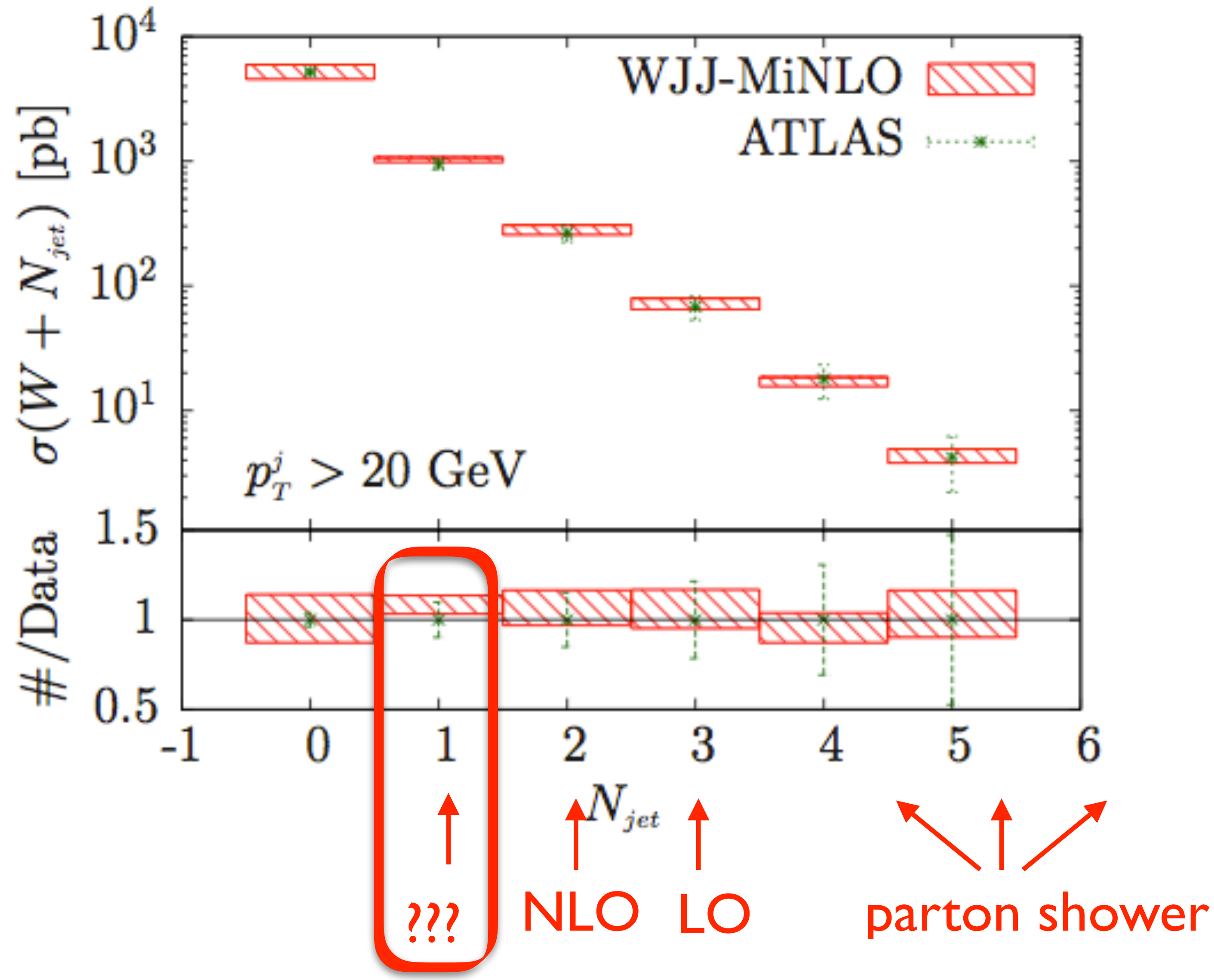
Which is equivalent to

$$\alpha_s^3(p_T)\exp\left[-\frac{C_A}{\pi b_0}\left\{\log\frac{\log\frac{Q^2}{\Lambda^2}}{\log\frac{Q_0^2}{\Lambda^2}}\log\frac{Q^2}{\Lambda^2}-\log\frac{Q^2}{Q_0^2}\right\}\right]$$ LL Sudakov

<u>Conclusion:</u> both choices are “fine” but one should not forget (most important) double logarithms in the Sudakov form factors

MiNLO - W + 2 jets versus data

Example: MiNLO on W/Z + 2 jets in POWHEG vs ATLAS data from 0 to 5 jets



The most interesting and surprising finding was that W + 2 jets with MiNLO also reproduces very accurately W + 1 jet data! (similar observation in other cases)

MiNLO-W+2jets versus data

What is the formal accuracy of MiNLO-W+2jets in the one jet region?

Insight from resummation becomes critical to answer the question

MiNLO-W+2jets versus data

NNLL$_\Sigma$ Higgs q_T resummation at fixed rapidity can be written as

$$\frac{d\sigma}{dy\,dq_T^2} = \sigma_0 \frac{d}{dq_T^2}\left\{\left[C_{ga}\otimes f_{a/A}\right](x_A, q_T) \times \left[C_{gb}\otimes f_{b/B}\right](x_B, q_T)\right\} \times \Delta_g^2(M_H, q_T) + R_f$$

Integrating in q_T one gets

$$\frac{d\sigma}{dy} = \sigma_0 \left\{\left[C_{ga}\otimes f_{a/A}\right](x_A, q_T) \times \left[C_{gb}\otimes f_{b/B}\right](x_B, q_T)\right\} \times \Delta_g^2(M_H, q_T) + \int dq_T^2 R_f$$

the formula is NLO accurate for Higgs production if $O(\alpha_s)$ corrections to the coefficient functions are included and R_f is LO accurate

Now, one can take the derivative explicitly, and see which terms are needed to maintain NLO accuracy after integration over q_T

Sudakov form factor for Higgs

The Sudakov form factor for Higgs production has the form

$$\Delta_g(M_H, q_T) = \exp\left\{ -\int_{q_T^2}^{M_H^2} \left[\frac{dq^2}{q^2} A(\alpha_s(q^2)) \ln \frac{M_H^2}{q^2} + B(\alpha_s(q^2)) \right] \right\}$$

$$A(\alpha_s) = \sum_i A_i \left(\frac{\alpha_s}{2\pi} \right)^i \qquad B(\alpha_s) = \sum_i B_i \left(\frac{\alpha_s}{2\pi} \right)^i$$

$$A_1 = C_A \qquad B_1 = -\frac{\beta_0}{2} \qquad A_2 = K_{\mathrm{CMW}}$$

Essential ingredient comes from coherent branching formalism of Catani, Marchesini and Webber

$$\boxed{K_{\mathrm{CMW}} = \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9} n_f}$$

Catani, Marchesini, Webber ’90

Counting rule

Use the simple gaussian integrals

$$\int_0^\infty dL e^{-\alpha L^2} = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}} \qquad \int_0^\infty dL L e^{-\alpha L^2} = \frac{1}{2} \qquad \ldots$$

To obtain

$$I(m,n) \equiv \int_{\Lambda^2}^{Q^2} \frac{\mathrm{d}q^2}{q^2}\left(\log\frac{Q^2}{q^2}\right)^m \alpha_s^n\left(q^2\right)\exp\left\{-\int_{q^2}^{Q^2}\frac{\mathrm{d}\mu^2}{\mu^2}A\,\alpha_s(\mu^2)\log\frac{Q^2}{\mu^2}\right\}$$

$$\approx \left[\alpha_s(Q^2)\right]^{n-\frac{m+1}{2}}$$

i.e. each log "counts" as a square-root of $1/\alpha_s$ after integration over the transverse momentum when the Sudakov weight is present

$$dL \sim L \sim \frac{1}{\sqrt{\alpha_s}}$$

Accuracy

Taking the derivative one gets

$$\sigma_0 \frac{dq_T^2}{q_T^2}\left[A_1\alpha_s L, B_1\alpha_s, A_2\alpha_s^2 L, B_2\alpha_s^2, C_1 \times C_1 \times A_1\alpha_s^3 L, \ldots\right]\exp\{\Delta_g(M_H, q_T)^2\}$$

After integration with the Sudakov weight, the counting is set by $L \sim dL \sim 1/\sqrt{\alpha_s}$. So these terms contribute, as

$$\int dL\, A_1\, \alpha_s L \exp\{\Delta_g(M_H, q_T)^2\} \sim A_1$$

$$\int dL\, B_1\, \alpha_s \exp\{\Delta_g(M_H, q_T)^2\} \sim B_1\sqrt{\alpha_s}$$

LO

$$\int dL\, A_2\, \alpha_s^2 L \exp\{\Delta_g(M_H, q_T)^2\} \sim A_2\alpha_s$$

$$\int dL\, B_2\, \alpha_s^2 \exp\{\Delta_g(M_H, q_T)^2\} \sim B_2\alpha_s^{3/2}$$

NLO

To claim NLO accuracy one needs to include B_2 in the Sudakov (neglected terms must be $O(\alpha_s^2)$ and not $O(\alpha_s^{3/2})$)

Merging with MiNLO

Conclusion:

☛ The original MiNLO prescription is less than NLO accurate for inclusive quantities, in that it neglects $O(\alpha_s^{3/2})$ terms

☛ achieve NLO accuracy from HJ also for inclusive Higgs by

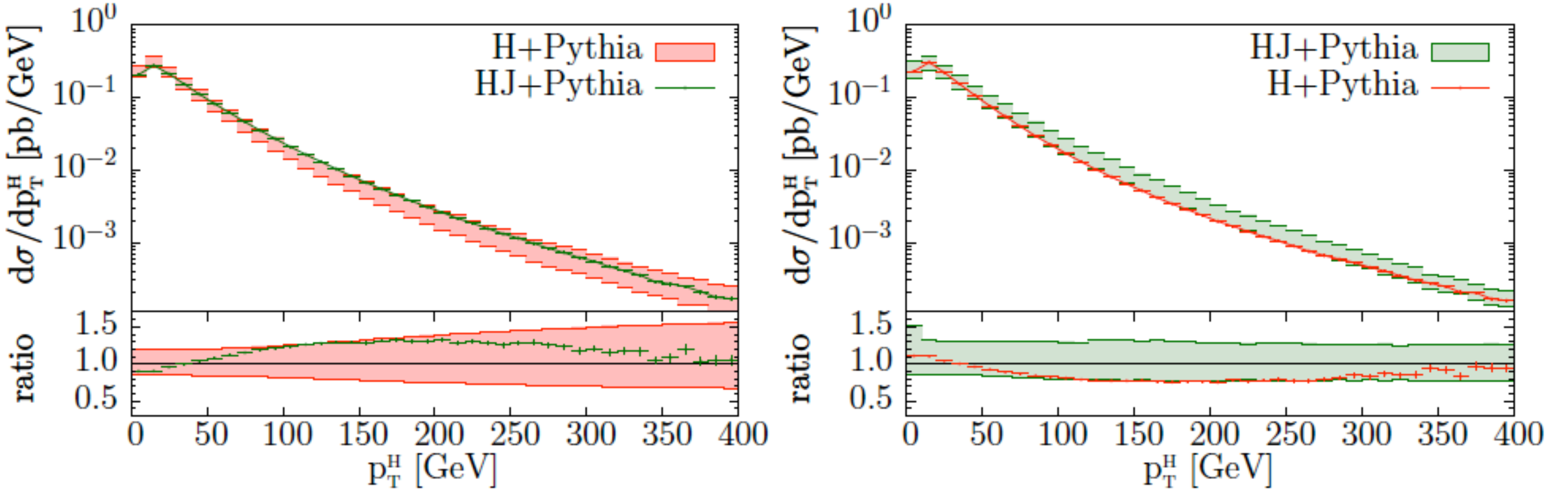
✔ including the B_2 term in the Sudakov form factors

✔ taking the scale in the coupling in the real and virtual equal to the Higgs transverse momentum (effect of same size as B_2)

Provided this is done, the HJ-MiNLO describes both H and H+jet at NLO, i.e. merging of H and H+jet is achieved without any merging scale

MiNLO: sample results

Higgs transverse momentum:



Nice agreement at intermediate values. At high transverse momenta H calculation is only LO accurate (band widens), while HJ-MiNLO remains NLO accurate throughout

Now we are ready to go back to the question of building an NNLOPS Monte Carlo

NNLOPS generator with MiNLO

Example:

- take H + 1 jet-MiNLO, as implemented in POWHEG (HJ-MiNLO)
- compare to the target generator that is NNLOPS accurate for Higgs production (H-NNLOPS)

	inclusive H	H + 1 jet	H + 2 jets	H + n jets
HJ-MiNLO	NLO	NLO	LO	parton shower
H-NNLOPS	NNLO	NLO	LO	parton shower

Conclusion: the HJ-MiNLO generator almost does the right job

NNLOPS generator with MiNLO

For inclusive Higgs the Born kinematics is fixed by the Higgs rapidity

$\left(\frac{d\sigma}{dy}\right)_{\rm NNLO}$ $\Longrightarrow$ inclusive Higgs rapidity computed at NNLO

$\left(\frac{d\sigma}{dy}\right)_{\rm HJ-MiNLO}$ $\Longrightarrow$ inclusive Higgs rapidity from HJ-MINLO generator

Since HJ-MINLO is NLO accurate, it follows that

$$\frac{\left(\frac{d\sigma}{dy}\right)_{\rm NNLO}}{\left(\frac{d\sigma}{dy}\right)_{\rm HJ\text{-}MiNLO}} = \frac{c_2\alpha_s^2 + c_3\alpha_s^3 + c_4\alpha_s^4}{c_2\alpha_s^2 + c_3\alpha_s^3 + d_4\alpha_s^4} \approx 1 + \frac{c_4 - d_4}{c_2}\alpha_s^2 + \mathcal{O}(\alpha_s^3)$$

This reweighing promotes HJ-MiNLO to NNLO. Since the re-weighting factor is 1+O(α_s^2) it does not spoil NLO accuracy (unlike usual re-weighting procedures), hence one obtains NNLOPS generator

More complicated processes?

Can one do more complicated processes?

- Yes, the only modification is that the reweighing should be differential in the Born variables
- Higgs, Drell-Yan and associated Higgs production have been computed with this method
- Exploiting the Collins-Soper parametrization of the decay of the vector boson reduces the number of degrees of freedom in the reweighing, but a treatment of more generic processes remains difficult …

Conclusions

- developing a Monte Carlo that “does the right job” was a big ambition of Pino
- this is still today a big, multi-lateral challenge
- an NNLO + parton shower accurate generator is one of the steps in this direction (and still not a solved problem …)
- Pino’s work is a backbone of Monte Carlos and their NNLOPS extension

A big thank you to the organizers for the opportunity to be here