The path towards automated resummations

Andrea Banfi

University of Sussex

Giuseppe Marchesini Memorial Conference

GGI Florence - 19 may 2017

Outline

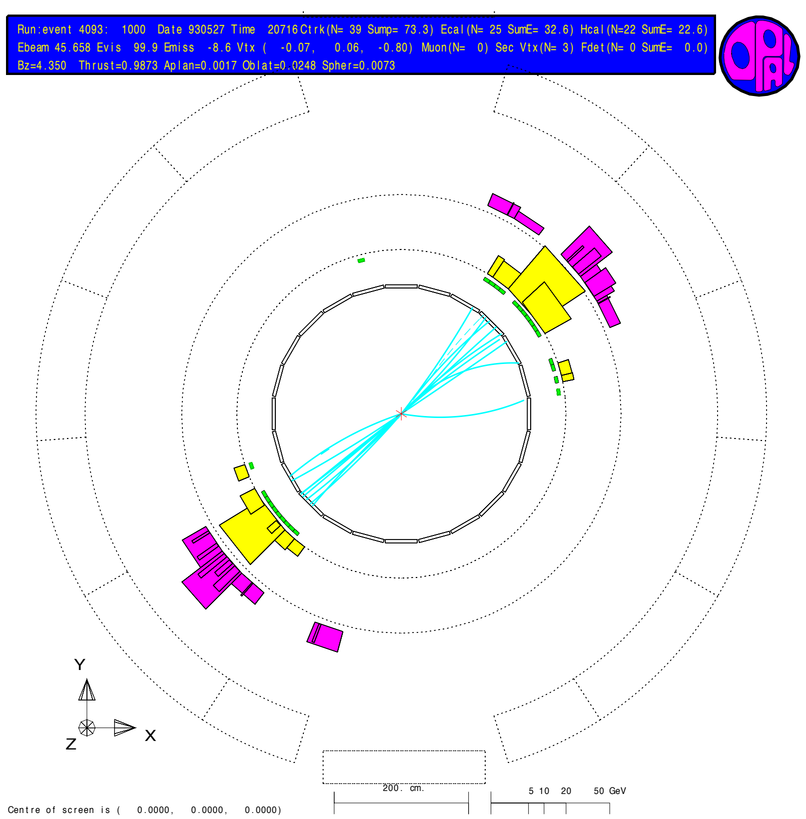
- Resummation of final-state observables
- Automated resummation: why?
- General NLL resummation with CAESAR
- Extension to NNLL with ARES
- Current work in progress and Pino's legacy

Final-state observables

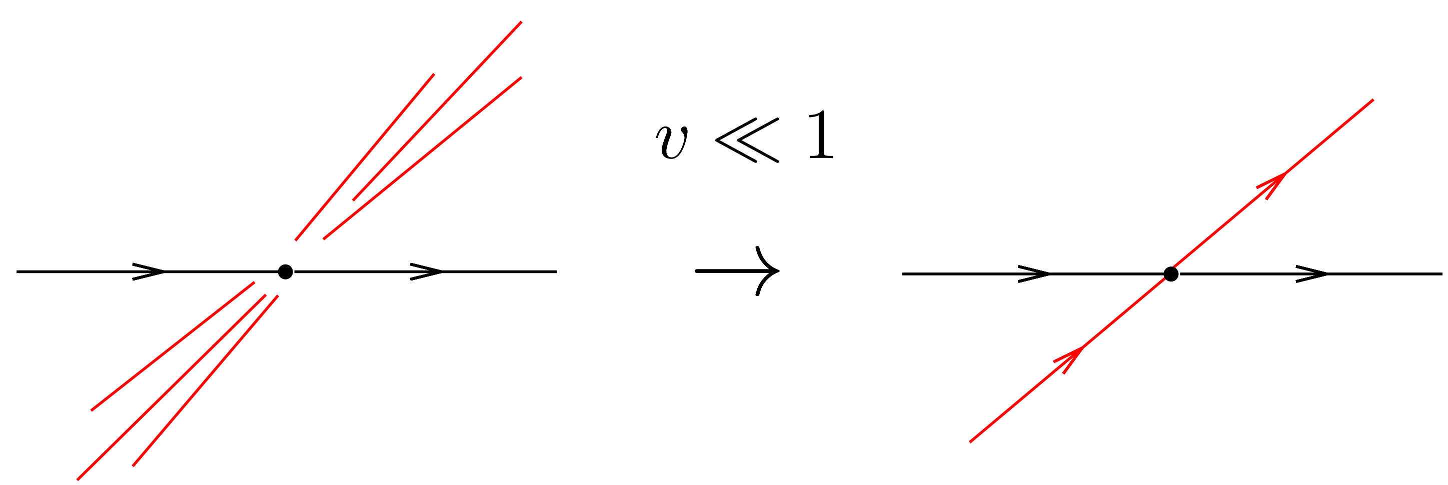
- We consider a generic final-state observable, a function $V(p_1, \ldots, p_n)$ of all possible final-state momenta $p_1, \ldots, p_n$

- Examples: leading jet transverse momentum in Higgs production or thrust in $e^+e^- \to \text{hadrons}$

$$\frac{p_{\text{t,max}}}{m_H} = \max_{j \in \text{jets}} \frac{p_{t,j}}{m_H} \qquad\qquad T \equiv \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$



$$\Sigma(v) = \text{Prob}[V(p_1, \ldots, p_n) < v]$$



The narrow-jet limit

- Selecting events close to the Born limit, i.e. $v \ll 1$, produces large logarithms of the resolution variable v due to incomplete real-virtual cancellations

$$\Sigma(v) \simeq 1 - C\,\frac{\alpha_s}{\pi}\ln^2\frac{1}{v} + \dots$$

LO NLO

breakdown of perturbation theory!

All-order resummation

- All-order resummation of large logarithms $\Rightarrow$ reorganisation of the PT series in the region $\alpha_s L \sim 1$, with $L = \ln(1/v)$

$$\Sigma(v) \simeq e^{\overbrace{\underbrace{Lg_1(\alpha_s L)}_{\text{LL}}}} \times \left(\overbrace{\underbrace{G_2(\alpha_s L)}_{\text{NLL}}}^{1} + \overbrace{\underbrace{\alpha_s G_3(\alpha_s L)}_{\text{NNLL}}}^{\alpha_s} + \overbrace{\dots}^{\dots} \right)$$

NLL resummation

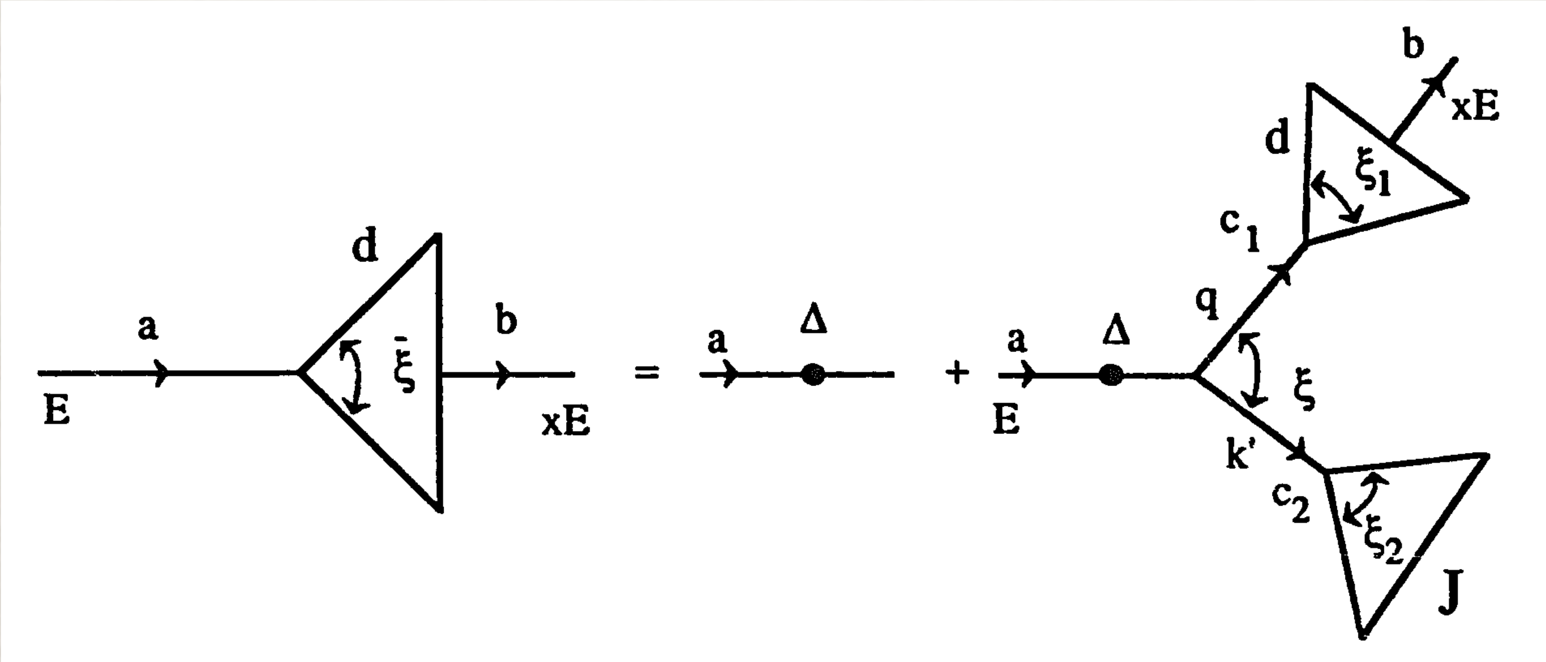
- At the end of the '90s, NLL resummations existed for a number of observables (extensive literature ~1 observable per article)
- Two main approaches
  - Branching algorithm

    [Bassetto Ciafaloni Marchesini '83, Catani Marchesini Webber et al.]
  - Collins-Soper-Sterman factorisation theorems

    [Collins Soper Sterman Kidonakis Laenen Magnea et al.]
- Resummation achieved by exploiting factorisation properties of QCD to derive evolution equations that can be solved analytically

Coherent branching

- Coherence properties of QCD emissions lead to the formulation a number of evolution equations [Bassetto Ciafaloni Marchesini ’83, Catani Marchesini Webber ‘90]

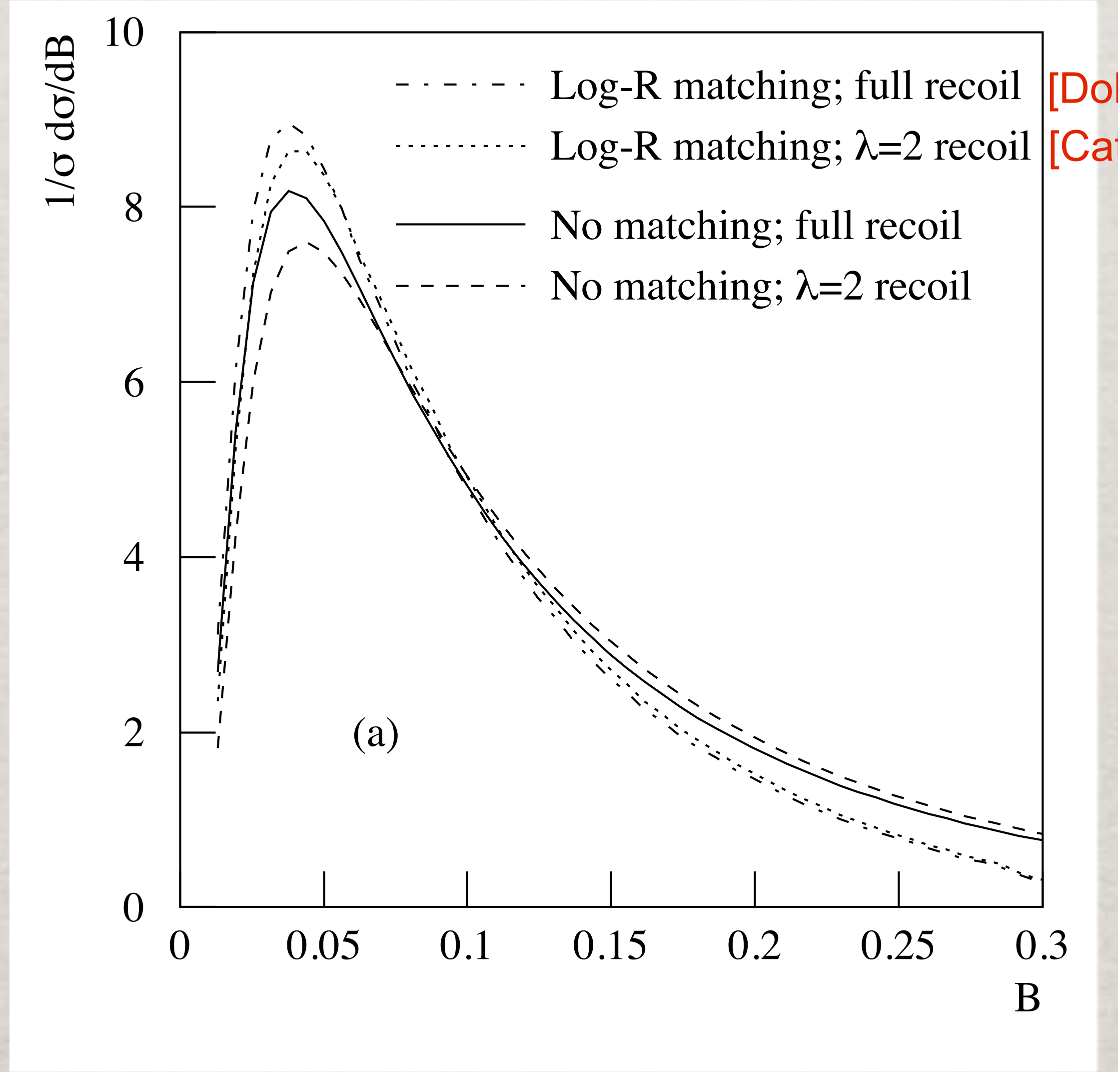


- Numerical iterative solution $\Rightarrow$ coherent branching algorithm for parton-showe event generators, especially HERWIG [Marchesini Webber ‘84]

- Approximate analytical solution $\Rightarrow$ NLL resummation of final-state observables (e.g. event shapes) [Catani Trentadue Turnock Webber ’91] [Catani Turnock Webber ’92]

Issues of evolution equations

- Evolutions equations for thrust and broadenings need to be carefully simplified to achieve NLL accuracy [Catani Trentadue Turnock Webber '91] [Catani Turnock Webber '92]
- Example: NLL treatment of recoil in jet-broadenings require a two-dimensional integral transform, overlooked in previous works [Dokshitzer Marchesini Lucenti Salam '98]

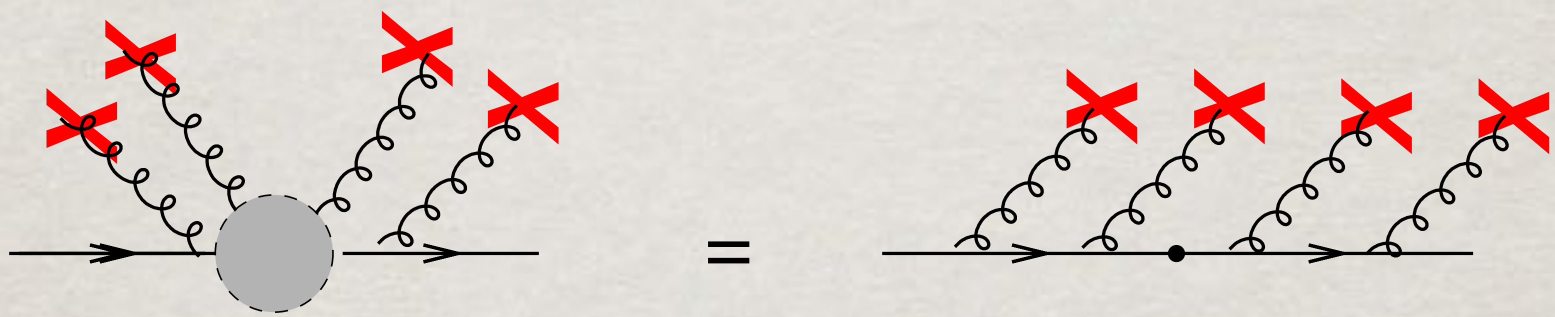


[Dokshitzer Marchesini Lucenti Salam]
[Catani Turnock Webber]

Independent emission

- The relevant emissions to achieve NLL accuracy were soft and collinear gluons widely separated in angle $\Rightarrow$ independent emission picture

[Dokshitzer Marchesini Lucenti Salam '98]



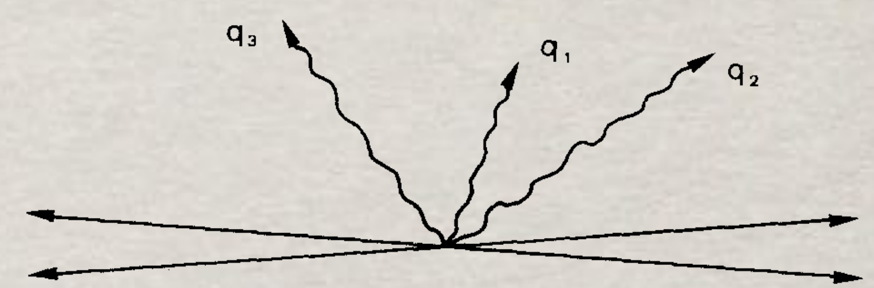
- Structure of resummation at NLL accuracy

[Bassetto Ciafaloni Marchesini '83]

$$\Sigma(v) = e^{-R_{\mathrm{NLL}}(v)}\mathcal{F}_{\mathrm{NLL}}(v)$$

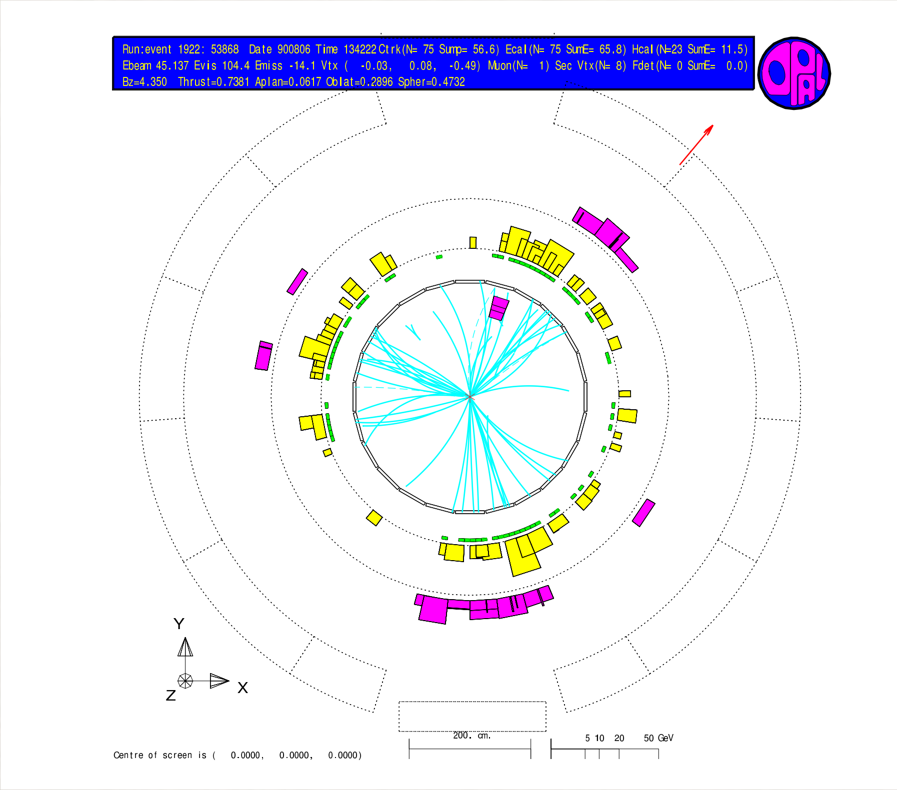
Sudakov form factor,
a.k.a. "radiator"

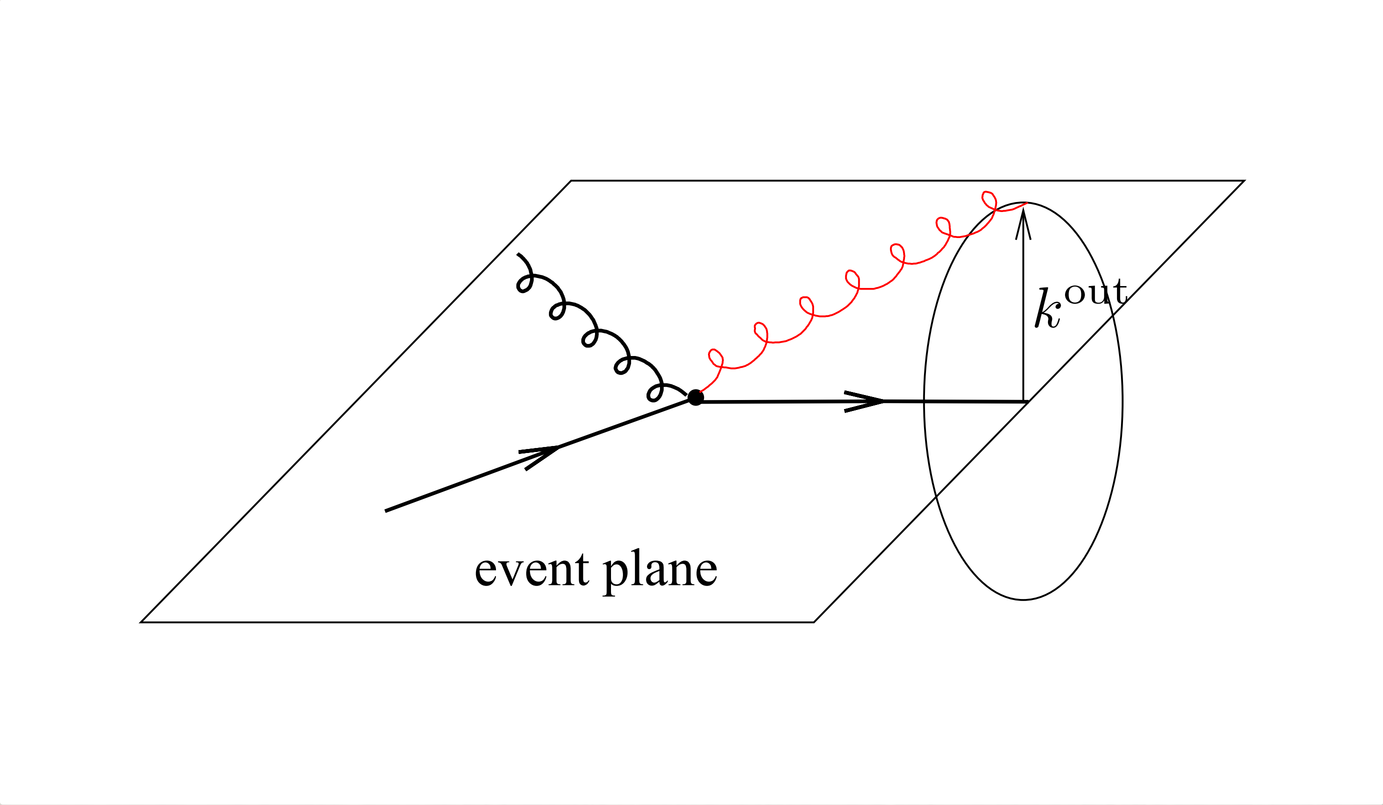
multiple emissions



Near-to-planar three-jet events

- Deviations from the Born limit (a planar $q\bar{q}g$ system) are probed by event shapes like the D-parameter or the thrust-minor, sensitive in different ways to the radiation out of the event plane [Banfi Dokshitzer Marchesini Zanderighi '01]





- Due to subtle kinematical effects, analytical treatment of multiple emissions is technically involved, requiring for the thrust-minor a five-dimensional integral transform

Semi-numerical resummation

- For all observables for which independent emission gives correct NLL resummation, multiple-emission effects depend only on kinematics
- A computer is better than humans at kinematics $\Rightarrow$ simulate multiple soft-collienear emissions with a Monte-Carlo procedure [Banfi Salam Zanderighi '01]

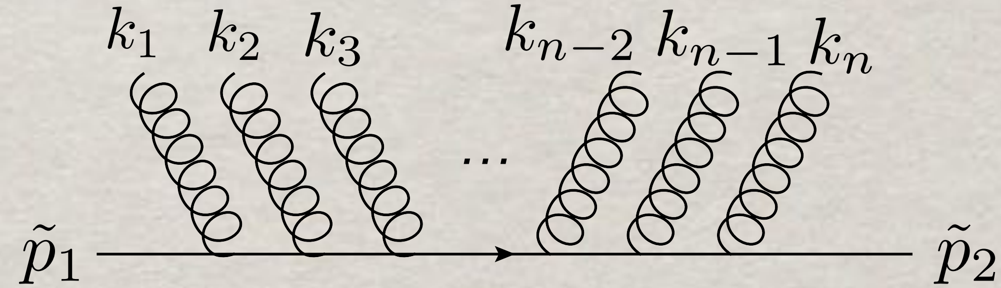
$$\mathcal{F}_{\mathrm{NLL}}(v) = \int \frac{dv_s}{v_s} \left(\frac{v}{v_s}\right)^{-R'} vP(v|v_s)$$

"simple" observable, having the same radiator as "true" observable

probability that the true observable has a value v, given a value v_s of the simple observable

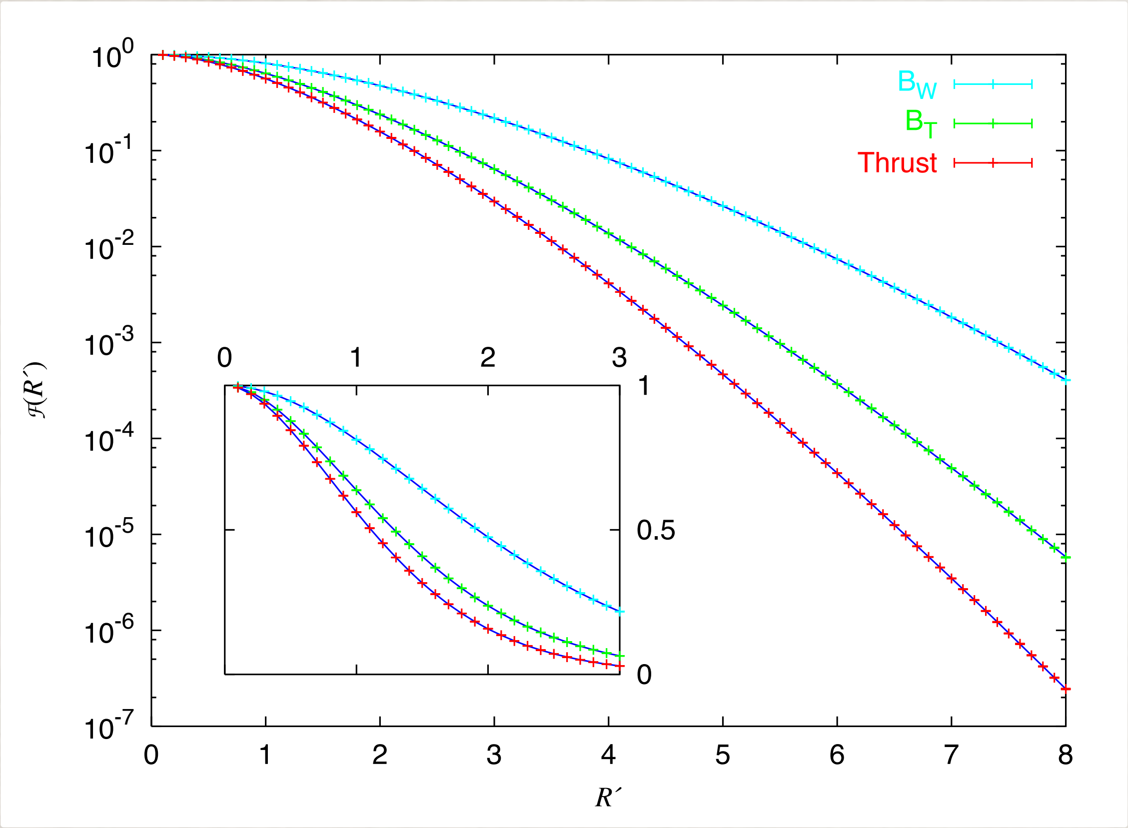
- For each final-state observable $V(\{\tilde{p}\}, k_1, \ldots, k_n)$, we chose as simple observable

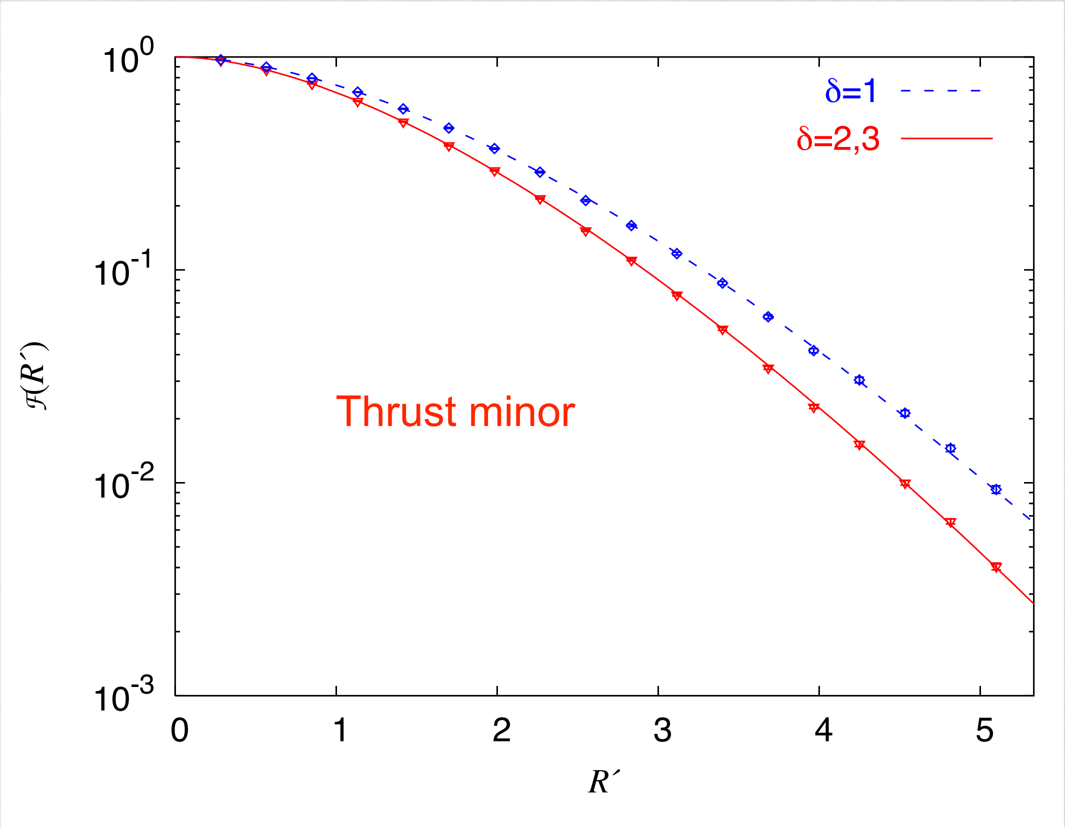
$$V_s(\{\tilde{p}\}, k_1, \ldots, k_n) = \max_i V(\{\tilde{p}\}, k_i)$$



Semi-numerical resummations

- The results of the Monte-Carlo procedure have the same quality as analytical predictions [Banfi Salam Zanderighi '01]





- First NLL resummation for thrust-major distribution and two-jet rate in Durham algorithm, which do not admit analytical representation

- The Monte Carlo we used back then had simplified emissions, without exact energy-momentum conservation, and used both approximated and actual observable subroutines

Automated NLL resummation

- It is possible to perform NLL resummations by just providing to a computer program (CEASAR, the Computer Expert Automated Semi-Analytical Resummer) the observable $V(\{\tilde{p}\}, k_1, \ldots, k_n)$ in the form of a computer subroutine, as happens for fixed-order calculations [Banfi Salam Zanderighi '05]

- Such automation requires understanding the general principles of final-state resummations in QCD

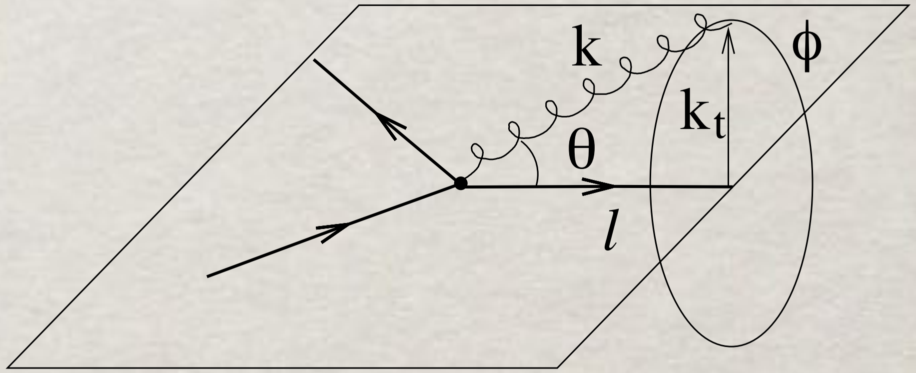
Automated NLL resummation

- Principles of NLL resummation in the CAESAR approach

  [Banfi Salam Zanderighi '05]

  - formulate the needed conditions so that independent soft-collinear emissions lead to correct NLL resummation $\Rightarrow$ recursive infrared and collinear safety
  - compute the Sudakov exponent form the behaviour of an observable in the soft and collinear limit, determined using multiple-precision arithmetics

$$V(\{\tilde{p}\}, k) = d_\ell \left(\frac{k_t^{(\ell)}}{Q} \right)^a e^{-b_\ell \eta^{(\ell)}} g_\ell(\phi^{(\ell)})$$



  - compute multiple-emission effects with a suitable Monte Carlo procedure

Multiple-emission effects

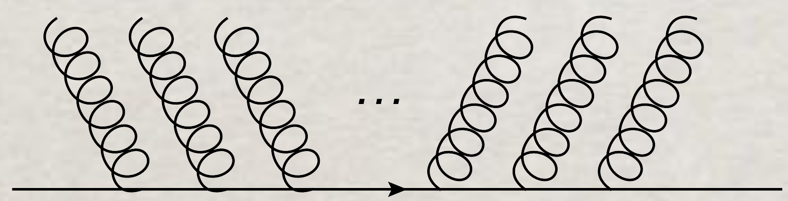
- General master formula for NLL resummation

[Banfi Salam Zanderighi '05]

$$\Sigma(v) = e^{-R_{\mathrm{NLL}}(v)} \mathcal{F}_{\mathrm{NLL}}(v)$$

- Multiple-emission effects at NLL computed by MC simulation of soft and collinear gluons independently emitted in a suitable single-logarithmic region

Measure defined by the soft-collinear ensemble



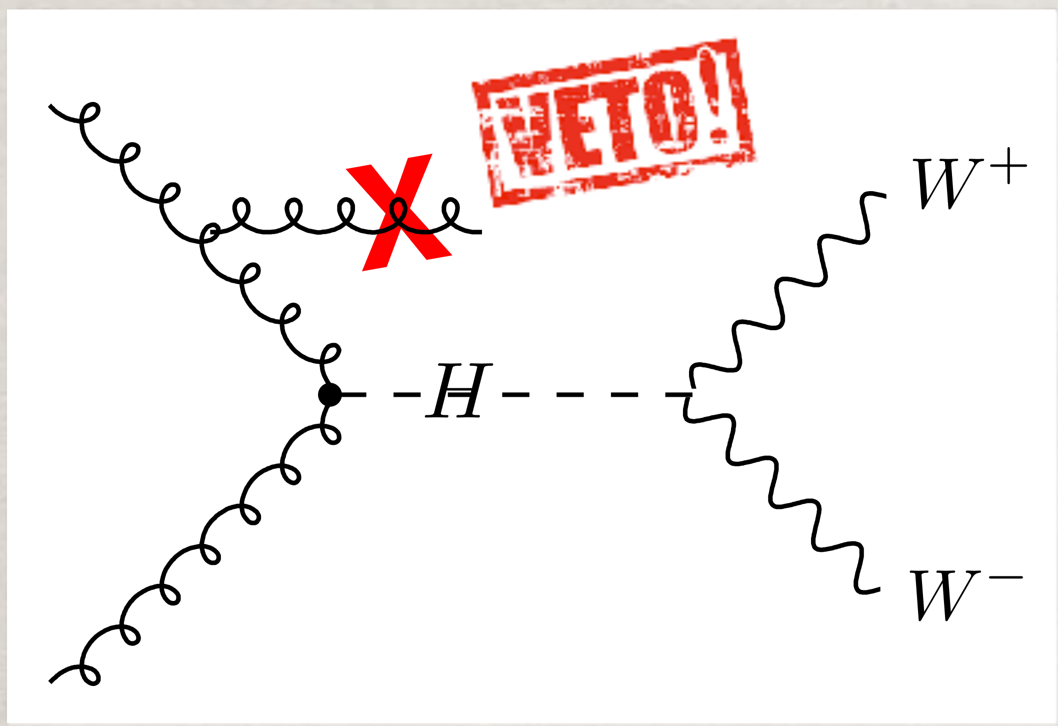
$$= \int d\mathcal{Z}[\{R'_{\ell_i}, k_i\}]$$

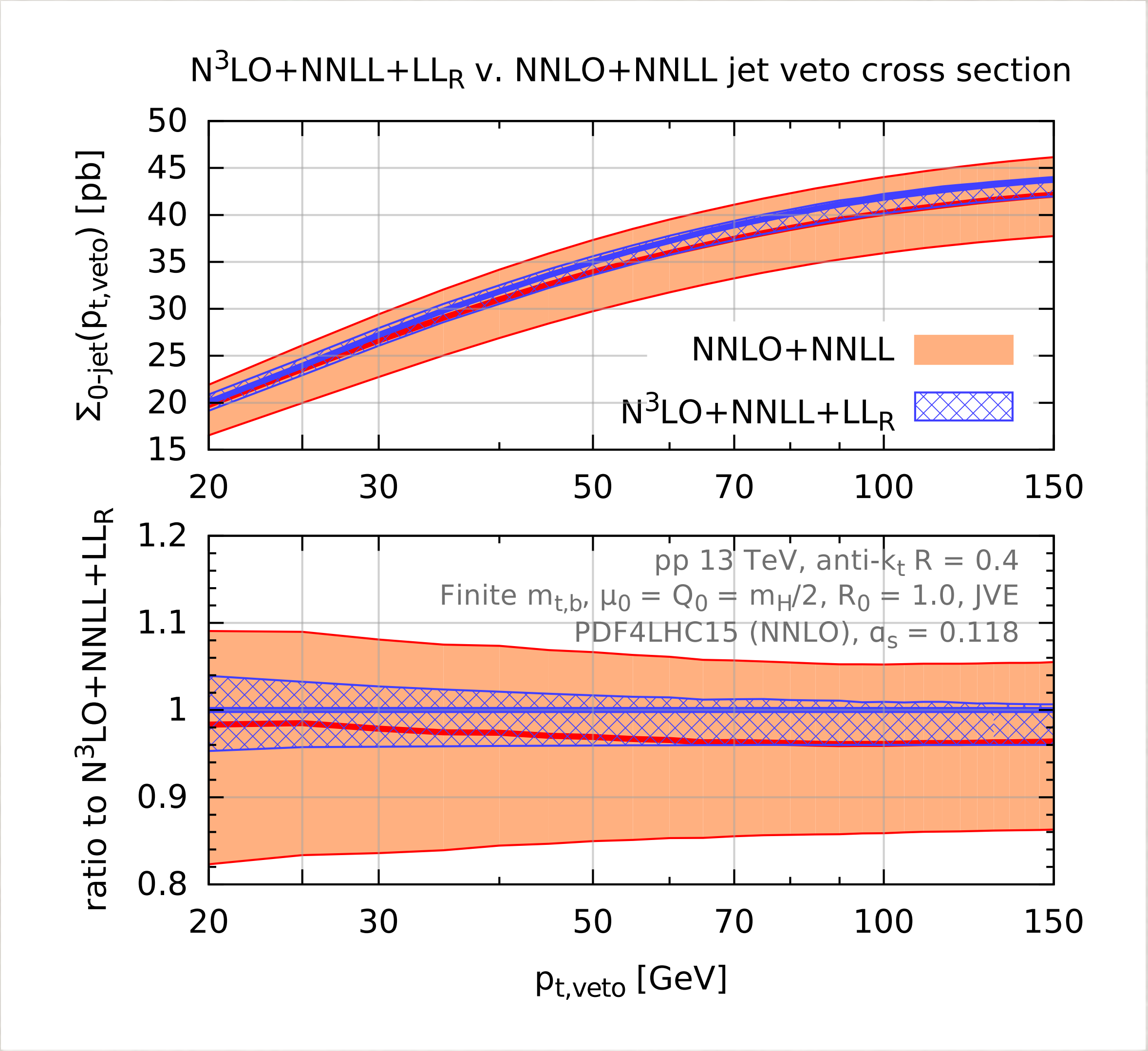
$$\mathcal{F}_{\mathrm{NLL}}(v) = \left\langle \Theta\left(1 - \lim_{v \to 0} \frac{V(\{\tilde{p}\}, \{k_i\})}{v}\right)\right\rangle$$

- CAESAR uses the actual observable's subroutine $V(\{\tilde{p}\}, \{k_i\})$ and eliminates subleading effects by numerically taking the limit $v \to 0$

Jet-veto efficiencies beyond NLL

- The CAESAR philosophy was extended to NNLL to compute cross section for Higgs production without extra jets [Banfi Monni Salam Zanderighi ’12] [Banfi Caola Dreyer Monni Salam Zanderighi Dulat ’16]





The ARES method

- NNLL corrections are often sizeable and important for precision physics
- The Automated Resummer for Event Shapes (ARES) is a novel semi-numerical approach that:
  - is fully general for rIRC safe observables (~ all that can be possibly resummed at NNLL accuracy)
  - is NNLL accurate and extendable to higher orders
  - is flexible and automated
- ARES uses as inputs the observable's subroutine in relevant soft and collinear limits, which have to be taken analytically by the user, e.g.

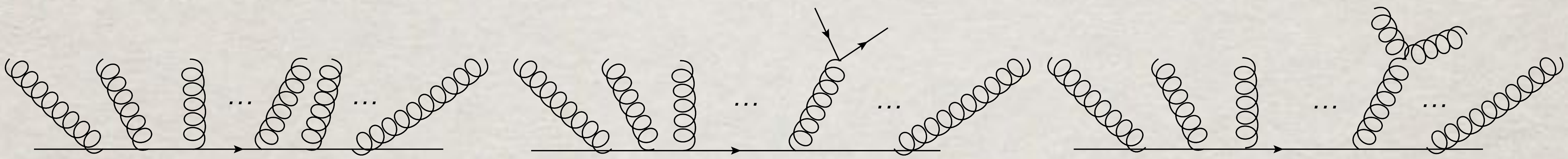
$$\Sigma(v) = e^{-R_{\mathrm{NLL}}(v)}\mathcal{F}_{\mathrm{NLL}}(v)$$

$$\mathcal{F}_{\mathrm{NLL}}(v) = \left\langle \Theta\left(1 - \lim_{v\to 0}\frac{V_{\mathrm{sc}}^{\mathrm{NLL}}(\{\tilde{p}\},\{k_i\})}{v}\right)\right\rangle$$

NNLL resummation

- The NNLL radiator, encoding the cancellation of real and virtual corrections, is known only for observables that scale like the jet mass or jet broadening

$$\Sigma(v) = e^{-R_{\rm NNLL}(v)}\left[\mathcal{F}_{\rm NLL}(v) + \frac{\alpha_s}{\pi}\delta\mathcal{F}_{\rm NNLL}(v)\right]$$

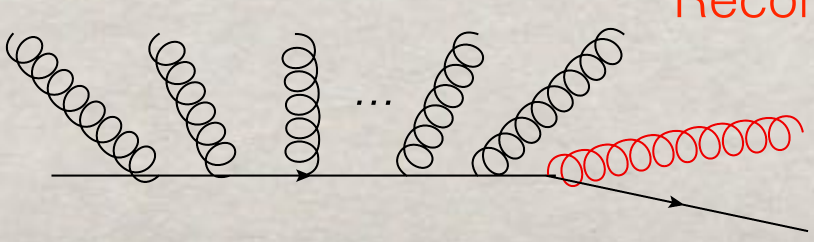


Clustering
(jet algorithms only)

Correlated emission

$$\delta\mathcal{F}_{\rm NNLL} = \delta\mathcal{F}_{\rm clust} + \delta\mathcal{F}_{\rm correl} + \delta\mathcal{F}_{\rm hc} + \delta\mathcal{F}_{\rm rec} + \delta\mathcal{F}_{\rm wa} + \delta\mathcal{F}_{\rm sc}$$

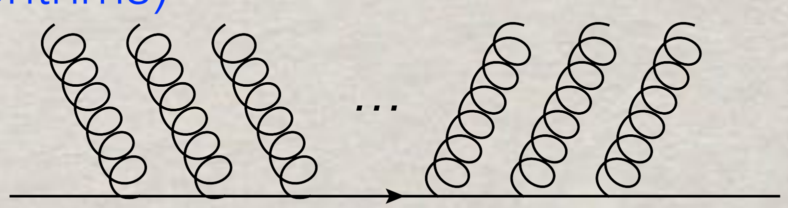
Hard collinear
Recoil



Running coupling
Rapidity
(jet algorithms)

Soft wide-angle

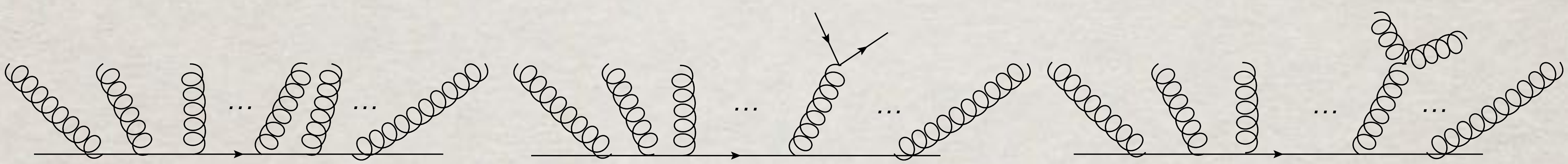
Soft-collinear
(event shapes)



NNLL resummation

- All NNLL corrections can be written in terms of finite integrals in four dimensions

$$\Sigma(v) = e^{-R_{\mathrm{NNLL}}(v)}\left[\mathcal{F}_{\mathrm{NLL}}(v) + \frac{\alpha_s}{\pi}\delta\mathcal{F}_{\mathrm{NNLL}}(v)\right]$$



Clustering
(jet algorithms only)

Correlated emission

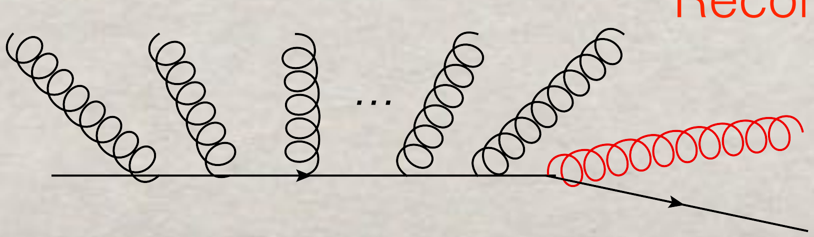
$$\delta\mathcal{F}_{\mathrm{NNLL}} = \delta\mathcal{F}_{\mathrm{clust}} + \delta\mathcal{F}_{\mathrm{correl}} + \delta\mathcal{F}_{\mathrm{hc}} + \delta\mathcal{F}_{\mathrm{rec}} + \delta\mathcal{F}_{\mathrm{wa}} + \delta\mathcal{F}_{\mathrm{sc}}$$

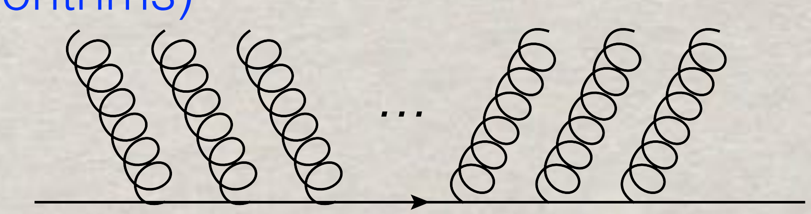
Hard collinear
Recoil

Running coupling
Rapidity
(jet algorithms)

Soft wide-angle

Soft-collinear
(event shapes)



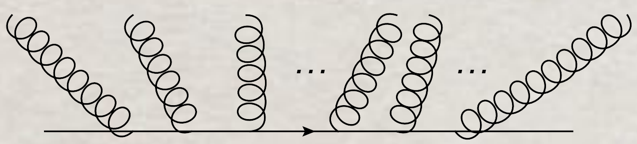


NNLL resummation

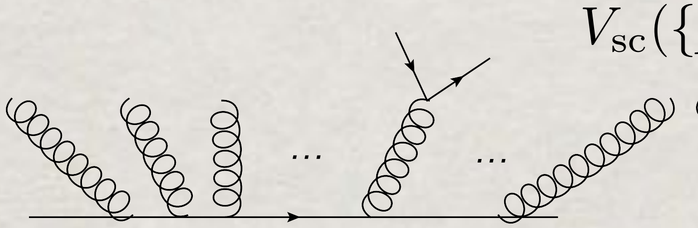
- Every NNLL correction requires to determine an approximate expression for the observable in the relevant kinematic limit

$$\Sigma(v) = e^{-R_{\mathrm{NNLL}}(v)}\left[\mathcal{F}_{\mathrm{NLL}}(v) + \frac{\alpha_s}{\pi}\delta\mathcal{F}_{\mathrm{NNLL}}(v)\right]$$

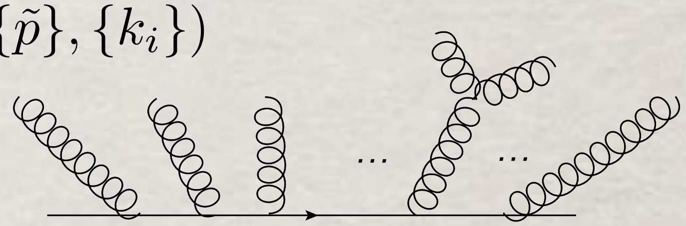
$V_{\mathrm{sc}}(\{\tilde{p}\},\{k_i\}) \neq V_{\mathrm{sc}}^{\mathrm{NLL}}(\{\tilde{p}\},\{k_i\})$



Clustering
(jet algorithms only)



$V_{\mathrm{sc}}(\{\tilde{p}\},\{k_i\})$



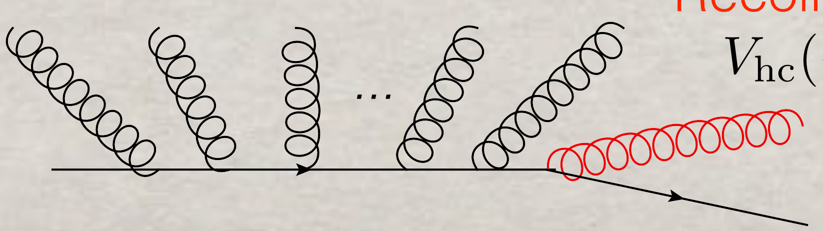
Correlated emission

$$\delta\mathcal{F}_{\mathrm{NNLL}} = \delta\mathcal{F}_{\mathrm{clust}} + \delta\mathcal{F}_{\mathrm{correl}} + \delta\mathcal{F}_{\mathrm{hc}} + \delta\mathcal{F}_{\mathrm{rec}} + \delta\mathcal{F}_{\mathrm{wa}} + \delta\mathcal{F}_{\mathrm{sc}}$$

$V_{\mathrm{sc}}(\{\tilde{p}\},\{k_i\})$

Hard collinear
Recoil

$V_{\mathrm{hc}}(\{\tilde{p}\},\{k_i\})$



$V_{\mathrm{sc}}^{\mathrm{NLL}}(\{\tilde{p}\},\{k_i\})$

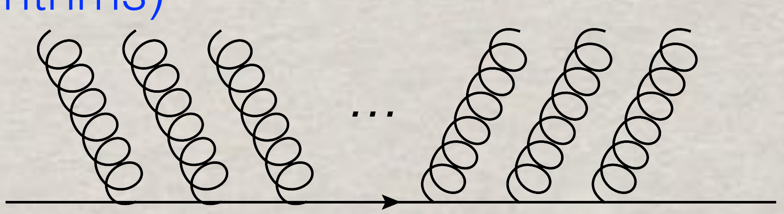
Running coupling
Rapidity
(jet algorithms)

$V_{\mathrm{wa}}(\{\tilde{p}\},\{k_i\})$

Soft wide-angle

$V_{\mathrm{sc}}(\{\tilde{p}\},\{k_i\})$

Soft-collinear
(event shapes)



CAESAR vs ARES

[Banfi Salam Zanderighi '05]

- Establishes the range in which actual emissions can be considered soft and collinear
- Uses the actual observable subroutine and computes its soft-collinear limit numerically
- Requires careful extrapolations to be extended at NNLL

[Banfi McAslan Monni Zanderighi '15]

- Generates emissions that are by construction soft and collinear (no energy-momentum conservation)
- Uses analytically determined soft and collinear limits of each observable
- Can be in principle extended to any logarithmic accuracy

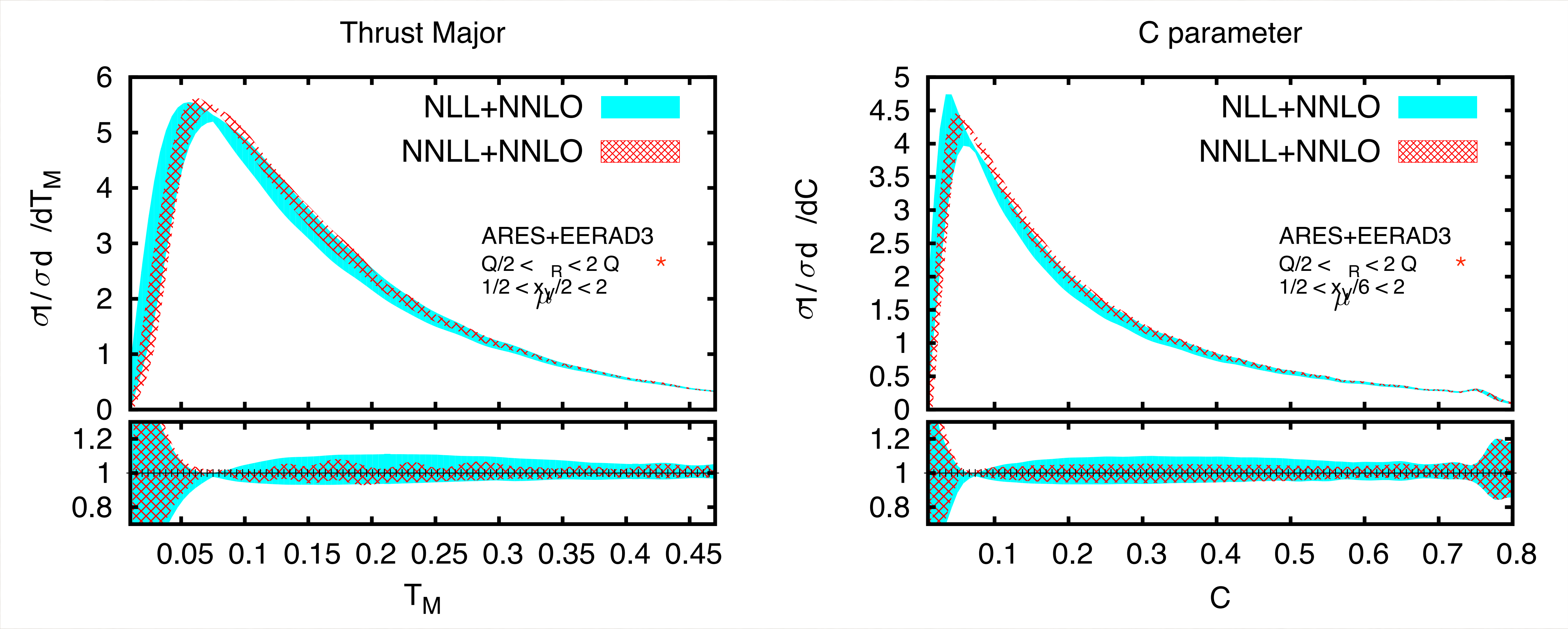
Event-shape variables

- Event-shapes distributions at NNLL matched to exact NNLO

[Banfi McAslan Monni Zanderighi ‘15]

- Reproduced existing results for thrust, heavy-jet mass and broadenings

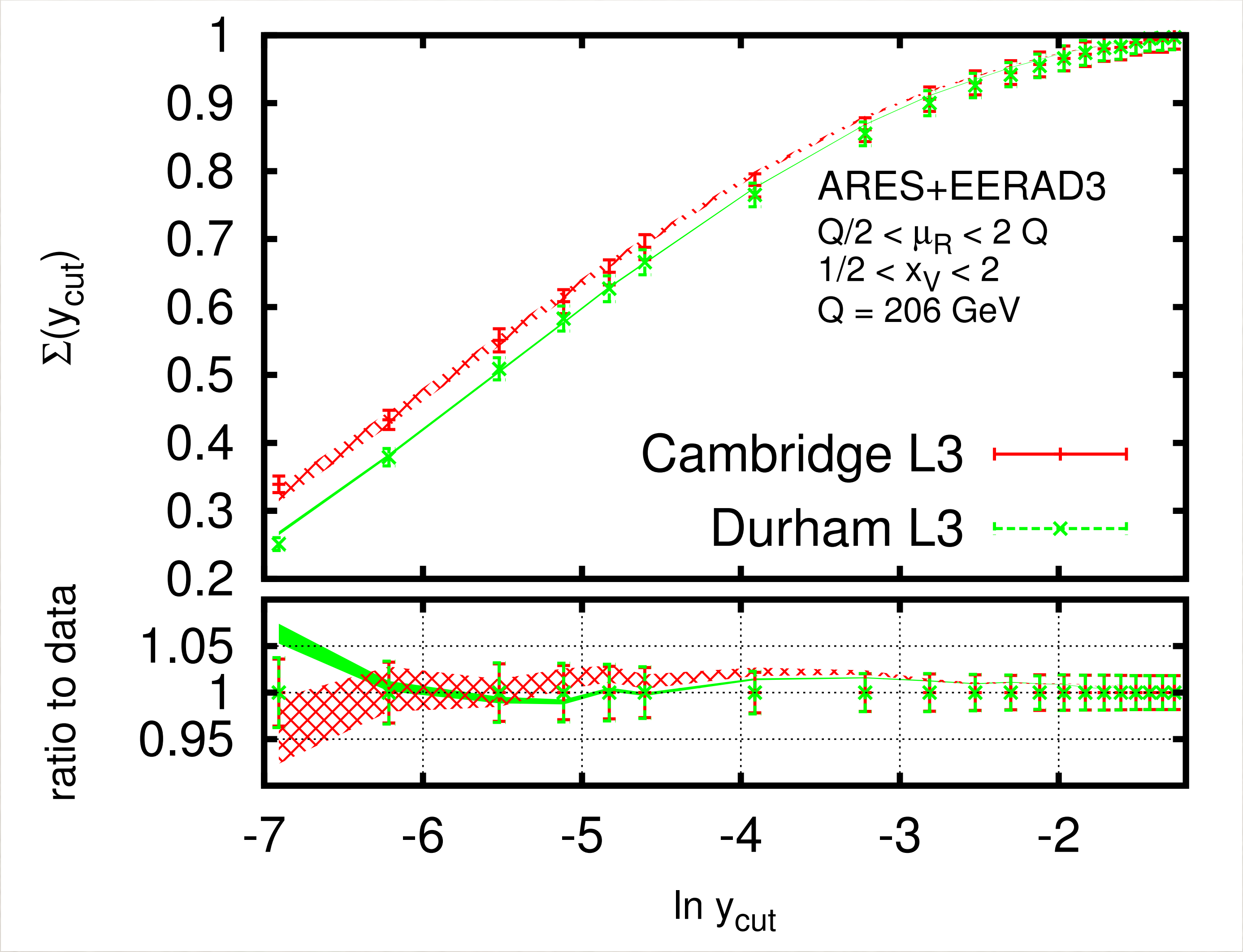
- New results for thrust-major, C-parameter and oblateness



[* Gehrmann-De Ridder Gehrmann Glover Heinrich]

Two-jet rate

- First-ever NNLL resummation of the two-jet rate for the Durham and Cambridge algorithms [Banfi McAslan Monni Zanderighi '16]



- Good agreement with LEP data $\Rightarrow$ fit of $\alpha_s(M_Z)$ in progress

Conclusions

- Recent years have seen an impressive progress towards the understanding of QCD dynamics behind soft-collinear resummations
- These developments encode many of Pino's ideas
  - factorisation properties of soft-emissions
  - QCD coherence and branching algorithms
  - Parton-shower event generators

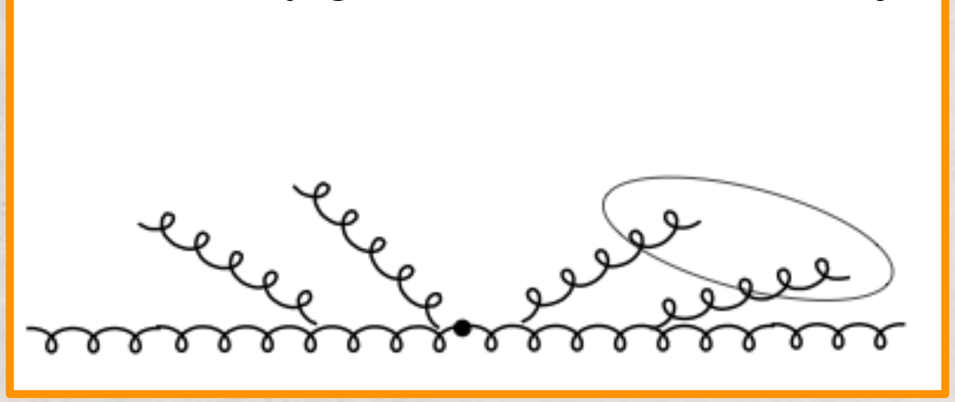
- Work in progress
  - New global fit of α_s from e^+e^- event shapes
  - More NNLL resummations in multi-jet events
- Pino's legacy: use solution of QCD evolution equations as the key ingredient to build the next generation of parton-shower event generators

Extra

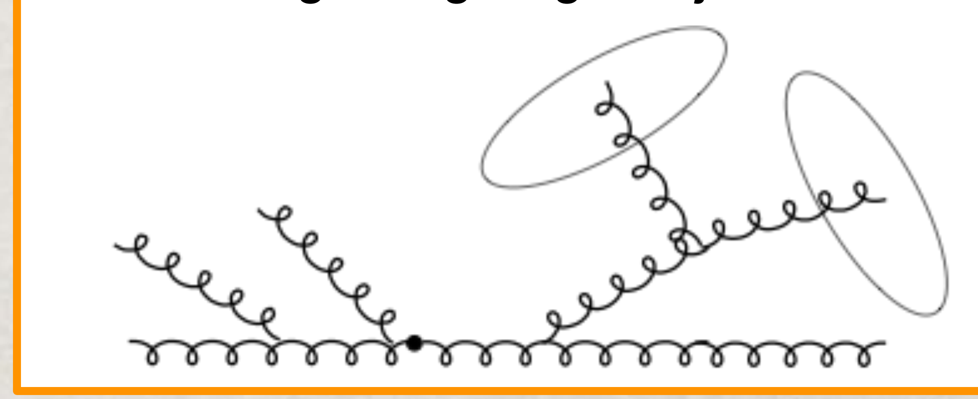
Higgs plus zero jets at NNLL

- The resummed jet-veto efficiency is a Sudakov form factor, with corrections due to the jet algorithm starting only at NNLL [Banfi Monni Salam Zanderighi '12]

Two nearby gluons clustered in one jet

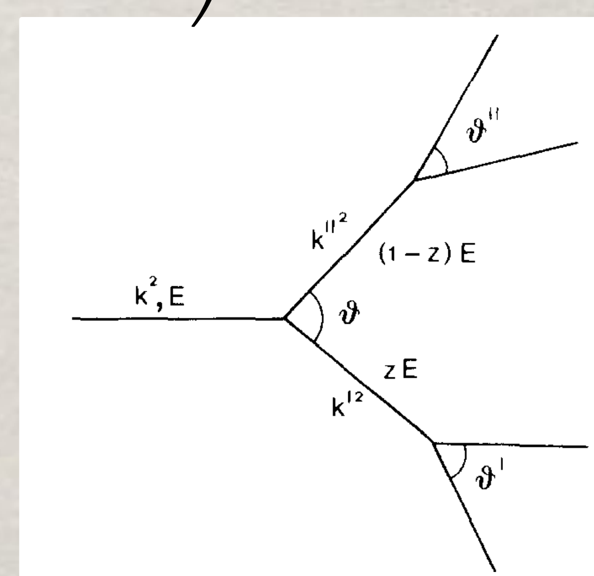


One gluon giving two jets



$$\sigma_{0-\text{jet}} \simeq \mathcal{L}_{gg}(p_{\text{t,veto}}) \left(1 + \underbrace{\alpha_s(p_{\text{t,veto}})\, R'(p_{\text{t,veto}})\, f(R)}_{\text{NNLL}} \right) e^{-R(p_{\text{t,veto}})}$$

- Logarithm of the jet radius appearing in can be again resummed through the Monte-Carlo solution of an evolution equation [Dasgupta Dreyer Salam Soyez '15]



[Bassetto Ciafaloni Marchesini '83]