Five-dimensional gauge theory via holography

Eric D'Hoker
Mani L. Bhaumik Institute for Theoretical Physics
Department of Physics and Astronomy, UCLA

Galileo Galilei Institute for Theoretical Physics, Arcetri, Florence
Supersymmetric Quantum Field Theories in the Non-Perturbative Regime
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Overview

- **Dynamics of branes in M-theory and Type II suggest**
  - *Existence of five-dimensional superconformal fixed points with 16 supercharges possessing Coulomb branch and Higgs branch deformations*
  - *Despite the lack of perturbative renormalizability of Yang-Mills theory*

- **Prior approaches**
  - *Field theory: approach from the Coulomb branch*
  - *D4/D8 branes in massive Type IIA, D5/NS5 brane webs in Type IIB*
  - *Superconformal phase difficult to access in either approach*

- **Holographic approach to the super-conformal phase**
  - *Type IIB supergravity on* $AdS_6 \times S^2$ *warped over Riemann surface* Σ
  - *Obtain exact local solutions to the BPS equations for 16 supersymmetries*
  - *Construct global solutions*
  - *Many open problems*

Bibliography

Key earlier work

- *Five-dimensional SUSY Field Theories, Non-trivial Fixed Points, and String Dynamics*, N. Seiberg, hepth/9608111;
- *Five-Dimensional Supersymmetric Gauge Theories and Degenerations of Calabi-Yau Spaces*, K. Intriligator, D.R. Morrison, N. Seiberg, hepth/9702198;
- *Branes, Superpotentials and Superconformal Fixed Points*, O. Aharony, A. Hanany, hepth/9704170;
- *The D4-D8 Brane System and Five Dimensional Fixed Points*, A. Brandhuber, Y. Oz, arXiv:9905148.

Our papers

- *Warped $AdS_6 \times S^2$ in Type IIB supergravity I: Local Solutions*, ED, Michael Gutperle, Andreas Karch, Christoph F. Uhlemann, arXiv:1606.01254;
- *Holographic duals for five-dimensional superconformal quantum field theories*, ED, Michael Gutperle, Christoph F. Uhlemann, arXiv:1611.09411;
- *Warped $AdS_6 \times S^2$ in Type IIB supergravity II: Global Solutions and five-brane webs*, ED, Michael Gutperle, Christoph F. Uhlemann, arXiv:1703.08186;
- *Warped $AdS_6 \times S^2$ in Type IIB supergravity III: Global Solutions with seven-branes*, ED, Michael Gutperle, Christoph F. Uhlemann, arXiv:1706.00433;

Five-dimensional supersymmetry

- Minimal Poincaré supersymmetry in five dimensions
  - has 2 supersymmetry spinor generators = 8 real supersymmetries
  - and thus $SU(2)_R$ symmetry

- Supermultiplets
  - gauge multiplet $\mathcal{A} = (A_\mu, \lambda_\alpha, \phi)$ with ϕ a real scalar;
  - hypermultiplet $\mathcal{H} = (\psi, H_a)$ with H_a four real scalars;

- Super-conformal symmetry
  - the conformal algebra in $d \geq 3$ dimensions is $SO(2,d)$
  - the superconformal algebra contains $SO(2,d)$, the R-symmetry algebra, and fermionic generators which are spinors under $SO(2,d)$

$d = 3$	$OSp(2m\|4)$	$SO(2,3) = Sp(4,\mathbb{R}),\ m = 1,2,3,4$
$d = 4$	$SU(2,2\|m)$	$SO(2,4) = SU(2,2),\ m = 1,2,3,4$
$d = 5$	$F(4)$	$SO(2,5) \oplus SU(2)$ max bosonic subalgebra
$d = 6$	$OSp(8^*\|2m)$	$SO(2,6) = SO(8^*),\ m = 1,2$

  - maximal 32 supersymmetries in $d = 3,4,6$ but only 16 in $d = 5$.

Five-dimensional supersymmetric gauge theory

- Five-dimensional gauge theory (e.g. $SU(N)$ gauge group)

$$\mathcal{L} \sim g^{-2}\,\mathrm{tr}(F^2) + \frac{c}{24\pi^2}\,\mathrm{tr}(A \wedge F \wedge F + \cdots)$$

  - $[g^{-2}] =$ mass and hence perturbatively non-renormalizable;
  - c quantized in integers by gauge invariance;

- Poincaré supersymmetric theories with gauge and hypermultiplets,
  - Coulomb branch: gauge scalars acquire vevs $\langle \phi \rangle \neq 0$
  - Higgs branch: hypermultiplet scalars acquire vevs $\langle H \rangle \neq 0$

- Reach super-conformal fixed point via Coulomb branch (Seiberg, 1996)
  - generically $SU(N) \to U(1)^{N-1}/\mathrm{Weyl}$
  - $U(1)$ gauge supermultiplets $\mathcal{A}^i = (A^i_\mu, \lambda^i_\alpha, \phi^i)$ with $i = 1, \cdots, N, \ \sum_i \mathcal{A}^i = 0$

The pre-potential

- Dynamics in the Coulomb branch is governed by a pre-potential $\mathcal{F}(\mathcal{A}^i)$
  - bosonic part of the effective Lagrangian dictated by supersymmetry

$$\mathcal{L} \sim \sum_{i,j} \partial_i \partial_j \mathcal{F}(\phi) \Big(F^i F^j + \partial\phi^i \partial\phi^j \Big) + \sum_{i,j,k} \partial_i \partial_j \partial_k \mathcal{F}(\phi) \Big(A^i \wedge F^j \wedge F^k + \cdots \Big)$$

  - Gauge invariance $A^i \to A^i + d\theta^i$ requires $\partial^3 \mathcal{F}$ to be constant.
  - Hence the pre-potential is at most cubic in $\phi^i, \mathcal{A}^i$.

- Exact pre-potential for $SU(N)$ with N_f hypermultiplets in the $\mathbf{N}$ of $SU(N)$

$$\mathcal{F}(\phi) = \frac{1}{2g_0^2} \sum_i \phi_i^2 + \frac{c}{6} \sum_i (\phi_i)^3 + \frac{1}{6} \sum_{i<j} |\phi_i - \phi_j|^3 - \frac{1}{12} \sum_{f=1}^{N_f} \sum_i |\phi_i + m_f|^3$$

  - the bare coupling g_0^2 is a UV cutoff, m_f are hypermultiplet masses.

Dynamics on the Coulomb branch

- Regularity requires the gauge kinetic energy to have positive sign,
  - $\partial_i \partial_j \mathcal{F}$ must be positive for $\phi \in \mathbb{R}^{N-1}/\mathrm{Weyl}$
- For $SU(2)$ gauge group $\phi = \phi_1 = -\phi_2$, and N_f hypermultiplets,

$$\frac{1}{g^2(\phi)} = \frac{1}{g_0^2} + 2\,|\phi| - \frac{1}{4}\sum_{f=1}^{N_f} |\phi - m_f| \qquad\qquad \frac{1}{g^2(\phi)} = \partial^2 \mathcal{F}(\phi)$$

- Regularity $g^2(\phi) > 0$ requires $N_f \leq 7$.
  - $g_0^2 \to \infty$ leaves UV finite theory on the Coulomb branch.
  - Super-conformal fixed point as $\phi, m_f \to 0$ is strongly coupled.
  - Exceptional global symmetries $E_8, E_7, E_6, SO(10), SU(5), \cdots$

Supersymmetric field theories from branes

- Standard cases have maximal supersymmetry
  - 16 Poincaré supercharges
  - in the near-horizon limit enhanced to 32 conformal supercharges

dim	theory	brane	near-horizon	asymptotic symmetry
d=3	M-theory	M2	$AdS_4 \times S^7$	$SO(2,3) \times SO(8)$
d=4	Type IIB	D3	$AdS_5 \times S^5$	$SO(2,4) \times SO(6)$
d=6	M-theory	M5	$AdS_7 \times S^4$	$SO(2,6) \times SO(5)$

- For $d = 5$, superconformal $F(4)$ is unique and has 16 supercharges (8 Poincaré)

- Brane approaches to five-dimensional gauge theory
  - D4 probe brane and parallel D8 branes in massive Type IIA
    (Seiberg, 1996) and (Brandhuber, Oz 1999)
  - D5 intersecting NS5 branes in Type IIB
    (Aharony, Hanany 1997)

- M-theory on 6-dim Calabi-Yau approach to five-dimensional fixed points
    (Morrison, Seiberg, 1996)

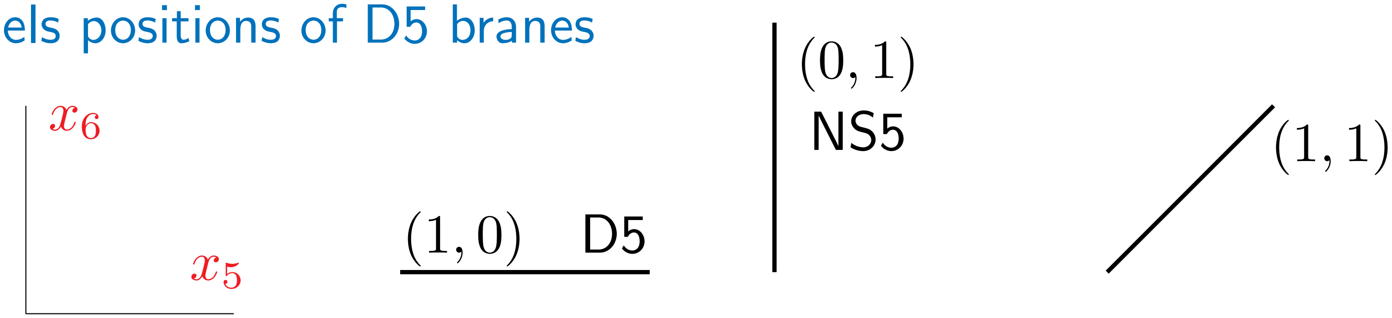
Five-branes in Type IIB string theory

- D5 and NS5 branes intersecting along a five-dimensional space-time

branes	0	1	2	3	4	5	6	7	8	9
D5	×	×	×	×	×	×				
NS5	×	×	×	×	×		×			

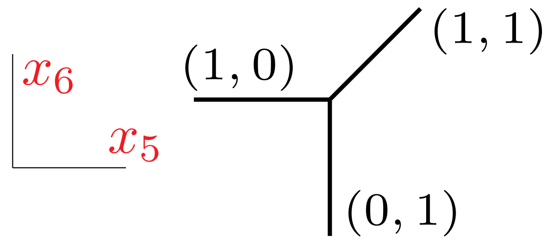
  - Poincaré $ISO(1,4)$ invariant along 01234 parallel directions
  - $SO(3)$ invariant along 789-transverse directions
  - has 8 Poincaré supersymmetries

- D5 and NS5 transform under $SL(2,\mathbb{Z})$ duality of Type IIB (Schwarz 1995)
  - (p,q) five-branes with $p,q \in \mathbb{Z}$
  - x_5 labels positions of NS5 branes,
  - x_6 labels positions of D5 branes

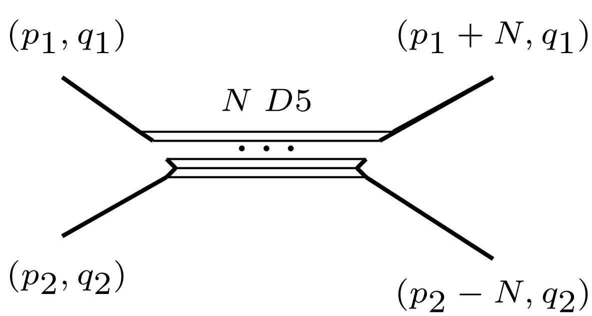


(p,q) **brane webs**

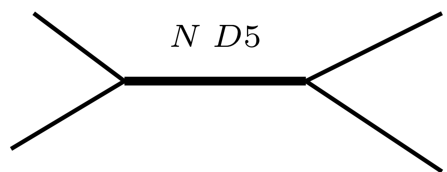
- (p,q)-brane intersections conserve p,q-charges due to $SL(2,\mathbb{Z})$ symmetry



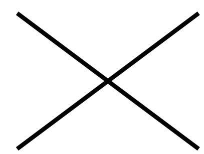
- N parallel D5 branes suspended between two semi-infinite branes
  - non-coincident: $U(1)^{N-1}$ gauge theory plus massive W-bosons
  - coincident: $SU(N)$ gauge theory
  - superconformal: web collapses to a single point



non-coincident



coincident



superconformal

Near-horizon limit

- Take the near-horizon limit of a (p, q) web configuration
  - with a large number N of coincident D5 branes

branes	0	1	2	3	4	5	6	7	8	9
D5	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$				
NS5	$\times$	$\times$	$\times$	$\times$	$\times$		$\times$			

  - radial coordinate in 789 direction combines with 01234 to AdS_6
  - remaining angular directions of 789 give S^2
  - with combined isometries $SO(2,5) \times SO(3)$

- Total space-time geometry

$$AdS_6 \times S^2 \times \Sigma$$

  - where $AdS_6 \times S^2$ is warped over the two-dimensional surface Σ
  - Σ contains the structure of the web in the near-horizon limit

- Our approach: obtain the Type IIB supergravity solutions directly
  - several earlier attempts (with unphysical singularities)

    Lozano et al, 2012; Apruzzi et al, 2014; Kim et al 2015; O'Colgain et al 2015

Type IIB supergravity

- The fields of Type IIB sugergravity are

g_{MN}	metric		
B	axion/dilaton	$P, Q \sim dB$	(contains χ, Φ)
C_2	complex 2-form	G	(contains NSNS, RR)
C_4	real 4-form	F_5	$\star F_5 = F_5$
ψ_M	gravitino	Weyl spinor	
λ	dilatino	Weyl spinor	

- Type IIB supergravity is invariant under global $SL(2,\mathbb{R}) = SU(1,1)$
  - Einstein-frame metric and F_5 are invariant,
  - dilaton/axion B in coset $SU(1,1)/U(1)$, complex C_2 transforms linearly,

$$B \to \frac{uB+v}{\bar{v}B+\bar{u}} \qquad C_2 \to uC_2 + v\bar{C}_2 \qquad |u|^2 - |v|^2 = 1$$

- Bianchi identities and field equations.

Supersymmetric solutions and BPS equations

- Susy variations in Type IIB at vanishing Fermi fields

$$\begin{aligned} \delta\lambda &= iP\cdot\Gamma\,\mathcal{B}^{-1}\varepsilon^* - \frac{i}{4}(G\cdot\Gamma)\varepsilon \\ \delta\psi_M &= D_M\varepsilon + \frac{i}{4}(F_5\cdot\Gamma)\,\Gamma_M\varepsilon - \frac{1}{16}\Big(\Gamma_M(G\cdot\Gamma) + 2(G\cdot\Gamma)\Gamma_M\Big)\mathcal{B}^{-1}\varepsilon^* \end{aligned}$$

– Γ_M are Dirac matrices, $\mathcal{B}$ effects charge conjugation.

– A configuration is supersymmetric if $\delta\psi_M = \delta\lambda = 0$ has solutions with $\varepsilon \neq 0$
– A configuration is half-BPS if there are 16 linearly independent solutions ε

- BPS equations remind of Lax equations in integrable systems

field equations $\Leftrightarrow$ integrability of system of linear differential eqs

– with 32 susys, BPS eqs imply all Bianchi and field equations;
– with ≥ 28 susy, several general results (Gran, Gutowski, Papadopoulos)
– with 16 susys, BPS eqs plus some Bianchi identities imply all the field eqs;

The supergravity Ansatz

- The $SO(2,5) \times SO(3)$ symmetry dictates the space-time structure,

$$AdS_6 \times S^2 \text{ warped over a Riemann surface } \Sigma$$

- The metric and flux fields are restricted by symmetry,

$$\begin{aligned} ds^2 &= f_6^2\, d\hat{s}^2_{AdS_6} + f_2^2\, d\hat{s}^2_{S^2} + ds^2_\Sigma \\ F_3 &= g_a\, e^a \wedge e^6 \wedge e^7 \\ P &= p_a\, e^a \\ Q &= q_a e^a \\ F_5 &= 0 \end{aligned}$$

  - $d\hat{s}^2_{AdS_6}$ and $d\hat{s}^2_{S^2}$ have unit radius "round" metrics;
  - e^A is orthonormal frame, $A = 6, 7$ for S^2 and $A = a = 8, 9$ for Σ
  - $ds^2_\Sigma = e^a \otimes e^b \delta_{ab}$ with $a, b = 8, 9$.

Reducing the BPS equations

- Use Killing spinors on $AdS_6 \times S^2$ as basis for the susy parameter ε,

$$\varepsilon = \sum_{\eta_1,\eta_2} \chi^{\eta_1,\eta_2} \otimes \zeta_{\eta_1,\eta_2}$$

  – χ^{η_1,η_2} fixed basis of Killing spinors, $\eta_1 = \pm$ and $\eta_2 = \pm$ independently;
  – ζ_{η_1,η_2} are 2-component spinors on Σ.

- The BPS equations reduce to a system of 4 spinor equations,

$$\begin{aligned}
0 &= 4p_a \gamma^a \gamma^9 \zeta^* - g_a \tau^3_{(2)} \gamma^a \zeta \\
0 &= -\frac{i}{2f_6} \tau^2_{(1)} \otimes \tau^1_{(2)} \zeta + \frac{D_a f_6}{2f_6} \gamma^a \zeta - \frac{1}{16} g_a \tau^3_{(2)} \gamma^a \gamma^9 \zeta^* \\
0 &= \frac{1}{2f_2} \tau^2_{(2)} \zeta + \frac{D_a f_2}{2f_2} \gamma^a \zeta + \frac{3}{16} g_a \tau^3_{(2)} \gamma^a \gamma^9 \zeta^* \\
0 &= \left(D_a + \frac{i}{2} \omega_a \sigma^3 - \frac{i}{2} q_a \right) \zeta + \frac{3}{16} g_a \tau^3_{(2)} \gamma^9 \zeta^* - \frac{1}{16} g_b \tau^3_{(2)} \gamma_a{}^b \gamma^9 \zeta^*
\end{aligned}$$

  – Derivative D_a and connection ω_a are defined with respect to the frame e^a,
  – $\tau_{(1,2)}$ are Pauli matrices acting on indices $\eta_{1,2}$.

Decoupling the reduced BPS equations

- Algebraic methods used to restrict range of ζ (ED, Estes, Gutperle 2007)

$$\begin{aligned} \bar\alpha &= \zeta_{+++} = -\zeta_{--+} = -i\nu\zeta_{+-+} = +i\nu\zeta_{-++} \qquad \nu^2 = 1 \\ \bar\beta &= \zeta_{---} = +\zeta_{++-} = -i\nu\zeta_{-+-} = -i\nu\zeta_{+--} \end{aligned}$$

- The radii f_6 and f_2 may be obtained algebraically in terms of α, β,

$$f_6 = 3(|\alpha|^2 + |\beta|^2) \qquad\qquad f_2 = -\nu(|\alpha|^2 - |\beta|^2)$$

  – Choose local complex coordinates $(w, \bar w)$ with $e^z = e^8 + ie^9 = \rho\, dw$
  – Use Bianchi identities to express the fields $p_z, q_z, p_{\bar z}, q_{\bar z}$ in terms of B

- Two of the four differential equations may be integrated exactly,

$$\begin{aligned} \rho\bar\alpha^2 &= f(\kappa_+ + B\,\kappa_-) \qquad\qquad \kappa_\pm = \partial_w \mathcal{A}_\pm \\ \rho\bar\beta^2 &= f(\bar B\,\kappa_+ + \kappa_-) \qquad\qquad f^{-2} = 1 - |B|^2 \end{aligned}$$

  – where $\mathcal{A}_\pm$ are arbitrary locally holomorphic functions on Σ.

The secret to integrability

- The remaining reduced equations for $B, \bar{B}, \rho$ are as follows

$$
\begin{aligned}
2\,\partial_w \ln\rho - f^2(\partial_w \bar{B})\,\frac{\kappa_+ + B\kappa_-}{\bar{B}\kappa_+ + \kappa_-} - 2f^2(\partial_w \bar{B})\,e^{+i\vartheta} &= \frac{\bar{B}\partial_w\kappa_+ + \partial_w\kappa_-}{\bar{B}\kappa_+ + \kappa_-} \\
2\,\partial_w \ln\rho - f^2(\partial_w B)\,\frac{\bar{B}\kappa_+ + \kappa_-}{\kappa_+ + B\kappa_-} - 2f^2(\partial_w B)\,e^{-i\vartheta} &= \frac{\partial_w\kappa_+ + B\partial_w\kappa_-}{\kappa_+ + B\kappa_-} \\
(\partial_w B)\,\frac{(\bar\kappa_+ + \bar{B}\bar\kappa_-)^{\frac{3}{2}}}{(B\bar\kappa_+ + \bar\kappa_-)^{\frac{1}{2}}} - (\partial_w \bar{B})\,\frac{(B\bar\kappa_+ + \bar\kappa_-)^{\frac{3}{2}}}{(\bar\kappa_+ + \bar{B}\bar\kappa_-)^{\frac{1}{2}}} + \frac{2\rho^2}{3f^3} &= 0
\end{aligned}
$$

  - where the phase angle ϑ is defined by,

$$
e^{2i\vartheta} = \left(\frac{\kappa_+ + B\kappa_-}{\bar\kappa_+ + \bar{B}\bar\kappa_-}\right)\left(\frac{B\bar\kappa_+ + \bar\kappa_-}{\bar{B}\kappa_+ + \kappa_-}\right)
$$

- This system is actually solvable,
  - Effectively a Lax system on Σ, and thus integrable in principle,
  - Three fields $(B, \bar{B}, \rho)$ version of the sine-Gordon-Liouville-Toda type

Local solutions to the BPS equations

- Metric components of the solution are given as follows,

$$\rho^4 = \frac{R(1+R)(\kappa^2)^3}{|\partial_w \mathcal{G}|^2(1-R)} \qquad f_2^2 = \frac{\kappa^2(1-R)}{\rho^2(1+R)} \qquad f_6^2 = \frac{9\kappa^2(1+R)}{\rho^2(1-R)}$$

  – in terms of the following combinations,

$$\kappa^2 = -|\partial_w \mathcal{A}_+|^2 + |\partial_w \mathcal{A}_-|^2 \qquad R + \frac{1}{R} = 2 + \frac{6\kappa^2 \mathcal{G}}{|\partial_w \mathcal{G}|^2}$$

$$\mathcal{G} = |\mathcal{A}_+|^2 - |\mathcal{A}_-|^2 + \mathcal{B} + \bar{\mathcal{B}} \qquad \partial_w \mathcal{B} = \mathcal{A}_+ \partial_w \mathcal{A}_- - \mathcal{A}_- \partial_w \mathcal{A}_+$$

- $SU(1,1)$ symmetry of Type IIB acts naturally,

$$B \to \frac{uB+v}{\bar{v}B+\bar{u}} \qquad \begin{pmatrix} \mathcal{A}_+ \\ \mathcal{A}_- \end{pmatrix} \to \begin{pmatrix} u & -v \\ -\bar{v} & \bar{u} \end{pmatrix} \begin{pmatrix} \mathcal{A}_+ \\ \mathcal{A}_- \end{pmatrix} \qquad |u|^2 - |v|^2 = 1$$

  – manifestly leaves $\kappa^2, \mathcal{G}$ and thus the Einstein frame metric invariant

- Positive metric functions f_6^2, f_2^2, ρ^4 requires κ^2, $\mathcal{G} > 0$ choosing $0 < R < 1$.

Strategy for global solutions

- Summary of the associated mathematical problem
  - Riemann surface Σ of unknown type (genus ? boundaries ?)
  - Locally holomorphic functions $\mathcal{A}_+, \mathcal{A}_-$ on Σ
    - $\star$ with linear transformation law under $SU(1,1)$ symmetry of Type IIB
    - $\star$ subject to positivity conditions

$$0 \quad < \quad \kappa^2 = -|\partial_w \mathcal{A}_+|^2 + |\partial_w \mathcal{A}_-|^2$$

$$0 \quad < \quad \mathcal{G} = |\mathcal{A}_+|^2 - |\mathcal{A}_-|^2 + \mathcal{B} + \bar{\mathcal{B}}$$

- No (regular) solutions when Σ is compact without boundary,

$$\partial_{\bar{w}} \partial_w \mathcal{G} = -\kappa^2 \qquad \Longrightarrow \qquad \int_\Sigma \kappa^2 = 0$$

- The boundary $\partial\Sigma$ of Σ has vanishing S^2 radius

$$\partial\Sigma : \quad f_2 \to 0 \qquad f_6 \neq 0$$

  - $\partial\Sigma$ is not a boundary of the solution's space-time manifold,
  - $\partial\Sigma$ corresponds to S^2 slice of S^3 cycle,
  - requires $\kappa^2 = \mathcal{G} = 0$ on $\partial\Sigma$.

Inspiration from Electro-statics

- Holomorphic $SU(1,1)$-vector bundles give unproductive hint.
- Map this onto an electro-statics problem.
  - Consider the locally meromorphic ratio λ on Σ (it can have poles)

$$\lambda = \frac{\partial_w \mathcal{A}_+}{\partial_w \mathcal{A}_-} \qquad \kappa^2 = -|\partial_w \mathcal{A}_+|^2 + |\partial_w \mathcal{A}_-|^2$$

    ⋆ in the interior of Σ the condition $\kappa^2 > 0$ requires $|\lambda|^2 < 1$
    ⋆ on the boundary of Σ the condition $\kappa^2 = 0$ requires $|\lambda|^2 = 1$

  - Consider the "electro-static potential"

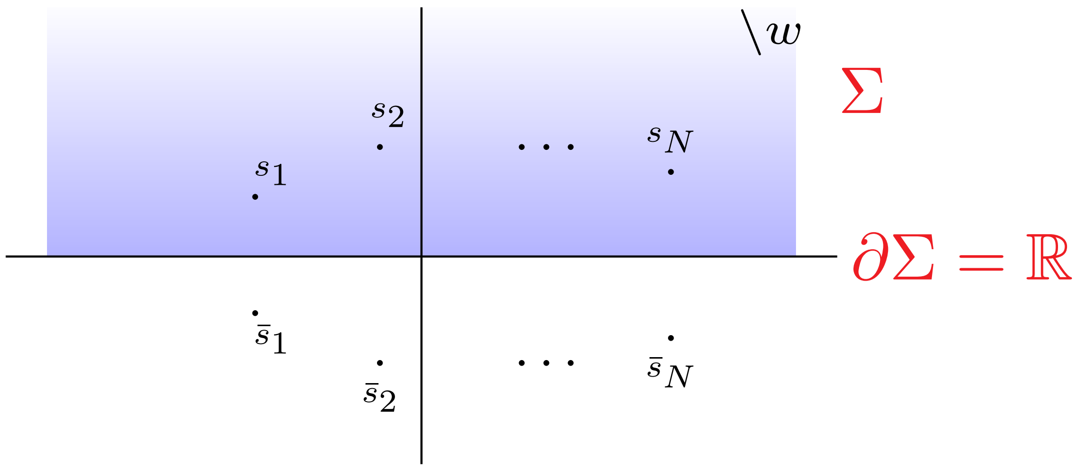
$$\Phi = -\ln |\lambda|^2$$

    ⋆ Φ is real harmonic on Σ
    ⋆ $\Phi > 0$ in the interior of Σ, and $\Phi = 0$ on the boundary of Σ

- Place an array of positive electric charges in the interior of Σ and opposite image charges in the mirror image of Σ

Σ of genus zero and one boundary component

- With a single boundary component, and genus zero,
  - $\partial\Sigma$ may be mapped onto the real line
  - Σ may be mapped onto the upper half plane



  - The general electro-static solution is immediate

$$\Phi(w) = -\ln|\lambda|^2 = -\sum_{n=1}^{N} q_n \Big(\ln|w - s_n|^2 - \ln|w - \bar s_n|^2 \Big) \qquad q_n > 0$$

  - for arbitrary N, q_n, s_n.

Solving for the differentials

- Regularity of the meromorphic function λ requires $q_n = 1$ for all n,

$$\lambda(w) = \prod_{n=1}^{N} \frac{w - s_n}{w - \bar{s}_n}$$

- Assuming $\partial_w \mathcal{A}_\pm$ meromorphic, $\partial_w \mathcal{A}_+ = \lambda \partial_w \mathcal{A}_-$ and regularity require,

$$\partial_w \mathcal{A}_+ = \frac{1}{R(w)} \prod_{n=1}^{N} (w - s_n)$$

$$\partial_w \mathcal{A}_- = \frac{1}{R(w)} \prod_{n=1}^{N} (w - \bar{s}_n)$$

  – $R(w)$ is polynomials with only real roots r_ℓ

$$R(w) = \prod_{\ell=1}^{\deg R} (w - r_\ell)$$

  – real zeros are also allowed but may be viewed as the limit of $\mathrm{Im}(s_n) \to 0$
  – regularity at ∞ requires $\deg R = N + 2$

Satisfying the regularity conditions

- Alternative form of $\partial_w \mathcal{A}_\pm$,

$$\partial_w \mathcal{A}_\pm(w) = \sum_{\ell=1}^{N+2} \frac{Z_\pm^\ell}{w - r_\ell} \qquad\qquad Z_+^\ell = (Z_-^\ell)^* = \frac{1}{P'(r_\ell)} \prod_{n=1}^{N} (r_\ell - \bar{s}_n)$$

  – allows us to integrate up to $\mathcal{A}_\pm$,

$$\mathcal{A}_\pm(w) = \sum_{\ell=1}^{N+2} Z_\pm^\ell \ln(w - r_\ell)$$

  – and to obtain $\mathcal{B}$ in terms of "dilogarithm integrals"

$$\mathcal{B}(w) = \sum_{\ell,\ell'=1}^{N+2} \left(Z_+^\ell Z_-^{\ell'} - Z_+^{\ell'} Z_-^\ell \right) \int_{w_0}^{w} dw \, \frac{\ln(w - r_\ell)}{w - r_{\ell'}}$$

  – judicious choice of branch cuts allows one to show that

$$\mathcal{G} = |\mathcal{A}_+|^2 - |\mathcal{A}_-|^2 + \mathcal{B} + \bar{\mathcal{B}}$$

    - obeys $\mathcal{G} = 0$ on the boundary of Σ
    - obeys $\mathcal{G} > 0$ in the interior of Σ

Asymptotics near pole = near (p,q) five-brane

- The solution is regular everywhere on Σ, except at the poles r_ℓ

$$w = r_\ell + u\, e^{i\theta}$$

- The dilaton diverges and the string-frame metric becomes,

$$ds^2 = (-\ln u) d\hat{s}^2_{AdS_6} + |Z_+^\ell - Z_-^\ell| \left(\frac{du^2}{u^2} + d\hat{s}^2_{S^3} \right)$$

  - AdS_6 expands to infinite radius, by rescaling tends to $\mathbb{R}^6$;
  - (p,q)-charges at the pole given by $p_\ell = \mathrm{Re}(Z_+^\ell)$ and $q_\ell = -\mathrm{Im}(Z_+^\ell)$

- Stack of N coincident NS5 branes produces string frame metric and dilaton,

$$ds^2 = dx^\mu dx_\mu + e^{2\Phi} d\mathbf{y}^2 \qquad\qquad e^{2\Phi(\mathbf{y})} = e^{2\Phi(\infty)} + N/\mathbf{y}^2$$

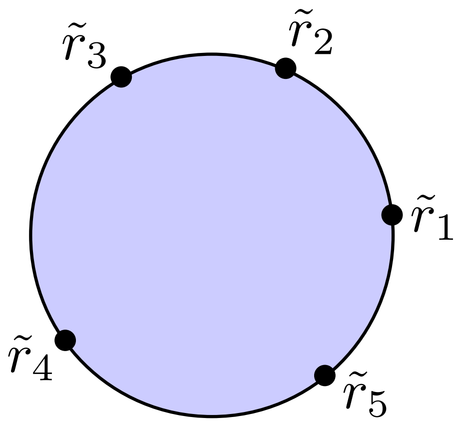
  - x^μ along 5-brane, $\mathbf{y}$ perpendicular to 5-brane, near-horizon $u^2 = \mathbf{y}^2 \to 0$

$$ds^2 \sim dx^\mu dx_\mu + \frac{du^2}{u^2} + ds^2_{S^3} \qquad\qquad e^{2\Phi(\mathbf{y})} \sim N/u^2$$

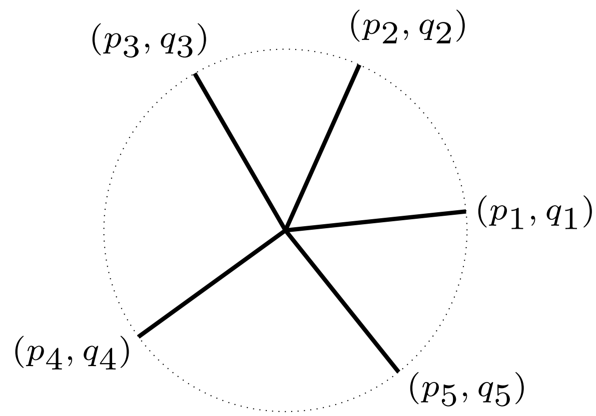
  - agrees with behavior near the poles of our solutions

Poles represent semi-infinite "heavy" branes

- Conformally map the upper half plane to the unit disc;
  - real axis to unit circle
  - points $r_\ell \in \mathbb{R}$ to points $\tilde{r}_\ell$ on unit circle



Riemann surface Σ



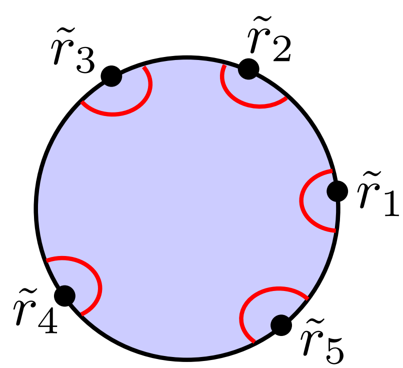
(p, q) five-brane web

Correlators holographically ?

- Key motivation for obtaining our Type IIB supergravity solutions
  - access the superconformal phase of five-dimensional SCFT
  - compute operator dimensions and correlators

- For standard cases, asymptotic region has enhanced symmetry
  - eg asymptotically $SU(2,2|4)$ for asymptotic $AdS_5 \times S^5$
  - In five dimensions superconformal algebra $F(4)$ throughout

- The "heavy" effectively six-dimensional branes are part of the solution (as poles)
  - Effects of warping persist to the holographic boundary
  - For five-dim holography one must prevent access to six-dim regions
  - impose boundary conditions on red "walls" ?



Outlook

- We constructed exactly a wealth of $AdS_6 \times S^2 \times \Sigma$ solutions in Type IIB
  - regular except for expected asymptotics of "heavy" (p,q) branes,
  - precise matching of parameters in brane and supergravity constructions,
  - solutions with D7-branes (ED, Gutperle, Uhlemann arXiv:1706.00433)
  - solutions to the "double analytic continuation" $AdS_2 \times S^6 \times \Sigma$
    (Corbino, ED, Uhlemann, arXiv:1712.04463)

- Largely open questions
  - spectrum of operator dimensions around the solutions ?
    - Entanglement entropy: Gutperle, Marasinou, Trivella, Uhlemann, arXiv: 1708.03404
    - probe (p,q) strings: Kaidi, arXiv: 1708.03404
  - Do these solutions exhibit exceptional global symmetries $E_8, E_7, E_6, \cdots$?
  - Can disc-solutions be extended to global solutions on higher surfaces ?
    - one would hope that such extensions may capture internal five-brane web structure
      ED, Gutperle, Uhlemann tried hard but were not successful (yet ?)