The Higgs branch of $6d$ $\mathcal{N} = (1, 0)$ theories at infinite coupling

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in the Non-perturbative Regime

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Based on the following work:

- [arXiv:1801.01129] with A. Hanany
- [arXiv:1707.05785] with K. Ohmori, H. Shimizu and A. Tomasiello
- [arXiv:1707.04370] with K. Ohmori, Y. Tachikawa and G. Zafrir
- [arXiv:1612.06399] with T. Rudelius and A. Tomasiello

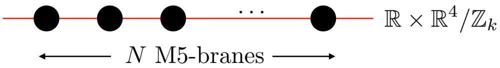
Plan

- $6d$ $\mathcal{N} = (1,0)$ theories on M5-branes on an ADE singularity
- Their T^2 compactification to $4d$ $\mathcal{N} = 2$ theories
- Use lower dimensional theories to learn about the Higgs branch moduli space of $6d$ $\mathcal{N} = (1,0)$ theories at infinite coupling
- Quantify the massless degrees of freedom at the SCFT fixed point of a large class of $6d$ $\mathcal{N} = (1,0)$ theories

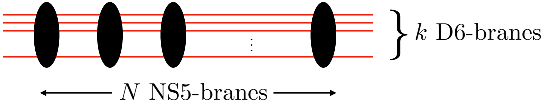
Part I: M5-branes on an ADE singularity

M5-branes on $\mathbb{C}^2/\Gamma_G$ singularity

- The worldvolume theory of N M5-branes on flat space is $6d$ $\mathcal{N} = (2,0)$ theory of Type A_{N-1}
- The presence of $\mathbb{C}^2/\Gamma_G$ breaks half of the amount of supersymmetry $\longrightarrow \quad 6d$ $\mathcal{N} = (1,0)$ theory on the worldvolume
- For $\Gamma_G = \mathbb{Z}_k$, one can conveniently find a description of the worldvolume theory in 2 steps.
  1. Separate the N M5-branes

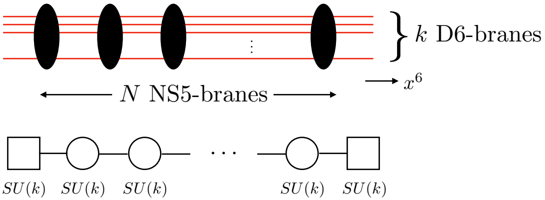


  2. Reduce to the Type IIA theory [Hanany, Zaffaroni '97; Brunner, Karch '97]



A description of the theory on M5-branes on $\mathbb{C}^2/\mathbb{Z}_k$

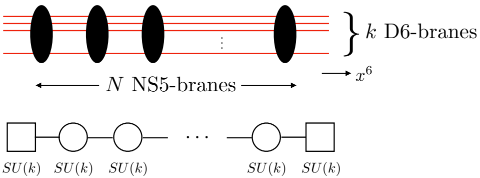
- From the Type IIA set-up, we can write down the quiver description [Hanany, Zaffaroni '97; Brunner, Karch '97; Ferrara, Kehagias, Partouche, Zaffaroni '98]



A circular node $\rightarrow$ an $SU(k)$ vector multiplet
A square node $\rightarrow$ an $SU(k)$ flavour symmetry
A line $\rightarrow$ a bi-fundamental hypermultiplet + a tensor multiplet

- Each hypermultiplet and each tensor multiplet contain a scalar component.
- The scalar VEVs in the h-plet parametrise the **Higgs branch** and those in the t-plet parametrise the **tensor branch** of the moduli space.

Important points

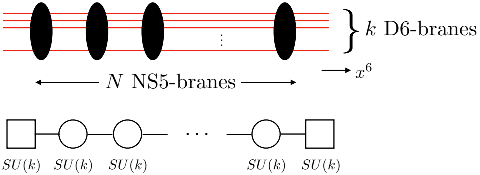


- Each NS5-brane carries a $6d$ $\mathcal{N} = (1,0)$ tensor multiplet (t-plet)
- The position of each NS5-brane in the x^6-direction

  $\equiv$ the VEV of the scalar ϕ in each t-plet
- There are $N-1$ independent t-plets (after fixing the CoM of NS5s)
  - The VEVs of their scalars parametrise the tensor branch of the moduli space
- The gauge coupling $1/g_i^2$ of the i-th gauge group $(i = 1, \dots, N-1)$

  $\equiv$ the relative VEV $\phi_{i+1} - \phi_i$ of the scalars in the adjacent t-plets.

The infinite coupling point: SCFT



- When all NS5-branes are coincident, all gauge couplings become infinity
  - This happens at the origin of the tensor branch, where all $\phi_{i+1} - \phi_i = 0$
  - **Tensionless strings:**

    The D2-branes inside the D6-branes become tensionless

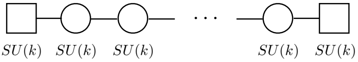
    (the D2-brane $\equiv$ the instanton to the gauge field on the D6-brane)

    $\rightarrow$ a critical point at the origin of the tensor branch
  - Non-trivial physics: This is believed to be an SCFT at infinite coupling

    [Hanany, Ganor '96; Seiberg, Witten '96]

The infinite coupling point: SCFT

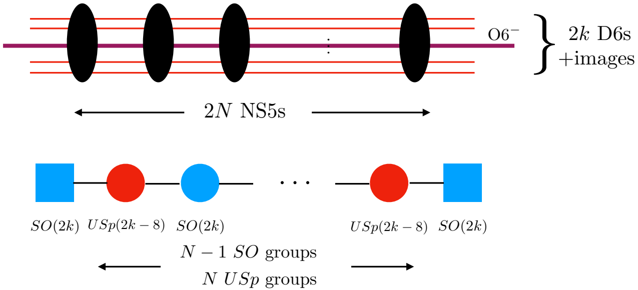
- It should be emphasised that the quiver



  provides a good description at finite coupling
  (*i.e.* generic VEVs of the scalars in the t-plets)
  $\equiv$ generic point of the tensor branch moduli space
- But the physics at infinite coupling may be different from that is described by the quiver!
- *The aim of this talk:*
  - Show that for a number of $\mathcal{N} = (1, 0)$ theories, the Higgs branch at infinite coupling is *different* from that at finite coupling
  - *Quantify* this difference, *e.g.* in terms of the dimensions of the Higgs branches

A description of the theory on M5-branes on $\mathbb{C}^2/\Gamma_G$ (continued)

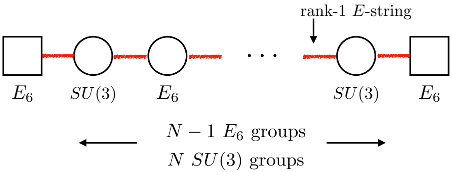
- For $G = SO(2k)$, the Type IIA description is
[Ferrara, Kehagias, Partouche, Zaffaroni '98]



- For $G = E_{6,7,8}$, there's no known Type IIA brane construction. We need a description from F-theory
[Aspinwall, Morrison '97; del Zotto, Heckman, Tomasiello, Vafa '14; etc.]

A description of the theory on M5-branes on $\mathbb{C}^2/\Gamma_G$ (continued)

- For $G = E_6$, the quiver looks something like this



  - The thick red line is *not* a fundamental hyper. It's a $6d$ $\mathcal{N} = (1,0)$ theory by itself, known as the **rank-1 E-string**

    [Hanany, Ganor '96; Seiberg, Witten '96; Morrison, Vafa '96; Witten '96]
  - A **rank-1 E-string** contains 1 tensor multiplet and at the origin of the tensor branch, it's an SCFT with E_8 global symmetry whose Higgs branch $\equiv$ the moduli space of one E_8 instanton
  - Here E_8 decomposes into $E_6 \times SU(3)$

A brief digression on F-theory quivers

- $6d$ theories can be constructed by F-theory on $\mathbb{R}^{1,5} \times$ elliptically fibred CY_3
- The base of the CY_3 is a non-compact complex 2-dimensional space with a collection of 2-cycles $\mathcal{C}^i$
- The size of the curves $\equiv$ the VEVs of the scalars in 6d $\mathcal{N} = (1, 0)$ t-plets
- The configuration of curves is determined by a matrix

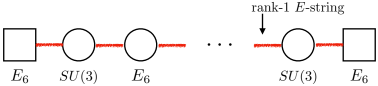
  $$\eta^{ij} = -(\text{the intersection number of } \mathcal{C}^i \text{ and } \mathcal{C}^j)$$

  This gives the kinetic term of tensor multiplets ϕ_i: $\eta^{ij}\partial_\mu\phi_i\partial^\mu\phi_j$
- Shrinking all curves $\mathcal{C}^i$ simultaneously to zero size

  $\Leftrightarrow$ taking the VEVS of the t-plets to zero $\Leftrightarrow$ 6d SCFT

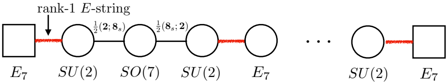
A description of the theory on M5-branes on $\mathbb{C}^2/\Gamma_G$ (continued)

- $G = E_6$:



$$[E_6]\ 1\ \overset{\mathfrak{su}(3)}{3}\ 1\ \overset{\mathfrak{e}_6}{6}\ 1\ \cdots\ 1\ \overset{\mathfrak{su}(3)}{3}\ 1\ [E_6]$$

- $G = E_7$:



$$[E_7]\ 1\ \overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{so}(7)}{3}\ \overset{\mathfrak{su}(2)}{2}\ 1\ \overset{\mathfrak{e}_7}{8}\ \cdots\ \overset{\mathfrak{su}(2)}{2}\ 1\ [E_7]$$

- $G = E_8$:

$$[E_8]\ 1\ 2\ \overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{f}_4}{5}\ 1\ \overset{\mathfrak{g}_2}{3}\ \overset{\mathfrak{su}(2)}{2}\ 2\ 1\ \overset{\mathfrak{e}_8}{12}\ \cdots\ \overset{\mathfrak{su}(2)}{2}\ 2\ 1\ [E_8]$$

M5-branes on $\mathbb{C}^2/\Gamma_G$ (continued)

- In the literature, the theory on N M5-branes on $\mathbb{C}^2/\Gamma_G$ is often referred to as the conformal matter of type (G, G). For $N = 1$, it's a.k.a. the minimal conformal matter. [del Zotto, Heckman, Tomasiello, Vafa '14]
- We have the quiver descriptions at a generic point on the tensor branch of these theories
  - But we want to know the physics at infinite coupling (*e.g.* extra massless degrees of freedom)
  - How do we extract such information from the quivers?

PART II: T^2 compactification

T^2 compactification

Aim: Study the Higgs branch of the $6d$ theory at infinite coupling using $4d$ theories from T^2 compactification

$$\boxed{\begin{array}{c}\text{Description of } 6d\ (1,0) \\ \text{theory at a generic pt.} \\ \text{on the tensor branch}\end{array}} \xrightarrow[\text{tensor branch}]{\text{origin of}} \boxed{\begin{array}{c}6d\ (1,0) \\ \text{SCFT}\end{array}} \xrightarrow{T^2} \boxed{\begin{array}{c}4d\ \mathcal{N}=2 \\ \text{field theory}\end{array}}$$

- The Higgs branch of the $6d$ $\mathcal{N}=(1,0)$ SCFT is the same as the Higgs branch of the $4d$ $\mathcal{N}=2$ theory from the T^2 compactification
- Can use the Higgs branch of the lower dimensional theories (*i.e.* that of the 4d $\mathcal{N}=2$ theory) to learn about the infinite coupling Higgs branch of the 6d theory

T^2 compactification of the min. conformal matter theory

The min. conformal matter of type (G, G) (*i.e.* the SCFT for 1 M5-brane on $\mathbb{C}^2/\Gamma_G$)	$\xrightarrow{T^2}$	A theory of class S of type G assoc. w/ a sphere with two max. punctures and one min. puncture

[Ohmori, Shimizu, Tachikawa, Yonekura (Part I) '15; del Zotto, Vafa, Xie '15]

Use this class S theory to study the infinite coupling Higgs branch of the $6d$ theory

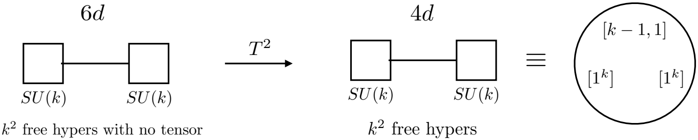
An argument using the chain of dualities

1 M5 on $\mathbb{C}^2/\Gamma_G$ on T^2	$\xrightarrow[\text{\& T-dual}]{\text{Type IIA}}$	Type IIB on $\mathbb{R}^{1,3} \times \mathbb{R} \times S^1 \times \mathbb{C}^2/\Gamma_G$ with the D3 filling $\mathbb{R}^{1,3}$

- Take the low energy limit & ignore the CoM mode of the D3
- Type IIB on $\mathbb{R} \times S^1 \times \mathbb{C}^2/\Gamma_G \quad \to \quad 6d\ (2,0)$ theory of type G on $\mathbb{R} \times S^1$
- The tension of the D3-brane becomes infinite
- The D3-brane $\equiv$ a co-dim.-2 defect of the $\mathcal{N} = (2,0)$ theory of type G

T^2 compactification of the min. conformal matter theory

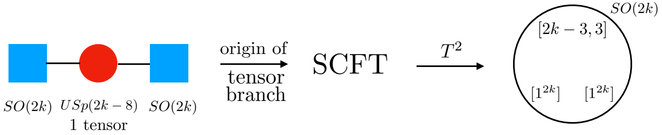
- The two infinities of  $\mathbb{R} \times S^1$ $\equiv$ two maximal punctures
- The $4d$ theory from the T^2 compactification of the $6d$ theory $\equiv$ a theory of class S assoc. w/ a sphere with 2 max. punctures and another puncture of type X
- To fix X, we look at $G = SU(k)$.



- Hence, X is a minimal puncture
- This can be shown more rigorously using geometric engineering

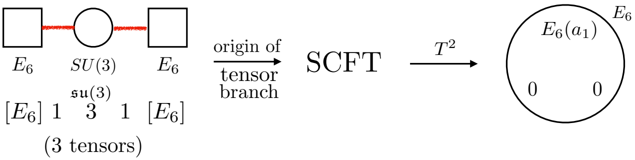
[del Zotto, Vafa, Xie '15]

Example I: 1 M5-brane on $\mathbb{C}^2/\Gamma_{D_k}$ (revisited)



- The Higgs branch dimension as computed from the quiver description:
  $d_{\text{Higgs}}(\text{6d quiver}) = (2k-8)(2k) - \frac{1}{2}(2k-8)(2k-7) = 2k^2 - k - 28$
- The Higgs branch dimension as computed from the *4d class* S *theory*:
  $d_{\text{Higgs}}(\text{4d class S}) = 2k^2 - k + 1 = d_{\text{Higgs}}(\text{6d SCFT})$
- But there is a mismatch of 29 (for all $k \geq 4$):
  $d_{\text{Higgs}}(\text{6d SCFT}) - d_{\text{Higgs}}(\text{6d quiver}) = 29$
- There are 29 extra DoFs on the Higgs branch when we go from a generic point (finite coupling) to the origin of the tensor branch (infinite coupling)
- One tensor multiplet becomes 29 hypermultiplets at infinite coupling

Example II: 1 M5-brane on $\mathbb{C}^2/\Gamma_{E_6}$



- The Higgs branch dimension as computed from the *4d class* S *theory*:

$$d_{\text{Higgs}}(\text{4d class S}) = 79 = d_{\text{Higgs}}(\text{6d SCFT})$$

- In the quiver, there's no hyper whose VEV higgses the gauge group $SU(3)$.
- But if we assume that ALL 3 tensors become 29×3 hypers at the origin of the tensor branch, we obtain the Higgs branch dimension to be

$$(29 \times 3) - 8 = 79 \ ,$$

in agreement with the above $d_{\text{Higgs}}(\text{6d SCFT})$.

General statements

- In the previous examples, we've seen that ALL n_T tensor multiplets become $29n_T$ hypermultiplets at the orgin of the tensor branch.
- This phenomenon is known as the small instanton transition

  [Hanany, Ganor '96; Seiberg, Witten '96; Intriligator '97; Blum, Intriligator '97; Hanany, Zaffaroni ' 97]
  - It was first discussed in the context of M5/M9 brane system
  - When an M5-brane is away from the M9-brane, there's one tensor multiplet (and no hypermultiplet)
  - When the M5 is on top of the M9, this system realises the reduced moduli space of one small E_8 instanton, whose dimension is 29.
  - Indeed, at this point, the E-string, which is an M2-brane, stretching between M5 and M9 becomes tensionless.
  - The tensor multiplet becomes 29 hypermultiplets in this set-up
- However, we'll see below that it's NOT true in general that *all* tensors turn into hypers at infinite coupling. There're cases in which only some of the tensors, or even none, turn into hypers.

Part III: The Higgs branch at infinite coupling

The SCFT Higgs branch dimension

- The **main claim** of this talk is that the Higgs branch dimension of the SCFT is given by [NM, Ohmori, Shimizu, Tomasiello '17]

$$\boxed{d_{\text{Higgs}}(\text{6d SCFT}) = 29 N_{T \to H} + n_H - n_V}$$

  where $N_{T \to H}$ is the number of the tensors that turn into hypers at the origin of the tensor branch:

$$N_{T \to H} = \sum_{i,j=1}^{n_T} \eta_{ij}^{-1} (2 - \eta^{ii})(2 - \eta^{jj}) \ ,$$

  with n_T, n_H, n_V the numbers of tensors, hypers and vectors and η the matrix of the intersection numbers of the curves in the F-theory quiver.
- This formula computes a quantity at the origin of the tensor branch using the information from a generic point of the tensor branch (*i.e.* the F-theory quiver).
- Indeed, we'll later support this formula by an anomaly argument: $N_{T \to H}$ actually comes from the Green-Schwarz-West-Sagnotti term.

Example III: N M5-brane on $\mathbb{C}^2/\Gamma_{D_k}$

- The F-theory quiver for this theory is

$$[SO(2k)] \quad \overset{\mathfrak{usp}(2k-8)}{1} \quad \overset{\mathfrak{so}(2k)}{4} \quad \cdots \quad \overset{\mathfrak{so}(2k)}{4} \quad \overset{\mathfrak{usp}(2k-8)}{1} \quad [SO(2k)] \ ;$$

  there are $n_T = 2N-1$ tensor multiplets.
- The matrix of the intersection numbers is

$$\eta = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 4 & -1 & 0 & 0 \\ & & \ddots & & \\ 0 & -1 & & -1 & 0 \\ 0 & 0 & -1 & 4 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix}$$

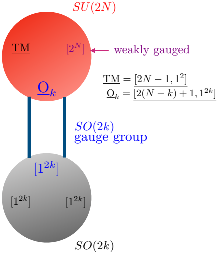
  Hence, $N_{T\to H} = \sum_{i,j=1}^{n_T} \eta_{ij}^{-1}(2-\eta^{ii})(2-\eta^{jj}) = N$
- Out of $2N-1$ tensors, only N tensors turn into hypers at infinite coupling

Example III: N M5-branes on $\mathbb{C}^2/\Gamma_{D_k}$

- The Higgs branch dimension at infinite coupling is

$$\begin{aligned} d_{\text{Higgs}}(\text{6d SCFT}) &= 29N_{T\to H} + n_H - n_V \\ &= 29N + 2N(2k)(2k-4) - \Big[(N-1)k(2k-1) \\ &\qquad + N(k-1)(2k-7)\Big] \\ &= N + \frac{1}{2}(2k)(2k-1) \end{aligned}$$

- This can be checked against the Higgs branch dimension of the $4d$ theory from T^2 compactification [Ohmori, Shimizu, Tachikawa, Yonekura (Part II) '15; Ohmori '16]



- The resulting $4d$ theory is

$$\frac{\mathsf{S}_{SU(2N)}\{\underline{\mathrm{TM}}, [2^N], \underline{\mathrm{O}_k}\} \times \mathsf{S}_{SO(2k)}\{[1^{2k}], [1^{2k}], [1^{2k}]\}}{SU(N) \times \mathrm{diag}(SO(2k) \times SO(2k))}$$

- The Higgs branch dimension is

$$d_{\text{Higgs}}(\text{4d theory}) = N + \frac{1}{2}(2k)(2k-1)$$

Matching of certain anomaly coefficients

- Why are we able to use the effective description at finite coupling to compute a quantity at infinite coupling?

$$\boxed{d_{\mathsf{Higgs}}(\text{6d SCFT}) = 29N_{T\to H} + n_H - n_V}$$

- This is because we can match the anomaly coefficients γ and δ in

$$I_8 = \alpha c_2(R)^2 + \beta c_2(R)p_1(T) + \gamma p_1(T)^2 + \delta p_2(T)$$

between the starting point and the end point of this diagram:

$$\text{6d quiver} \xrightarrow{\text{origin}} \text{6d SCFT} \xrightarrow{\text{Higgs flow}} d_{\mathsf{Higgs}}(\text{6d SCFT}) \text{ hypers} + \mathfrak{n} \text{ tensors}$$

  - Matching δ gives

$$d_{\mathsf{Higgs}}(\text{6d SCFT}) + 29\mathfrak{n} = 29n_T + n_H - n_V$$

  - Matching γ gives [Green, Schwarz, West '85; Sagnotti '92]

$$n_T = n_{\mathsf{GSWS}} + \mathfrak{n} = \sum_{i,j=1}^{n_T} \eta_{ij}^{-1}(2-\eta^{ii})(2-\eta^{jj}) + \mathfrak{n}$$

where $\quad n_{\mathsf{GSWS}} = \sum_{i,j=1}^{n_T} \eta_{ij}^{-1}(2-\eta^{ii})(2-\eta^{jj}) = N_{T\to H}$

Conclusions

- In general, the Higgs branch at infinite coupling can be different from that at finite coupling.
- A certain number of tensor multiplets become hypermultiplets at the origin of the tensor branch. We have quantified how many.
- The Higgs branch dimension of the SCFT at the infinite coupling point can be computed using the quiver data at a generic point of tensor branch.
- **Applications:**
  - $\mathbb{T}^3$ compactification to 3d $\mathcal{N} = 4$ theories & mirror symmetry
  - T-brane theories
  - Theories associated with (partially or completely) frozen singularities

Backup slides

Various applications

Application I: $\mathbb{T}^3$ compactification to 3d $\mathcal{N}=4$ theories

$$\boxed{\begin{array}{c} 6d\ (1,0) \\ \text{SCFT} \end{array}} \xrightarrow{\mathbb{T}^3} \boxed{\begin{array}{c} 3d\ \mathcal{N}=4 \\ \text{field theory} \end{array}} \xrightarrow{\text{mirror sym.}} \boxed{\begin{array}{c} 3d\ \mathcal{N}=4 \text{ theory } (\mathcal{T}), \\ \text{possibly with a Langrangian} \\ \text{description} \end{array}}$$

- The Coulomb branch of $\mathcal{T}$ = The Higgs branch of the 6d SCFT
- $\dim_{\mathbb{H}}$ Higgs$(\mathcal{T})$ $=$ $n_T +$ total rank of the gauge groups in 6d theory
- Some new theories $\mathcal{T}$ for conformal matter theories (N M5s on $\mathbb{C}^2/\Gamma_G$)
  - For $G = SU(k)$ $(\Gamma_G = \mathbb{Z}_k)$, $\mathcal{T}$ is [Hanany, Zafrir '18]

$$\begin{array}{ccccccccccccccccc} &&&&&&&& \overset{\cap}{\underset{N}{\circ}} &&&&&&&& \\ &&&&&&&& | &&&&&&&& \\ \underset{1}{\circ} & - & \underset{2}{\circ} & - & \cdots & - & \underset{k-1}{\circ} & - & \underset{k}{\circ} & - & \underset{k-1}{\circ} & - & \cdots & - & \underset{2}{\circ} & - & \underset{1}{\circ} \end{array}$$

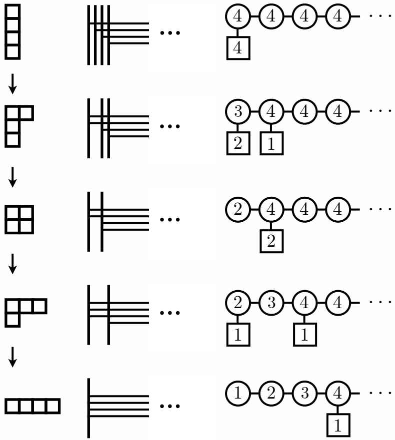
  - For $G = SO(2k)$ $(\Gamma_G = D_k)$, $\mathcal{T}$ is [Hanany, NM '18]

$$\begin{array}{ccccccccccccccccccccccccc} &&&&&&&&&&&& \overset{A'}{\overset{\cap}{\underset{2N}{\color{blue}\bullet}}} &&&&&&&&&&&& \\ &&&&&&&&&&&& | &&&&&&&&&&&& \\ \underset{2}{\color{red}\bullet} & - & \underset{2}{\color{blue}\bullet} & - & \underset{4}{\color{red}\bullet} & - & \underset{4}{\color{blue}\bullet} & - & \cdots & - & \underset{2k-2}{\color{red}\bullet} & - & \underset{2k-2}{\color{blue}\bullet} & - & \underset{2k}{\color{red}\bullet} & - & \underset{2k-2}{\color{blue}\bullet} & - & \underset{2k-2}{\color{red}\bullet} & - & \cdots & - & \underset{4}{\color{blue}\bullet} - \underset{4}{\color{red}\bullet} - \underset{2}{\color{blue}\bullet} - \underset{2}{\color{red}\bullet} \end{array}$$

$\underset{m}{\color{blue}\bullet} = USp(m)$, $\underset{n}{\color{red}\bullet} = SO(n)$, and A' is an rank 2 antisymmetric traceless hyper

Application II: T-brane theories

- Start with a theory on M5-branes on $\mathbb{C}^2/\Gamma_G$: Flavour symmetry $G \times G$
- Can turn on the nilpotent VEV to Higgs each flavour symmetry G
- **Example:** The case of $G = SU(4)$ [Kraft, Procesi '82; Gaiotto, Witten '08; del Zotto, Heckman, Tomasiello, Vafa '14; Heckman, Rudelius, Tomasiello '14; Cabrera, Hanany '16, '17]



Application II: T-brane theories (continued)

- The Higgsing is labelled by a nilpotent orbit Y of G
  - For $G = SU(k)$, Y is specified by a partition of k
  - For $G = SO(2k)$, Y is specified by a D-partition of $2k$
  - For G an exceptional group, Y is specified by a Bala-Carter label
- Suppose that we Higgs $G \times G$ with the orbit Y_L for the first G and with the orbit Y_R for the second G.
  - The resulting theory is known as a **T-brane theory**, $T_G(Y_L, Y_R)$
  - **Example:** $G = SU(4)$, $Y_L = [2, 1^2]$ **and** $Y_R = [2^2]$

$$\underset{[SU(2)]}{\overset{\mathfrak{su}_3}{2}} \quad \underset{[N_f=1]}{\overset{\mathfrak{su}_4}{2}} \quad \overset{\mathfrak{su}_4}{2} \quad \cdots \quad \overset{\mathfrak{su}_4}{2} \quad \underset{[SU(2)]}{\overset{\mathfrak{su}_4}{2}} \quad \overset{\mathfrak{su}_2}{2}$$

  - **Example:** $G = E_6$, $Y_L = E_6$ **(principal orbit) and** $Y_R = 0$ **(trivial orbit)**

$$2 \;\; \overset{\mathfrak{su}_2}{2} \;\; \overset{\mathfrak{g}_2}{3} \;\; 1 \;\; \overset{\mathfrak{f}_4}{5} \;\; 1 \;\; \overset{\mathfrak{su}_3}{3} \;\; 1 \;\; \overset{\mathfrak{e}_6}{6} \;\; 1 \;\; \overset{\mathfrak{su}_3}{3} \;\; 1 \;\; \ldots \;\; [E_6]$$

Application II: T-brane theories (continued)

- The Higgs branch dimension at infinite coupling of $T_G(Y_L, Y_R)$ is

$$\boxed{d^{\text{CFT}}_{\text{Higgs}}\ T_G(Y_L, Y_R) = \mathfrak{n} + \dim(G) + 1 - d_{Y_L} - d_{Y_R}}$$

  - $\mathfrak{n} = \#$ of the (-2)-curves after blowing down all (-1)-curves
    - **Blowing down a (-1)-curve:** $x\ 1\ y\ \to\ (x-1)\ (y-1)$
    - **Field theoretically:** No matter how we try to higgs the theory at a generic point of tensor branch, there still remain $\mathfrak{n}$ tensor multiplets which remain un-higgsed.
  - d_{Y_L}, d_{Y_R} are the dimension of the orbits Y_L and Y_R
  - Here, $N_{T\to H} = n_T - \mathfrak{n}$, and
    $29n_T + n_H - n_V = 30\mathfrak{n} + \dim(G) + 1 - d_{Y_L} - d_{Y_R}$

- **Example.** $G = E_6$, $Y_L = E_6$ and $Y_R = 0$:

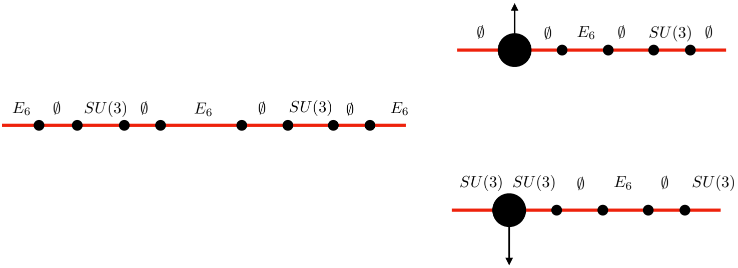
$$2\ \overset{\mathfrak{su}_2}{2}\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{f}_4}{5}\ 1\ \overset{\mathfrak{su}_3}{3}\ 1\ [E_6]$$

  - Blow down the (-1)-curves:
    $22315131 \quad \to \quad 2224131 \quad \to \quad 222321 \quad \to \quad 22231 \quad \to \quad 2222$
  - We have $\mathfrak{n} = 4$, $\dim(G) = 78$, $d_{Y_L} = 36$, $d_{Y_R} = 0$
  - $d^{\text{CFT}}_{\text{Higgs}}\ T_{E_6}(E_6, 0) = 4 + 78 + 1 - 36 - 0 = 47$

Application III: Frozen singularities

[de Boer, Dijkgraaf, Hori, Keurentjes, Morgan, Morrison, Sethi '01; Atiyah, Witten '01; Tachikawa '15]

- One can combine fractional M5-branes on a singularity in different ways
- **Example:** 2 M5-branes on $\mathbb{R} \times \mathbb{R}^4/\Gamma_{E_6}$.
  Each of the individual fractions is $1/4$ an ordinary M5



- From the E_6 conformal matter theory, we can obtain

$$[1] \overset{\mathfrak{su}(3)}{3} \; 1 \; \overset{\mathfrak{e}_6}{6} \; [1]\,, \qquad\qquad [SU(3)] \; 1 \; \overset{\mathfrak{e}_6}{6} \; 1 \; [SU(3)]\,.$$

  - In the first case, E_6 is said to be completely frozen to $G_{\text{fr}} = \{1\}$
  - In the second case, E_6 is said to be partially frozen to $G_{\text{fr}} = SU(3)$

Application III: Frozen singularities (continued)

- The Higgs branch dimension at infinite coupling is

$$\boxed{\dim^{\mathsf{CFT}}_{\mathsf{Higgs}}\ \mathcal{T}_{G\to G_{\mathsf{fr}}} = \mathfrak{n} + \dim(G_{\mathsf{fr}}) + 1}$$

- Let's focus on the minimal case: $\mathfrak{n} = 0$ (*i.e.* the case of a single M5-brane)
- When G_{fr} is trivial (G is completely frozen), the Higgs branch dim. is 1
  - The Higgs branch is $\mathbb{C}^2/\Gamma_G$
  - When $\mathcal{T}_{G\to\emptyset}$ compactified on T^3 to 3d, the Coulomb branch dim. is $h^\vee_G - 1$. This is equal to (# tensors + total rank of the gauge groups) in $\mathcal{T}_{G\to G_{\mathsf{fr}}}$
  - $\mathcal{T}_{G\to\emptyset} \quad \xrightarrow{T^3} \quad$ 3d $\mathcal{N} = 4$ quiver theory given by an affine Dynkin diagram of G with unitary gauge groups of ranks equal to the Coxeter labels
  - **Example:** $G = E_6$

$$\begin{array}{ccccccccc} & & & & \circ\, U(1) & & & & \\ & & & & | & & & & \\ & & & & \circ\, U(2) & & & & \\ & & & & | & & & & \\ \underset{U(1)}{\circ} & - & \underset{U(2)}{\circ} & - & \underset{U(3)}{\circ} & - & \underset{U(2)}{\circ} & - & \underset{U(1)}{\circ} \end{array}$$

with an overall $U(1)$ modded out

Application III: Frozen singularities (continued)

- **Another application:** "New" conformal matter theories of type (G, G) with G non-simply-laced. For example, starting from one M5 on $\mathbb{C}^2/\Gamma_{E_8}$

$$[E_8]\ 1\ 2\ \overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{f}_4}{5}\ 1\ \overset{\mathfrak{g}_2}{3}\ \overset{\mathfrak{su}(2)}{2}\ 2\ 1\ [E_8]$$

one can obtain the following (G_2, G_2) and (F_4, F_4) conformal matter theories by partially freezing E_8:

$$[G_2]\ \overset{\mathfrak{su}_2}{2}\ 2\ 1\ \overset{\mathfrak{e}_8}{12}\ 1\ 2\ \overset{\mathfrak{su}_2}{2}\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{f}_4}{5}\ 1\ [G_2]\ ,$$

$$[F_4]\ 1\ \overset{\mathfrak{g}_2}{3}\ \overset{\mathfrak{su}_2}{2}\ 2\ 1\ \overset{\mathfrak{e}_8}{12}\ 1\ 2\ \overset{\mathfrak{su}_2}{2}\ \overset{\mathfrak{g}_2}{3}\ 1\ [F_4]$$