Dark Quarkonium Formation in the Early Universe BSM: Where Do We Go from Here? Galileo Galilei Institute, Arcetri, IT

Gabriel Lee

Cornell University/Korea University JHEP 1806 (2018) 135 [arXiv:1802.07720] with M. Geller, S. Iwamoto, Y. Shadmi, O. Telem

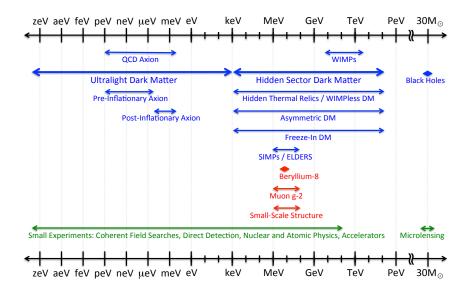
Sep 13, 2018

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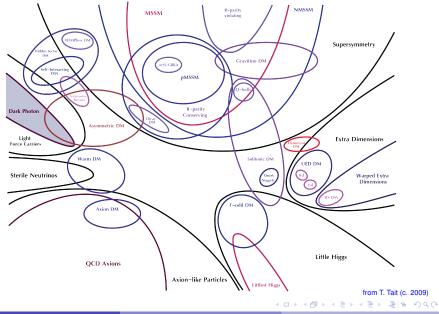
Dark Quarkonium Formation in the Early Universe

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DM Mass Parameter Space



Tim Tait's DM Venn Diagram



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Beyond One-Particle Dark Sectors

Motivations:

- asymmetric DM (connecting DM to baryon asymmetry),
- excited or inelastic DM (novel methods for direct detection),
- different cosmology (more possibilities for obtaining present relic density).

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Confining dynamics give us nontrivial spectra!

- Generic in dark matter models with nonabelian hidden sectors.
- Below some confinement scale, "coloured" particles must combine to form singlets.
- The hadrons can now have qualitatively different interactions than the constituents (at the very least, nonperturbative).
- Generic in BSM model building, e.g., strong dynamics, mirror sectors,

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"Colour"-Singlet States in Nonabelian Dark Matter

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►	Quirks (heavy fund	amentals)	Kribs, Roy, Terning, Zurek 0909.2034
►	Mesons and baryo	ns (light fundamental	Kang, Luty, Nasri 0611322; Appelquist et al. 1503.04203
►	Baryons (heavy fur	ndamentals)	Harigaya et al. 1606.00159; Mitridate et al. 1707.05380
►	R-hadrons (heavy a	adjoints)	Arvanitaki et al. 0812.2075; Feng & Shadmi 1102.0282
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►	Nuclear DM	e.g., Hardy et al. 1504.05419; W	ise & Zhang 1407.4121 + An 1604.01776; Gresham, Hou, Zurek 1707.02313
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Parameter space in (coupling, mass) varies widely!

Cosmology deviates from standard freeze-out scenario.

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Second-Stage Annihilation

Cosmology deviates from standard freeze-out scenario.

- Below confinement scale, bound states have finite size dictated by the light coloured states ("brown muck").
- Hadrons can undergo a second stage of annihilation, reducing the relic density (and therefore allowing heavier dark matter masses).
- Argument: no symmetry dictates that the cross section should be smaller than the naïve geometric cross section.

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Q: Which bound state formation (BSF) processes allow for a geometric cross section?

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Outline

Toy Model







Comments on Cosmology and Phenomenology

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Matter Content and Symmetries

Field	$\mathrm{SU}(N)$	U(1)	Mass
X, \bar{X}	N, \bar{N}	1, -1	$m_X \gg \Lambda_D$
q,ar q	N, \bar{N}	0	$m_q \sim \Lambda_D$

- Add a flavour symmetry to make X stable.
- The nonabelian gauge group is confining at a scale Λ_D .
- Below confinement temperature, X, \bar{X} will form heavy-light mesons

$$H_X \equiv X\bar{q}, \quad \bar{H}_X \equiv \bar{X}q.$$

Thermal History

 $---- m_X$ X, g, q $---- T_{f,X} \sim m_X/20$

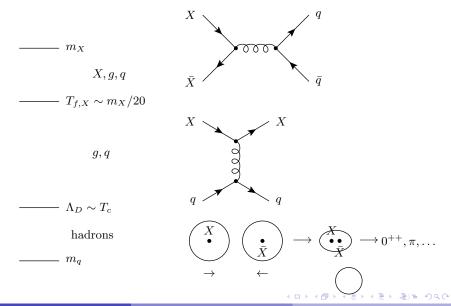
g,q

 $---- \Lambda_D \sim T_c$

hadrons

 $---- m_q$

Thermal History



Scales in Confined QCD

$$\begin{array}{c} m_{\pi}, f_{\pi}, m_{q} \\ f_{\pi}^{2} = \frac{V^{3}}{f_{\pi}^{2}}(m_{u} + m_{d}) \\ T_{c} \sim 165 \text{-}195 \text{ MeV} \text{ (lattice)} & V \sim 230 \pm 30 \text{ MeV} \\ \hline V \sim 230 \pm 30 \text{ MeV} \\ \text{(QCD sum rules)} \\ \sigma \sim 0.16 \text{-}0.19 \text{ GeV}^{2} \longleftrightarrow \sigma \sim \frac{C_{F}\Lambda_{\text{QCD}}^{2}}{2\beta_{0}} \xrightarrow{\Lambda_{\text{QCD}}} \Lambda_{\text{QCD}} \sim 332 \pm 17 \text{ MeV} \\ \text{(Regge, pot'n model)} & \sigma \sim \frac{C_{F}\Lambda_{\text{QCD}}^{2}}{2\beta_{0}} \xrightarrow{\Lambda_{\text{QCD}}} \text{(4L \overline{0}\text{MS RG running})} \end{array}$$

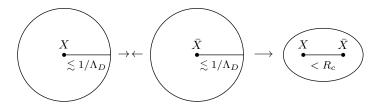
Teper 0812.0085, Brambilla et al. 1010.5827, Simolo 0807.1501, RPP

- ▶ In our toy model, Λ_D is determined by $\alpha_D(m_X), m_X$, both of which are free parameters.
- We will be concerned with the two ratios

$$rac{T}{\Lambda_D} \lesssim rac{T_c}{\Lambda_D} \lesssim 1 \,,$$
 $rac{m_X}{\Lambda_D} \gtrsim \mathcal{O}(10^{2-6}) \,.$

Spectrum of Dark Quarkonia

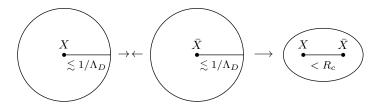
We can calculate the spectrum of $X-\bar{X}$ dark quarkonia using QM, modelling the interaction using a Cornell potential in analogy to QCD.



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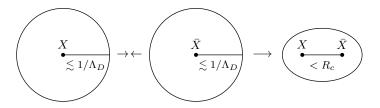
With light coloured states, the nonabelian force has finite range in the confined phase. To enforce this, we add a cutoff to the Cornell potential:

$$V(R) = \begin{cases} -\bar{\alpha}_D \left(\frac{1}{R} - \frac{1}{R_c}\right) + \Lambda_D^2(R - R_c) + V_0 & R < R_c \,, \\ V_0 & R > R_c \,. \end{cases}$$

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How to determine R_c ?

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Heavy-Light Mesons in QCD

Consider D and B mesons in QCD.

Q	m_Q	$m_{\rm meson}$
c	$1.3{ m GeV}$	$1865{ m MeV}$
b	$4.65 {\rm GeV} \ (1S)$	$5280{ m MeV}$

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In both cases, we have

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or about twice the "constituent mass" of a light quark. Writing this parametrically as

$$m_{\rm meson} - m_Q \sim \kappa_\Lambda \Lambda \,,$$

with $\Lambda \sim \sqrt{\sigma} \sim 400 \,\mathrm{MeV} \Rightarrow \kappa_{\Lambda} \sim 1.5$.

Heavy-Light Mesons and the Potential Cutoff

A natural way to define the cutoff: the threshold for open production of two H_X hadrons. If this occurs in the linear regime of the potential, then

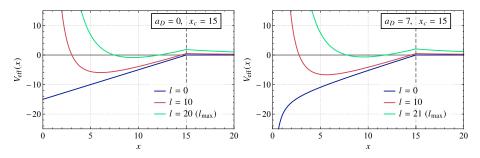
$$\begin{split} E_b^{\max} &= \Lambda_D^2 R_c = 2 \kappa_\Lambda \Lambda_D \,, \\ R_c &= 2 \frac{\kappa_\Lambda}{\Lambda_D} \sim \frac{3}{\Lambda_D} \,. \end{split}$$

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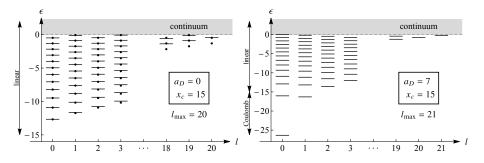
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Spectrum Snapshot



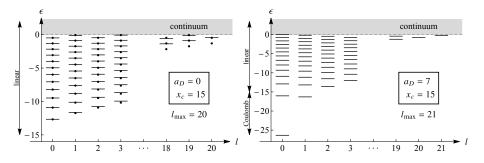
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Spectrum Snapshot



• hydrogenic:
$$E_{nl} \sim -\frac{\bar{\alpha}_D^2 \mu}{2n^2}$$

- linear: $E_{nl} \propto \Lambda_D \left(\frac{\Lambda_D}{2\mu}\right)^{1/3} \left[\frac{3}{2}\pi \left(n + \frac{l}{2} \frac{1}{4}\right)\right]^{2/3}$
- Size of linear bound states determined by virial theorem: $\langle R \rangle = E_{nl} / \Lambda_D^2$.

e.g., Quigg & Rosner, Phys.Rept. 56, 167 (1979); Hall & Saad 1411.2023

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3 Radiative Emission

4 Comments on Cosmology and Phenomenology

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Setup

 $H_X + \bar{H}_X \rightarrow (X\bar{X}) + (q\bar{q})$

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- Rigorous calculation for $m_q \lesssim \Lambda_D$ requires (p)NRQCD machinery (relativistic quarks).
- Regime with $m_q \gtrsim \Lambda_D$ can be treated using scattering theory in nonrelativistic QM.
- Here, calculation is analogous to H-H rearrangement at low temperatures (e.g., for CPT tests)

$$\mathrm{H}(1s) + \bar{\mathrm{H}}(1s) \to \mathrm{Pn}(NLM) + \mathrm{Ps}(nlm) \,.$$

Kolos et al., PRA 11, 1792 (1975)

Froelich et al., PRL 84, 4577 (2000); Jonsell et al., PRA 64, 052712 (2001)

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From QM, the cross section in COM frame is

$$\frac{d\sigma}{d\Omega} = (2\pi)^2 \frac{k_f}{k_i} m_X m_q |\mathcal{M}|^2 \,,$$

with matrix element

$$\mathcal{M} = 2\pi \left\langle \Psi_f \left(\mathbf{R}, \mathbf{r}_q, \mathbf{r}_{\bar{q}} \right) \right| \mathcal{H}_{\mathrm{tr}} \left| \Psi_i \left(\mathbf{R}, \mathbf{r}_q, \mathbf{r}_{\bar{q}} \right) \right\rangle \,.$$

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(Semi)classical Arguments

Kang, Luty, Nasri hep-ph/0611322

For $m_q > \Lambda_D$, distance b/w X and \bar{q} in H_X and the average force between them are

$$a_q \sim \frac{1}{\bar{\alpha}_D m_q}, \ F \sim \frac{\bar{\alpha}_D}{a_q^2} \sim \bar{\alpha}_D^3 m_q^2.$$

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• The velocity is set by the temperature, which is assumed to be of order the binding energy $v \sim \sqrt{T/m_X} \sim \bar{\alpha}_D \sqrt{m_q/m_X}$.

• When a free X with initial velocity v comes with a distance a_q of a \bar{q} , its velocity changes by

$$\frac{\Delta v}{v} \sim \frac{1}{v} \frac{F}{m_X} \Delta t \sim \frac{1}{v} \frac{F}{m_X} \frac{a_q}{v} \sim 1.$$

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The change in the position of X due to the force is

$$\frac{\Delta r}{a_q} \sim \frac{1}{a_q} \frac{F \Delta t^2}{m_X} \sim 1 \,.$$

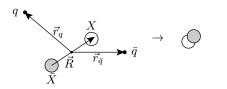
Can we justify this in the QM calculation?

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Hamiltonian

The full interacting Hamiltonian includes terms that couple all heavy and light dof:

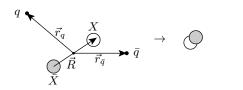
$$\begin{split} \mathcal{H}_{\text{tree}} &= -\frac{1}{m_X} \nabla_R^2 - \frac{1}{2m_q} \nabla_{r_q}^2 - \frac{1}{2m_q} \nabla_{r_{\bar{q}}}^2 \,, \\ \mathcal{H}_{\text{int}} &= V_{X\bar{X}} \left(R \right) + V_{q\bar{q}} \left(|\mathbf{r}_q - \mathbf{r}_{\bar{q}}| \right) + \mathcal{H}_{\text{tr}} \,, \\ \mathcal{H}_{\text{tr}} &= V_{q\bar{X}} \left(|\mathbf{r}_q + \mathbf{R}/2| \right) + V_{\bar{q}X} \left(|\mathbf{r}_{\bar{q}} - \mathbf{R}/2| \right) \\ &- V_{\bar{q}\bar{X}} \left(|\mathbf{r}_{\bar{q}} + \mathbf{R}/2| \right) - V_{qX} \left(|\mathbf{r}_q - \mathbf{R}/2| \right) \end{split}$$



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NB: we have taken the states to be colour-singlets in the Coulombic regime, so $V(r) \propto \pm \frac{\bar{\alpha}_D}{r}$.

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Multichannel Scattering

$$H_X + \bar{H}_X \to (X\bar{X}) + (q\bar{q})$$

- Final: eigenstate of free Hamiltonian and $V_{X\bar{X}} + V_{q\bar{q}}$.
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- Want to separate the total wavefunction for the initial and final states into products of wavefunctions for light and heavy dof:

$$\Psi = \psi^{q\bar{q}} \left(\mathbf{R}, \mathbf{r}_{q}, \mathbf{r}_{\bar{q}} \right) \cdot \psi^{X\bar{X}} \left(\mathbf{R} \right).$$

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Easy to see for final state: \u03c8 \u03c4 X \u03c8 and \u03c8 \u03c4 q \u03c9 are standard bound state solutions (for Coulomb potential), multiplied by a plane wave for the translational motion of the \u03c9 \u03c8.

Asymptotically at large R for initial states, the residual interaction terms in the Hamiltonian produce van der Waals-like forces since the q, \bar{q} are bound in well-separated H_X, \bar{H}_X .

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- First, we solve for light dof with fixed positions for the heavy dof:

$$\left(-\frac{1}{2m_q}\nabla_{r_q}^2 - \frac{1}{2m_q}\nabla_{r_{\bar{q}}}^2 + \underbrace{V_{q\bar{q}} + \mathcal{H}_{\mathrm{tr}}}_{\mathcal{H}_{\mathrm{tot}} - V_{X\bar{X}}}\right)\psi_i^{q\bar{q}} = V_{\mathrm{BO}}(\mathbf{R})\psi_i^{q\bar{q}}.$$

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Solve for the heavy dof with distorted potential from light dof ("potential surface"):

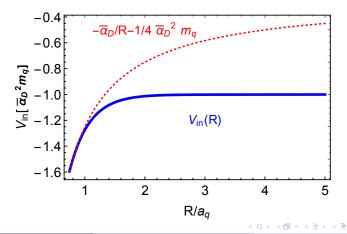
$$\left(-\frac{1}{m_X}\nabla_R^2 + \underbrace{V_{X\bar{X}}(R) + V_{\rm BO}(\mathbf{R})}_{V_{\rm in}}\right)\psi_i^{X\bar{X}} = E_i\psi_i^{X\bar{X}}$$

Potentials

- $V_{\rm BO}({f R})$ should interpolate between
 - at large R (initial state): twice the binding energy of H_X , $E_b = \bar{\alpha}_D^2 m_q/2$ ("Hartree energy"),
 - at small R (final state): the q-onium binding energy, $-\bar{\alpha}_D^2 m_q/4$.
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Potentials

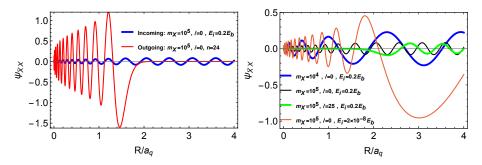
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Wavefunctions

Asymptotics determine phase matching of $\psi_i^{X\bar{X}}$ beyond the screening distance $R \ge 4a_q$:

$$\psi_i^{X\bar{X}} \to \sum_l i^l \sqrt{(2l+1)} e^{i\delta_l} \left[\cos \delta_l j_l(kR) - \sin \delta_l n_l(kR)\right] Y_{l0}(\theta_R) \,.$$



- Left: free initial-state and near-threshold bound final-state wavefunction for same hierarchy.
- Right: free initial-state wavefunctions for different m_X, l, E_i.

> After separating heavy and light dof, the matrix element in position space is

$$\mathcal{M} = \int d^{3}\mathbf{R} \ \psi_{f}^{X\bar{X}*}\left(\mathbf{R}\right) \ \psi_{i}^{X\bar{X}}\left(\mathbf{R}\right) \ T\left(\mathbf{R}\right) ,$$
$$T\left(\mathbf{R}\right) = \int d^{3}\mathbf{r}_{q} \ d^{3}\mathbf{r}_{\bar{q}} \ \psi_{f}^{q\bar{q}*}\left(\mathbf{r}_{q},\mathbf{r}_{\bar{q}}\right) \ \mathcal{H}_{\mathrm{tr}} \ \psi_{i}^{q\bar{q}}\left(\mathbf{R};\mathbf{r}_{q},\mathbf{r}_{\bar{q}}\right) .$$

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- Again T(R) is independent of m_X in the BO approximation, so we can use the results from numerical studies of H– \overline{H} rearrangement. Jonsell et al., J. Phys. B 37 (2004) 1195

$$T(R) = \begin{cases} \beta \left(E_f + \frac{1}{4} \bar{\alpha}_D m_q - V_{\rm BO}(R) \right) & R \gtrsim 3/4 \, a_q \\ 0 & R \lesssim 3/4 \, a_q \end{cases}$$

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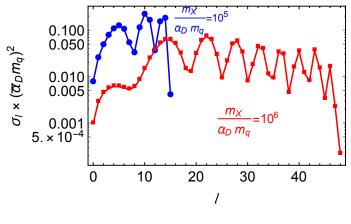
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- Actually, only the result near $R \approx a_q$ is needed.
- The dependence on the final bound-state energy is encapsulated in E_f, the kinetic energy of the final state.

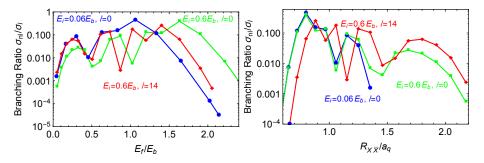
Results for Partial Waves



Cross section shuts off at classical $l_{\rm max} \sim k_i a_q$. (Calculation for $\bar{\alpha}_D \approx 1/137$)

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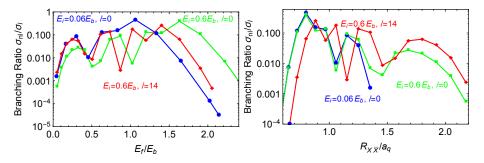
Exothermic Process



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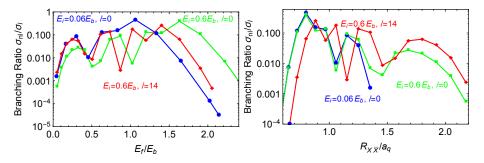
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Exothermic Process



- Left: preferentially form quarkonia states with binding energies of $\sim E_b$.
- Right: cross section dominated by formation of quarkonia states with size $\approx a_q$.
- True for an order of magnitude in incoming energy (or temperature).

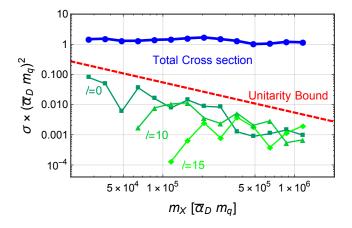
Exothermic Process



- Left: preferentially form quarkonia states with binding energies of $\sim E_b$.
- Right: cross section dominated by formation of quarkonia states with size $\approx a_q$.
- True for an order of magnitude in incoming energy (or temperature).

The process is exothermic, and irreversible at the temperatures of interest.

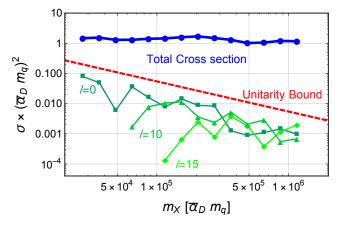
Results



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Results



- blue: total σ_{rearr} for $E_i = (0.6)\bar{\alpha}_D^2 m_q$ is indeed geometric.
- red: unitarity bound $4\pi/k_i^2$ (for given mass m_X at above E_i).
- greens: partial waves normalized by 2l + 1.

In the rearrangement process, each partial wave approximately saturates the unitarity bound with

$$\sigma_l \sim \frac{1}{k_i^2} (2l+1) \,.$$

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which depends only on the scale of the light dof.

As we dial m_q below the confinement scale $a_q \to \Lambda_D^{-1}$, and the geometric scaling becomes

$$\sigma \sim \frac{1}{\Lambda_D^2}$$

Outline

Toy Model





Comments on Cosmology and Phenomenology

Gabriel Lee (Cornell/Korea)

Dark Quarkonium Formation in the Early Universe

Sep 13, 2018 28 / 40

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BSF by Emission of Light Bosons

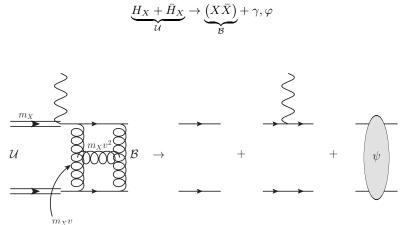
- There are extensive examples in the literature of bound-state formation via emission of light bosons, e.g., dark photons or (pseudo)-scalars.
- Together with Sommerfeld enhancement from long-range forces, can produce nontrivial effect in halos and structure formation.
 - Dissipation affects the size and morphology of halos.
 - Compton scattering off hidden photons delays kinetic decoupling, modifying small-scale structure.
 - Enhance DM annihilation in halos.

Feng, Kaplinghat, Tu, Yu 0905.3039; Petraki, Pearce, Kusenko 1403.1077; Agrawal, Cyr-Racine, Randall, Scholtz 1610.04611

- Bound states that use a U(1) force have a small effect on present relic density (at most an order of magnitude).
- Is this still true if the bound state is governed by a confining force, but formed through perturbative emission of a vector?

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Schematically



von Harling & Petraki 1407.7874; Petraki, Postma, Wiechers 1505.00109 Petraki, Postma, de Vries 1611.01394; Cirelli et al. 1612.07295

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Solving for the Wavefunctions

Dimensionless Schrödinger equation and effective potential:

$$-\chi_{nl}''(x) + V_{\text{eff}}(x)\chi_{nl}(x) = \epsilon_{nl}\chi_{nl}(x),$$
$$V_{\text{eff}}(x) = \frac{l(l+1)}{x^2} + \Theta(x-x_c)\left(-a_D\left(\frac{1}{x} - \frac{1}{x_c}\right) + x - x_c\right),$$

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with scalings $\epsilon = E/E_0, x = r/r_0$:

$$r_0 = \frac{1}{\Lambda_D} \left(\frac{\Lambda_D}{m_X}\right)^{1/3}, \quad E_0 = \Lambda_D \left(\frac{\Lambda_D}{m_X}\right)^{1/3}.$$

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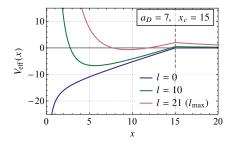
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- Effective potential has minimum at $x_{\min} = (2l(l+1))^{1/3} \, .$
- Then l_{max} for bound states is governed by condition that

$$x_{\min}(l_{\max}) \le x_c$$
.

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Employ standard methods in QFT for computing 2–2 cross section.

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$$v_{\rm rel} \,\sigma_{k\hat{\mathbf{z}}\to nl} = \frac{2e_X^2}{m_X^2} \left(\frac{\Lambda_D}{m_X}\right)^{2/3} \left(\epsilon_k - \epsilon_{nl}\right)^3 \,\left[(l+1) \left| I_{k,l+1\to nl} \right|^2 + l \left| I_{k,l-1\to nl} \right|^2 \right] \,,$$

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where ϵ are energies in units of E_0 and I is the radial wavefunction overlap integral

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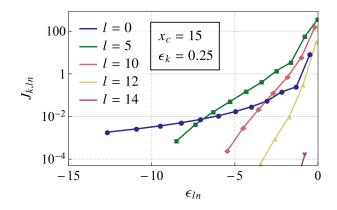
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$$I_{k,l\pm 1\to nl} = \int dx \, x \, \chi_{nl}^*(x) \chi_{k,l\pm 1}(x) \, .$$

Overlap integral prefers near-threshold states.

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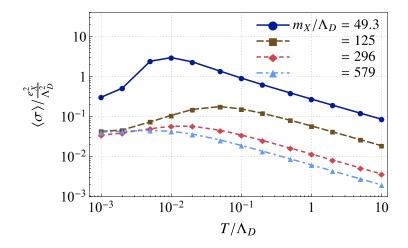
Radiative BSF: Overlap Integral



(Linear only) Shallowest bound states give the largest contribution to overlap.

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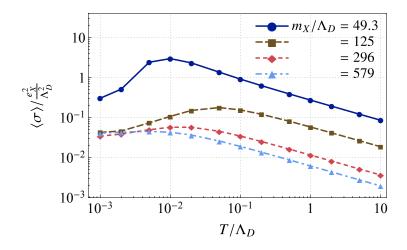
Results



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Results



For large hierarchies, the cross section is not geometric. Agrees with semiclassical expectation from Larmor: $m_{\chi}^{-3/2}$ scaling.

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Summary

- For larger mass hierarchies, the cross section is not geometric.
- The cross section approaches an asymptotic value for lower temperatures.
- Then $\langle \sigma v \rangle$ will decrease at lower temperatures, so this process will never dominate.

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Summary

- For larger mass hierarchies, the cross section is not geometric.
- The cross section approaches an asymptotic value for lower temperatures.
- Then $\langle \sigma v \rangle$ will decrease at lower temperatures, so this process will never dominate.
- Our calculation was largely independent of spin of light state, so should hold for emission of scalars as well.
- Major difference with rearrangement: light dof are spectators in the perturbative process, only entered parametrically through the final-state wavefunctions.

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Outline





Comments on Cosmology and Phenomenology

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 Second stage should be over fairly quickly, as depletion is faster nearer to confinement temperature (larger number density).

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- Standard freeze-out relic density parameter:

$$\Omega_X h^2 \sim \frac{10^{-9} \,\mathrm{GeV}^{-2}}{\langle \sigma_X^{\mathrm{ann}} v \rangle} \sim \left(\frac{m_X}{10 \,\mathrm{TeV}}\right)^2 \frac{1}{\alpha_D^2(m_X)}$$

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.

• With large hierarchy, can dilute the relic density of H_X, \bar{H}_X .

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- Second stage should be over fairly quickly, as depletion is faster nearer to confinement temperature (larger number density).
- Standard freeze-out relic density parameter:

$$\Omega_X h^2 \sim \frac{10^{-9} \,\mathrm{GeV}^{-2}}{\langle \sigma_X^{\mathrm{ann}} v \rangle} \sim \left(\frac{m_X}{10 \,\mathrm{TeV}}\right)^2 \frac{1}{\alpha_D^2(m_X)}$$

After second stage of annihilation:

$$\Omega_{H_X} \sim \sqrt{\frac{\Lambda_D}{m_X}} \left(\frac{m_X}{100 \text{ TeV}}\right)^2$$
.

- With large hierarchy, can dilute the relic density of H_X, \bar{H}_X .
- Opens up possibility for very heavy DM.

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Take $N_c = 3$, like QCD.

Other hadronic states in the spectrum with multiple X's, but some amount of light q's, have geometric rearrangement cross sections.

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- e.g., *Xqq* participates in the chain:
 - $\blacktriangleright Xqq + X\bar{q} \to XXq + q\bar{q},$
 - $\blacktriangleright XXq + X\bar{q} \to XXX + q\bar{q}.$
- Since these interactions all involve the brown muck, we expect them to have geometric cross sections.

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- Future work: solve Boltzmann equation with these dynamics to find equilibrium yields.

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This should apply to other DM candidates in QCD: e.g., Dirac adjoints.

- ▶ $Xg + Xg \rightarrow XX + gg$, or same process with \bar{X} , has a geometric cross section.
- ▶ If accidental symmetries protect *X*, then *XX* is stable and makes up DM.
- Some model building done in this regime: long-lived gluinos, "gluequarks" (see CERN-TH talk by M. Redi).

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- Relaxation to ground state before annihilation or dissociation is model-dependent:
 - toy model like QCD: pions vs. photons,
 - light (pseudo-)scalars,
 - dark glueballs if adjoints.

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1503.00009 + Mishra-Sharma 1707.05326; Park, Zhang 1712.09279

Schwaller, Stolarski, Weiler 1502.05409; Cohen, Lisanti, Lou

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 - Self-interaction with large cross sections can yield interesting halo dynamics (and are constrained by such).
 - If there is a coupling to photons, could see many transition lines from spectrum of bound states. Boddy et al. 1408.6532
 - Neutrino telescope signals for very heavy DM?

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- Amusing aside: in an ideal world, we could test this via $B\bar{B}$ scattering.

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Nonabelian, confining hidden sectors with both heavy and light states are natural frameworks for rich dark matter sectors that include bound states.

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- Nonabelian, confining hidden sectors with both heavy and light states are natural frameworks for rich dark matter sectors that include bound states.
- Below the confinement temperature, the heavy-light meson relics can deviate from standard cosmology by undergoing an additional annihilation phase.
- To achieve substantial differences from standard freeze-out relic density, we need a large geometric cross section for annihilation in the second stage.

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... and in the darkness bind them.

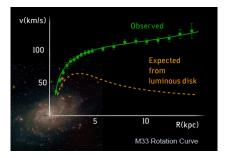
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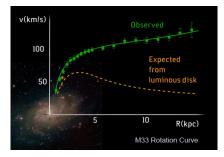
Thank you.

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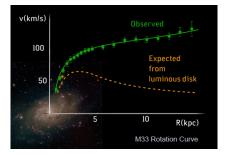
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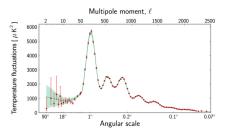
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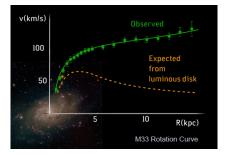
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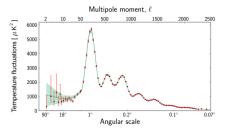
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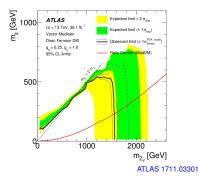
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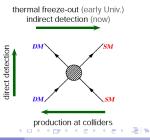


Gravitational evidence

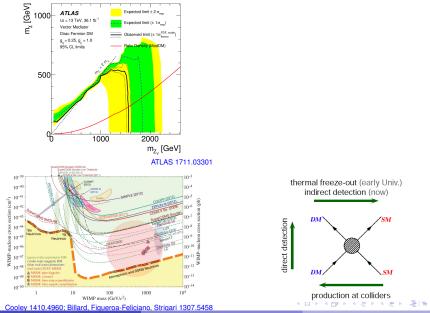
- galactic
- cluster
- cosmological

Catch Me If You Can





Catch Me If You Can

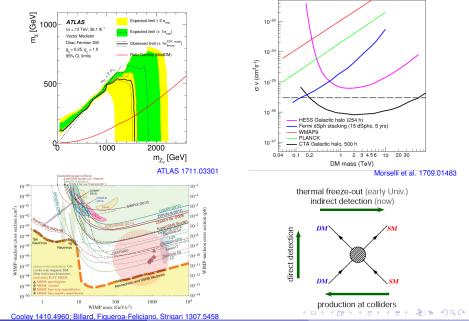


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Dark Quarkonium Formation in the Early Universe

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Catch Me If You Can

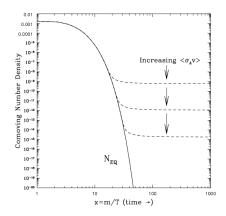


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Dark Quarkonium Formation in the Early Universe

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WIMP Miracle



If DM was in thermal contact with the SM in the early universe, the observed DM relic density parameter is

$$0.11 \sim \Omega h^2 \sim 10^{-9} \,\mathrm{GeV}^{-2} \frac{m^2}{\alpha^2} \,,$$

if *s*-wave scattering dominates annihilation cross section.

For weak-scale masses and couplings $m\sim 3\times 10^3~{\rm GeV}, \alpha\sim 0.6^2/(4\pi)\sim 0.3,$

$$10^{-9} \frac{10^7}{0.1} \sim 0.1$$
.

Unitarity Bound on DM Mass

Griest & Kamionkowski, Phys.Rev.Lett. 64, 615 (1990)

Upper limit on mass of DM once in thermal equilibrium with SM from partial-wave unitarity:

$$(\sigma_J) < \frac{4\pi(2J+1)}{m_X^2 v_{\rm rel}^2}$$

At freeze-out $m/T_f = 20$, $v_{\rm rel} \sim 2v \sim 2/\sqrt{10}$, and cross sections higher-order in J are suppressed by powers of $v_{\rm rel}^2/4 \sim 1/6$. Then using J = 0,

$$\begin{split} \Omega_X h^2 &\sim 0.1 \gtrsim \frac{2.5 \times 10^{-10} \,\mathrm{GeV}^{-2}}{4\pi} m_X^2 v_{\mathrm{rel}}^2 \,, \\ \Rightarrow \left(\frac{m_X}{\mathrm{TeV}}\right)^2 \lesssim 10^4 \end{split}$$

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Results from $H-\overline{H}$

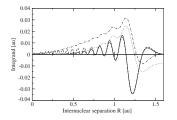


Figure 1. The integrand in equation (6) calculated using the full R dependence of the leptonic matrix element (solid line), and using the approximation that the overlap between initial and final is constant leptonic wavefunctions (see equation (9)) (dashed line). We also show their integrals from 0 to R (multiplied by 10) as the dotted line and the dash–dotted line, respectively. The dash–dotted line shows that the constant overlap approximation yields a vanishing T matrix, while the dotted line shows that the small change in the integrand induced by including the R dependence of the overlap results an on-vanishing T matrix.

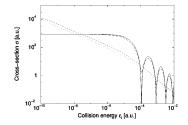
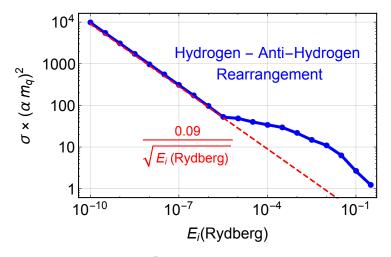


FIG. 6. Cross sections for the H-H system: elastic cross section obtained from the real part of the phase shift only (solid), elastic cross section including correction for the presence of inelastic scattering (long dashed), rearrangement cross section (dotted), and proton-antiproton annihilation in flight (dashed). At low energies, the elastic cross section is 823 without the correction for inelastic scattering, and 829 including this correction, while the low-energy behavior of σ^{rearr} is $0.09/\sqrt{\tilde{\epsilon_i}}$, and $\sigma_{\mu}^{p\bar{\rho}} \sim 0.14/\sqrt{\epsilon_i}$.

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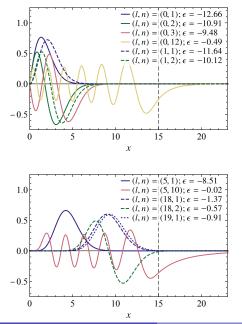
Rearrangement at Low Temperatures: $H-\overline{H}$

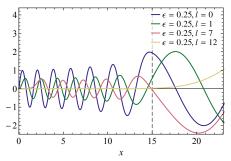


Agrees with $H-\overline{H}$ results at very low temperatures.

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Wavefunctions in Radiative BSF





Near-threshold states (bound [left col.] and scattering [top]) have largest overlap.

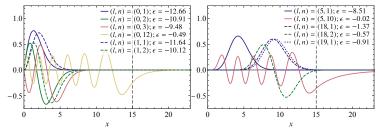
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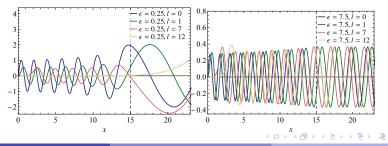
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More Wavefunctions in Radiative BSF

- Bound states with "radial number" n have n 1 nodes.
- Shallower bound states with smaller l tend to penetrate into the region beyond $x > x_c$.

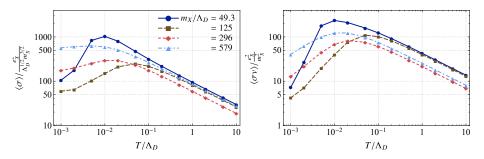


Scattering states with larger ϵ and smaller l have larger penetration to $x < x_c$.



Gabriel Lee (Cornell/Korea)

Radiative BSF Results with $m_{X}^{-3/2}\ {\rm Scaling}$



For $T\Lambda_D \sim 10^{-2}$ –1, results are $\mathcal{O}(1)$ within each other for various hierarchies. Agrees with semiclassical expectation from Larmor: $m_{\chi}^{-3/2}$ scaling.