QCD at high density
and the fate of the (tri-)critical point

Dirk H. Rischke

"Teaching through research: remembering Raoul Gatto"

GGI, Firenze, Sep. 28, 2018

Raoul Gatto and high-density QCD

Raoul's research interests in high-density QCD:

- QCD phase diagram
- Chiral phase transition, (tri-)critical point
- Color superconductivity at high n_B
- Inhomogeneous/crystalline phases at high n_B
- Effect of magnetic fields on QCD phase diagram

Raoul's research works on high-density QCD:

- 36 (of 283) papers ($\cong$ 13%)
- 2055 (of 15803) citations ($\cong$ 13%)
- 12 papers with +50 citations
- 7 papers with +100 citations

"Pre-historic" QCD phase diagram

G. Baym, "Closing remarks" of "Quark Matter 1983" @ BNL, NPA 418 (1984) 433

QCD phase diagram in the $T - n_B$ plane:

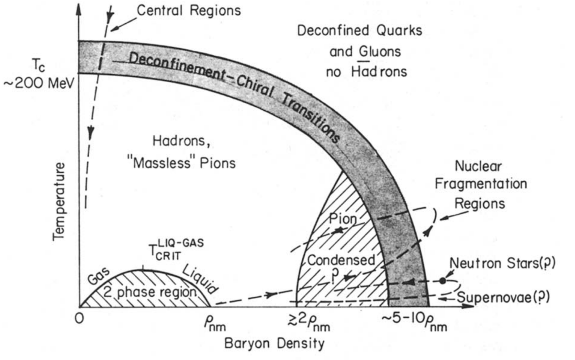


FIGURE 2
Phase diagram of nuclear matter.

Note: "cross-over" transition at $n_B = 0$ predicted by universality arguments (s. next page) and confirmed by lattice-QCD calculations!

Note: inhomogeneous pion-condensed phase predicted by recent model calculations!

Order of the QCD transition at $n_B = 0$

B. Svetitsky, L. Yaffe, NPB 210 (1982) 423
pure SU(3) Yang–Mills theory ($m_q \to \infty$) $\Longrightarrow$ Deconfinement transition is of first order

R.D. Pisarski, F. Wilczek, PRD 29 (1984) 338
QCD with N_f flavors of massless quarks ($m_q = 0$)

$\Longrightarrow$ If $\mathrm{U}(1)_\mathrm{A}$ is restored, chiral transition is of first order for $N_f \geq 2$

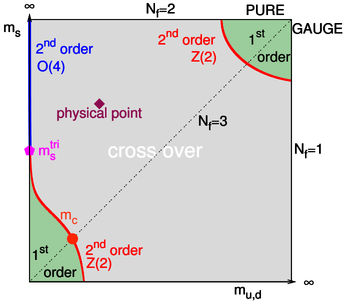
$\Longrightarrow$ If $\mathrm{U}(1)_\mathrm{A}$ is broken, chiral transition: – can be of second order for $N_f = 2$
– is of first order for $N_f \geq 3$

Finite quark masses ($m_q > 0$) act like a "magnetic field", wash out the transition

$\Longrightarrow$ "cross-over" transition!

All this is confirmed by lattice-QCD calculations!

H.-T. Ding, F. Karsch, S. Mukherjee, IJMP E24 (2015) 1530007



Order of the QCD transition for $n_B > 0$

Lattice-QCD calculations for $n_B > 0$ plagued by infamous sign problem

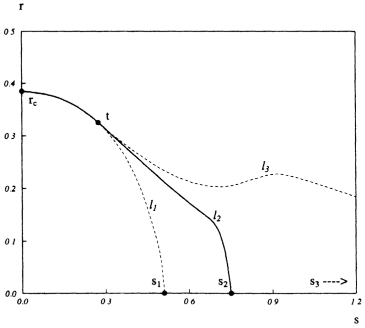
$\Longrightarrow$ QCD transition difficult to investigate via first-principle methods

$\Longrightarrow$ Model calculations predict first-order transition line at $n_B > 0$:

$N_f = 2, m_q = 0$:

$\exists$ tri-critical point between 1st order line and 2nd order line extending from $n_B = 0$

A. Barducci, R. Casalbuoni, S. De Curtis, R. Gatto, G. Pettini, PRD 41 (1990) 1610

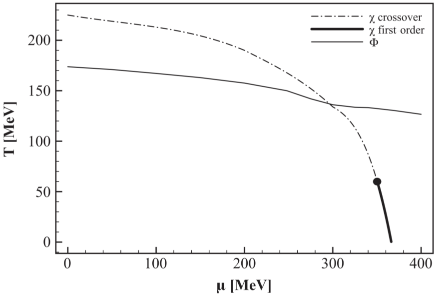


$N_f \geq 2, m_q \neq 0$:

$\exists$ 2nd order critical endpoint of 1st order line (cf. phase diagram of water)

M. Asakawa, K. Yazaki, NPA 504 (1989) 668

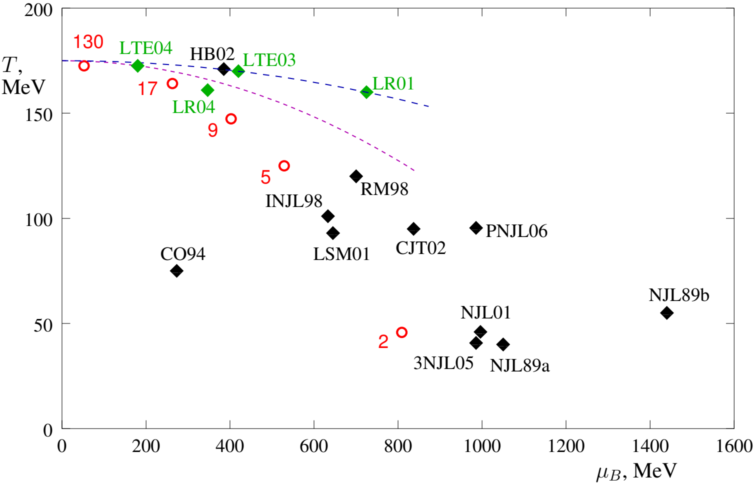
H. Abuki, R. Anglani, R. Gatto, G. Nardulli, M. Ruggieri, PRD 78 (2008) 034034



Location of the critical endpoint

Problem: model calculations have uncontrolled systematic errors!

M.A. Stephanov, PoS LAT2006 (2006) 024



$\Longrightarrow$ Predictions for location of critical endpoint vary all over phase diagram!

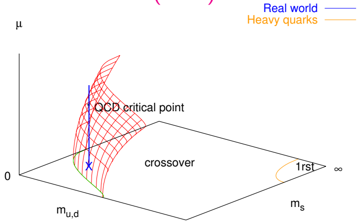
Critical endpoint – is it there or not?

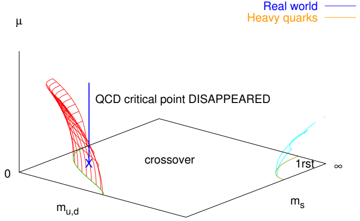
Lattice-QCD calculations can be done at:

– $\mu_B = 0$ and then extrapolated (Taylor expansion, reweighting techniques) to $\mu_B > 0$

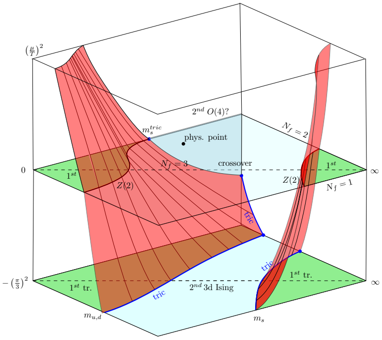
– $\mu_B^2 \leq 0$ (imaginary chemical potential) and then extrapolated to $\mu_B > 0$

P. de Forcrand, S. Kim, O. Philipsen, PoS LATTICE2007 (2007) 178

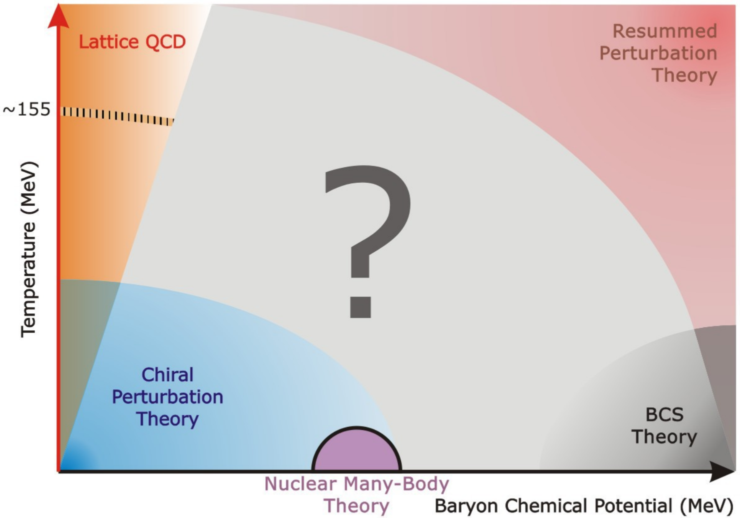




O. Philipsen, C. Pinke, PRD 93 (2016) 114507



$\Longrightarrow$ at present, (some) lattice-QCD calculations find no indication for critical endpoint!



$\Longrightarrow$ Goal of DFG-funded CRC-TR 211: reduce size of uncharted region!

Experimental efforts to find the critical endpoint

CRC-TR 211

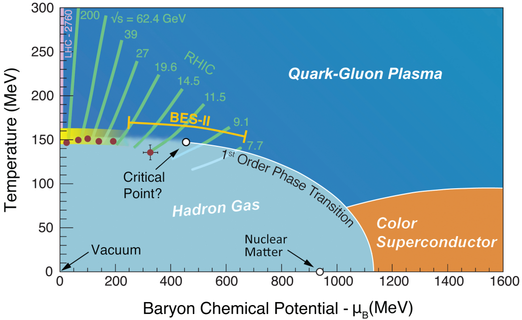
QCD phase diagram can be explored in heavy-ion collisions:

– Collision energy $\sqrt{s_{AA}} \sim S/N_B$ entropy per baryon number $\sim T/\mu_B$

– Expansion of hot and dense system (approximately) isentropic

$\Longrightarrow$ Expansion along lines of constant T/μ_B in QCD phase diagram

$\Longrightarrow$ Varying $\sqrt{s_{AA}}$ allows to explore different regions of QCD phase diagram!



$\Longrightarrow$ Beam-energy scan (BES) program at RHIC (BNL),

new accelerator facilities FAIR (Darmstadt), NICA (Dubna), J-PARC (Tokai)

Fluctuations near the critical endpoint

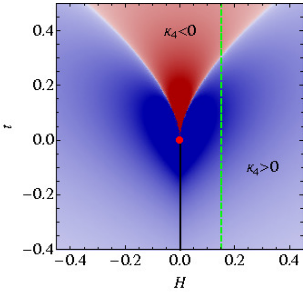
M.A. Stephanov, PRL 107 (2011) 052301

Fluctuations $\sigma(X)$ of order-parameter field

Zero-mode in volume V: $\sigma_V = \int_V \mathrm{d}^3\mathbf{x}\, \sigma(X)$

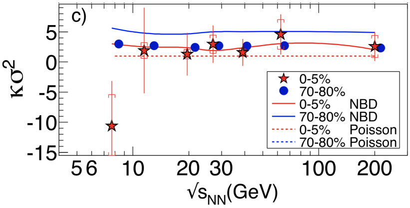
Near critical endpoint:

Curtosis $\kappa \equiv \dfrac{\langle \sigma_V^4 \rangle - 3\langle \sigma_V^2 \rangle^2}{\langle \sigma_V^2 \rangle^2} < 0$



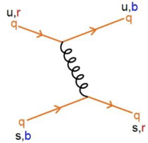
RHIC-BES program seems to see signs of $\kappa < 0$ (in net-charge fluctuations)!

X. Luo (STAR collaboration), PoS CPOD2014 (2015) 019



QCD is asymptotically free theory

$\Longrightarrow$ At high temperatures and densities, QCD thermodynamics is (in principle) computable by (resummed) perturbation theory



However: one-gluon exchange is attractive in the (antisymmetric) $[\bar{3}]_c$ channel

$\Longrightarrow$ At sufficiently low temperatures, condensation of quark Cooper pairs occurs

$\Longrightarrow$ Color superconductivity! (non-perturbative phenomenon)

D. Bailin, A. Love, Phys. Rept. 107 (1984) 325

M.G. Alford, A. Schmitt, K. Rajagopal, T. Schäfer, RMP 80 (2008) 1455

Color wave function of color-superconducting order parameter: $[\bar{3}]_c$

Pairing is usually favored in (antisymmetric) spin $J = 0$ channel: $[1]_J$

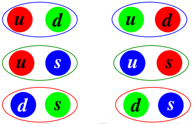
$\Longrightarrow$ Pauli principle: flavor wave function needs to be antisymmetric, too, e.g. for 3 quark flavors (u,d,s) of (approximately) equal mass: $[\bar{3}]_f$

$\Longrightarrow$ Color-superconducting order parameter: Δ_i^f, $i = \bar{r}, \bar{g}, \bar{b}$, $f = \bar{u}, \bar{d}, \bar{s}$

Condensation of Cooper pairs: $\langle \Delta_i^f \rangle = \delta_i^f \Delta \neq 0$

$\Longrightarrow$ breaks $\mathrm{SU(3)_c} \times \mathrm{SU(3)_f} \to \mathrm{SU(3)_{c+f}}$

$\Longrightarrow$ color-flavor locking (CFL)!



Effective theories for color-superconducting quark matter

For $\mu_q \gg m_q$ quarks can be considered (approximately) massless

$\Longrightarrow$ Flavor symmetry $\mathrm{SU(3)_f} \longrightarrow \mathrm{SU(3)_L} \times \mathrm{SU(3)_R}$ chiral symmetry

$\Longrightarrow$ Left- and right-handed color-superconducting order parameters: $\Delta^f_{L,i}, \Delta^f_{R,i}$

$\Longrightarrow$ CFL breaks $\mathrm{SU(3)_c} \times \mathrm{SU(3)_L} \times \mathrm{SU(3)_R} \to \mathrm{SU(3)_{c+L+R}}$

$\Longrightarrow$ Chiral symmetry broken, just as in the QCD vacuum!

$\Longrightarrow$ R. Casalbuoni, R. Gatto, PLB4 64 (1999) 111
Effective low-energy theory for the Goldstone modes in CFL phase (similar to χPT)

Broken $\mathrm{SU(3)_c}$ $\Longrightarrow$ gluons acquire Meissner mass

$\Longrightarrow$ R. Casalbuoni, R. Gatto, M. Mannarelli, G. Nardulli, PLB 524 (2002) 144
Computed gluon Debye and Meissner masses and dispersion relations
(in 2-flavor color superconductor - 2SC phase) using effective theory
for quarks near Fermi surface

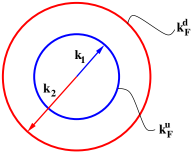
$\Longrightarrow$ Confirmed results of DHR, PRD 62 (2000) 034007;
DHR, D.T. Son, M.A. Stephanov, PRL 87 (2001) 062001

gluon color a	$1-3$	$4-7$	8
Debye	0	$\frac{3}{2} m_g^2$	$3 m_g^2$
Meissner	0	$\frac{1}{2} m_g^2$	$\frac{1}{3} m_g^2$

$$m_g^2 \equiv \frac{g^2 \mu_q^2}{3\pi^2}$$

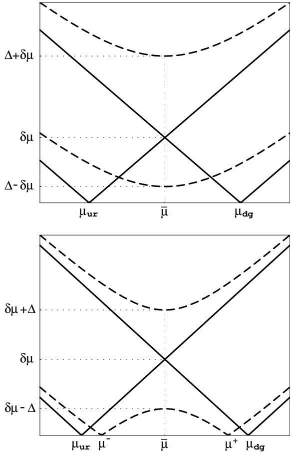
"Stressing" the Cooper pairs

Fermi surfaces of paired quarks are displaced by:
– difference in quark masses
– constraints of electric neutrality and β-equilibrium (neutron stars)



M. Huang, I.A. Shovkovy, NPA 729 (2003) 835

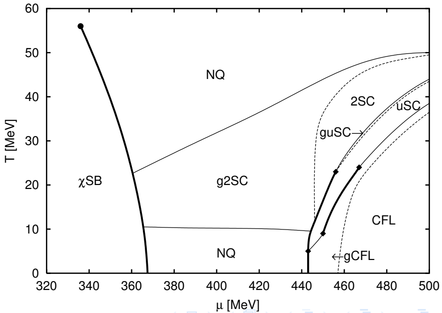
Quasiparticle dispersion relations:



$\Longrightarrow$ gapless excitations when $\Delta \leq \delta\mu$

$\Longrightarrow$ gapless color-superconducting phases!

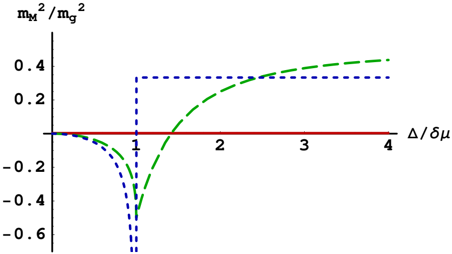
S.B. Ruester, V. Werth, M. Buballa, I.A. Shovkovy, DHR, PRD 72 (2005) 034004



(Some) gapless color-superconducting phases exhibit imaginary Meissner masses,

– g2SC phase: gluons with colors $a = 4 - 8$

M. Huang, I.A. Shovkovy, PRD 70 (2004) 051501



– CFL phase: gluons with colors $a = 1, 2, 3, 8$

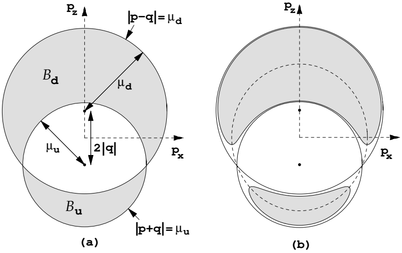
R. Casalbuoni, R. Gatto, M. Mannarelli, G. Nardulli, M. Ruggieri, PLB 605 (2005) 362

$\Longrightarrow$ "Chromomagnetic (CM) instability" $\Longleftrightarrow$ gapless phase is not true groundstate!

$\Longrightarrow$ Inhomogeneous ("crystalline") and/or gluon-condensed ("gluonic") phases

"Stress relief" by giving Cooper pairs nonzero momentum $2\mathbf{q}$

M.G. Alford, J.A. Bowers, K. Rajagopal, PRD 63 (2001) 074016



$\Longrightarrow$ Condensate varies in space:

– plane wave: $\Delta(\mathbf{x}) = \Delta \, \exp\left(2i\mathbf{q}\cdot\mathbf{x}\right)$
  P. Fulde, R.A. Ferrell, PR 135 (1964) A550

– standing wave: $\Delta(\mathbf{x}) = \Delta \, \cos\left(2\mathbf{q}\cdot\mathbf{x}\right)$
  A.I. Larkin, Yu.N. Ovchinnikov, Zh. Eksp. Teor. Fiz. 47 (1964) 136

– multiple crystal structures
  J.A. Bowers, K. Rajagopal, PRD 66 (2002) 065002

$\Longrightarrow$ "LOFF" phases

Electrically neutral 2-flavor FF phase: CM unstable

E.V. Gorbar, M. Hashimoto, V.A. Miransky, PRL 96 (2006) 022005

3-flavor FF phase $\Delta^{ij}_{fg}(\mathbf{x}) = \sum_{k=1}^{3} \Delta_k \exp\left(2i\mathbf{q}_k\cdot\mathbf{x}\right)\epsilon^{ijk}\epsilon_{fgk}$: CM stable

M. Ciminale, G. Nardulli, M. Ruggieri, R. Gatto, PLB 636 (2006) 317

Face-centered cubic (FCC) and body-centered cubic (BCC) crystals: CM stable

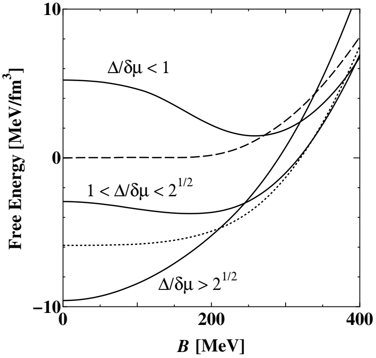
R. Gatto, M. Ruggieri, PRD 75 (2007) 114004

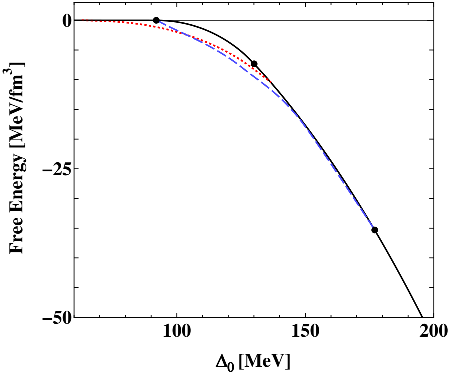
E.V. Gorbar, M. Hashimoto, V.A. Miransky, PLB 611 (2005) 207

CM instability of the 2SC phase cured by gluon condensation $g\langle \mathbf{A}_a \rangle \equiv \mathbf{e}_z \delta_{a6} B \neq 0$

$\Longrightarrow$ breaks spatial isotropy

O. Kiriyama, DHR, I.A. Shovkovy, PLB 643 (2006) 331

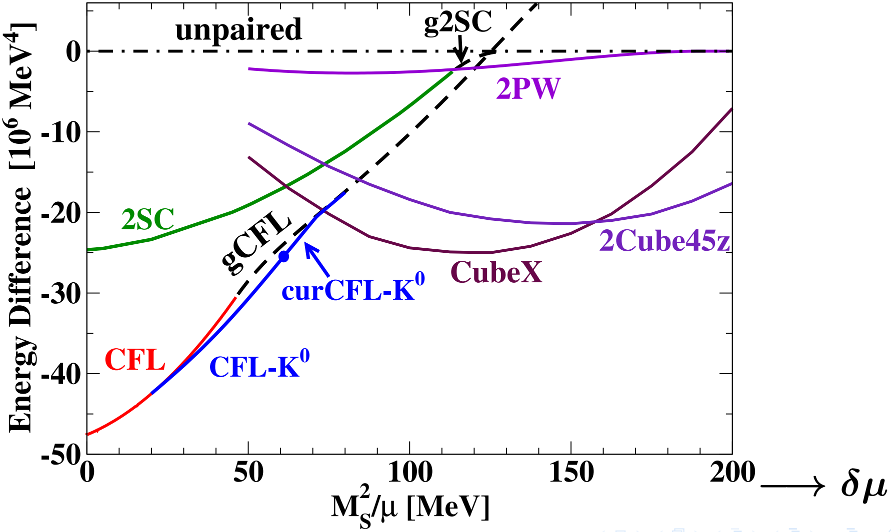




...not all possible gluon-condensation patterns were studied, and the corresponding free energies compared...

...but crystals seem to win energetically:

M.G. Alford, A. Schmitt, K. Rajagopal, T. Schäfer, RMP 80 (2008) 1455



Phase diagram of the Gross-Neveu model

1+1-dimensional Gross-Neveu model

$\Longrightarrow$ Crystal phase: mass (order parameter) exhibits spatial modulation:

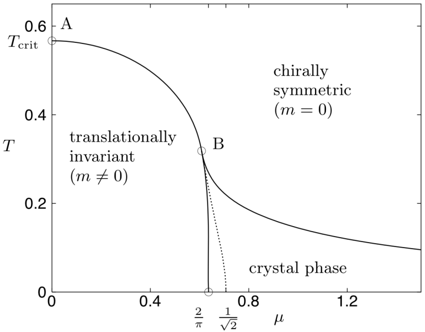
$m(z) = \Delta\sqrt{\nu}\,\mathrm{sn}(\Delta z|\nu)$,

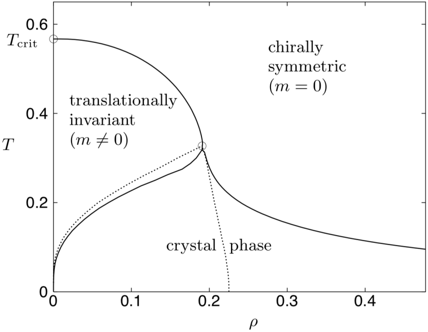
with Jacobi elliptic function $\mathrm{sn}(\alpha|\nu)$ of elliptic modulus $\nu \in [0,1]$,

$\mathrm{sn}(\alpha|0) = \sin\alpha$, $\mathrm{sn}(\alpha|1) = \tanh\alpha$

$\Longrightarrow$ "Real-kink crystal (RKC)" or "solitonic crystal"

M. Thies, K. Urlichs, PRD 67 (2003) 125015





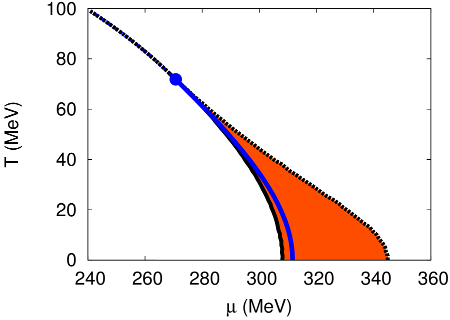
3+1-dimensional Nambu–Jona-Lasinio (NJL) model

$\Longrightarrow$ "Chiral density wave (CDW)" ansatz for mass:

$m(z) = \Delta e^{iqz}$ (cf. FF ansatz for color superconductivity!)

E. Nakano, T. Tatsumi, PRD 71 (2005) 114006

M. Buballa, S. Carignano, Prog. Part. Nucl. Phys. 81 (2015) 39



Since mass $m \sim g\,(\sigma + i\gamma_5\,\boldsymbol{\tau}\cdot\boldsymbol{\pi}\,)$

$\Longrightarrow$ $\langle\sigma(z)\rangle = \dfrac{\Delta}{g}\,\cos(qz)\,,$

$\langle\pi_3(z)\rangle = \dfrac{\Delta}{g}\,\sin(qz)$

$\Longrightarrow$ σ "rotates" into π_3 as one moves along z

$\Longrightarrow$ "Chiral spiral"

(similar to pion condensation in hadronic phase!)

$\Longrightarrow$ CDW phase has lower free energy, supersedes 1st order phase transition!

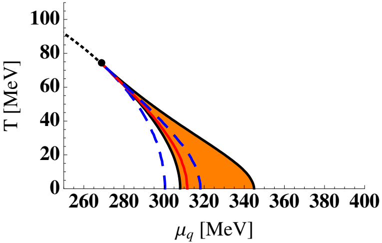
$\Longrightarrow$ 2nd order critical endpoint becomes Lifshitz point!

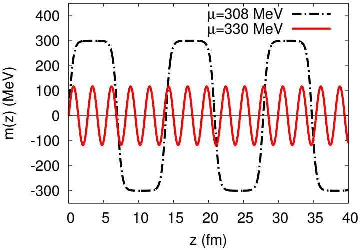
Real-kink crystals in the NJL model

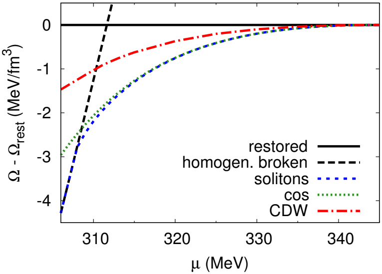
3+1-dimensional NJL model $\Longrightarrow$ RKC ansatz for mass

D. Nickel, PRD 80 (2009) 074025

M. Buballa, S. Carignano, Prog. Part. Nucl. Phys. 81 (2015) 39



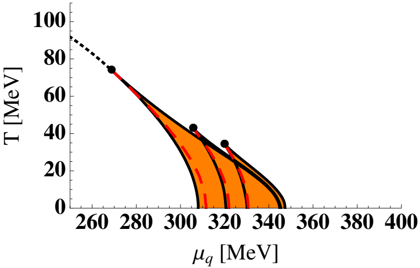




$\Longrightarrow$ For 1-dimensional spatial modulations,
RKC wins over CDW

D. Nickel, PRD 80 (2009) 074025

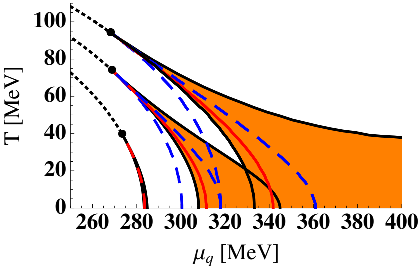
Dependence on current quark mass:



RKC phase for $m_q = 0, 5, 10$ MeV

$\Longrightarrow$ RKC phase shrinks with increasing m_q

Dependence on constituent quark mass:

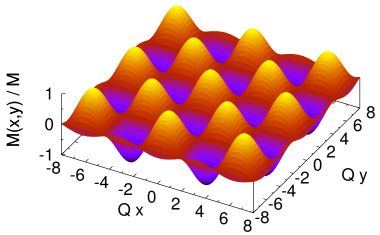


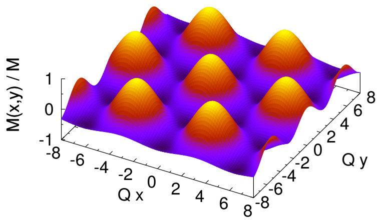
RKC phase for $M_q = 250, 300, 350$ MeV

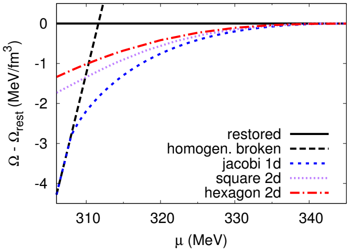
$\Longrightarrow$ RKC phase grows with increasing M_q

S. Carignano, M. Buballa, PRD 86 (2012) 074018

"egg carton" (square 2d) "honeycomb" (hexagon 2d)







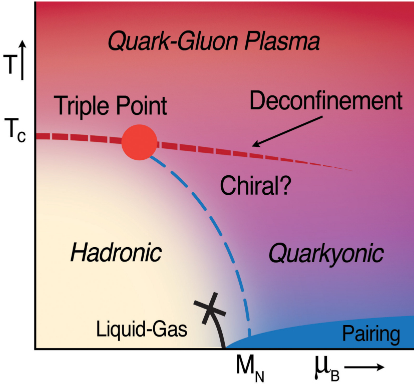
$\Longrightarrow$ 1-dimensional RKC still wins over 2-dimensional modulations!

QCD at large N_c (fixed N_f)

$\Longrightarrow$ gluons: $\varepsilon_g \sim N_c^2$, quarks: $\varepsilon_q \sim N_c$, hadrons: $\varepsilon_h \sim N_c^0$

$\Longrightarrow$ 3 phases, separated by 1st order transitions, meeting at triple point

L. McLerran, R.D. Pisarski, NPA 796 (2007) 83



Deconfinement transition temperature $T_c \sim N_c^0$ independent of μ_B

$\Longrightarrow$ (nearly) horizontal transition line in $T - \mu_B$ diagram!

For $T \lesssim T_c$ and $\mu_B \gtrsim M_N$:

Fermi sea of (chirally restored?) quarks, but excitations around Fermi surface confined!

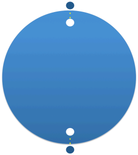
$\Longrightarrow$ "Quarkyonic" phase

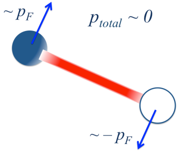
Excitons vs. Overhauser pairs

T. Kojo, Y. Hidaka, L. McLerran, R.D. Pisarski, NPA 843 (2010) 37

Exciton: particle-hole pair near the Fermi surface

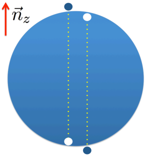
$\Longrightarrow$ Pair momentum $p_{\text{total}} \sim 0$, relative momentum large

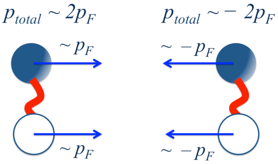




Overhauser pairing: particle-hole pair at opposite edges of the Fermi surface

$\Longrightarrow$ Pair momentum $p_{\text{total}} \sim 2p_F$, relative momentum small





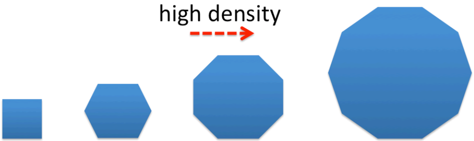
$\Longrightarrow$ 2 Overhauser pairs with momenta $p_{\text{total}} = \pm 2p_F \mathbf{n}_z$ create chiral spiral

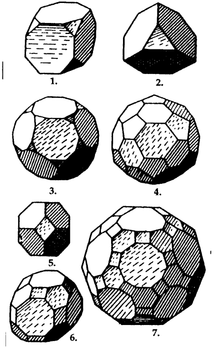
T. Kojo, Y. Hidaka, L. McLerran, R.D. Pisarski, NPA 843 (2010) 37

Quarkyonic phase:

– Fermi surface covered with patches

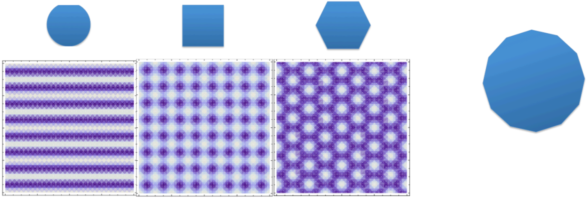
– No. of patches grows with density

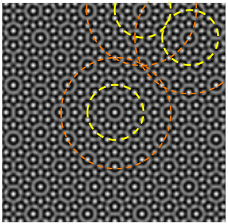




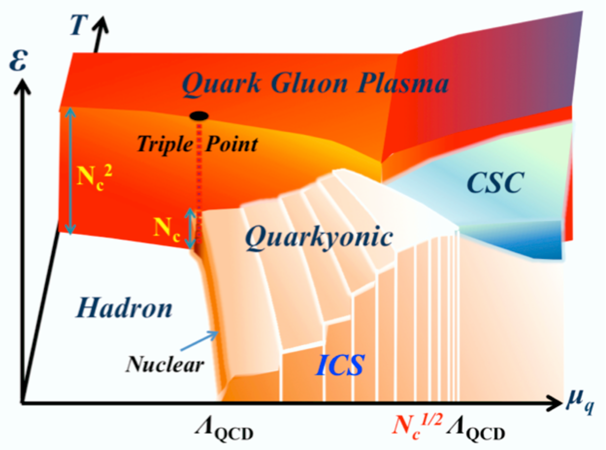
– Chiral spirals from Overhauser pairs at opposite patches

$\Longrightarrow$ Crystal structures from "interweaving chiral spirals" (ICS)





T. Kojo, Y. Hidaka, L. McLerran, R.D. Pisarski, NPA 843 (2010) 37



Lifshitz regime

Back to the real world of $N_c = 3$, $0 < m_q < \infty$:

– 1^{st} order deconfinement transition $\Longrightarrow$ crossover chiral transition

– Triple point $\Longrightarrow$ Lifshitz point

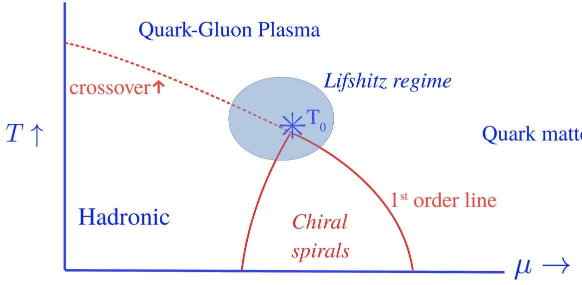
However: dispersion relation of critical fluctuations: $\omega^2(\mathbf{k}) = m^2 + Z\mathbf{k}^2 + \dfrac{(\mathbf{k}^2)^2}{\Lambda^2} + \ldots$

(NB: similar to inhomogeneous polymers)

Lifshitz point: $Z = m^2 = 0 \quad \Longrightarrow \quad \omega(\mathbf{k}) = \dfrac{\mathbf{k}^2}{\Lambda}$

$\Longrightarrow$ Lifshitz point washed out by long-range fluctuations $\Longrightarrow$ "Lifshitz regime"

R.D. Pisarski, V.V. Skokov, A.M. Tsvelik, arXiv:1801.08156 [hep-ph]



$\Longrightarrow$ Critical fluctuations ✓

$\Longrightarrow$ Critical point ✘

Conclusions

Raoul has made many important contributions towards our understanding of QCD at high density and a putative critical point

Current status of knowledge:

– Color superconductivity at asymptotically large μ_q

– Crystalline color superconductors in neutral and β-equilibrated matter at smaller μ_q

– Models and large–N_c QCD suggest existence of inhomogeneous phase(s) (RKC or ICS phase?)

– Critical point $\Longrightarrow$ Lifshitz point

– Lifshitz point likely washed out by fluctuations $\Longrightarrow$ Lifshitz regime

Open questions:

– Competition of color-superconducting and crystal phases

– How to detect Lifshitz regime experimentally?

– First-principles (lattice-QCD) calculations at nonzero μ_q