QCD - properties

confinement, Higgs mechanism

more on confinement

(i) the absence of free quarks in Nature

[but quarks could combine with a fundamental coloured scalar]

(ii) observable particles are colour singlets

[but this confuses confinement and screening (Higgs phase)]

(iii) quarks interact with a long range linear interaction

[obfuscated by string breaking]

(iv) the work required to separate quarks grows linearly as one takes the quark mass to infinity

[ok, but removes quarks from the definition!]

more on confinement

$$H = \frac{1}{2}\int d\mathbf{x}\,(E^2 + B^2) - \frac{1}{2}\int d\mathbf{x} d\mathbf{y}\, f^{abc} E^b(\mathbf{x}) A^c(\mathbf{x}) \langle \mathbf{x} a | \frac{g}{\nabla \cdot D} \nabla^2 \frac{g}{\nabla \cdot D} | \mathbf{y} d \rangle f^{def} E^e(\mathbf{y}) A^f(\mathbf{y})$$

$$K(\mathbf{x} - \mathbf{y}; \mathbf{A})$$

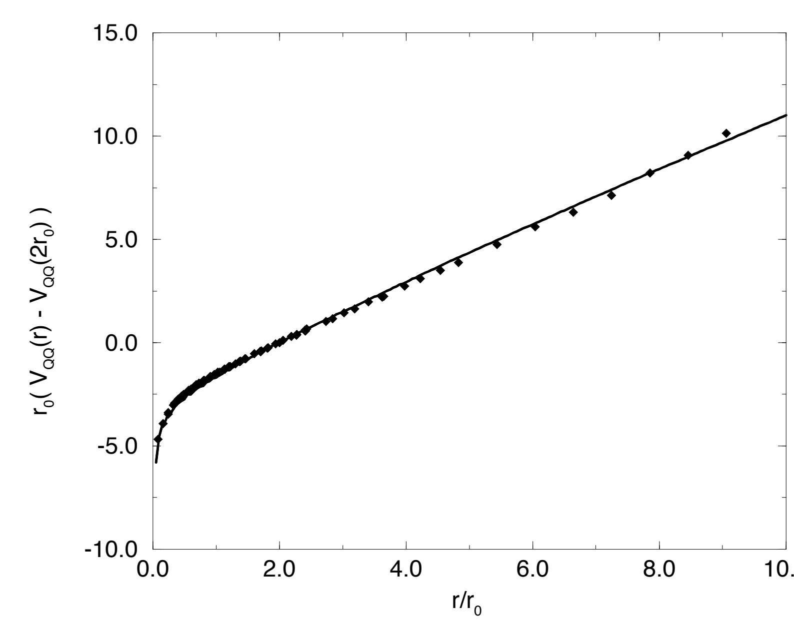
an instantaneous potential that depends on the gauge potential

K generates the beta function

K is renormalization group invariant

K is an upper limit to the Wilson loop potential

K is infrared enhanced at the Gribov boundary

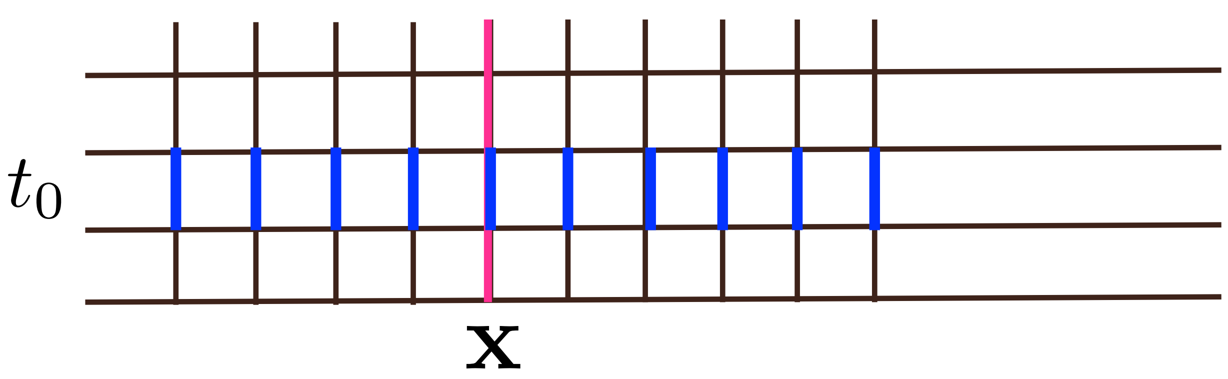


Szczepaniak & Swanson

more on confinement

$$U_t(t_0, \mathbf{x}) \to z U_t(t_0, \mathbf{x}) \quad \forall \mathbf{x}$$

a global symmetry of QCD



$$P(\mathbf{x}) \to z P(\mathbf{x})$$

more on confinement

either $\langle P(\mathbf{x}) \rangle = 0$ symmetric Z_N phase
or $\langle P(\mathbf{x}) \rangle \neq 0$ broken Z_N phase

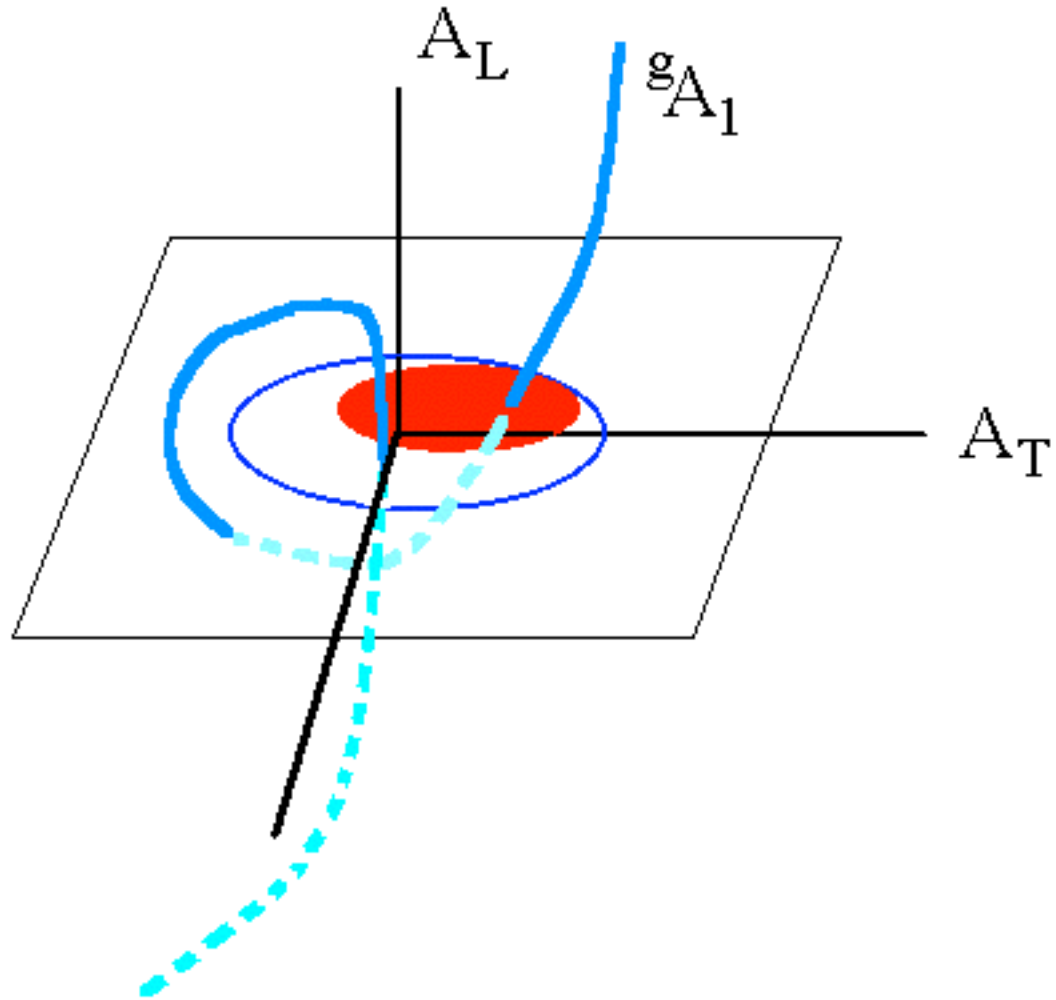
$$\langle P(\mathbf{x}) \rangle = \mathrm{e}^{-F_q T}$$

confinement iff QCD is in the symmetric Z_N phase

more on confinement

Coulomb gauge and the Gribov problem

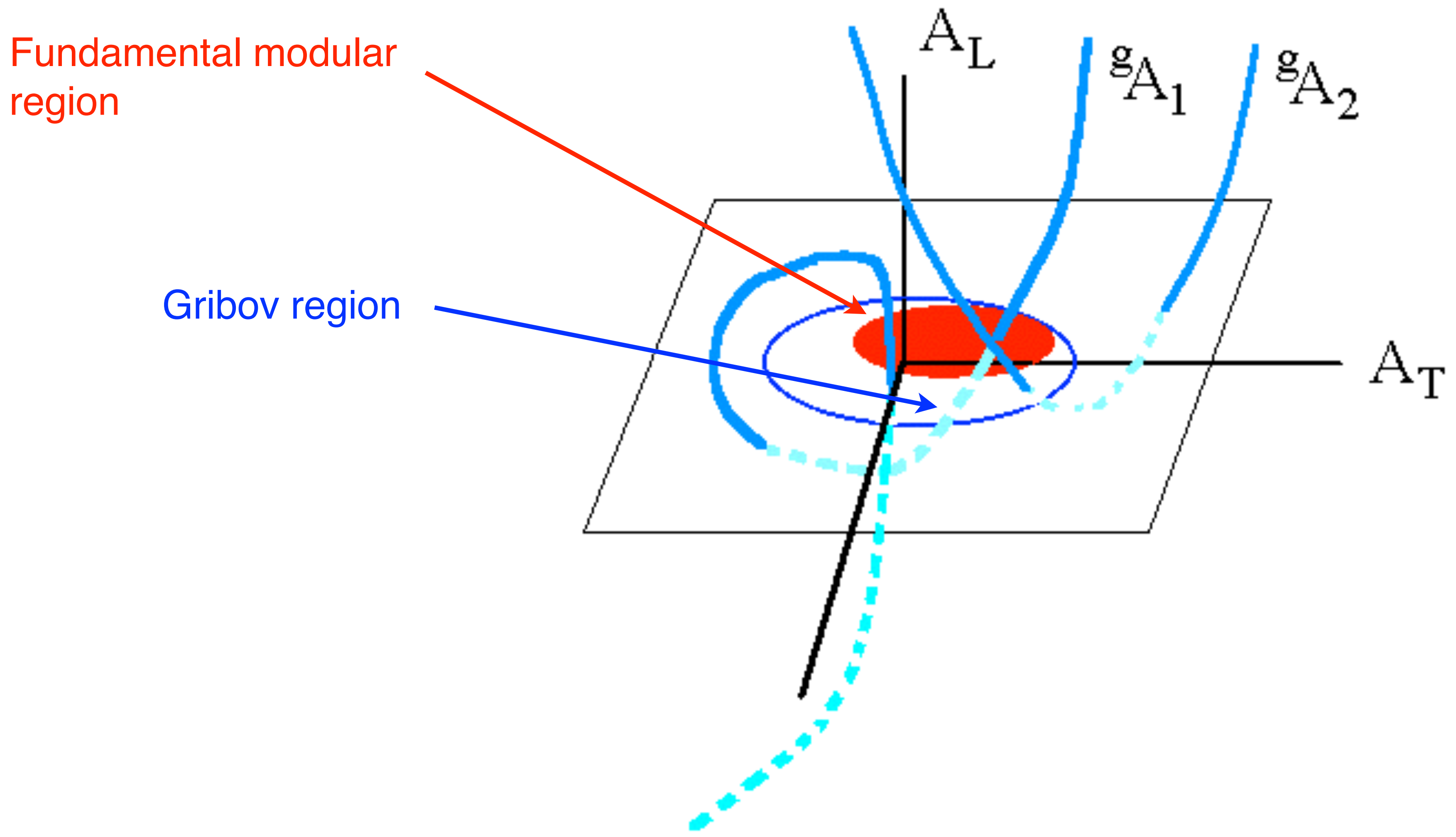
$$\nabla \cdot \vec{A}^a = 0 \qquad \det(\nabla \cdot D) = 0$$



more on confinement

Coulomb gauge and the Gribov problem

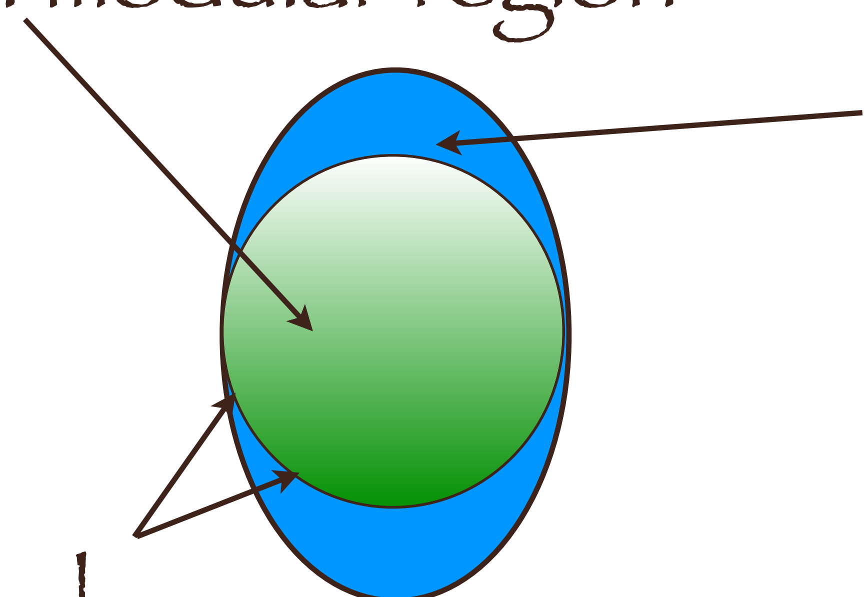
$$\nabla \cdot \vec{A}^a = 0 \qquad \det(\nabla \cdot D) = 0$$



more on confinement

Coulomb gauge and the Gribov problem

Fundamental modular region



Gribov region $\det(\nabla \cdot D) = 0$

identify boundary configurations

FMR is convex

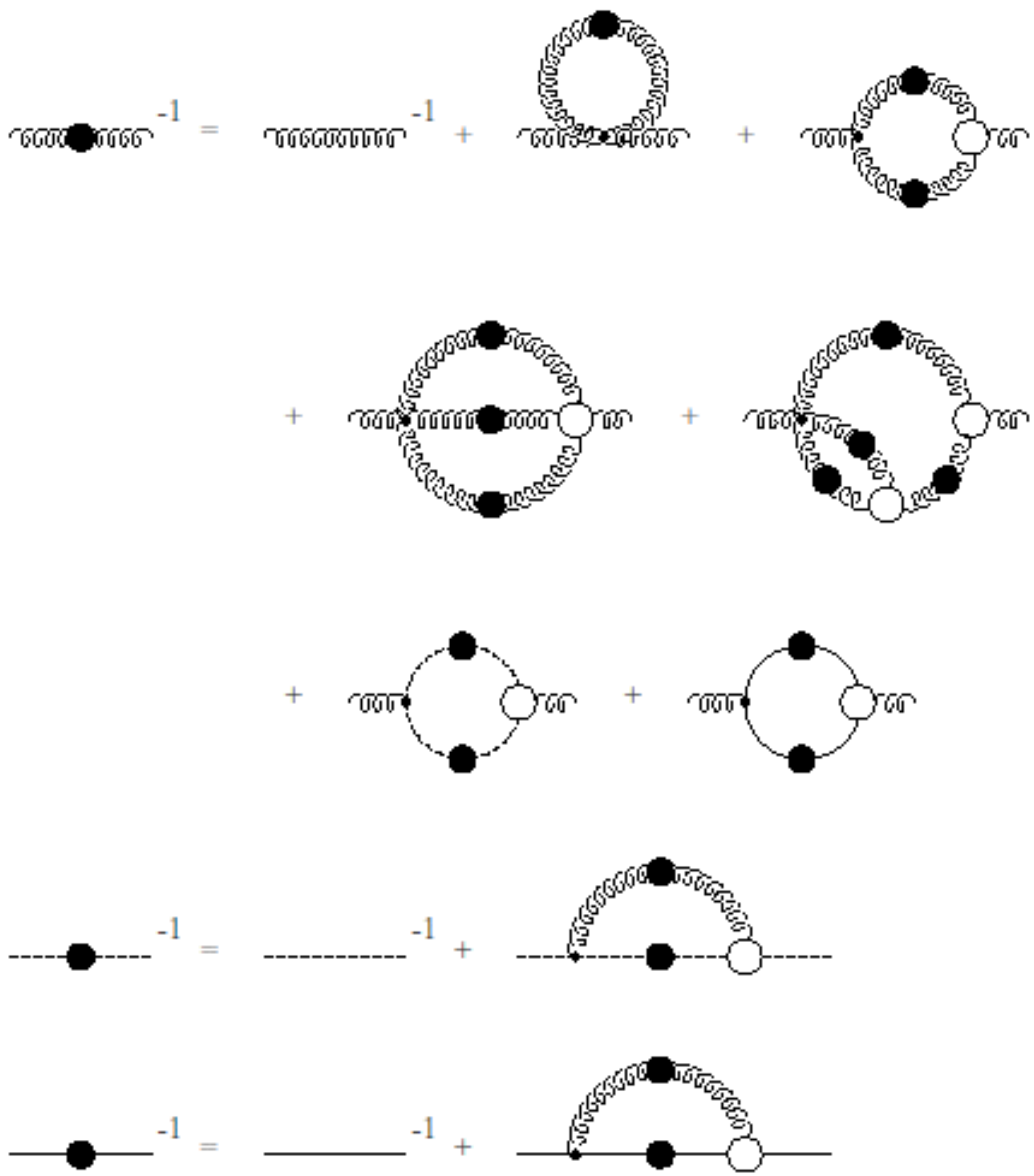
GR contains the FMR

FMR contains A=0

physics lies at the intersection of the FMR and GR

Zwanziger; van Baal

more on confinement



$$D(p) = -\frac{1}{p^2}\frac{1}{1+u(p)}$$

$$u(p) \to -1 \qquad p \to 0$$

Kugo-Ojima confinement criterion

Alkofer, von Smekal, Fischer

Z_N transformations are an example of "singular gauge transformations" (which are *not* gauge transformations!); namely they can be generated by performing a gauge transformation that is periodic modulo a Z_N phase factor:

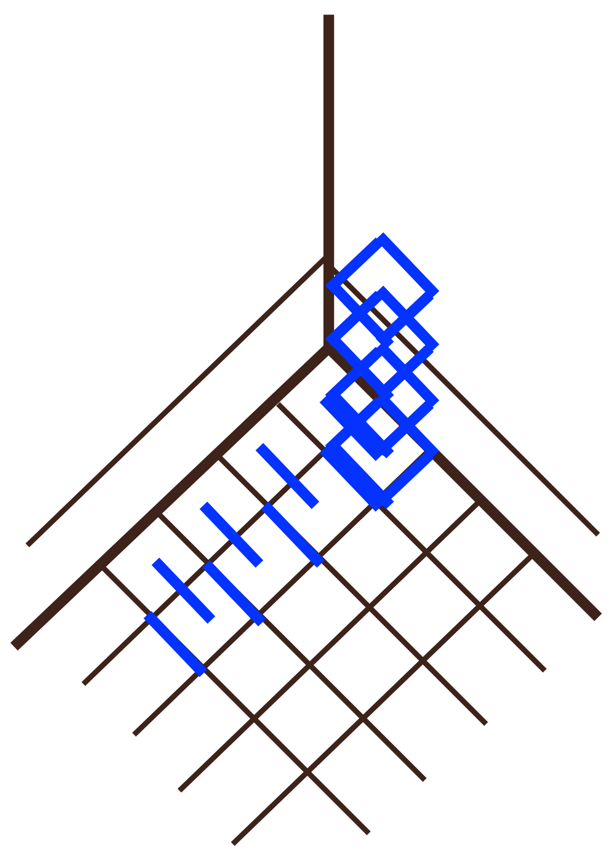
$$L(t, \vec{x}, \hat{t}) \rightarrow g(t, \vec{x})U(t, \vec{x}, \hat{t})g^{\dagger}(t+1, \vec{x}) \tag{24.90}$$

with $g(L, \vec{x}) = z^* g(0, \vec{x})$. For example the symmetry transformation $L(t_0, \vec{x}, \hat{t}) \rightarrow zL(t_0, \vec{x}, \hat{t})$ can be achieved by setting $g(t, \vec{x}) = 1$ for $t \leq t_0$ and $g(t, \vec{x}) = z^*$ for $t > t_0$. Singular gauge

It has been postulated that vortices drive confinement ('t Hooft, 1979) because they are localized field configurations which percolate (Chapter 8!) the lattice. The argument is quite general and relies on the fact that localized field distributions contribute independently to the expectation value of the Wilson loop operator, and hence yield an area law interaction.

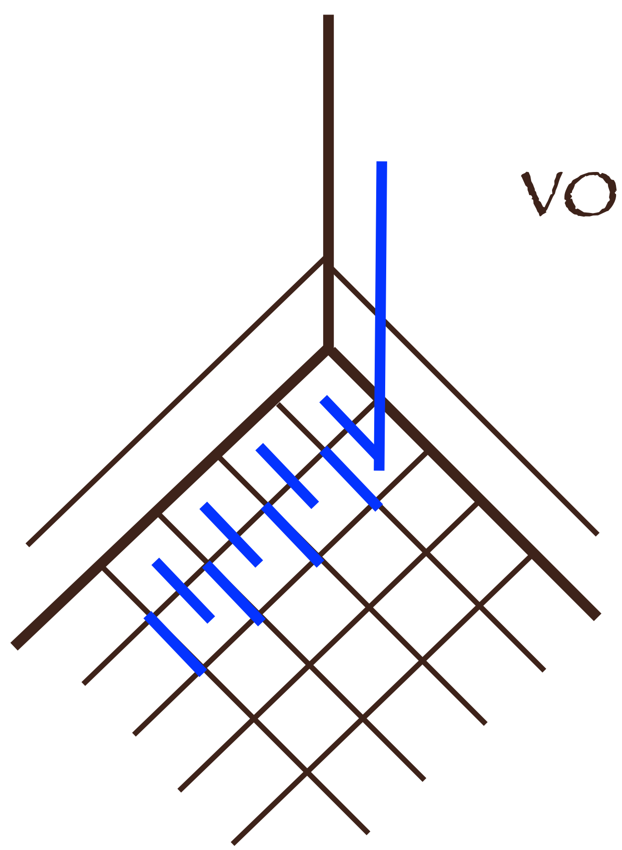
Vortex driven confinement

make a singular gauge transformation



Vortex driven confinement

make a singular gauge transformation



vortex (locates infinite field strength caused by the sgt)

Vortex driven confinement

$$\langle \mathrm{tr} U_{LR}(C) \rangle = \langle \mathrm{tr} \prod_{i=1}^{A/A_0} Z(i) \rangle$$

$$\langle Z(1) Z(2) \rangle \sim \langle Z(1) \rangle \langle Z(2) \rangle$$

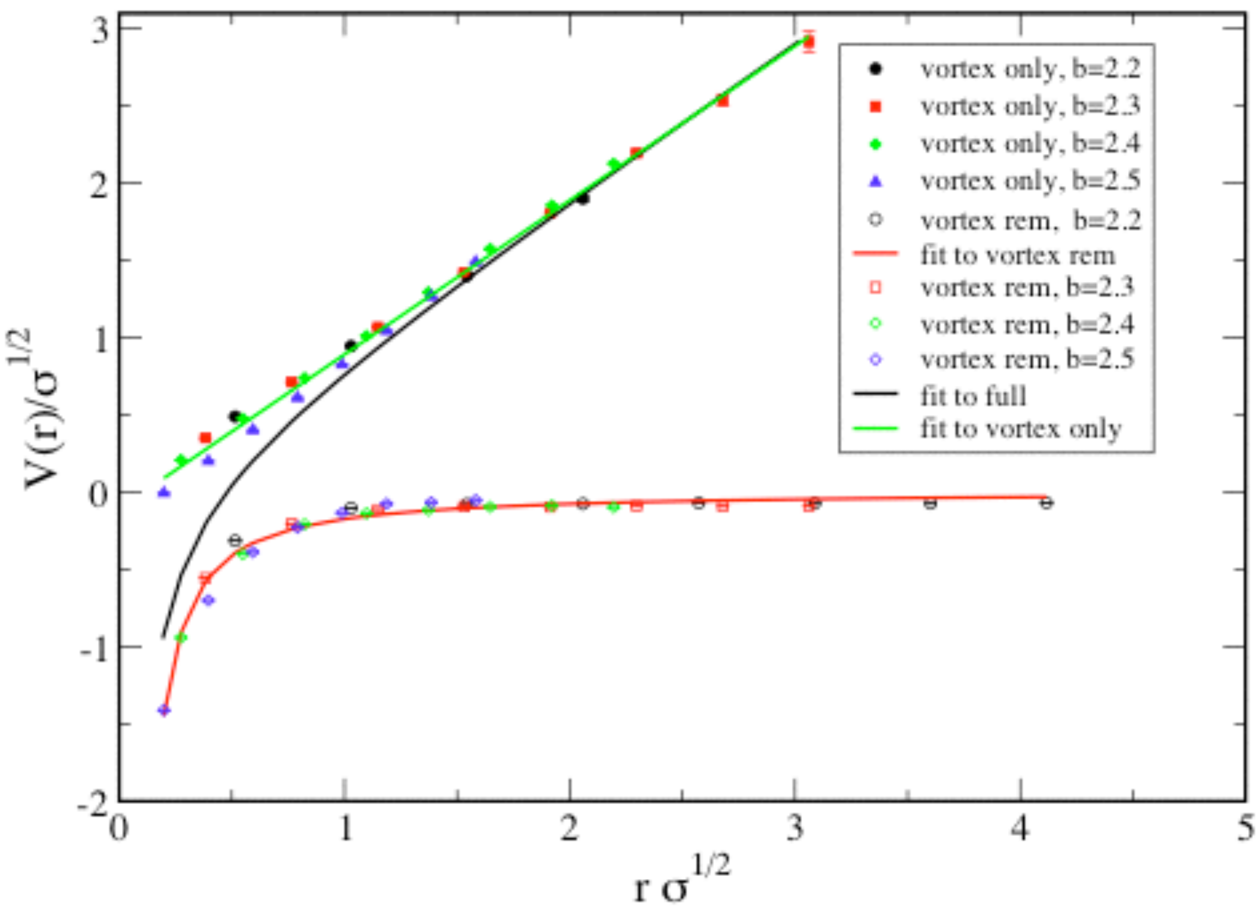
$$= \mathrm{tr} \prod_{i=1}^{A/A_0} \langle Z(i) \rangle$$

$$= \mathrm{e}^{-\sigma A}$$

$$\sigma = -\frac{\log \langle Z(1) \rangle}{A_0}$$

more on confinement

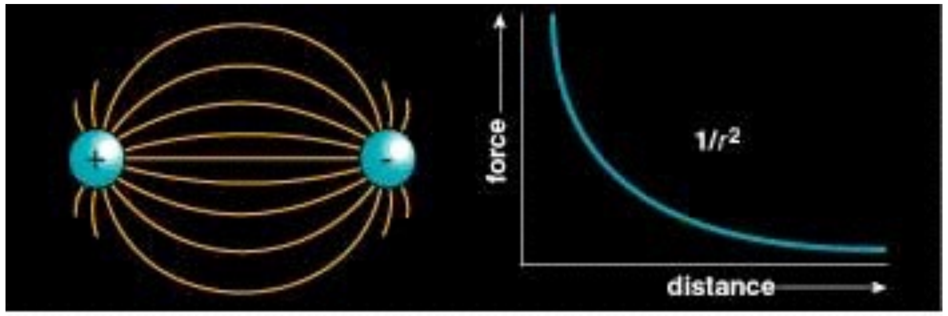
vortices!

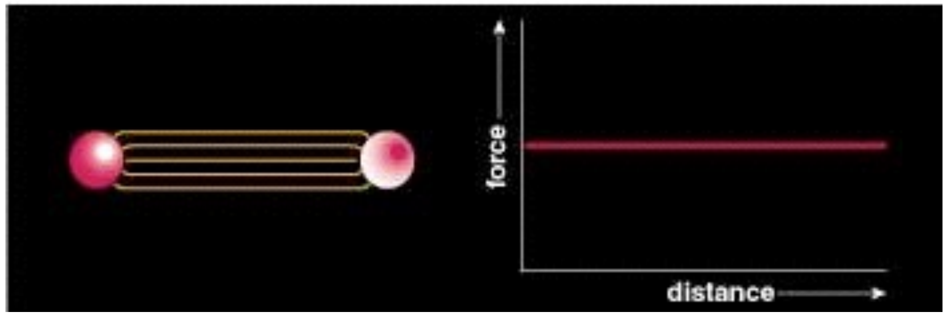


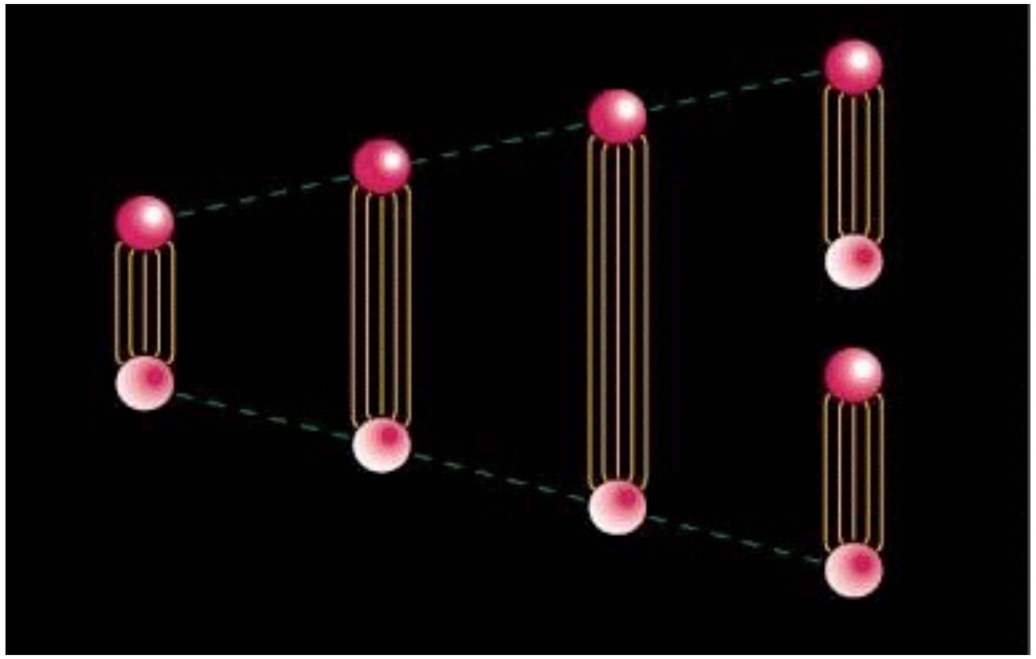
Greensite; Weise; de Forcrand; Reinhardt; Langfeld;...

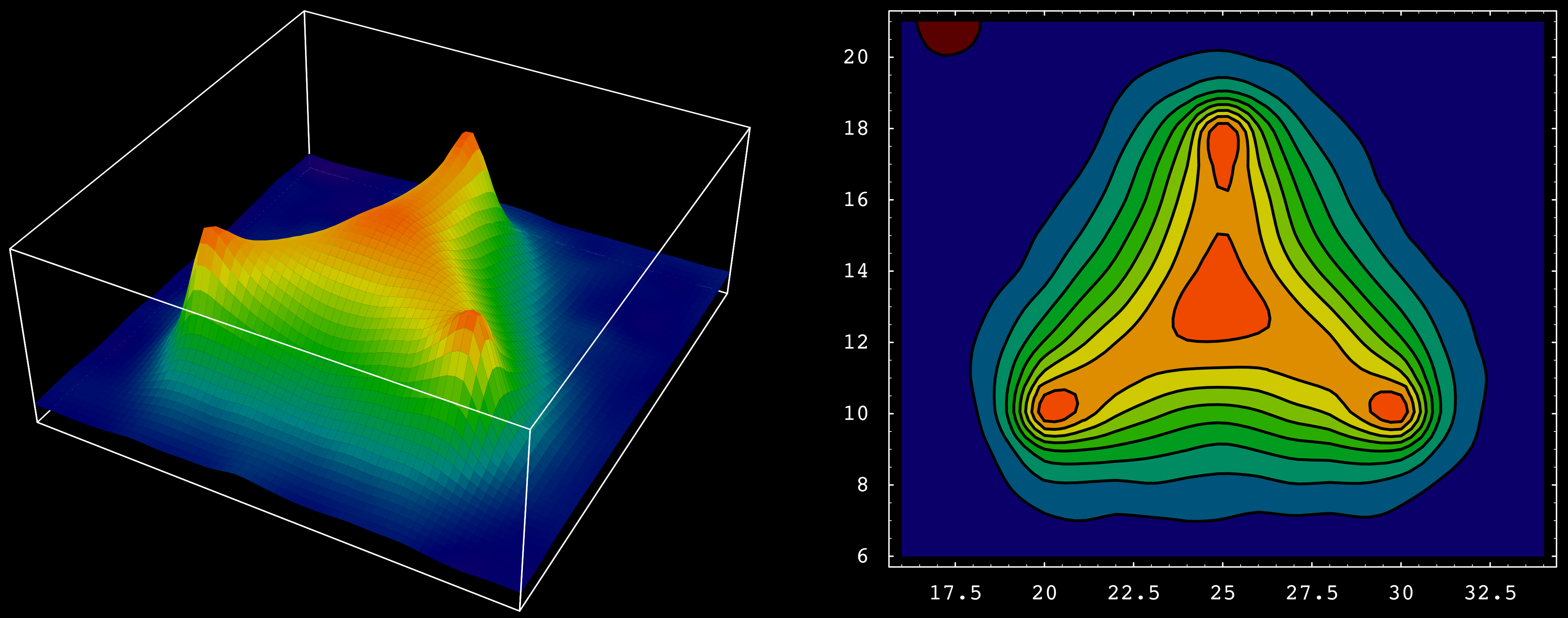
Lattice Gauge Theory

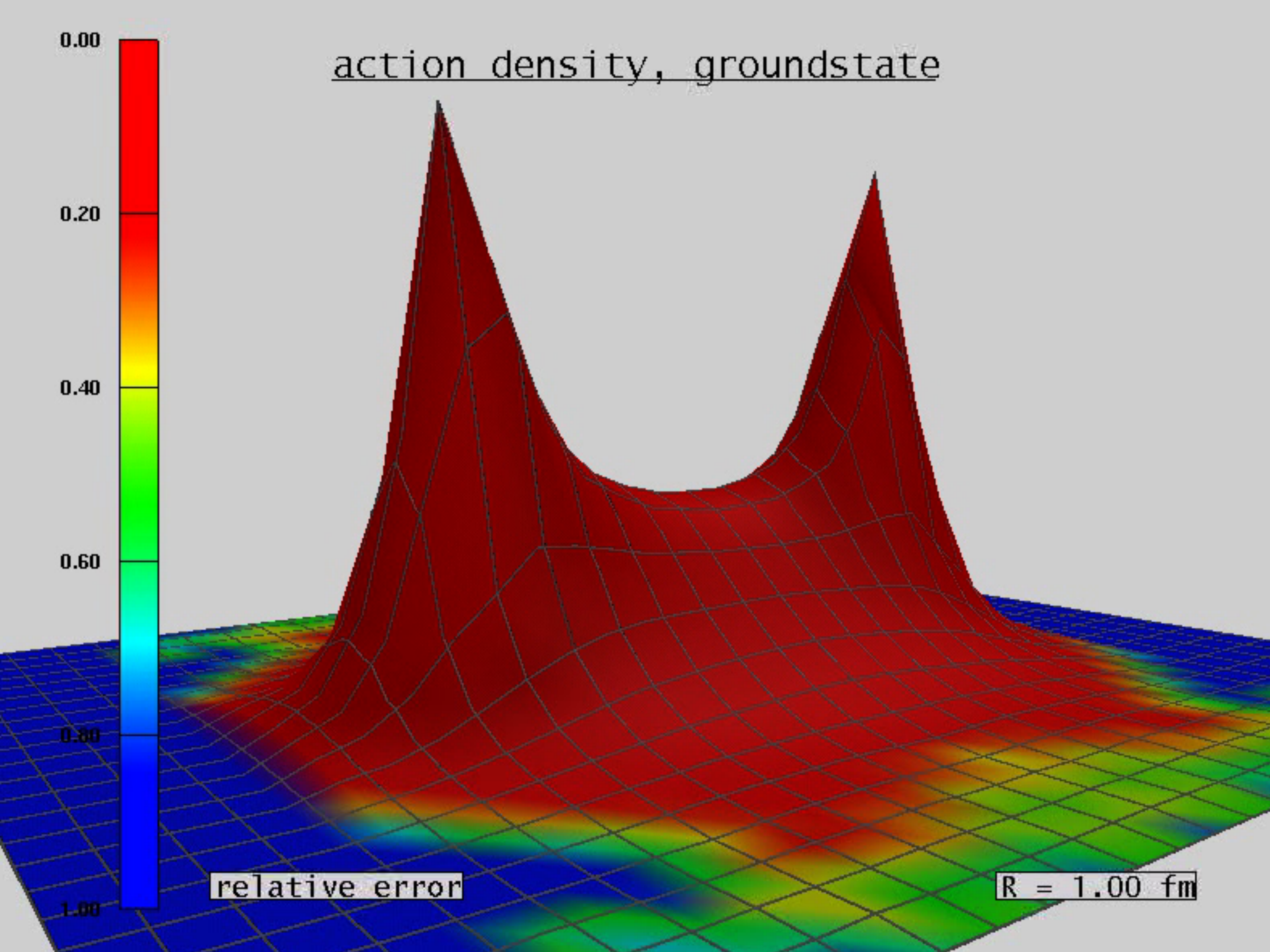
confinement cartoons



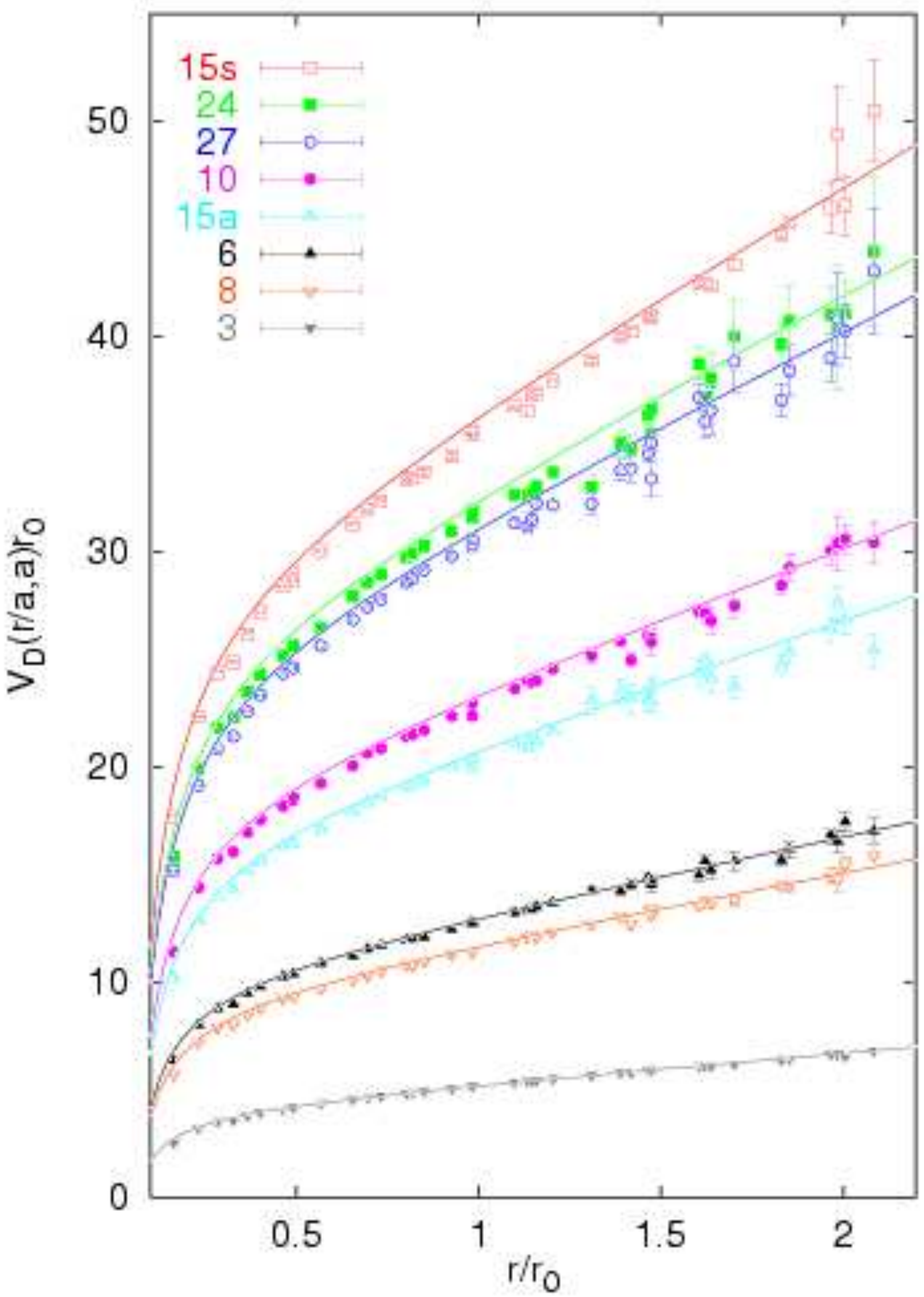





20
18
16
14
12
10
8
6
17.5
20
22.5
25
27.5
30
32.5


action density, groundstate
0.00
0.20
0.40
0.60
0.80
1.00
relative error
R = 1.00 fm

Lattice Gauge Theory



`Casimir scaling'

c= 18/3

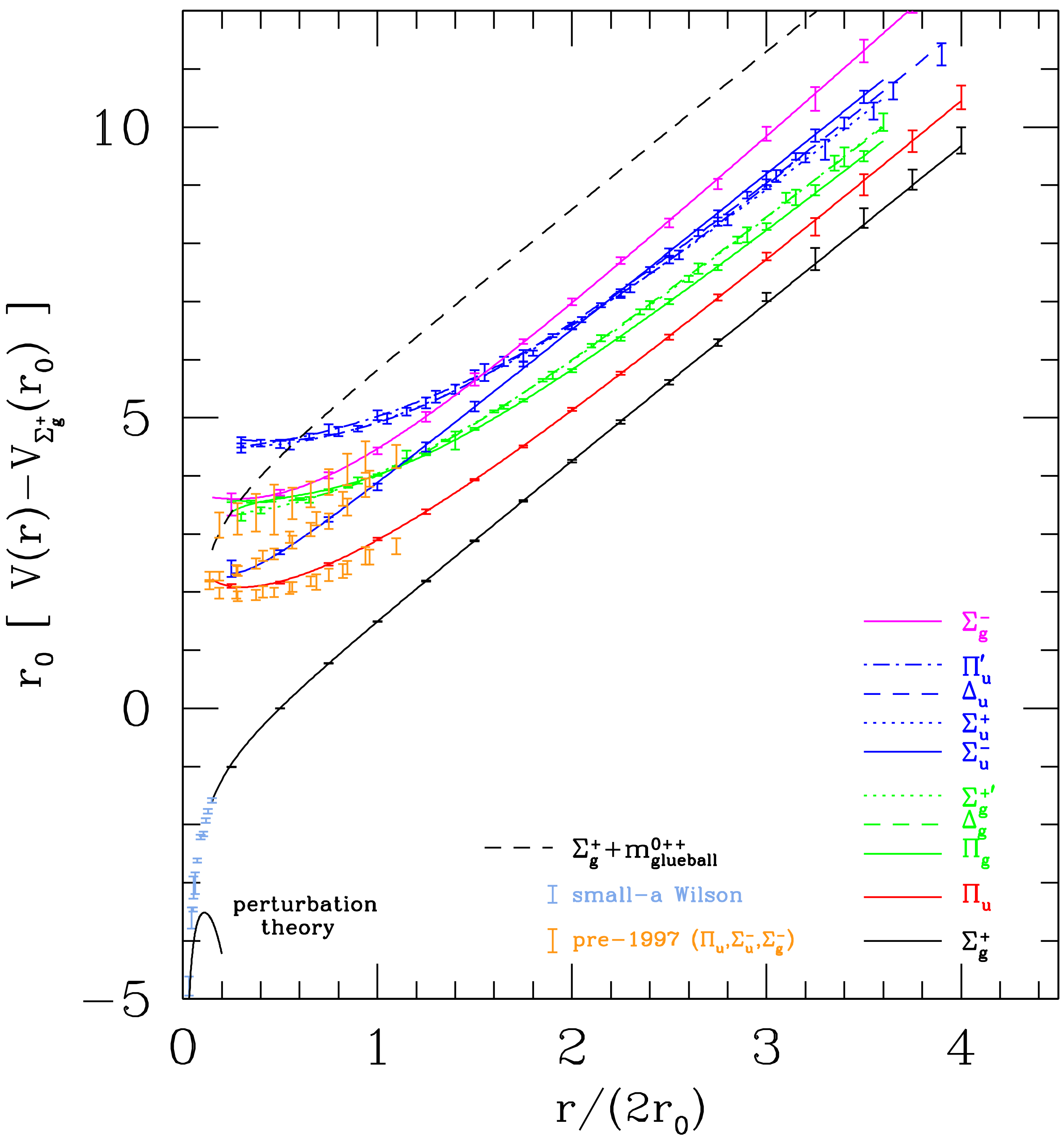
c= 16/3

c= 10/3

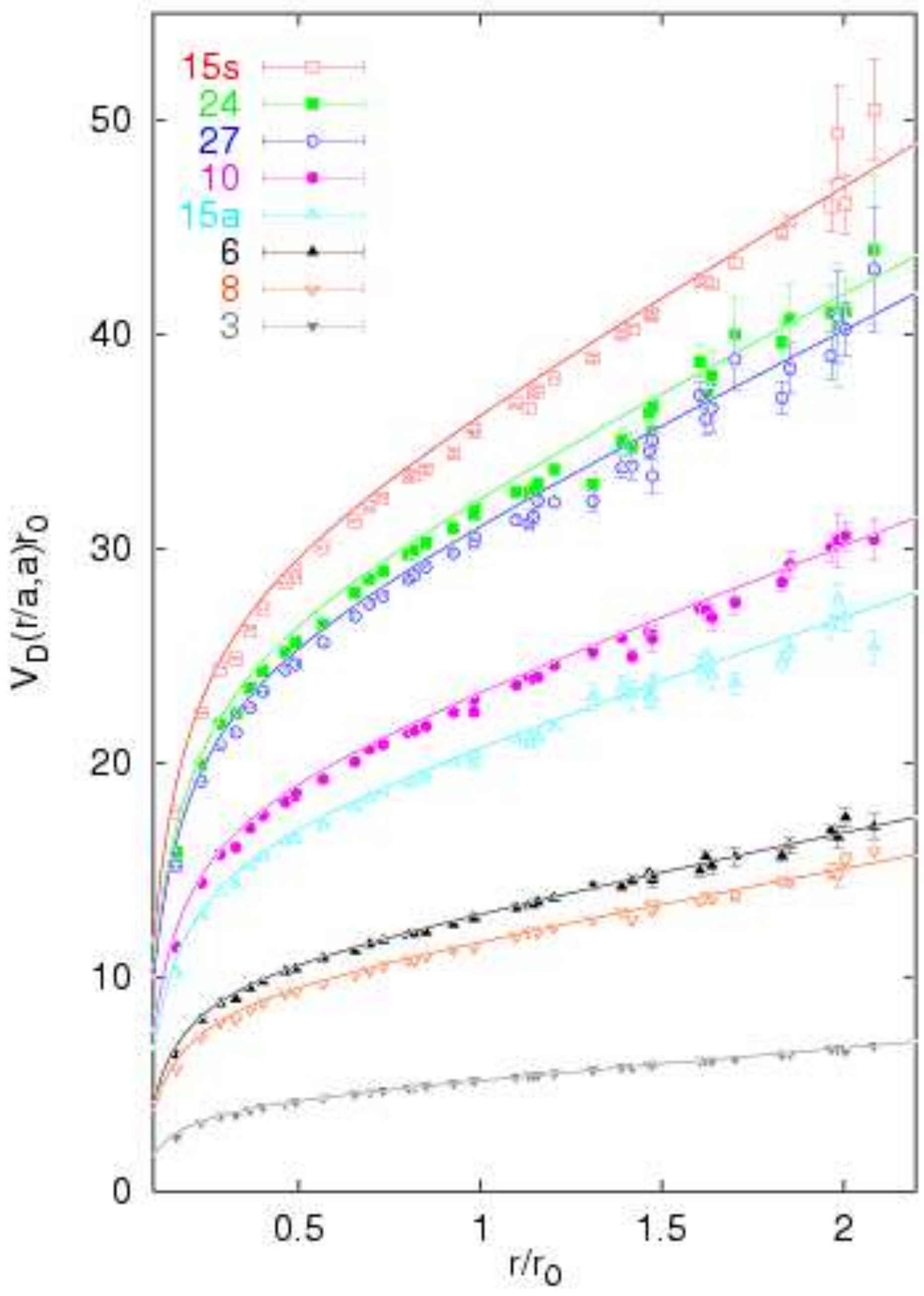
c= 3

c= 4/3

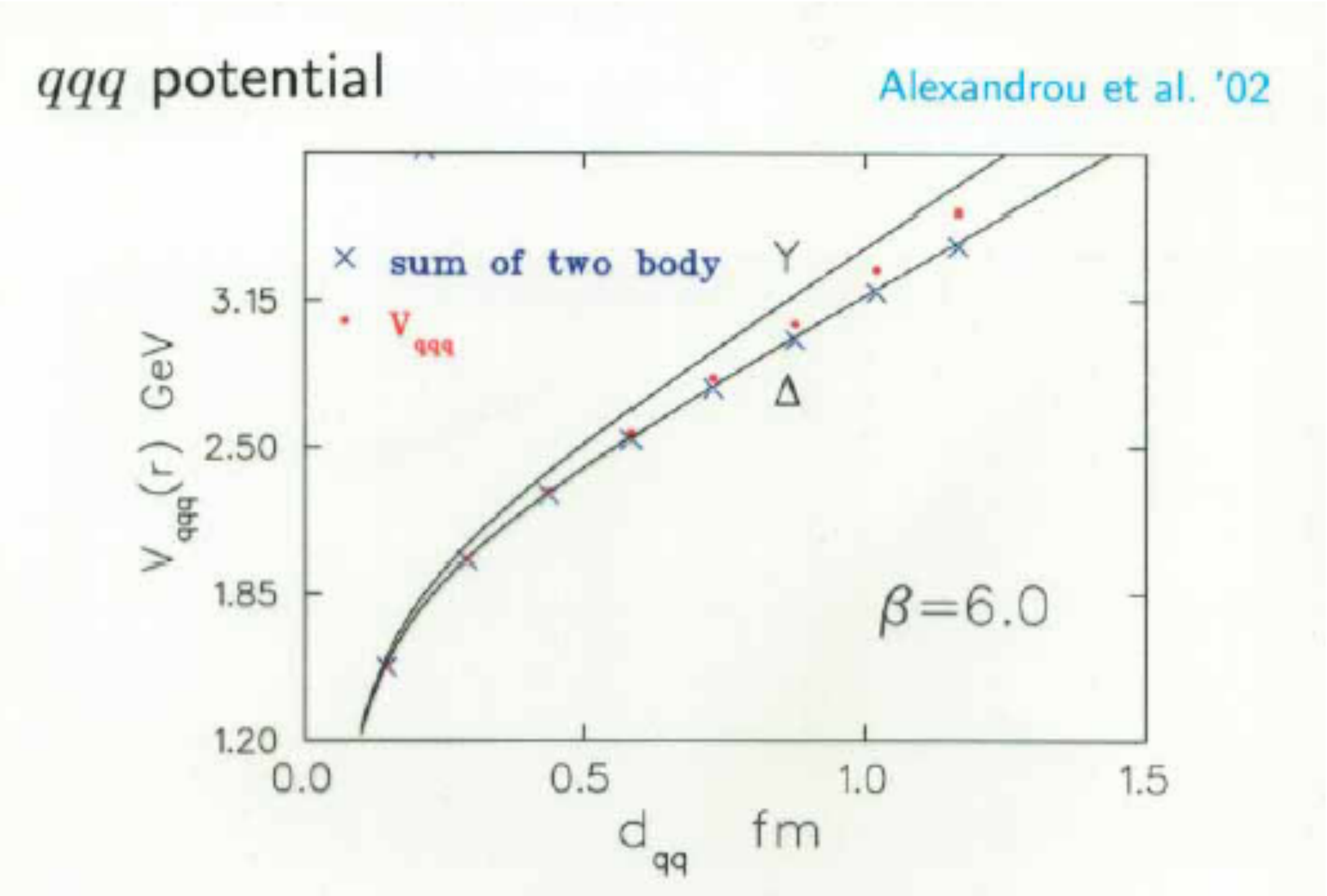
Lattice Gauge Theory



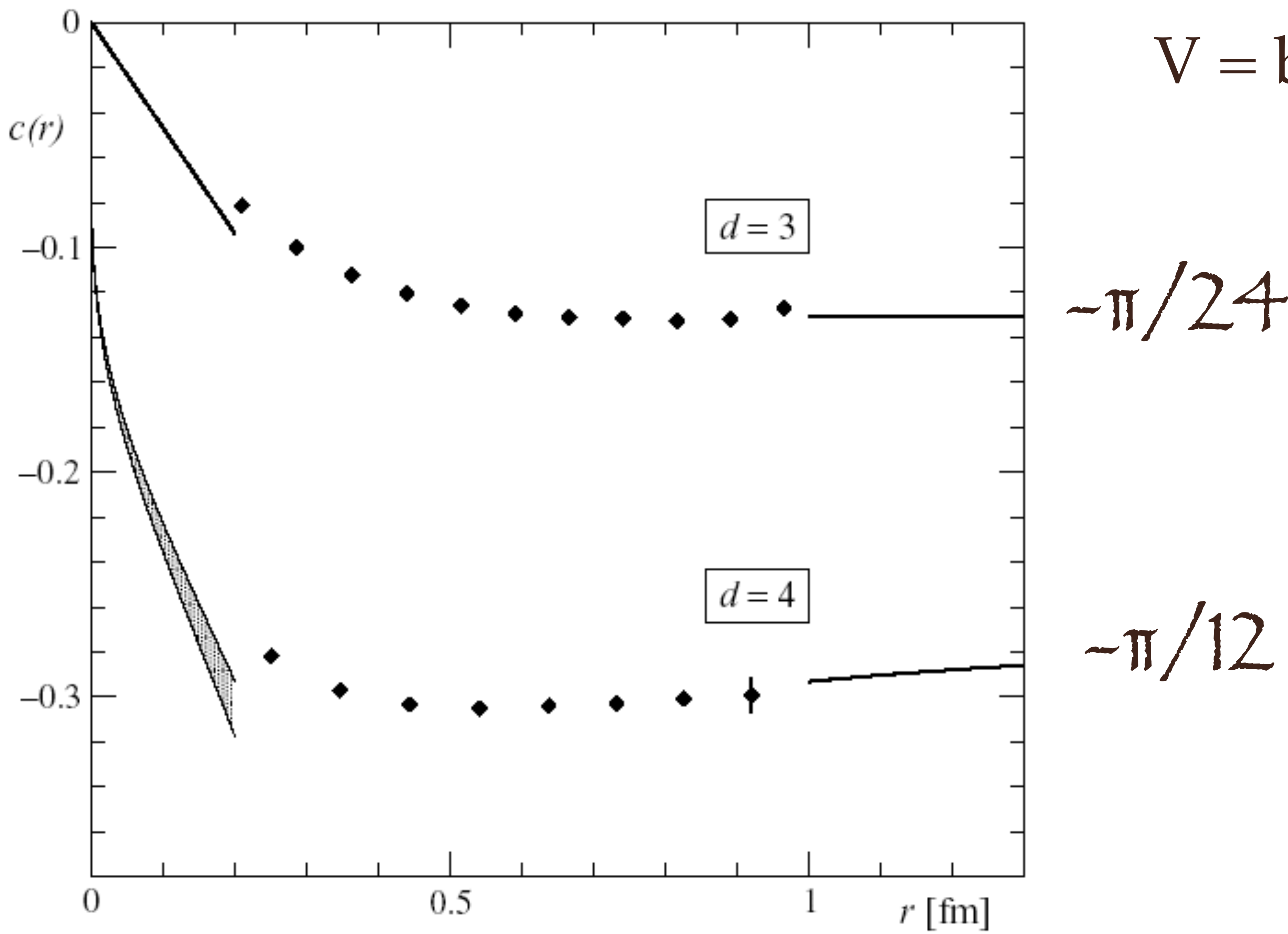
Properties: Confinement



Lattice Gauge Theory



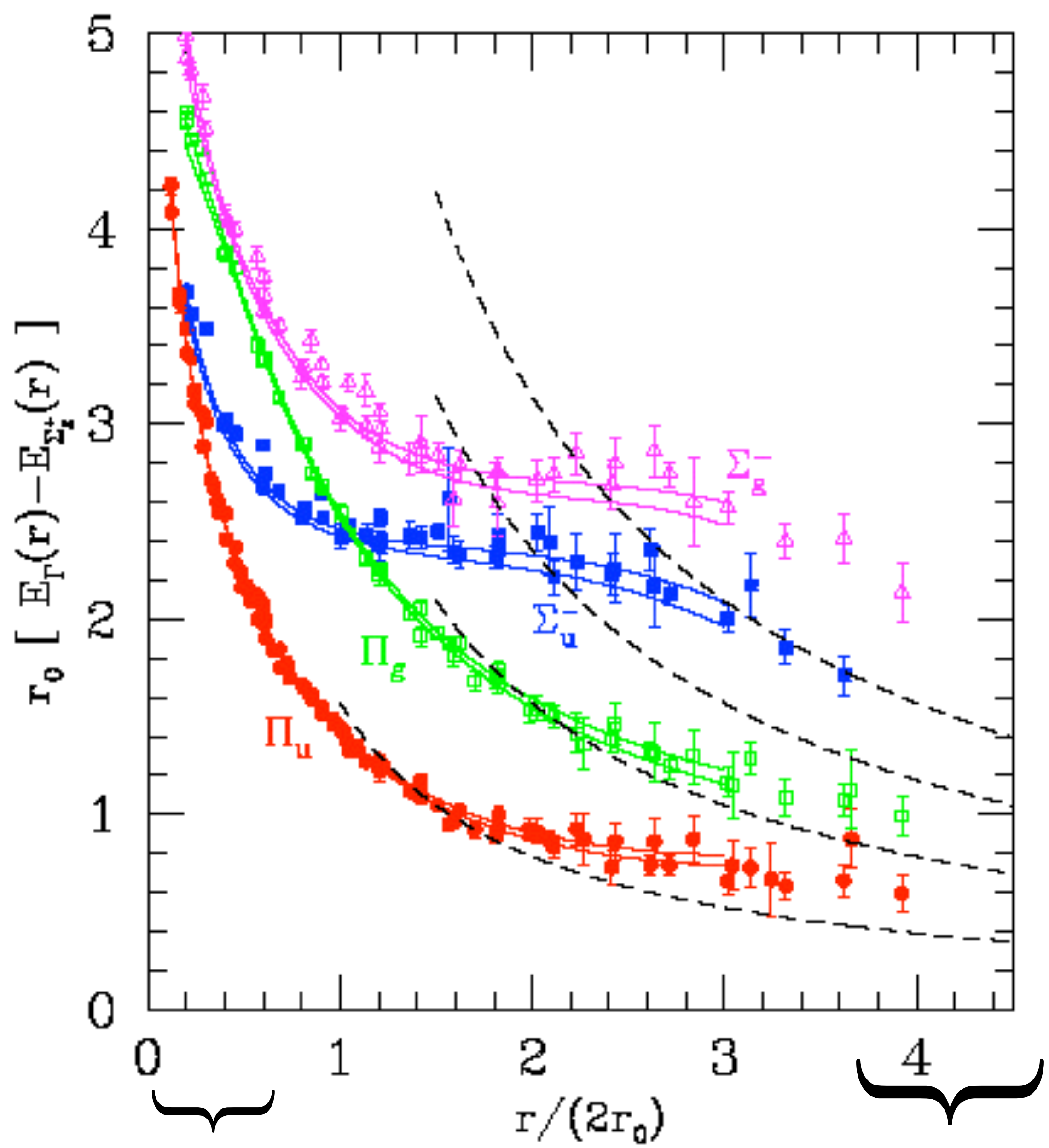
Lattice Gauge Theory



$V = br + c/r$

Lüscher & Weisz

Lattice Gauge Theory



short range intermediate long range

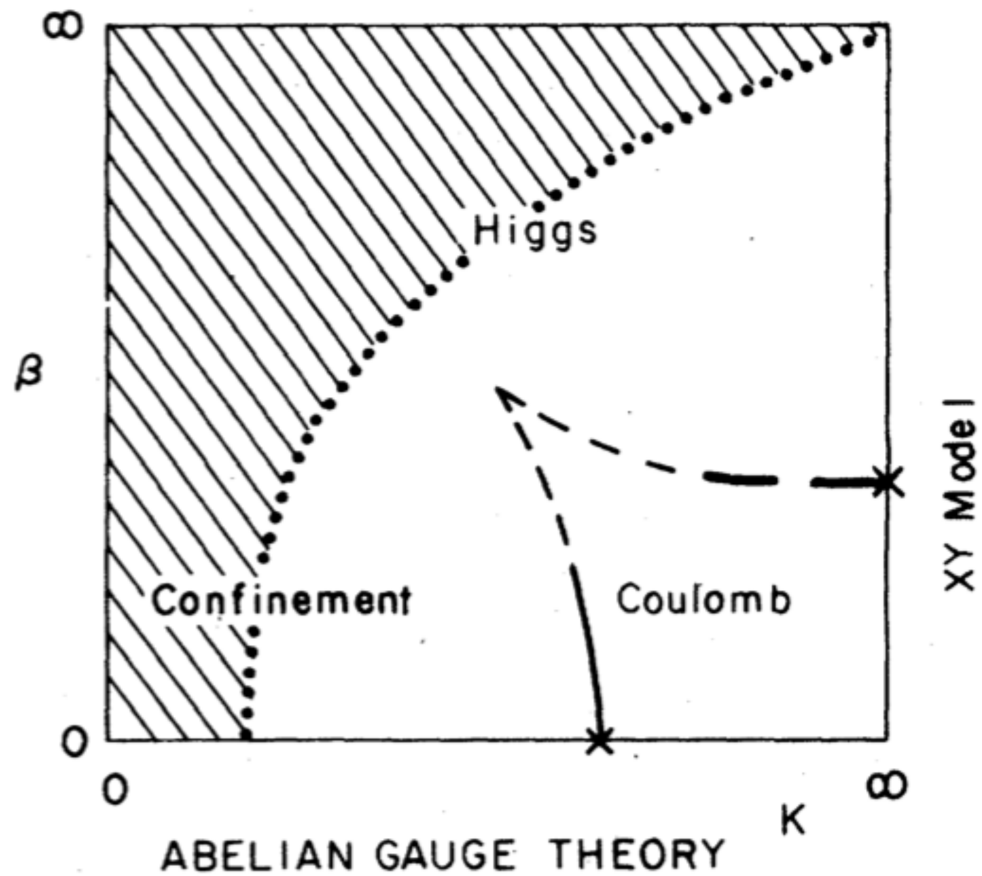
Juge, Kuti, & Morningstar

Confinement vs. Higgs Mechanism

Recall that Wegner's motivation in developing Z_2 gauge theory was an exploration of phase transitions in systems that do not magnetize. This point is underscored by ***Elitzur's theorem***, which states that it is impossible for a local gauge symmetry to break spontaneously (Elitzur, 1975). It is clear that the expectation value of a gauge-variant quantity like a link variable must be zero in the same way that an integral such as

$$\int d^3x\, \vec{x}\, f(|\vec{x}|)$$

must be zero because of the rotational symmetry of the function f and the measure. However, this statement applies to *finite* systems. What Elitzur succeeded in doing was showing that the matrix element remains zero even in the thermodynamic limit.



E. Fradkin and S. Shenker, PRD19, 3682 (1979)

QCD - functional approaches

What is Nonperturbative?

$$F = 1 + x + x^2 + \ldots$$

$$F \equiv \frac{1}{1-x}$$

not this...

$$F = \mathrm{e}^{-1/x}$$

$$F = 0 + 0 + 0 + \ldots$$

$$x^2 F' = F$$

NONPERTURBATIVE QUANTITIES

- electron mass in a massless theory
- confinement
- symmetry breaking

THE GENERATING FUNCTIONAL

$$Z[J] = \int D\varphi \mathrm{e}^{iS+i\int d^4x\, J(x)\varphi(x)}$$

$$\frac{\delta}{\delta f(x)} f(y) = \delta(y-x)$$

Z and its derivatives contain all the information in this quantum field theory

THE GENERATING FUNCTIONAL

$$Z[J] = \int D\varphi \mathrm{e}^{iS + i\int d^4x\, J(x)\varphi(x)}$$

$$S = \int d^4x\, \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2 - \frac{\lambda}{24}\varphi^4$$

$$\int D\varphi \mathrm{e}^{\frac{i}{2}\int\int \varphi_1 M(12)\varphi_2 + i\int J\varphi} = (\det M)^{-1/2}\mathrm{e}^{-\frac{i}{2}\int\int J_1(M^{-1})(12)J_2}$$

(the only integral we can do!)

THE TWO-POINT FUNCTION

$$Z[J] = \int D\varphi \mathrm{e}^{iS + i\int d^4x\, J(x)\varphi(x)}$$

$$\begin{aligned}
\langle \varphi_1 \varphi_2 \rangle_J &= \frac{1}{Z[0]} \int D\varphi \varphi_1 \varphi_2 \mathrm{e}^{iS + i\int J\varphi} \\
&= \frac{\langle 0| T[\varphi_I(x_1)\varphi_I(x_2)\mathrm{e}^{-i\int V_I}]|0\rangle}{\langle 0|\mathrm{e}^{-i\int V_I}|0\rangle}
\end{aligned}$$

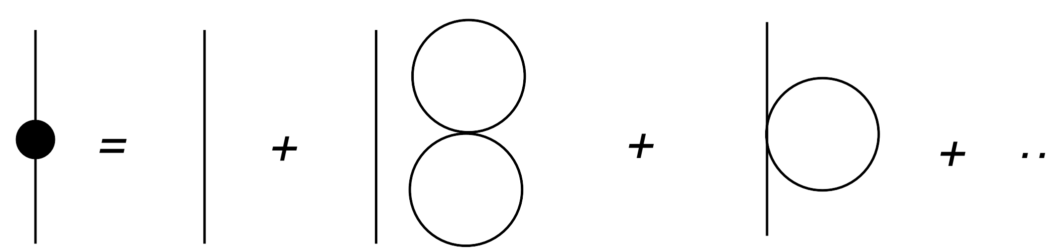
THE TWO-POINT FUNCTION

$$\langle\varphi_1\varphi_2\rangle^{(0)}_{J=0} = i\int \frac{d^4k}{(2\pi)^4}\,\frac{\mathrm{e}^{-ik\cdot(x_1-x_2)}}{k^2-m^2+i\epsilon} \equiv \overset{(0)}{\Delta}(x_1-x_2)$$

$$(\partial_x^2+m^2)\overset{(0)}{\Delta}(x-y) = -i\delta^4(x-y)$$

CONNECTED GREENS FUNCTIONS

$$\Delta(x-y) = \mathrm{e}^{-i\int V[\frac{\delta}{i\delta J}]} Z^{(0)}[J]$$



$$\mathrm{e}^{iF[J]} = Z[J]$$

$$\int D\varphi \mathrm{e}^{iS} = \langle\Psi_0|\mathrm{e}^{-iHT}|\Psi_0\rangle$$

$$F = -E_0$$

LEGENDRE TRANSFORMATION

$$\varphi_{cl}(x) \equiv \frac{\langle\varphi(x)\rangle}{\langle 1\rangle} = \frac{1}{Z[0]}\frac{\delta}{i\delta J(x)}Z[J] = \frac{\delta}{\delta J(x)}F[J]$$

$$\varphi_{cl}^{(0)}(x) = i\Delta(x-y)J_y$$

$$(\partial_x^2 + m^2)\varphi_{cl}^{(0)}(x) = J_x$$

$$\Gamma^{(0)}[\varphi_{cl}^{(0)}] = F^{(0)}[J] - J_x\varphi_{cl}^{(0)}(x)$$

$$\Gamma^{(0)}[\varphi_{cl}^{(0)}] = \frac{1}{2}\int d^4x\ \left(\partial_\mu\varphi_{cl}\partial^\mu\varphi_{cl} - m^2\varphi_{cl}^2\right)$$

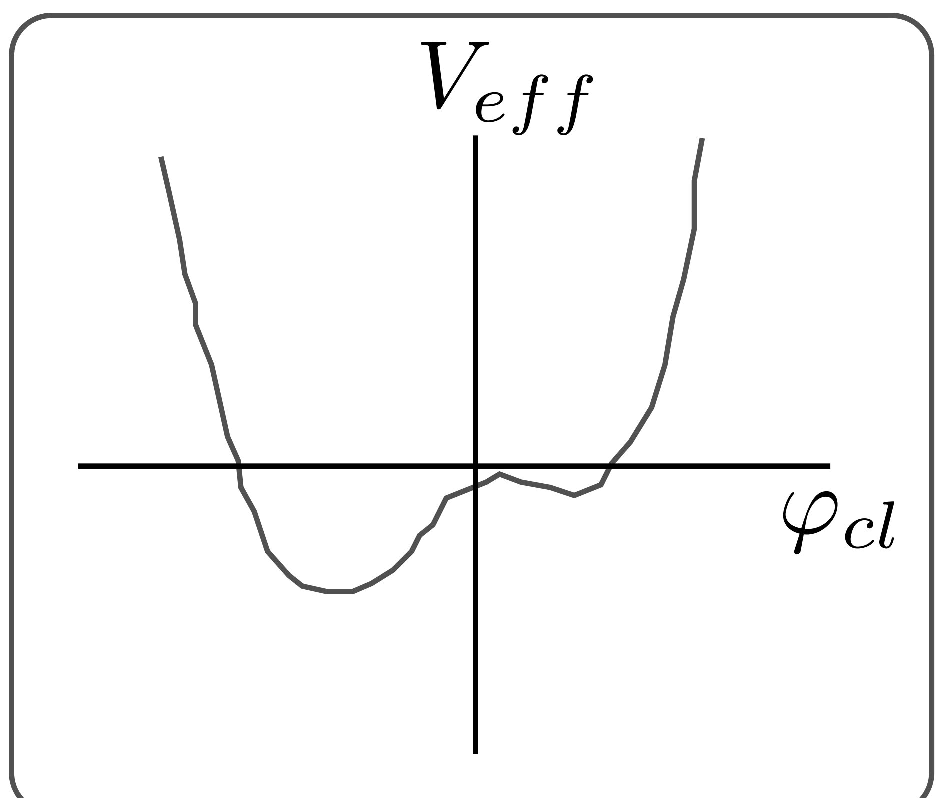
LEGENDRE TRANSFORMATION

$$\begin{aligned} \varphi_{cl} &\equiv \frac{\delta}{\delta J} F[J] \\ \Gamma[\varphi_{cl}] &\equiv F[J] - J_x \varphi_{cl}(x) \\ \frac{\delta \Gamma}{\delta \varphi_{cl}(x)} &= -J(x) \end{aligned}$$

THE EFFECTIVE ACTION

$$\Gamma[\varphi_{cl}] = -(TL^3)V_{eff}(\varphi_{cl})$$

$$\frac{\partial V_{eff}}{\partial \varphi_{cl}}|_{J=0} = 0.$$

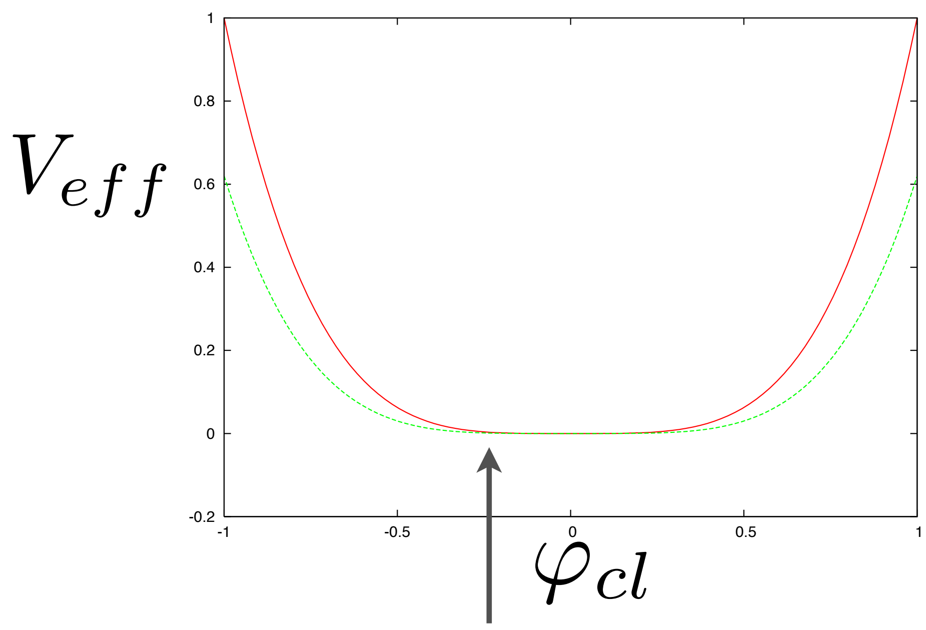


THE EFFECTIVE ACTION

$$\Gamma[\varphi_{cl}] = -(TL^3)V_{eff}(\varphi_{cl})$$

$$\frac{\partial V_{eff}}{\partial \varphi_{cl}}|_{J=0} = 0.$$

$$V_{eff}^{(0)} = \frac{1}{4}\lambda\varphi_{cl}^4$$



$$V_{eff} = \frac{1}{4}\varphi_{cl}^4\left[\lambda + \frac{\lambda^2}{16\pi^2}((N+8)\log(\lambda\varphi_{cl}^2/M^2) - \frac{3}{2}) + 9\log 3\right].$$

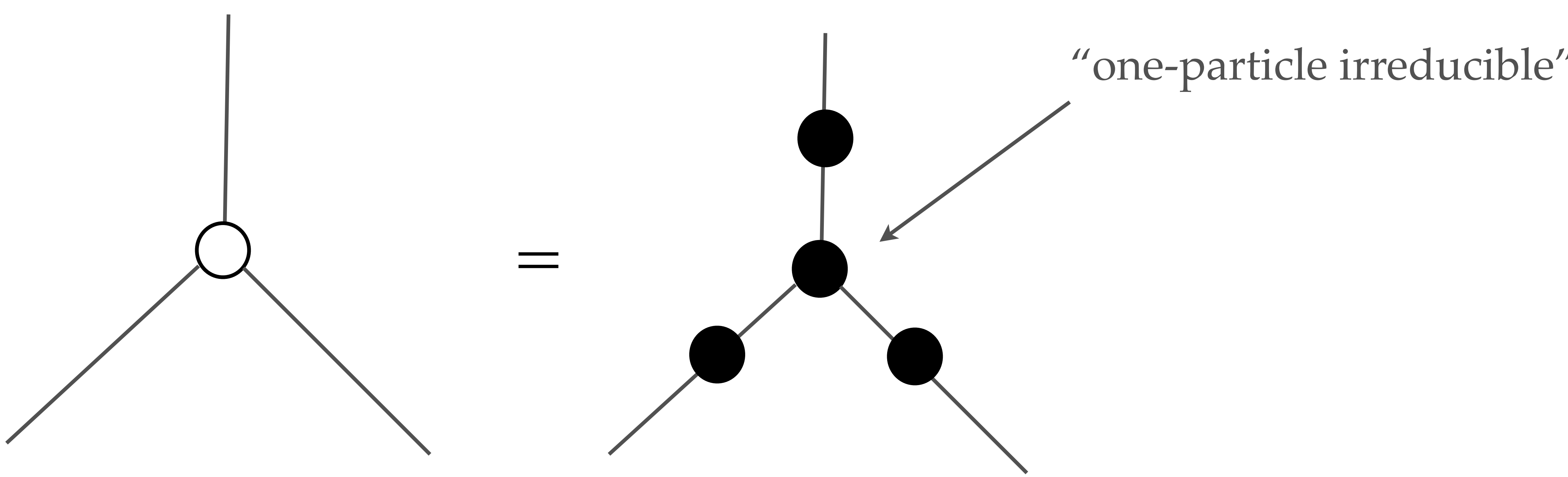
1PI greens functions

$$\delta(x-z) = \frac{\delta\phi_c(x)}{\delta\phi_c(z)} = \int d^4y \frac{\delta^2 F}{\delta J(x)\delta J(y)} \frac{J(y)}{\delta\phi_c(z)} = -\int d^4y \frac{\delta^2 F}{\delta J(x)\delta J(y)} \frac{\delta^2\Gamma}{\delta\phi_c(z)\delta\phi_c(y)}$$

$$\frac{\delta^2\Gamma}{\delta\phi_c(x)\delta\phi_c(z)} = -\left(\frac{\delta^2 F}{\delta J(x)\delta J(z)}\right)^{-1}$$

$$\frac{\delta^2\Gamma}{\delta\phi_c(x)\delta\phi_c(z)}\Big|_{\phi_c=0} = iS^{-1}(x-z)$$

1PI Greens functions



Schwinger-Dyson Master Equation

$$\int D\varphi \left[\frac{\delta S}{\delta\varphi} + J\right] \mathrm{e}^{iS+iJ_x\varphi_x} = 0$$

$$\mathrm{e}^{-iF} \frac{\delta S}{\delta\varphi(x)} \left[\frac{\delta}{i\delta J}\right] \mathrm{e}^{iF} = -J(x)$$

$$\frac{\delta S}{\delta\varphi(x)} \left[\frac{\delta}{i\delta J} + \frac{\delta F}{\delta J}\right] \cdot 1 = -J(x)$$

$$\frac{\delta}{i\delta J} = \frac{\delta\varphi_{cl}(z)}{i\delta J} \frac{\delta}{\delta\varphi_{cl}(z)} \qquad\qquad \frac{\delta\varphi_{cl}(z)}{\delta J} = \frac{\delta^2 F}{\delta J \delta J(z)}$$

Schwinger-Dyson Master Equation

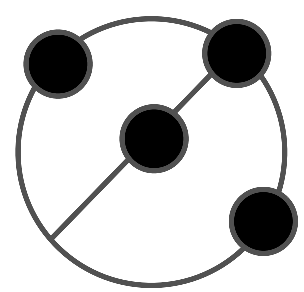
$$\frac{\delta S}{\delta \varphi}\left[\varphi_{cl} + i\left(\frac{\delta^2 \Gamma}{\delta\varphi_{cl}\delta\varphi_{cl}}\right)^{-1}_{.z}\frac{\delta}{\delta\varphi_{cl}(z)}\right]\cdot 1 = \frac{\delta\Gamma}{\delta\varphi_{cl}}$$

$$\frac{\delta S}{\delta \varphi} = -(\partial^2 + m^2)\varphi - \frac{\lambda}{6}\varphi^3$$

$$-(\partial^2 + m^2)\varphi_{cl}(x) - \frac{\lambda}{6}(\varphi_{cl}(x) + \Delta_{xz}\frac{\delta}{\delta\varphi_{cl}(z)})^3 \cdot 1 = \frac{\delta\Gamma}{\delta\varphi_{cl}(x)}$$

Schwinger-Dyson Master Equation

$$-(\partial^2 + m^2)\varphi_x - \frac{\lambda}{6}\left(\varphi_x^3 + 3\Delta_{xx}\varphi_x + \Delta_{xa}\Delta_{xb}\Delta_{xc} i\Gamma_{abc}\right) = \frac{\delta\Gamma}{\delta\varphi_x}$$



$= 0$

$$\Gamma_{abc} = 0$$

Propagator

$$-(\partial^2 + m^2)\varphi_x - \frac{\lambda}{6}\left(\varphi_x^3 + 3\Delta_{xx}\varphi_x + \Delta_{xa}\Delta_{xb}\Delta_{xc} i\Gamma_{abc}\right) = \frac{\delta\Gamma}{\delta\varphi_x}$$

$$\frac{\delta^2\Gamma}{\delta\varphi_x\delta\varphi_y}|_{J=0} = -(\partial_x^2 + m^2)\delta_{xy} - \frac{\lambda}{2}\Delta_{xx}\delta_{xy} - \frac{\lambda}{2}\Delta_{xb}\Delta_{xc}\Delta_{xd} i\Gamma_{abc}\Delta_{ea} i\Gamma_{dye} - \frac{\lambda}{6}\Delta_{xa}\Delta_{xb}\Delta_{xc} i\Gamma_{abcy}$$

no sum over x, sum over a,b,c

$$\frac{\delta}{\delta\varphi(z)}\Delta_{xy} = i\frac{\delta}{\delta\varphi(z)}\left(\frac{\delta^2\Gamma}{\delta\varphi\delta\varphi}\right)^{-1}_{xy} = \Delta_{xa}(i\Gamma_{azb})\Delta_{by}$$

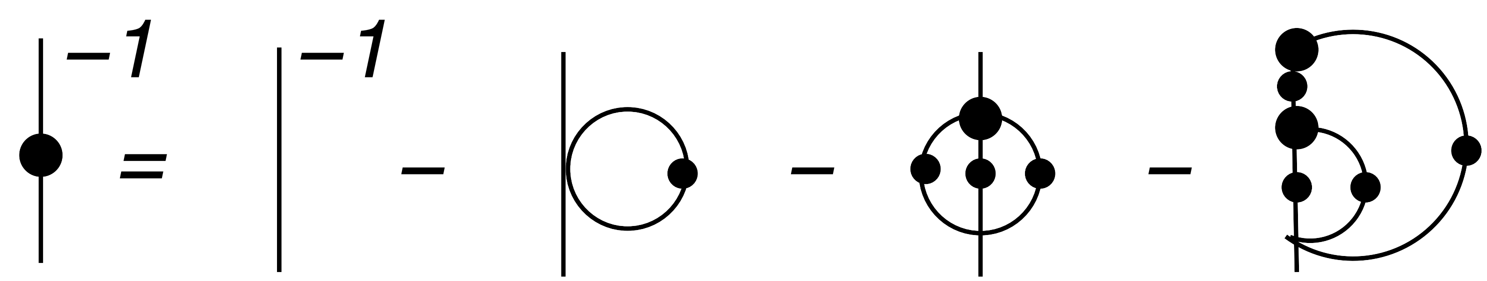
REVIEW

$$\frac{\delta S}{\delta \varphi}\left[\varphi_{cl} + i\left(\frac{\delta^2\Gamma}{\delta\varphi_{cl}\delta\varphi_{cl}}\right)^{-1}_{.z}\frac{\delta}{\delta\varphi_{cl}(z)}\right]\cdot 1 = \frac{\delta\Gamma}{\delta\varphi_{cl}}$$

$$\frac{\delta}{\delta\varphi(z)}\Delta_{xy} = i\frac{\delta}{\delta\varphi(z)}\left(\frac{\delta^2\Gamma}{\delta\varphi\delta\varphi}\right)^{-1}_{xy} = \Delta_{xa}(i\Gamma_{azb})\Delta_{by}$$

Propagator

$$\frac{\delta^2\Gamma}{\delta\varphi_x\delta\varphi_y}|_{J=0} = -(\partial_x^2 + m^2)\delta_{xy} - \frac{\lambda}{2}\Delta_{xx}\delta_{xy} - \frac{\lambda}{2}\Delta_{xb}\Delta_{xc}\Delta_{xd}i\Gamma_{abc}\Delta_{ea}i\Gamma_{dye} - \frac{\lambda}{6}\Delta_{xa}\Delta_{xb}\Delta_{xc}i\Gamma_{abcy}$$



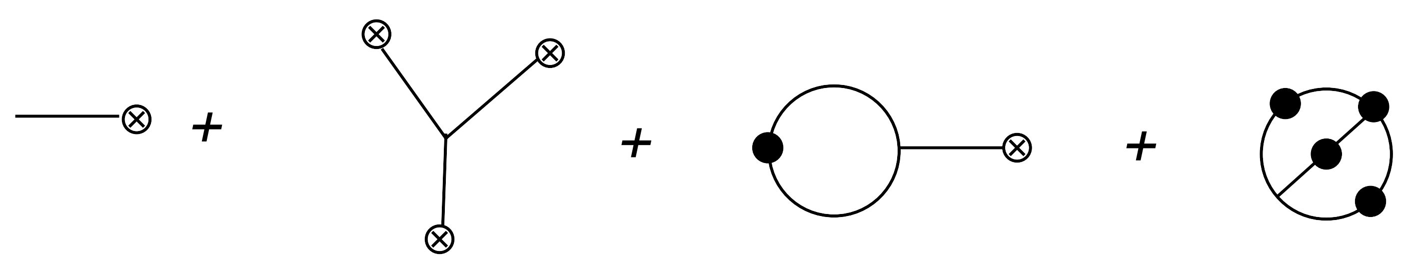
OBSERVATIONS

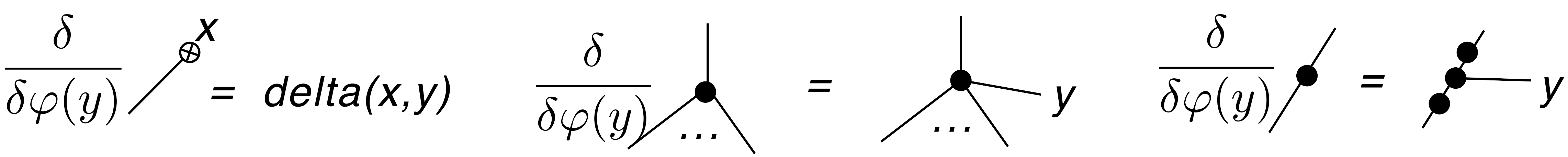
- diagram topology is specified by following indices in the Schwinger-Dyson equation
- factors of i, 1/2, -1, etc are absorbed into the definition of the diagrams *except* as indicated. This is because the perturbative Feynman rules are sufficient to determine all of these factors uniquely. The explicit minus signs then make everything work out.
- the master equation implies that there must be exactly one bare vertex in every Schwinger-Dyson equation
- the master equation restricts the form of possible diagrams that appear in Schwinger-Dyson equations.

Diagrammatic Approach

$$-(\partial^2 + m^2)\varphi_x - \frac{\lambda}{6}\left(\varphi_x^3 + 3\Delta_{xx}\varphi_x + \Delta_{xa}\Delta_{xb}\Delta_{xc} i\Gamma_{abc}\right) = \frac{\delta\Gamma}{\delta\varphi_x}$$

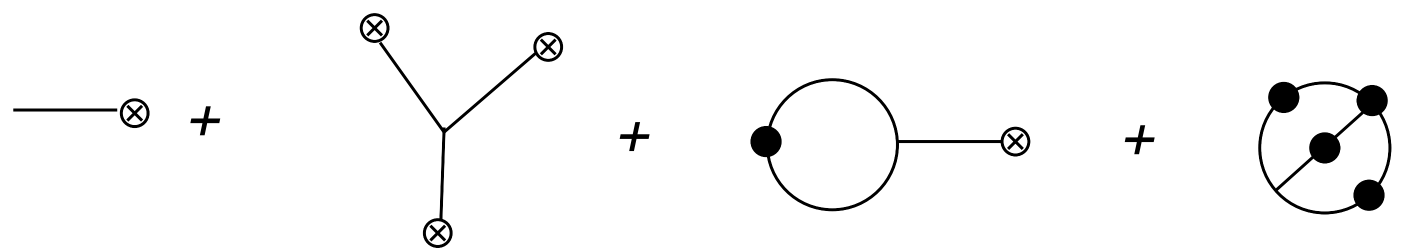
$$\frac{\delta\Gamma}{\delta\varphi} =$$



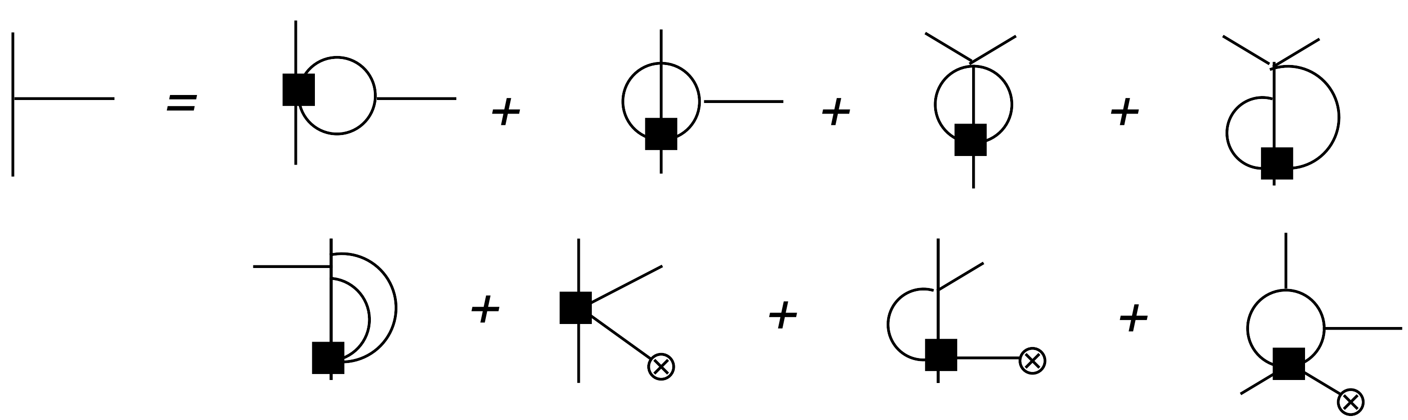


N-POINT FUNCTIONS

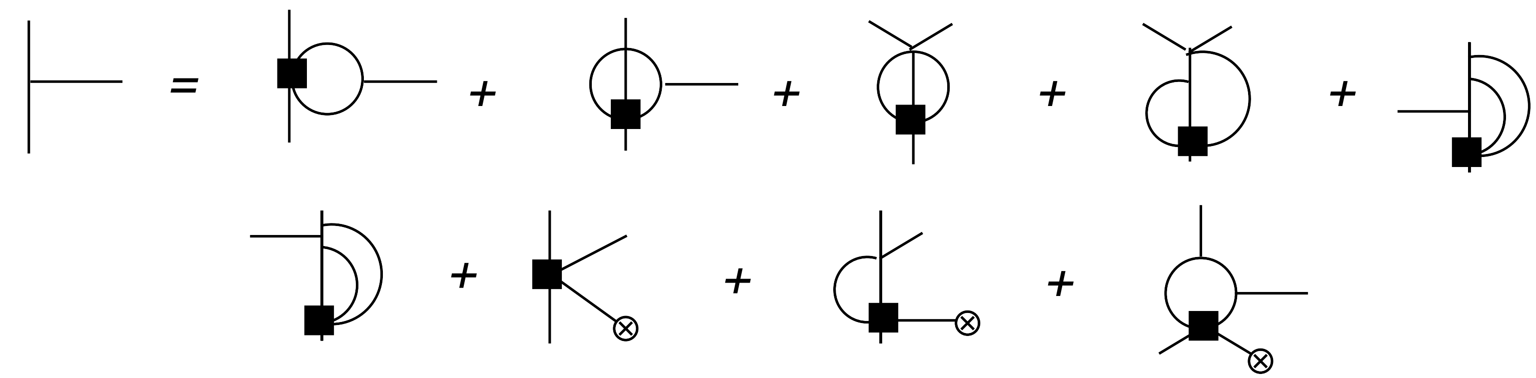
$$\frac{\delta \Gamma}{\delta \varphi} =$$

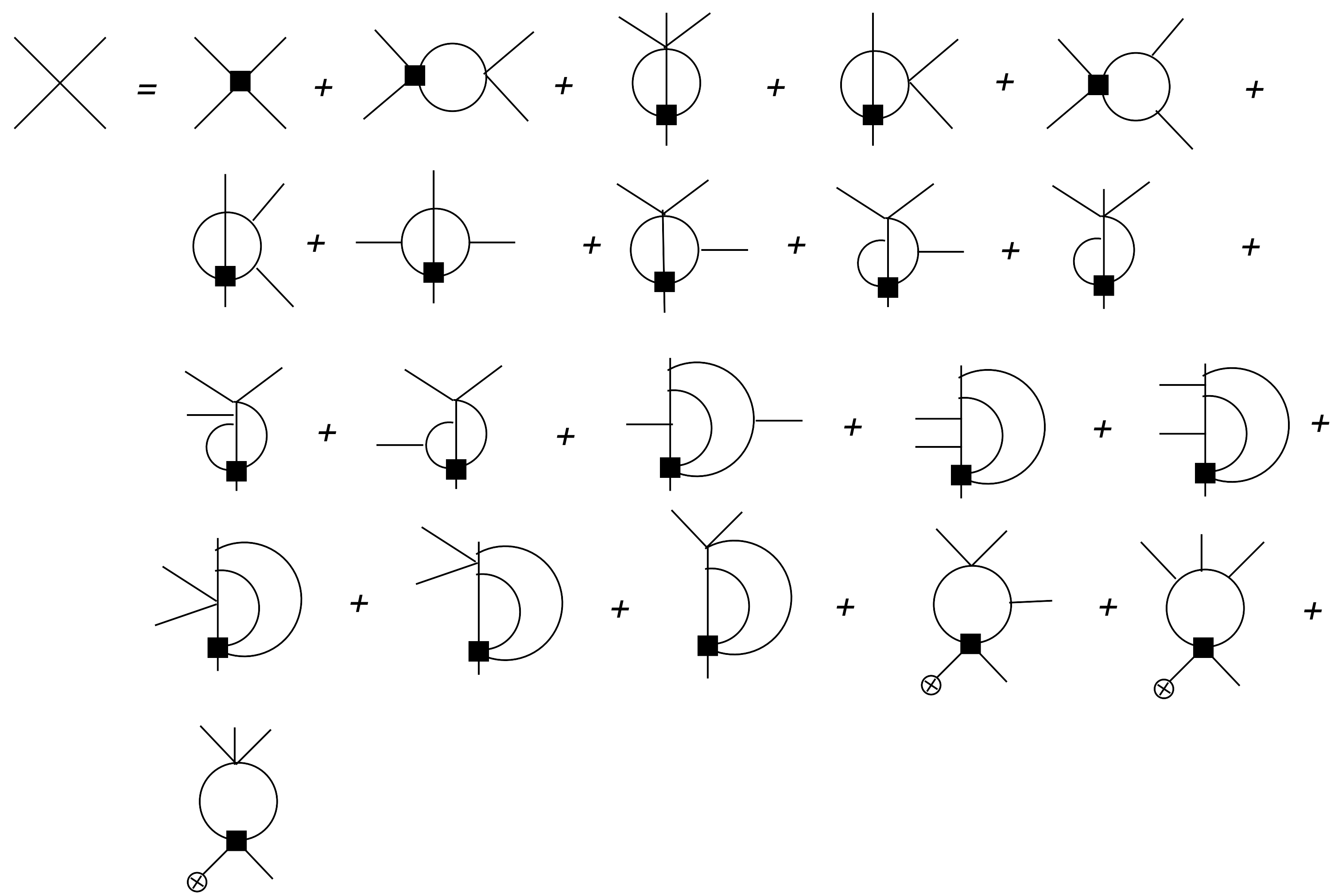


notation shift!

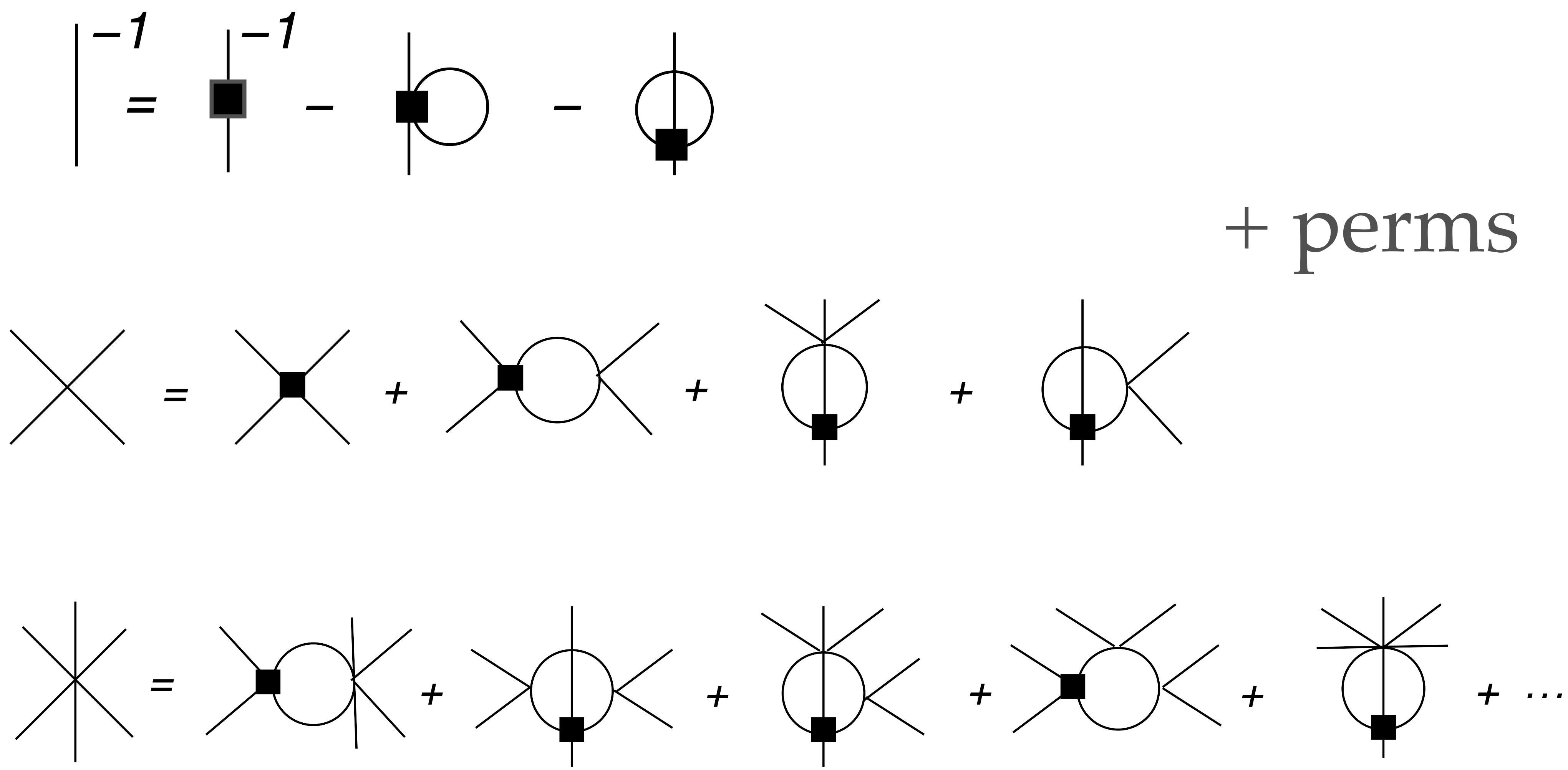


N-POINT FUNCTIONS

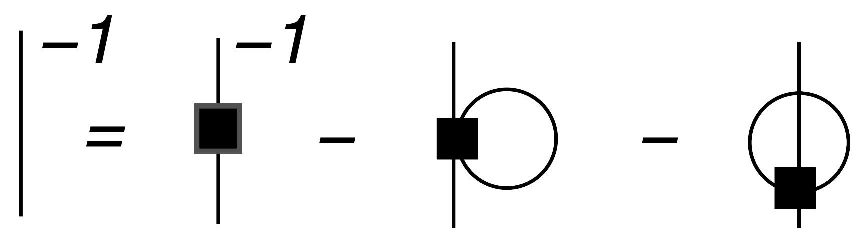




J=0 n-point functions



Solving Schwinger-Dyson Equations



$$\Delta_p = \frac{i}{A(p^2)p^2 - B(p^2)}$$

$$\Gamma(\ell_1, \ell_2, \ell_3, \ell_4)(2\pi)^4 \delta(\ell_1 + \ell_2 + \ell_3 + \ell_4)$$

$$\begin{aligned}\frac{Ap^2 - B}{i} \quad = \quad & \frac{p^2 - m^2}{i} - i\frac{\lambda}{2}\int \frac{d^4 q}{(2\pi)^4}\frac{i}{Aq^2 - B} - \\ & i\frac{\lambda}{6}\int \frac{d^4\ell_1}{(2\pi)^4}\frac{d^4\ell_2}{(2\pi)^4}\frac{d^4\ell_3}{(2\pi)^4}\frac{i}{A_1\ell_1^2 - B_1}\frac{i}{A_2\ell_2^2 - B_2}\frac{i}{A_3\ell_3^2 - B_3}\,\Gamma(\ell_1, \ell_2, \ell_3, -p)(2\pi)^4\delta(\ell_1 + \ell_2 + \ell_3 -\end{aligned}$$

Solving Schwinger-Dyson Equations

- we need to deal with divergent integrals
- we need an expression for Γ. Approximating this as the bare vertex is a typical.
- we need to evaluate some pretty nasty integrals
- we need to solve a(many) nonlinear integral equation(s).

Solving Schwinger-Dyson Equations

Wick rotate to Euclidean space $q_0 \to iq_4$

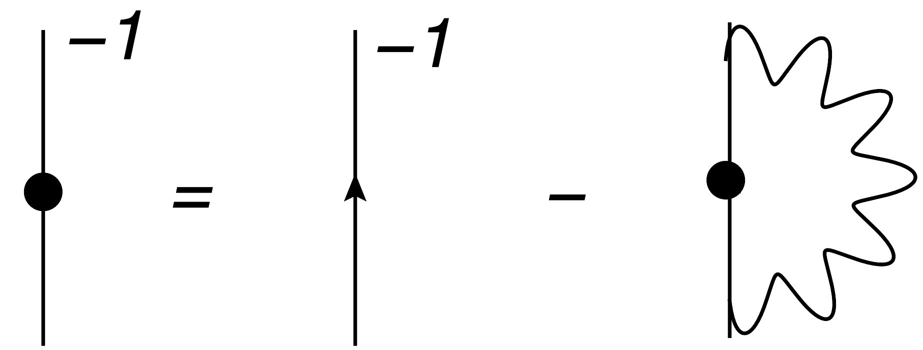
$$-A(-p_E^2)p_E^2 - B(-p_E^2) = -p_E^2 - \frac{\lambda}{2}\int \frac{d^4q_E}{(2\pi)^4}\frac{1}{Aq_E^2 + B}$$

$\Rightarrow$ $A = 1$

$$\alpha = 2\pi^2 \int^{\Lambda} \frac{q_E^3 dq_E}{(2\pi)^4}\frac{1}{q_E^2 + m_0^2 + \lambda\alpha/2}$$

$\Rightarrow$ $$\frac{\alpha}{\Lambda^2} = \frac{\pi^2}{1 - \frac{1}{2}\lambda\pi^2} \qquad m^2 = m_0^2 + \frac{\frac{1}{2}\lambda\pi^2\Lambda^2}{1 - \frac{1}{2}\lambda\pi^2}$$

Ladder QED



$$D_{\mu\nu} = \frac{-i}{k^2 + i\epsilon}\left(g_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right)$$

$$S(p) = \frac{i}{A(p^2)\not{p} - B(p^2)}$$

$$A\not{p} - B = \not{p} - m - e^2 \int \frac{d^4q}{(2\pi)^4}\frac{\gamma_\mu D_{\mu\nu}(p+q)(A\not{q} + B)\gamma_\nu}{A^2q^2 - B^2}$$

Ladder QED

$$A\not{p} - B = \not{p} - m - e^2 \int \frac{d^4q}{(2\pi)^4} \frac{\gamma_\mu D_{\mu\nu}(p+q)(A\not{q} + B)\gamma_\nu}{A^2 q^2 - B^2}$$

$$A(\xi = 0) = 1$$

$$B(p^2) = m - ie^2 \int \frac{d^4q}{(2\pi)^4} \frac{(3+0)B(q^2)}{(q^2 - B^2)(p-q)^2}$$

$$\int d\Omega_4 \frac{1}{(p-q)^2} = 2\pi^2 \left[\frac{1}{q^2}\theta(q > p) + \frac{1}{p^2}\theta(p > q)\right]$$

LADDER QED

$$B(-p_E^2) = \frac{3e^2}{8\pi^2}\left[\frac{1}{p_E^2}\int_0^{p_E} dq q^3 \frac{B(q)}{q^2+B^2} + \int_{p_E}^{\Lambda} dq q \frac{B(q)}{q^2+B^2}\right]$$

$$p_E^4 B' = -\frac{3e^2}{8\pi^2}\int_0^{p_E} dq q^3 \frac{B}{q^2+B^2} \qquad ' = \frac{d}{dp_E^2}$$

$$(p_E^4 B')' = -\frac{3e^3}{16\pi^2} p_E^2 \frac{B}{p_E^2+B^2}$$

LADDER QED

$$(p_E^4 B')' = -\frac{3e^3}{16\pi^2} p_E^2 \frac{B}{p_E^2 + B^2}$$

$$B(-p_E^2) = \frac{3e^2}{8\pi^2} \left[\frac{1}{p_E^2} \int_0^{p_E} dq q^3 \frac{B(q)}{q^2 + B^2} + \int_{p_E}^{\Lambda} dq q \frac{B(q)}{q^2 + B^2} \right]$$

⇨ $(p^4 B')|_{p=0} = 0$ $\qquad$ $(B + p^2 B')|_{p=\Lambda} = 0$

⇨ $B \to p^{-1 \pm \sqrt{1 - 3e^2/(4\pi^2)}}$ $\qquad$ $\alpha > \alpha_\star \equiv \frac{\pi}{3}$

NUMERICAL METHODS

expand A(p) in a convenient basis

$$A(p) = \sum_i c_i T_i(p)$$

discretise

$$A(p) \to A_i = A(p_i)$$

$$x_i = f_i(\{x\})$$

NUMERICAL METHODS

$$x_i = f_i(\{x\})$$

(i) iterate $\quad x_i|_{n+1} = f_i(\{x\}_n)$

(ii) iterative Newton-Raphson

$$\sum_j \left(-\delta_{ij} + \frac{\partial f_i}{\partial x_j}|_{x_i} \right) \delta x_j = x_i - f_i(\{x\})$$

$$x_i|_{n+1} = x_i|_n + \delta x_i$$

(iii) minimise

$$G(\{x\}) = \sum_i (x_i - f_i(\{x\}))^2$$

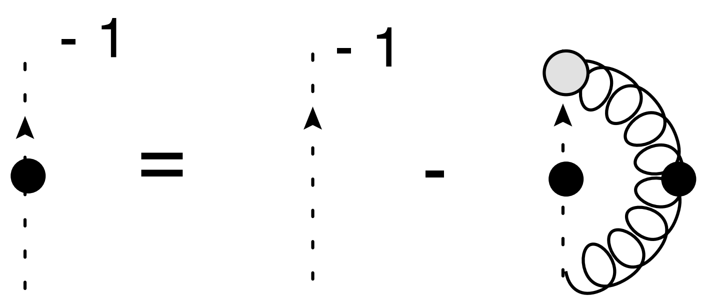
FINAL WORDS

Thus it is vital that the practitioner not abandon theoretical investigations too early. One must carefully track and deal with singularities in the equations, understand asymptotic behaviour, and develop decent analytic approximations to have any hope

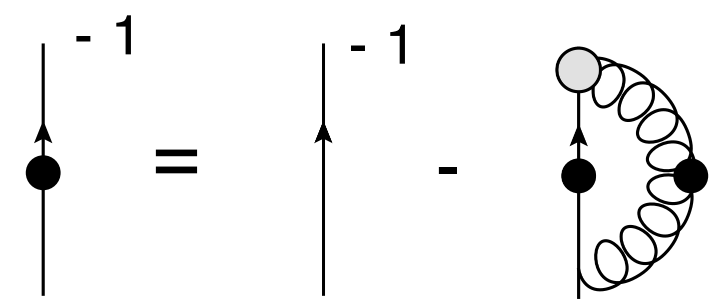
- All of the techniques discussed here will fail miserably unless one starts *very* close to the solution.
- how does one truncate SD equations (beyond convenience)?
- be prepared for heartbreak

QCD

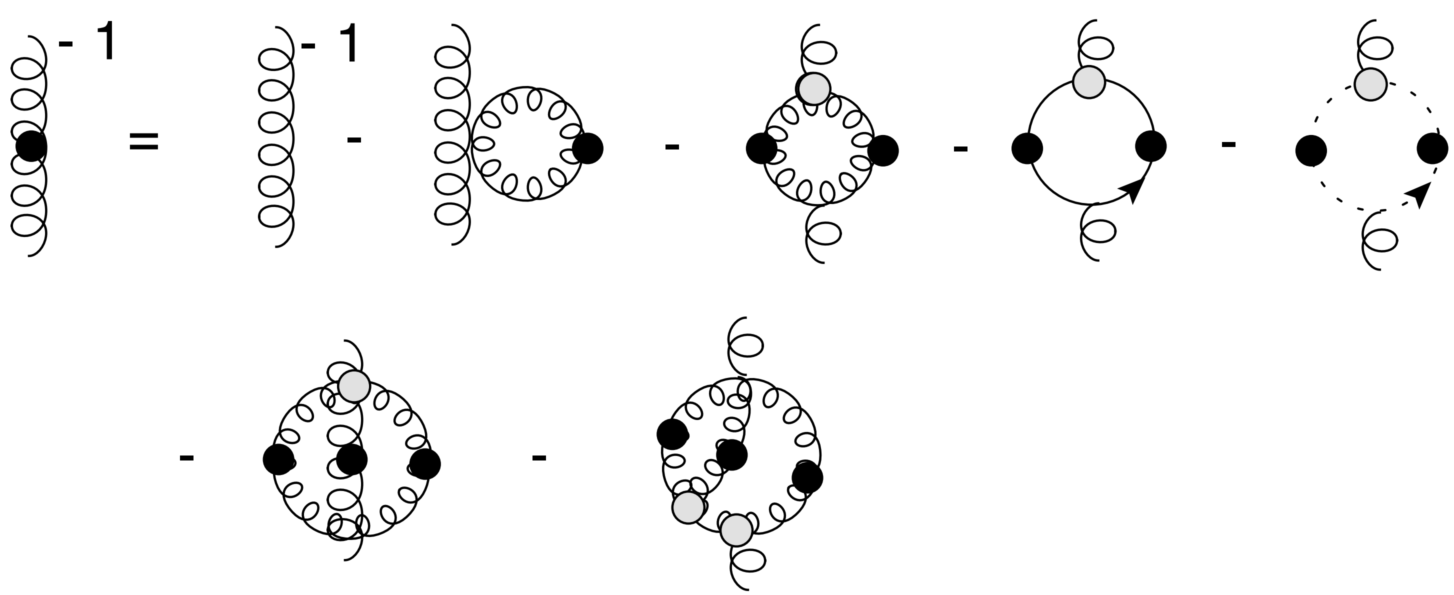
ghost



quark

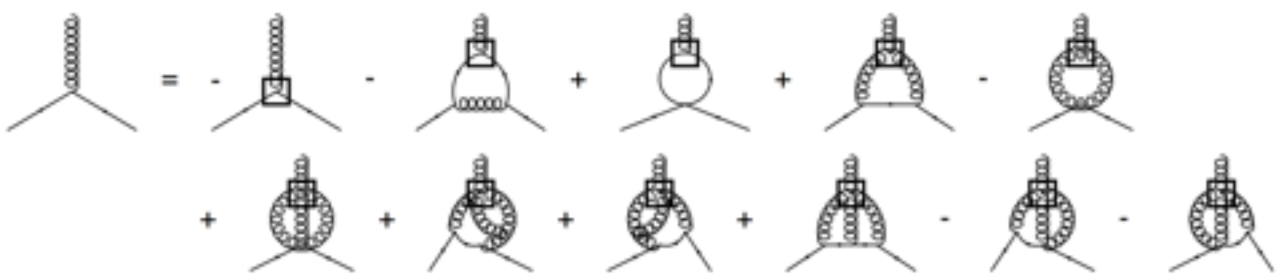


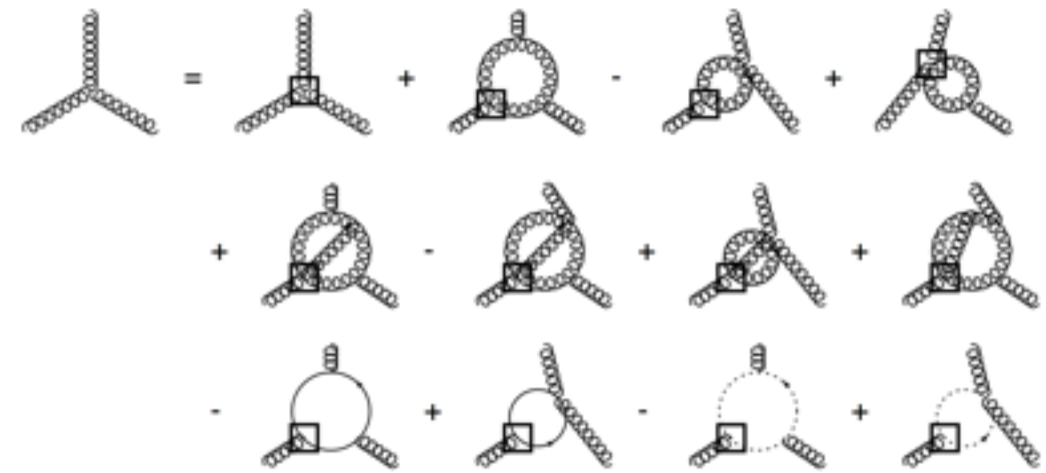
gluon

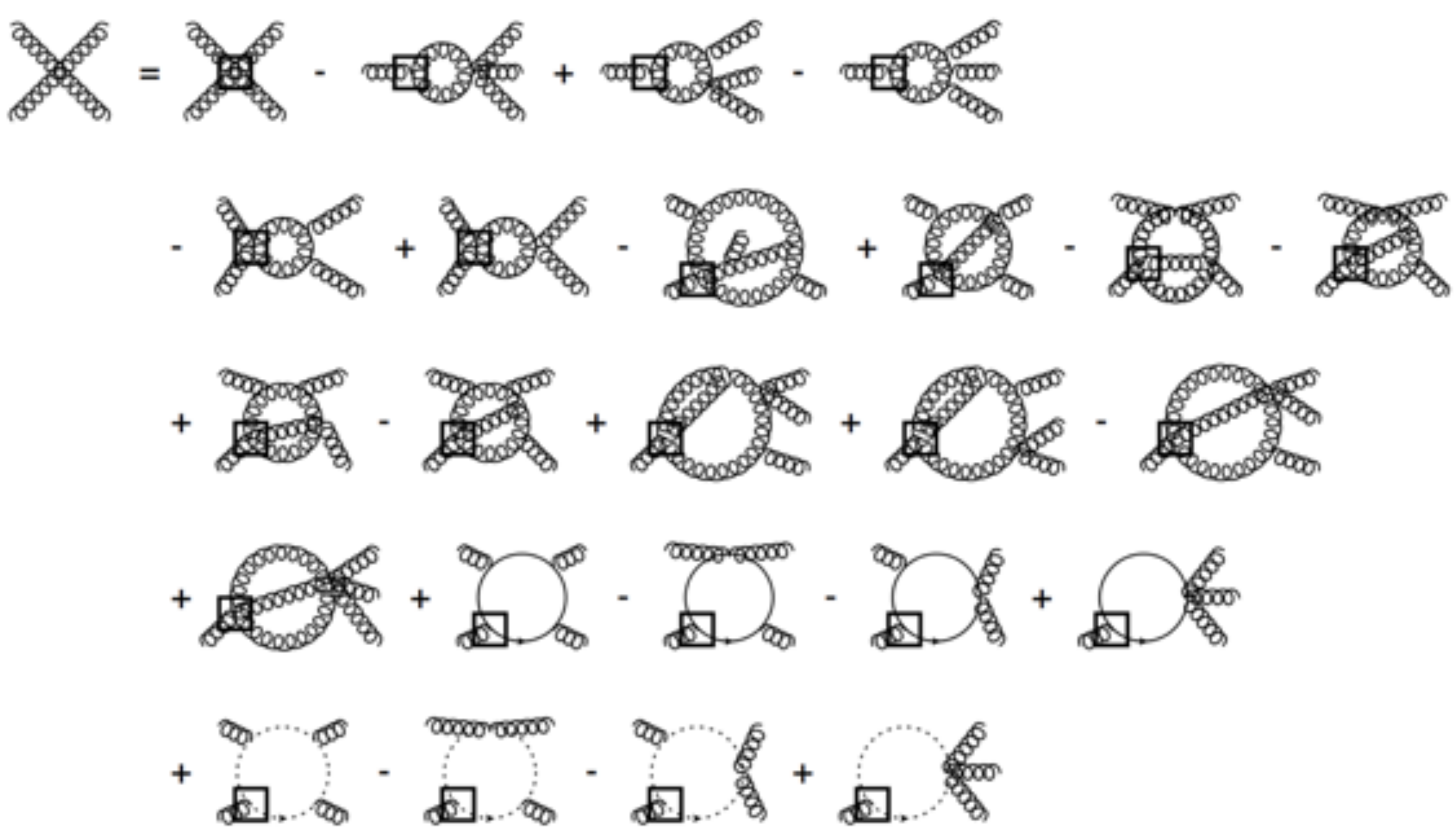


Exotic Theory: Schwinger-Dyson Equations

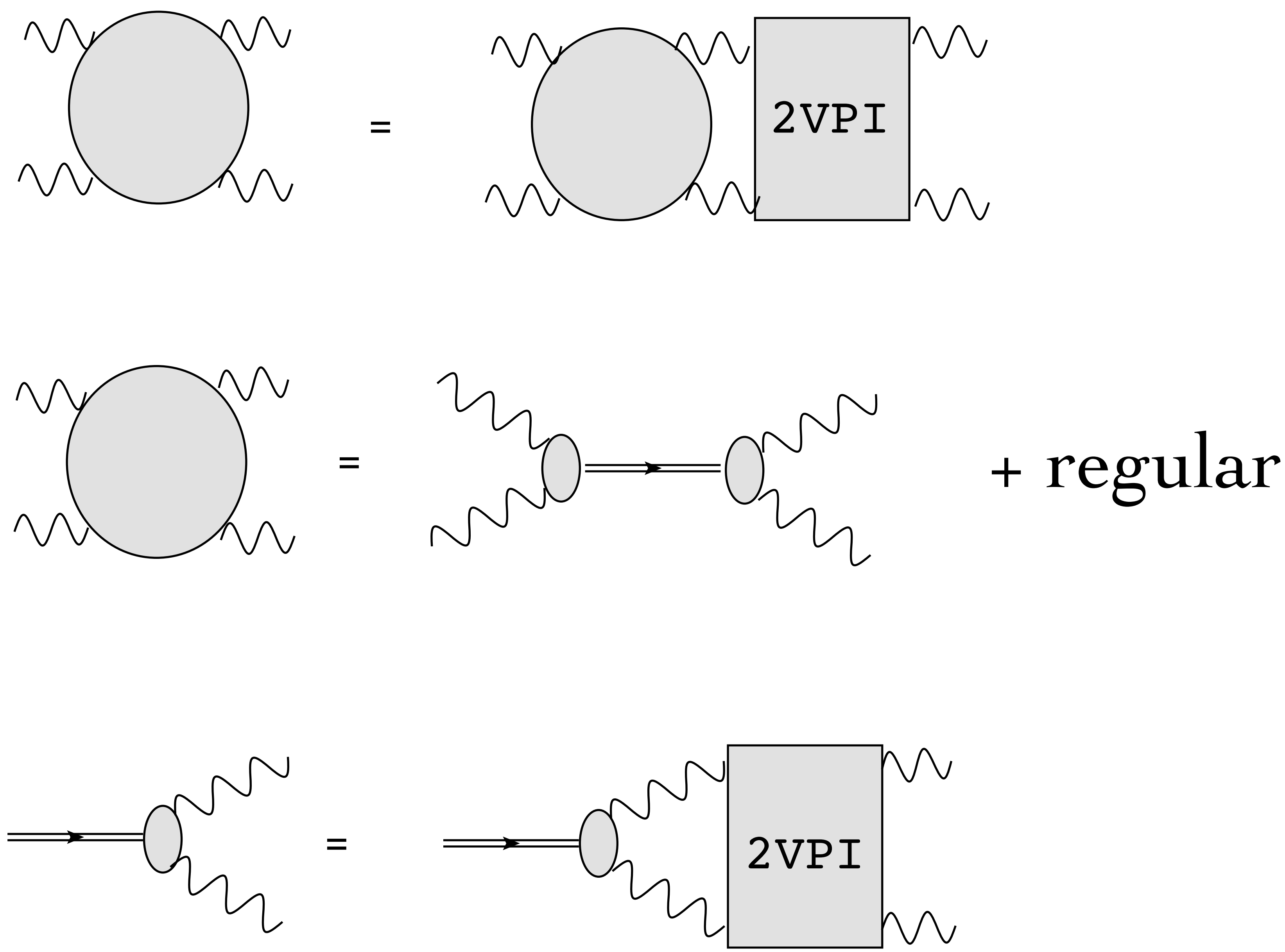
J. Meyers, PhD Thesis, Pittsburgh, 2014.





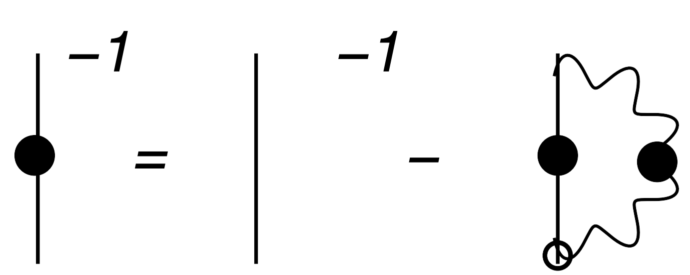


Bethe-Salpeter

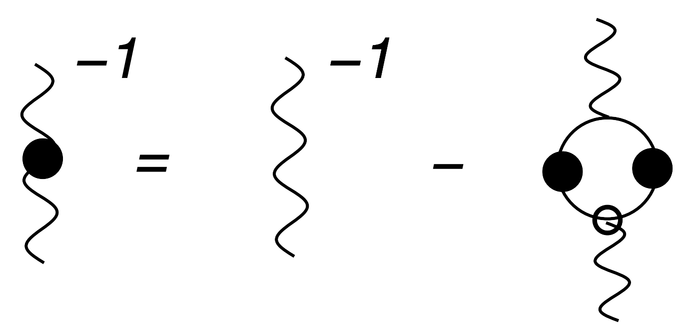

=
2VPI
=
+ regular
=
2VPI

Exotic Theory: Schwinger-Dyson Equations

electron



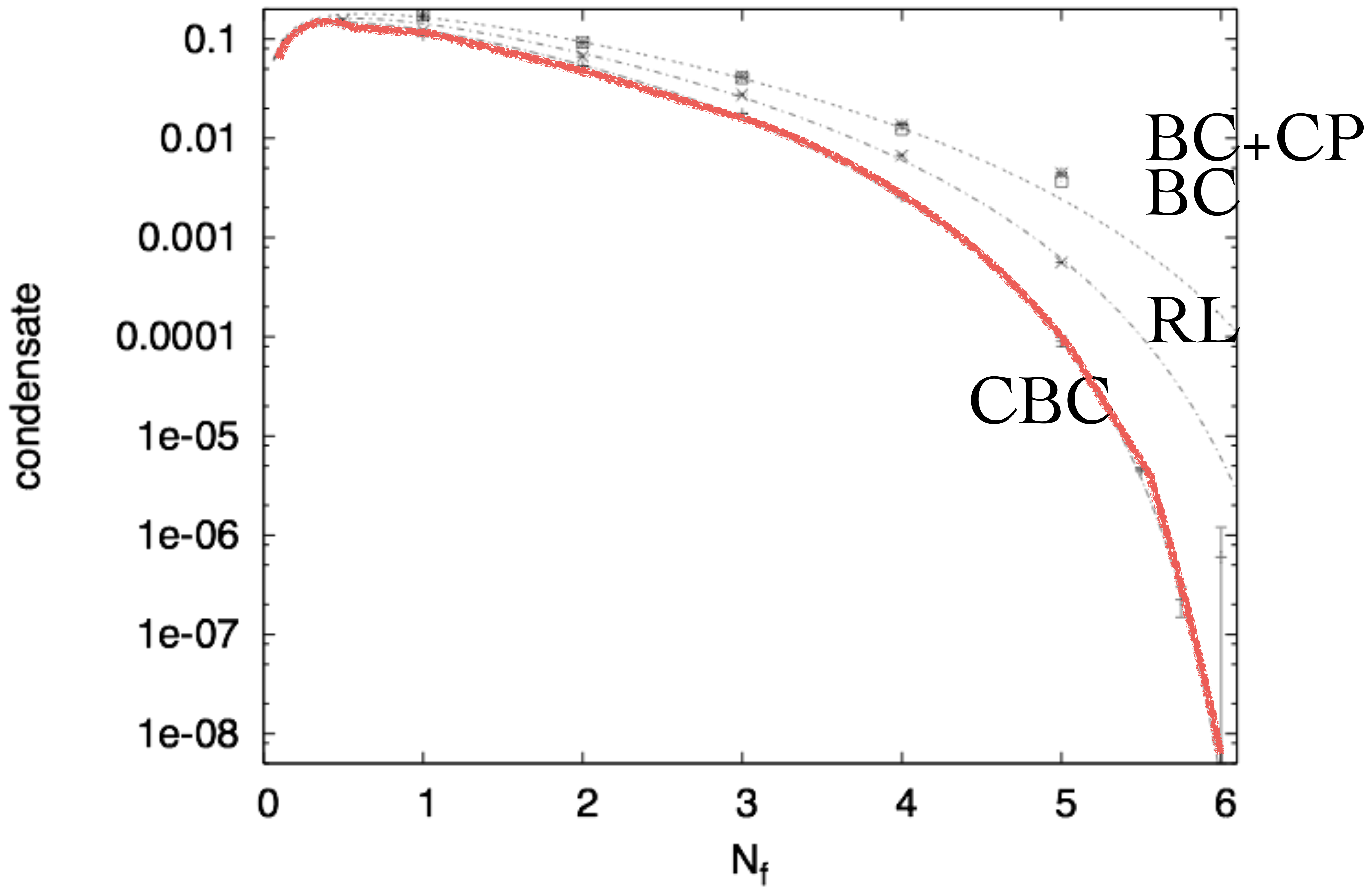
photon



E.S. Swanson, arXiv 1008.4337

Results: QED3

P.M. Lo, E.S. Swanson, PRD83, 065006 (2011)
P.M. Lo, E.S. Swanson, PRD81, 034030 (2010)



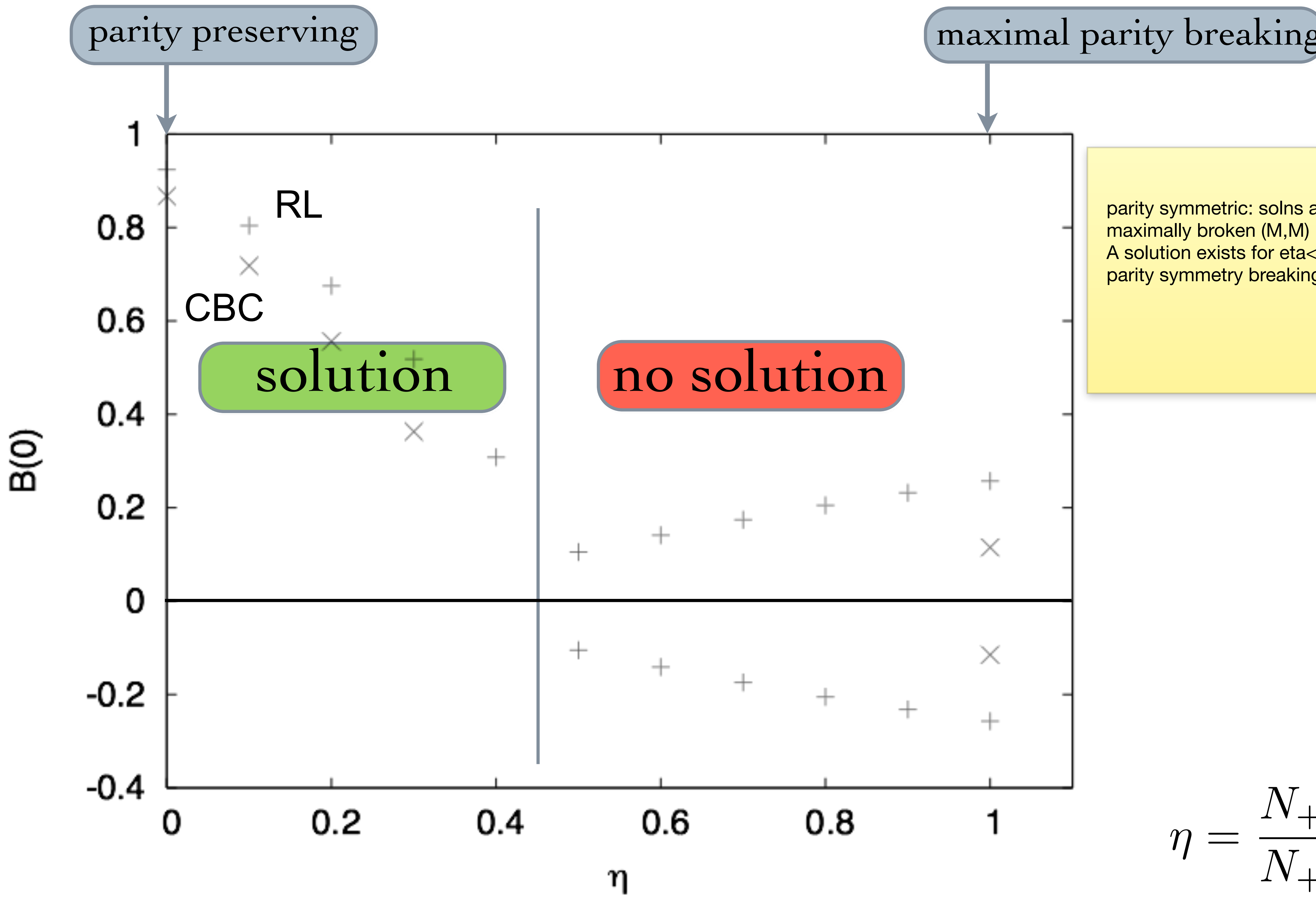
$$
\begin{aligned}
N_\star(\mathrm{CBC}) &= 1.00 \cdot N_\star^A \\
N_\star(\mathrm{RL}) &= 1.10 \cdot N_\star^A \\
N_\star(\mathrm{CP}) &= N_\star(\mathrm{BC}) = 1.21 \cdot N_\star^A
\end{aligned}
$$

$$
\langle \bar{\psi}\psi \rangle (N_f) = a N_f \exp\left(\frac{-2\pi}{\sqrt{N_\star/N_f - 1}} \right)
$$

Results: QED3

P.M. Lo, E.S. Swanson, PRD83, 065006 (2011)

P.M. Lo, E.S. Swanson, PRD81, 034030 (2010)



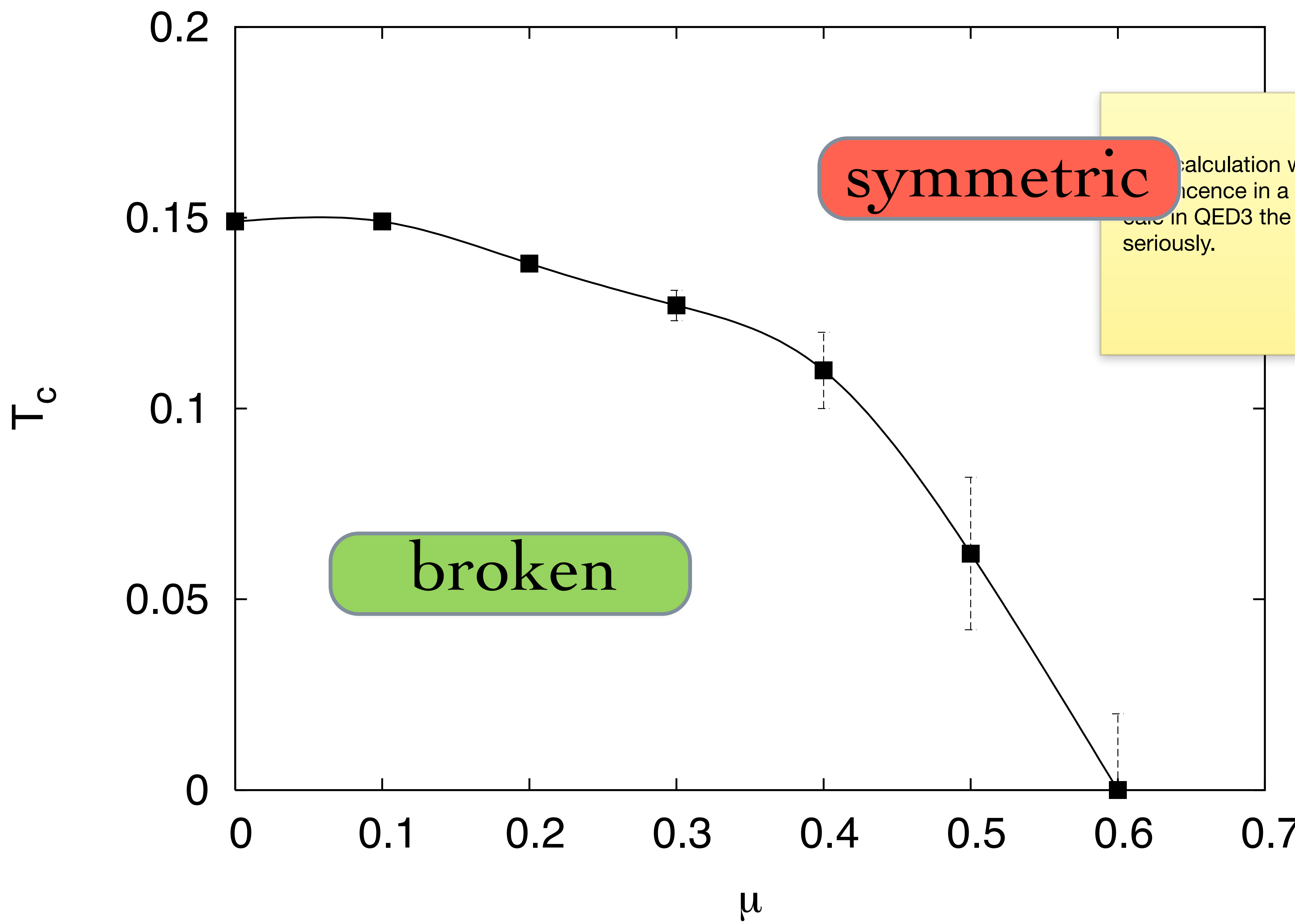
parity symmetric: solns are (M,-M)
maximally broken (M,M)
A solution exists for eta<0.4, implying parity symmetry breaking!

$$\eta = \frac{N_+ - N_-}{N_+ + N_-}$$

Results: QED3, finite temperature and density

P.M. Lo, E.S. Swanson, PRD89, 025015 (2014)

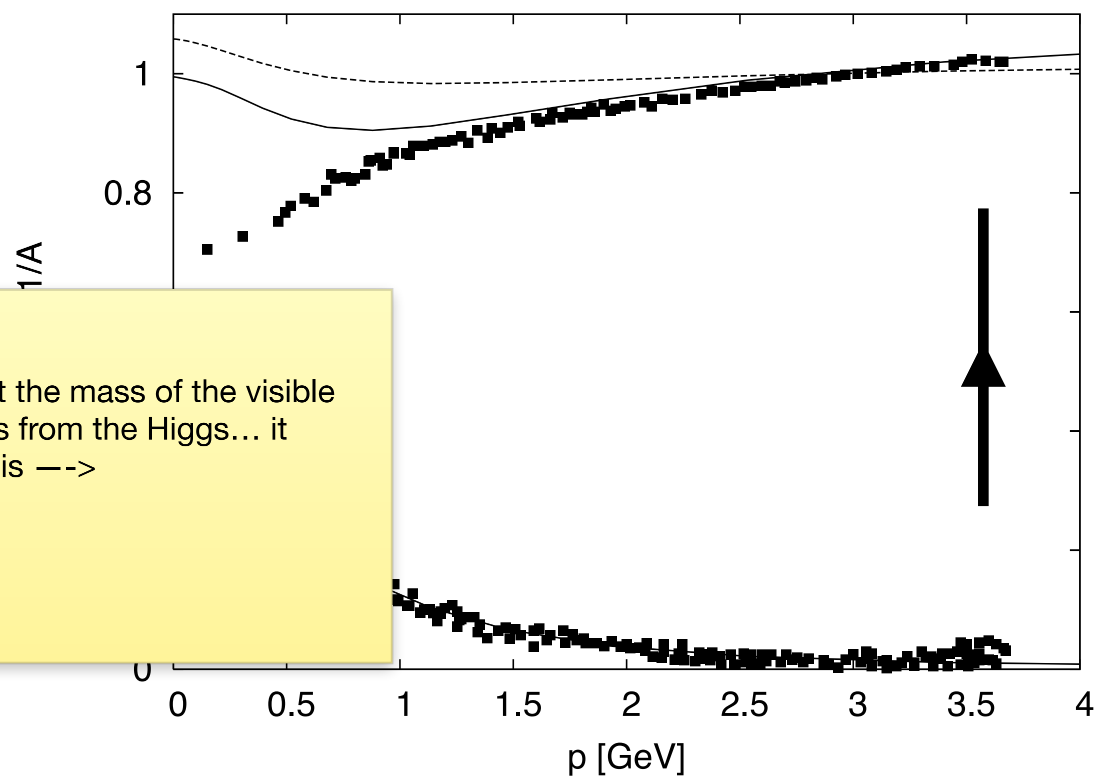
P.M. Lo, E.S. Swanson, PLB697, 164 (2011)



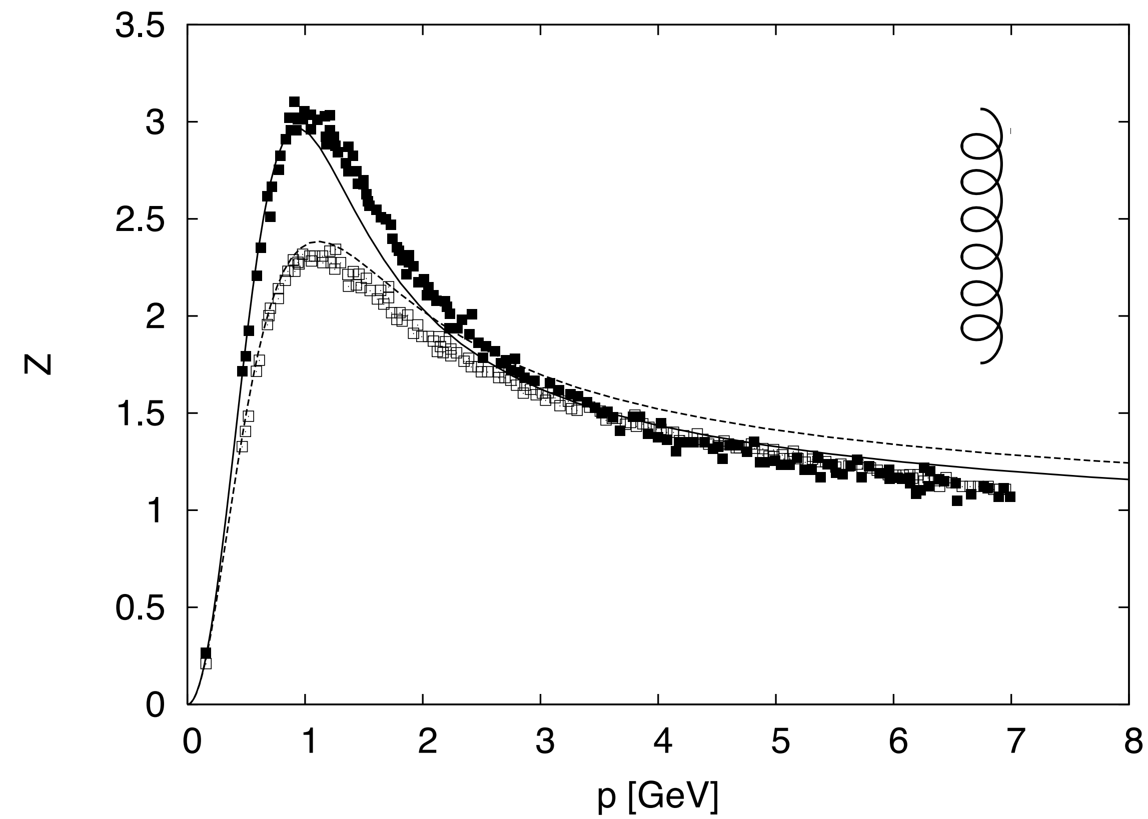
alculation with full frequency
ncence in a gauge theory. First
ale in QED3 the treat the IR-div
seriously.

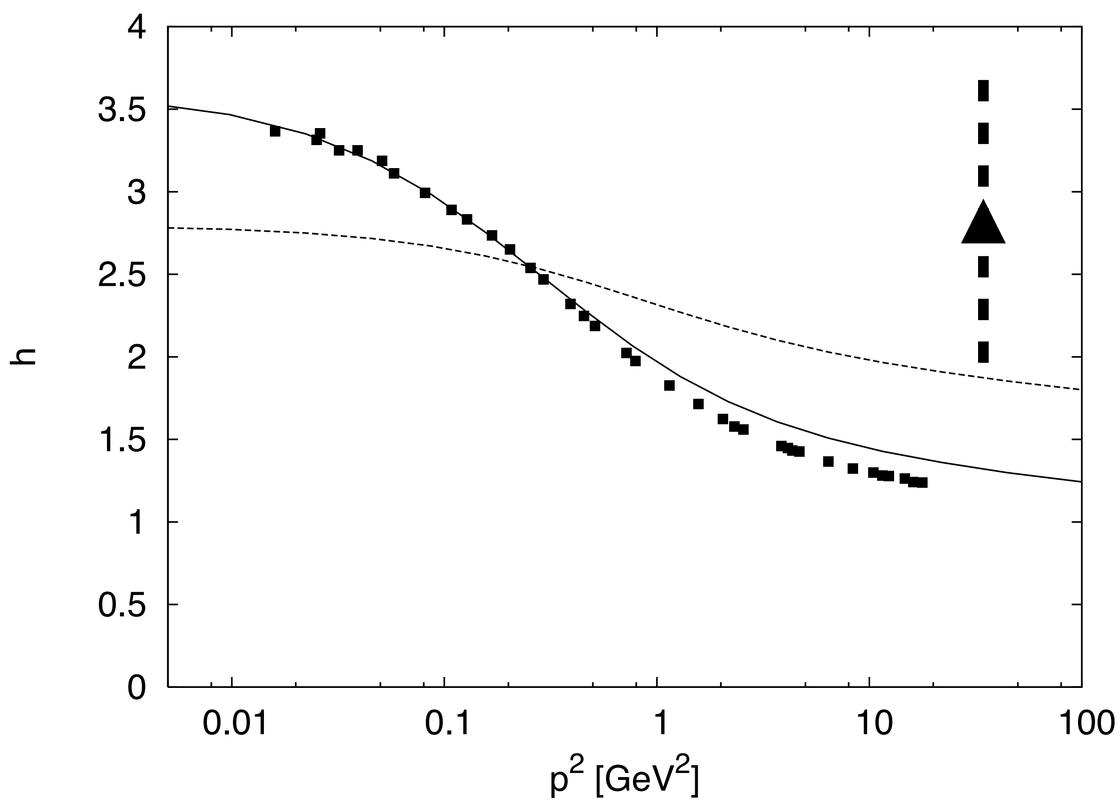
Results: QCD propagators

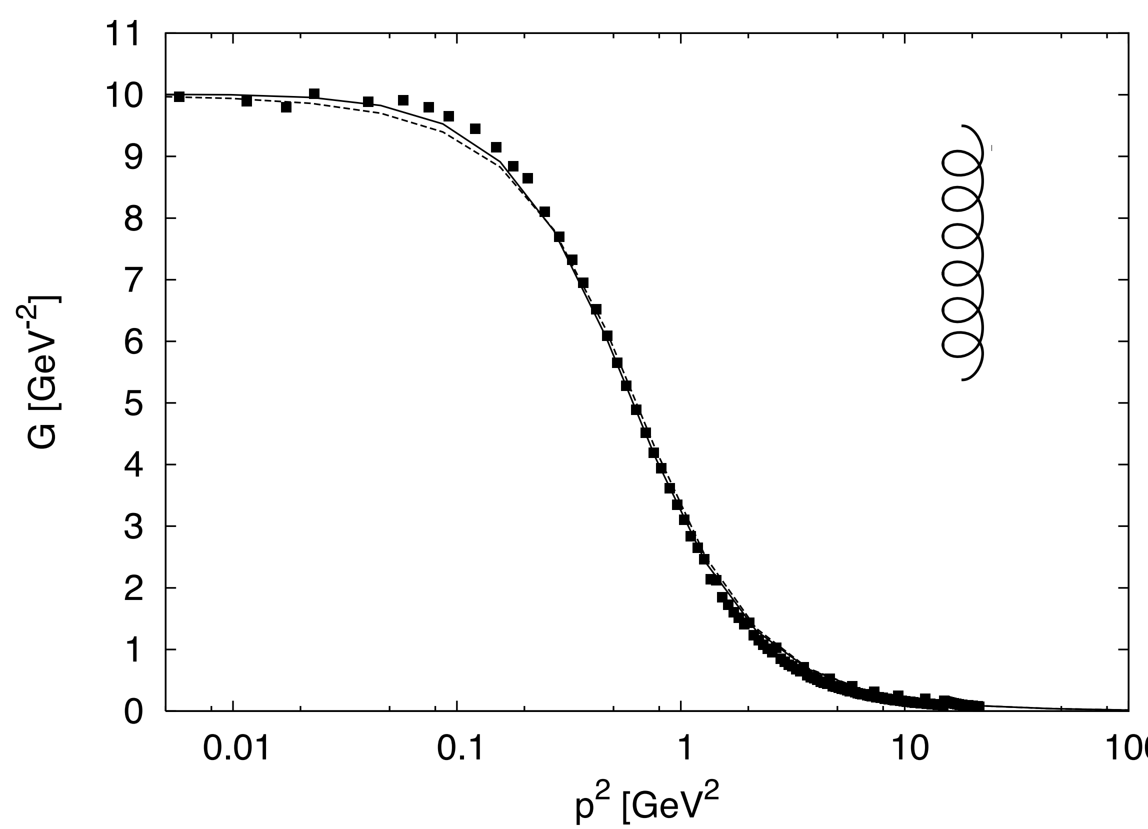
J. Meyers, E.S. Swanson, PRD90, 045037 (2014)



RUE that the mass of the visible
se comes from the Higgs… it
from this —->

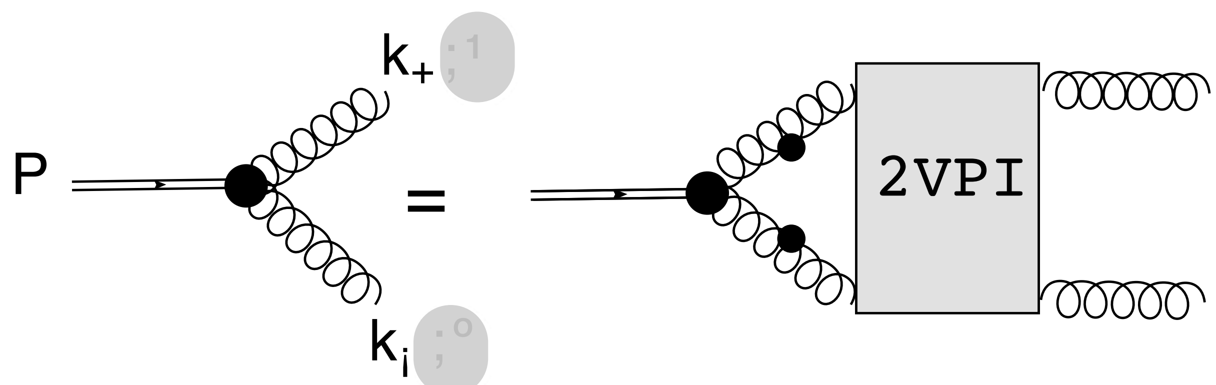


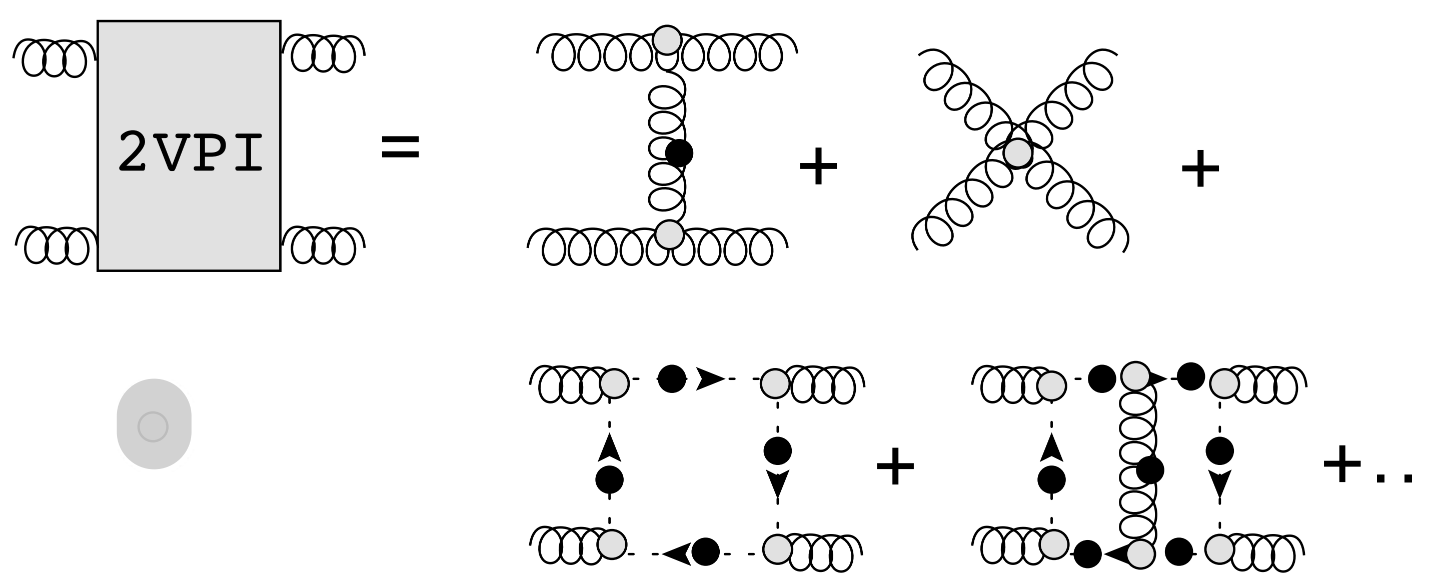




Exotic Theory: Schwinger-Dyson and Bethe-Salpeter Equations

J. Meyers, E.S. Swanson, PRD87, 036009 (2013)



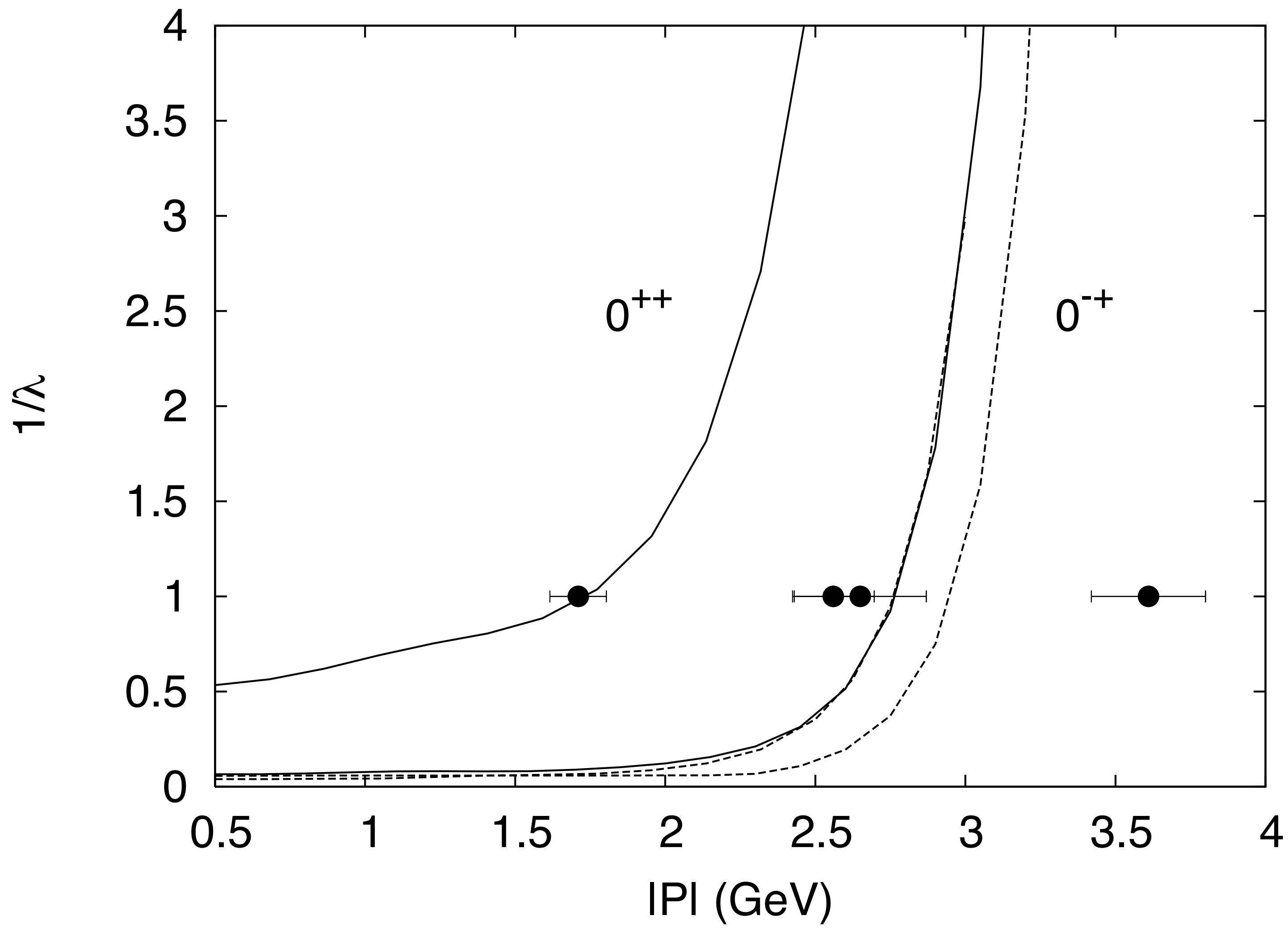


Results: Glueballs

$$
\begin{aligned}
\chi_{\mu\nu}(k_+,k_-) &= ig^2N\int\frac{d^4q}{(2\pi)^4}\,\chi_{\alpha\beta}(q_+,q_-)\,\mathcal{C}^{\alpha\beta}_{..\mu\nu}\,G(q_+)G(q_-)\\
&+ig^2N\int\frac{d^4q}{(2\pi)^4}\,\chi_{\alpha\beta}(q_+,q_-)\,\mathcal{T}^{\alpha\beta}_{..\mu\nu}(q_+,q_-,k_+,k_-)\,G(q_+)G(q_-)G(Q)\\
&+ig^2N\int\frac{d^4q}{(2\pi)^4}\,\chi(q_+,q_-)\,\mathcal{G}_{\mu\nu}(q_+,q_-,k_+,k_-)\,H(q_+)H(q_-)H(Q)\\
&+i\frac{g^2}{2}\int\mathrm{tr}\left[\gamma_\mu S(q_+)\mathbb{X}(q_+,q_-)S(q_-)\gamma_\nu S(Q)\right]\\
\chi(k_+,k_-) &= ig^2N\int\frac{d^4q}{(2\pi)^4}\,\chi(q_+,q_-)\,\mathcal{H}(q_+,q_-,k_+,k_-)\,H(q_+)H(q_-)G(Q)\\
&+ig^2N\int\frac{d^4q}{(2\pi)^4}\,\chi_{\alpha\beta}(q_+,q_-)\,\mathcal{B}^{\alpha\beta}(q_+,q_-,k_+,k_-)\,G(q_+)G(q_-)H(Q)\\
\mathbb{X}(k_+,k_-) &= g^2C_F\int\gamma_\alpha S(k_-+q_-)\gamma_\beta G(q_+)G(q_-)\chi_{\alpha\beta}(q_+,q_-)\\
&+ig^2C_F\int\gamma_\mu S(q_+)\mathbb{X}(q_+,q_-)S(q_-)\gamma_\nu G(Q)P_{\mu\nu}(Q)
\end{aligned}
$$

Results: Glueballs

J. Meyers, E.S. Swanson, PRD87, 036009 (2013)



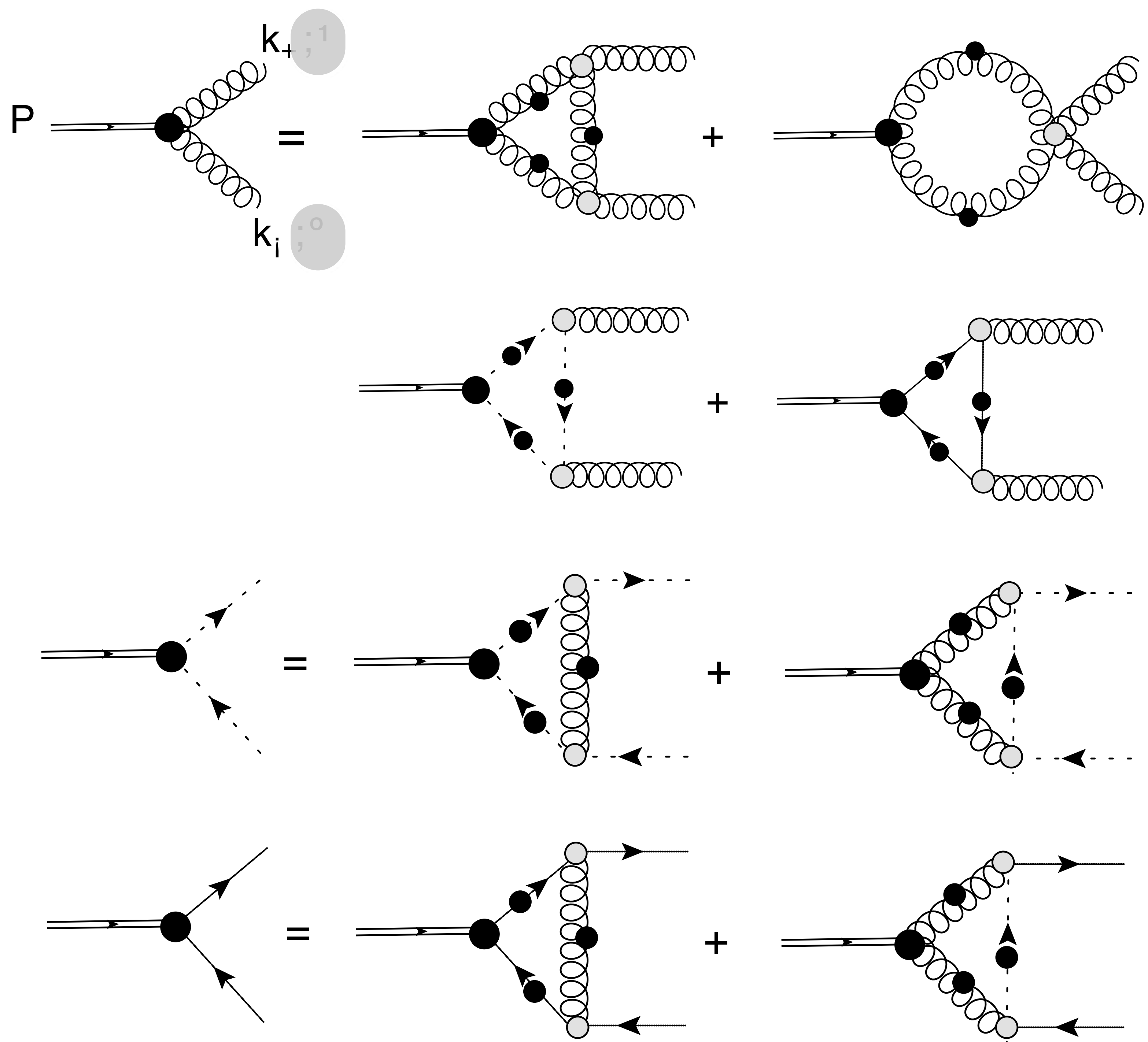
Schwinger-Dyson Equations

Vertex Ansätze

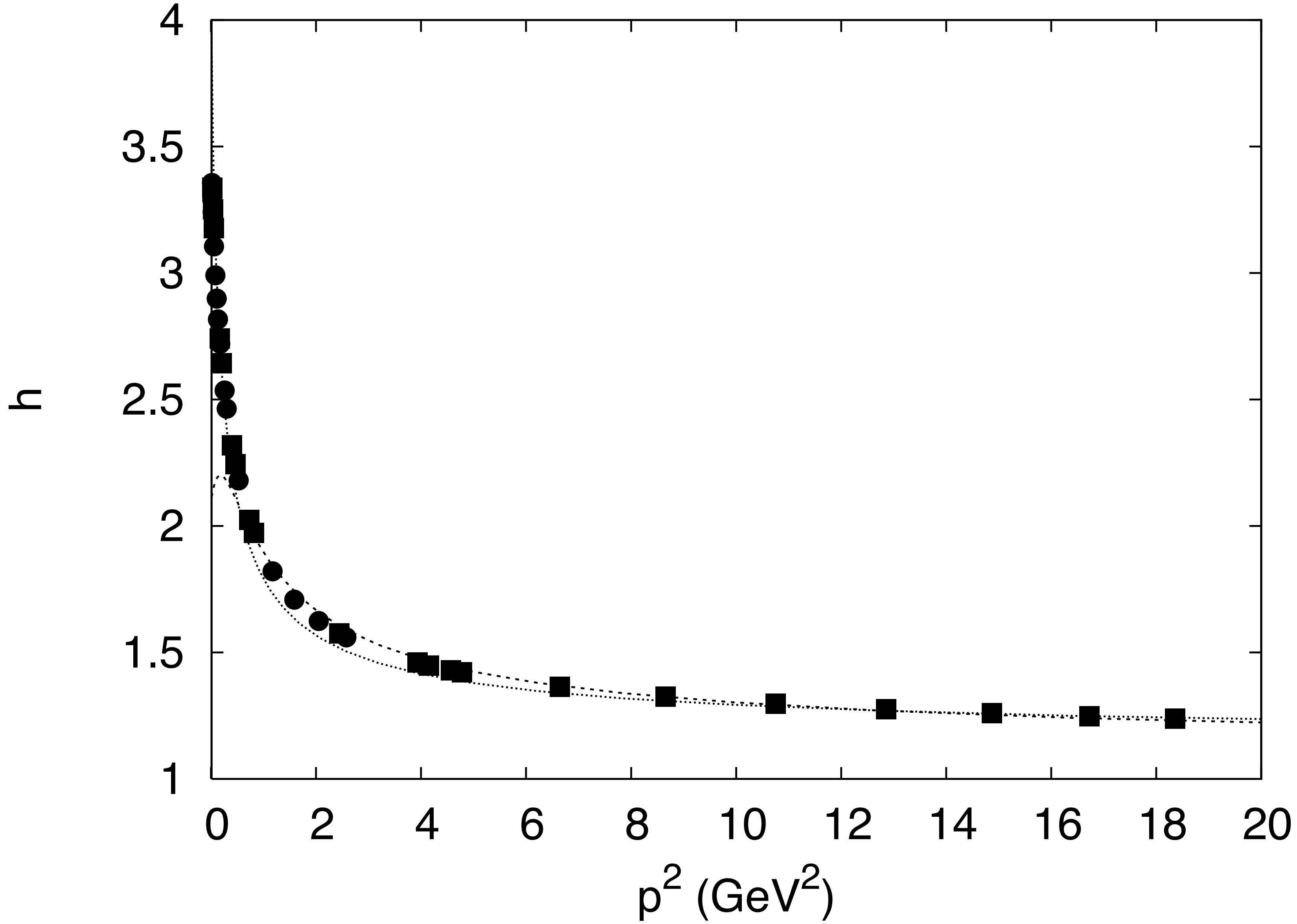
$$iS^{-1} = A\not{p} - B$$

- $i\Gamma^\mu_{RL}(k,p) = \gamma^\mu$
- $i\Gamma^\mu_{CBC}(k,p) = \frac{1}{2}(A(k)+A(p))\gamma^\mu$
- $i\Gamma^\mu_{BC}(k,p) = \frac{1}{2}\left(A(k)+A(p)\right)\gamma^\mu + \frac{1}{2}\frac{A(k)-A(p)}{k^2-p^2}(\not{k}+\not{p})(k^\mu+p^\mu) - \frac{B(k)-B(p)}{k^2-p^2}(k^\mu+p^\mu)$
- $i\Gamma^\mu_{CP}(k,p) = \frac{1}{2}\frac{A(k)-A(p)}{d(k,p)}\left[\gamma^\mu(k^2-p^2)-(k+p)^\mu\left(\not{k}-\not{p}\right)\right]$

$$d(k,p) = \frac{(k^2-p^2)^2+(M(k)^2+M(p)^2)^2}{k^2+p^2}$$


P
k+
ki

results: ghost

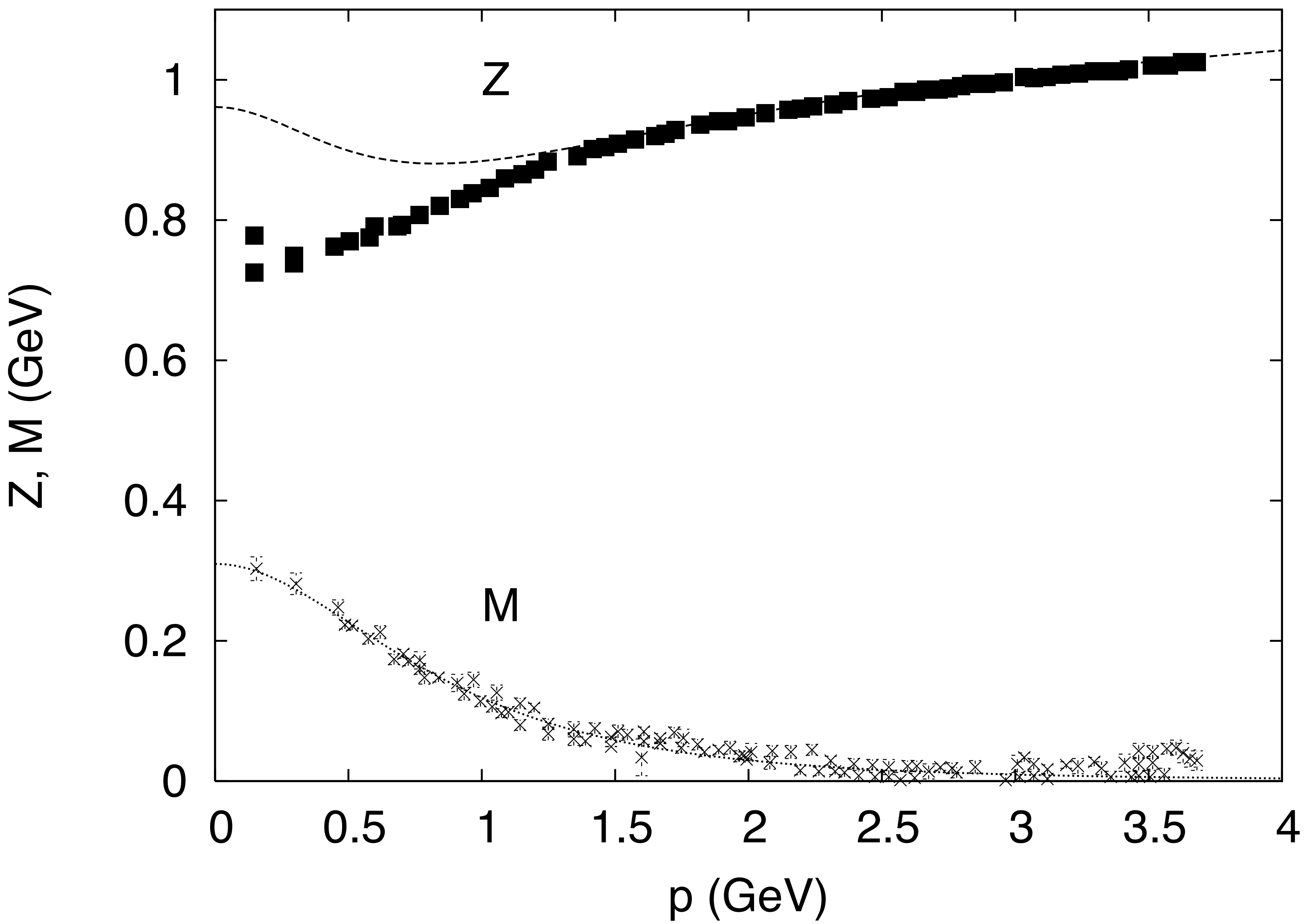


I.L. Bogolubsky, E.M. Ilgenfritz, M. Muller-Preussker and A. Sternbeck, Phys. Lett. **B676**, 69 (2009)

results: quark

$$f_\pi = 240 \text{ MeV}$$

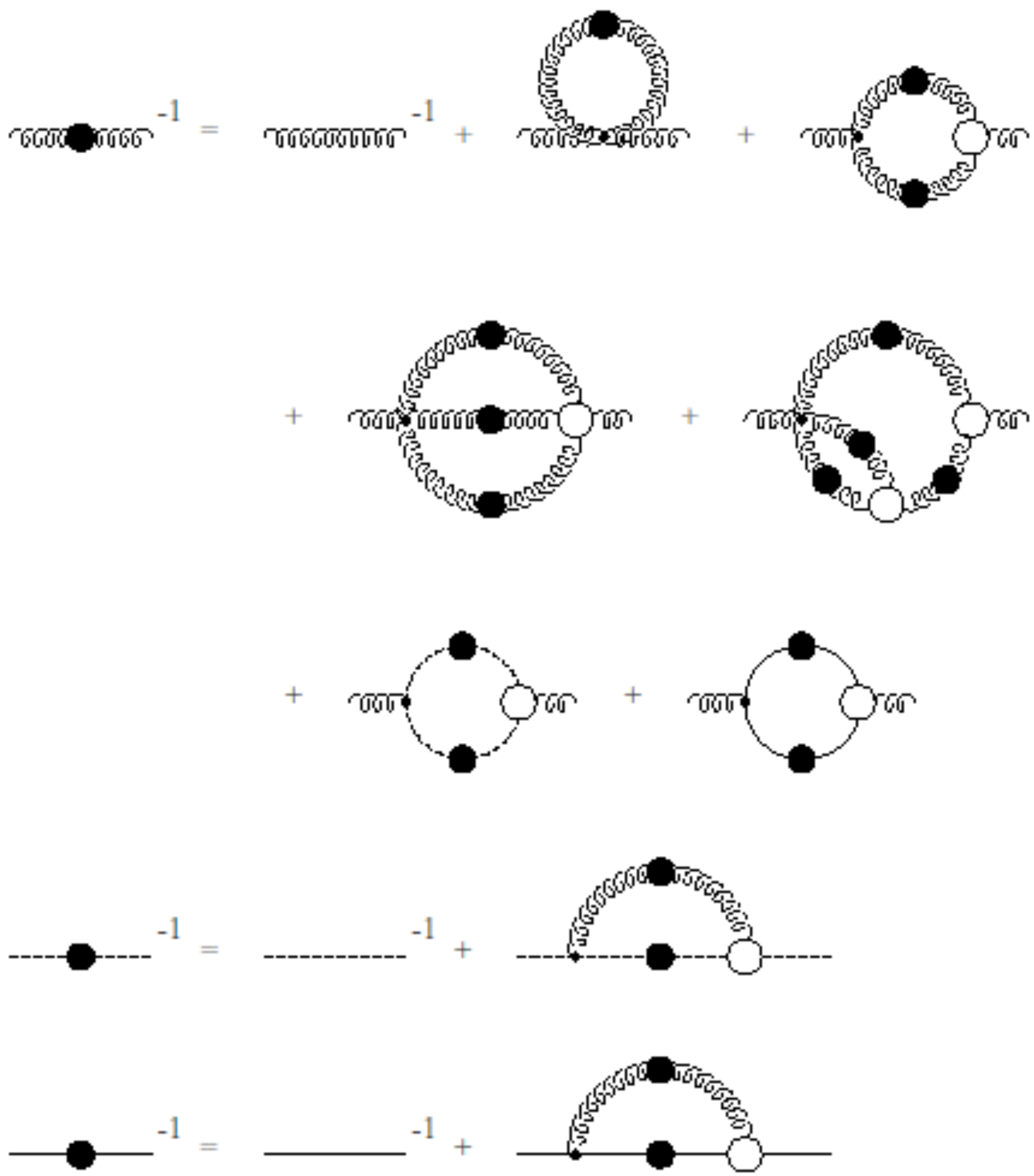
$$\langle\bar{\psi}\psi\rangle(1 \text{ GeV}) = (-251 \text{ MeV})^3$$



$$S(k) = i\frac{Z(k)}{\not{k} - M(k)}$$

P.O. Bowman et al., Phys. Rev. **D71**, 054507 (2005).

more on confinement



$$D(p) = -\frac{1}{p^2}\frac{1}{1+u(p)}$$

$$u(p) \to -1 \qquad p \to 0$$

Kugo-Ojima confinement criterion

Alkofer, von Smekal, Fischer

Heavy Quarks

modelling, quarks, heavy quarks

CHALLENGES

QCD is

- many body
- relativistic
- strong coupling (contrast to QED)
- quantum
- nonlinear

Modelling QCD

- physical pictures can change depending on the
  - scale

    quark mass, glue
  - observables

    ρ decay vs. ρ scattering
  - gauge

    confinement in Coulomb gauge vs. Weyl gauge

Modelling QCD

- we seek to understand low energy hadronic physics -- we are fortunate that we have the theory, but it is not always helpful!
- cf. the theory of DNA:

$$H = \sum_i \frac{-\hbar^2}{2m}\nabla_i^2 + \sum_{i<j} \frac{q_i q_j}{r_{ij}}$$

- to make progress we need to identify the appropriate effective degrees of freedom and their interactions

ex: the bag model

fermions ➡ current quarks

bosons ➡ bag pressure (+ perturbative one gluon exchange)

ex: the flux tube model

fermions ➡ constituent quarks

bosons ➡ flux tubes

Modelling QCD

- spontaneous chiral symmetry breaking implies both the existence of Goldstone bosons and constituent quarks

current quarks evolve into constituent quarks at scales $< \Lambda_{QCD}$

- it is the structure of the vacuum that gives chiral symmetry breaking and confinement

it is desirable to incorporate the physics of the vacuum and chiral symmetry breaking into the model from the beginning

- effective degrees of freedom should be derived from QCD to the extent possible

only in this way can we recover perturbative QCD in the high energy regime

Constituent Quarks

- pre-QCD quarks: $m \sim 5$ GeV
- Copley, Karl, & Obryk: $m \sim 330$ MeV
- QCD: $m(2\text{ GeV}) \sim 4$ MeV
- but recall that quarks are not observable $\Rightarrow$ different kinds of quark masses exist: current/constituent

PHYSICAL REVIEW D VOLUME 12, NUMBER 1 1 JULY 1975

Hadron masses in a gauge theory*

A. De Rújula, Howard Georgi,† and S. L. Glashow

FIG. 9. Detail of the singly charmed $S=-1$ baryons.

X. AFTERWORD

There are several purposes to this perhaps too ambitious work: We are not merely interested in explaining hadron masses. We feel that many of the wilder theoretical oats sown in recent years, and nourished by new and exciting experimental discoveries, are yielding a truly bountiful harvest. The naive quark model, supplemented by color gauge theory, asymptotic freedom, and infrared slavery, is turning out to be not so naive, and more than just a model. The demand for charm coming from abstract arguments about selection rules and triangle anomalies in unified models of weak and electromagnetic interactions may soon be met by nature. We can see coming a time when the subject of hadron spectroscopy as it is now known will be generally recognized to be interesting, but no longer truly fundamental. Hadron masses, widths, and cross sections may soon be "understood" if not precisely calculable. This optimistic view may yet be mere illusion, for it depends crucially on the discovery of charmed hadrons, and on the continued development of our theoretical tools. Remember, for example, that arguments for quark confinement, perhaps plausible, are certainly not yet rigorous.

But, if the time does come that hadron physics becomes mere spectroscopy, what are the remaining fundamental questions?

Why are the masses of quarks and leptons what they are? What is the significance of the Cabibbo angle, and can it be computed? Who violates CP? And, most profoundly, what is the unified system containing the weak, electromagnetic, and strong interactions? We presume that this will be a gauge theory based on a large but simple local symmetry group. We have seen[53] how such a picture necessarily involves particles with masses comparable to the Planck mass. It has been suggested[54] that it is gravitational attraction that supplies the missing force leading to the binding of Goldstone bosons necessary for the spontaneous breaking of the gauge group, and it is at the Planck mass that this force becomes relevant.

A disquieting aspect of current particle theory is the appearance of exact global symmetries which are not local and not associated (as electric charge is) with massless gauge fields. Corresponding to these symmetries are exact conservation laws for baryon number, electron number, and muon number. These conservation laws are familiarly deduced by imagining the effects of infinitesimal operations performed over all of space-time–not only in the laboratory today, but behind the moon next week. We find such a theoretical construct to be *a priori* absurd, and are therefore relieved that conservation laws of this kind, in truly unified theories, are only approximate.[53]

The conjectured existence of black holes pre-

$$|\mathrm{meson}\rangle = \phi_2|q\bar{q}\rangle + \phi_3|q\bar{q}g\rangle + \phi_4|q\bar{q}q\bar{q}\rangle + \ldots$$

$$= \tilde{\phi}_2|Q\bar{Q}\rangle + \tilde{\phi}_3|Q\bar{Q}G\rangle + \tilde{\phi}_4|Q\bar{Q}Q\bar{Q}\rangle + \ldots$$

$$P b^{\dagger}_{k,m} P^{\dagger} = \eta^{*} b^{\dagger}_{-k,m} \qquad P d^{\dagger}_{k,m} P^{\dagger} = -\eta\, d^{\dagger}_{-k,m}$$

$$P|nJM[LS]\rangle = (-)^{L+1}|nJM[LS]\rangle$$

$$C|nJM[LS]\rangle = (-)^{L+S}|nJM[LS]\rangle$$

$$Cb_{pm}C^\dagger = \eta_C d_{pm}$$

$$Cd^\dagger_{pm}C^\dagger = \eta_C b^\dagger_{pm}$$

$$|\mathbf{P}; nJM[LS]; II_z\rangle = \int d^3k d^3\bar{k}\, \delta(\mathbf{k} + \bar{\mathbf{k}} - \mathbf{P})\, \phi_{nL}(\mathbf{k}, \bar{\mathbf{k}}) X_{c,s,f;\bar{c},\bar{s},\bar{f}} Y_{LM_L}(\hat{k})\, b^\dagger_{c,s,f}(\mathbf{k}) d^\dagger_{\bar{c},\bar{s},\bar{f}}(\bar{\mathbf{k}})|0\rangle$$

$$\begin{aligned} P|nJM[LS]; II_z\rangle &= \ldots b^\dagger_{-\mathbf{r}}(-)d^\dagger_{+\mathbf{r}}|0\rangle \\ &= -Y_{LM_L}(-\hat{r}) \ldots b^\dagger_{+\mathbf{r}} d^\dagger_{-\mathbf{r}}|0\rangle \\ &= (-)^{L+1}|nJM[LS], II_z\rangle \end{aligned}$$

$$C|nJM[LS]; I0\rangle = \ldots d^\dagger_{+\mathbf{r},s} b^\dagger_{-\mathbf{r},\bar{s}}|0\rangle$$

$$\begin{aligned} &= (-)(-)^L \ldots b^\dagger_{+\mathbf{r},\bar{s}} d^\dagger_{-\mathbf{r},s}|0\rangle \\ &= (-)^{L+S}|nJM[LS]; I0\rangle \end{aligned}$$

$$\langle j_1 m_1, j_2 m_2 | JM\rangle = (-)^{J-j_1-j_2}\langle j_2 m_2; j_1 m_1 | JM\rangle.$$

$$... = X_{c_1,s_1,f_1;c_2,s_2,f_2;c_3,s_3,f_3} \int d^3k_1 d^3k_2 d^3k_3 \, \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 - \mathbf{P}) \, \phi(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \, b^\dagger_{c_1,s_1,f_1}(\mathbf{k}_1) b^\dagger_{c_2,s_2,f_2}(\mathbf{k}_2) b^\dagger_{c_3,s_3,f_3}(\mathbf{k}_3)|0\rangle$$

$$P = (-)^{L+1}$$

$$C = (-)^{L+S}$$

ular momentum and spin of the meson. It is now a simple matter to categori

L	S	J^{PC}	$^{(2S+1)}L_J$	example
0	0	0^{-+}	$^1S_0$	π
0	1	1^{--}	$^3S_1$	ρ
1	0	1^{+-}	$^1P_1$	h_1
1	1	$(0,1,2)^{++}$	$^3P_{(0,1,2)}$	a_0, a_1, a_2
2	0	2^{-+}	$^1D_2$	π_2
2	1	$(1,2,3)^{--}$	$^3D_{(1,2,3)}$	ρ, ρ_2, ρ_3

L	S	J^{PC}	$^{(2S+1)}L_J$	example
0	0	0^{-+}	$^1S_0$	π
0	1	1^{--}	$^3S_1$	ρ
1	0	1^{+-}	$^1P_1$	h_1
1	1	$(0,1,2)^{++}$	$^3P_{(0,1,2)}$	a_0, a_1, a_2
2	0	2^{-+}	$^1D_2$	π_2
2	1	$(1,2,3)^{--}$	$^3D_{(1,2,3)}$	ρ, ρ_2, ρ_3

not in the list: 0^{--}, $(\text{even})^{+-}$, $(\text{odd})^{-+}$

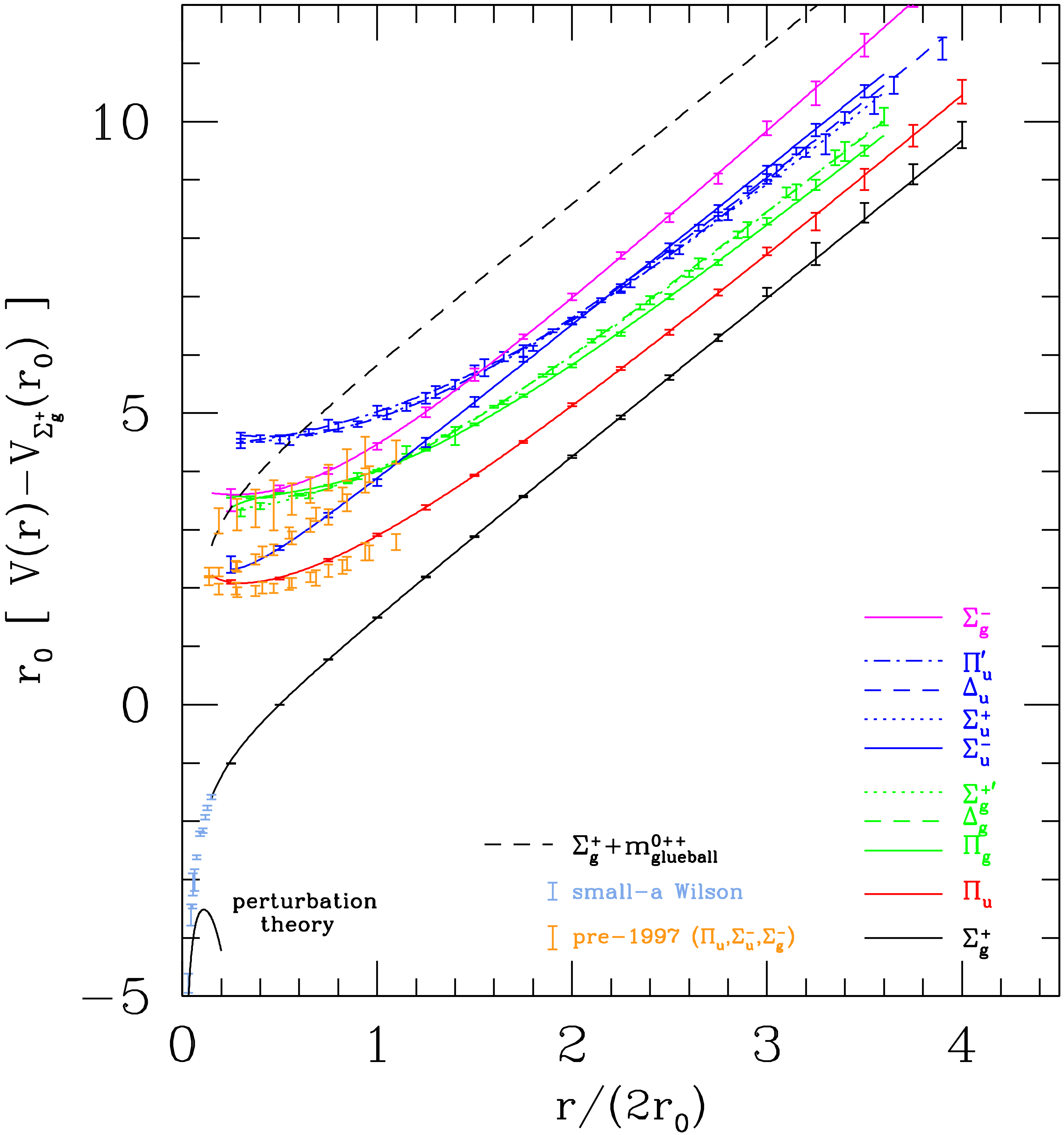
'(quantum number) exotics'

discovering such a state would be the first time a meson has been observed with no $q\bar{q}$ content

Heavy Quarkonia

ψ, Υ

- Bohr levels with a Bohr radius $(2/3m\alpha_s) \sim 0.01$ fm
- $^3L_{L+1}, {}^3L_L, {}^3L_{L-1}$ & ^1L_L splittings are due to tensor and spin-orbit interactions
- $^3S_1$ - $^1S_0$ splittings are due to the contact interaction



$r_0\,[\,V(r)-V_{\Sigma_g^+}(r_0)\,]$
$r/(2r_0)$
10
5
0
−5
0
1
2
3
4
Σ_g^-
Π_u'
Δ_u
Σ_u^+
Σ_u^-
$\Sigma_g^{+\prime}$
Δ_g
Π_g
Π_u
Σ_g^+
$\Sigma_g^+ + m_{\text{glueball}}^{0++}$
small−a Wilson
pre−1997 ($\Pi_u,\Sigma_u^-,\Sigma_g^-$)
perturbation theory

Constituent Quarks (heavy)

spatial regimes:

$r < 1/10$ fm	$r \sim 1/2$ fm	$r > 1$ fm
one gluon exchange	confinement	one pion exchange

Lorentz structure:

sources of spin-dependence are

(i) gluon exchange

(ii) corrections to the static potential

(iii) meson exchange (Fock sector mixing)

(iv) instanton forces

make a (field-theoretic) Foldy-Wouthuysen transformation

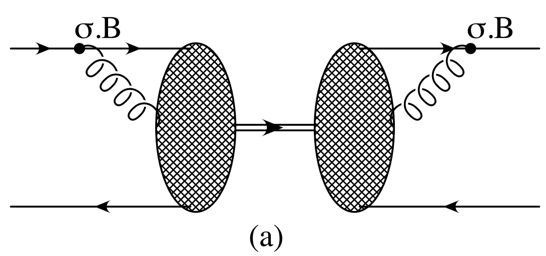
$$H_{\mathrm{QCD}} \to H_{\mathrm{FW}} = \int d\mathbf{x}[m_q h^\dagger(\mathbf{x}) h(\mathbf{x}) - m_{\bar{q}} \chi^\dagger(\mathbf{x}) \chi(\mathbf{x})] + H_{\mathrm{YM}} + V_C + H_1 + H_2 + \cdots, \quad (9)$$

$$H_1 = \frac{1}{2m_q} \int d\mathbf{x} h^\dagger(\mathbf{x})(\mathbf{D}^2 - g\boldsymbol{\sigma}\cdot\mathbf{B}) h(\mathbf{x}) - (h \to \chi; m_q \to m_{\bar{q}}), \quad (10)$$

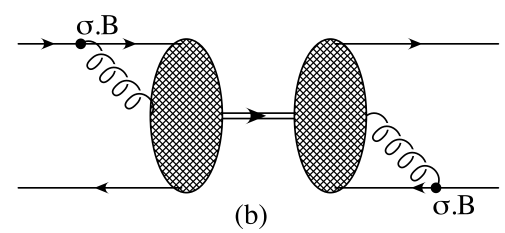
$$H_2 = \frac{1}{8m_q^2} \int d\mathbf{x} h^\dagger(\mathbf{x}) g\boldsymbol{\sigma}\cdot[\mathbf{E}, \times \mathbf{D}] h(\mathbf{x}) + (h \to \chi; m_q \to m_{\bar{q}}). \quad (11)$$

Interactions

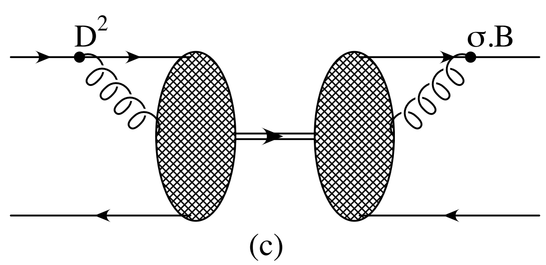
sig1_i*sig1_j = del_ij+ eps_ijk sig k so = B^2 so not spin-dependent



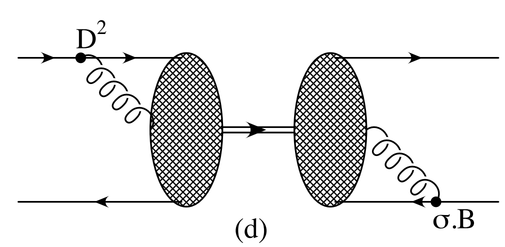
zero



hyperfine + tensor



V_1



V_2

spin-dependence in the confinement potential

examine in Coulomb gauge via the Foldy-Wouthuysen transformation

$$H_{QCD} \to H_{FW} = \int dx \left(m_q h^\dagger(x)h(x) - m_{\bar{q}}\chi^\dagger(x)\chi(x)\right) + H_{YM} + \\ +V_C + H_1 + H_2 + \ldots$$

$$H_1 = \frac{1}{2m_q}\int dx h^\dagger(x)\left(D^2 - g\sigma\cdot B\right)h(x) - (h \to \chi; m_q \to m_{\bar{q}})$$

$$H_2 = \frac{1}{8m_q^2}\int dx h^\dagger(x) g\sigma\cdot[E, \times D]h(x) + (h \to \chi; m_q \to m_{\bar{q}})$$

$$D = i\nabla + gA$$

$$E^a = -\Pi^a + E^a_\ell$$

$$E^a_\ell = -\nabla A^a_0 - g\nabla\nabla^{-2} f^{abc} A^b \cdot \nabla A^c_0$$

$$A^a_0(x) = g\int dy V^{ab}(x, y; A)\rho^b(y)$$

model building — more later

$$V_{SI}(r) = -\frac{3}{4}\frac{\alpha_s}{r} + br$$

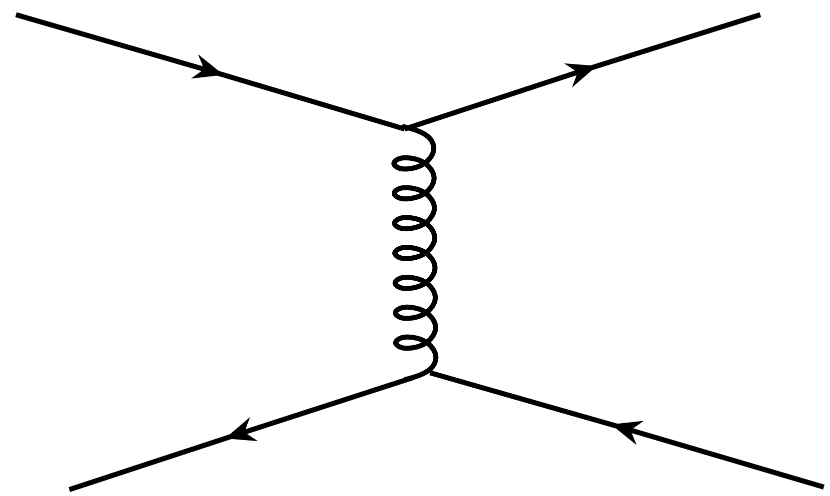
$$\begin{aligned} V_{SD}(r) \quad = \quad & \left(\frac{\sigma_q}{4m_q^2} + \frac{\sigma_{\bar{q}}}{4m_{\bar{q}}^2}\right) \cdot \mathbf{L} \left(\frac{1}{r}\frac{dV_{conf}}{dr} + \frac{2}{r}\frac{dV_1}{dr}\right) + \left(\frac{\sigma_{\bar{q}} + \sigma_q}{2m_q m_{\bar{q}}}\right) \cdot \mathbf{L} \left(\frac{1}{r}\frac{dV_2}{dr}\right) \\ & + \frac{1}{12m_q m_{\bar{q}}}\left(3\sigma_q \cdot \hat{\mathbf{r}}\, \sigma_{\bar{q}} \cdot \hat{\mathbf{r}} - \sigma_q \cdot \sigma_{\bar{q}}\right) V_3 + \frac{1}{12m_q m_{\bar{q}}}\sigma_q \cdot \sigma_{\bar{q}} V_4 \\ & + \frac{1}{2}\left[\left(\frac{\sigma_q}{m_q^2} - \frac{\sigma_{\bar{q}}}{m_{\bar{q}}^2}\right) \cdot \mathbf{L} + \left(\frac{\sigma_q - \sigma_{\bar{q}}}{m_q m_{\bar{q}}}\right) \cdot \mathbf{L}\right] V_5. \end{aligned} \tag{1}$$

Eichten & Feinberg

Ng, Pantaleone, & Tye

oge approximation / model

spin dependence



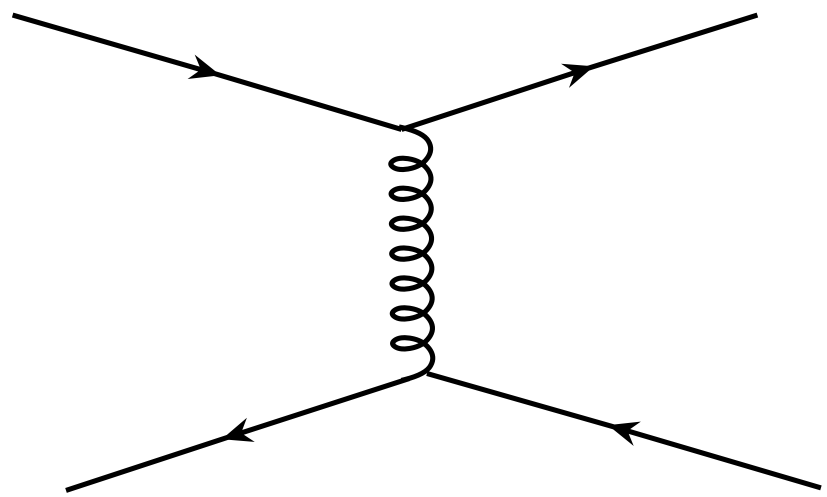
$$U = (V_C + V_{so} + V_{hyp}) \frac{\vec{\lambda}_1}{2} \cdot \frac{-\vec{\lambda}_2^{\,*}}{2}$$

$$V_C = \frac{\alpha}{r} - \frac{\alpha\pi}{2}\left(\frac{1}{m_1^2} + \frac{1}{m_2^2}\right)\delta(\vec{r})$$

$$V_{hyp} = \frac{\alpha}{4m_1m_2}\left(\frac{\vec{\sigma}_1\cdot\vec{\sigma}_2}{r^3} - \frac{3(\vec{\sigma}_1\cdot\vec{r})(\vec{\sigma}_2\cdot\vec{r})}{r^5} - \frac{8\pi}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2\,\delta(\vec{r})\right)$$

$$V_{so} = -\frac{\alpha}{2m_1m_2r}\left(\vec{p}_1\cdot\vec{p}_2 + \frac{\vec{r}\cdot(\vec{r}\cdot\vec{p}_1)\vec{p}_2}{r^2}\right) - \frac{\alpha}{4r^3}\left(\frac{\vec{r}\times\vec{p}_1\cdot\vec{\sigma}_1}{m_1^2} - \frac{\vec{r}\times\vec{p}_2\cdot\vec{\sigma}_2}{m_2^2}\right)$$
$$- \frac{\alpha}{2m_1m_2r^3}\left(\vec{r}\times\vec{p}_1\cdot\vec{\sigma}_2 - \vec{r}\times\vec{p}_2\cdot\vec{\sigma}_1\right)$$

One Gluon Exchange



$F_i \cdot F_j$

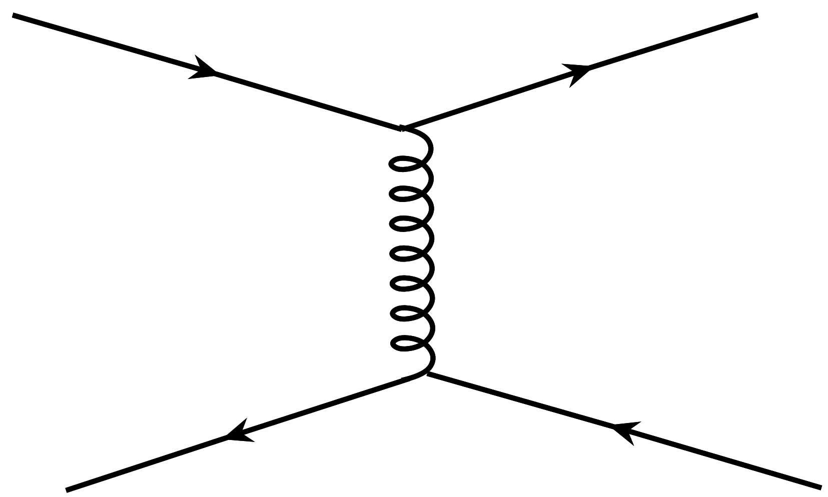
$$U = (V_C + V_{so} + V_{hyp})\frac{\vec{\lambda}_1}{2} \cdot \frac{-\vec{\lambda}_2{}^*}{2}$$

$$V_C = \frac{\alpha}{r} - \frac{\alpha\pi}{2}\left(\frac{1}{m_1^2} + \frac{1}{m_2^2}\right)\delta(\vec{r})$$

$$V_{hyp} = \frac{\alpha}{4m_1 m_2}\left(\frac{\vec{\sigma}_1\cdot\vec{\sigma}_2}{r^3} - \frac{3(\vec{\sigma}_1\cdot\vec{r})(\vec{\sigma}_2\cdot\vec{r})}{r^5} - \frac{8\pi}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2\,\delta(\vec{r})\right)$$

$$V_{so} = -\frac{\alpha}{2m_1 m_2 r}\left(\vec{p}_1\cdot\vec{p}_2 + \frac{\vec{r}\cdot(\vec{r}\cdot\vec{p}_1)\vec{p}_2}{r^2}\right) - \frac{\alpha}{4r^3}\left(\frac{\vec{r}\times\vec{p}_1\cdot\vec{\sigma}_1}{m_1^2} - \frac{\vec{r}\times\vec{p}_2\cdot\vec{\sigma}_2}{m_2^2}\right)$$
$$- \frac{\alpha}{2m_1 m_2 r^3}\left(\vec{r}\times\vec{p}_1\cdot\vec{\sigma}_2 - \vec{r}\times\vec{p}_2\cdot\vec{\sigma}_1\right)$$

One Gluon Exchange



Darwin term

$$U = (V_C + V_{so} + V_{hyp}) \frac{\vec{\lambda}_1}{2} \cdot \frac{-\vec{\lambda}_2^{\,*}}{2}$$

$$V_C = \frac{\alpha}{r} - \frac{\alpha\pi}{2}\left(\frac{1}{m_1^2} + \frac{1}{m_2^2}\right)\delta(\vec{r})$$

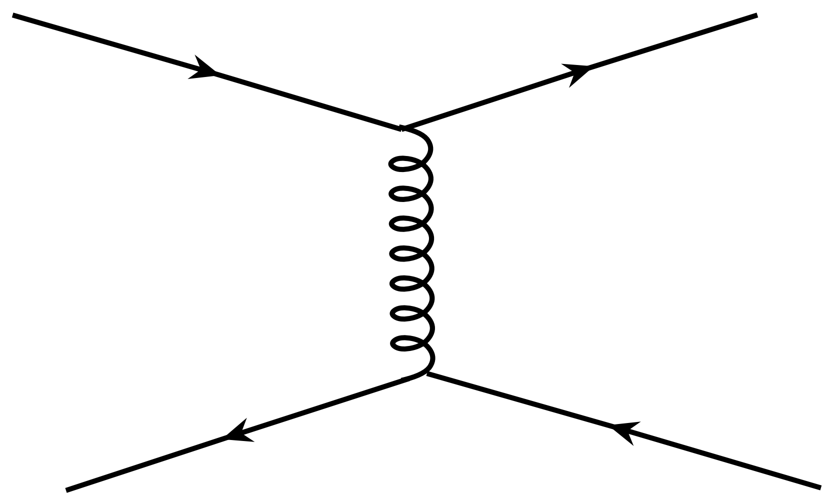
contact hyperfine term

$$V_{hyp} = \frac{\alpha}{4m_1 m_2}\left(\frac{\vec{\sigma}_1\cdot\vec{\sigma}_2}{r^3} - \frac{3(\vec{\sigma}_1\cdot\vec{r})(\vec{\sigma}_2\cdot\vec{r})}{r^5} - \frac{8\pi}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2\,\delta(\vec{r})\right)$$

tensor hyperfine term

$$V_{so} = \frac{\alpha}{2m_1 m_2 r}\left(\vec{p}_1\cdot\vec{p}_2 + \frac{\vec{r}\cdot(\vec{r}\cdot\vec{p}_1)\vec{p}_2}{r^2}\right) - \frac{\alpha}{4r^3}\left(\frac{\vec{r}\times\vec{p}_1\cdot\vec{\sigma}_1}{m_1^2} - \frac{\vec{r}\times\vec{p}_2\cdot\vec{\sigma}_2}{m_2^2}\right)$$
$$- \frac{\alpha}{2m_1 m_2 r^3}\left(\vec{r}\times\vec{p}_1\cdot\vec{\sigma}_2 - \vec{r}\times\vec{p}_2\cdot\vec{\sigma}_1\right)$$

One Gluon Exchange



$$U = (V_C + V_{so} + V_{hyp})\,\frac{\vec{\lambda}_1}{2}\cdot\frac{-\vec{\lambda}_2^{\,*}}{2}$$

$$V_C = \frac{\alpha}{r} - \frac{\alpha\pi}{2}\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}\right)\delta(\vec{r})$$

$$V_{hyp} = \frac{\alpha}{4m_1m_2}\left(\frac{\vec{\sigma}_1\cdot\vec{\sigma}_2}{r^3} - \frac{3(\vec{\sigma}_1\cdot\vec{r})(\vec{\sigma}_2\cdot\vec{r})}{r^5} - \frac{8\pi}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2\,\delta(\vec{r})\right)$$

$$V_{so} = -\frac{\alpha}{2m_1m_2r}\left(\vec{p}_1\cdot\vec{p}_2 + \frac{\vec{r}\cdot(\vec{r}\cdot\vec{p}_1)\vec{p}_2}{r^2}\right) - \frac{\alpha}{4r^3}\left(\frac{\vec{r}\times\vec{p}_1\cdot\vec{\sigma}_1}{m_1^2} - \frac{\vec{r}\times\vec{p}_2\cdot\vec{\sigma}_2}{m_2^2}\right)$$
$$- \frac{\alpha}{2m_1m_2r^3}\left(\vec{r}\times\vec{p}_1\cdot\vec{\sigma}_2 - \vec{r}\times\vec{p}_2\cdot\vec{\sigma}_1\right)$$

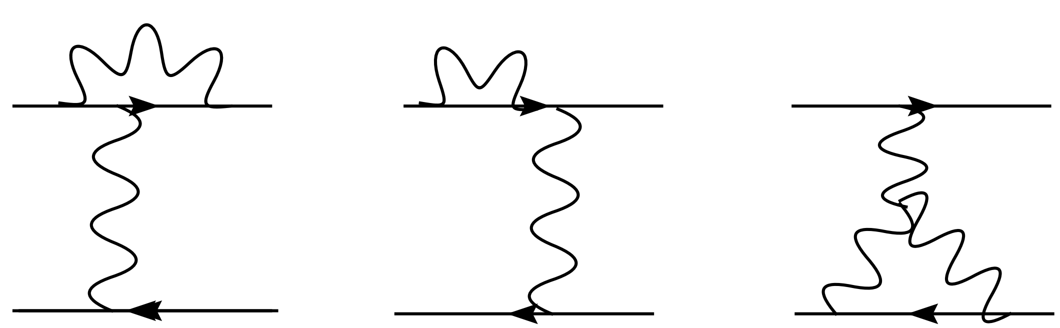
spin independent term

spin orbit term

Constituent Quarks (heavy)

perturbative potentials

Gupta & Radford, PRD33, 777 (86)



Constituent Quarks (heavy)

perturbative potentials

$$
\begin{aligned}
V_1(m_q, m_{\bar{q}}, r) &= -br - C_F \frac{1}{2r} \frac{\alpha_s^2}{\pi} \left(C_F - C_A \left(\ln \left[(m_q m_{\bar{q}})^{1/2} r \right] + \gamma_E \right) \right) \\
V_2(m_q, m_{\bar{q}}, r) &= -\frac{1}{r} C_F \alpha_s \left[1 + \frac{\alpha_s}{\pi} \left[\frac{b_0}{2} [\ln(\mu r) + \gamma_E] + \frac{5}{12} b_0 - \frac{2}{3} C_A + \frac{1}{2} \left(C_F - C_A \left(\ln \left[(m_q m_{\bar{q}})^{1/2} r \right] + \gamma_E \right) \right) \right] \right] \\
V_3(m_q, m_{\bar{q}}, r) &= \frac{3}{r^3} C_F \alpha_s \left[1 + \frac{\alpha_s}{\pi} \left[\frac{b_0}{2} [\ln(\mu r) + \gamma_E - \frac{4}{3}] + \frac{5}{12} b_0 - \frac{2}{3} C_A + \right. \right. \\
&\quad \left. \left. + \frac{1}{2} \left(C_A + 2C_F - 2C_A \left(\ln \left[(m_q m_{\bar{q}})^{1/2} r \right] + \gamma_E - \frac{4}{3} \right) \right) \right] \right] \\
V_4(m_q, m_{\bar{q}}, r) &= \frac{32 \alpha_s \sigma^3 e^{-\sigma^2 r^2}}{3\sqrt{\pi}} \\
V_5(m_q, m_{\bar{q}}, r) &= \frac{1}{4r^3} C_F C_A \frac{\alpha_s^2}{\pi} \ln \frac{m_{\bar{q}}}{m_q} \qquad (1)
\end{aligned}
$$

Constituent Quarks (heavy)

model the 1/c^2 spin-dependence in the confinement interaction…

$$V = \frac{1}{2}\int d^3x\, d^3y\, \bar{\psi}\Gamma\psi(y) K(x-y) \bar{\psi}\Gamma\psi(x)$$

$\Gamma = \mathbb{1}$ scalar

$\Gamma = \gamma_\mu$ vector

$\Gamma = \gamma_5$ pseudoscalar

NB: there is no reason for QCD to take on this simple form!

spin-dependence in the confinement potential

$$V_{conf} \to \epsilon + V_{SD} + \ldots$$

Γ	ϵ_Γ	V_1	V_2	V_3	V_4
scalar	S	$-S$	0	0	0
vector	V	0	V	$V'/r - V''$	$2\nabla^2 V$
pseudoscalar	0	0	0	$P'' - P'/r$	$\nabla^2 P$

Dieter Gromes

Constituent Quarks (heavy)

compare to experimental spin splittings

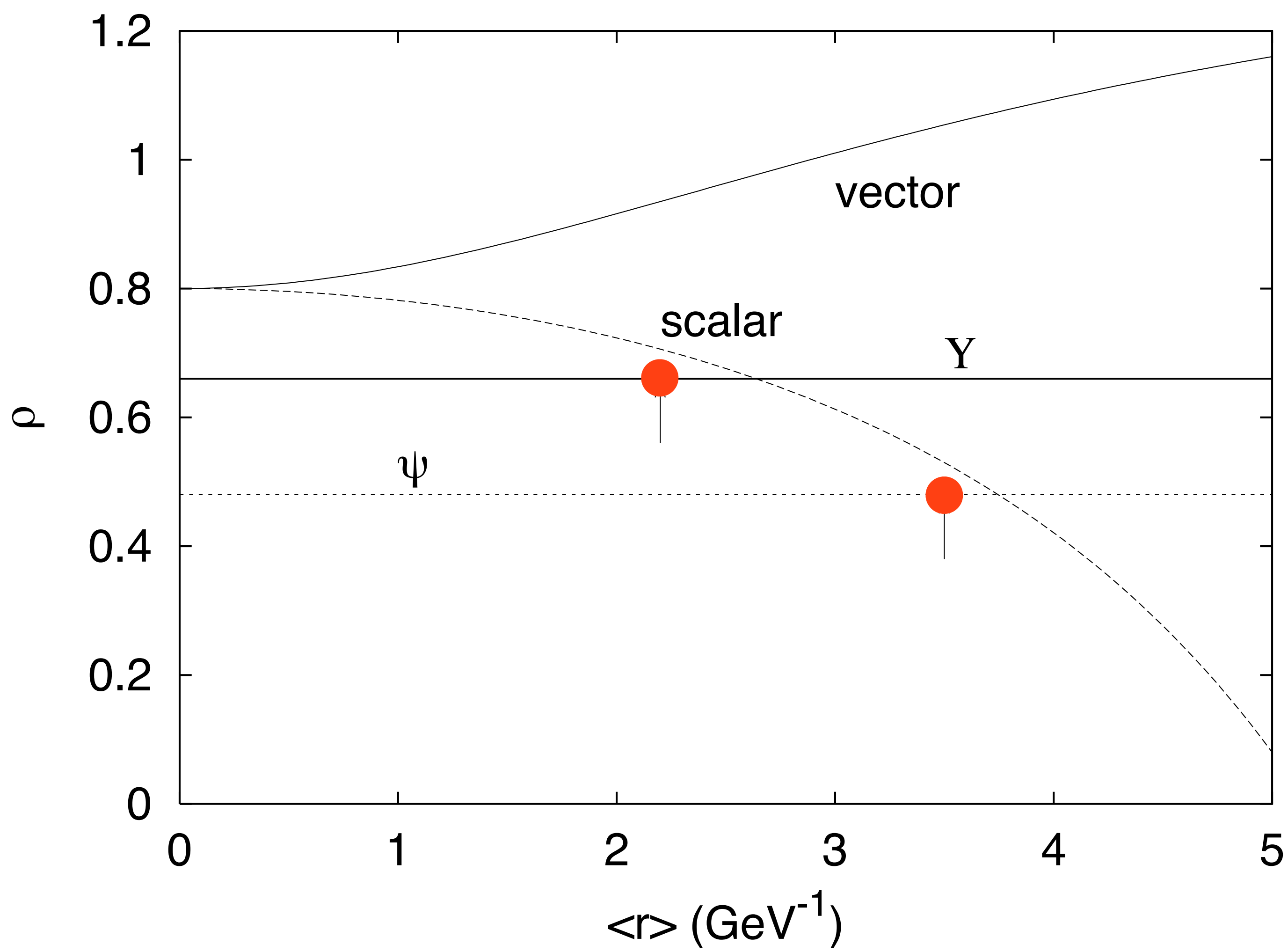
$$\rho_{\substack{V\\S}} \equiv \frac{M(^3P_2) - M(^3P_1)}{M(^3P_1) - M(^3P_0)} = \frac{\langle \frac{16\alpha_s}{5r^3} \pm A\frac{b}{r}\rangle}{\langle \frac{4\alpha_s}{r^3} \pm B\frac{b}{2r}\rangle},$$

$$\rho = \frac{2^{++} - 1^{++}}{1^{++} - 0^{++}}$$

$A = 14$ (1) and $B = 4$ (1) for the vector (scalar) case.

'scalar' confinement
is preferred

Constituent Quarks (heavy)



Howard Schnitzer

Thus we are in a position to guess that the full spatial, colour, and Lorentz structure of the long range confining interaction is

$$V_{conf} = \frac{1}{2} \int \bar{\psi}(x) T^a \psi(x) V(x-y) \bar{\psi}(y) T^a \psi(y) \tag{52}$$

The nonrelativistic reduction of this operator yields

$$V_{conf} = V(r) - \frac{1}{2r} \frac{dV(r)}{dr} \left(\frac{S_1}{m_1^2} + \frac{S_2}{m_2^2} \right) \cdot \boldsymbol{L} \tag{53}$$

and hence we identify V as $-3/4br$. This is same form as the Thomas precession contribution to the spin-orbit interaction of the one gluon exhange potential. Thus the assumed Lorentz structure for the confinement term gives rise to a spin-dependent Thomas precession spin orbit interaction which tends to cancel the spin orbit term.

quarks. However one should be aware that it has many problems. Among these are

(i) Eq. 52 necessarily yields a Δ confinement potential for baryons. We have seen that this is not in accord with flux tube ideas or (possibly) with lattice data.

(ii) A scalar interaction breaks chiral symmetry and so can never be used to obtain the chirally broken vacuum, chiral pions, or and other chiral properties of QCD. The best one can hope is that it captures important aspects of the broken phase of QCD (recall that it does agree with lattice computations of the long range spin dependent interquark potential).

(iii) Scalar potentials do not admit stable vacua.

(iv) If the nonrelativistic reduction of Eq. 52 yields confinement for quark-antiquark interactions, it necessarily gives anticonfinement for quark-quark interactions. Thus either mesons or baryons do not exist in this model and it must be fixed by hand in going from one hadron sector to the other.

> Use the rules of Appendix XXX to show this explicitly.

(v) Eq. 52 implies that the long distance interaction between colour singlet states is a power law. This is the colour analogue of the van der Waals force. Of course the actual long distance behaviour is described by the Yukawa potential of one pion exchange. Thus the physical interaction is exponentially smaller than the colour van der Waals force predicted by the $\vec{F}_i \cdot \vec{F}_j$ model and is strongly ruled out by experiment. Of course the problem is that the $\vec{F}_i \cdot \vec{F}_j$ model does not know enough about string dynamics.

Constituent Quarks (heavy)

$$H = \sum_i m_i + \sum_i \frac{p_i^2}{2m_i} + C + \sum_{i<j}\left[-\left(-\frac{\alpha_s}{r_{ij}} + \frac{3}{4}br_{ij}\right)\vec{F}_i\cdot\vec{F}_j + V_{SD}^{oge}(r_{ij}) + V_{SD}^{conf}(r_{ij})\right]$$

variants:

running coupling
smeared delta functions
relativized
perturbative corrections
Mercedes baryon potential
instanton potential
flip flop potential
apply to light quarks?

$$V_0^{(c\bar{c})}(r) = -\frac{4}{3}\frac{\alpha_s}{r} + br + \frac{32\pi\alpha_s}{9m_c^2}\,\tilde{\delta}_\sigma(r)\,\vec{\mathrm{S}}_c \cdot \vec{\mathrm{S}}_{\bar{c}}$$

$$\tilde{\delta}_\sigma(r) \;=\; (\sigma/\sqrt{\pi})^3\, e^{-\sigma^2 r^2}.$$

$$V_{spin-dep} = \frac{1}{m_c^2}\left[\left(\frac{2\alpha_s}{r^3} - \frac{b}{2r}\right)\vec{\mathrm{L}}\cdot\vec{\mathrm{S}} + \frac{4\alpha_s}{r^3}\,\mathrm{T}\,\right].$$

$$\langle {}^3\mathrm{L_J}|\mathrm{T}|{}^3\mathrm{L_J}\rangle = \begin{cases} -\frac{\mathrm{L}}{6(2\mathrm{L}+3)}, & \mathrm{J} = \mathrm{L}+1 \\ +\frac{1}{6}, & \mathrm{J} = \mathrm{L} \\ -\frac{(\mathrm{L}+1)}{6(2\mathrm{L}-1)}, & \mathrm{J} = \mathrm{L}-1 \end{cases}$$

L*S requires L=J and S=0,1 so 1P1+3P1; 1D2+3D2; 1F3+3F3

L*S = [J(J+1) - L(L+1)/2 - S(S+1)]/2

for tensor: 3S1-3D1; 3P2-3F2; 3D3-3G3

Heavy Quarkonia

$$E(2++) = E_0 + S/4 - T/5 + L$$

$$E(1++) = E_0 + S/4 + T - L$$

$$E(0++) = E_0 + S/4 - 2T - 2L$$

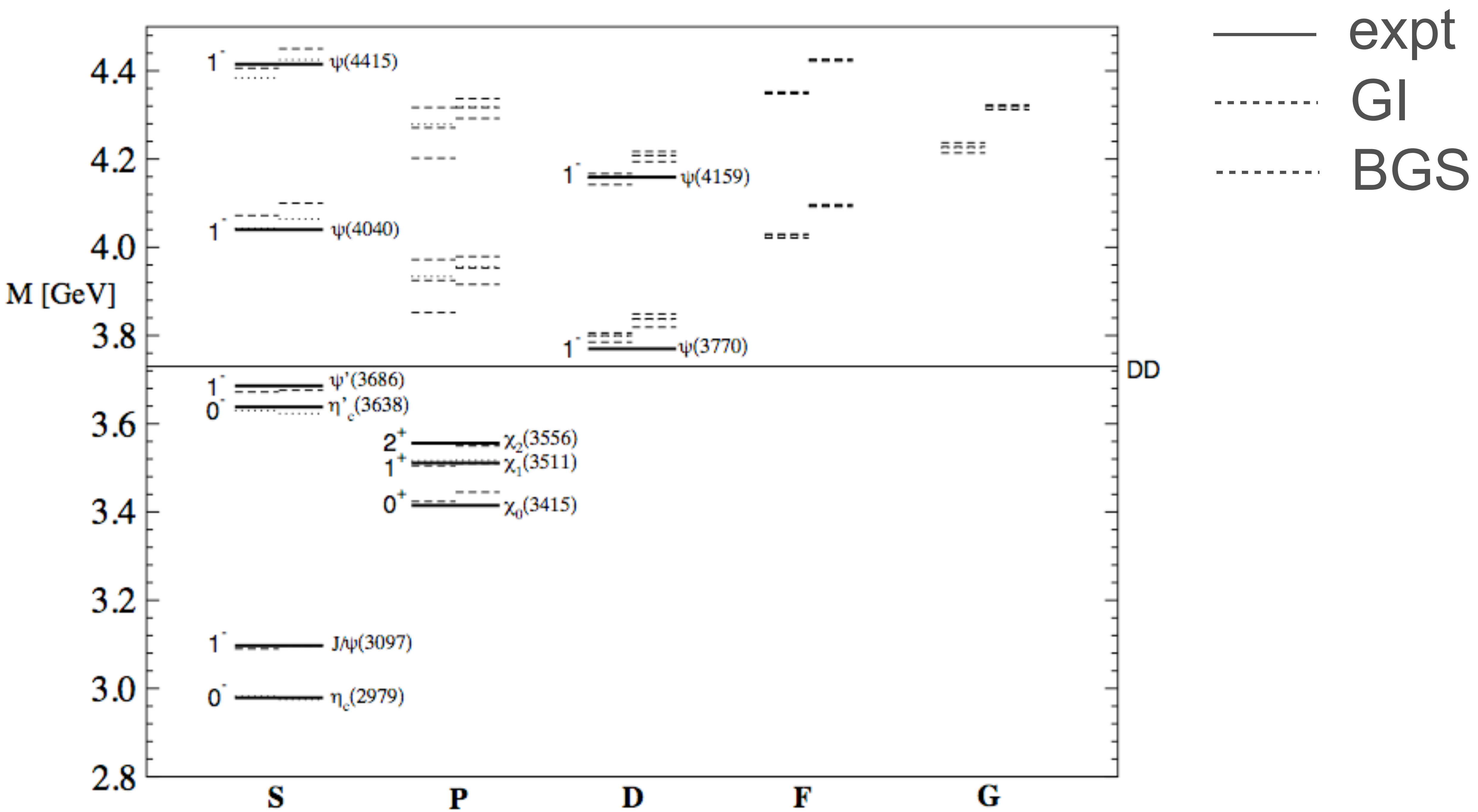
$$E(1+-) = E_0 - 3S/4$$

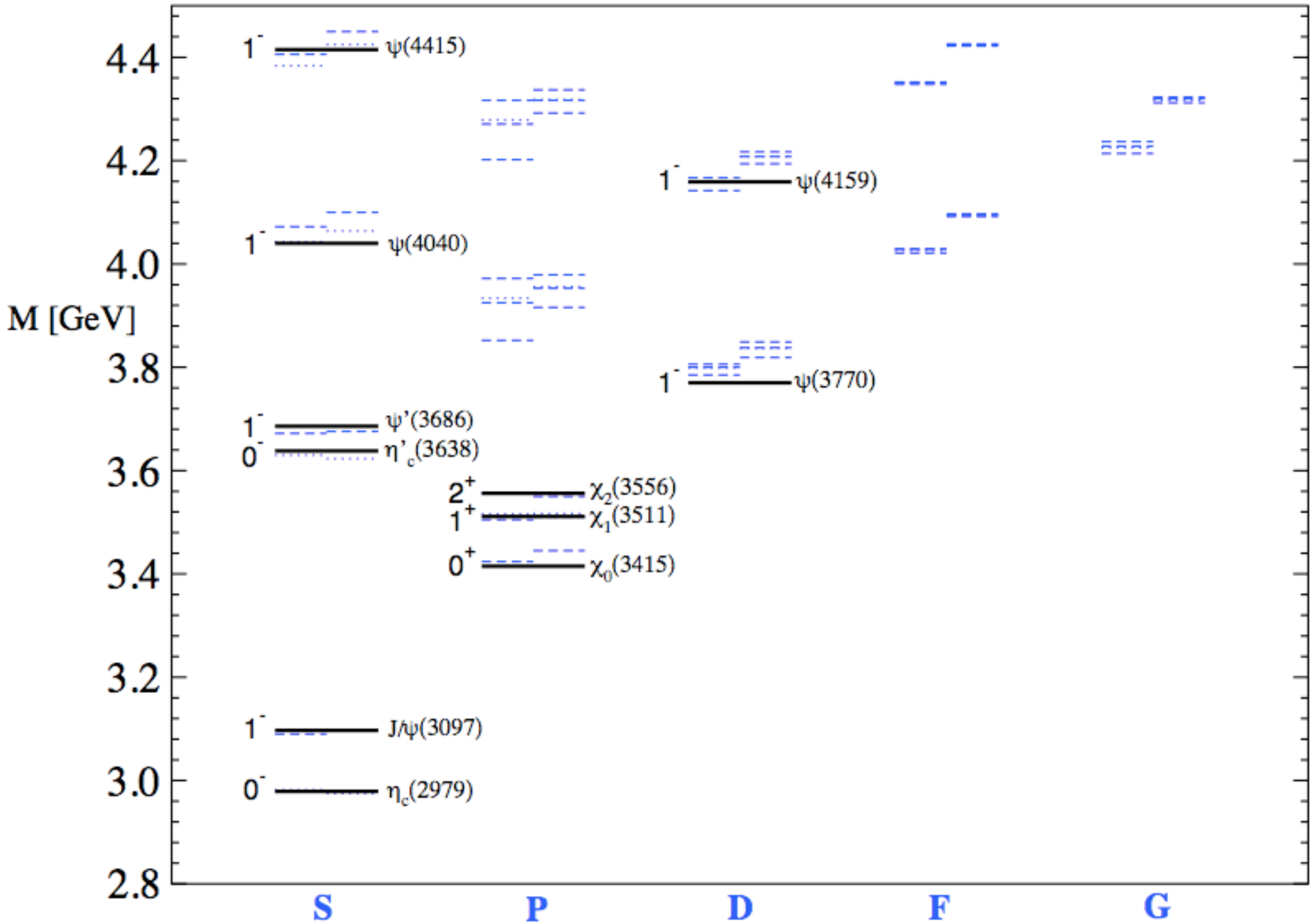
$S = \langle V_{con} \rangle$ $T = \langle V_{ten} \rangle$ $L = \langle V_{so} \rangle$

T= 13 MeV, L=28 MeV T= 20(2) MeV, L=34(3)MeV

Charmonium Spectrum

nonrelativistic quark models





M [GeV]
4.4
4.2
4.0
3.8
3.6
3.4
3.2
3.0
2.8
1⁻ ψ(4415)
1⁻ ψ(4040)
1⁻ ψ'(3686)
0⁻ η'c(3638)
1⁻ J/ψ(3097)
0⁻ ηc(2979)
2⁺ χ2(3556)
1⁺ χ1(3511)
0⁺ χ0(3415)
1⁻ ψ(4159)
1⁻ ψ(3770)
S
P
D
F
G

Charmonium Spectrum

lattice gauge theory

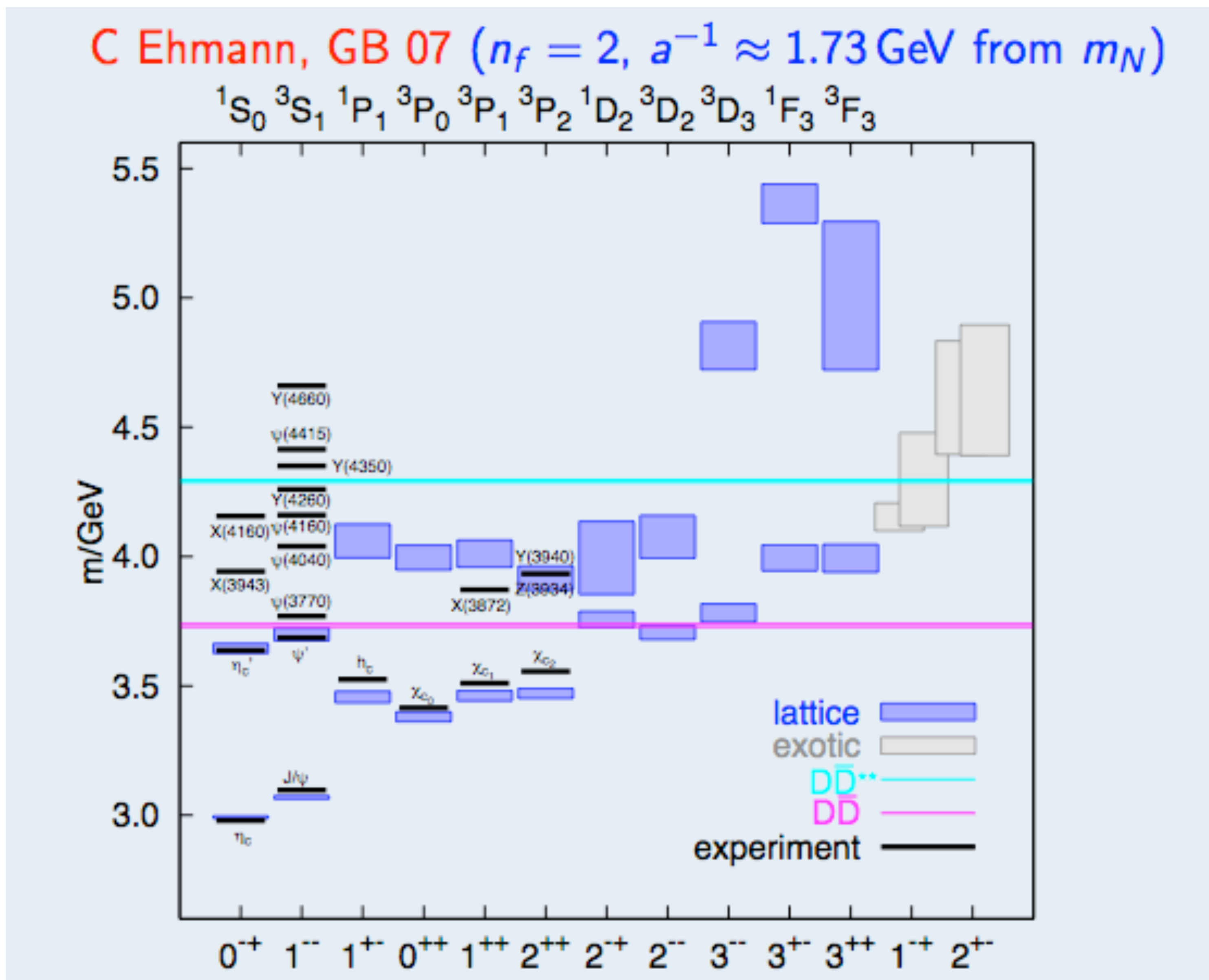
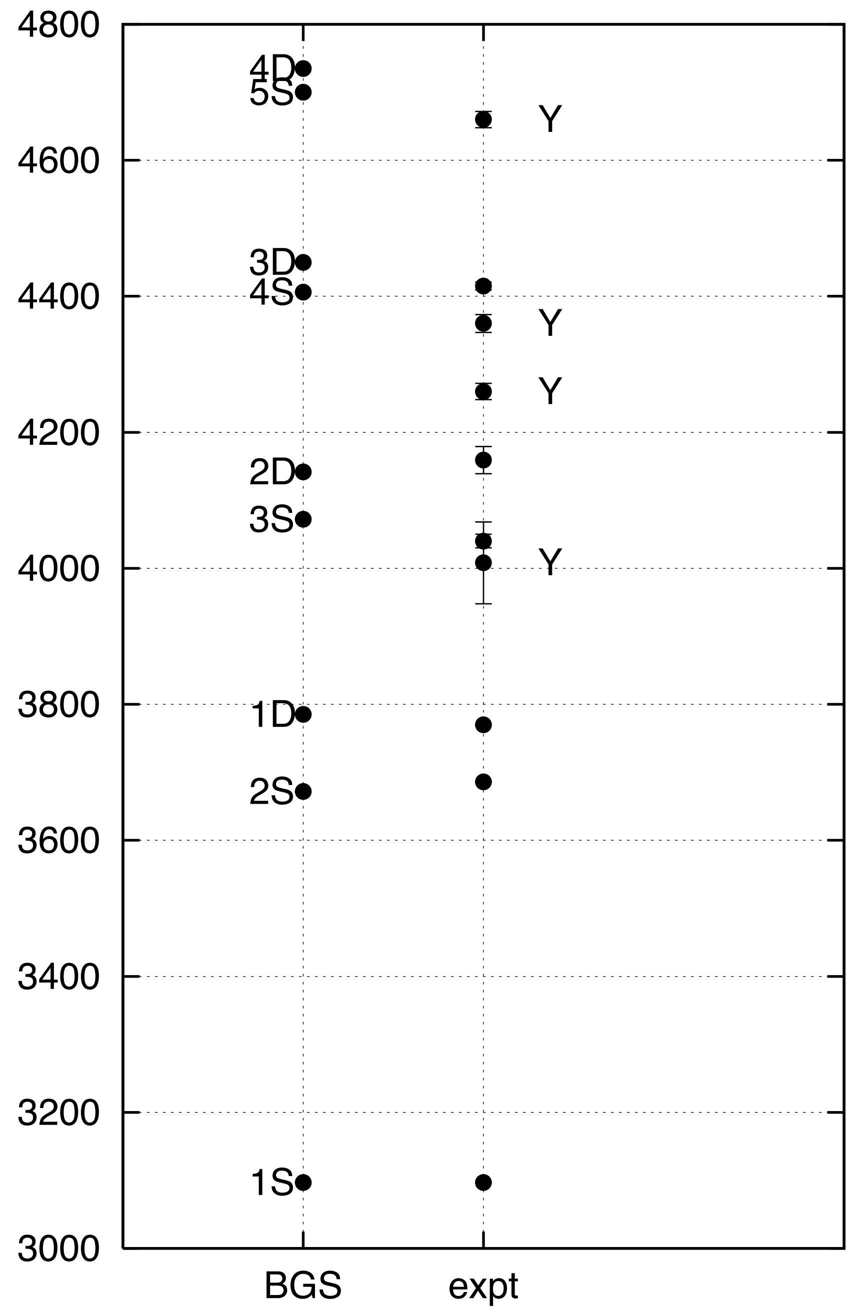
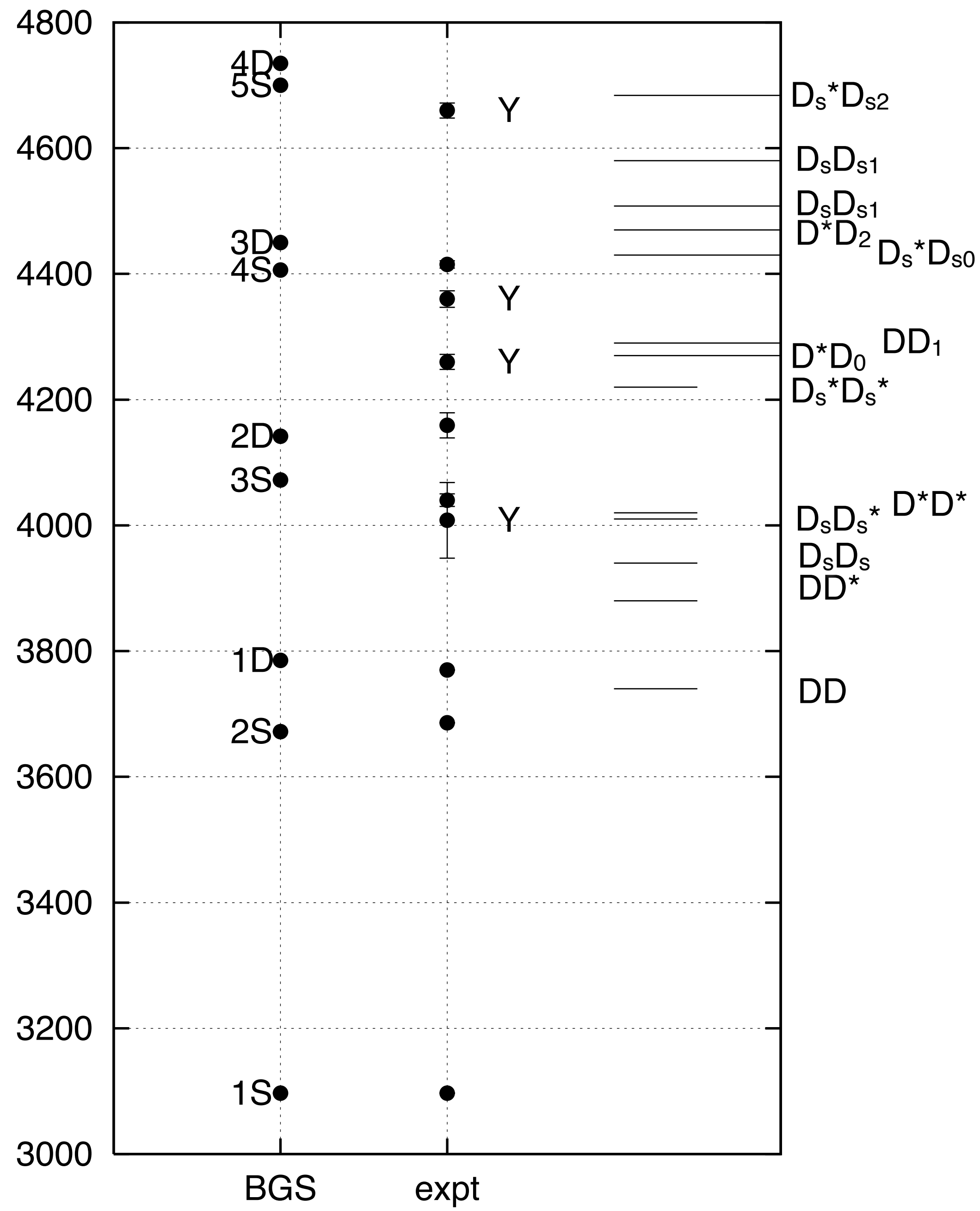


FIG. 1: Predicted and observed spectrum of charmonium states (Table I). The solid lines are experiment, and the broken lines are theory (NR model left, GI right). Spin triplet levels are dashed, and spin singlets are dotted. The DD open-charm threshold at 3.73 GeV is also shown.

Charmonium Vectors -- Constituent Quark Model

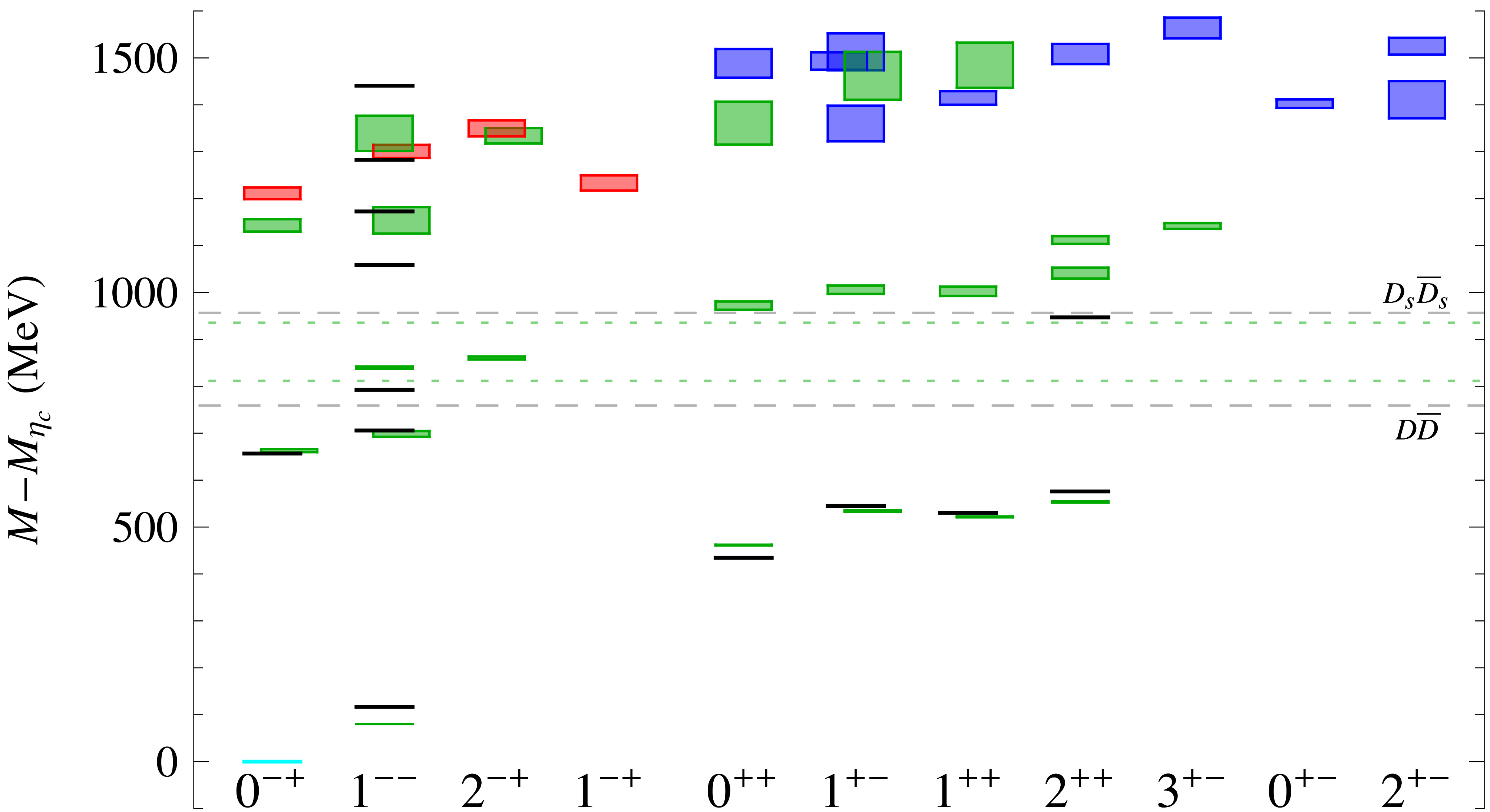


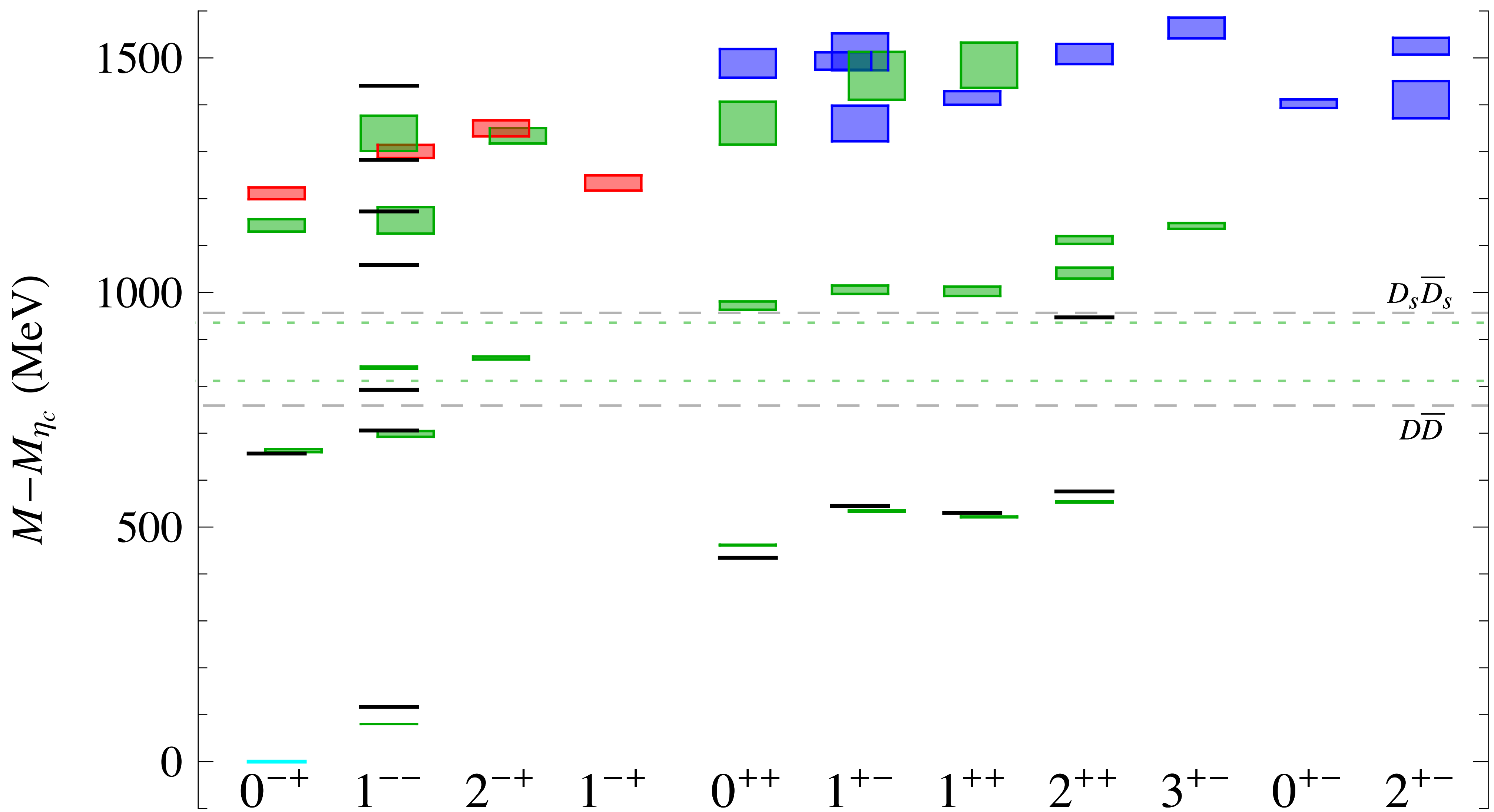
Charmonium Vectors -- Constituent Quark Model



$(J^{P\tilde{C}})_g = 1^{+-}$

orange is hybrids (later)
black = expt





1500
1000
500
0
$M - M_{\eta_c}$ (MeV)
$D_s\overline{D}_s$
$D\overline{D}$
0^{-+}
1^{--}
2^{-+}
1^{-+}
0^{++}
1^{+-}
1^{++}
2^{++}
3^{+-}
0^{+-}
2^{+-}

PERTURBATIVE QCD

M. Shepherd [CLEO], GHP09
T. Pedlar [CLEO], Moriond, 2009

$$\frac{Bf(J/\psi \to \gamma\eta)}{Bf(J/\psi \to \gamma\eta')} = \frac{11.01 \pm 0.29 \pm 0.22}{52.4 \pm 1.2 \pm 1.1} = 0.21 \pm 0.04$$

$*10^{-4}$

$$\frac{Bf(\psi(2S) \to \gamma\eta)}{Bf(\psi(2S) \to \gamma\eta')} = \frac{< 0.02}{1.19 \pm 0.08 \pm 0.03} < 0.018$$

why the difference? Speculate that it is due to interference with hybrids?

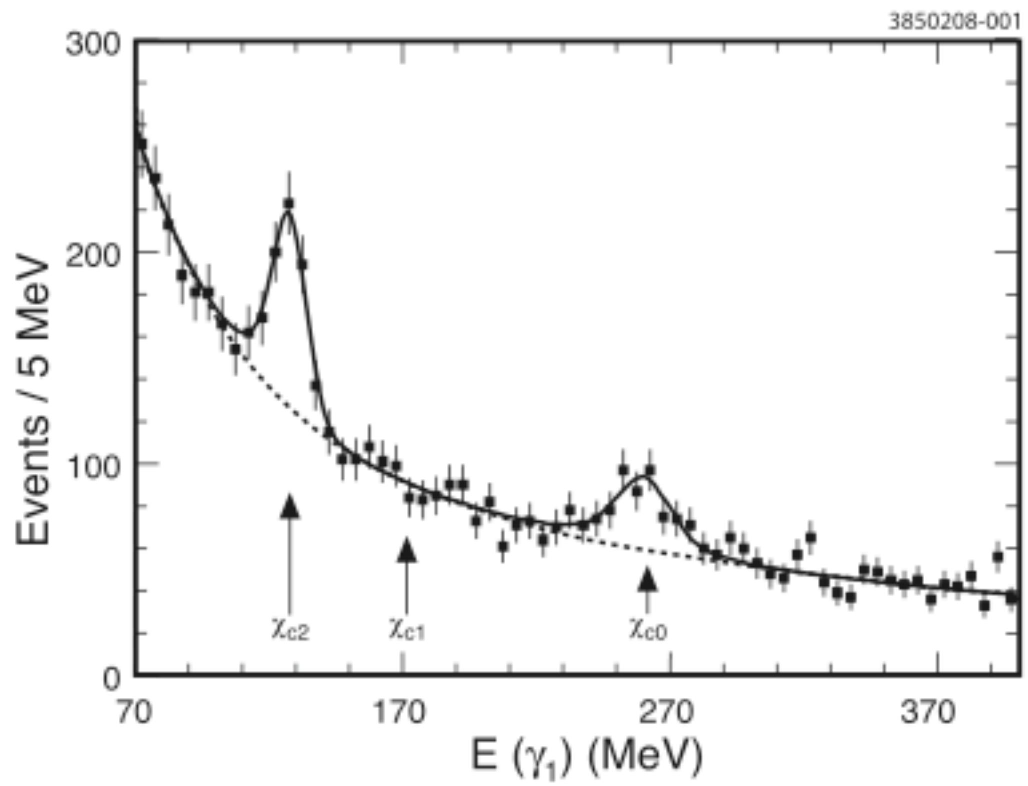
PERTURBATIVE QCD

$$R = \frac{\Gamma(\chi_{c2} \to \gamma\gamma)}{\Gamma(\chi_{c0} \to \gamma\gamma)} = \frac{4}{15}(1 - 1.76\alpha_s) \;= 0.12 \;(\alpha_s = 0.32)$$

W. Bardeen et al. PRD18, 3998 (78)

$$R = \frac{0.66 \pm 0.07 \pm 0.04 \pm 0.05 \text{ keV}}{2.36 \pm 0.35 \pm 0.11 \pm 0.19 \text{ keV}} = 0.278 \pm 0.050 \pm 0.018 \pm 0.031$$

CLEO, PRD78, 091501 (2008



note: $4/15 = 0.27$!

PERTURBATIVE QCD

e^+e^- widths

van Royen and Weisskopf

$$\Gamma(^3S_1 \to e^+e^-) = 16\alpha_s^2 Q^2 \frac{|\psi(0)|^2}{M^2}$$

$$\Gamma(^3D_1 \to e^+e^-) = 50\alpha_s^2 Q^2 \frac{|\psi''(0)|^2}{M^2 m_c^4}$$

state	qn	thy (keV)	expt (keV)	
J/ψ	1^3S_1	12	5.40(17)	
ψ'	2^3S_1	5	2.12(12)	} mixing?
$\psi(3770)$	1^3D_1	0.06	0.26(4)	} mixing?
$\psi(4040)$	3^3S_1	3.5	0.75(15)	
$\psi(4159)$	2^3D_1	0.1	0.77(23)	
$\psi(4415)$	4^3S_1	2.6	0.47(10)	

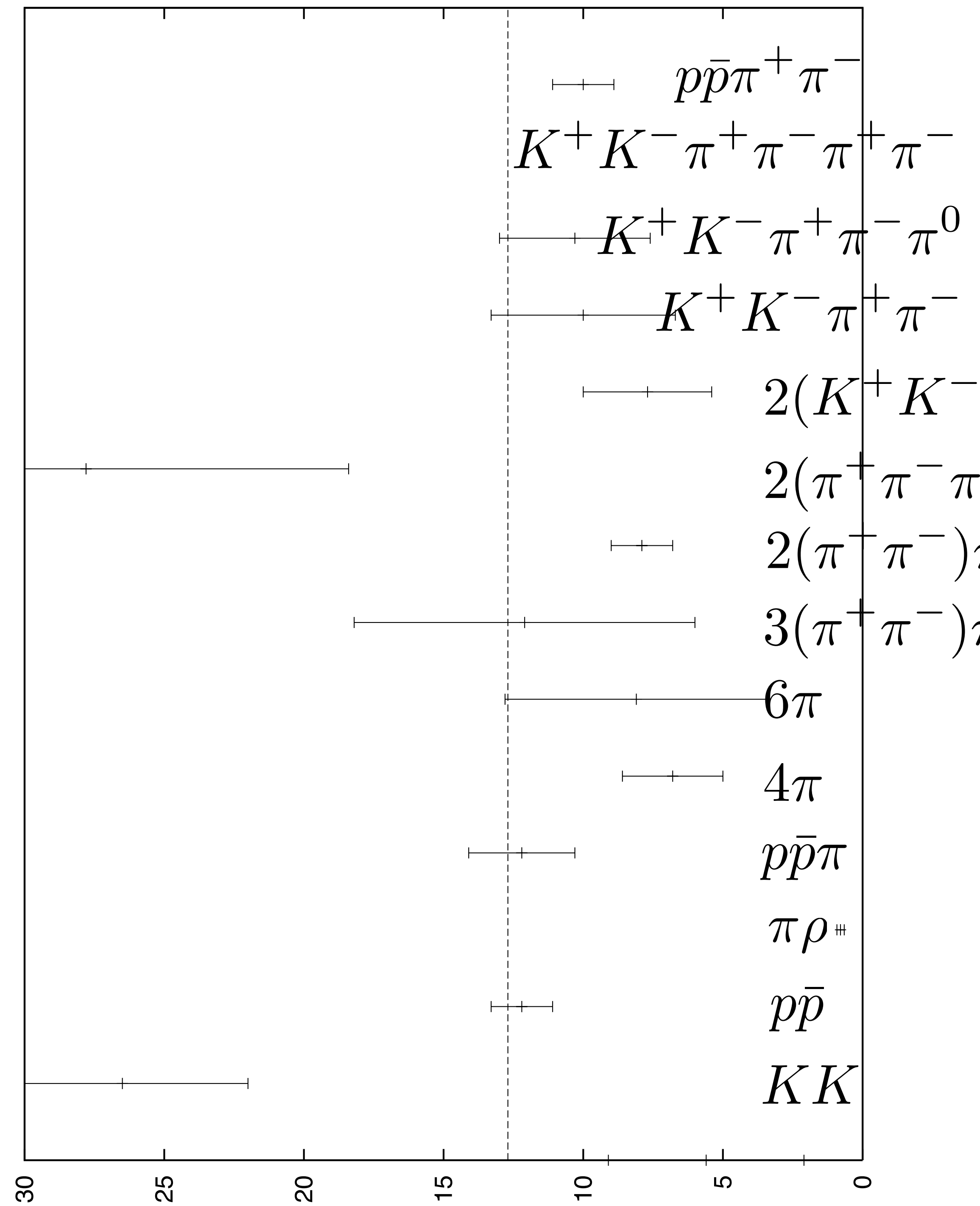
PERTURBATIVE QCD

the pi-rho puzzle

$$Q_h \equiv \frac{Bf(\psi' \to h)}{Bf(J/\psi \to h)} = \frac{Bf(\psi' \to e^+e^-)}{Bf(J/\psi \to e^+e^-)} \approx 12.7\%$$

Appelquist and Politzer, PRL34, 43 (75)

Mo et al., hep-ph/0611214



PERTURBATIVE QCD

T. Pedlar [CLEO], Moriond, 2009

PRL101, 101801 (2008)

$$BF(J/\psi \to \gamma\gamma\gamma) = (1.17 \pm 0.3 \pm 0.1) \cdot 10^{-5}$$

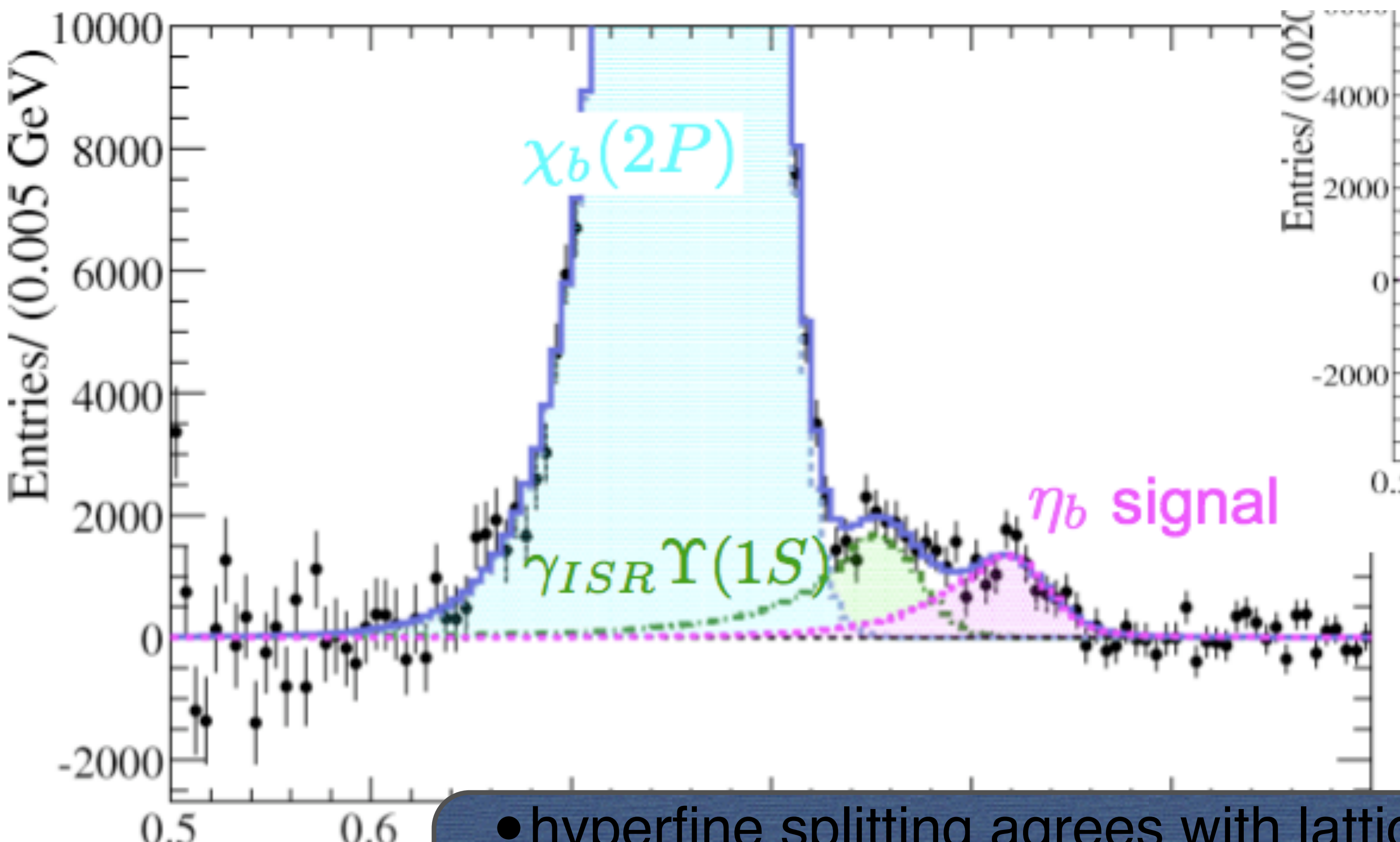
agrees with LO pQCD, but NLO is negative

A WORD ON THE η_b

BaBar, PRD78, 091501 (2008)

$\Upsilon(3S) \to \gamma\eta_b$

$M = 9388.9 \pm 3 \pm 3$



now confirmed

arXiv:0903.1124

$\Upsilon(2S) \to \gamma\eta_b$

$M = 9392.9 \pm 5 \pm 2$

$M_{\text{comb}} = 9390.4 \pm 3$

$\Delta M_{\text{hyp}} = 69.9 \pm 3$

- hyperfine splitting agrees with lattice
- hyperfine splitting disagrees with pNRQCD
- hyperfine splitting gives $\alpha_s(M_{\eta_b})$
- test NRQCD
- test mixing with A_0