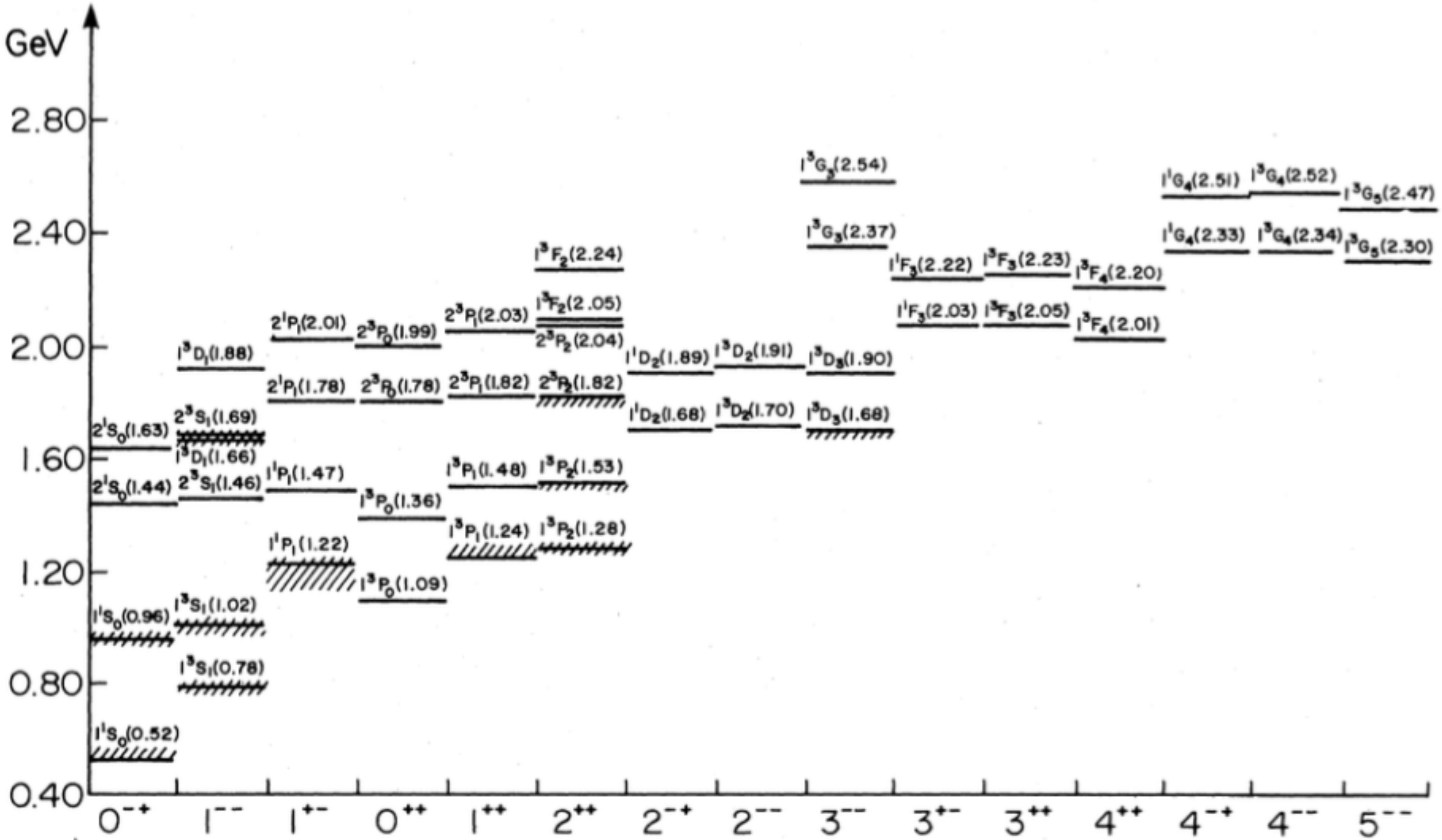
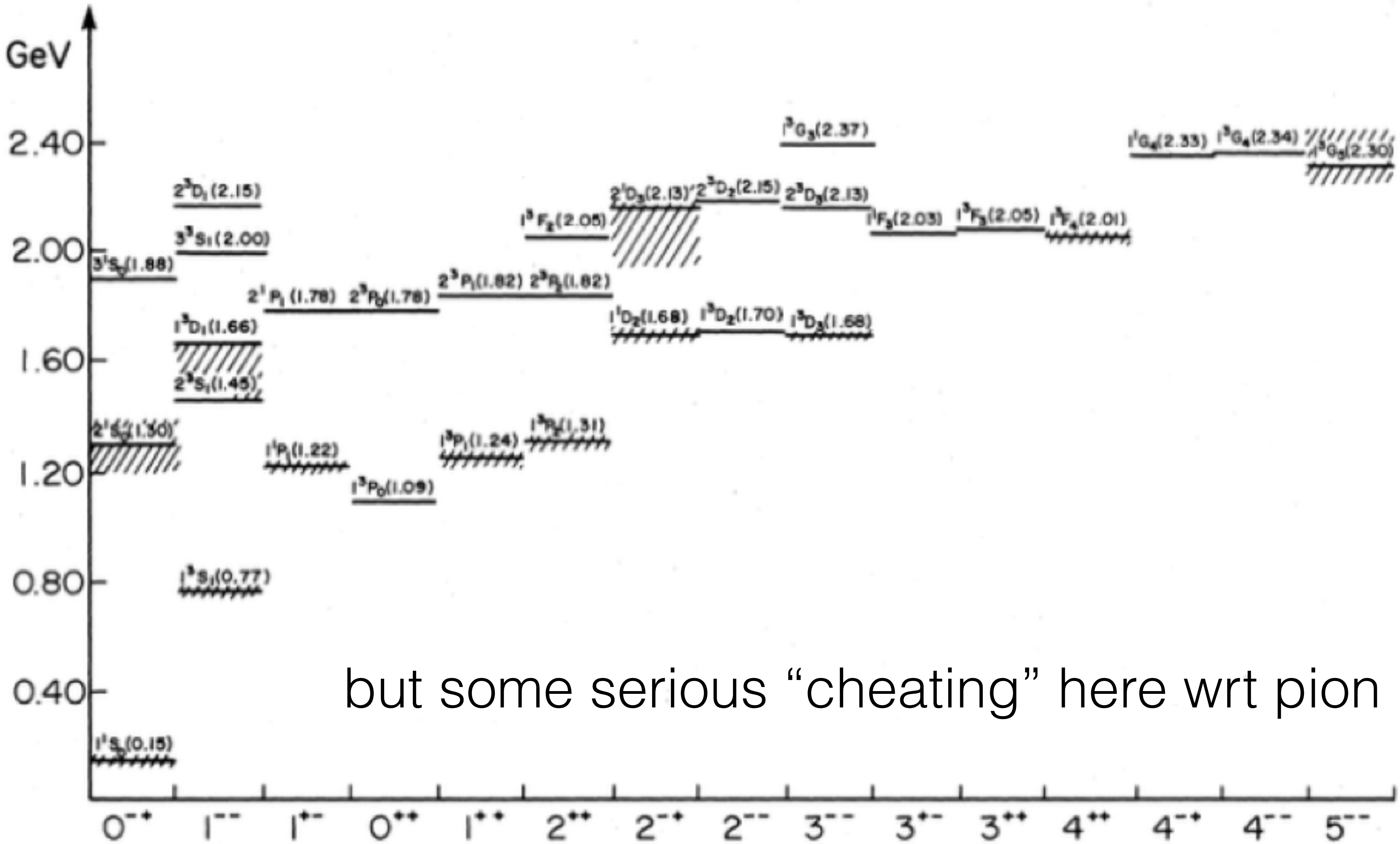
Light Quarks

models, pions, quasiparticles,

apply HQ model to light quarks … how do we do?

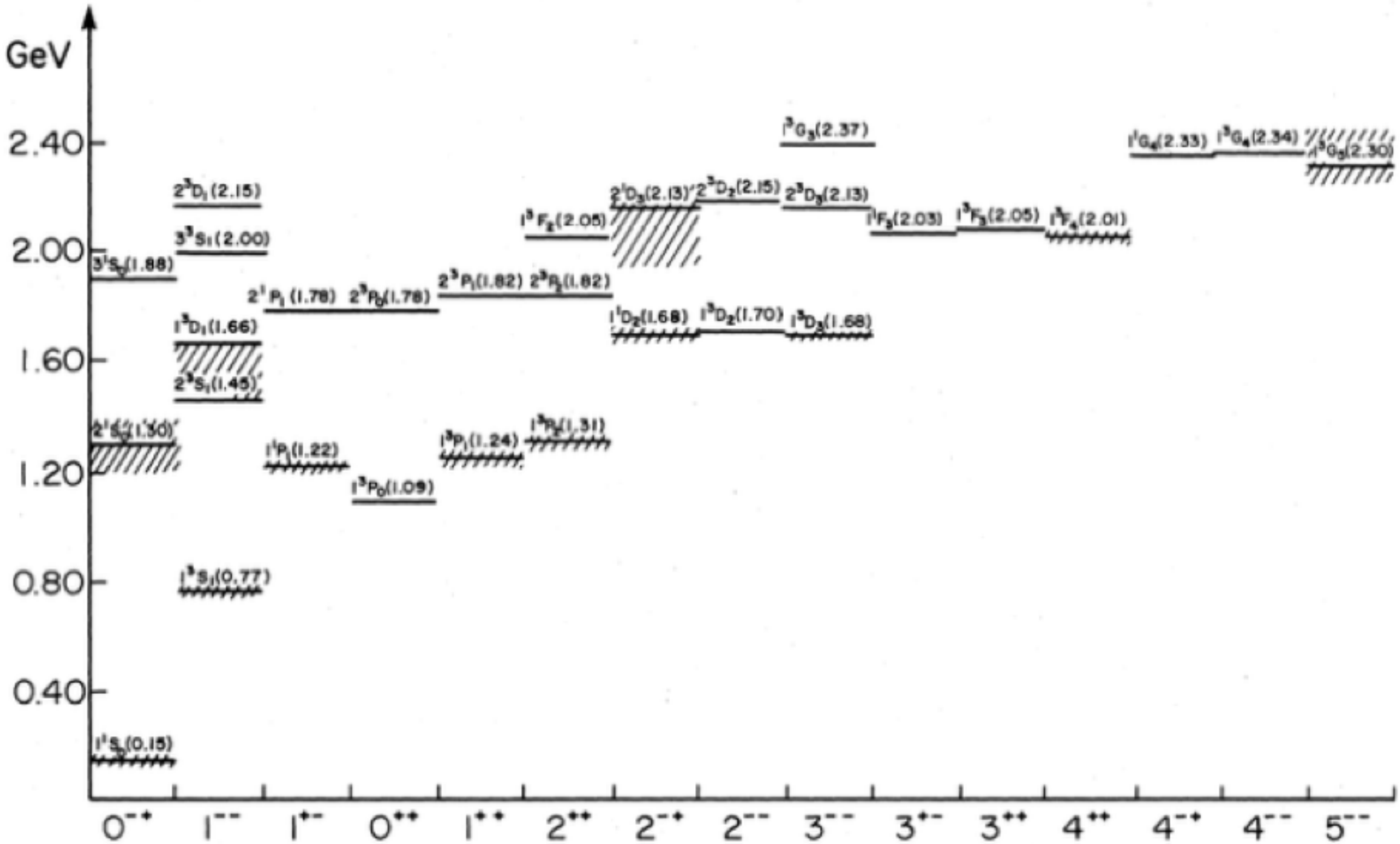


isoscalar uu+dd+ss



but some serious “cheating” here wrt pion

isovector



isovector

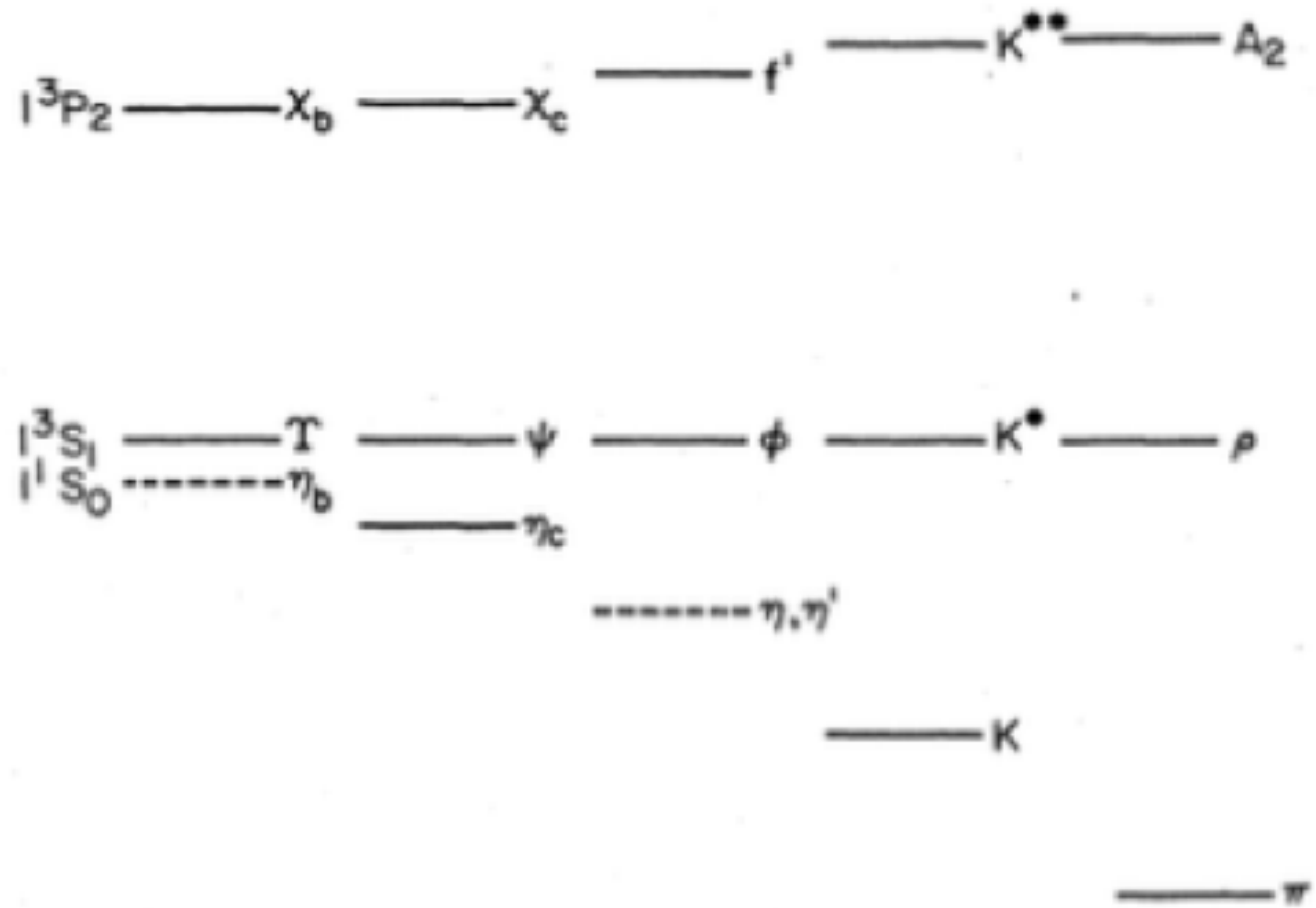


FIG. 19. A graphic illustration of the universality of meson dynamics from the π to the Υ, showing the splittings of $^3P_2$ and $^1S_0$ from $^3S_1$ in the $b\bar{b}$, $c\bar{c}$, $s\bar{s}$, $u\bar{s}$, and $u\bar{d}$ families.

"Isgur plot" — not very convincing!

Constituent Quarks (light)

an example:

Szczepaniak & Swanson, PRL87,072001 (01)

$$\mathcal{L} = \int d^4x\bar{\psi}\left(i\gamma^\mu\partial_\mu - m\right)\psi - \frac{\lambda}{2\Lambda^2}\int^\Lambda d^4x\,\psi^\dagger(x)T^a\psi(x)\psi^\dagger(x)T^a\psi(x)$$

$$\begin{aligned} H &= p\dot{q} - L \\ \gamma^\mu\partial_\mu &= \gamma^0\partial_t - \gamma^i\partial_i \\ &= \beta\partial_t + \vec{\gamma}\cdot\nabla \\ &= \beta\partial_t + \beta\vec{\alpha}\cdot\nabla \end{aligned}$$

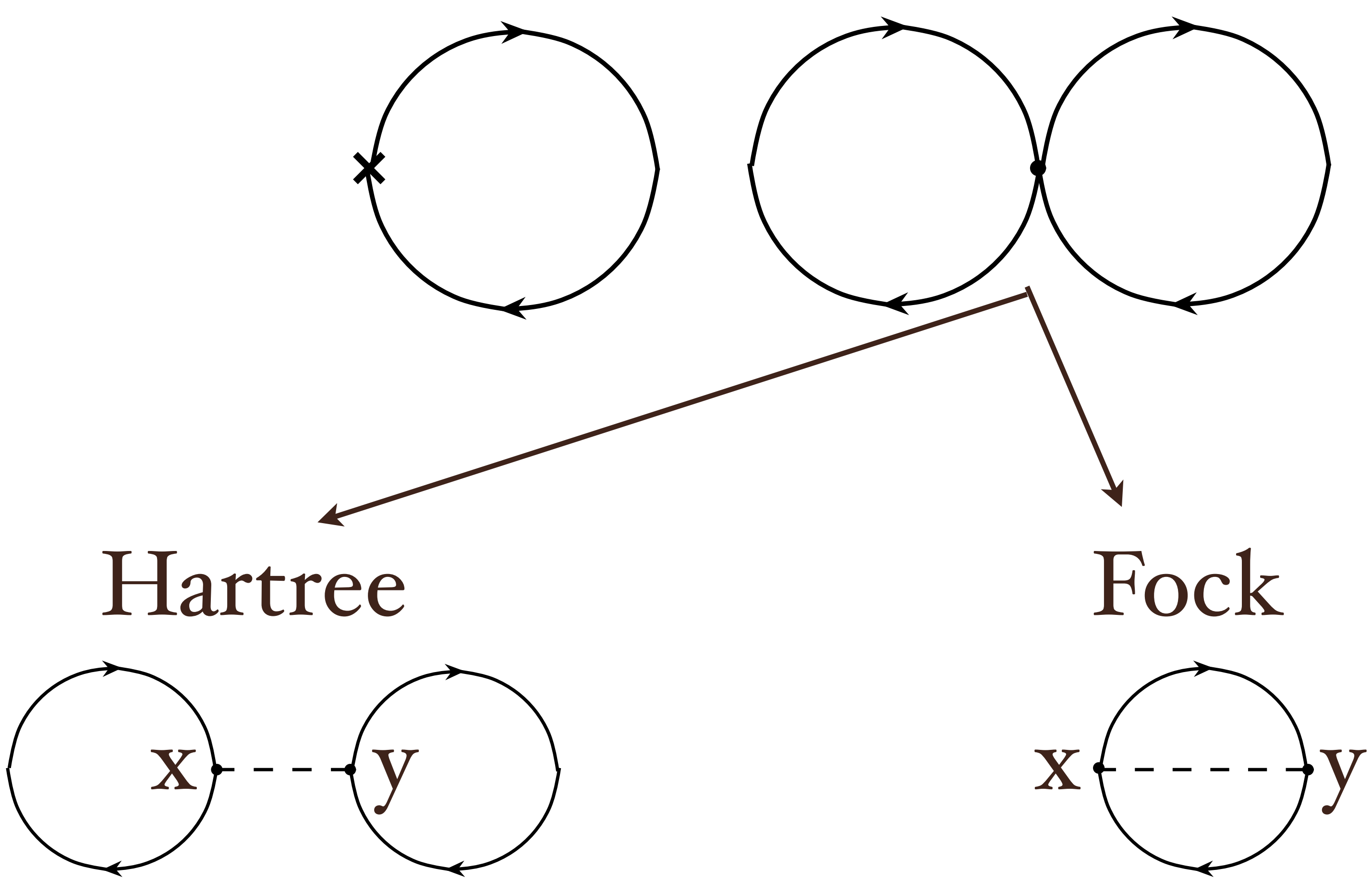
$$H = \int d^3x\psi^\dagger\left(-i\alpha\cdot\nabla + \beta m\right)\psi + \frac{\lambda}{2\Lambda^2}\int^\Lambda d^3x\,\psi^\dagger(x)T^a\psi(x)\psi^\dagger(x)T^a\psi(x)$$

Constituent Quarks (light)

$$\langle H \rangle = \int d^3x \psi^\dagger \left(-i\alpha \cdot \nabla + \beta m\right) \psi +$$

$$\frac{\lambda}{2\Lambda^2} \int^\Lambda d^3x\, \psi^\dagger(x) T^a \psi(x) \psi^\dagger(x) T^a \psi(x)$$

x x y y



Quark Field Expansion

$$\psi_{a,\alpha,f}(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3}[u(\mathbf{k},s)_\alpha b_{a,s,f}(\mathbf{k}) + v(\mathbf{k},s)_\alpha d_{a,s,f}(-\mathbf{k})^\dagger]\mathrm{e}^{-\mathbf{k}\cdot\mathbf{x}}$$

where $a = 1\ldots8$, $f = u,d,c,s,t,b$, $\alpha = 0\ldots3$, and $s = +1/2, -1/2$.

It is convenient to normalize the spinors as

c=k/E

s=M/E

$$u(\mathbf{k},s) = \sqrt{\frac{1+s(k)}{2}}\begin{pmatrix} \chi_s \\ \frac{c(k)}{1+s(k)}\sigma\cdot\hat{k}\chi_s \end{pmatrix}$$

$$v(\mathbf{k},s) = \sqrt{\frac{1+s(k)}{2}}\begin{pmatrix} -\frac{c(k)}{1+s(k)}\sigma\cdot\hat{k}\tilde{\chi}_s \\ \tilde{\chi}_s \end{pmatrix}$$

Constituent Quarks (light)

$$\langle H \rangle = -2N_c \int^\Lambda \frac{d^3k}{(2\pi)^3} \left(s(k)m + c(k)k\right) + \frac{\lambda}{2\Lambda^2}\frac{N_c^2-1}{2}\int^\Lambda \frac{d^3k}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\left(1 - s(k)s(q) - c(k)c(q)\hat{k}\cdot\hat{q}\right)$$

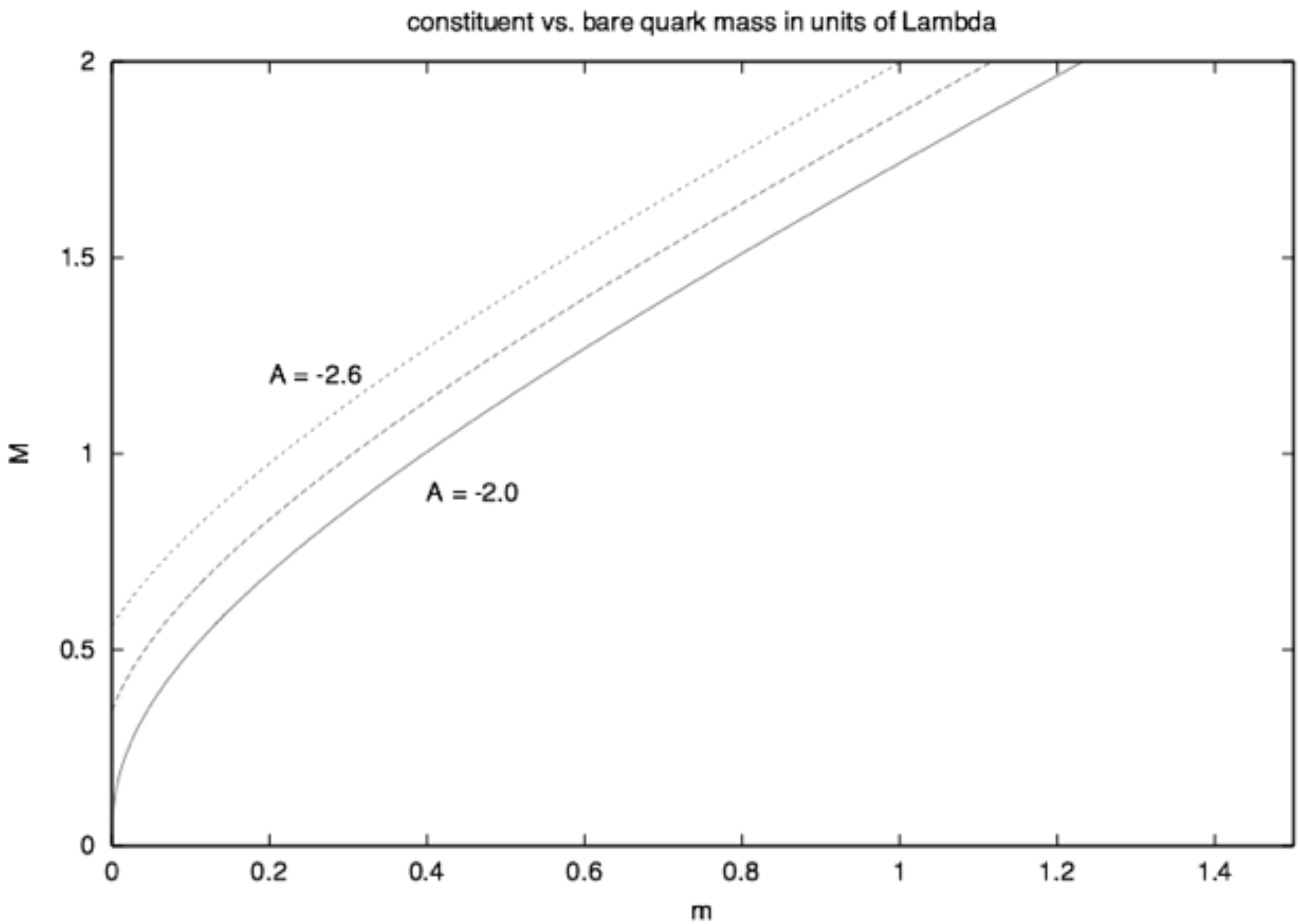
$$s(k) = \sin\phi(k)$$

$$\frac{\delta}{\delta\phi}\langle H \rangle = 0$$

$$M(p) = m(\Lambda) + \frac{C_F\lambda}{4\pi^2\Lambda^2}\int^\Lambda q^2 dq\, \frac{M(q)}{\sqrt{M(q)+q^2}}$$

$$M(p) = \frac{ps(p)}{c(p)}$$

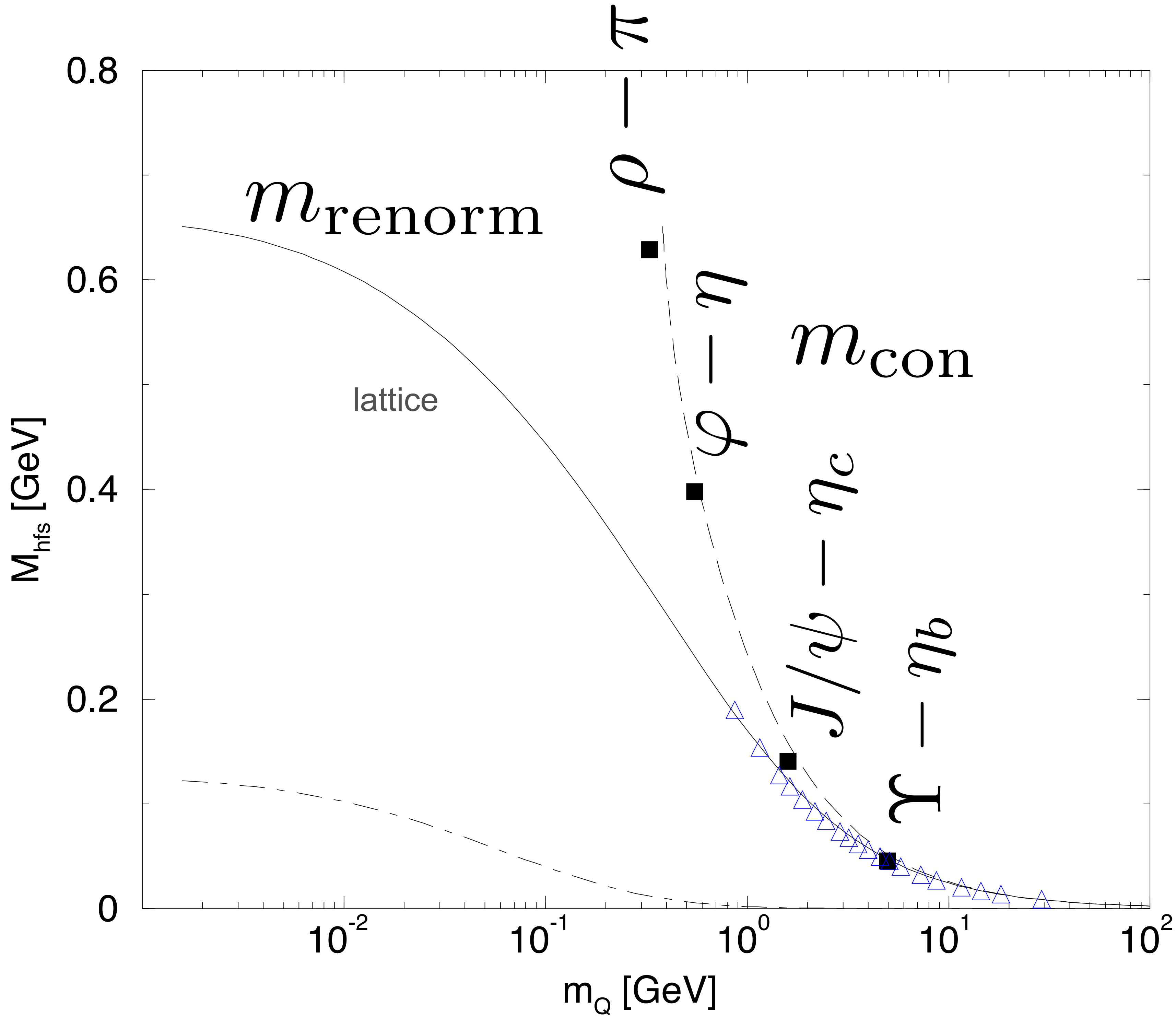
Constituent Quarks (light)



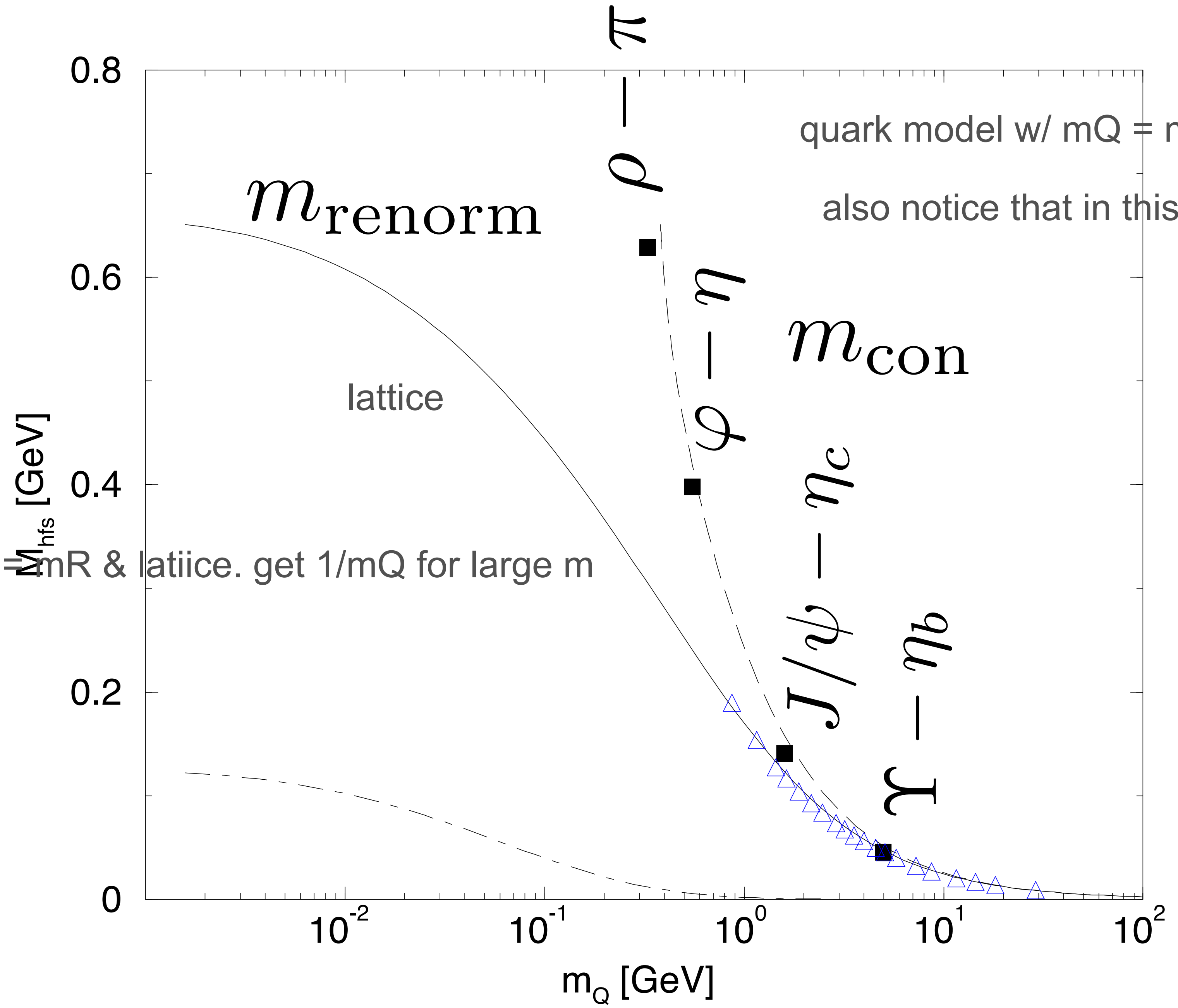
Constituent Quarks (light)

$$\langle M|[H,Q_M^\dagger]|BCS\rangle = (E_M - E_{BCS})\langle M|Q_M^\dagger|BCS\rangle$$

$$Q_M^\dagger = \Sigma_{\alpha\beta}(\psi^+_{\alpha\beta} B^\dagger_\alpha D^\dagger_\beta - \psi^-_{\alpha\beta} D_\beta B_\alpha)$$



$m_{\rm renorm}$
$\rho - \pi$
$\varphi - \eta$
$m_{\rm con}$
$J/\psi - \eta_c$
$\Upsilon - \eta_b$
lattice
M_{hfs} [GeV]
m_Q [GeV]
0.8
0.6
0.4
0.2
0
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}



quark model w/ mQ = m_const => m_running
also notice that in this "basis" 1/M works everywhere
thy w/ mQ = mR & latiice. get 1/mQ for large m
$m_{\rm renorm}$
$m_{\rm con}$
lattice
$\rho - \pi$
$\varphi - \eta$
$J/\psi - \eta_c$
$\Upsilon - \eta_b$
M_{hfs} [GeV]
m_Q [GeV]
0.8
0.6
0.4
0.2
0
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}

Constituent Quarks (light)

chiral symmetry breaking generates
Goldstone bosons *and* constituent quarks

and underpins applicability of the NCQM to light hadrons

$$
\begin{aligned}
(E_\pi - E_{BCS})\psi^+(k) = 2[ms_k + kc_k + \Sigma(k)]\psi^+(k) \\
- \frac{C_F}{2}\int \frac{p^2dp}{(2\pi)^3}[V_0(k,p)(1 + s_k s_p) \\
+ V_1(k,p)c_k c_p]\psi^+(p) \\
- \frac{C_F}{2}\int \frac{p^2dp}{(2\pi)^3}[V_0(k,p)(1 - s_k s_p) \\
- V_1(k,p)c_k c_p]\psi^-(p)\,.
\end{aligned}
$$

$$u_s(k) = \sqrt{\frac{1+s_k}{2}}\begin{pmatrix} \chi_s \\ \frac{c_k}{1+s_k}\boldsymbol{\sigma}\cdot\hat{k}\chi_s \end{pmatrix}$$

$$\langle M|[H,Q_M^\dagger]|RPA\rangle = (E_M - E_{BCS})\langle M|Q^\dagger|RPA\rangle$$

$$Q_M^\dagger = \Sigma_{\alpha\beta}(\psi_{\alpha\beta}^{+}\tilde{B}_\alpha^\dagger D_\beta^\dagger - \psi_{\alpha\beta}^{-}D_\beta \tilde{B}_\alpha)$$

$$E\psi_{PC}(k) = 2[ms_k + kc_k + \Sigma(k)]\psi_{PC}(k)$$
$$- \frac{C_F}{2}\int \frac{p^2 dp}{(2\pi)^3} K_J^{PC}(k,p)\psi_{PC}(p)$$

$$\Sigma(k) = \frac{C_F}{2}\int \frac{p^2 dp}{(2\pi)^3}(V_0 s_k s_p + V_1 c_k c_p)$$

$$(\text{kinetic} + \text{self-energy}) = 2[E(k) + \Gamma(k)],$$

$$\Gamma(k) = \frac{C_F}{2} \int \frac{p^2 dp}{(2\pi)^3} V_1 \frac{c_p}{c_k}$$

$$E(k) = \sqrt{k^2 + \mu(k)^2}.$$

Nonrelativistic models

$$\langle p \rangle << m$$

L and S separately conserved

different parity corresponds to different waves

$$0^{-+} = {}^1S_0 \qquad 0^{++} = {}^3P_0$$

Relativistic models

$$\langle p \rangle >> m$$

L and S are *not* separately conserved

$$V(0^{++}) = V_0 c_p c_k + V_1 (1 + s_p s_k)$$

wave

$$V(0^{-+}) = V_0 (1 + s_p s_k) + V_1 c_p c_k$$

$$V(0^{++}) = V_0 c_p c_k + V_1(1 + s_p s_k)$$

$$V(0^{-+}) = V_0(1 + s_p s_k) + V_1 c_p c_k$$

NonRel

$$c_p = \frac{p}{E(p)} \to \frac{p}{m} \qquad s_p = \frac{\mu(p)}{E(p)} \to 1$$

$$V(0^{++}) \to 2V_1 + \mathcal{O}(\frac{1}{m^2})$$ P-wave

$$V(0^{-+}) \to 2V_0 + \mathcal{O}(\frac{1}{m^2})$$ S-wave

$$V(0^{++}) = V_0 c_p c_k + V_1(1 + s_p s_k)$$

$$V(0^{-+}) = V_0(1 + s_p s_k) + V_1 c_p c_k$$

Rel

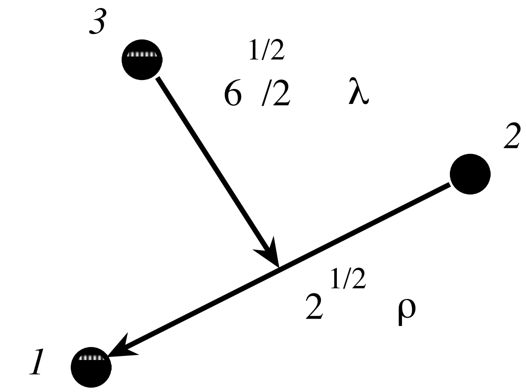
$$c_p = \frac{p}{E(p)} \to 1 \qquad s_p = \frac{\mu(p)}{E(p)} \to 0$$

$$V(0^{++}) \to V_0 + V_1$$

$$V(0^{-+}) \to V_0 + V_1$$

BARYONS

Isgur-Karl Model



$$H_{IK} = \sum_{i=1}^{3} \left(m_i + \frac{p_i^2}{2m_i} \right) + \sum_{i<j} \frac{1}{2} k r_{ij}^2$$

$$H_{IK} = M_{tot} + \frac{P^2}{2M_{tot}} + \frac{p_\rho^2}{2m_\rho} + \frac{p_\lambda^2}{2m_\lambda} + \frac{3}{2} k\rho^2 + \frac{3}{2} k\lambda^2$$

$$m_\rho = m_1 = m_2 \qquad m_\lambda = 3\frac{m_1 m_3}{M_{tot}}$$

Isgur-Karl Model

$$E = (N_\rho + \frac{3}{2})\omega_\rho + (N_\lambda + \frac{3}{2})\omega_\lambda$$

$$\omega_\rho = \sqrt{\frac{3k}{m_\rho}} \qquad \omega_\lambda = \sqrt{\frac{3k}{m_\lambda}}$$

proton:

$$\Psi = C_A uud \left(\frac{\alpha_\rho \alpha_\lambda}{\pi}\right)^{3/2} \mathrm{e}^{-\frac{1}{2}(\alpha_\rho^2 \rho^2 + \alpha_\lambda^2 \lambda^2)} \chi$$

$$C_A = \frac{1}{\sqrt{6}} \left(rbg - brg + bgr - gbr + grb - rgb\right)$$

$$\chi = -\frac{1}{\sqrt{6}} \left(|\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle - 2|\uparrow\uparrow\downarrow\rangle\right)$$

Baryons

baryon flavour wavefunctions

State	++	+	0	-
N		uud	ddu	
Δ	uuu	uud	ddu	ddd
Λ			$\frac{1}{\sqrt{2}}$(ud -du)s	
Σ		uus	$\frac{1}{\sqrt{2}}$(ud+du)s	
Ξ			ssu	ssd
Ω				sss
Λ_c		$\frac{1}{\sqrt{2}}$(ud-du)c		
Σ_c	uuc	$\frac{1}{\sqrt{2}}$(ud+du)c	ddc	
Λ_b			$\frac{1}{\sqrt{2}}$(ud-du)b	
Σ_b		uub	$\frac{1}{\sqrt{2}}$(ud+du)b	ddb

Baryons

magnetic moments

$$\begin{aligned}\mu_p &= \langle\chi^\lambda_{1/21/2}|\sum_i \frac{e_i}{2m_i}\sigma^z_i|\chi^\lambda_{1/21/2}\rangle \\ &= \frac{4}{3}\mu_u - \frac{1}{3}\mu_d.\end{aligned}$$

$$\mu_n = 4/3\mu_d - 1/3\mu_u$$

$$\mu_u = -2\mu_d$$

$$\frac{\mu_n}{\mu_p} = -\frac{2}{3}$$

expt: -0.6849

Baryons

hyperfine splitting

$$\Delta m = \frac{4\pi\alpha_s}{9}|\psi(0)|^2 \sum_{i<j} \frac{\langle \vec{\sigma}_i \cdot \vec{\sigma}_j \rangle}{m_i m_j}.$$

$$\begin{aligned} \Delta N &= \frac{4\pi\alpha_s}{9m_u^2}(-3)|\psi(0)|^2 \equiv \frac{-3}{m_u^2}K \\ \Delta\Delta &= \frac{3}{m_u^2}K \\ \Delta\Sigma &= (\frac{1}{m_u^2} - \frac{4}{m_u m_s})K. \end{aligned}$$

K = 0.0066 GeV^3

$$\begin{aligned} \langle \chi^\lambda | \vec{\sigma}_1 \cdot \vec{\sigma}_2 | \chi^\lambda \rangle &= 1 \\ \langle \chi^\lambda | \vec{\sigma}_1 \cdot \vec{\sigma}_3 | \chi^\lambda \rangle &= -2 \\ \langle \chi^\lambda | \vec{\sigma}_2 \cdot \vec{\sigma}_3 | \chi^\lambda \rangle &= -2 \end{aligned}$$

baryon(mass)	composition	$\Delta E/K$	predicted mass
N(939)	nnn	$-3/m_n^2$	939
Λ(1116)	nns	$-3/m_n^2$	1114
Σ(1193)	nns	$1/m_n^2 - 4/(m_n m_s)$	1179
Ξ(1318	nss	$1/m_s^2 - 4/(m_n m_s)$	1327
Δ(1232)	nnn	$3/m_n^2$	1239
Σ(1384)	nns	$1/m_n^2 + 2/(m_n m_s)$	1381
Ξ(1533)	nss	$1/m_s^2 + 2/(m_n m_s)$	1529
Ω(1672)	sss	$3/m_s^2$	1682

Hyperfine Splitting in P-wave Baryons

S-wave P-wave

contact in λ, tensor in ρ

$$
\begin{aligned}
m_\Delta - m_N &= A\frac{8\pi}{3}\langle\psi_{00}|\delta(\vec\rho)|\psi_{00}\rangle\left[\langle\chi^S_{3/2}|\vec S_1\cdot\vec S_2|\chi^S_{3/2} - \langle\chi^\lambda_{1/2}|\vec S_1\cdot\vec S_2|\chi^\lambda_{1/2}\right] \\
&= A\frac{8\pi}{3}\frac{\beta^3}{\pi^{3/2}}\left[\frac{3}{4}-\frac{-3}{4}\right] \\
&= 4A\frac{\beta^3}{\sqrt{\pi}} \\
&= 300\mathrm{MeV}.
\end{aligned}
$$

notation

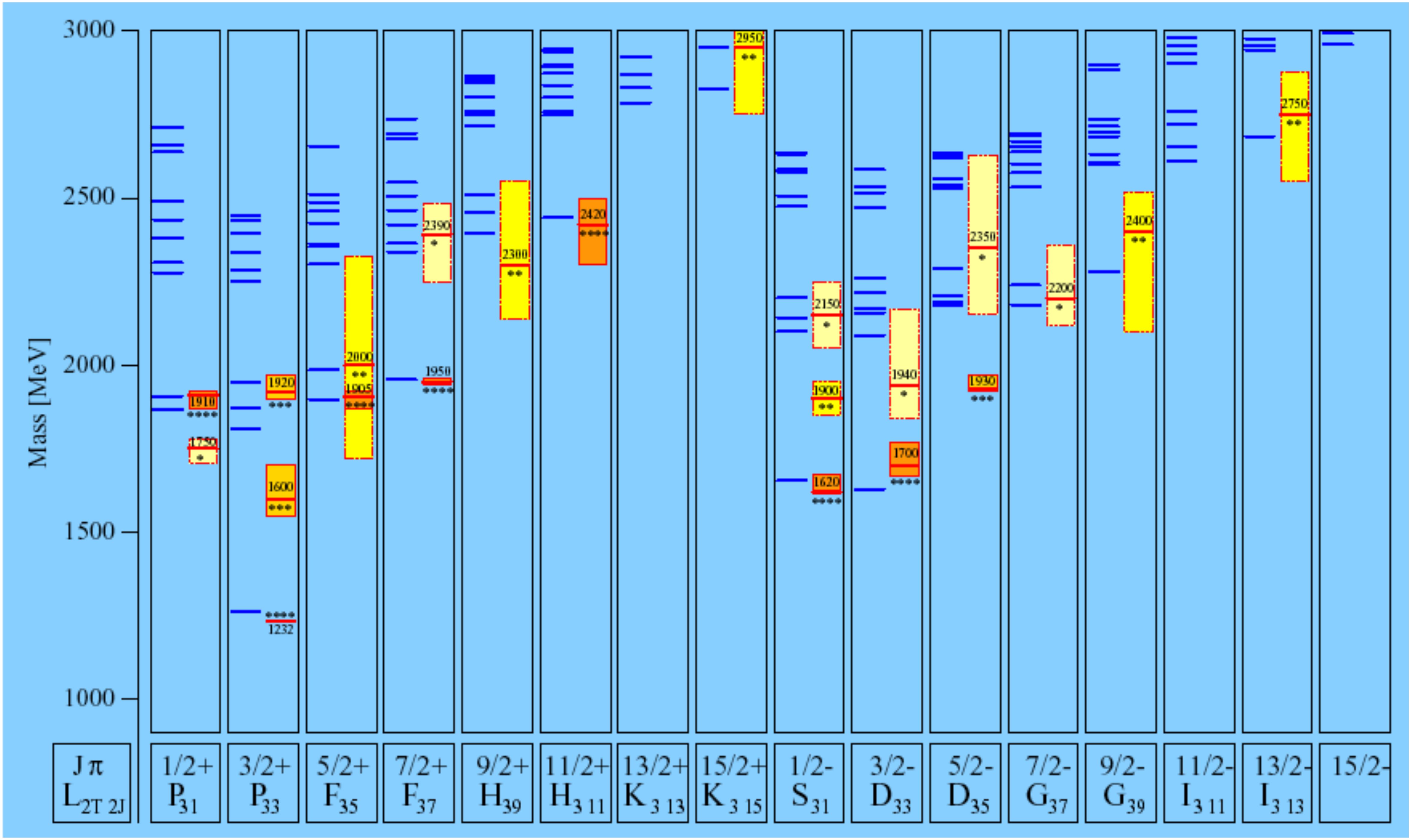
$|\Xi S L J^P\rangle$

$$
\begin{aligned}
|N\,1/2\,P\,3/2^-\rangle &= C_A\frac{1}{2}\left[\chi^\rho_{1/2 1/2}\phi^\rho_N\psi^\lambda_{11} + \chi^\rho_{1/2 1/2}\phi^\lambda_N\psi^\rho_{11} + \chi^\lambda_{1/2 1/2}\phi^\rho_N\psi^\rho_{11} - \chi^\lambda_{1/2 1/2}\phi^\lambda_N\psi^\lambda_{11}\right] \\
|N\,3/2\,P\,5/2^-\rangle &= C_A\chi^S_{3/2}\frac{1}{\sqrt{2}}\left[\phi^\rho_N\psi^\rho_{11} + \phi^\lambda_N\psi^\lambda_{11}\right] \\
|\Delta\,1/2\,P\,3/2^-\rangle &= C_A\,\phi^S_\Delta\frac{1}{\sqrt{2}}\left[\chi^\rho_{1/2 1/2}\psi^\rho_{11} + \chi^\lambda_{1/2 1/2}\psi^\lambda_{11}\right].
\end{aligned}
$$

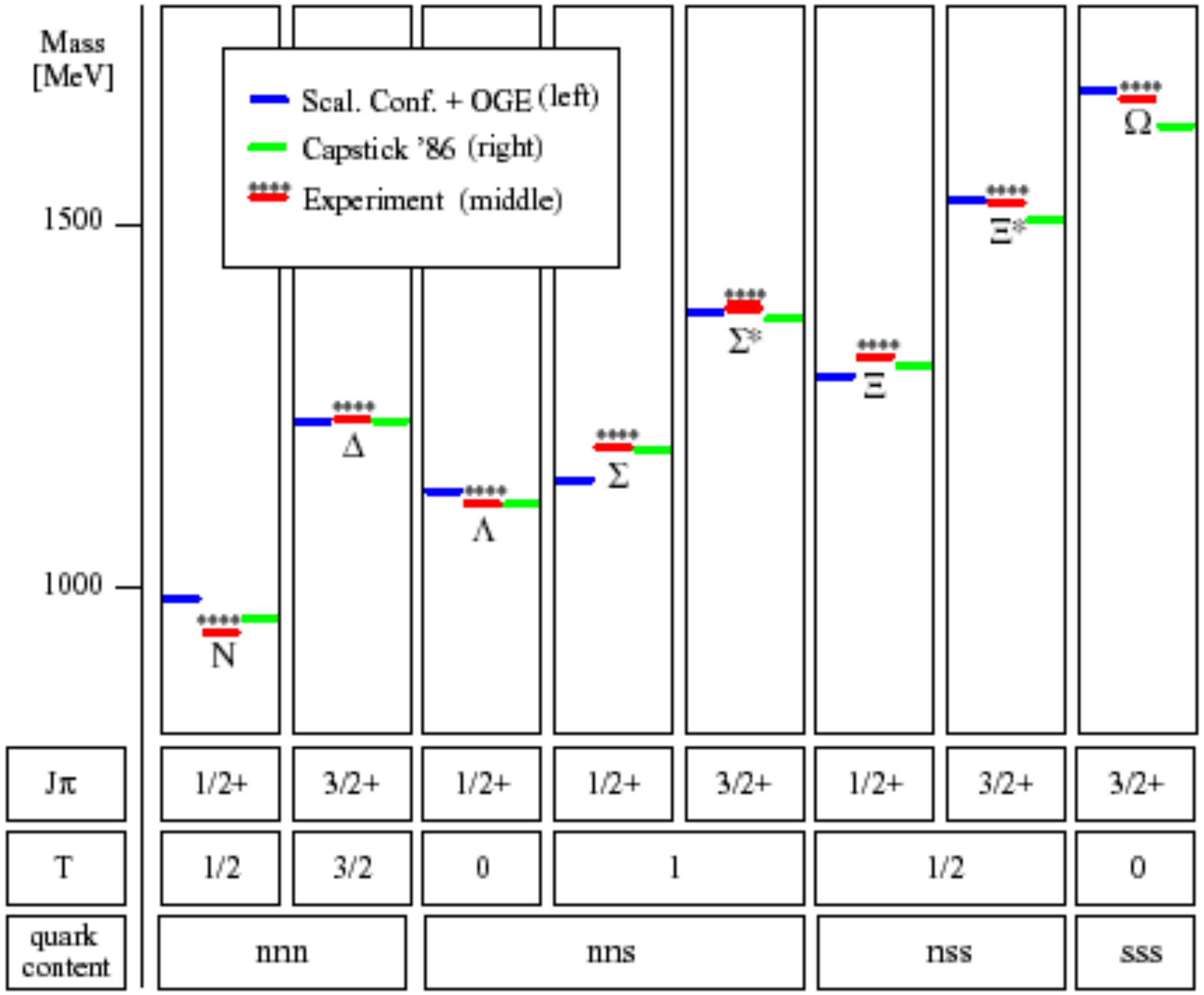
$$
\begin{aligned}
\langle\Delta\,1\,1/2\,3/2|V_{hyp}|\Delta\,1\,1/2\,3/2\rangle &= 1 \\
\langle\Delta\,1\,1/2\,1/2|V_{hyp}|\Delta\,1\,1/2\,1/2\rangle &= 1 \\
\langle N\,1\,3/2\,5/2|V_{hyp}|N\,1\,3/2\,5/2\rangle &= \frac{4}{5}
\end{aligned}
$$

$$
V_{hyp}\begin{pmatrix}|N\,1\,3/2\,3/2\rangle \\ |N\,1\,1/2\,3/2\rangle\end{pmatrix} = \begin{pmatrix}\frac{9}{5} & \frac{1}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} & -1\end{pmatrix}\begin{pmatrix}|N\,1\,3/2\,3/2\rangle \\ |N\,1\,1/2\,3/2\rangle\end{pmatrix} \Rightarrow \theta = 6.3^\circ \quad \text{(expt)}\ \theta = 10^\circ
$$

$$
V_{hyp}\begin{pmatrix}|N\,1\,3/2\,1/2\rangle \\ |N\,1\,1/2\,1/2\rangle\end{pmatrix} = \begin{pmatrix}0 & -1 \\ -1 & -1\end{pmatrix}\begin{pmatrix}|N\,1\,3/2\,1/2\rangle \\ |N\,1\,1/2\,1/2\rangle\end{pmatrix} \Rightarrow \theta = -31.7^\circ \quad \text{(expt)}\ \theta = -32^\circ
$$



3000
2500
2000
1500
1000
Mass [MeV]
J π
L2T 2J
1/2+ P31
3/2+ P33
5/2+ F35
7/2+ F37
9/2+ H39
11/2+ H3 11
13/2+ K3 13
15/2+ K3 15
1/2- S31
3/2- D33
5/2- D35
7/2- G37
9/2- G39
11/2- I3 11
13/2- I3 13
15/2-
1910
1750
1920
1600
1232
2000
1905
2390
1950
2300
2420
2950
2150
1900
1620
1940
1700
2350
1930
2200
2400
2750


Mass [MeV]
1500
1000
Scal. Conf. + OGE (left)
Capstick '86 (right)
Experiment (middle)
N
Δ
Λ
Σ
Σ*
Ξ
Ξ*
Ω
Jπ
1/2+
3/2+
1/2+
1/2+
3/2+
1/2+
3/2+
3/2+
T
1/2
3/2
0
1
1/2
0
quark content
nnn
nns
nss
sss

Traditional constituent quark models (CQM) adopted one-gluon exchange (OGE) [1] as the interaction between constituent quarks (Q). Over the years it has become evident that CQM relying solely on OGE Q-Q interactions face some intriguing problems in light-baryon spectroscopy [2,3]. Most severe are:

(i) the wrong level orderings of positive- and negative-parity excitations in the N, Δ, Λ, and Σ spectra;

(ii) the missing flavour dependence of the Q-Q interaction necessary, e.g., for a simultaneous description of the correct level orderings in the N and Λ spectra; and

(iii) the strong spin-orbit splittings that are produced by the OGE interaction but not found in the empirical spectra.

Feynman Rules

Feynman Rules

consider interactions of the type:

$$H_{int} = \int d^3x \psi^\dagger(\mathbf{x})\Gamma(\mathbf{x})\psi(\mathbf{x})$$

$$H_{int} = \frac{1}{2}\int d^3x d^3y \, \psi^\dagger(\mathbf{x})\Gamma\psi(\mathbf{x})V(\mathbf{x}-\mathbf{y})\psi^\dagger(\mathbf{y})\Gamma\psi(\mathbf{y})$$

Feynman Rules

(i) label all lines and wavefunctions with momenta flowing in the time direction

(ii) conserve momenta at each vertex, extract a factor of $\delta\,(P_f - P_i)$

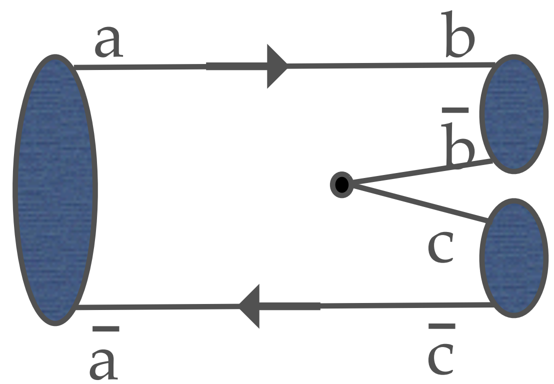
(iii) allow for $(x \leftrightarrow y)$ interchange

(iv) fermion loops get a minus sign

(v) through going interacting antifermion lines get a minus sign

(vi) order spinors against charge flow using u(k), v(-k).

Hadronic Decays

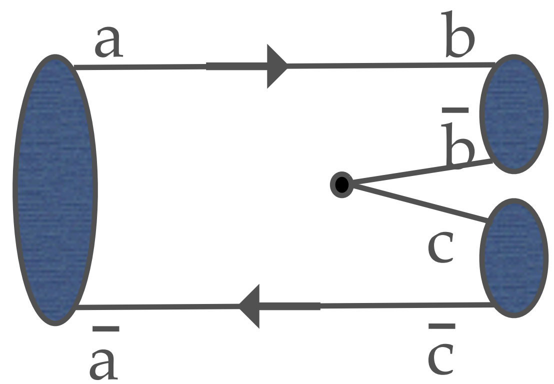


colour

$$\frac{\delta_{a\bar{a}}}{\sqrt{3}}\,\frac{\delta_{b\bar{b}}}{\sqrt{3}}\,\frac{\delta_{c\bar{c}}}{\sqrt{3}}\,\cdot\Gamma^{C}_{d,\bar{d}}\,\delta_{ab}\,\delta_{\bar{a}\bar{c}}\,\delta_{\bar{b}\bar{d}}\,\delta_{dc}$$

$$\Rightarrow\quad \frac{1}{3\sqrt{3}}\Gamma^{C}_{dd}\qquad (=\frac{1}{\sqrt{3}})$$

Hadronic Decays

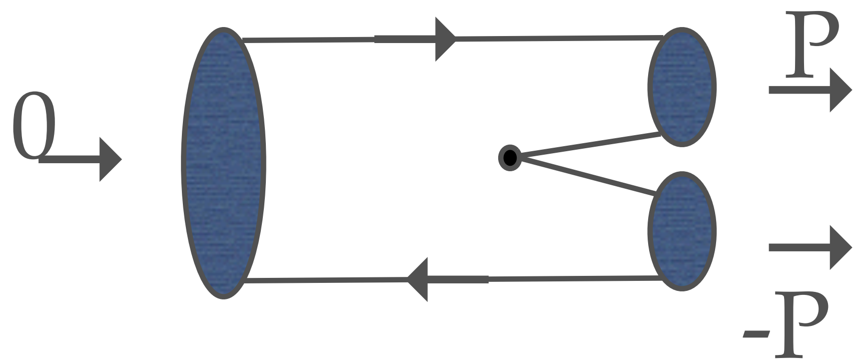


spin

$$\chi^{A}_{a\bar{a}}\,\chi^{B*}_{b\bar{b}}\,\chi^{C*}_{c\bar{c}}\,\Gamma^{S}_{d,\bar{d}}\,\delta_{ab}\,\delta_{\bar{a}\bar{c}}\,\delta_{\bar{b}\bar{d}}\,\delta_{dc}$$

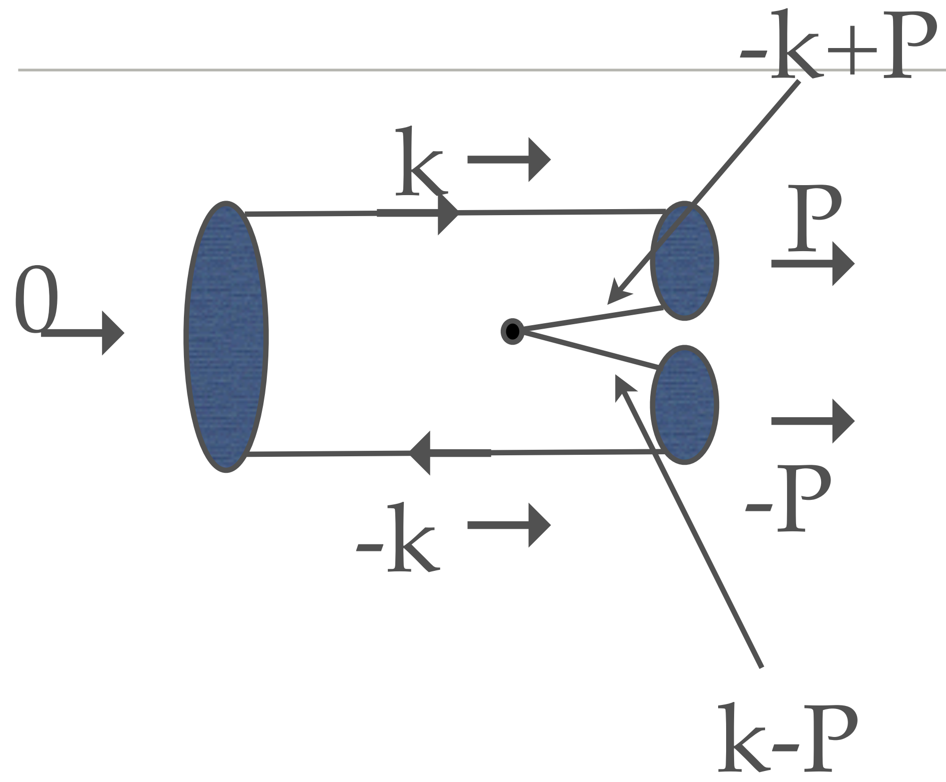
$$\Gamma^{S}_{d\bar{d}} \propto (\sigma)_{d\bar{d}}$$

Hadronic Decays



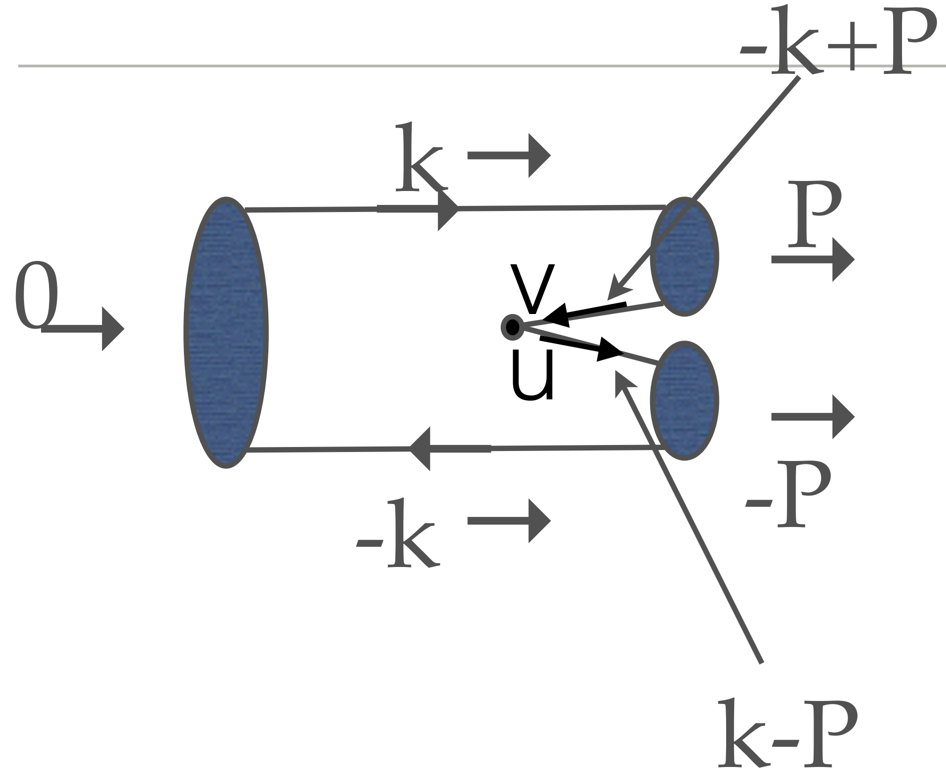
momentum flow

Hadronic Decays



momentum flow

Hadronic Decays



momentum flow

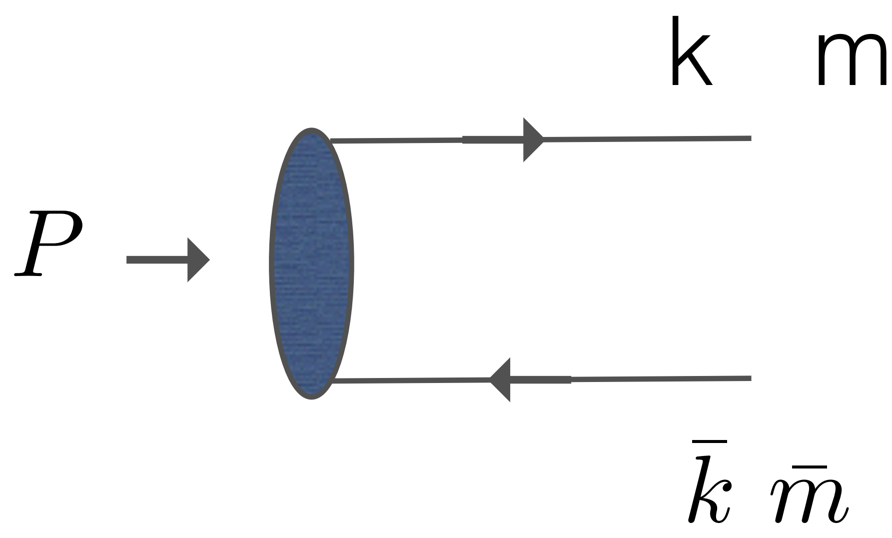
v(-P)

$$\mathcal{A} = \int \frac{d^3k}{(2\pi)^3} \phi_A(k)\phi_B^*(k-P/2)\phi_C^*(k-P/2)u^\dagger(k-P)\Gamma v(k-P)$$

$$u(q)^\dagger \gamma_0 v(q) \propto \chi_s(\sigma\cdot\mathbf{q} + \sigma\cdot\mathbf{q})\tilde{\chi}_{s'}$$

$$u(q)^\dagger v(q) \propto \chi_s(\sigma\cdot\mathbf{q} - \sigma\cdot\mathbf{q})\tilde{\chi}_{s'}$$

general case w/ meson wavefunctions

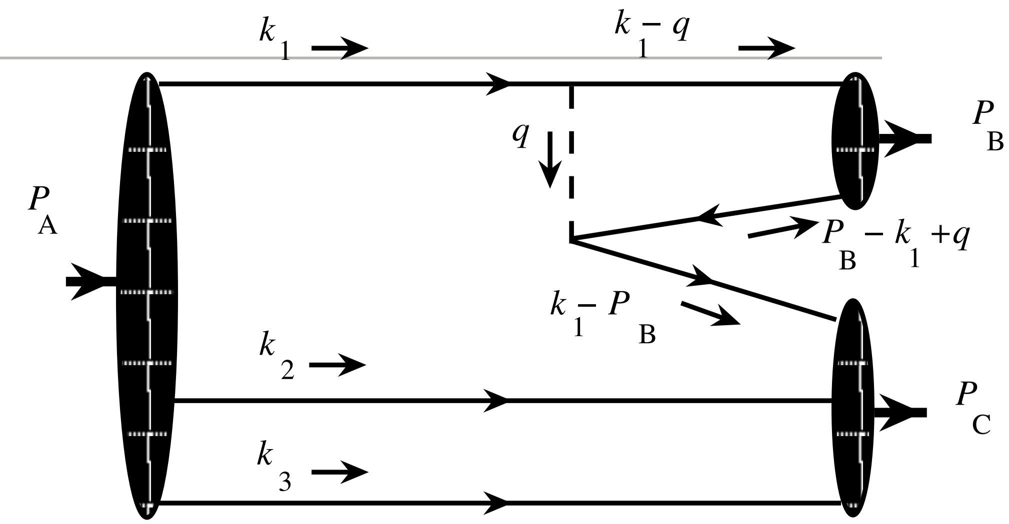


$$\phi(\frac{\bar{m}k - m\bar{k}}{m + \bar{m}})$$

$$P = k + \bar{k}$$

Feynman Rules

example: baryon decay



$$\mathcal{A} = tr(\phi_A V \phi_B \phi_C)$$

flavour:

$$\Xi^A(f_1, f_2, f_3)\Xi^B(f_1, f)\Xi^C(f, f_2, f_3)$$

colour:

$$\mathcal{C}^A(a_1, a_2, a_3)T^A_{b_1,a_1}T^A_{c_1,b_2}\mathcal{C}^{B*}(b_1, b_2)\mathcal{C}^{C*}(c_1, c_2, c_3)\delta_{a_2,c_2}\delta_{a_3,c_3}$$

spin:

$$\chi^A(a_1, a_2, a_3)\chi^{B*}(b_1, b_2)\chi^{C*}(c_1, c_2, c_3)\delta_{a_2,c_2}\delta_{a_3,c_3}\cdot$$
$$u^\dagger(k_1 - q)_{b_1}\Gamma u(k_1)_{a_1} \cdot u^\dagger(k_1 - P_B)_{c_1}\Gamma v(k_1 - q - P_B)_{b_2}$$

momentum:

$$\int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{d^3k_3}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\,\phi^A(k_1, k_2, k_3)\phi^{B*}(k_1 - q, P_B - k_1 - q)\phi^{C*}(k_1 - P_B, k_2, k_3)$$
$$\cdot V(q) \cdot \texttt{spin} \cdot \delta(P_A - k_1 - k_2 - k_3)\delta(P_A - P_B - P_C)$$

Feynman Rules

mesons:

$$X_{c,s,f;\bar{c},\bar{s},\bar{f}} = \frac{\delta_{c,\bar{c}}}{\sqrt{3}} \, \Xi^{I,I_z}_{f,\bar{f}} \, \langle \tfrac{1}{2}s, \tfrac{1}{2}\bar{s}|SM_S\rangle \, \langle SM_S, LM_L|JM\rangle$$

$$|\mathbf{P}; nJM[LS]; II_z\rangle = \int \frac{d^3k}{(2\pi)^3} \frac{d^3\bar{k}}{(2\pi)^3} \, (2\pi)^3 \delta(\mathbf{k} + \bar{\mathbf{k}} - \mathbf{P}) \, \phi_{nLM}(\mathbf{k}, \bar{\mathbf{k}}) X_{c,s,f;\bar{c},\bar{s},\bar{f}} b^\dagger_{c,s,f}(\mathbf{k}) d^\dagger_{\bar{c},\bar{s},\bar{f}}(\bar{\mathbf{k}})|0\rangle$$

$$\phi_{nLM}(\mathbf{k}, \bar{\mathbf{k}}) = \phi_{nLM}\left(\frac{m_{\bar{q}}\mathbf{k} - m_q\bar{\mathbf{k}}}{m_q + m_{\bar{q}}}\right) \qquad \phi_{nLM}(\mathbf{q}) = \phi_{nL}(q) Y_{LM}(\hat{q})$$

$$\langle \mathbf{P}'; n'J'M'[l'S']|\mathbf{P}; nJM[LS]\rangle = (2\pi)^3 \delta(\mathbf{P}' - \mathbf{P}) \delta_{nn'} \delta_{JJ'} \delta_{MM'} \delta_{SS'} \delta_{LL'}$$

$$\int \frac{k^2 dk}{(2\pi)^3} |\phi_{nL}(k)|^2 = 1$$

Feynman Rules

baryons:

$$P = p_1 + p_2 + p_3$$

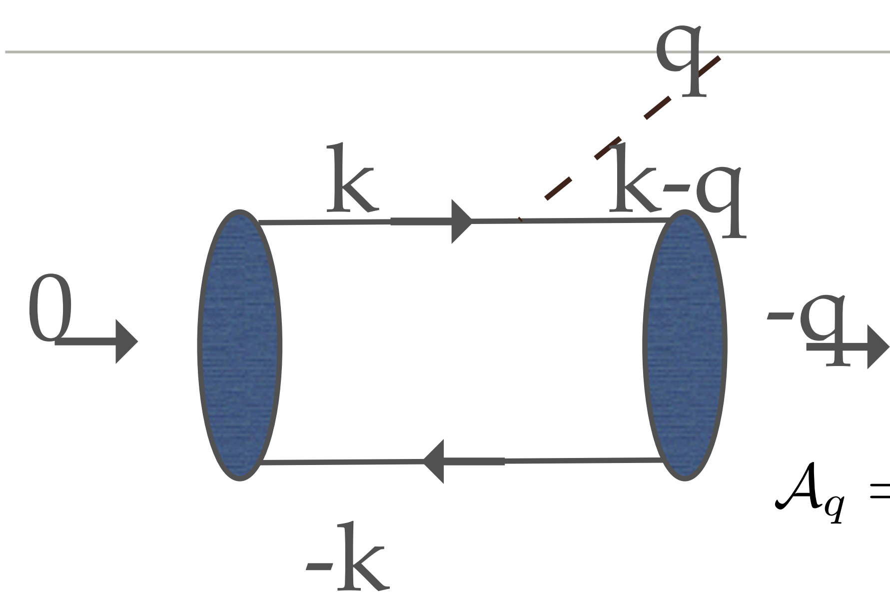
$$p_\lambda = \frac{\sqrt{6}}{2M}\,(m_3 p_1 + m_3 p_2 - (m_1 + m_2) p_3)$$

$$p_\rho = \frac{1}{2M}\,((m_3 + 2m_2) p_1 - (m_3 + 2m_1) p_2 + (m_2 - m_1) p_3)$$

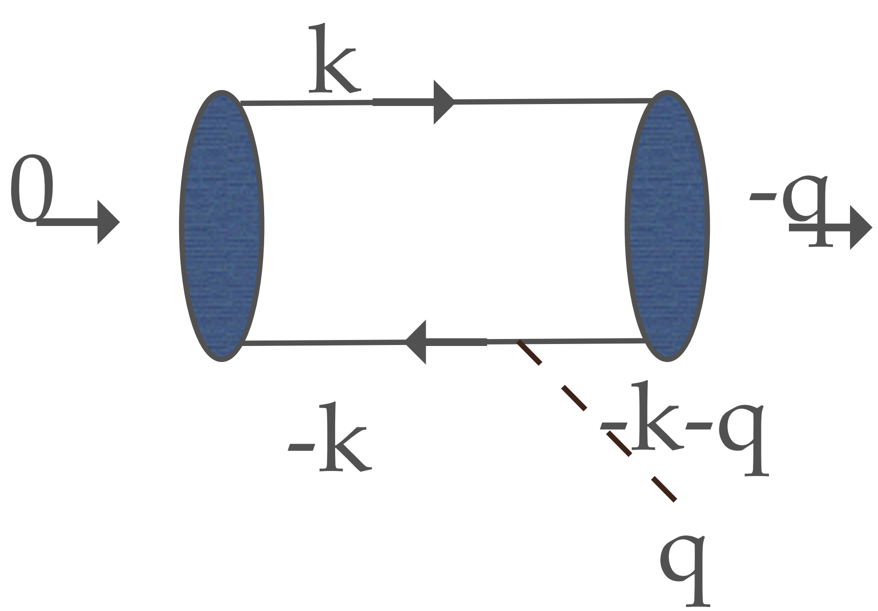
$$\psi(p_\rho, p_\lambda) = \int d^3\rho d^3\lambda\, \mathrm{e}^{-ip_\rho\cdot\rho}\, \mathrm{e}^{-ip_\lambda\cdot\lambda} \psi(\rho, \lambda)$$

$$(2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 - \mathbf{P})\, \phi(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)\, b^\dagger_{c_1,s_1,f_1}(\mathbf{k}_1) b^\dagger_{c_2,s_2,f_2}(\mathbf{k}_2) b^\dagger_{c_3,s_3,f_3}(\mathbf{k}_3)|0\rangle$$

RADIATIVE TRANSITIONS



$$\mathcal{A}_q = \int \frac{d^3k}{(2\pi)^3} \phi_A(k)\phi_B^*(k-q/2)u^\dagger(k-q)Q_q\alpha^i u(k)\epsilon^{i*}(q)$$



$$\mathcal{A}_{\bar{q}} = -\int \frac{d^3k}{(2\pi)^3} \phi_A(k)\phi_B^*(k+q/2)v^\dagger(k)Q_{\bar{q}}\alpha^i v(k+q)\epsilon^{i*}(q)$$

$$H_{em} = -\frac{e\epsilon^*_{\mathbf{q}\lambda}}{\sqrt{2q}}\left(Q_q\, u^\dagger_{s_2}(\mathbf{k}_2)\vec{\alpha}u_{s_1}(\mathbf{k}_1) + Q_{\bar{q}}\, v^\dagger_{\bar{s}_1}(\bar{\mathbf{k}}_1)\vec{\alpha}v_{\bar{s}_2}(\bar{\mathbf{k}}_2)\right) \quad (1)$$

$$H^{(q)}_{em} = \frac{1}{2m_q}\chi^\dagger_{s_2}\left((2\mathbf{k}-\mathbf{q}) + \imath\mathbf{q}\times\sigma\right)\chi_{s_1}$$

$$H^{(\bar{q})}_{em} = -\frac{1}{2m_{\bar{q}}}\tilde{\chi}^\dagger_{\bar{s}_1}\left((2\mathbf{k}+\mathbf{q}) + \imath\mathbf{q}\times\sigma\right)\tilde{\chi}_{\bar{s}_2}$$

$$H_{em} = \frac{e\epsilon^*_{\mathbf{q}\lambda}}{\sqrt{2q}}\left(Q_q H^{(q)}_{em} + Q_{\bar{q}} H^{(\bar{q})}_{em}\right)$$

$$H^{(q)}_{e} = \frac{1}{2m_q}\chi^\dagger_{s_2}(2\mathbf{k}-\mathbf{q})\chi_{s_1} = \frac{2\mathbf{k}-\mathbf{q}}{2m_q}\delta_{s_1 s_2} \quad (\qquad \text{E1}$$

$$H^{(q)}_{m} = \frac{1}{2m_q}\chi^\dagger_{s_2}(\imath\,\mathbf{q}\times\sigma)\chi_{s_1} = \frac{\imath\,\mathbf{q}}{2m_q}\times(\chi^\dagger_{s_2}\,\sigma\,\chi_{s_1})(\qquad \text{M1}$$

$$A^{(q)} = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \Phi_2^* \left(\mathbf{k} - \frac{\mathbf{q}}{2}\right) \Phi_1(\mathbf{k}) H_{em}^{(q)}(\mathbf{k}, \mathbf{q})$$

$$A^{(\bar{q})} = -\int \frac{d^3\mathbf{k}}{(2\pi)^3} \Phi_2^* \left(\mathbf{k} + \frac{\mathbf{q}}{2}\right) \Phi_1(\mathbf{k}) H_{em}^{(\bar{q})}(\mathbf{k}, \mathbf{q})$$

$$A = \frac{e\epsilon^*_{\mathbf{q}\lambda}}{\sqrt{2q}} \left(Q_q A^{(q)} + Q_{\bar{q}} A^{(\bar{q})}\right)$$

$$\Gamma = \frac{q^2 E_2}{4\pi^2 m_1} \int \sum_\lambda |A|^2 d\Omega$$

$$= \frac{q^2 E_2}{\pi m_1} \frac{1}{2J_1 + 1} \sum_\lambda |A|^2$$

For the decay ${}^3S_1 \to {}^1S_1\gamma$ it could be shown that the electric part of the operator does not give any contribution to the amplitude because of the cancellations between quark and antiquark interaction amplitudes ($A_e^{(q)} = -A_e^{(\bar{q})}$). The transition is pure magnetic and called $M1$ if we only consider the first term in the

	γ(MeV)	Analitical	Gaussian		Coulomb+linear		Experiment
			nonrel	rel	nonrel	rel	
$J/\psi \to \gamma\eta_c$	115	2.84	2.85	2.52	2.82	2.11	2 ~~1.18 ± 0.09~~
$X_{C0} \to \gamma J/\psi$	303	193	194	167	349	276	119 ± 25
$X_{C1} \to \gamma J/\psi$	389	221	221	193	422	325	288 ± 75
$X_{C2} \to \gamma J/\psi$	430	135	137	114	352	260	426 ± 71
$\Psi(2S) \to \gamma\eta_c$	639		5.95	3.21	8.15	1.41	0.79 ± 0.23
$\Psi(2S) \to \gamma X_{C0}$	261		29.1	22.1	19.8	11.5	24.2 ± 3.5
$\Psi(2S) \to \gamma X_{C1}$	171		60.8	45.3	39.6	22.6	23.6 ± 3.8
$\Psi(2S) \to \gamma X_{C2}$	127		76.0	57.4	49.6	29.1	18.0 ± 2.9
$h_c \to \gamma\eta_c$	496		189	162	497	363	

M1

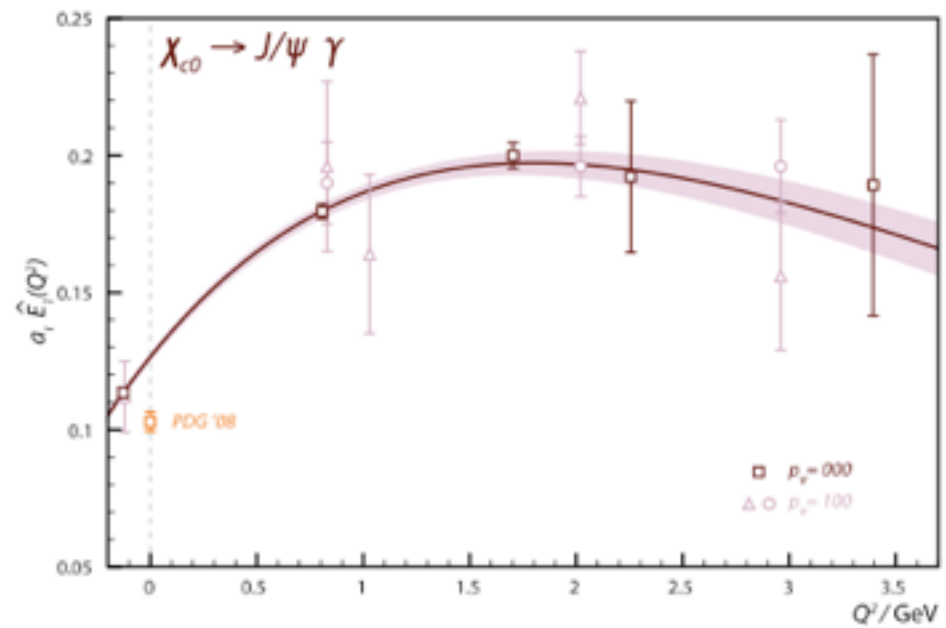
E1

Multiplets	Initial meson	Final meson	E_γ (MeV)		Γ_{thy} (keV)		Γ_{expt} (keV)
			NR	GI	NR	GI	
$2S \to 1P$	$\psi'(2^3S_1)$	$\chi_2(1^3P_2)$	128.	128.	38.	24.	$27. \pm 4.$
		$\chi_1(1^3P_1)$	171.	171.	54.	29.	$27. \pm 3.$
		$\chi_0(1^3P_0)$	261.	261.	63.	26.	$27. \pm 3.$
	$\eta_c'(2^1S_0)$	$h_c(1^1P_1)$	111.	119.	49.	36.	
$1P \to 1S$	$\chi_2(1^3P_2)$	$J/\psi(1^3S_1)$	429.	429.	424.	313.	$426. \pm 51.$
	$\chi_1(1^3P_1)$		390.	389.	314.	239.	$291. \pm 48.$
	$\chi_0(1^3P_0)$		303.	303.	152.	114.	$119. \pm 19.$
	$h_c(1^1P_1)$	$\eta_c(1^1S_0)$	504.	496.	498.	352.	

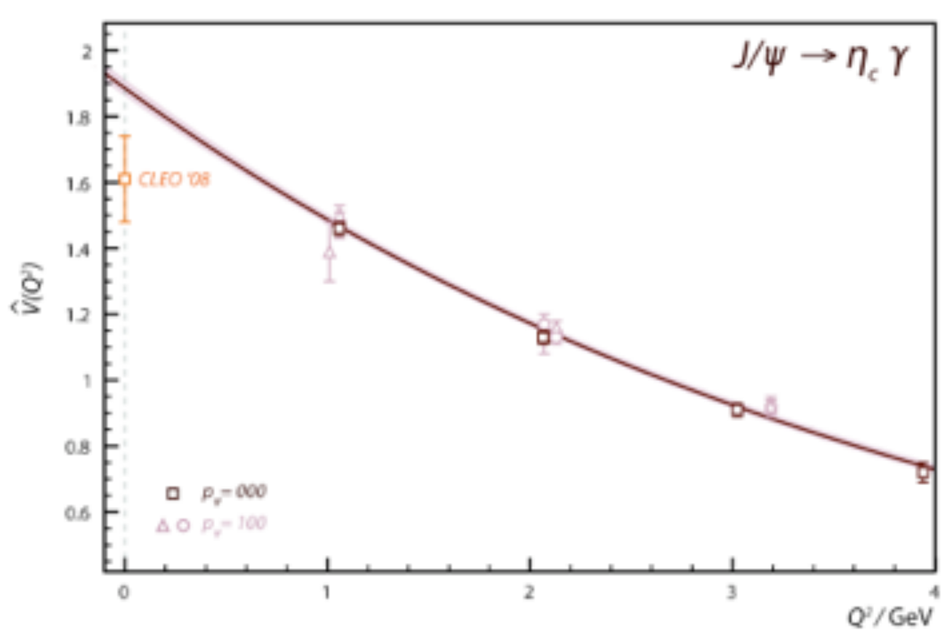
JLAB LATTICE RESULTS

Dudek, Edwards, Thomas, 0902.2241

sink level	suggested transition	$a_t \hat{E}_1(0)$	β/MeV λ/GeV^{-2}	Γ_{lat}/keV	Γ_{expt}/keV
0	$\chi_{c0} \to J/\psi\gamma$	0.127(2)	409(12) 1.14(5)	199(6)	131(14)
1	$\psi' \to \chi_{c0}\gamma$	0.092(19)	164(55) 0[fixed]	26(11)	30(2)
3	$\psi'' \to \chi_{c0}\gamma$	0.265(33)	324(77) 0.58(56)	265(66)	199(26)
5	$Y_{\text{hyb.}} \to \chi_{c0}\gamma$	0.00(3)	linear fit	$\lesssim 20$	-



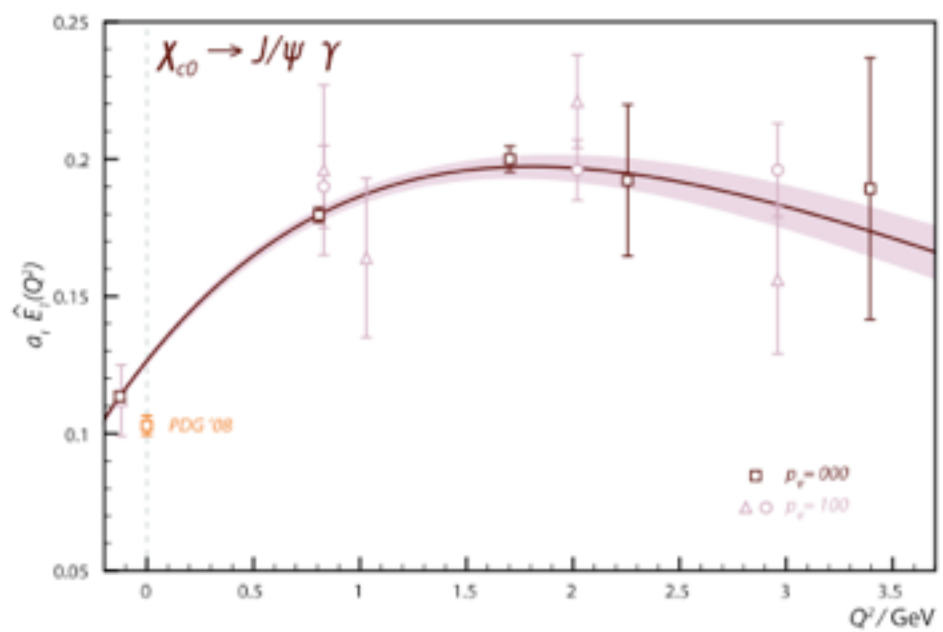
sink level	suggested transition	$\hat{V}(0)$	β/MeV λ/GeV^{-2}	Γ_{lat}/keV	Γ_{expt}/keV
0	$J/\psi \to \eta_c\gamma$	1.89(3)	513(7) 0[fixed]	2.51(8)	1.85(29)
1	$\psi' \to \eta_c\gamma$	0.062(64)	530(110) 4(6)	0.4(8)	0.95(16) 1.37(20)
3	$\psi'' \to \eta_c\gamma$	0.27(15)	367(55) −1.25(30)	10(11)	-
5	$Y_{\text{hyb.}} \to \eta_c\gamma$	0.28(6)	250(200) 0[fixed]	42(18)	-



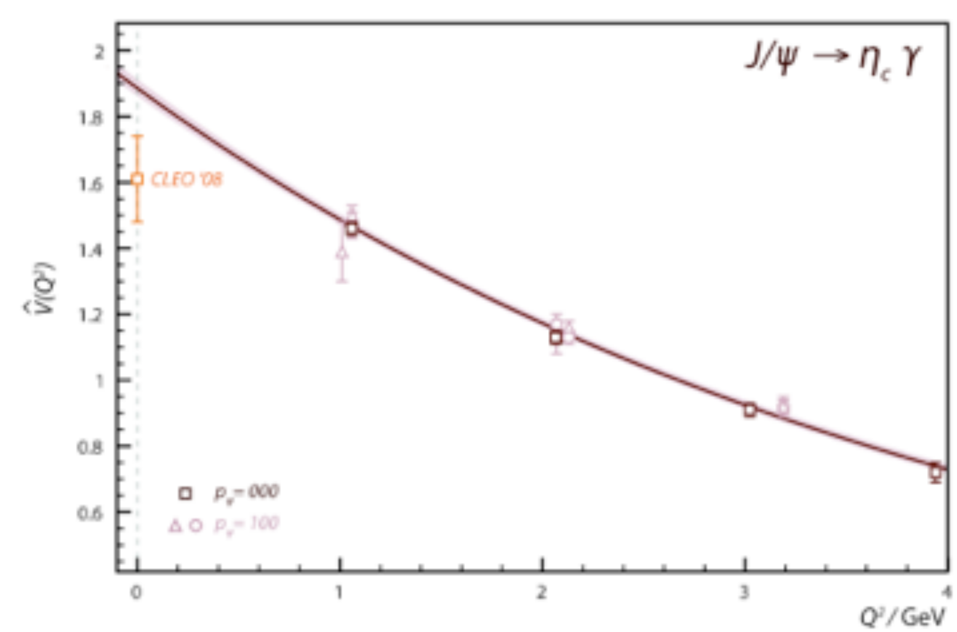
JLab lattice results

Dudek, Edwards, Thomas, 0902.2241

sink level	suggested transition	$a_t\hat{E}_1(0)$	β/MeV λ/GeV^{-2}	Γ_{lat}/keV	Γ_{expt}/keV
0	$\chi_{c0} \to J/\psi\gamma$	0.127(2)	409(12) 1.14(5)	199(6)	131(14)
1	$\psi' \to \chi_{c0}\gamma$	0.092(19)	164(55) 0[fixed]	26(11)	30(2)
3	$\psi'' \to \chi_{c0}\gamma$	0.265(33)	324(77) 0.58(56)	265(66)	199(26)
5	$Y_{\text{hyb.}} \to \chi_{c0}\gamma$	0.00(3)	linear fit	$\lesssim 20$	-



sink level	suggested transition	$\hat{V}(0)$	β/MeV λ/GeV^{-2}	Γ_{lat}/keV	Γ_{expt}/keV
0	$J/\psi \to \eta_c\gamma$	1.89(3)	513(7) 0[fixed]	2.51(8)	1.85(29)
1	$\psi' \to \eta_c\gamma$	0.062(64)	530(110) 4(6)	0.4(8)	0.95(16) 1.37(20)
3	$\psi'' \to \eta_c\gamma$	0.27(15)	367(55) $-1.25(30)$	10(11)	-
5	$Y_{\text{hyb.}} \to \eta_c\gamma$	0.28(6)	250(200) 0[fixed]	42(18)	-



	γ(MeV)	Analitical	Gaussian		Coulomb+linear		Experiment
			nonrel	rel	nonrel	rel	
$\rho^0 \to \gamma\pi^0$	376	50.1	51.1	20.9	41.6	13.1	90.2 ± 19.8
$\rho^\pm \to \gamma\pi^\pm$	375	50.0	50.9	20.9	41.5	13.1	67.6 ± 8.3
$\rho \to \gamma\eta$	195	53.4	55.9	26.1	41.7	14.9	45.1 ± 6.6
$w \to \gamma\pi^0$	380	468.	470.	192.	384.	121.	$757. \pm 31.$
$w \to \gamma\eta$	200	6.65	6.64	3.09	4.97	1.78	4.16 ± 0.47
$\eta' \to \gamma\rho^0$	165		114.	54.2	84.5	31.2	59.6 ± 6.9
$\eta' \to \gamma w$	159		11.5	5.51	8.55	3.16	6.12 ± 1.16
$f_0(980) \to \gamma\rho^0$	183		518.	233.	591.	256.	
$f_0(980) \to \gamma w$	178		55.8	25.1	63.8	27.6	
$a_0(980) \to \gamma\rho$	187		59.3	26.6	67.4	29.2	
$h_1 \to \gamma a_0(980)$	171		28.3	10.5	28.4	10.4	
$h_1 \to \gamma f_0(980)$	175		3.35	1.24	3.36	1.22	
$h_1 \to \gamma\eta'$	193		24.2	10.3	42.8	13.0	
$h_1 \to \gamma\eta$	457		30.5	11.1	63.9	17.0	
$h_1 \to \gamma\pi^0$	577		459.	152.	1097.	266.	
$\phi \to \gamma\eta$	363	45.5	43.0	27.1	44.5	21.4	55.2 ± 1.7
$b_1 \to \gamma\pi^\pm$	607		50.5	16.2	124.5	29.5	$227. \pm 75.$
$f_1(1285) \to \gamma\rho^0$	406		1066.	459.	1216.	489.	1326 ± 388
$a_2 \to \gamma\pi^\pm$	652		324.	144.	93.4	64.4	287.

$$\Gamma_{\rm E1}({\rm n}\,^{2{\rm S}+1}{\rm L}_{\rm J} \to {\rm n}'\,^{2{\rm S}'+1}{\rm L}'_{{\rm J}'} + \gamma) = \frac{4}{3}\,C_{fi}\,\delta_{\rm SS'}\,e_c^2\,\alpha\,|\,\langle\psi_f|\,r\,|\,\psi_i\rangle\,|^2\,{\rm E}_\gamma^3\,\frac{{\rm E}_{\rm f}^{(c\bar{c})}}{{\rm M}_{\rm i}^{(c\bar{c})}}$$

$$C_{fi} = \max({\rm L},\ {\rm L}')\ (2{\rm J}'+1)\left\{\begin{matrix}{\rm L}' & {\rm J}' & {\rm S}\\ {\rm J} & {\rm L} & 1\end{matrix}\right\}^2 .$$

$$\Gamma_{\rm M1}({\rm n}\,^{2{\rm S}+1}{\rm L}_{\rm J} \to {\rm n}'\,^{2{\rm S}'+1}{\rm L}'_{{\rm J}'} + \gamma) = \frac{4}{3}\,\frac{2J'+1}{2L+1}\,\delta_{\rm LL'}\,\delta_{{\rm S},{\rm S}'\pm1}\,e_c^2\,\frac{\alpha}{m_c^2}\,|\,\langle\psi_f|\,\psi_i\rangle\,|^2\,{\rm E}_\gamma^3\,\frac{{\rm E}_{\rm f}^{(c\bar{c})}}{{\rm M}_{\rm i}^{(c\bar{c})}}\ .$$

tricks/ problems with these formulae

DECAY CONSTANTS

$$m_V f_V \epsilon^\mu = \langle 0|\bar{\Psi}\gamma^\mu \Psi|V\rangle$$

$$\Gamma_{V\to e^+e^-} = \frac{e^4 Q^2 f_V^2}{12\pi m_V} = \frac{4\pi\alpha^2}{3}\frac{Q^2 f_V^2}{m_V}.$$

$$f_V = \sqrt{\frac{3}{m_V}}\int \frac{d^3k}{(2\pi)^3}\Phi(\vec{k})\sqrt{1+\frac{m_q}{E_k}}\sqrt{1+\frac{m_{\bar{q}}}{E_{\bar{k}}}}\left(1+\frac{k^2}{3(E_k+m_q)(E_{\bar{k}}+m_{\bar{q}})}\right)$$

$$f_V = 2\sqrt{\frac{3}{m_V}}\int \frac{d^3k}{(2\pi)^3}\Phi(\vec{k}) = 2\sqrt{\frac{3}{m_V}}\tilde{\Phi}(r=0).$$ nonrel

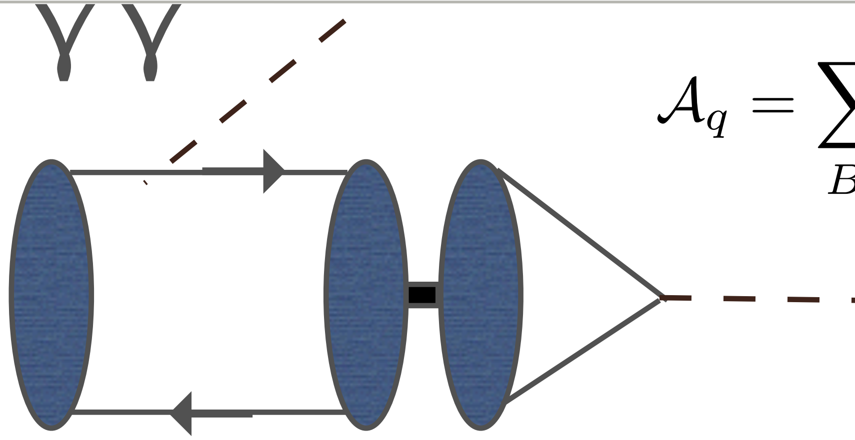
van Royen Weisskopf (1967!)

TABLE II: Charmonium Decay Constants (MeV).

Meson	BGS NonRel	BGS Rel	BGS log $\Lambda = 0.4$ GeV	BGS log $\Lambda = 0.25$ GeV	lattice	experiment
η_c	795	493	424	402	$429 \pm 4 \pm 25$	335 ± 75
η_c'	477	260	243	240	$56 \pm 21 \pm 3$	
η_c''	400	205	194	193		
J/ψ	615	545	423	393	399 ± 4	411 ± 7
ψ'	431	371	306	293	143 ± 81	279 ± 8
ψ''	375	318	267	258		174 ± 18
χ_{c1}	145	103	97	93		
χ_{c1}'	196	132	125	120		
χ_{c1}''	223	142	134	130		

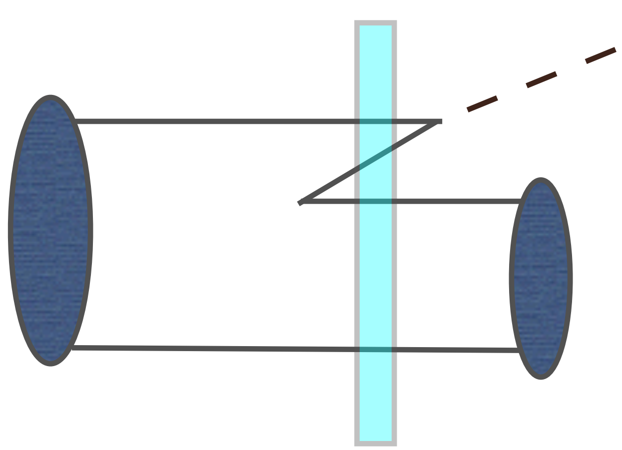
S/Ps-> gamma gamma

A → γ γ



$$\mathcal{A}_q = \sum_{B\gamma} \langle A|H_{EM}|B\gamma(q_1)\rangle \frac{1}{E_A - E_{B\gamma}} \langle B\gamma|H_{EM}|\gamma\gamma\rangle$$

A → B γ



$$\mathcal{A}_q = \sum_{BC} \langle A|H_{^3P_0}|BC\rangle \frac{1}{E_A - E_{BC}} \langle BC|H_{EM}|\gamma C\rangle$$

general structure

$$\mathcal{A}(\lambda_1 p_1; \lambda_2 p_2) = \epsilon^*_\mu(\lambda_1, p_1)\epsilon^*_\nu(\lambda_2, p_2)\mathcal{M}^{\mu\nu}$$

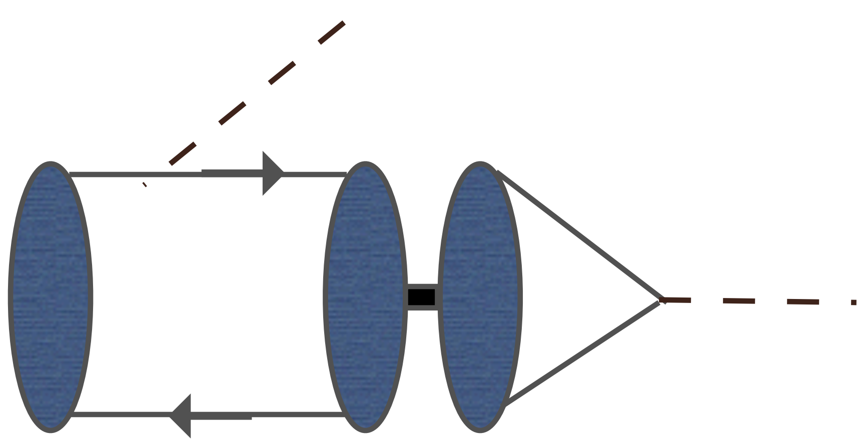
$$\mathcal{M}^{\mu\nu}_{Ps} = iM_{Ps}(p_1^2, p_2^2, p_1 \cdot p_2)\, \epsilon^{\mu\nu\alpha\beta}\, p_{1\alpha} p_{2\beta}$$

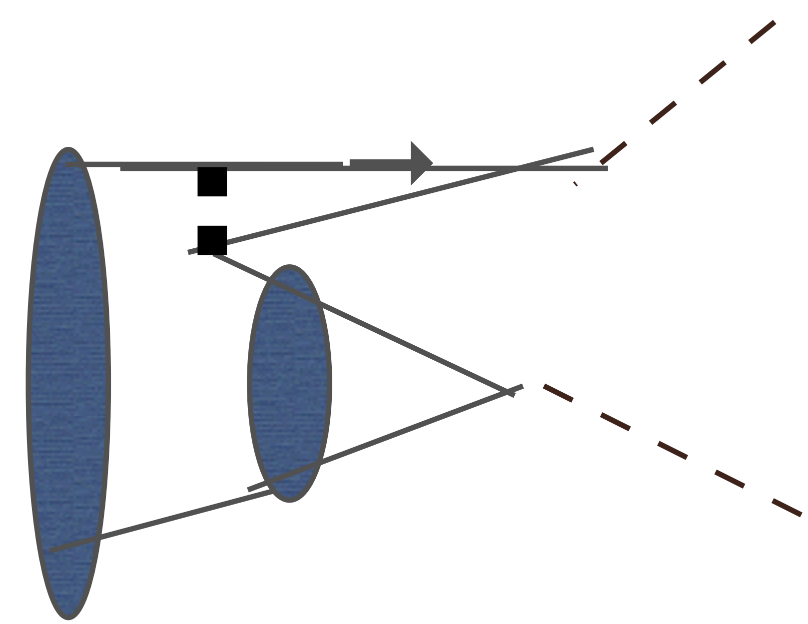
$$\mathcal{M}^{\mu\nu}_{S} = M_S(p_1^2, p_2^2, p_1 \cdot p_2)\, g^{\mu\nu}$$

$$\Gamma(Ps \to \gamma\gamma) = \frac{m^3_{Ps}}{64\pi}|M_{Ps}(0,0)|^2 \text{ or } \Gamma(S \to \gamma\gamma) = |M_S(0,0)|^2/(8\pi m$$

quark model. Other time ordering is higher order, so ignored.

$$\mathcal{A} = \sum_{\gamma,V} \frac{\langle \gamma(\lambda_1, p_1)\gamma(\lambda_2, p_2)|H|\gamma, V\rangle \langle \gamma, V|H|Ps\rangle}{(m_{Ps} - E_{\gamma V})}$$





$$M_{Ps} = \sum_V Q^2 \sqrt{\frac{m_V}{E_V}} f_V \frac{F^{(V)}(q)}{m_{Ps} - E_{\gamma V}(q)}$$

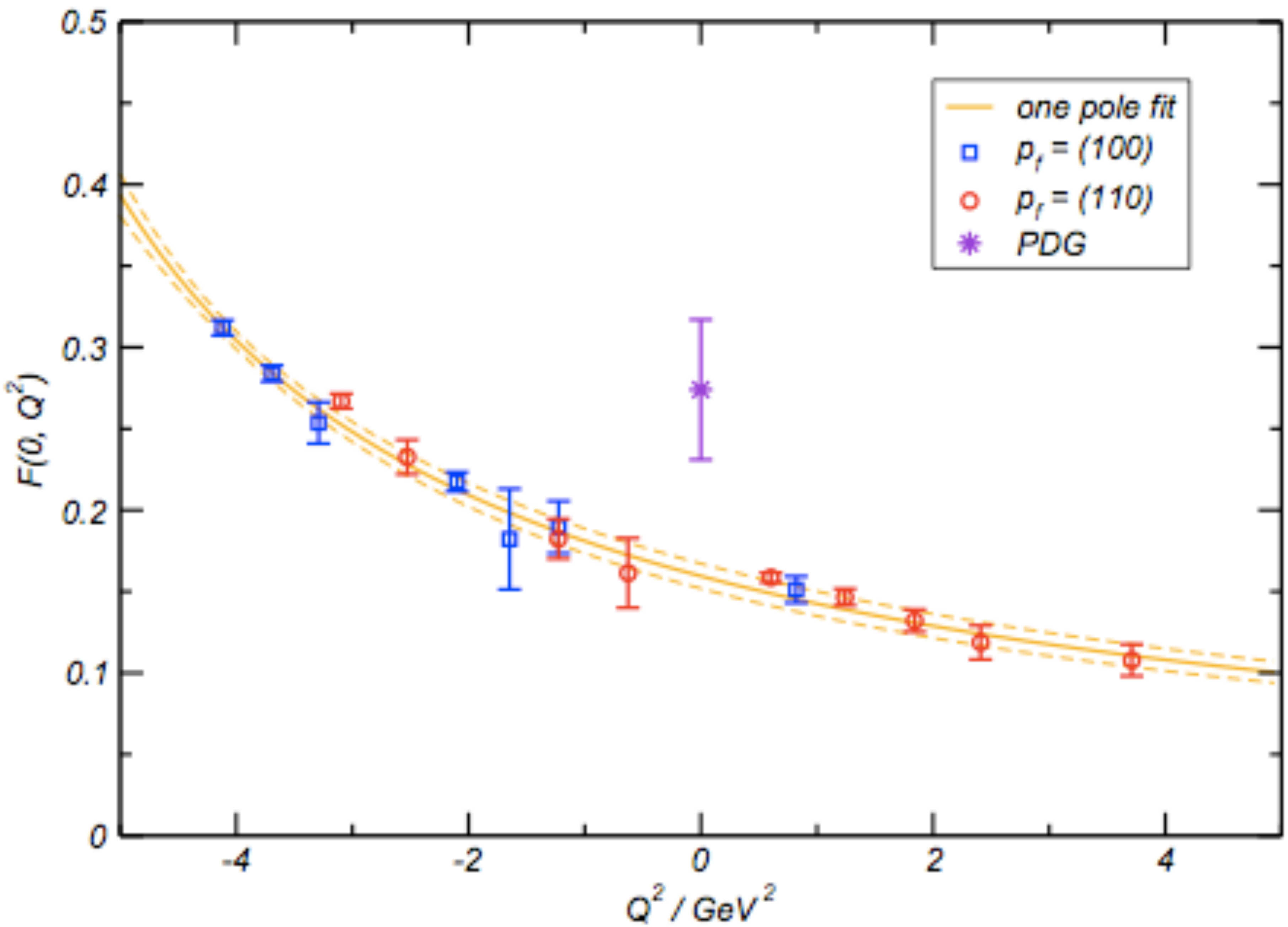
$$M_S = \sum_V Q^2 \sqrt{\frac{m_V}{E_V}} f_V \frac{E_1^{(V)}(q)}{m_S - E_{\gamma V}(q)}.$$

process	BGS	BGS log ($\Lambda = 0.25$ GeV)	G&I[4]	HQ[30]	A&B[31]	EFG[32]	Munz[33]	Chao[34]	CWV[35]	PDG[a]
$\eta_c \to \gamma\gamma$	14.2	7.18	6.76	7.46	4.8	5.5	3.5(4)	6-7	6.18	7.44 ± 2.8
$\eta_c' \to \gamma\gamma$	2.59	1.71	4.84	4.1	3.7	1.8	1.4(3)	2	1.95	1.3 ± 0.6
$\eta_c'' \to \gamma\gamma$	1.78	1.21	–	–	–	–	0.94(23)	–	–	–
$\chi_{c0} \to \gamma\gamma$	5.77	3.28	–	–	–	2.9	1.39(16)	–	3.34	2.63 ± 0.5

JLab lattice results

Dudek & Edwards, hep-ph/0607140

$$\eta_c \to \gamma\gamma^*$$



FORM FACTORS

$$\langle P_2(p_2)|\bar{\Psi}\gamma^\mu\Psi|P_1(p_1)\rangle = f(Q^2)(p_2+p_1)^\mu + g(Q^2)(p_2-p_1)^\mu$$

$$g(Q^2) = f(Q^2)\frac{M_2^2 - M_1^2}{Q^2}.$$

conserved current

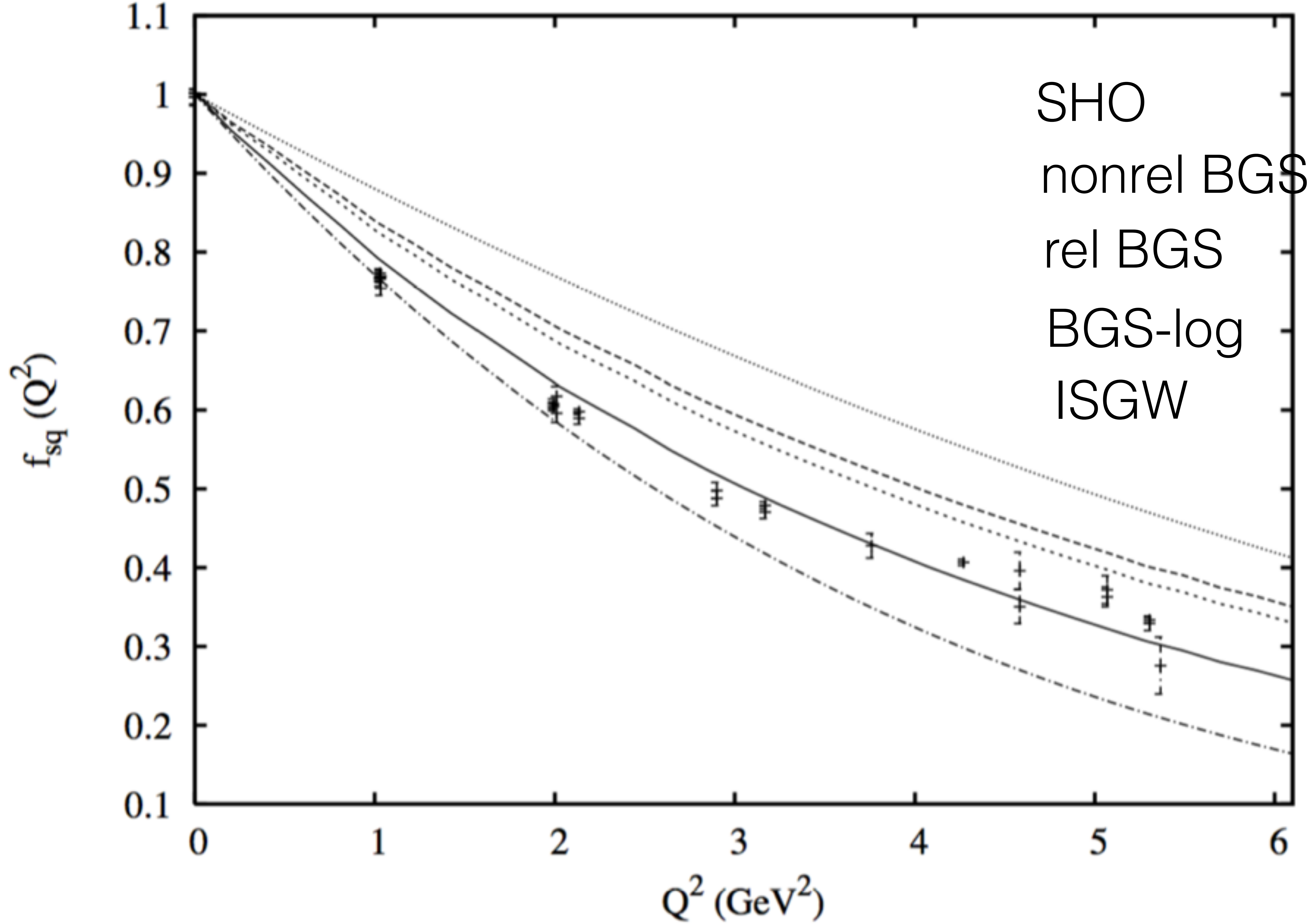
$$f(Q^2) = \frac{\sqrt{M_1 E_2}}{(E_2+M_1) - \frac{M_2^2-M_1^2}{q^2}(E_2-M_1)} \times \int \frac{d^3k}{(2\pi)^3}\Phi(\vec{k})\Phi^*\left(\vec{k}+\frac{\vec{q}}{2}\right)\sqrt{1+\frac{m_q}{E_k}}\sqrt{1+\frac{m_q}{E_{k+q}}}\left(1+\frac{(\vec{k}+\vec{q})\cdot\vec{k}}{(E_k+m_q)(E_{k+q}+m_q)}\right)$$

$$f(Q^2) = \frac{2\sqrt{M_1 E_2}}{E_2+M_1}\int \frac{d^3k}{(2\pi)^3}\Phi(\vec{k})\Phi^*\left(\vec{k}+\frac{\vec{q}}{2}\right)$$

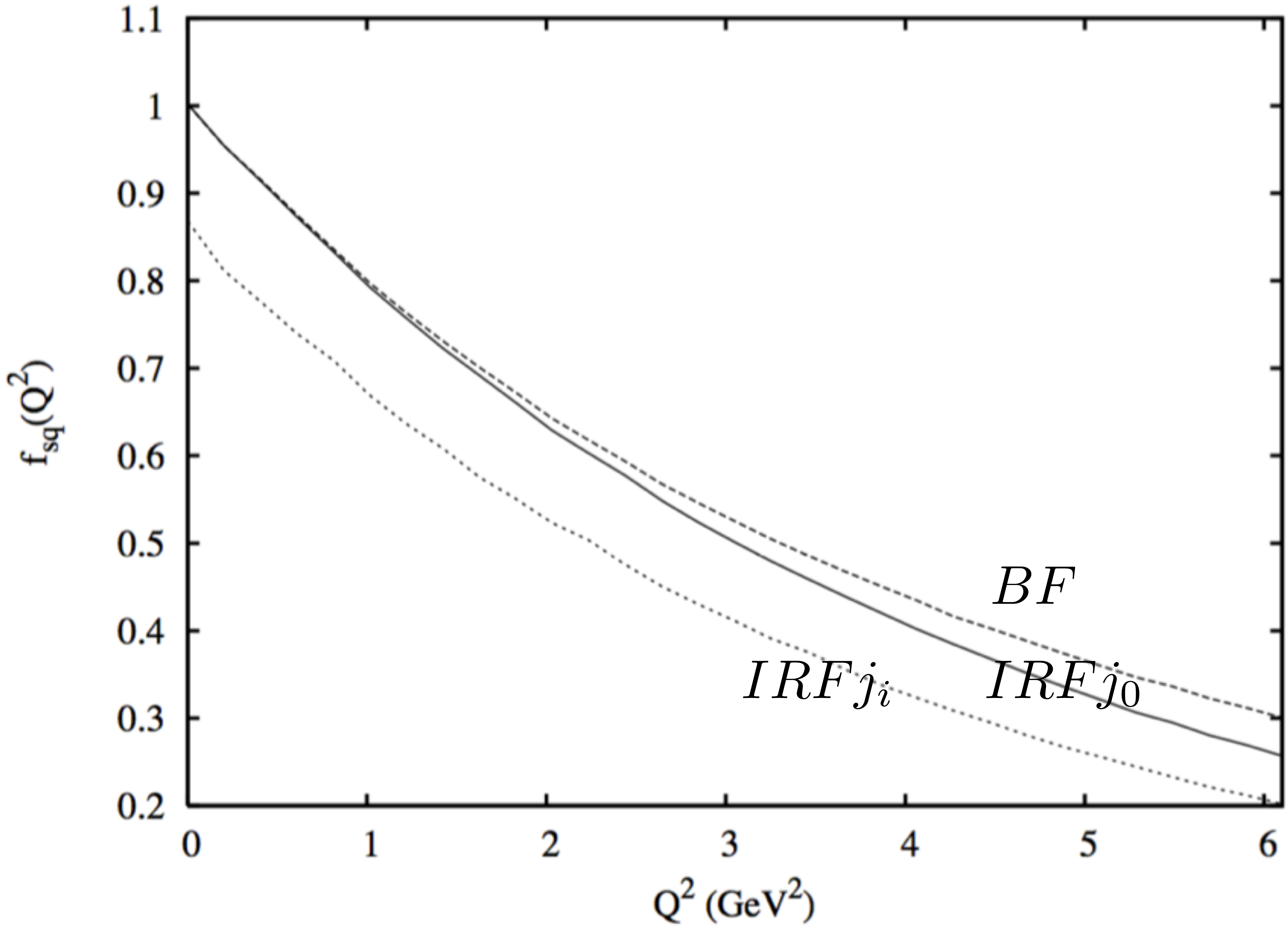
nonrel

FT: $\propto \int d^3x\, |\phi(x)|^2 e^{-iqx/2}$

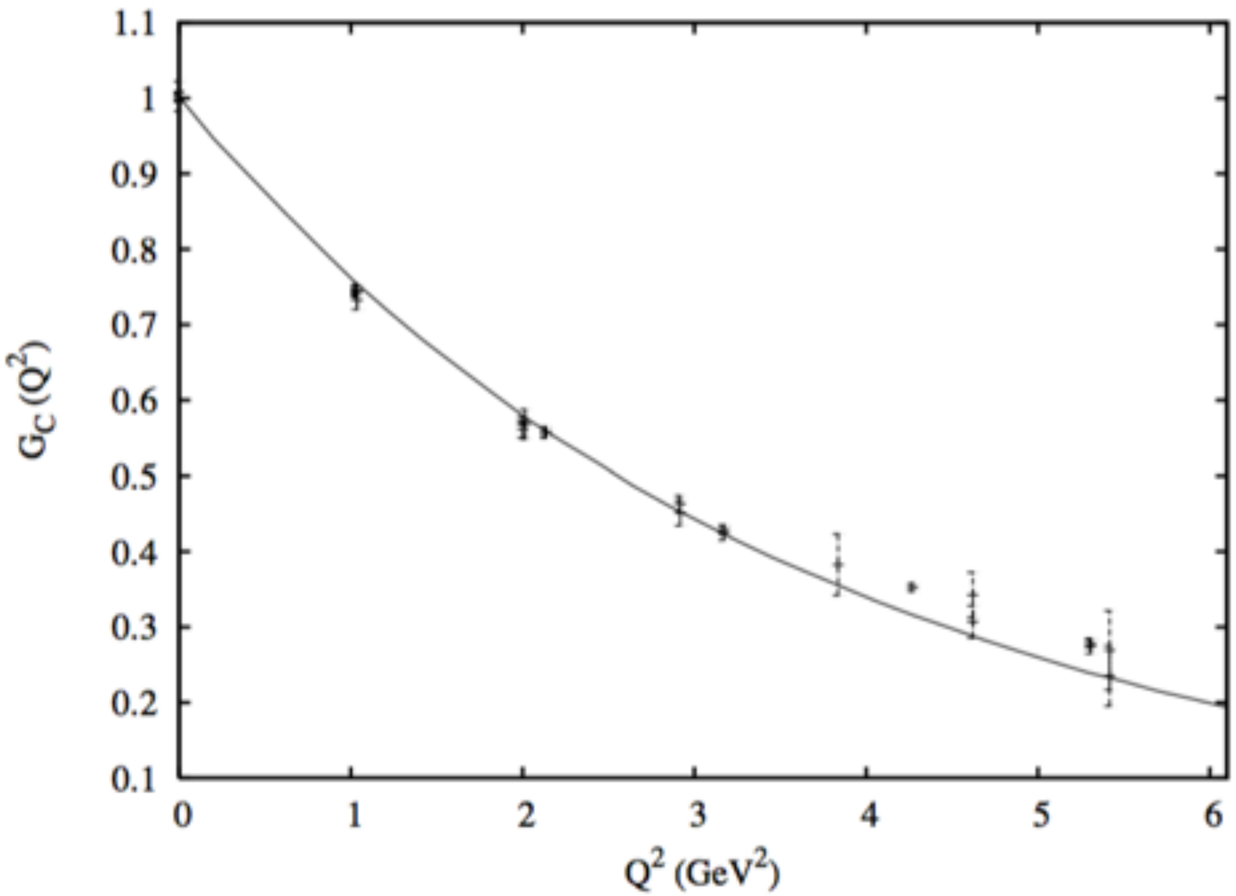
η_c "single quark" FF

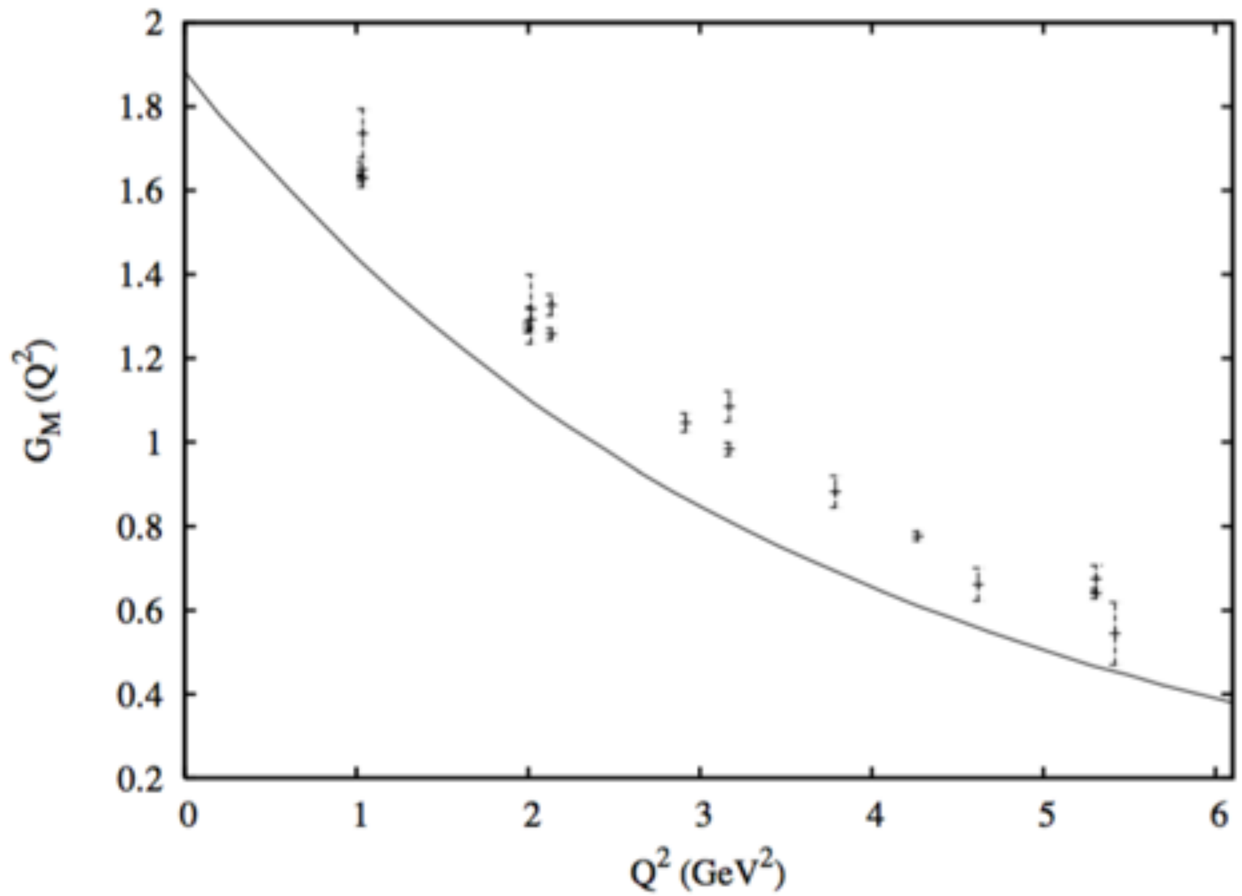


testing covariance BF=Breit frame, IRF = initial rest frame

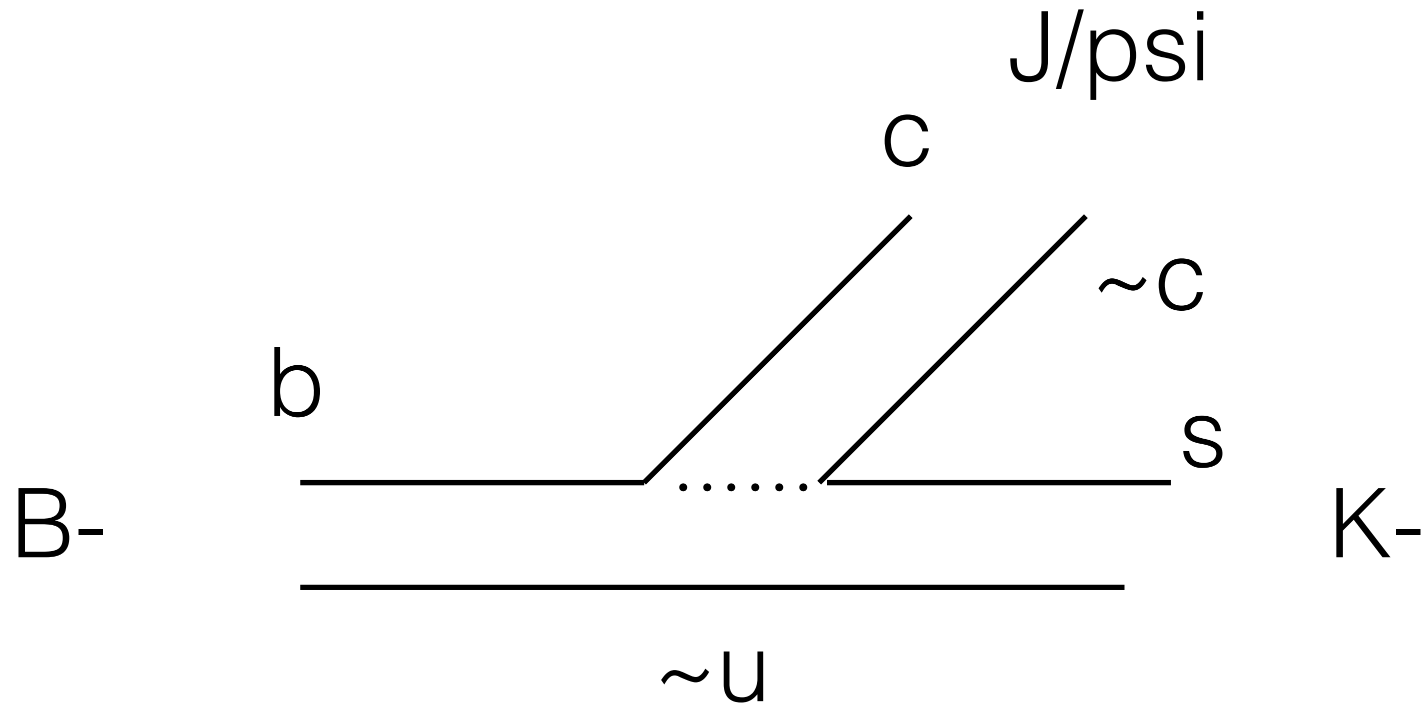


LGT psi : etac gamma





Electroweak transitions



$$iM = \frac{1}{N_c}\frac{G_F}{\sqrt{2}}V_{bc}V_{cs}^{*}m_{\psi}f_{\psi}\epsilon_{\mu}^{*}(p_{\psi},\lambda)\left[f^{(+)}(p_B+p_K)^{\mu}+f^{(-)}\cdot(p_B-p_K)^{\mu}\right]$$

p_{ψ}

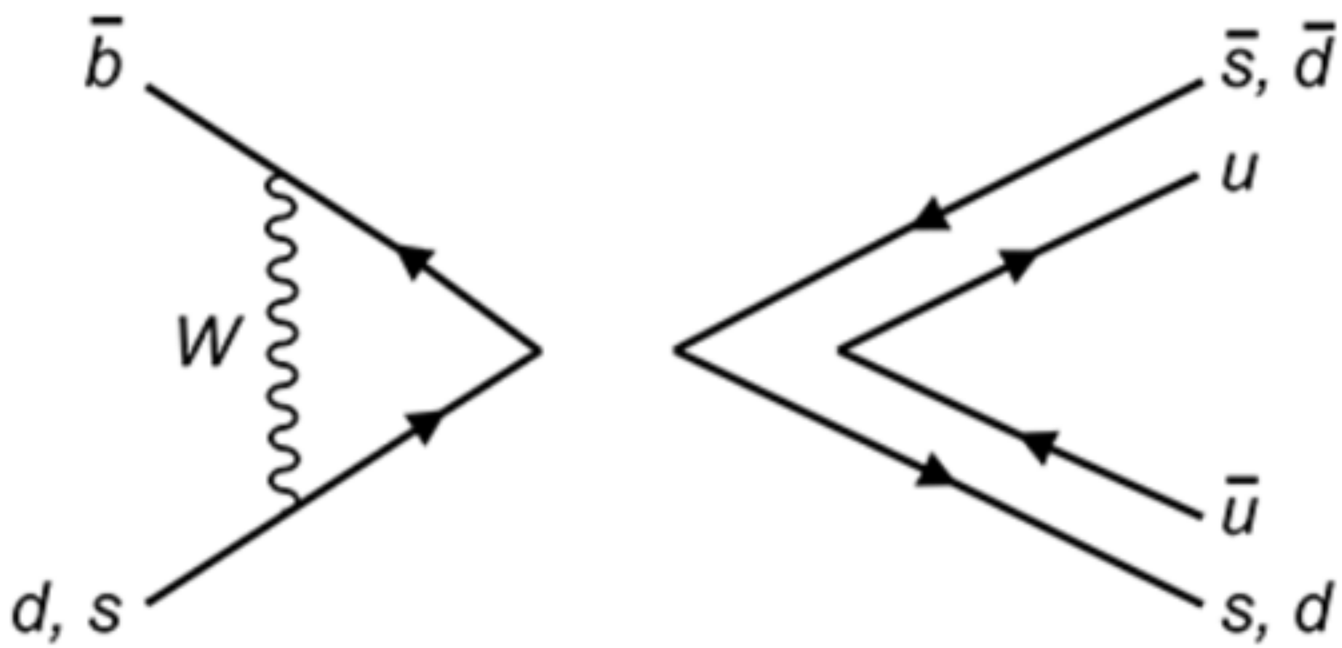
$2p_K$

$$\xi(w) = \left(\frac{2}{1+w}\right)^2$$

$$w = \frac{m_B^2+m_K^2-m_{\psi}^2}{m_B m_K}$$

$$\Gamma = \frac{q}{32\pi^2 m_B^2}\int d\Omega |M|^2$$

average initial; sum final



rarest B decay ever observed, PRL 118, 081801 (17) add nonfact Lambda decay…

STRONG DECAYS

Decay Models

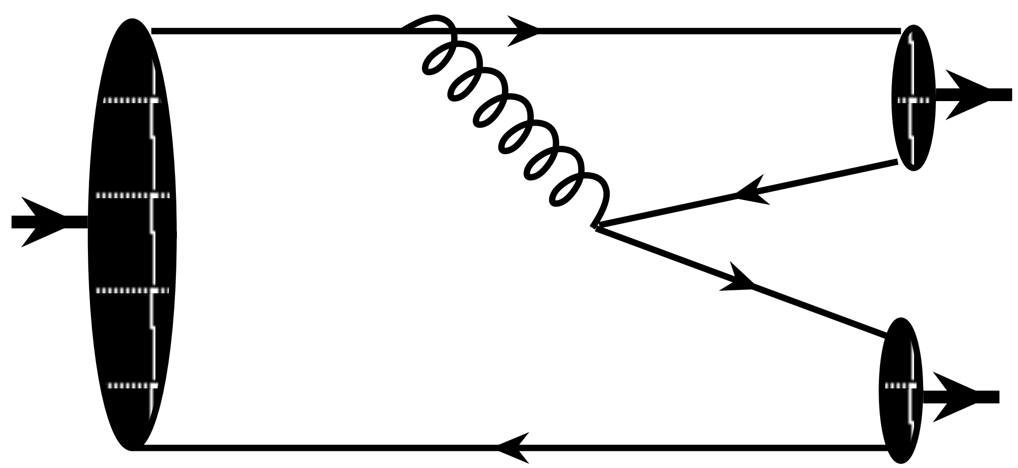
why we need them:

they give coupled channels (FSIs, mass shifts)

they provide diagnostic information on the parent states

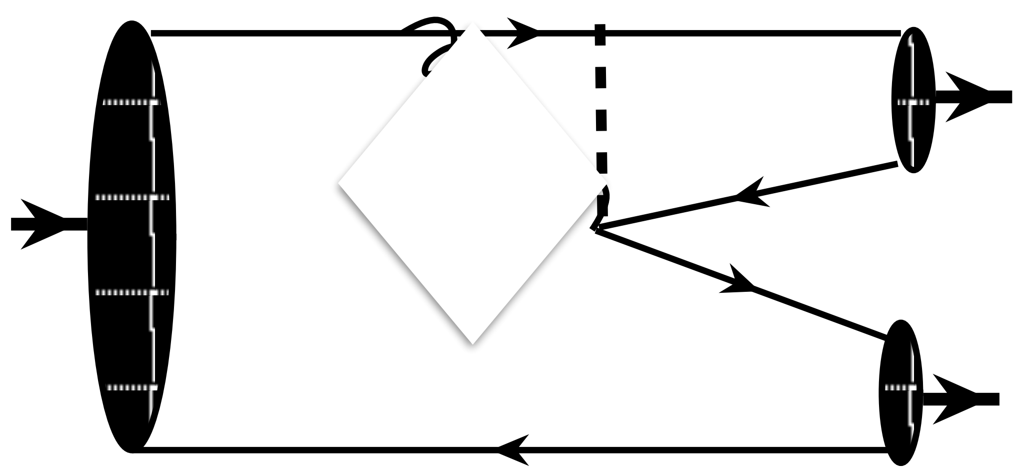
they probe nonperturbative gluodynamics in a new regime

$^3S_1$ Model

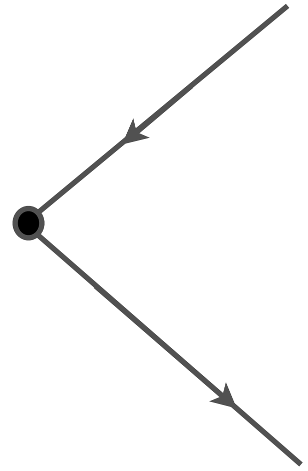


Cornell Model

$$V = \frac{1}{2}\int d^3x d^3y\, \psi^\dagger(\mathbf{x})\psi(\mathbf{x})V(\mathbf{x}-\mathbf{y})\psi^\dagger(\mathbf{y})\psi(\mathbf{y})$$



$^3P_0$ Model



$$H_{int} = g \int d^3x \, \bar{\psi}(\mathbf{x})\psi(\mathbf{x}) \quad \text{or} \quad \int d^3x \, b^\dagger(\mathbf{x})\alpha \cdot \nabla d^\dagger(\mathbf{x})$$

IKP Flux Tube Decay Model

quark creation operator

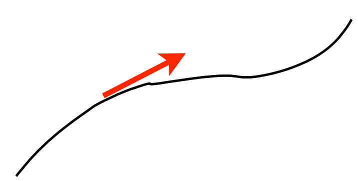
Kokoski & Isgur, PRD35, 907 (87)

Isgur, Kokoski, & Paton, PRL54, 869 (85)

$$H = \frac{g^2}{2a}\sum_\ell E^a_\ell E_{a\ell} + \sum_n m\bar{\psi}_n\psi_n + \frac{1}{a}\sum_{n,\mu}\psi^\dagger_n\alpha_\mu U_\mu(n)\psi_{n+\mu} + \frac{1}{ag^2}\sum_P \mathrm{tr}(N - U_P - U^\dagger_P)$$

$$H_{int} \sim \psi^\dagger_n \alpha\cdot\mu\psi_{n+\mu}$$

$$\sim \psi^\dagger_n\alpha\cdot\mu\psi_n + a\psi^\dagger_n\alpha\cdot\mu\,\mu\cdot\nabla\psi_n$$



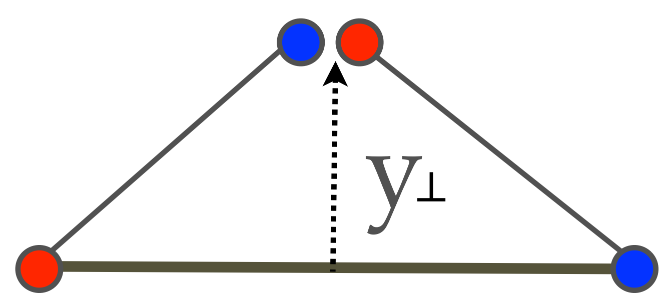
$\psi^\dagger_n\alpha\cdot\mu\psi_n$ $\psi^\dagger_n\alpha\cdot\nabla\psi_n$

$^3S_1$ $^3P_0$

IKP Flux Tube Decay Model

meson decay

$$<\{0...0\}bd;\ \{0...0\}bd\ |\ O\ |\ \{0...0\}b^\dagger d^\dagger> \ \sim\ <bd;\ bd\ |\ {}^3P_0\ |\ b^\dagger d^\dagger> \cdot$$
$$<\{0...0\};\ \{0...0\}\ |\ \{0...0\}>$$



$$\downarrow$$

$$e^{-fby_\perp^2}$$

hybrid decay

$$<\{0...0\}bd;\ \{0...0\}bd\ |\ O\ |\ \{1,0...0\}b^\dagger d^\dagger> \ \sim\ <bd;\ bd\ |\ {}^3P_0\ |\ b^\dagger d^\dagger> \cdot$$
$$<\{0...0\};\ \{0...0\}\ |\ \{1,0...0\}>$$

$$y_\perp\, e^{-fby_\perp^2}$$

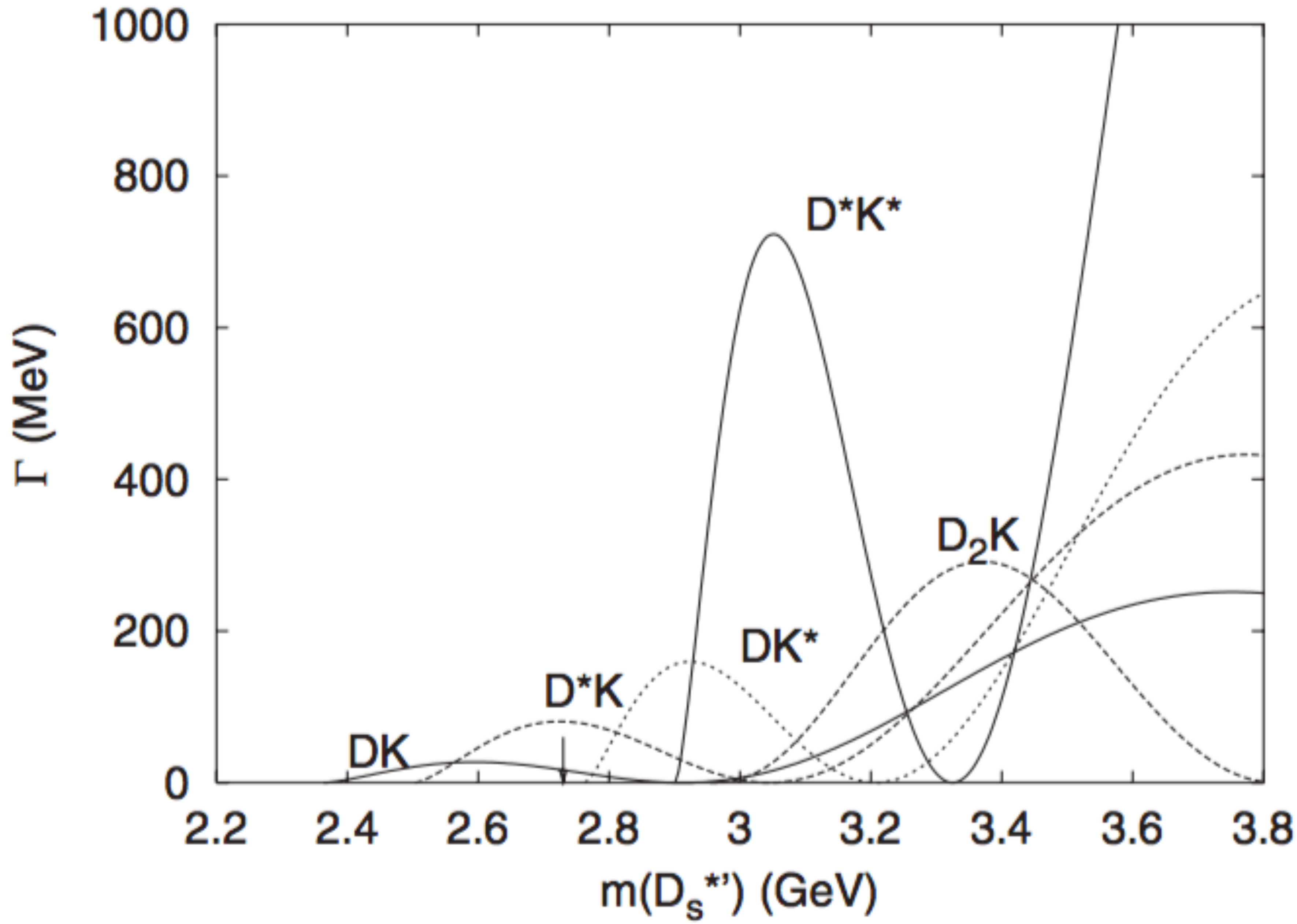
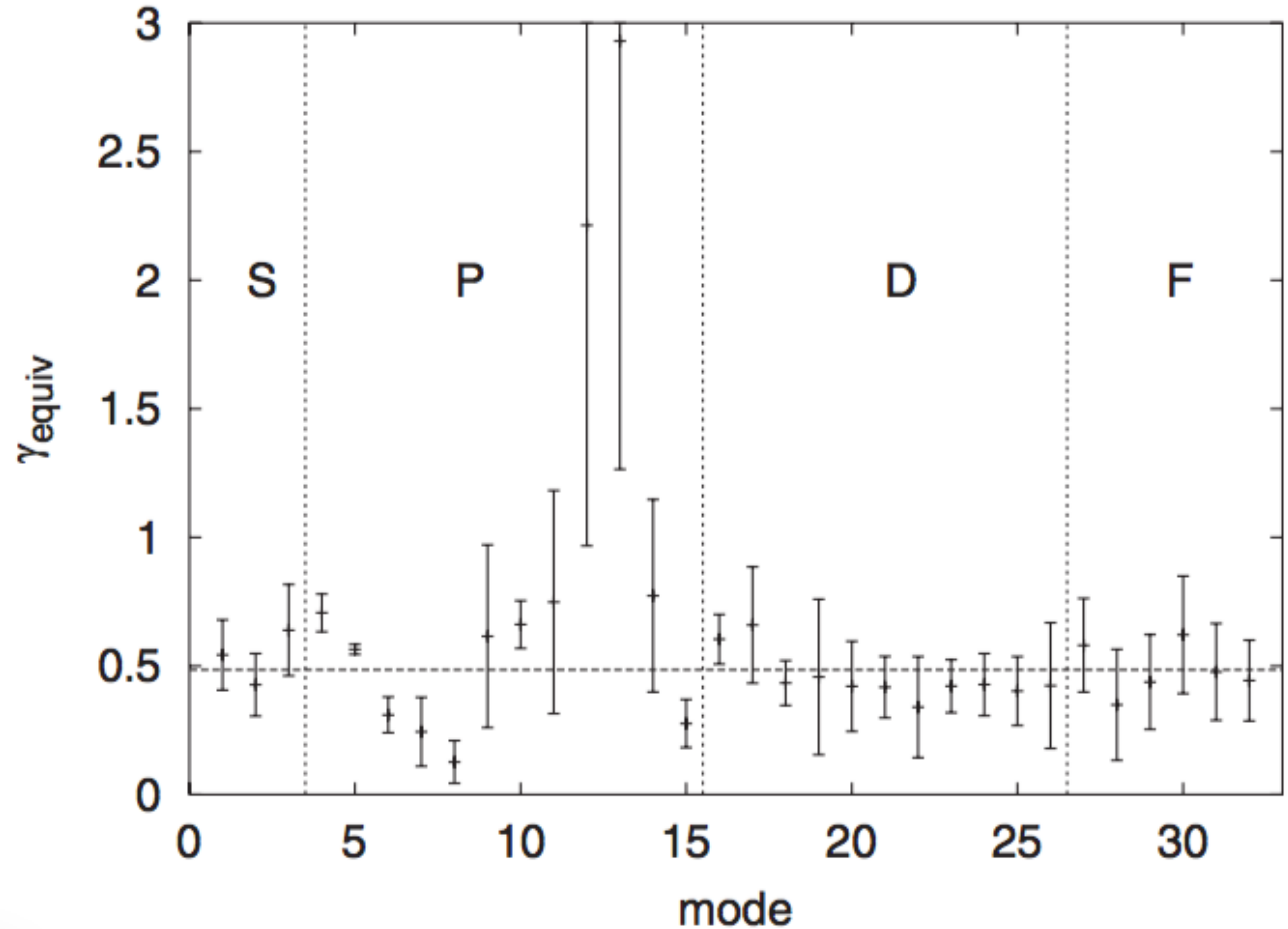


FIG. 3. $D_s^{*\prime}$ partial widths vs mass. The arrow shows the nominal mass of the $D_s^{*\prime}$.



3
2.5
2
1.5
1
0.5
0
γ_{equiv}
S
P
D
F
0
5
10
15
20
25
30
mode

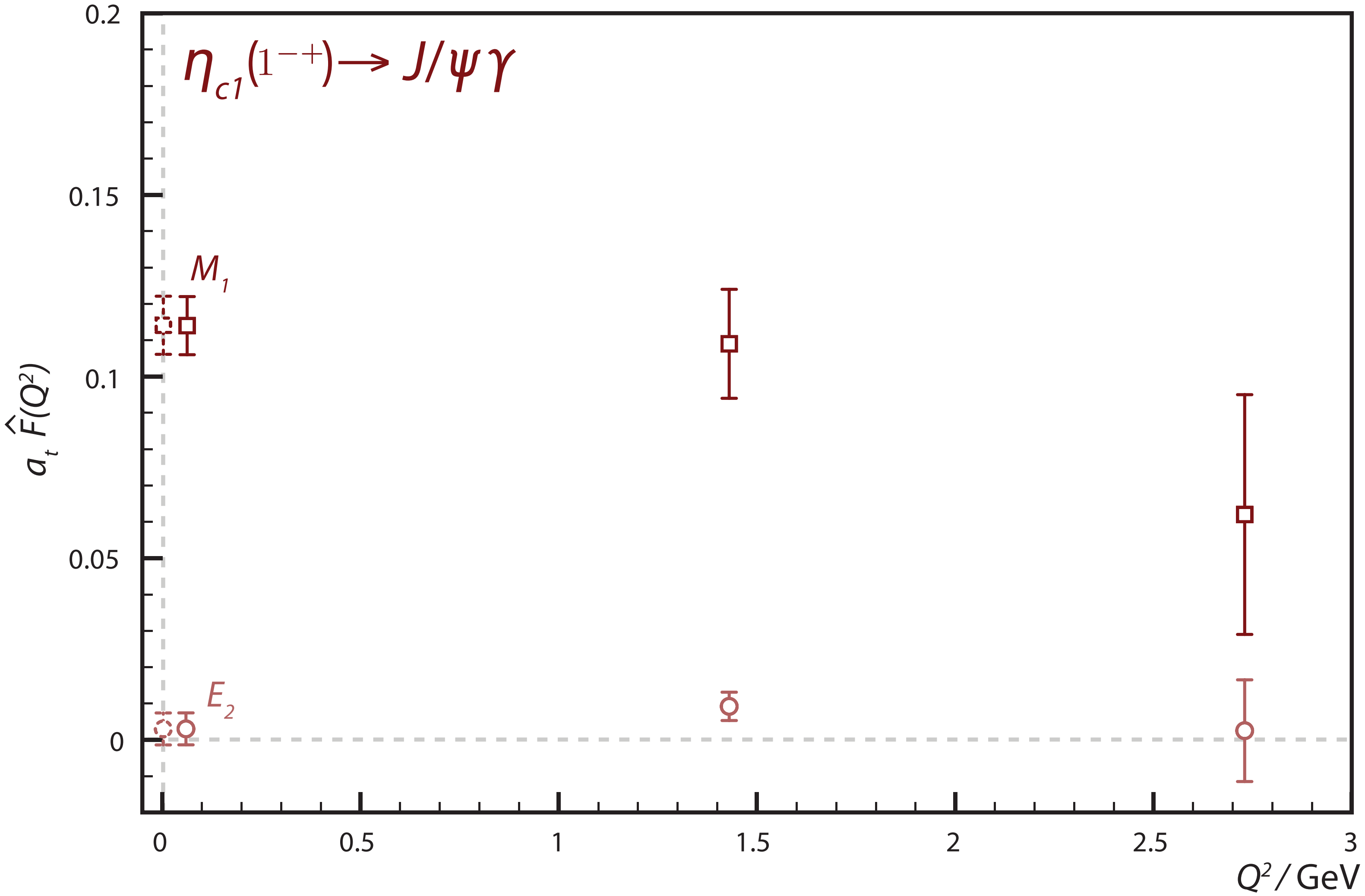
The decay modes of Fig. 7 are as follows: [1] $b_1 \to \omega\pi$, [2] $\pi_2 \to f_2\pi$, [3] $K_0 \to K\pi$, [4] $\rho \to \pi\pi$, [5] $\phi \to K\bar{K}$, [6] $\pi_2 \to \rho\pi$, [7] $\pi_2 \to K^*\bar{K} + cc$, [8] $\pi_2 \to \omega\rho$, [9] $\phi(1680) \to K^*\bar{K} + cc$, [10] $K^* \to K\pi$, [11] $K^{*\prime} \to K\pi$, [12] $K^{*\prime} \to \rho K$, [13] $K^{*\prime} \to K^*\pi$, [14] $D^{*+} \to D^0\pi^+$, [15] $\psi(3770) \to D\bar{D}$, [16] $f_2 \to \pi\pi$, [17] $f_2 \to K\bar{K}$, [18] $a_2 \to \rho\pi$, [19] $a_2 \to \eta\pi$, [20] $a_2 \to K\bar{K}$, [21] $f_2' \to K\bar{K}$, [22] $D_{s2} \to DK + D^*K + D_s\eta$, [23] $K_2 \to K\pi$, [24] $K_2 \to K^*\pi$, [25] $K_2 \to \rho K$, [26] $K_2 \to \omega K$, [27] $\rho_3 \to \pi\pi$, [28] $\rho_3 \to \omega\pi$, [29] $\rho_3 \to K\bar{K}$, [30] $K_3 \to \rho K$, [31] $K_3 \to K^*\pi$, [32] $K_3 \to K\pi$.

Hybrid Photocoupling

JLab, PRD79, 094504 (09)

$$\Gamma(H(1^{--}) \to \eta_c \gamma) = 42 \pm 18 \text{ keV}$$

$$\Gamma(H(1^{-+}) \to J/\psi \gamma) \approx 100 \text{ keV}$$



this is an M1 decay that is comparable to an E1 decay!

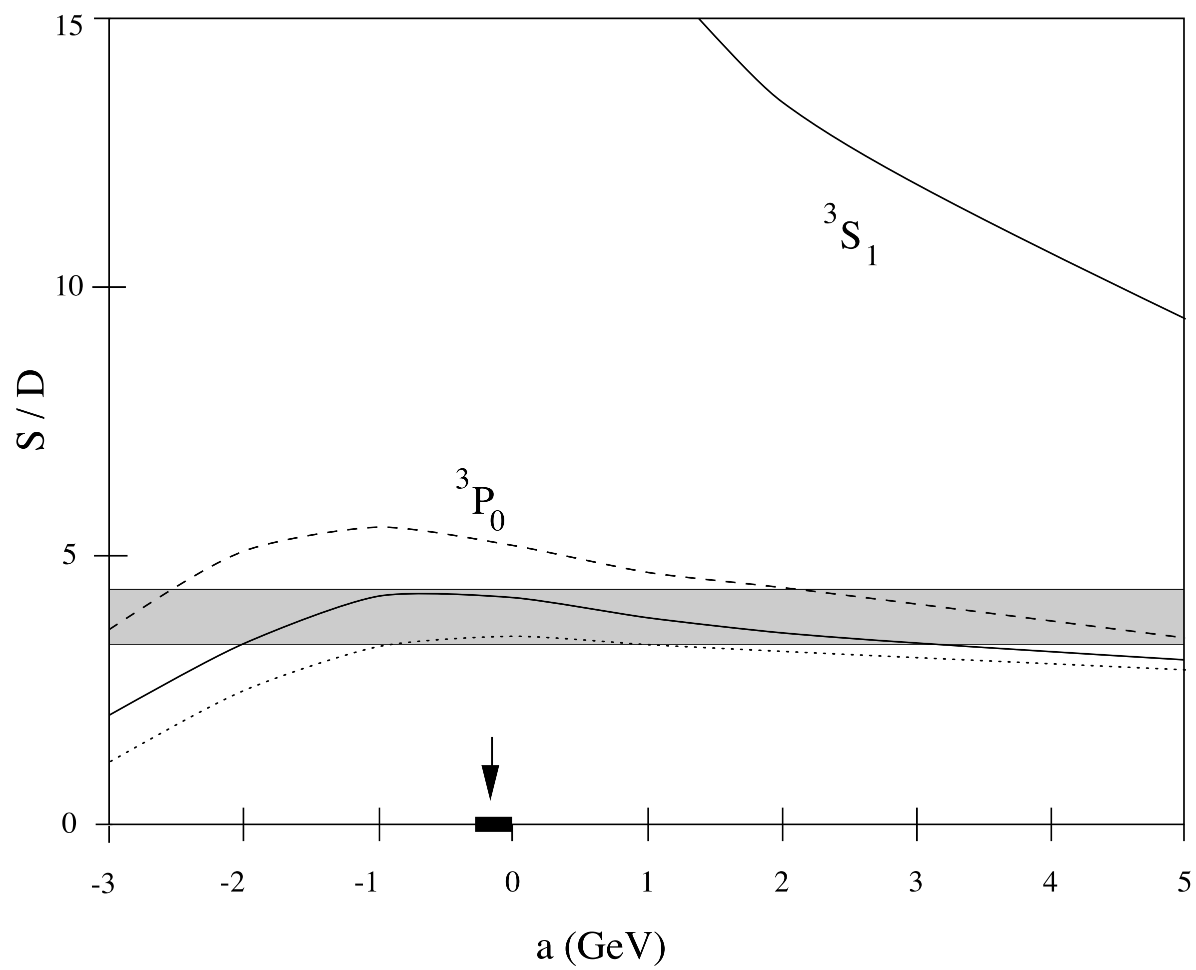
Supports the idea that 1-+ is Sqq = 1 (as in FTM)

flux tube computation finds similar results (30-60 keV for 1-+)

F. Close and J.J. Dudek, PRL91, 142001 (03)

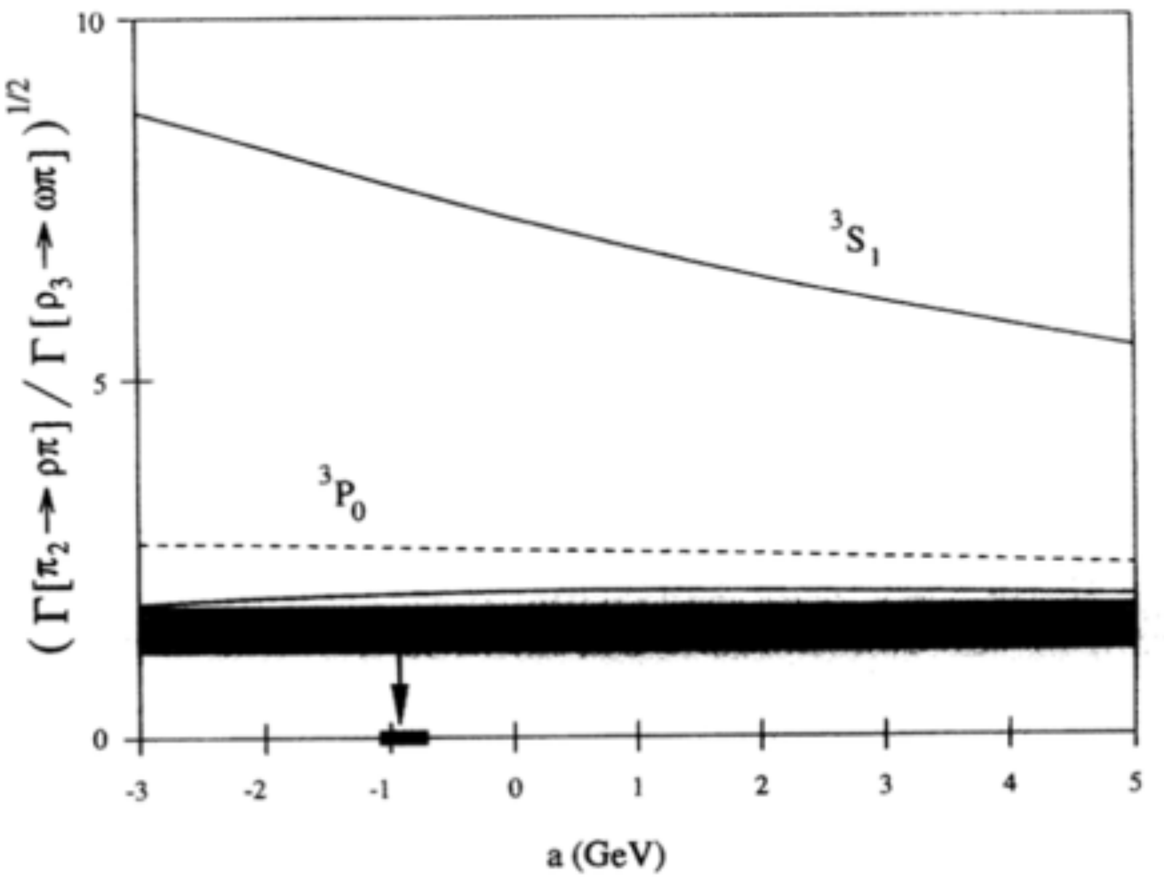
model comparison

$b_1 \rightarrow \omega\ \pi$



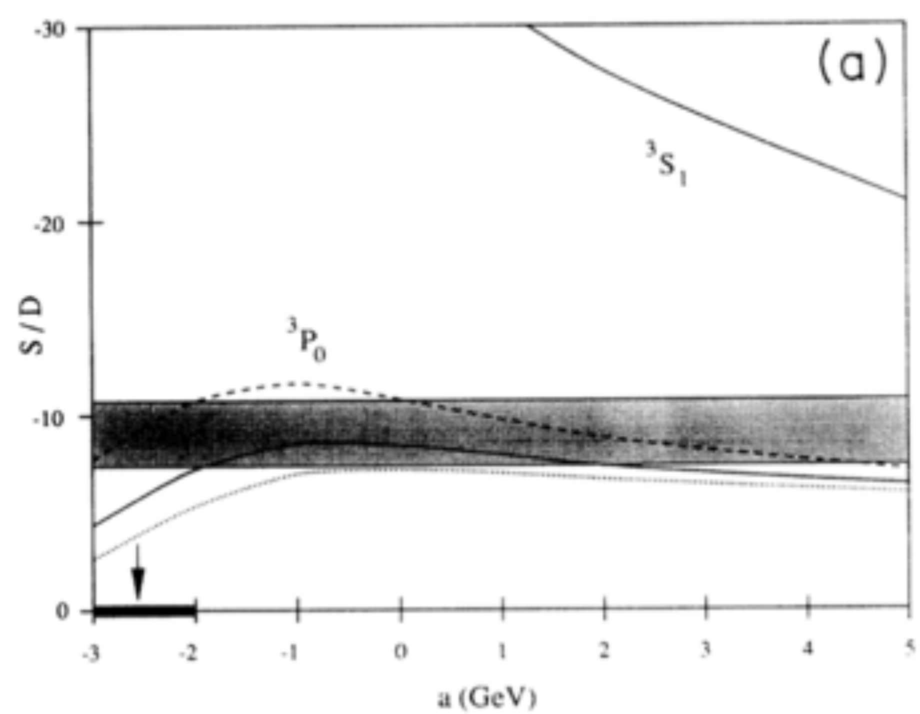
same for

1+ -> 1- 0- = 1+(S),3,2,1+ (D)



$(\Gamma[\pi_2 \to \rho\pi] / \Gamma[\rho_3 \to \omega\pi])^{1/2}$
$^3S_1$
$^3P_0$
a (GeV)

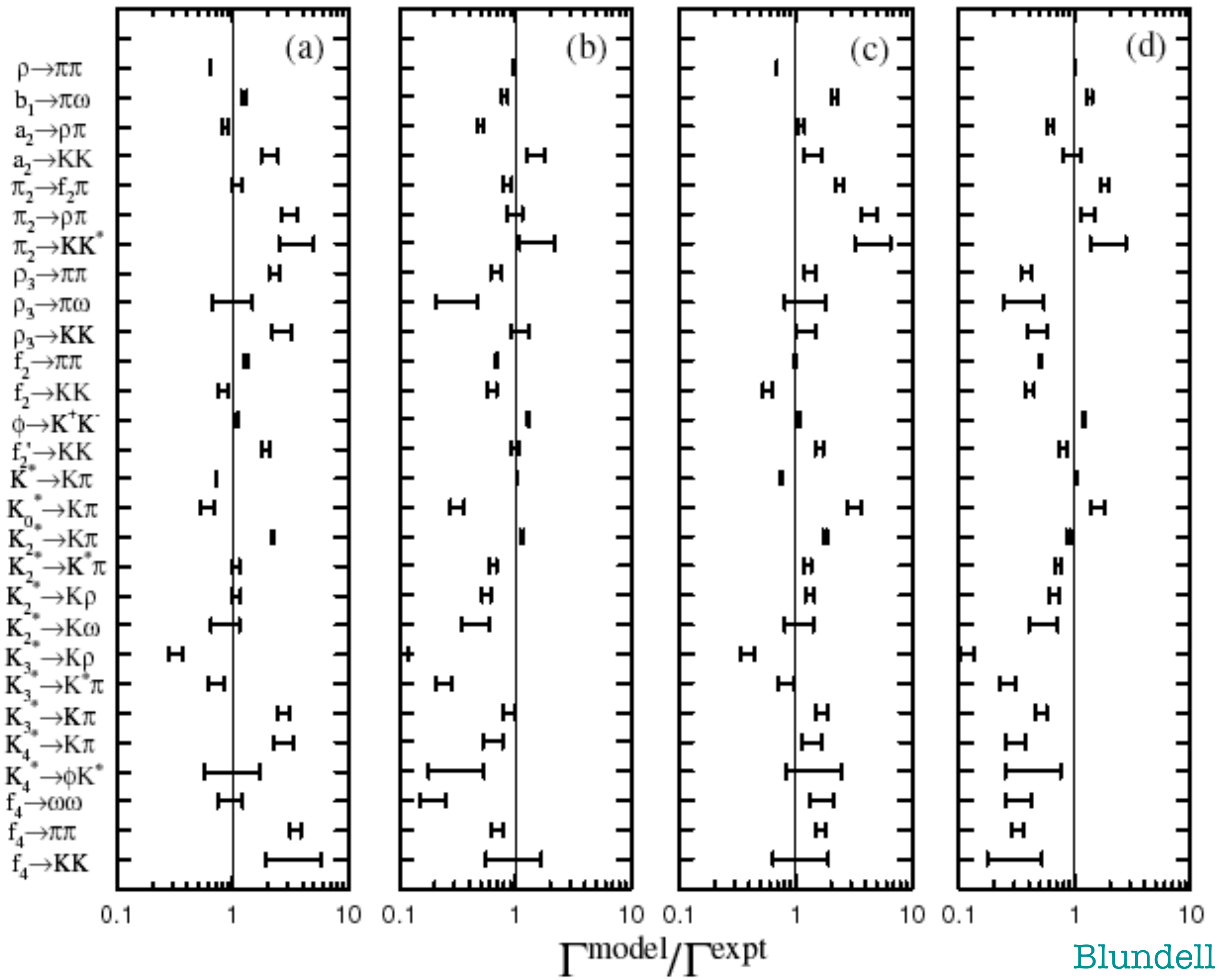
a1 : pi rho



(a)
S / D
$^3S_1$
$^3P_0$
a (GeV)

mesons

model:	$^3P_0$	$^3P_0$	Flux Tube	Flux Tube
wavefunction:	SHO	SHO	RQM	RQM
phase space:	Rel	K&I	Rel	K&I



Blundell & Godfrey,

baryons

$N\pi$ decay widths Γ[MeV] | $\Delta\pi$ decay widths Γ[MeV]

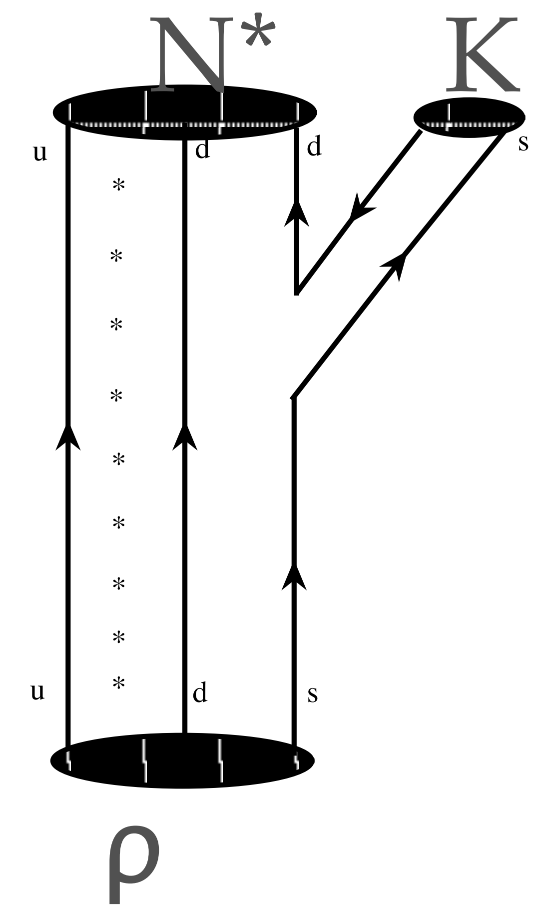
Decay	Calc	$^3P_0$	PDG	Decay	Calc	$^3P_0$	PDG
$S_{11}(1535) \to N\pi$	33	216	$(68 \pm 15)^{+45}_{-23}$	$\to \Delta\pi$	1	2	< 2
$S_{11}(1650) \to N\pi$	3	149	$(109 \pm 26)^{+29}_{-4}$	$\to \Delta\pi$	5	13	$(6 \pm 5)^{+2}_{0}$
$D_{13}(1520) \to N\pi$	38	74	$(66 \pm 6)^{+8}_{-5}$	$\to \Delta\pi$	35	35	$(24 \pm 6)^{+3}_{-2}$
$D_{13}(1700) \to N\pi$	0.1	34	$(10 \pm 5)^{+5}_{-5}$	$\to \Delta\pi$	88	778	seen
$D_{15}(1675) \to N\pi$	4	28	$(68 \pm 7)^{+14}_{-5}$	$\to \Delta\pi$	30	32	$(83 \pm 7)^{+17}_{-6}$
$P_{11}(1440) \to N\pi$	38	412	$(228 \pm 18)^{+65}_{-65}$	$\to \Delta\pi$	35	11	$(88 \pm 18)^{+25}_{-25}$
$P_{33}(1232) \to N\pi$	62	108	$(119 \pm 0)^{+5}_{-5}$				
$S_{31}(1620) \to N\pi$	4	26	$(38 \pm 7)^{+8}_{-8}$	$\to \Delta\pi$	72	18	$(68 \pm 23)^{+14}_{-14}$
$D_{33}(1700) \to N\pi$	2	24	$(45 \pm 15)^{+15}_{-15}$	$\to \Delta\pi$	52	262	$(135 \pm 45)^{+45}_{-45}$

$^3P_0$: S. Capstick, W. Roberts, Phys.Rev. D49 (1994) 4570-4586

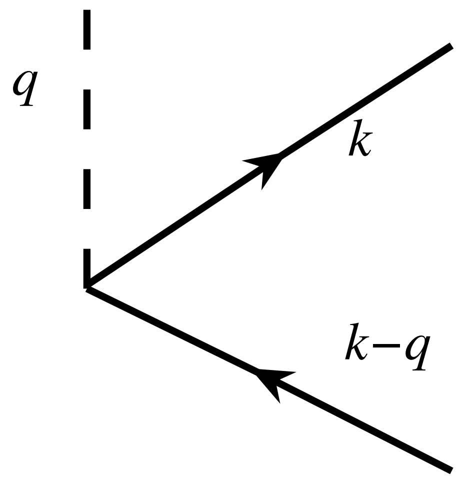
Calc: Bonn group BS computation

aside on 'missing resonances'

$N^* \rightarrow N\ K$



Coulomb vertex

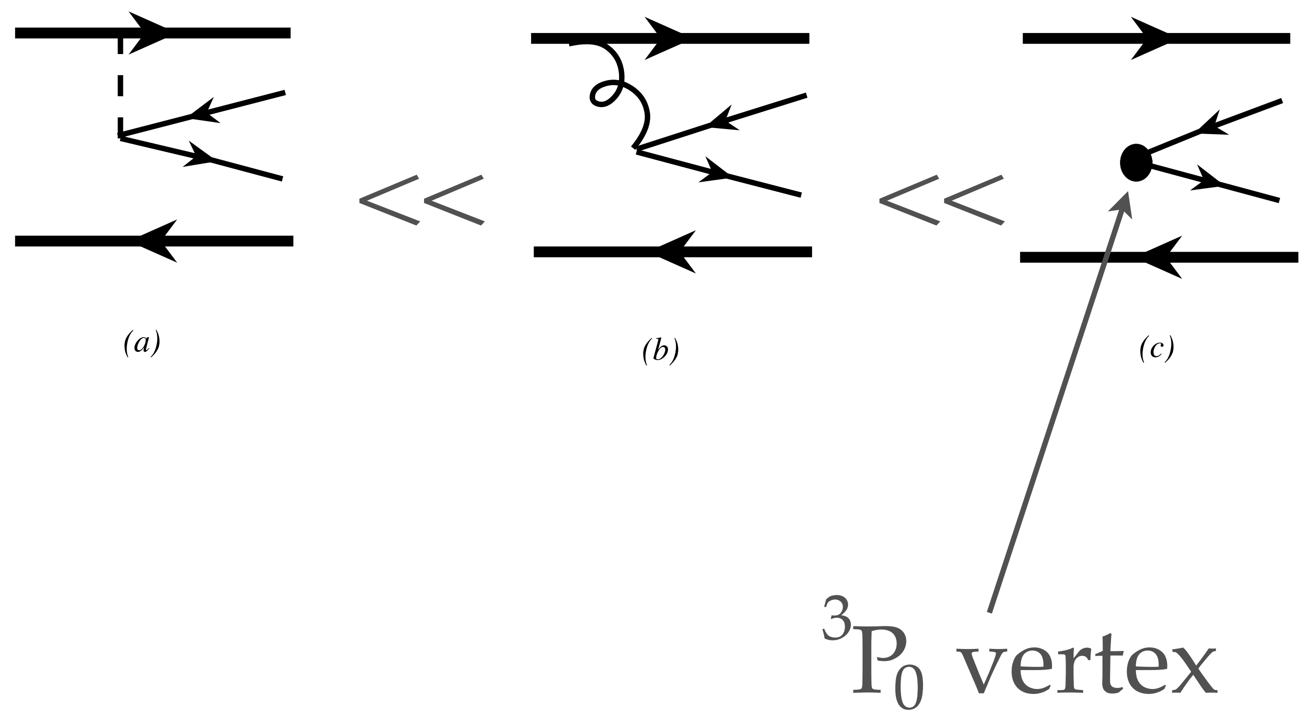


$$K^{(0)}\psi^\dagger\psi \to \frac{b}{q^4}\frac{1}{m}\boldsymbol{\sigma}\cdot\mathbf{q}\, b_k^\dagger d_{k-q}^\dagger$$

$$K^{(0)}\bar{\psi}\psi \to \frac{b}{q^4}\frac{1}{m}\boldsymbol{\sigma}\cdot(2\mathbf{k}-\mathbf{q})\, b_k^\dagger d_{k-q}^\dagger$$

confinement severely damps the integral over *q*

Decay Mechanism Hierarchy



Ackleh, Barnes, & ES, PR**D54,** 6811 (96)

Hybrid Decays

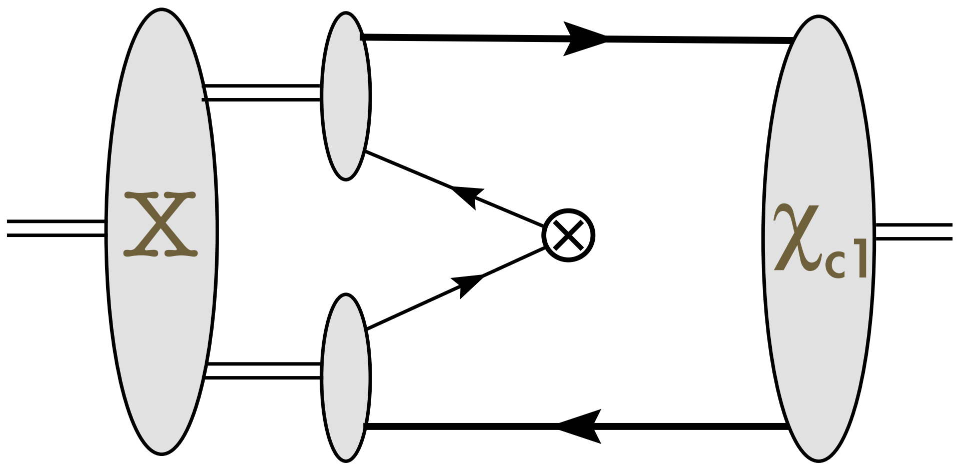
Γ	$\pi\rho$	$\omega\rho$	$\rho(1465)\pi$	$f_0(1300)\pi$	$f_2\pi$	$K^*\bar{K}$	total
$\pi_{3S}(1800)$	30	74	56	6	29	36	231
$\pi_H(1800)$	30	0	30	170	6	5	241

Glueball Decays

	$\pi\pi$	$K\bar{K}$	$\eta\eta$	$\eta'\eta$	$\eta'\eta'$	$\sigma\sigma$
$\mathcal{A}$	1	ρ	$\frac{1+\rho^2}{2}$	$\frac{1-\rho^2}{2}$	$\frac{1+\rho^2}{2}$	large
$\Gamma(\rho = 1, PS = 1)$	3	4	1	0	1	
$\Gamma(\rho = 1, M_G = 1.5\,\mathrm{GeV})$	4.3	4.4	1	–	–	–
$\Gamma(\rho = \frac{m_u}{m_s}, M_G = 1.5\,\mathrm{GeV})$	9.4	3.4	1	–	–	–
$\Gamma(f_0'; \mathrm{mixed})$	4.4	10	1	2	–	–
$\Gamma(f_0'(s\bar{s}); {}^3P_0\mathrm{model})$	–	3.0	1	1.5	–	–
$\Gamma(f_0(1500); \mathrm{expt})$	4.39(16)	1.1(4)	1	1.42(96)	–	14.9(32)

MIXING

Mixing



$$a_\chi = \sqrt{2} Z_{00}^{1/2} \int d^3k\, \psi_X(k) \mathcal{A}(-k)$$

state	E_B (MeV)	a (fm)	Z_{00}	a_χ (MeV)	prob
χ_{c1}	0.1	14.4	93%	94	5%
	0.5	6.4	83%	120	10%
χ'_{c1}	0.1	14.4	93%	60	100%
	0.5	6.4	83%	80	$>$ 100%

Coupled Channels

unquenching, cusps

"Oakes-Yang Problem"

R.J. Oakes and C.N. Yang, PRL 11, 174 (63)

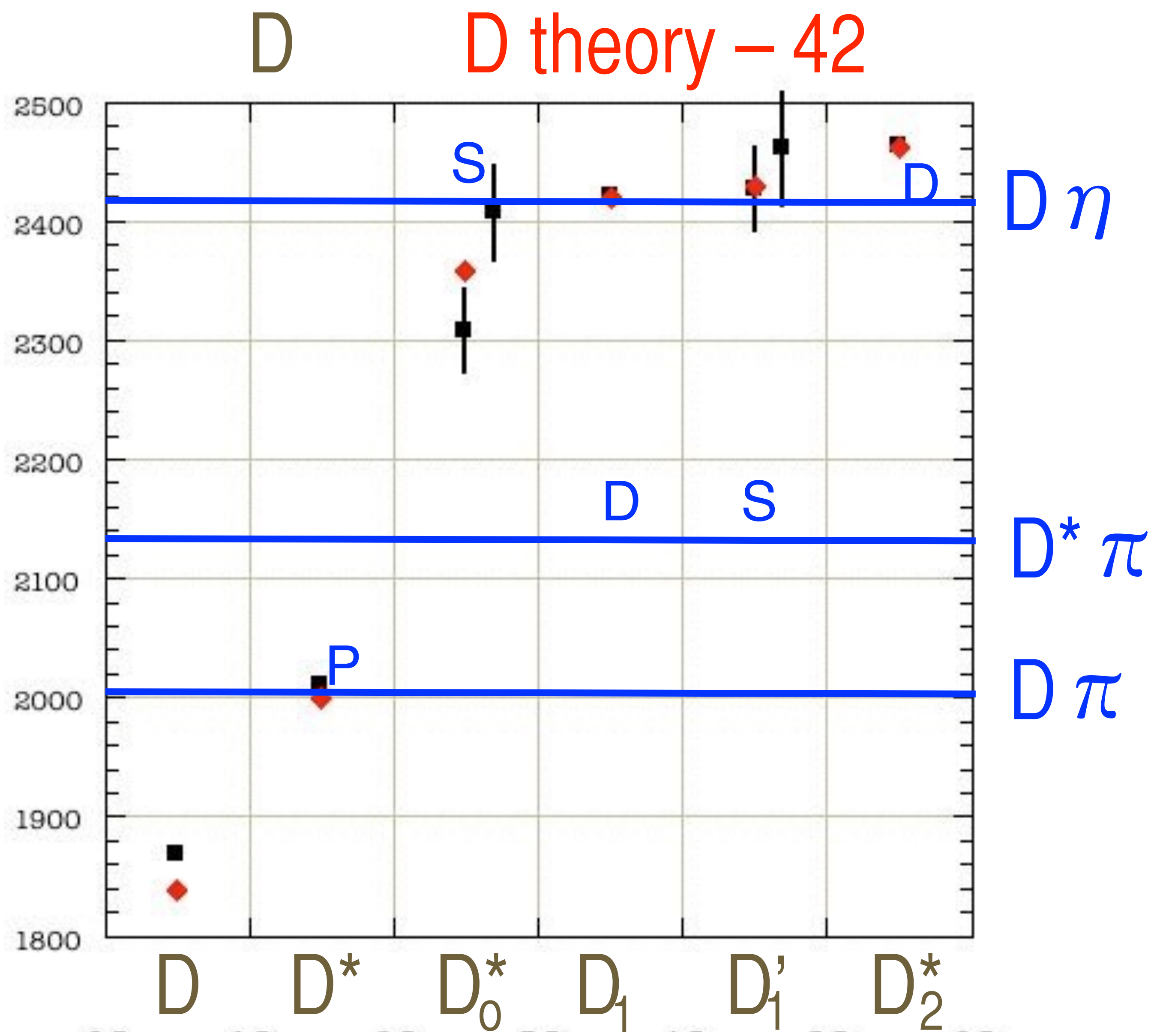
Why does the Gell-Mann--
Okubo mass formula work?
Thresholds affect the decuplet
states differently!

$$M = a_0 + a_1 S + a_2 \left[I\left(I+1\right) - \frac{1}{4} S^2 \right]$$

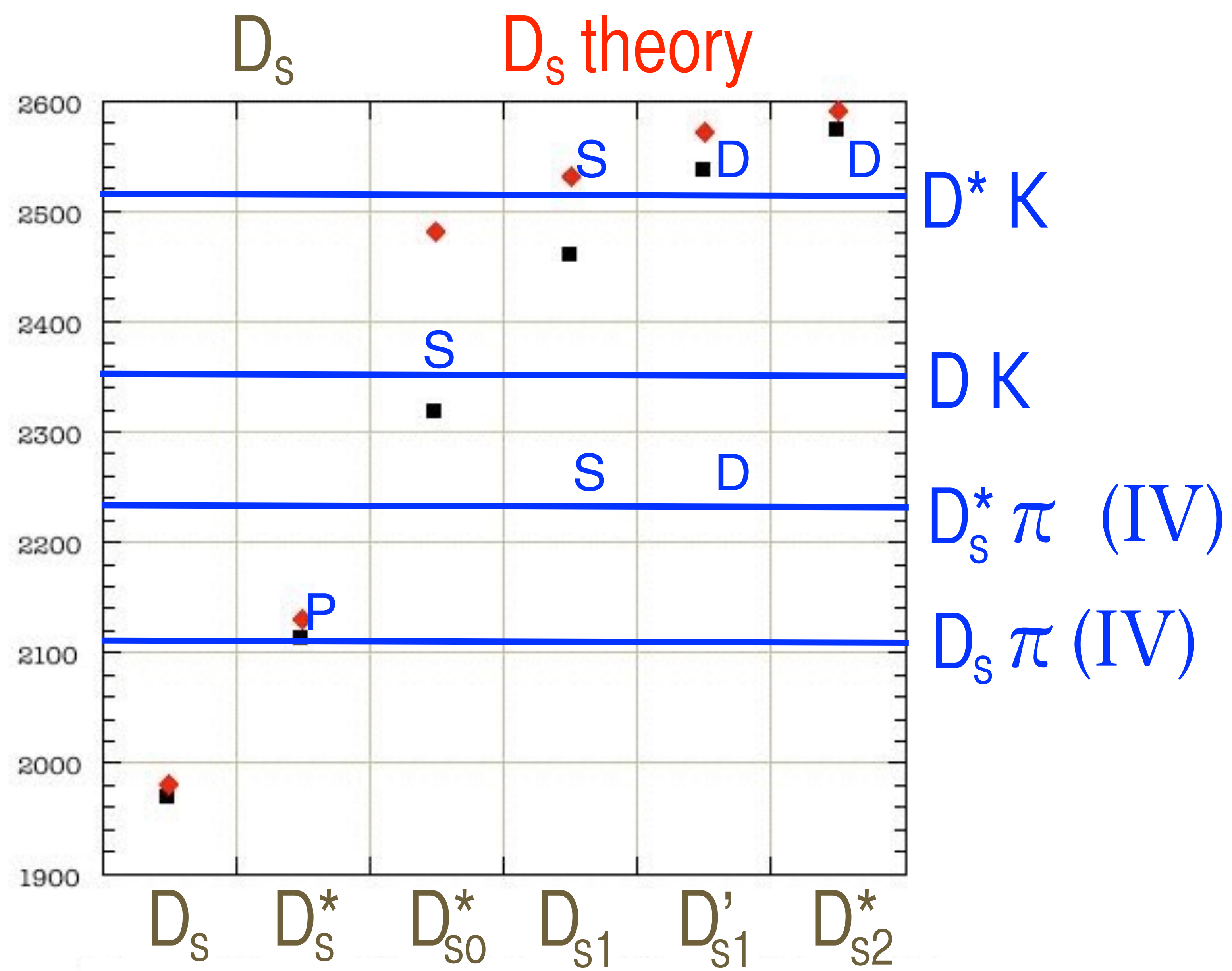
how does one include ‘loop effects’ in the quark model?

what does it mean?

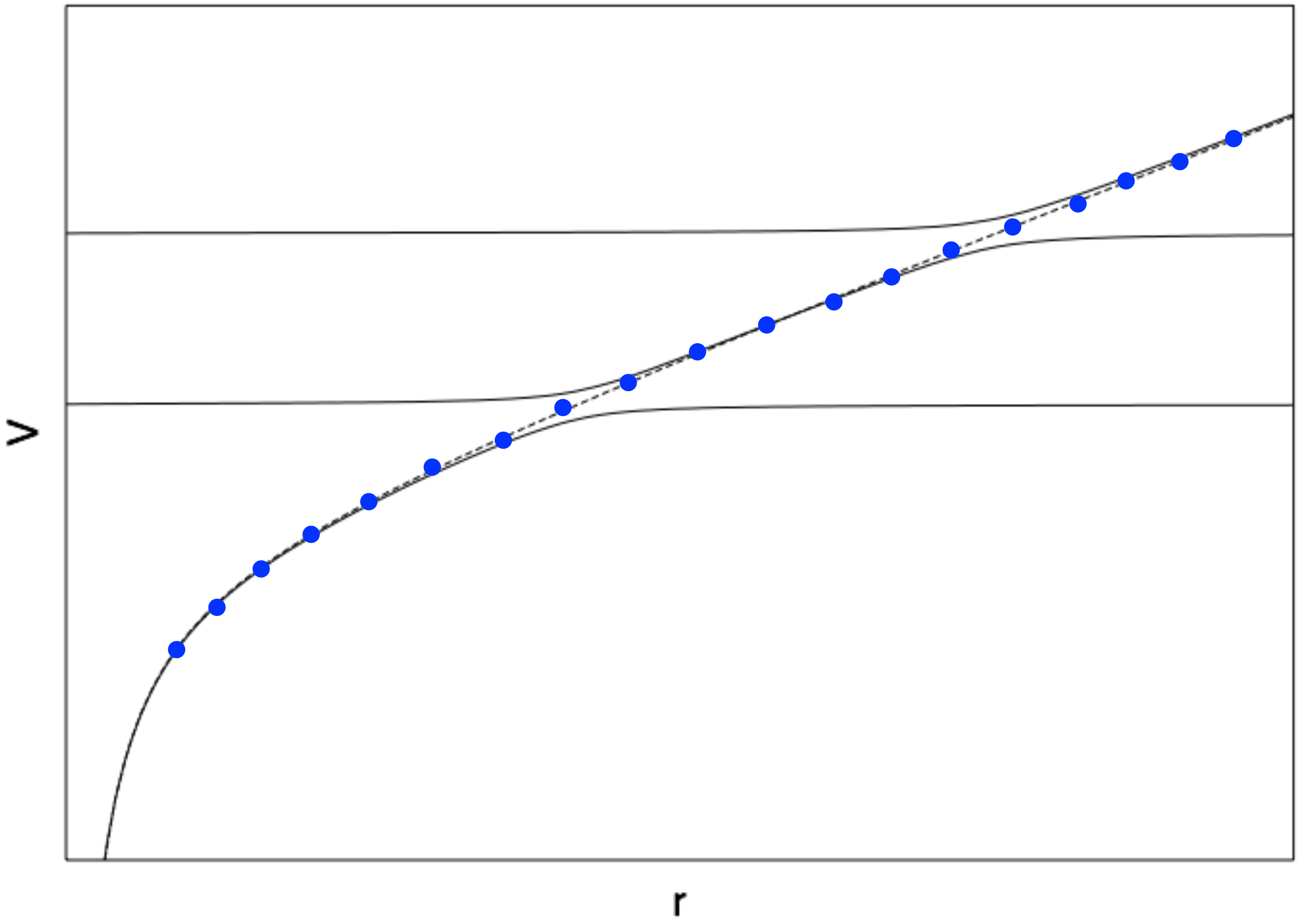
Thresholds in D



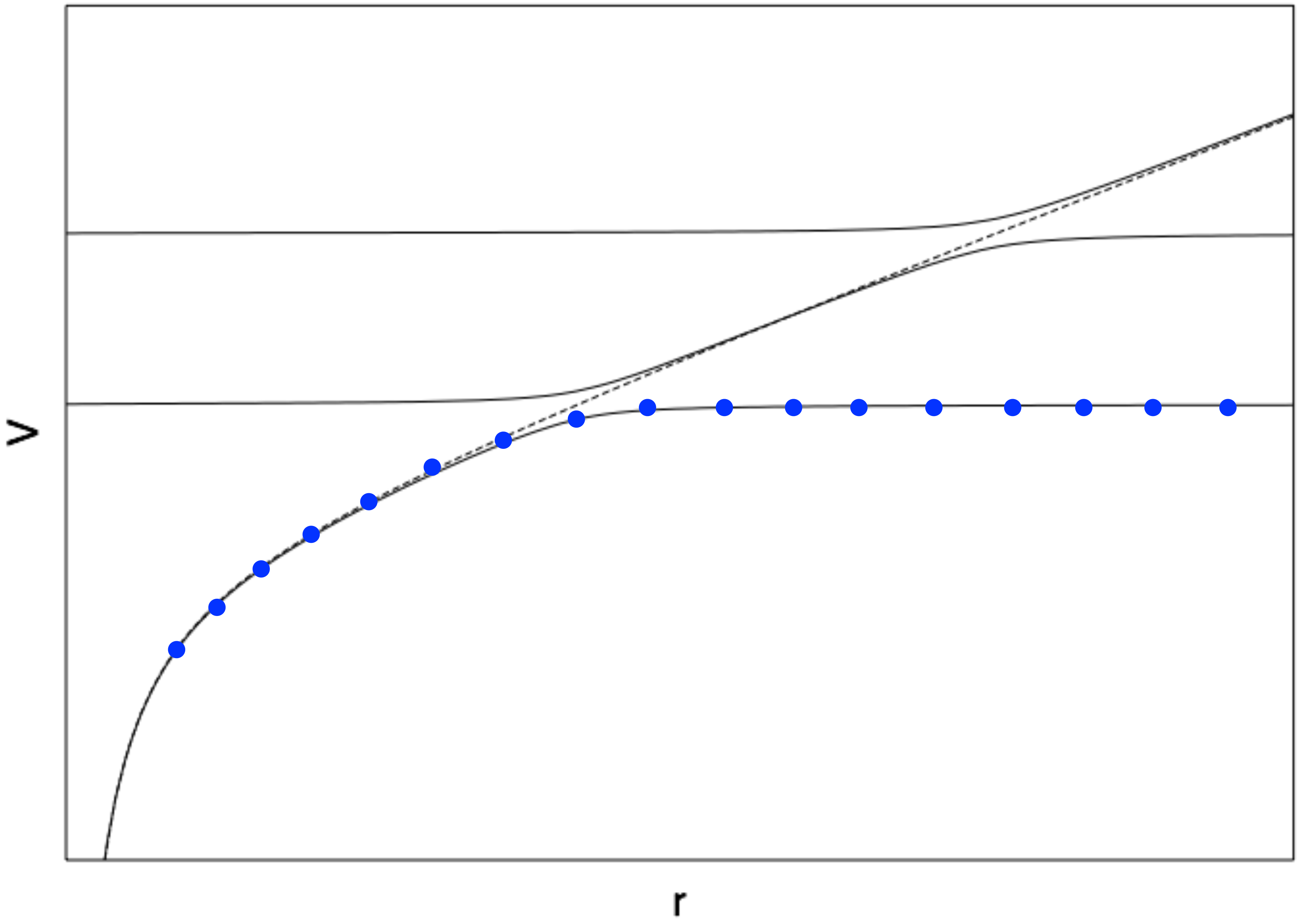
Thresholds in Ds



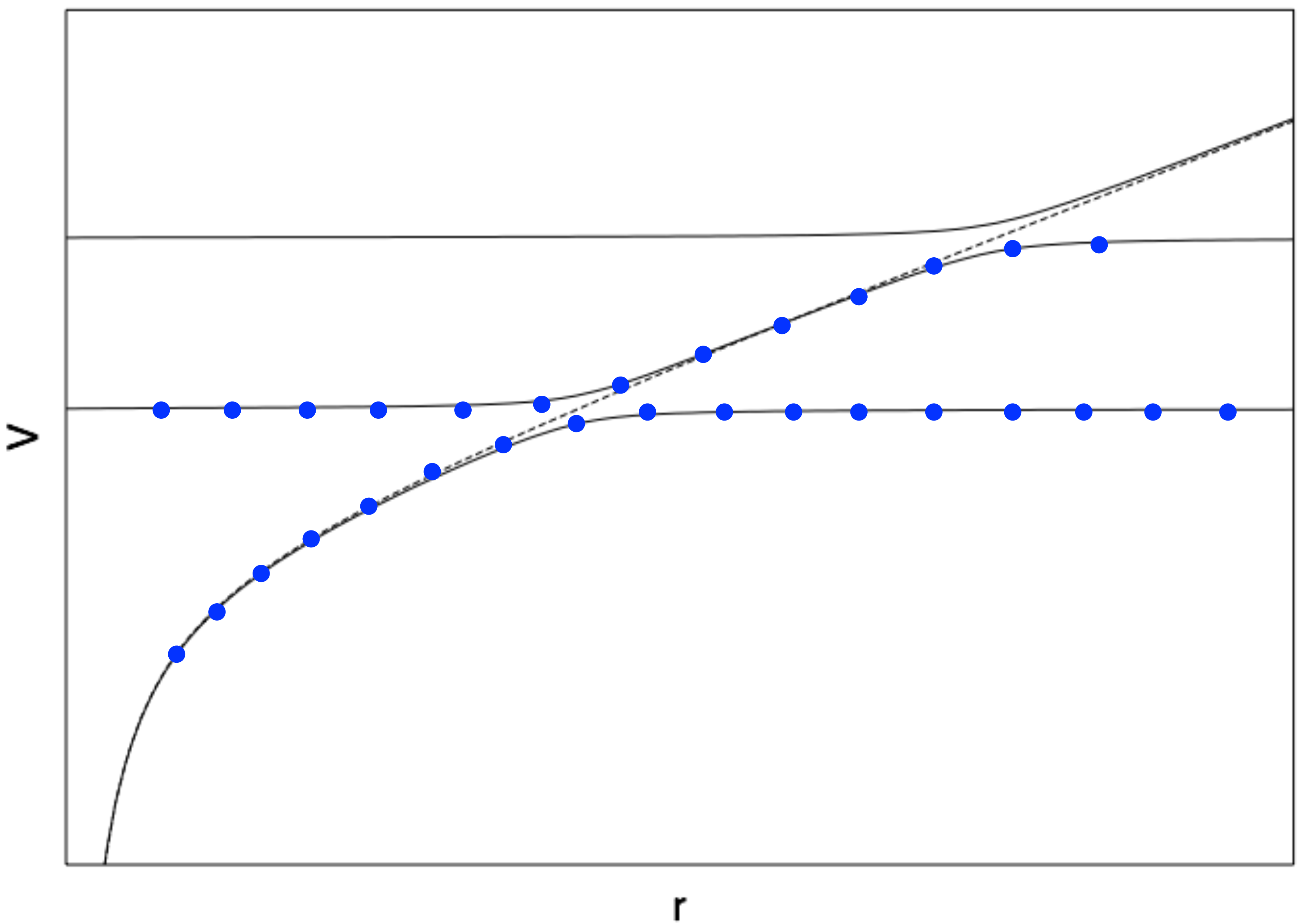
Screened Potentials



Screened Potentials



Screened Potentials

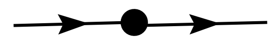


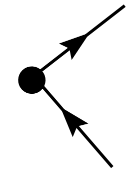
A Simple Model

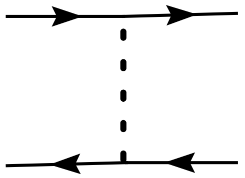
E.S. Swanson, JPG31, 845 (2005)

A Non-relativistic Quantum Field Theory

$$\hat{H} = -\int d^3x \hat{\psi}_f^\dagger \tau_3 \left(m_f - \frac{\nabla^2}{2m_f} \right) \hat{\psi}_f + g \int d^3x \hat{\psi}_f^\dagger \tau_1 \hat{\psi}_f + \frac{1}{2} \int d^3x d^3y \hat{\psi}_{f'}^\dagger(x) \hat{\psi}_{f'}(x) V(x-y) \hat{\psi}_f^\dagger(y) \hat{\psi}_f(y).$$







Non-relativistic Quantum Field Theory

$$\hat{H} = -\int d^3x \hat{\psi}_f^\dagger \tau_3 \left(m_f - \frac{\nabla^2}{2m_f} \right) \hat{\psi}_f + g \int d^3x \hat{\psi}_f^\dagger \tau_1 \hat{\psi}_f + \frac{1}{2} \int d^3x d^3y \hat{\psi}_{f'}^\dagger(x) \hat{\psi}_{f'}(x) V(x-y) \hat{\psi}_f^\dagger(y) \hat{\psi}_f(y).$$

$$|\Psi\rangle = \phi_{QQ} |Q\bar{Q}\rangle + \psi |Q\bar{q}q\bar{Q}\rangle$$

$$H_0 \phi_{QQ}(r) + \Omega(r) \psi(\frac{M}{m+M} r) = E \phi_{QQ}(r)$$

$$H_1 \psi(\rho) + (\frac{M}{m+M})^{-3} \Omega(\frac{m+M}{M} \rho) \phi_{QQ}(\frac{m+M}{M} \rho) = E \psi(\rho)$$

Non-relativistic Quantum Field Theory

$$H_0 \phi_{QQ}(r) + \Omega(r)\psi(\frac{M}{m+M} r) = E\phi_{QQ}(r)$$

$$H_1 \psi(\rho) + (\frac{M}{m+M})^{-3}\Omega(\frac{m+M}{M}\rho)\phi_{QQ}(\frac{m+M}{M}\rho) = E\psi(\rho)$$

$$\Omega(r) = g\int d^3x \, \phi_{Qq}(r/2 - x)\phi_{Qq}(r/2 + x)$$

an ‘unquenched’ quark model

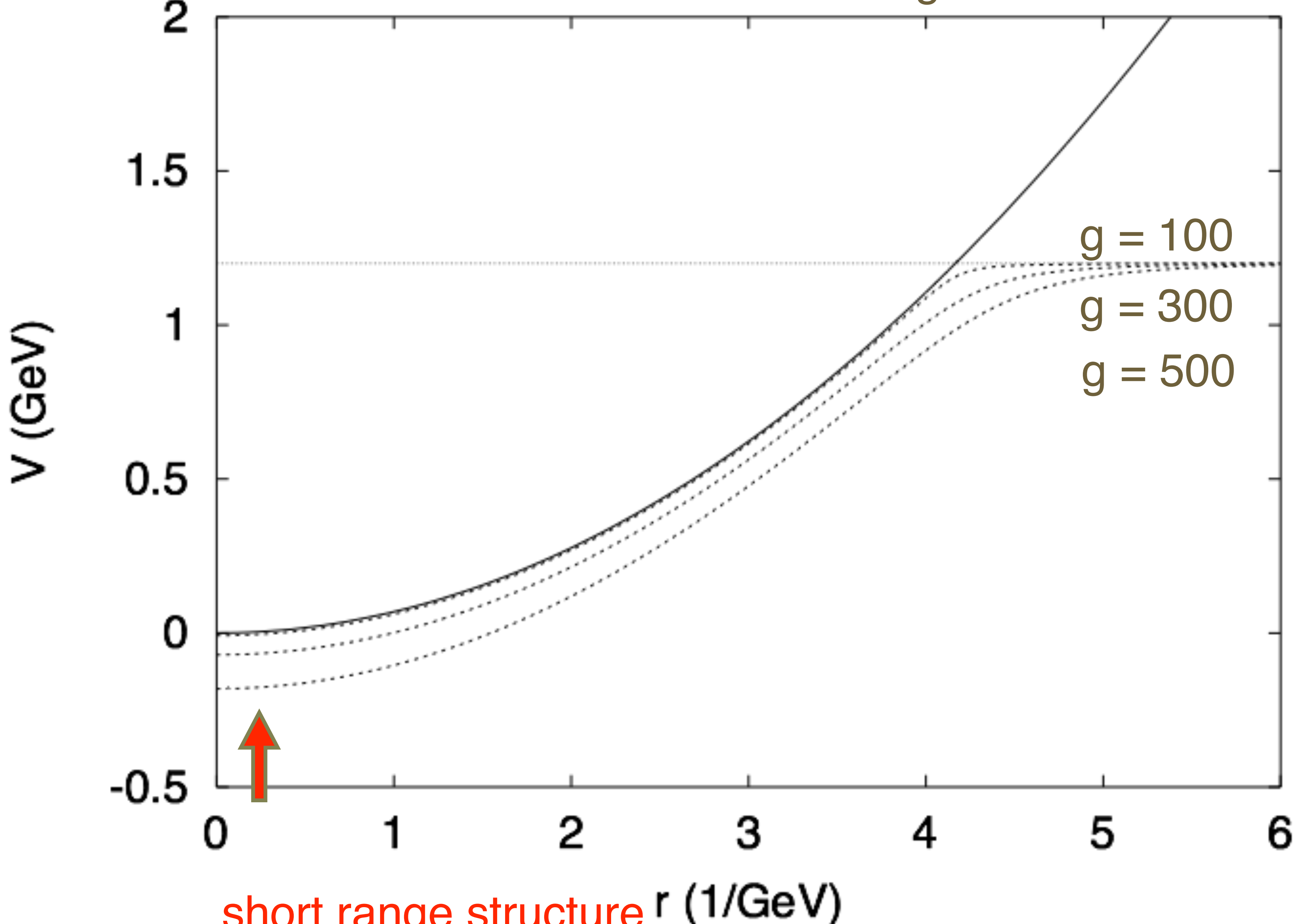
$$\hat{H} = \int dx \, (- \frac{\nabla^2}{2m_q} b_x^\dagger b_x - \frac{\nabla^2}{2m_{\bar{q}}} d_x^\dagger d_x) + \gamma \int dx \, (b_x^\dagger \sigma \cdot \overleftrightarrow{\nabla} d_x^\dagger + \text{H.c.})$$
$$+ \frac{1}{2} \int dx \, dy \, (b_x^\dagger b_y^\dagger + d_x^\dagger d_y^\dagger) V(x-y)(b_y b_x + d_y d_x).$$

$$|\Psi\rangle = \int \varphi_A(r_1 - r_2) b_1^\dagger d_2^\dagger |0\rangle$$
$$+ \int \sum_{BC} \Psi_{BC}(\frac{r_2 + r_4 - r_1 - r_3}{2}) \varphi_B(r_1 - r_3) \varphi_C(r_2 - r_4) b_1^\dagger d_3^\dagger b_2^\dagger d_4^\dagger |0\rangle$$

$$E\varphi_A(r) = H_{q\bar{q}}(r)\varphi_A(r)$$
$$-\gamma \int \vec{\Sigma} \cdot (\nabla_B + \nabla_C + \nabla_{BC}) \varphi_{0B}(r/2 - x) \varphi_{0C}(r/2 + x) \Psi_{BC}(-r/2), \quad (1)$$
$$\frac{-1}{2\mu_{13,24}} \nabla_R^2 + \int\int K_E(x,y,R)\Psi_{BC}(R') + \int\int V_E(x,y,R)\Psi_{BC}(R')$$
$$- 8\gamma \int \vec{\Sigma} \cdot (\nabla_B + \nabla_C + \nabla_{BC}) \varphi_{0B}\varphi_{0C}\varphi_A(-2R)$$
$$= E\Psi_{BC}(R) + E\int N_E(x,y,R)\Psi_{BC}(R')$$

Adiabatic Potentials

diagonalise in the heavy quark limit



short range structure

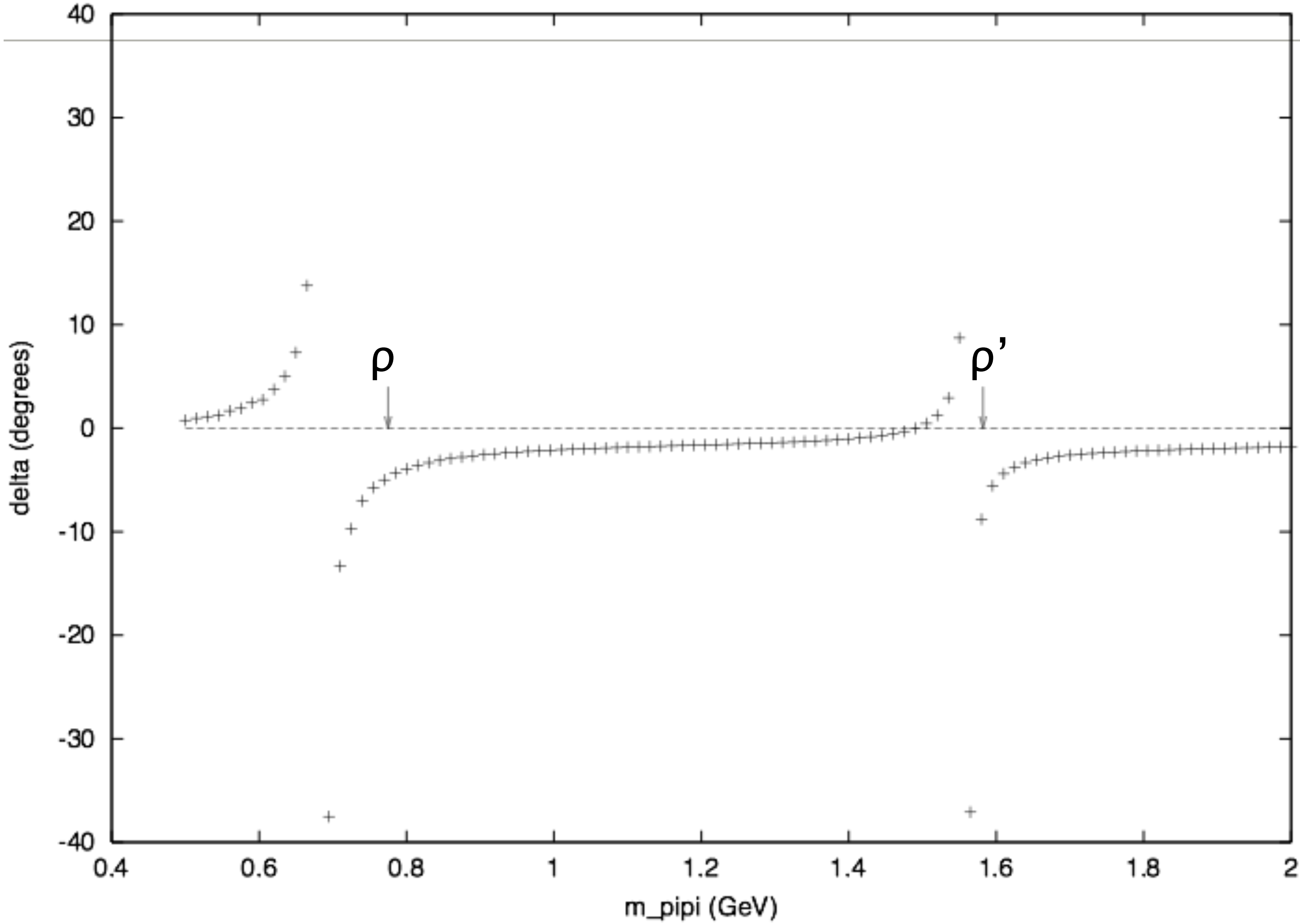
Coupled Channel Bethe-Heitler Equation

$$T(k,k') = V_{eff}(k,k') + \int d^3q\, V_{eff}(k,q)G_E(q)T(q,k')$$

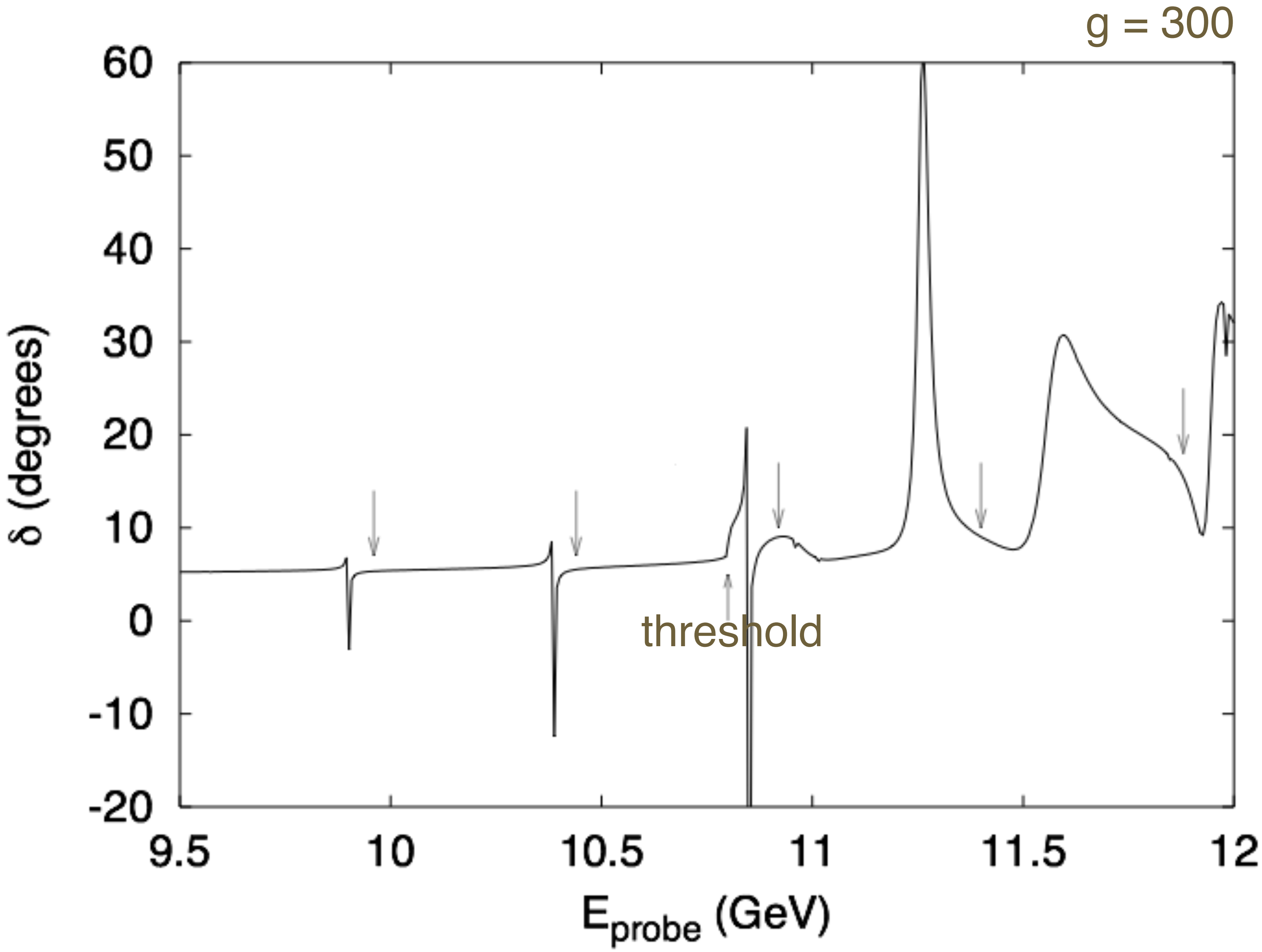
$$\langle k|V_{eff}|k'\rangle = 2\pi^2 \sum_i \frac{\omega_i^*(k)\omega_i(k')}{E - E_i}$$

$$\omega_i(k) = \langle \phi_{QQ}^{(i)}|\hat{\Omega}|k\rangle$$

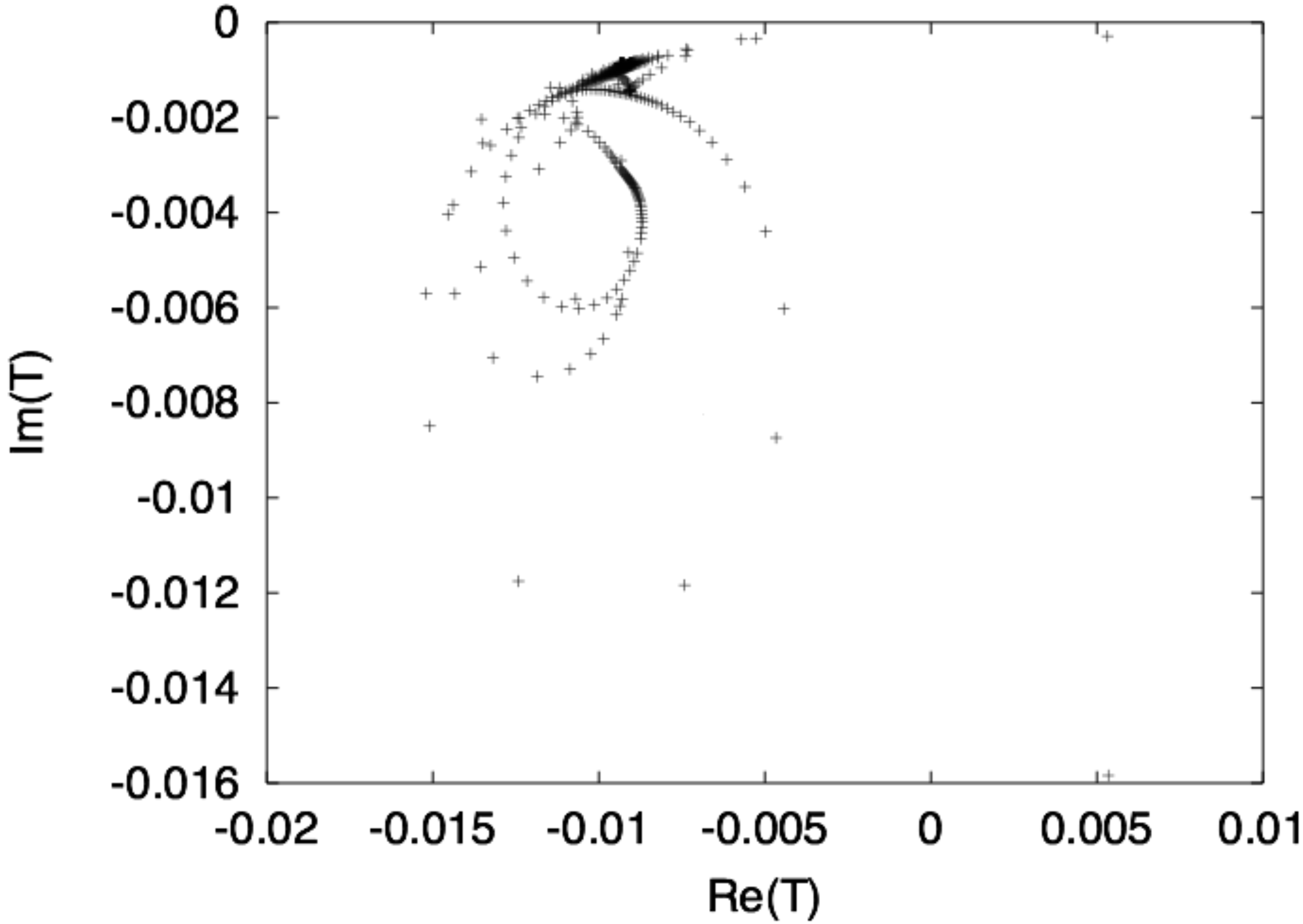
$\pi\pi$ I=1 L=1 scattering



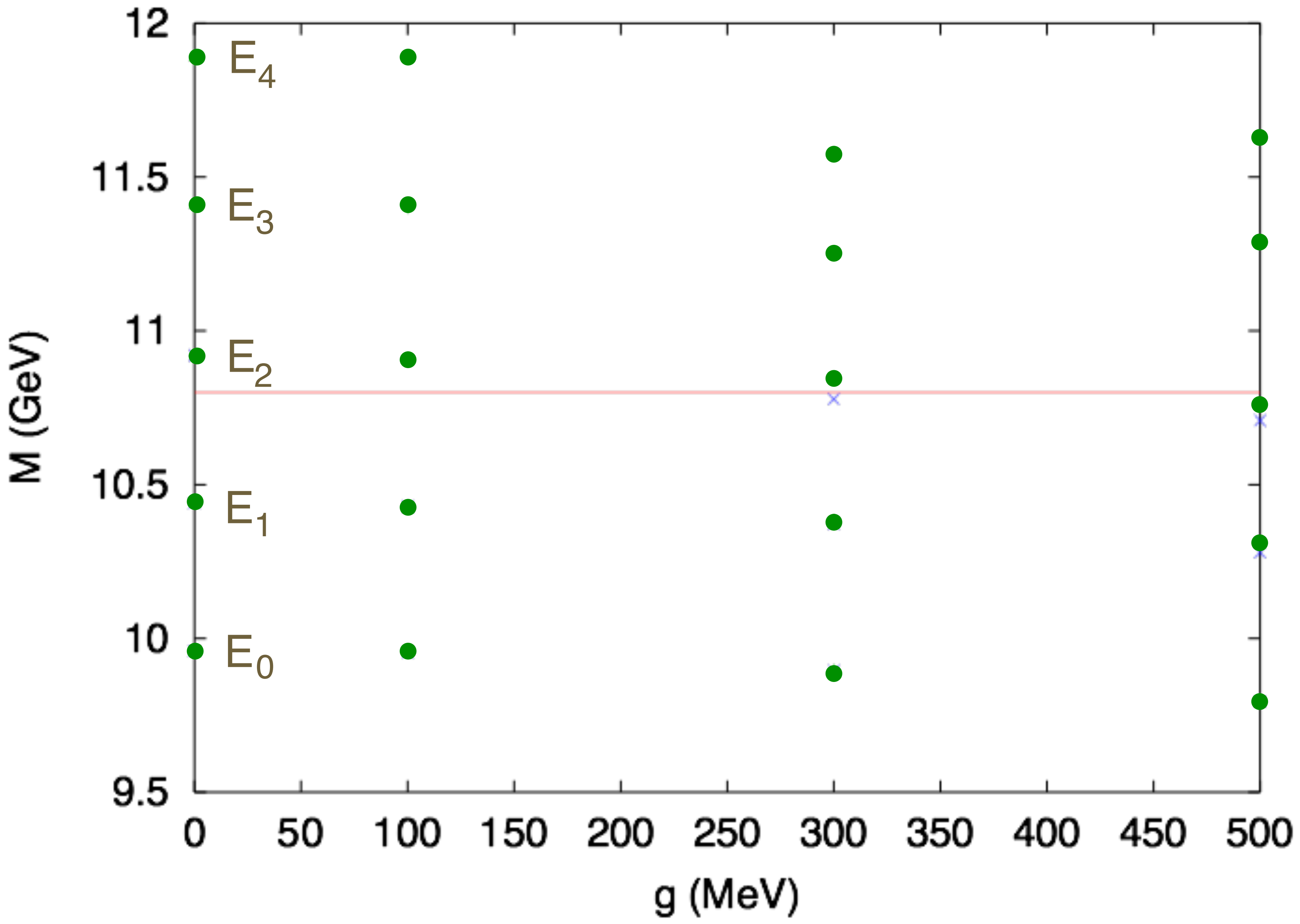
Couple to a Probe Channel



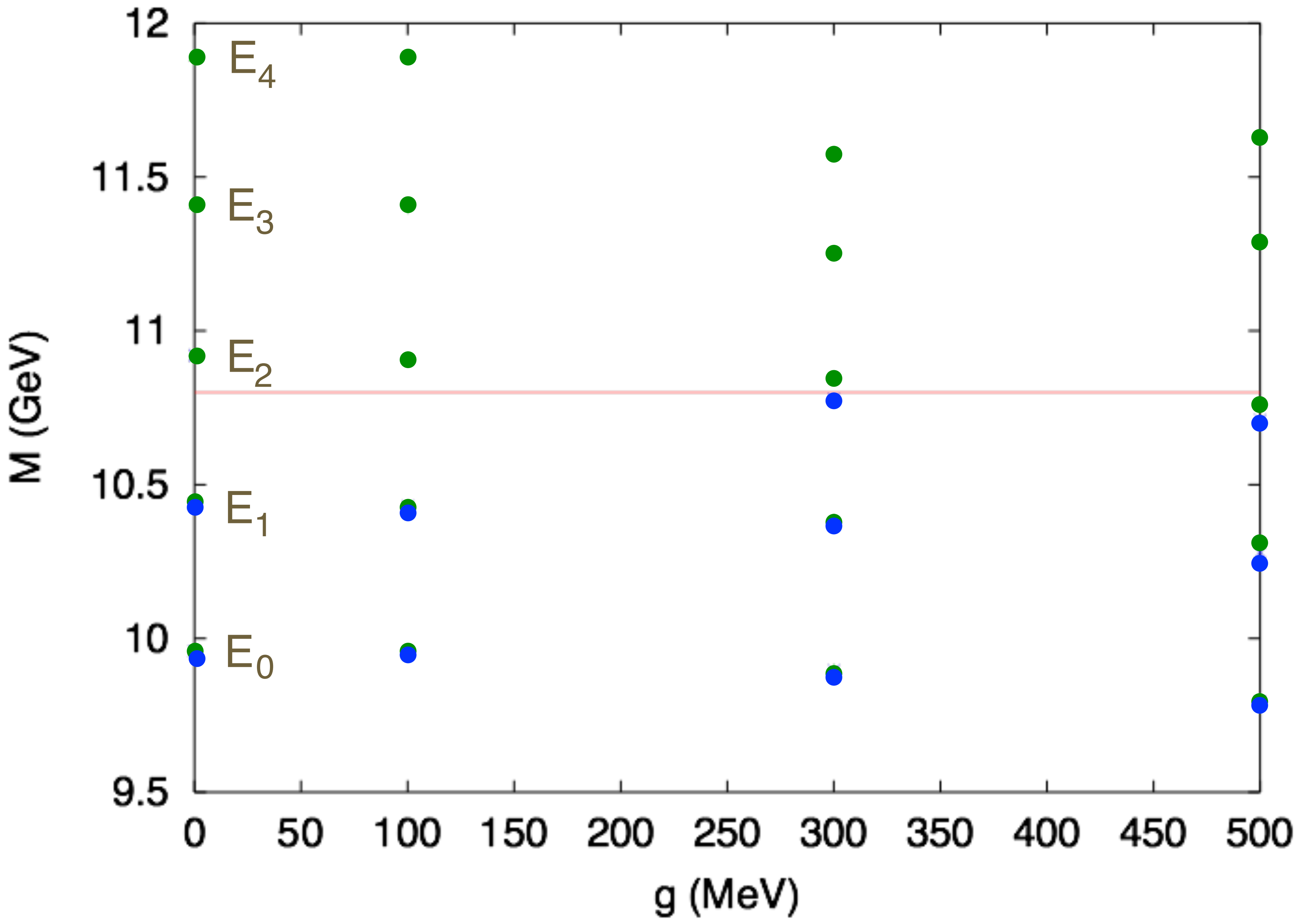
Argand Diagram



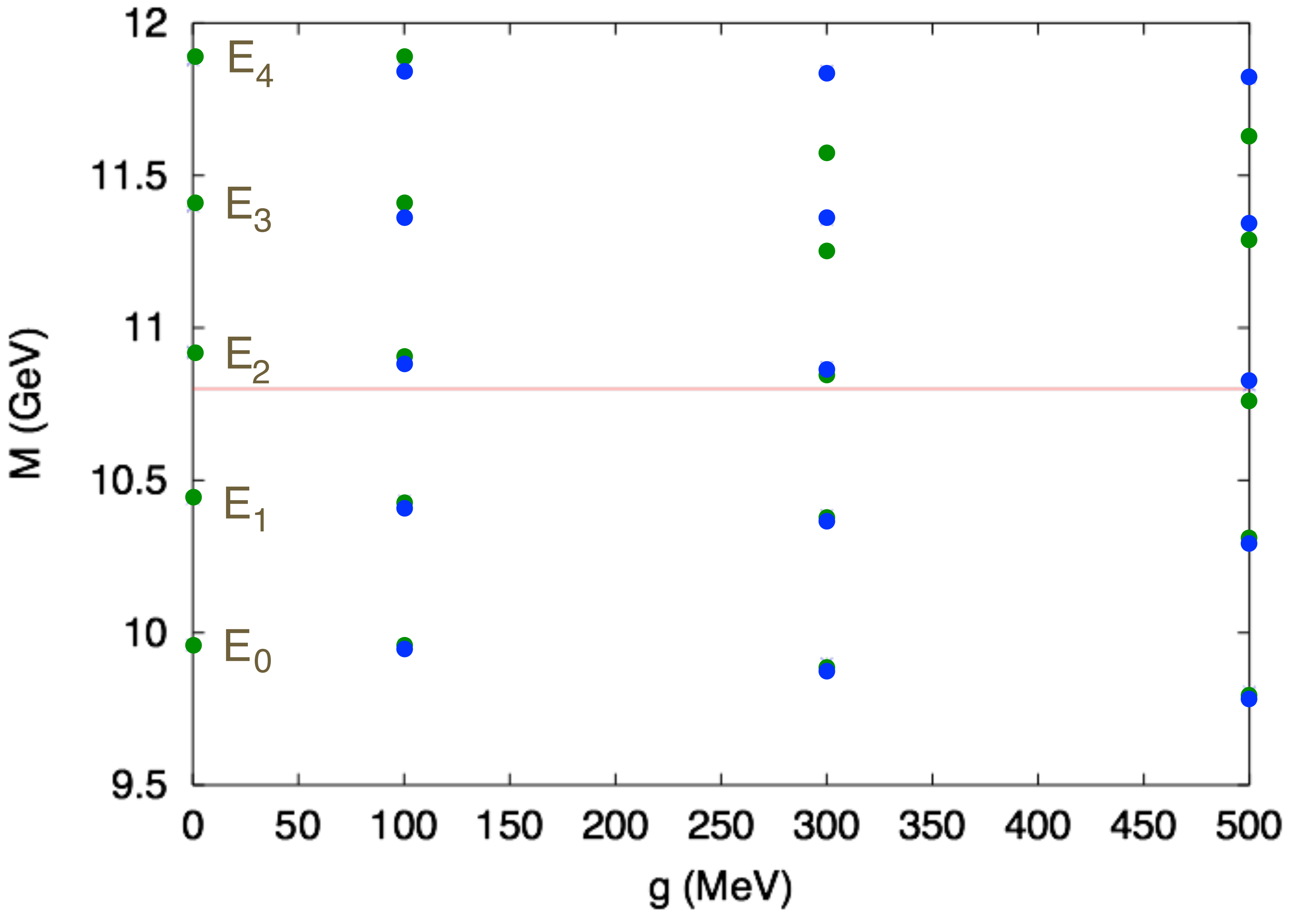
Full Spectrum



Screened Spectrum



Renormalised Spectrum



screened potential is not sensible
using a potential fit to the data (renormalised) is

Some Loop Theorems

T. Barnes and E.S. Swanson, PRC77, 055206, (2008)

$$-iG(s) = \frac{1}{(s - M^2 - \Sigma(s))}$$

$$\sqrt{s}\Gamma(s) = -\mathrm{Im}(\Sigma(s))$$

$$2\sqrt{s}\delta M(s) = \mathrm{Re}(\Sigma(s))$$

for a general class of decay models mixing via degenerate multiplets of states...

- Loop mass shifts are identical for all states in an N,L multiplet
- these states have the same open flavour decay widths
- loop-induced valence configuration mixing vanishes if $L_i <> L_f$ or $S_i <> S_f$

$$\langle J_{\rm A}[Lj_{\rm BC}];j_{\rm BC}[j_{\rm B}j_{\rm C}];j_{\rm B}[s_{\rm B}\ell\ _B]\,j_{\rm C}[s_{\rm C}\ell_{\rm C}]|\sigma\psi|J_{\rm A}[s_{\rm A}\ell_{\rm A}]\rangle =$$

$$\sum_{s_{\rm BC}\ell_{\rm BC}L_f} (-)^\eta \hat{1}\hat{L}_f \hat{s}_{\rm BC}\hat{\ell}_{\rm BC}\hat{j}_{\rm B}\hat{j}_{\rm C}\hat{j}_{\rm BC}\hat{s}_{\rm A}\hat{s}_{\rm B}\hat{s}_{\rm C}\hat{s}_{\rm BC} \cdot mber$$

$$\langle L_f[\text{Ł}\ell_{\rm BC}];\ell_{\rm BC}[\ell_{\rm B}\ell_{\rm C}]||\underline{\rm m}\psi||\ell_{\rm A}\rangle \cdot$$

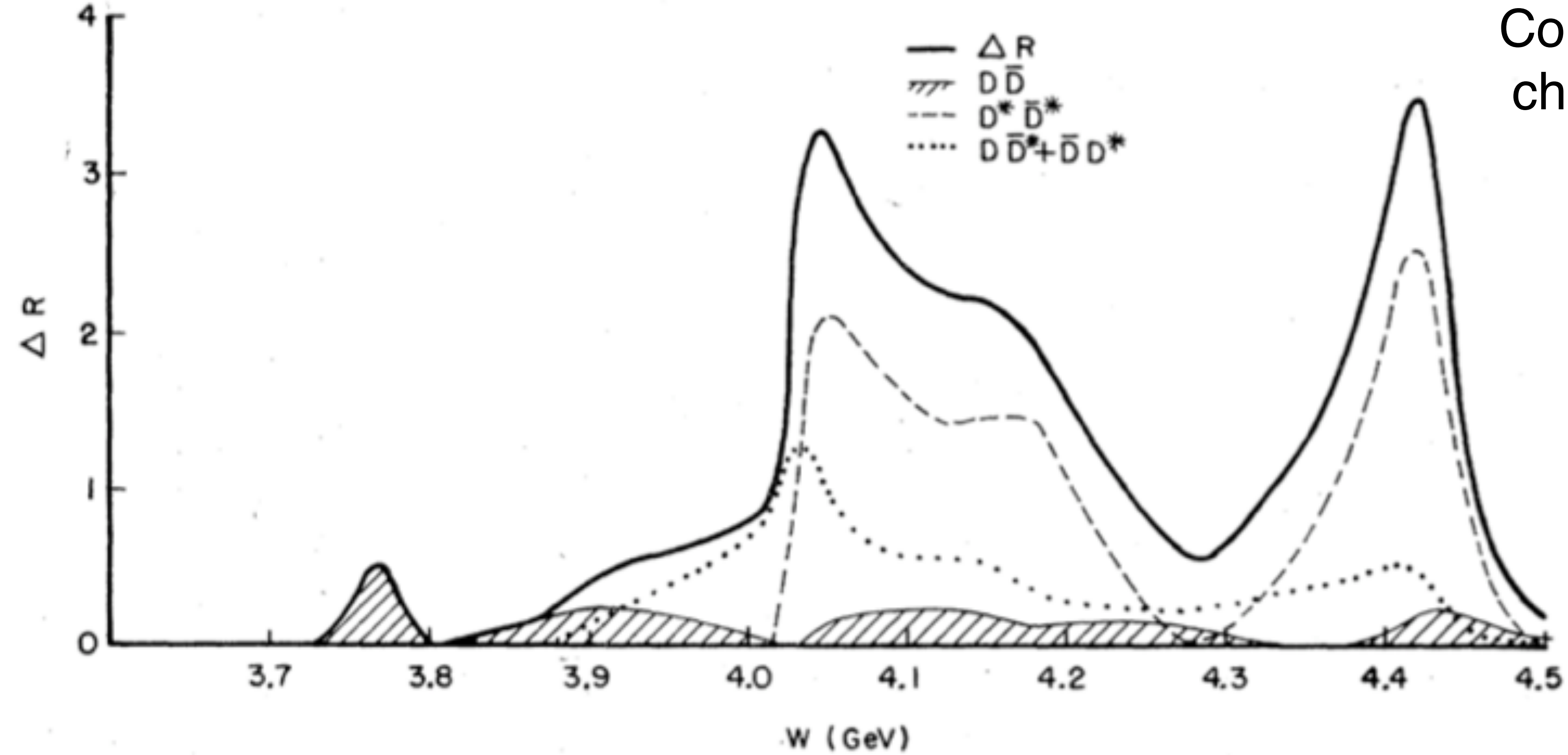
$$\left\{ \begin{array}{ccc} s_{\rm B} & \ell_{\rm B} & j_{\rm B} \\ s_{\rm C} & \ell_{\rm C} & j_{\rm C} \\ s_{\rm BC} & \ell_{\rm BC} & j_{\rm BC} \end{array} \right\} \left\{ \begin{array}{ccc} 1/2 & 1/2 & s_{\rm B} \\ 1/2 & 1/2 & s_{\rm C} \\ s_A & 1 & s_{\rm BC} \end{array} \right\} \cdot$$

$$\left\{ \begin{array}{ccc} s_{\rm BC} & \ell_{\rm BC} & j_{\rm BC} \\ \text{Ł} & j_{\rm A} & L_f \end{array} \right\} \left\{ \begin{array}{ccc} s_{\rm BC} & s_{\rm A} & 1 \\ \ell_{\rm A} & L_f & j_{\rm A} \end{array} \right\}$$

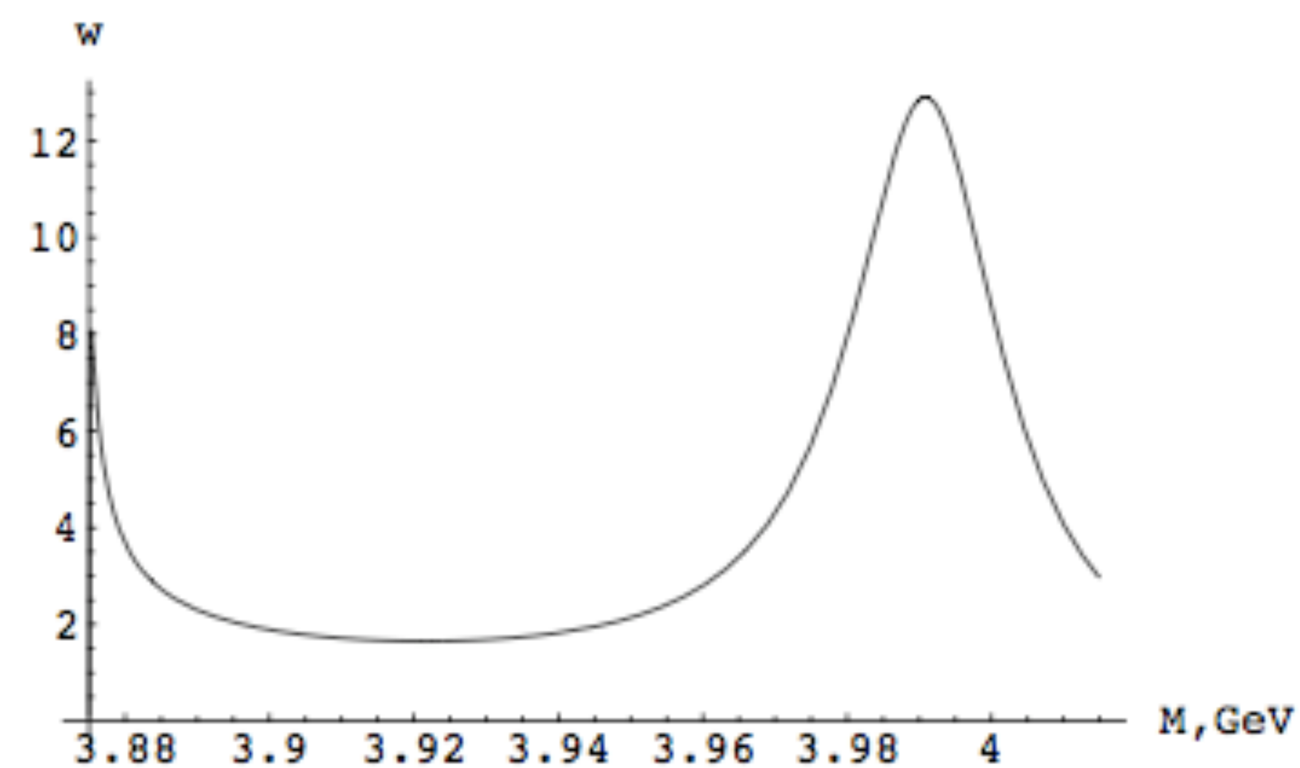
- deviations from the symmetry limit will either be driven by δH or will reflect δH

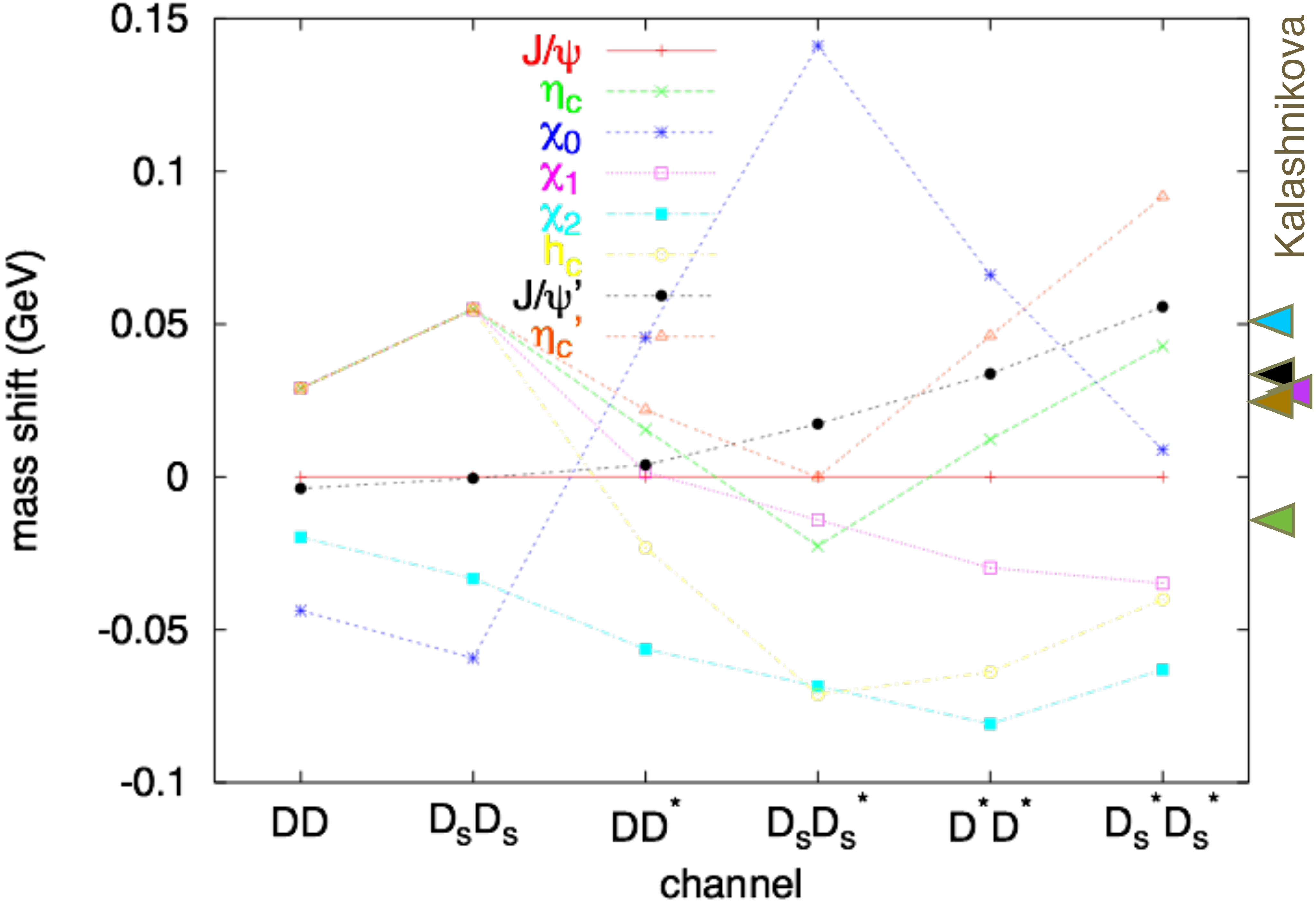
- there is thus some hope that the constituent quark model is robust (thereby resolving the Oakes-Yang problem)

Cornell coupled channel model

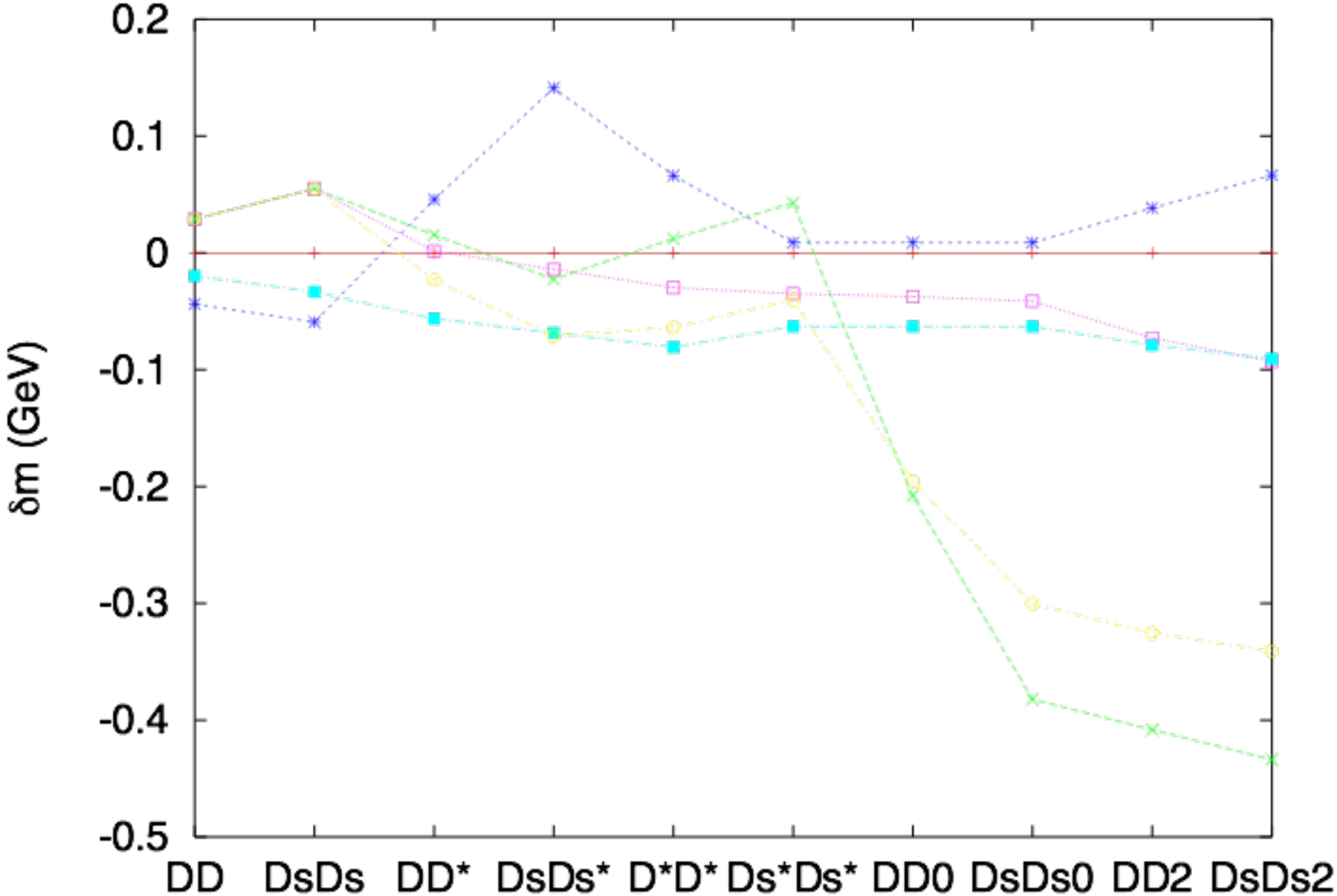


2^3P_1 spectral density (Kalshnikova)





0.15
0.1
0.05
0
-0.05
-0.1
mass shift (GeV)
J/ψ
η_c
χ_0
χ_1
χ_2
h_c
J/ψ'
η_c'
Kalashnikova
DD
D_sD_s
DD*
D_sD_s*
D*D*
D_s*D_s*
channel



0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
δm (GeV)
DD
DsDs
DD*
DsDs*
D*D*
Ds*Ds*
DD0
DsDs0
DD2
DsDs2

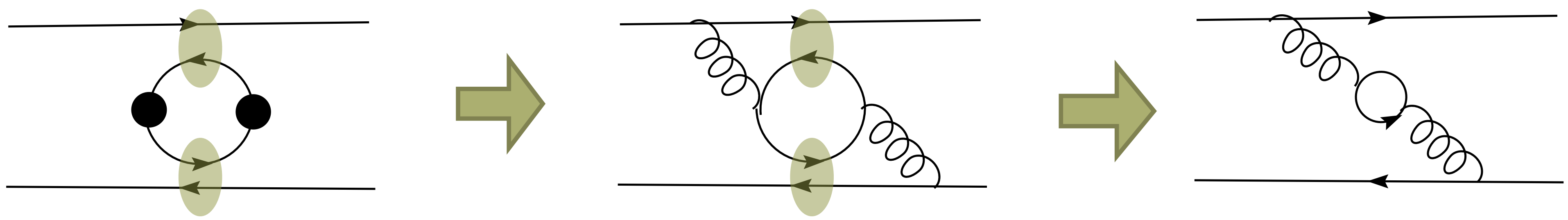
Issues

renormalisation-1

of course the ‘bare’ quark model must have its parameters refit to yield the experimental spectrum

renormalisation-2

summing the continuum



$q^2 < \Lambda^2 \approx 1\mathrm{GeV}^2$

$1\mathrm{GeV}^2 < q^2 < \Lambda^2 \approx 4\mathrm{GeV}^2$

$4\mathrm{GeV}^2 < q^2 < \Lambda^2 \to \infty$

how does one bridge the renormalisation gap between QCD and a model of QCD?

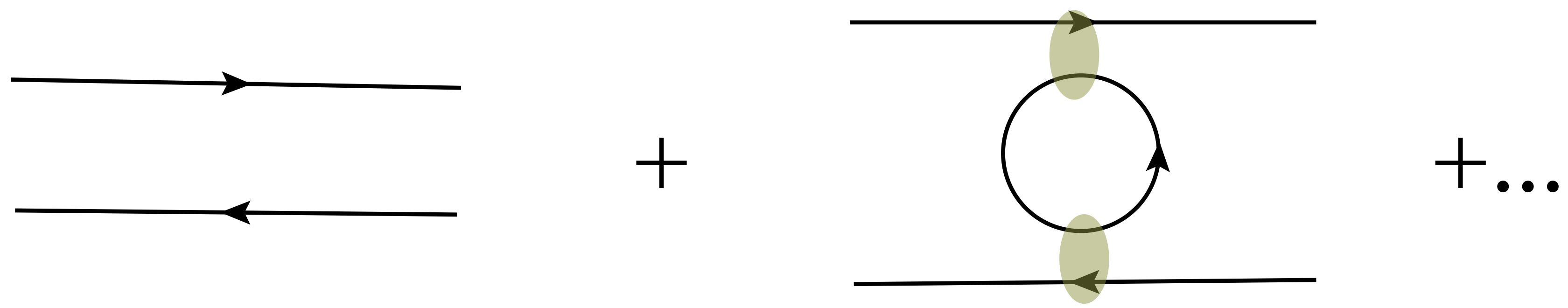
renormalisation-3

$$J/\psi \rightarrow \eta_c \gamma$$

$$A^{(HO)} = A^{(0)}(1 + 0.334 + 0.036) = 0.197 \text{ GeV}$$

$$A^{imp} = |\vec{q}|\sqrt{M_\psi E_\eta}\frac{eQ_q + eQ_{\bar{q}}}{m_q}\mathrm{e}^{-q^2/16\beta^2} \approx 0.095 \text{ GeV}$$

renormalisation-3

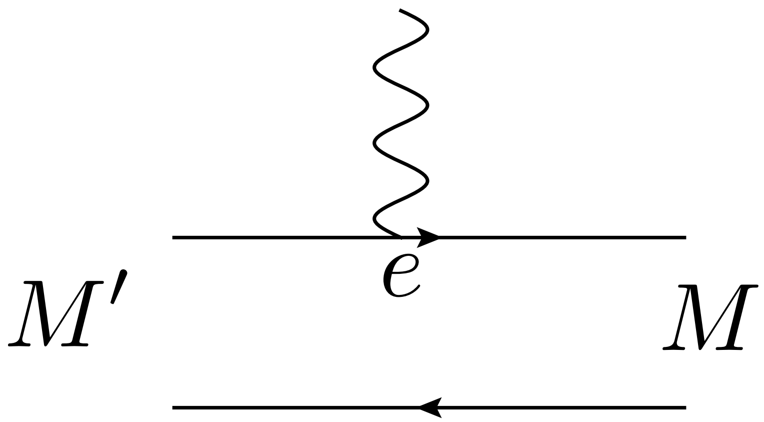


$$Z_{q\bar{q}} < 1 \qquad \Gamma_{ee} = Z_{q\bar{q}}\Gamma_0$$

$\Rightarrow$ a disaster for the quark model

renormalisation-4

what is the quark model?



$$\frac{e^2}{4\pi} = \frac{1}{137}$$

$$m_q = m_N/3$$

the defining characteristic of the quark model

renormalisation-4

- the quark model should be treated as a standard model ... there are no 'external parameters'
- an unquenched quark model is a field theory and needs to be properly renormalised

$$e \to e_R = \frac{e}{\sqrt{Z}} \qquad \frac{e_R^2}{4\pi} = \frac{1}{137}$$

and while we're at it...

- nonperturbative gluodynamics
- multipion intermediate states
- chiral restoration
- emergence of the string regime

Cusps

$$\Pi(s) = \int \frac{d^3q}{(2\pi)^3} \frac{q^{\ell_i+\ell_f}\, \mathrm{e}^{-2q^2/\beta_{AB}^2}}{\sqrt{s} - m_A - m_B - \frac{q^2}{2\mu_{AB}} + i\epsilon}$$

$$\Pi(s) = -\frac{\mu_{AB}\beta_{AB}}{\sqrt{2}\pi^2} \cdot I(Z)$$

relate to rel formula…

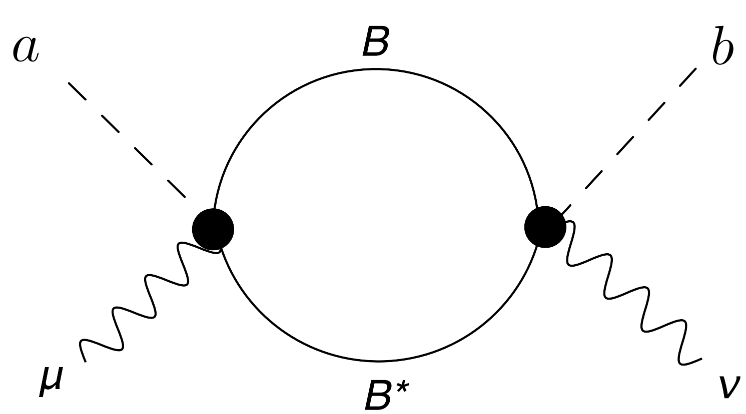
$$Z = \frac{4\mu_{AB}}{\beta_{AB}^2}(m_A + m_B - \sqrt{s})$$

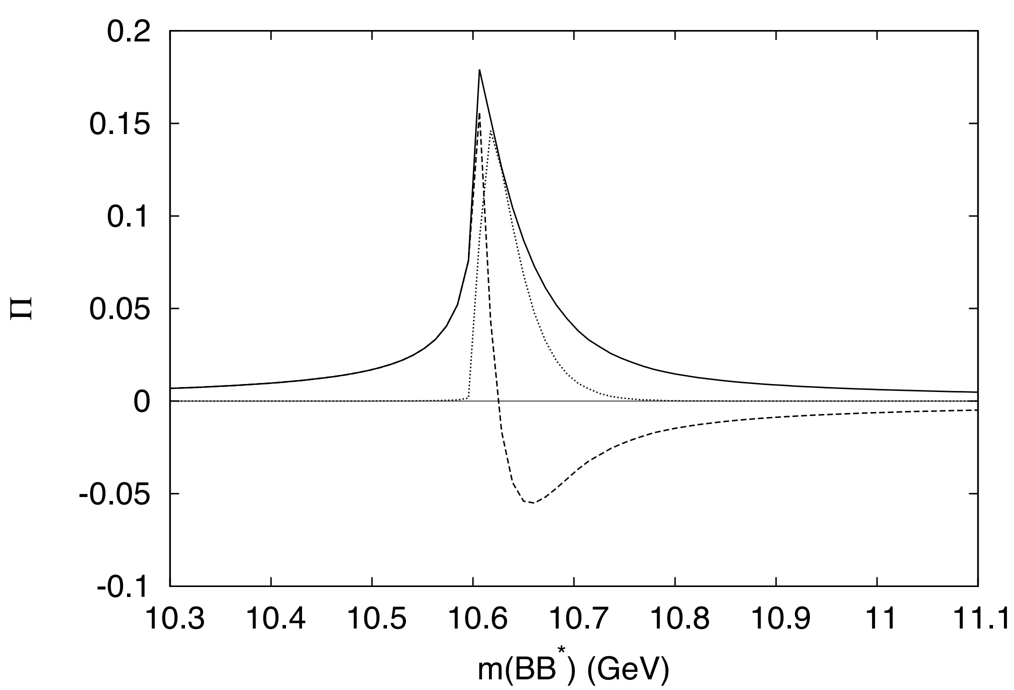
$$I(\ell_i + \ell_f = 0) = \frac{1}{2}\sqrt{\pi}[1 - \sqrt{\pi Z}\,\mathrm{e}^Z\,\mathrm{erfc}(\sqrt{Z})],$$

$$I(\ell_i + \ell_f = 1) = \frac{1}{2} - \frac{Z}{2}\mathrm{e}^Z\Gamma(0, Z),$$

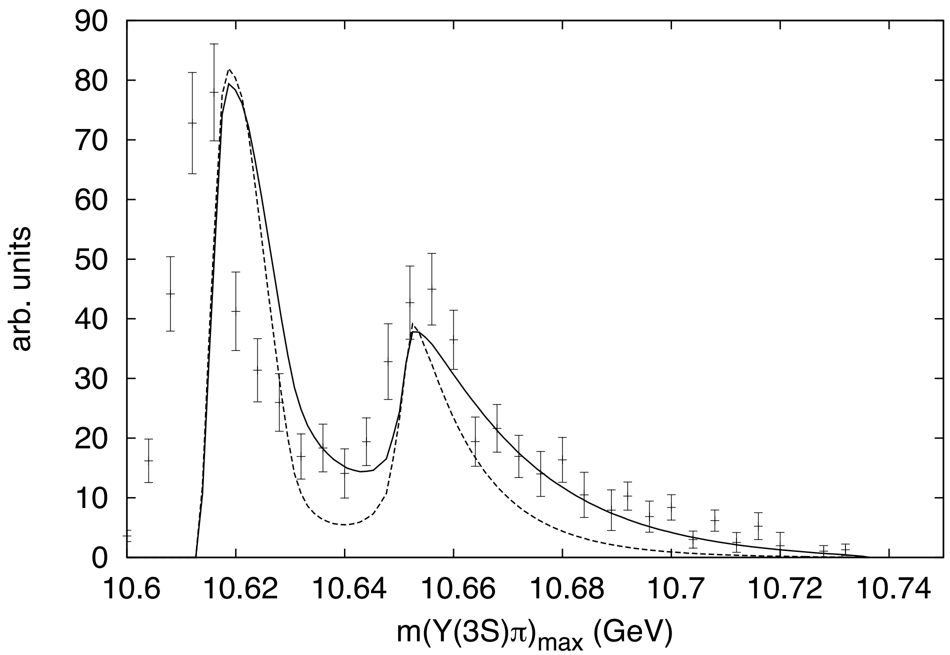
Z_b and Z_c as Threshold Cusps

bubble has Argand phase motion and is difficult to distinguish from a Breit-Wigner

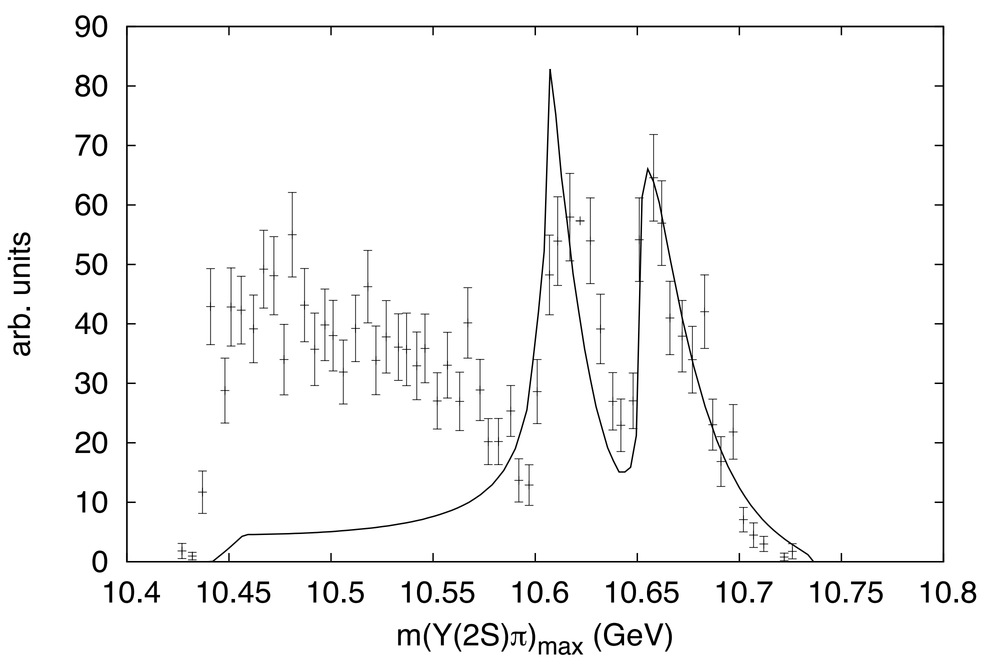


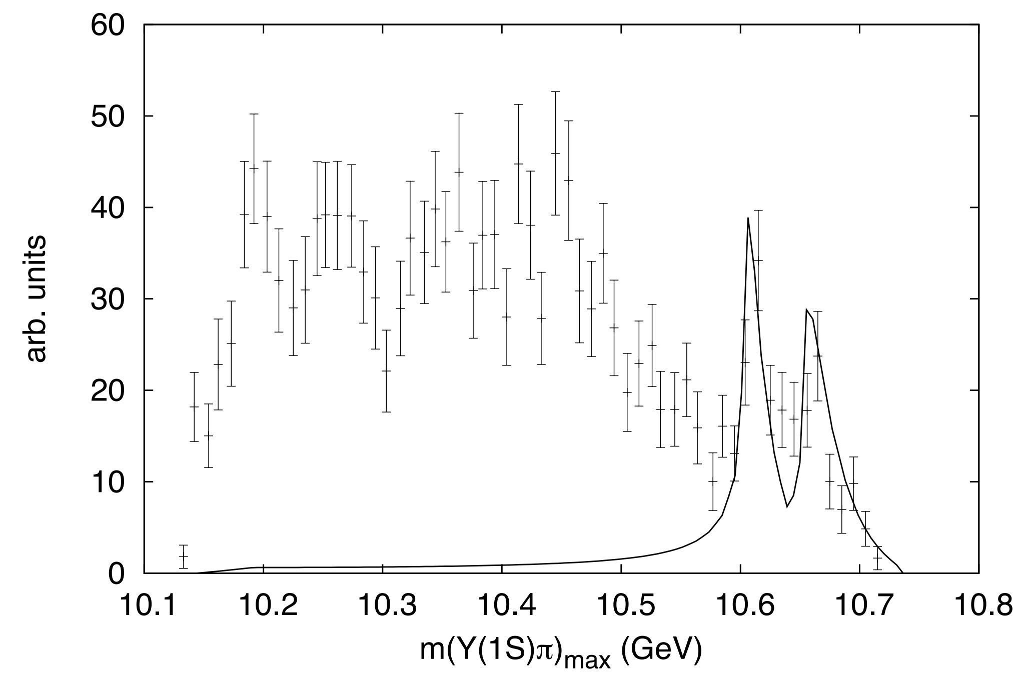


$\Upsilon(5S) \to \Upsilon(nS)\pi\pi$

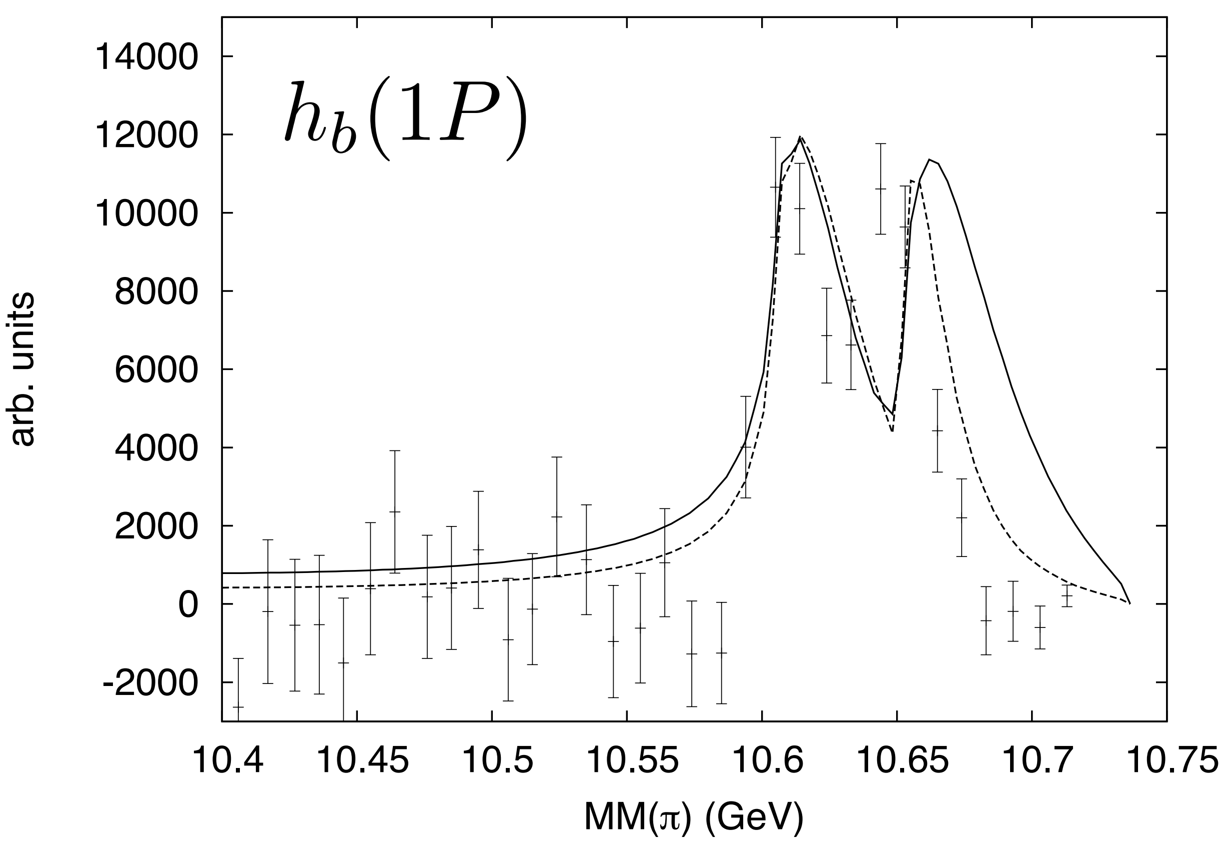


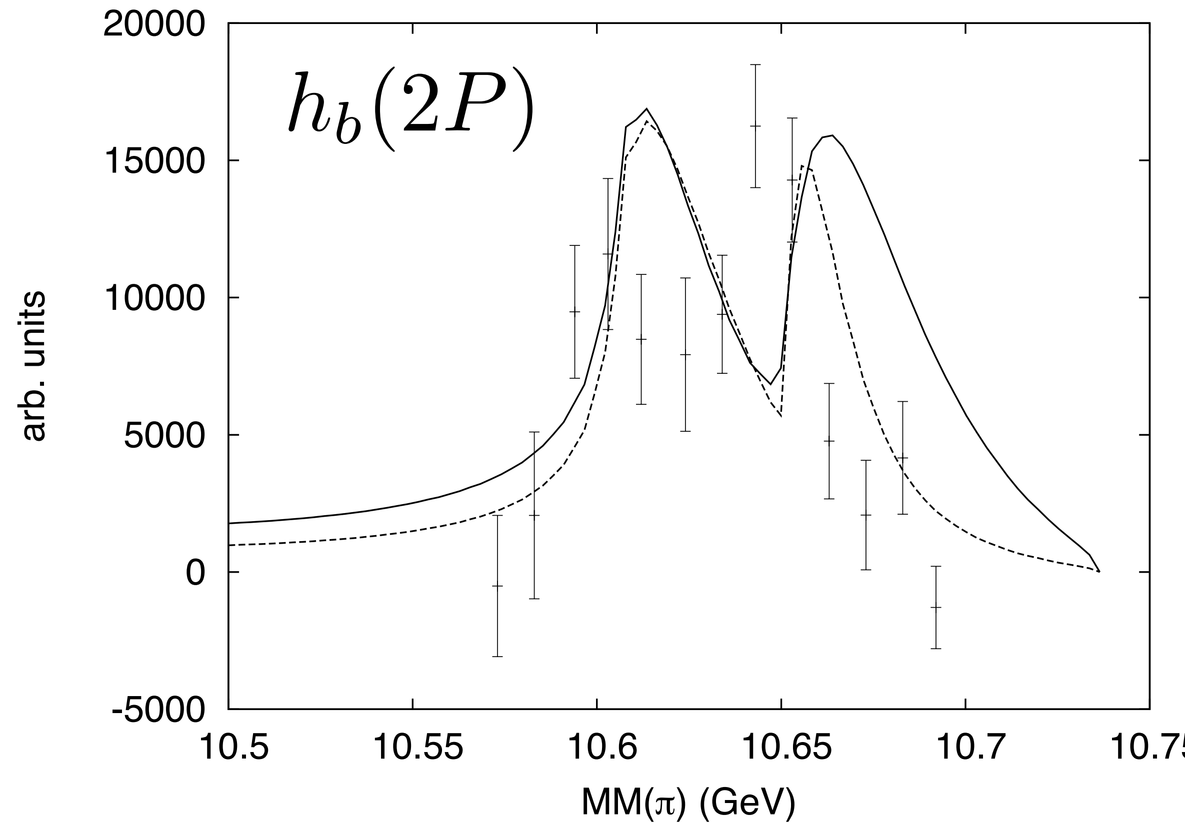
fix couplings and scales with Y(3S) — relatively little pipi dynamics. Get Y(2S) with same couplings! Y(1S) requires 70% smaller coupling BB*:piY(1S)





$$\Upsilon(5S) \to h_b(nP)\pi\pi$$





$Y(4260)$

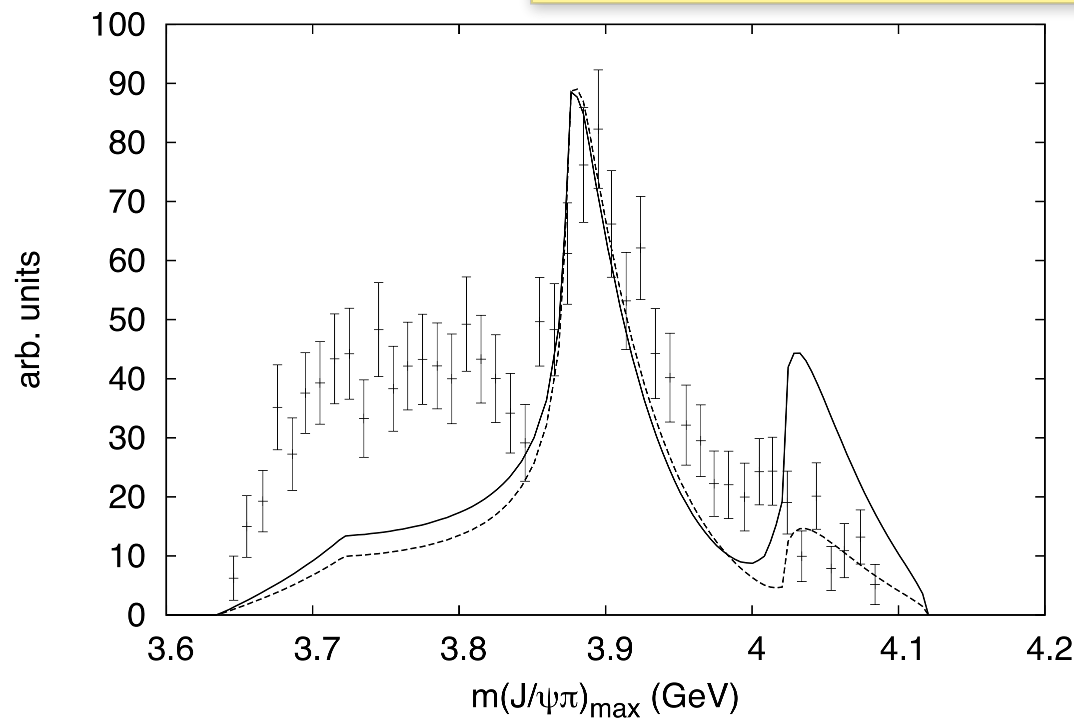
note that a 4025 should be visible??

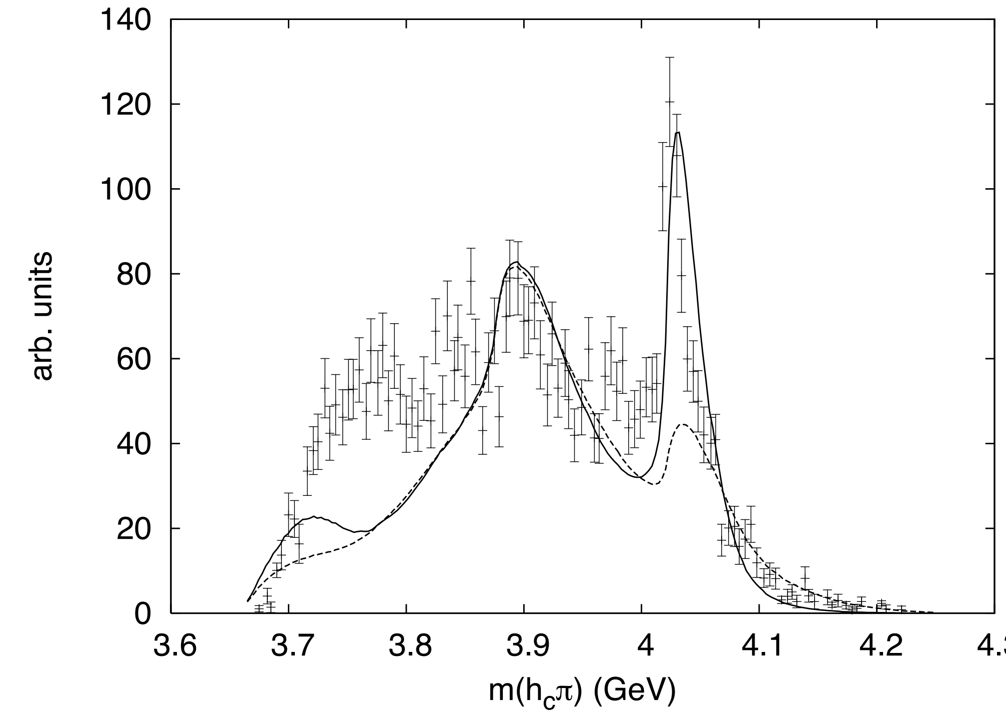
Zc(3900)

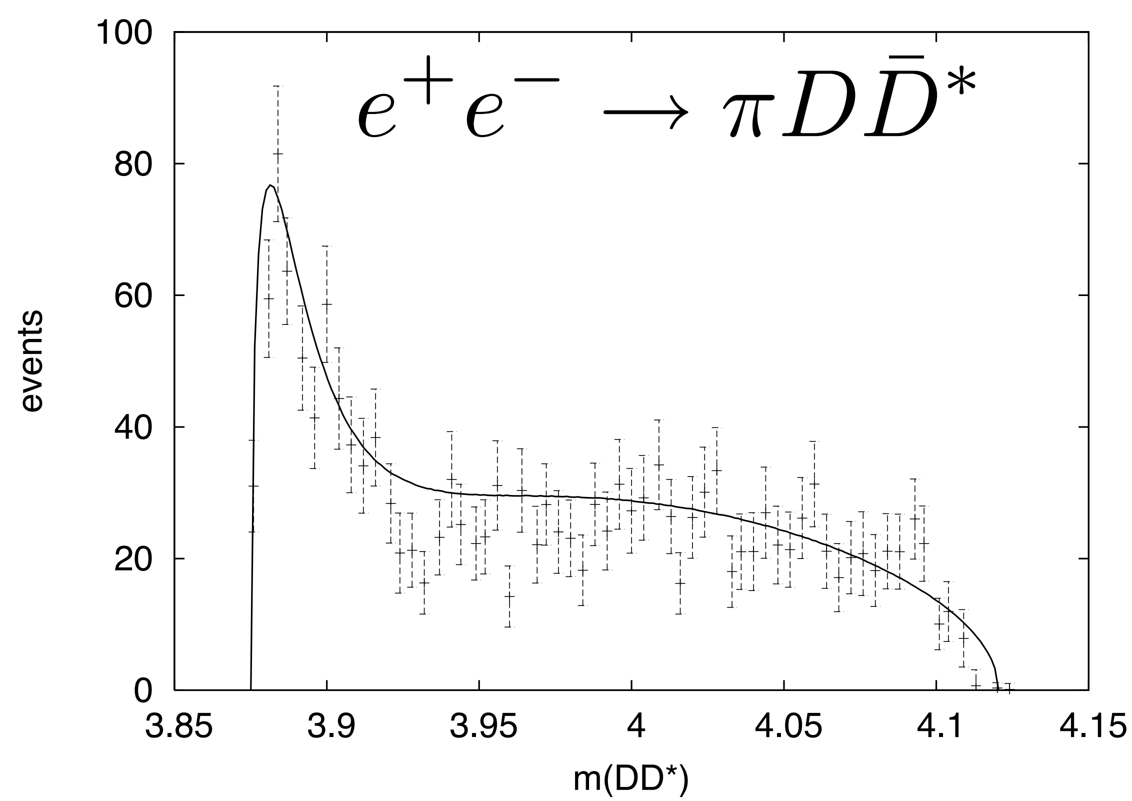
e^+e^- -

wieghted average of measurements at 13 sqrt(s)

Zc(4025)







This is a vertex model (beta=0.18) —- it needs to be verified in bubble diagrams to attempt a comprehensive model

Cusp Diagnostics

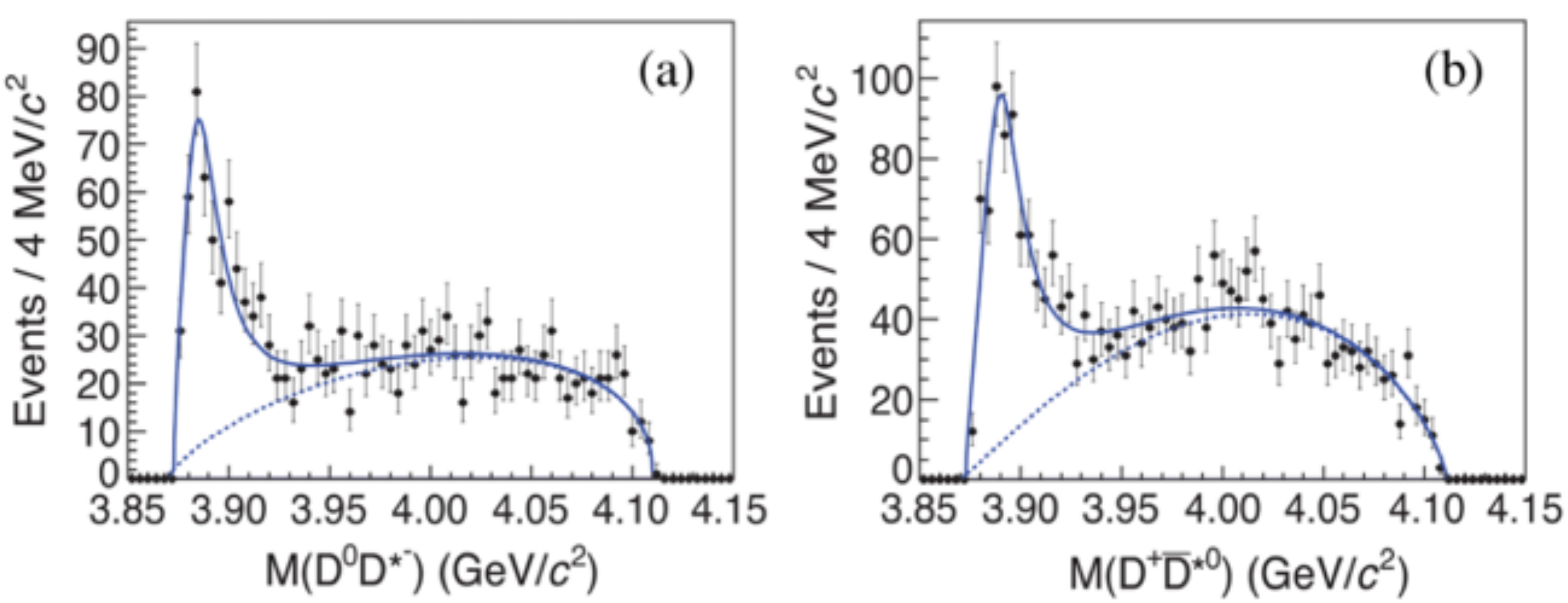
- lie just above thresholds
- S-wave quantum numbers
- partner states of similar width – widths will depend on channel
- the reaction $\Upsilon(5S) \to K\bar{K}\Upsilon(nS)$ should reveal "states" at 10695 ($B\bar{B}_s^* + B^*\bar{B}_s$) and 10745 ($B^*\bar{B}_s^*$)

Zc(3900)

$$e^+e^- \to \pi D\bar{D}^* \qquad \sqrt{s} = 4.26$$

$$M = 3883.9 \pm 1.5 \pm 4.2$$

$$\Gamma = 24.8 \pm 3.3 \pm 11.0$$

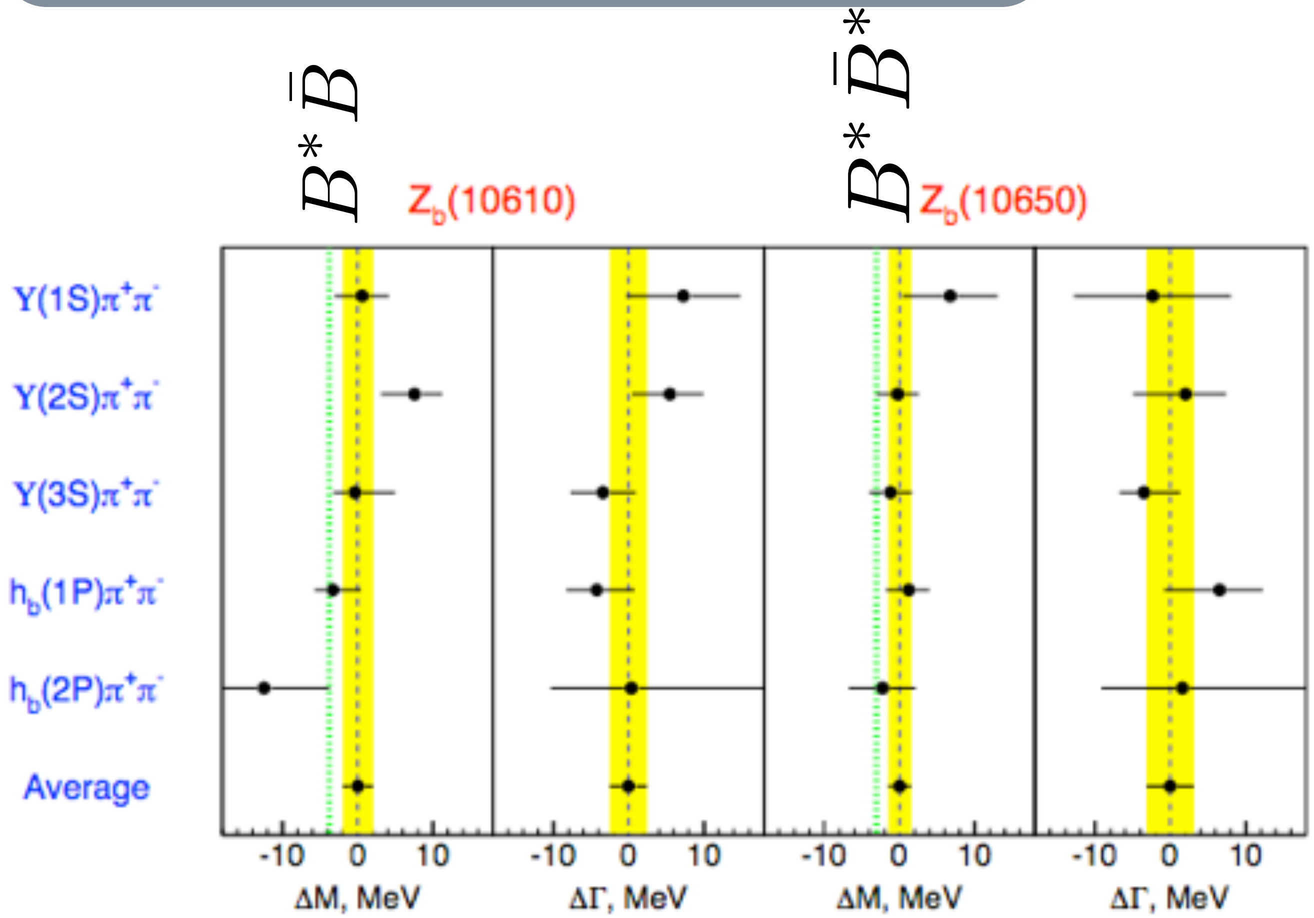


BESIII PRL112 022001 (14)

$Z_b^+(10610)$ $Z_b^+(10650)$

Adachi et al. [Belle] 1105.4583

$$I^G J^P = 1^+ 1^+$$



1+1+ B*B* is 5D1 and mildly attractive so likely a channel opening effect

isovector 1++ BB* is repulsive

note that both states are above threshold

narrow (15 MeV)

$\Upsilon(2S)$ $h_b(1P)$ $h_b(2P)$