Nested soft-collinear subtractions for NNLO color singlet production

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Theoretical tools for precision physics

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Precision physics at the LHC

- Discovery of Higgs boson and absence of enduring evidence for new physics at LHC → **precision physics programme** (NNLO and beyond).
  - ➢ Extensive studies of Higgs boson: fully understand the nature of EWSB.
  - ➢ Search for BSM physics through subtle deviations from SM background.
  - ➢ Determine fundamental parameters of nature.
- ~ 1% level theoretical precision for a hadron collider extremely challenging!
  - – Evaluating amplitudes with two (or more) loops → this workshop.
  - – Handling complicated IR divergences from two-parton emissions.

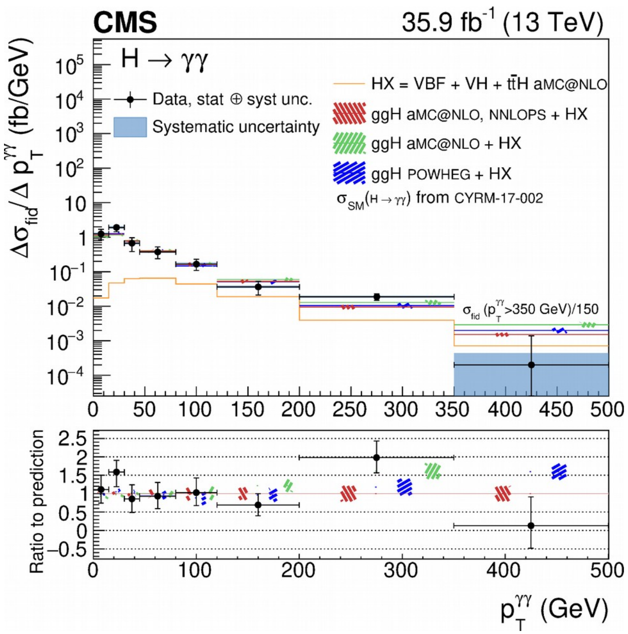
Infrared singularities in pQCD

- Infrared structure of multi-loop QCD amplitudes is **well-understood:**
  - Check on calculation.
- *What happens to the IR poles?*

  Cancel against poles from soft/collinear real radiation [Kinoshita '62, Lee & Nauenberg '64].
- For **inclusive** quantities (integrate over all final states – e.g. cross section) cancellation is straightforward:
  - Reverse unitarity: real and virtual radiation ~ different unitarity cuts.
  - Cf. talk by B. Mistlberger.

Infrared singularities in pQCD

- LHC phenomenology requires more **exclusive** observables:
  - e.g. differential cross sections in transverse momentum, rapidity, …
  - Enable theoretical predictions for realistic experimental setup.
  - Kinematic distributions vital for understanding underlying physics.



Don’t integrate over final state radiation

Cancellation of IR singularities between real and virtual corrections in exclusive calculations is more complicated.

Infrared singularities in pQCD

- Appearance of IR singularities is different in real and virtual corrections:
- **Virtual corrections:**
  - Born-like phase space.
  - **Explicit** IR singularities in *amplitudes* from loop integration (soft/collinear virtual particles).
- **Real corrections:**
  - Radiative phase space.
  - Emitted particle(s) may be soft and/or collinear.

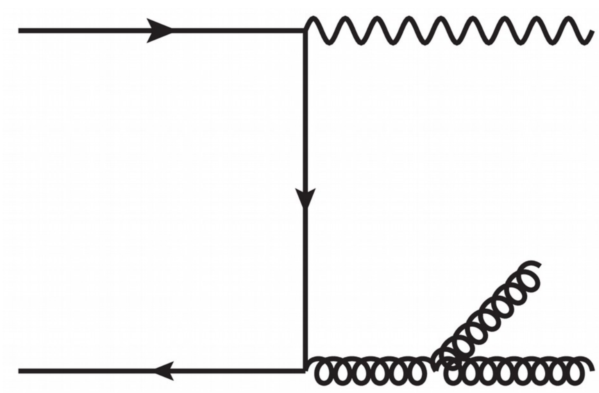
➡ IR singularities **after phase space integration.**

To get **fully differential** results from **numerical** (Monte Carlo) **integration:**

Extract and cancel all singularities *prior* to integration.

IR singularities at NLO and NNLO

- **Solved** at NLO (Catani-Seymour, Frixione-Kunszt-Signer,...).
  - Fully local.
  - Explicit, analytic cancellation of poles.
  - Applicable to any process at the LHC.
  - **Essential precursor** to “NLO revolution” & automation of NLO calculations.
- **Highly non-trivial at NNLO:** multiple soft/collinear limits which may **overlap** – can approach a limit in different ways.
- Consider real-real corrections to Drell-Yan (DY): $q\bar{q} \to V + gg$.



- Singularities arise when:
  - *Either* gluon or *both* gluons → soft.
  - *Either* gluon or *both* gluons → collinear to either initial state quark.
  - Gluons → collinear to each other.
  - **Any combination of above – overlap.**

Handling IR singularities at NNLO

- SLICING
  - qT [Catani, Grazzini ‘07]
  - N-jettiness [Gaunt *et al* ‘15; Boughezal *et al* ‘15]
- SUBTRACTION
  - Antenna [Gehrmann-de Ridder, Gehrmann, Glover ‘05, …]
  - STRIPPER [Czakon ‘10, ‘11]
  - Projection-to-Born [Cacciari *et al* ‘15]
  - CoLoRFulNNLO [Somogyi, Trócsányi, Del Duca ‘05, …]

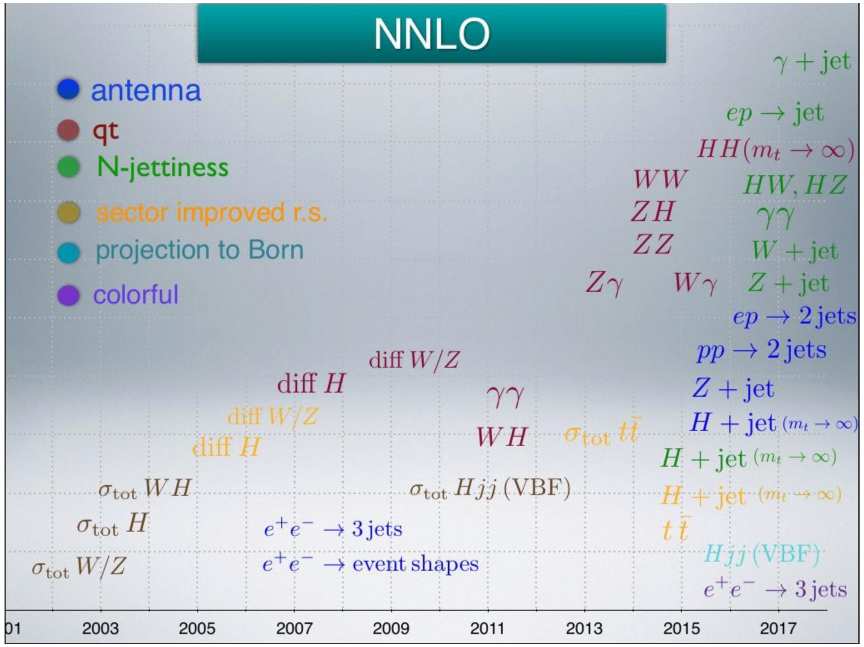
    “ESTABLISHED”
  - Unsubtraction [Sborlini, Driencourt-Mangin, Hernandez-Pinto, Rodrigo ‘16]
  - **Nested soft-collinear [Caola, Melnikov, R.R. ‘17]**
  - Geometric [Herzog ‘18] [F. Herzog]
  - Local analytic sector [Magnea *et al* ‘18] [L. Magnea]

    “2ND GEN.”

The NNLO Revolution

Great progress in subtraction & slicing methods:

All 2→2 process and a few 2→3 process (with special kinematics) known at **NNLO**.



Slide from Gudrun Heinrich, LHCP2017

Problem solved, but solutions **not optimal** – room for improvement. Current subtraction schemes

- Are **complicated** – difficult to implement.
- Obscure the **physical origin of singularities** in intermediate steps.
- Are sometimes **process-dependent.**
- Require **large computational times** and **fast scaling:**
  - ~100 CPU hrs for *V* (differential)
  - ~100k CPU hrs for *V+j* (differential).
  - 2→3 processes, e.g. *H+2j*?

Improving NNLO subtractions

Goal: Replicate success of NLO subtraction methods (FKS/CS).

A “better” subtraction scheme should:

- Be fully local
  - *avoid large numerical cancellations in intermediate steps.*
- Have a minimal structure displaying a clear origin of physical singularities
  - *easier for others to implement.*
- Have explicit, analytic cancellation of poles
  - *control over singular structures.*
- Allow four-dimensional evaluation of amplitudes
  - *improved numerical efficiency.*
- Be process-independent.
- Be flexible
  - *allow freedom in phase-space parametrization/mapping.*

Nested soft-collinear subtraction

[Caola, Melnikov, R.R. '17]

- Extension of FKS subtraction to NNLO.
- Independent subtraction of soft and collinear divergences **(color coherence):**
  - Overlapping soft singularities separated by energy ordering (trivial).
  - Overlapping collinear singularities separated using sectors (as in STRIPPER).
    - Natural splitting by rapidity.

[Czakon '10, '11]

- Fully local. ✓
- Clear physical origin of singularities (soft & collinear). ✓
- **Recombination** of sectors leading to simplifications in integrated subtraction terms.
  - Final IR structure very transparent.
  - Explicit (not yet fully analytic) pole cancellation (independent of matrix element). ✓
- Allows four-dimensional evaluation of matrix elements. ✓
- Process-independent *in principle* – details only worked out for color singlet hadroproduction & color singlet decay. ✓
- Not tied to phase space parametrization (currently using STRIPPER parametrization of angular phase space). ✓

Current status and outline

- Color singlet production:
  - ✓ Corrections to $q\bar{q} \to V$ (e.g. DY, *VH*, *VV*,...)
  - ✓ Corrections to $gg \to V$ (e.g. *H*, *HH*, ...)
- Color singlet decay:
  - ✓ Corrections to $V \to q\bar{q}$ (e.g. $H \to b\bar{b}$)
- Extension to initial & final states with color conceptually straightforward.
- Discuss corrections to $q\bar{q} \to V + ng$
  - **Most complicated** singular structure.

FKS subtraction at NLO: Notation

Consider real corrections to color singlet production

$$q(p_1)\bar{q}(p_2) \to V + g(p_4):$$

$$\mathrm{d}\sigma^{\mathrm{R}} = \frac{1}{2s}\int [\mathrm{d}g_4] F_{LM}(1,2,4) \equiv \langle F_{LM}(1,2,4)\rangle.$$

$$F_{LM}(1,2,4) = \mathrm{dLips}_V \; |\mathcal{M}(1,2,4,V)|^2 \; \mathcal{F}_{\mathrm{kin}}(1,2,4,V) \qquad [\mathrm{d}g_4] = \frac{\mathrm{d}^{d-1}p_4}{(2\pi)^d 2E_4}\theta(\sqrt{s}/2 - E_4)$$

Lorentz-inv. Phase space for V **(incl. delta-fn)**

Matrix-element sq.

IR-safe observable

Integration in partonic CoM frame

Define soft and collinear operators:

$$S_i A = \lim_{E_i \to 0} A \qquad C_{ij} A = \lim_{\rho_{ij} \to 0} A \qquad \rho_{ij} = 1 - \cos\theta_{ij}$$

FKS subtraction at NLO: Subtraction

Remove singular limits and add back as subtraction terms:

$$
\begin{aligned}
\langle F_{LM}(1,2,4)\rangle = & \langle (I - C_{41} - C_{42})(I - S_4) F_{LM}(1,2,4)\rangle + \\
& \langle S_4 F_{LM}(1,2,4)\rangle + \\
& \langle (C_{41} + C_{42})(I - S_4) F_{LM}(1,2,4)\rangle
\end{aligned}
$$

- First term: finite, can be integrated numerically in 4-dimensions.
- Second term: soft subtraction term – gluon decouples completely (need upper bound: $\sqrt{s}/2$).
- Third term: collinear and soft+collinear subtraction terms – gluon decouples partially or completely.
- Singularities made explicit by integrating subtraction terms over phase space of unresolved gluon.

FKS subtraction at NLO: finite result

- Combining with virtual corrections and pdf renormalization → cancel poles.
- Take $\epsilon \to 0$ limit to get finite remainder – NLO correction:

$$2s \cdot \mathrm{d}\hat{\sigma}^{\mathrm{NLO}} = \left\langle F_{LV}^{\mathrm{fin}}(1,2) + \frac{\alpha_s(\mu)}{2\pi}\left[\frac{2}{3}\pi^2 C_F F_{LM}(1,2)\right]\right\rangle + \langle \hat{O}_{\mathrm{NLO}} F_{LM}(1,2,4)\rangle +$$

$$-\frac{\alpha_s(\mu)}{2\pi}\int_0^1 \mathrm{d}z \left[\hat{P}_{qq}^{(0)}(z)\ln\left(\frac{\mu^2}{s}\right) + \mathcal{P}'_{qq}(z)\right]\left\langle \frac{F_{LM}(z\cdot 1,2)}{z} + \frac{F_{LM}(1,z\cdot 2)}{z}\right\rangle.$$

Sum of:

- **LO-like terms,** with or without convolutions with splitting functions.
- Real emission term, with singular configurations removed by iterated subtraction.
- Finite remainder of virtual corrections.
- $\hat{P}_{qq}^{(0)}$: Altarelli-Parisi splitting function; $\hat{O}_{\mathrm{NLO}} = (I - C_{41} - C_{42})(I - S_4)$
- $\mathcal{P}'_{qq}(z) = -C_F\left[-4D_1(z) - (1-z) + 2(1+z)\log(1-z)\right]$

Real-real subtractions at NNLO

Aim to replicate NLO results as much as possible at NNLO.

Consider real-real correction to color singlet production

$$q(p_1)\bar{q}(p_2) \to V + g(p_4) + g(p_5):$$

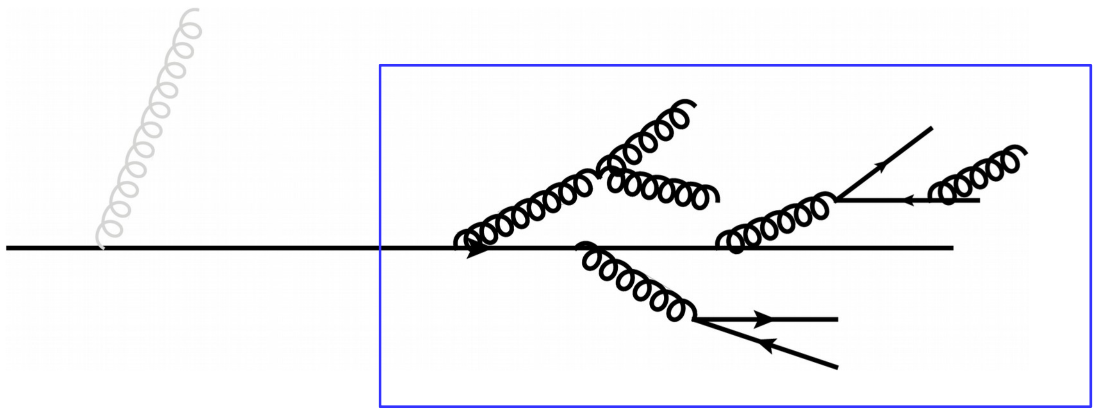
$$\mathrm{d}\sigma^{\mathrm{RR}} = \frac{1}{2s}\int [\mathrm{d}g_4][\mathrm{d}g_5] F_{LM}(1,2,4,5)$$

Recall: IR singularities from

- g_4 and/or g_5 → soft.
- g_4 or g_5 → collinear to initial state partons.
- g_4 or g_5 → collinear to each other.
- g_4 and g_5 collinear to same initial state parton (triple collinear limit).

Color coherence

- On-shell, gauge-invariant QCD scattering amplitudes : color coherence.
- Soft gluon cannot resolve details of later splittings; only sees total color charge.



Soft and collinear emissions can be treated **independently**:

- Regularize soft singularities first, then collinear singularities.
- No need for energy-angle ordering – energies and angles can be independently parametrized.

Treatment of real-real singularities

- **Step 1:** Limit operators.
  - Recall $S_i A = \lim_{E_i \to 0} A \qquad C_{ij} A = \lim_{\rho_{ij} \to 0} A. \qquad (\rho_{ij} = 1 - \cos\theta_{ij})$
  - NNLO like:

$$\mathbb{S} A = \lim_{E_4, E_5 \to 0} A, \text{ at fixed } E_5/E_4,$$

$$\mathbb{C}_i A = \lim_{\rho_{4i}, \rho_{5i} \to 0} A, \text{ with non vanishing } \rho_{4i}/\rho_{5i}, \rho_{45}/\rho_{4i}, \rho_{45}/\rho_{5i},$$

- **Step 2:** Order gluon energies $E_4 > E_5$.

$$2\,\mathrm{s}\cdot \mathrm{d}\sigma^{\mathrm{RR}} = \int [\mathrm{d}g_4][\mathrm{d}g_5]\theta(E_4 - E_5) F_{LM}(1,2,4) \equiv \left\langle F_{LM}(1,2,4,5) \right\rangle.$$

  - Gluon energies bounded by $\sqrt{s}/2$
  - Energies defined in CoM frame.
  - Soft singularities: either double soft or g_5 soft.

Soft singularities

- **Step 3:** Regulate the *soft* singularities:

$$\left\langle F_{LM}(1,2,4,5)\right\rangle = \left\langle \not{S} F_{LM}(1,2,4,5)\right\rangle + \left\langle S_5 (I-\not{S}) F_{LM}(1,2,4,5)\right\rangle$$
$$+ \left\langle (I-S_5)(I-\not{S}) F_{LM}(1,2,4,5)\right\rangle.$$

- First term: both g_4 and g_5 soft.
- Second term: g_5 soft, soft singularities in g_4 are regulated.
- Third term: regulated against all soft singularities,
- All three terms contain **(potentially overlapping)** collinear singularities.

Phase-space partitioning

- **Step 4:** Introduce **phase-space partitions**

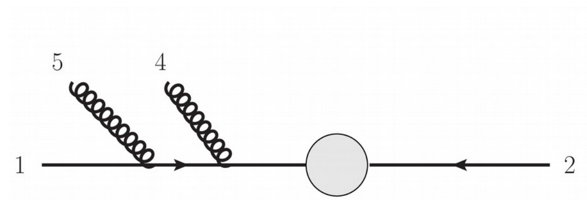
$$1 = w^{14,15} + w^{24,25} + w^{14,25} + w^{15,24}.$$

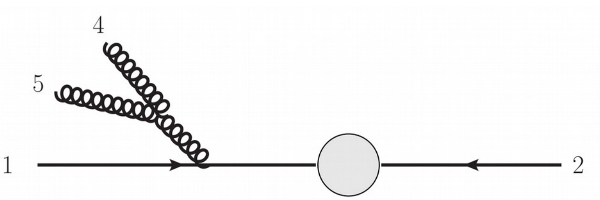
with

$$C_{42} w^{14,15} = C_{52} w^{14,15} = 0 \quad \Rightarrow \quad w^{14,15} \text{ contains } C_{41},\; C_{51},\; C_{45}$$

$$C_{41} w^{24,25} = C_{51} w^{24,25} = 0 \qquad\quad w^{24,25} \text{ contains } C_{42},\; C_{52},\; C_{45}$$

Triple collinear
partition



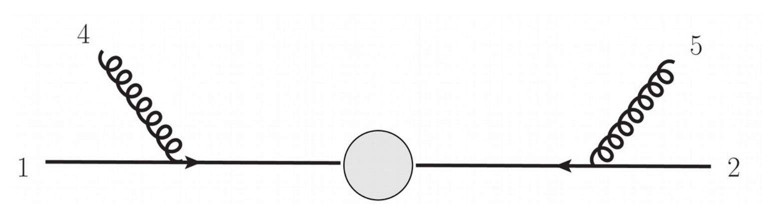


and

$$C_{42} w^{14,25} = C_{51} w^{14,25} = C_{45} w^{14,25} = 0 \quad \Rightarrow \quad w^{14,25} \text{ contains } C_{41},\; C_{52}$$

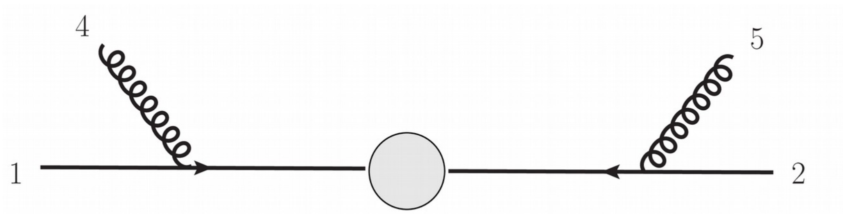
$$C_{41} w^{15,24} = C_{52} w^{15,24} = C_{45} w^{15,24} = 0 \qquad\quad w^{15,24} \text{ contains } C_{42},\; C_{51}$$

Double collinear
partition



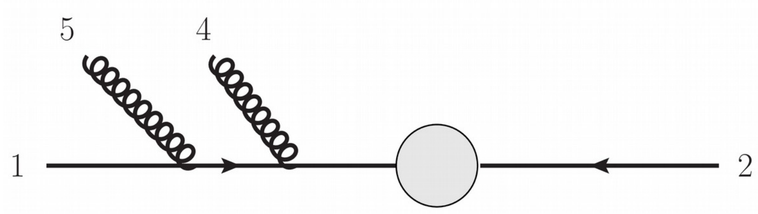
Phase-space partitioning

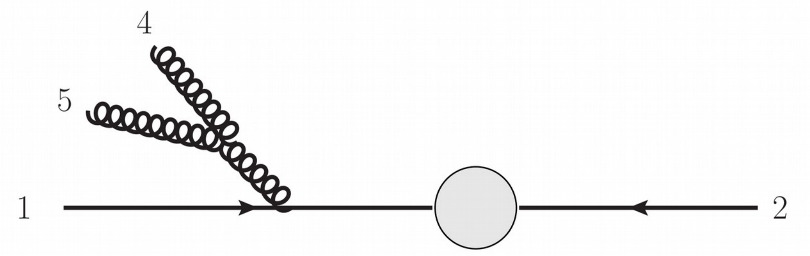
- Double collinear partition – **large** rapidity difference.



~ NLO x NLO → simple

- Triple collinear partition – **large/small** rapidity difference.





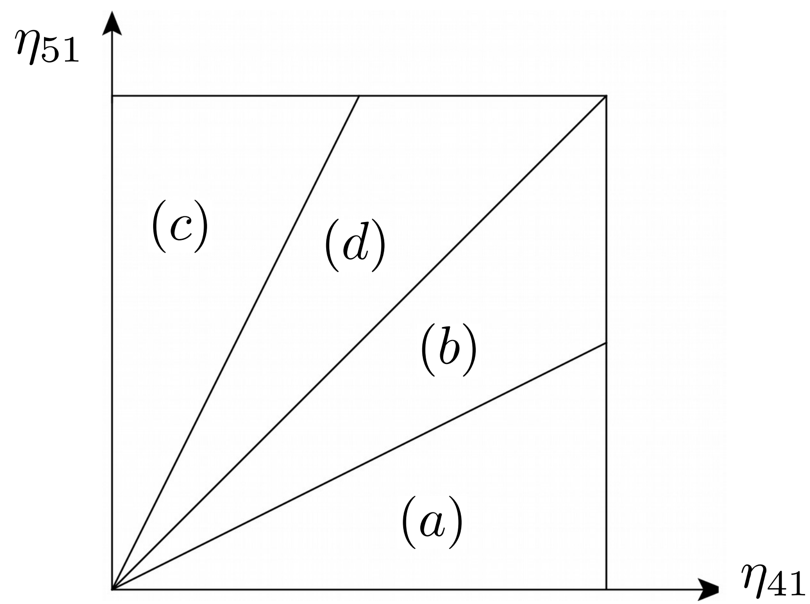
Overlapping singularities **remain** – need one last step to separate these.

Sector Decomposition

- **Step 5: Sector decomposition:**
- Define angular ordering to separate singularities.

$$\eta_{ij} = \rho_{ij}/2$$

$$\begin{aligned} 1 &= \theta\left(\eta_{51} < \frac{\eta_{41}}{2}\right) + \theta\left(\frac{\eta_{41}}{2} < \eta_{51} < \eta_{41}\right) \\ &+ \theta\left(\eta_{41} < \frac{\eta_{51}}{2}\right) + \theta\left(\frac{\eta_{51}}{2} < \eta_{41} < \eta_{51}\right) \\ &\equiv \theta^{(a)} + \theta^{(b)} + \theta^{(c)} + \theta^{(d)}. \end{aligned}$$



- Thus the limits are

$$\theta^{(a)} : C_{51} \qquad \theta^{(b)} : C_{45}$$
$$\theta^{(c)} : C_{41} \qquad \theta^{(d)} : C_{45}$$

- Sectors *a,c* and *b,d* same to *4* ↔ *5*, but recall energy ordering.
- Angular phase space parametrization [Czakon '10].

Removing collinear singularities

Then we can write soft-regulated term as

$$\Big\langle (I - S_5)(I - \mathbb{S}) F_{LM}(1,2,4,5) \Big\rangle = \Big\langle F_{LM}^{s_r c_s}(1,2,4,5) \Big\rangle + \Big\langle F_{LM}^{s_r c_t}(1,2,4,5) \Big\rangle + \Big\langle F_{LM}^{s_r c_r}(1,2,4,5) \Big\rangle,$$

$\langle F_{LM}^{s_r c_r}(1,2,4,5) \rangle$

- All singularities removed through iterated subtractions – evaluated in 4-dimensions.
- Only term involving fully-resolved real-real matrix element.

$\langle F_{LM}^{s_r c_{s,t}}(1,2,4,5) \rangle$

- Contain (soft-regulated) single and triple collinear singularities.
- Matrix elements of lower multiplicity.
- Partitioning factors and sectors: one collinear singularity in each term.

Treating singular limits

We have four singular subtraction terms:

$$\left\langle \mathcal{S}\!\!\!S F_{LM}(1,2,4,5) \right\rangle \quad \left\langle S_5 (I - \mathcal{S}\!\!\!S) F_{LM}(1,2,4,5) \right\rangle \quad \left\langle F_{LM}^{s_r c_s}(1,2,4,5) \right\rangle \quad \left\langle F_{LM}^{s_r c_t}(1,2,4,5) \right\rangle$$

We know how to treat them:

- Gluon(s) decouple partially or completely.
- Decouple completely:
  - Integrate over gluonic angles and energy.
- Decouple partially:
  - Integrate over gluonic angles.
  - Integral(s) over energy → integrals over splitting function in *z.*
- Significant analytic simplifications on recombining sectors after integration.
- Integration for first **three** subtraction terms done *analytically*, last one *numerically*

  (very promising ongoing work to compute this analytically).

Integrated double soft term

$$\left\langle \mathbb{S} F_{LM}(1,2,4,5)\right\rangle = F_{LM}(1,2)\int [\mathrm{d}g_4][\mathrm{d}g_4]\theta(E_4-E_5)\mathrm{Eik}_2(1,2,4,5)$$

- Computed recently [Caola, Delto, Frellesvig, Melnikov '18]
- Relatively simple result (shown here for ~ N_f term)

$$\begin{aligned}
\mathbb{S}_{ij}^{(q\bar{q})} = (2E_{\max})^{-4\epsilon}\left[\frac{1}{8\pi^2}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\right]^2 \Bigg\{ &-\frac{1}{3\epsilon^3}+\frac{1}{\epsilon^2}\left[\frac{2}{3}\ln(s^2)-\frac{4}{3}\ln 2\right.\\
&\left.+\frac{13}{18}\right]+\frac{1}{\epsilon}\left[-\frac{4}{3}\mathrm{Li}_2(c^2)-\frac{2}{3}\ln^2(s^2)+\ln(s^2)\left(\frac{8}{3}\ln 2-\frac{13}{9}\right)+\frac{\pi^2}{9}\right.\\
&\left.+\frac{4}{3}\ln^2 2+\frac{35}{9}\ln 2-\frac{125}{54}\right]-\frac{8}{3}\mathrm{Ci}_3(2\delta)-\frac{2}{3\tan(\delta)}\mathrm{Si}_2(2\delta)-\frac{4}{3}\mathrm{Li}_3(c^2)\\
&-\frac{8}{3}\mathrm{Li}_3(s^2)+\mathrm{Li}_2(c^2)\left[\frac{29}{9}-\frac{8}{3}\ln 2\right]+\frac{4}{9}\ln^3(s^2)+\ln^2(s^2)\left[-\frac{4}{3}\ln(c^2)\right.\\
&\left.-\frac{8}{3}\ln 2+\frac{13}{9}\right]+\ln(s^2)\left[-\frac{8}{3}\ln^2 2-\frac{70}{9}\ln 2+\frac{2}{9}\pi^2+\frac{107}{27}\right]+9\zeta_3\\
&+\frac{2\pi^2}{3}\ln 2-\frac{8}{9}\ln^3 2-\frac{23}{108}\pi^2-\frac{35}{9}\ln^2 2-\frac{223}{27}\ln 2+\frac{601}{162}+\mathcal{O}(\epsilon)\Bigg\}.
\end{aligned}$$

$$c=\cos\delta;\quad s=\sin\delta$$
$$\delta=\theta/2$$

Treating singular limits

After integration: subtraction terms written as lower multiplicity terms:

- LO-like:

  $\langle F_{LM}(z\cdot 1,\bar{z}\cdot 2)\rangle\,,\ \langle F_{LM}(z\cdot 1,2)\rangle\,,\ \langle F_{LM}(1,z\cdot 2)\rangle\,,\ \langle F_{LM}(1,2)\rangle$

- NLO-real-like (regulated by iterative subtraction):

  $\langle \mathcal{O}_{NLO}F_{LM}(z\cdot 1,2,4)\rangle\,,\ \langle \mathcal{O}_{NLO}F_{LM}(1,z\cdot 2,4)\rangle\,,\ \langle \mathcal{O}_{NLO}F_{LM}(1,2,4)\rangle$

convoluted with splitting functions with explicit singularities

- Pole cancellation within each structure

  (to $1/\epsilon^2$ analytically, $1/\epsilon$ numerically).

Finite remainders

- **Relatively compact** expressions for finite remainders for each *lower-multiplicity structure.*
- Familiar structures appear, e.g.

$$\mathrm{d}\sigma_{z1,2,4} = \frac{\alpha_s(\mu^2)}{2\pi} \int\limits_0^1 \mathrm{d}z \Bigg\{ \hat{P}_{qq}^{(0)}(z) \left\langle \log \frac{\rho_{41}}{4} \mathcal{O}_{\mathrm{NLO}} \left[\tilde{w}_{5||1}^{41,51} \frac{F_{\mathrm{LM}}(z\cdot 1,2,4)}{z} \right] \right\rangle$$

$$+ \left[\mathcal{P}'_{qq}(z) - \hat{P}_{qq}^{(0)}(z) \log\left(\frac{\mu^2}{s}\right) \right] \mathcal{O}_{\mathrm{NLO}} \frac{F_{\mathrm{LM}}(z\cdot 1,2,4)}{z} \Bigg\}$$

$$\mathrm{d}\sigma_{z1,\bar{z}2} = \left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^2 \int\limits_0^1 \mathrm{d}z\mathrm{d}\bar{z} \left[\mathcal{P}'_{qq}(z) - \log\left(\frac{\mu^2}{s}\right) \hat{P}_{qq}^{(0)}(z) \right]$$

$$\times \left[\mathcal{P}'_{qq}(\bar{z}) - \log\left(\frac{\mu^2}{s}\right)) \hat{P}_{qq}^{(0)}(z) \right] \frac{F_{LM}(z\cdot 1, \bar{z}\cdot 2)}{z\bar{z}}$$

- Same functions that appeared at NLO (as expected...)

Finite remainders

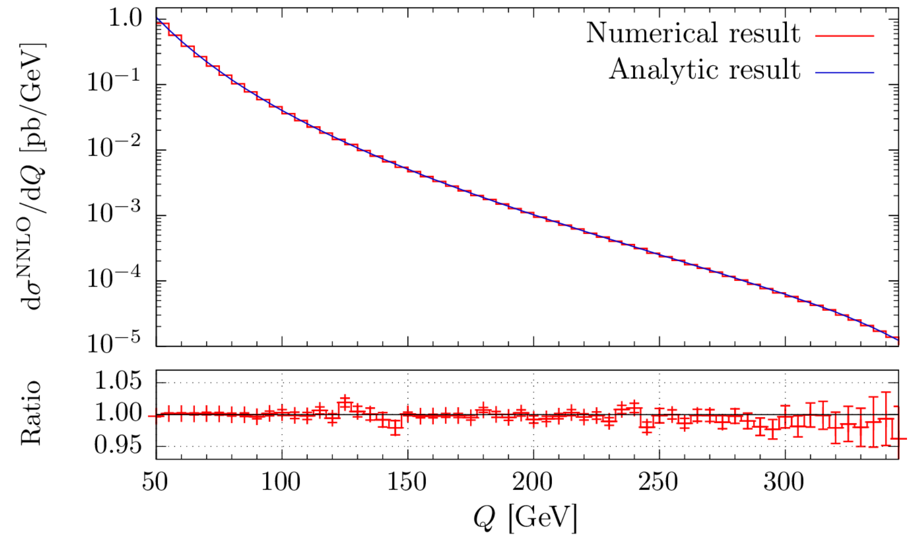
- New functions are relatively simple...
- Extension of NLO calculation to NNLO:
  - LO and NLO results convoluted with known functions.
  - Nested subtraction for real-real contribution.

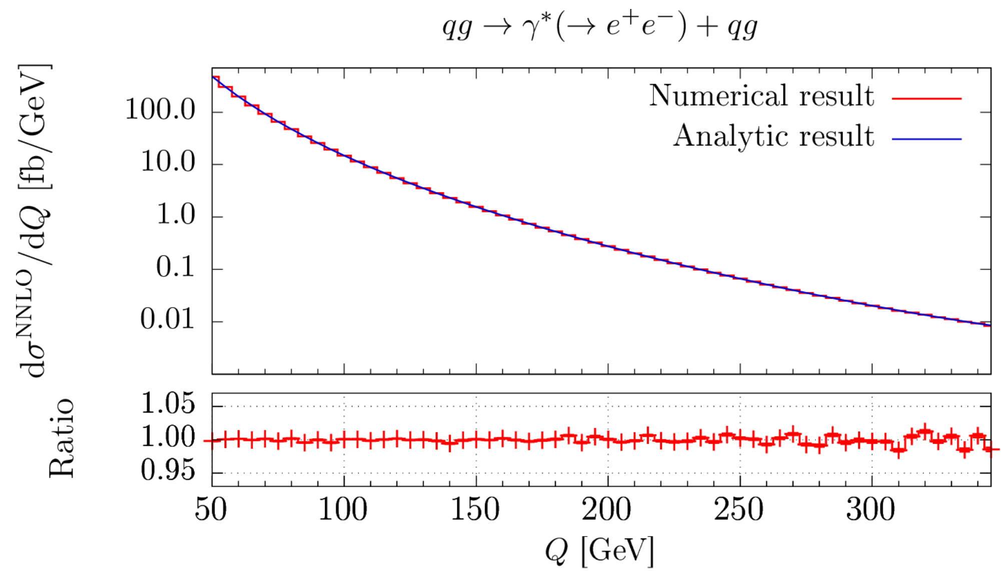
$$
\begin{aligned}
&\mathrm{d}\hat{\sigma}^{\mathrm{NNLO}}_{F_{LM}(z\cdot 1,2)}(\mu^2 = s) = \\
&\left[\frac{\alpha_s(\mu)}{2\pi}\right]^2 \int\limits_0^1 \mathrm{d}z \Bigg\{ C_F^2 \Bigg[8\bar{\mathcal{D}}_3(z) + 4\bar{\mathcal{D}}_1(z)(1+\ln 2) + 4\bar{\mathcal{D}}_0(z)\left[\frac{\pi^2}{3}\ln 2 + 4\zeta_3\right] \\
&+ \frac{5z-7}{2} + \frac{5-11z}{2}\ln z + (1-3z)\ln 2\ln z + \ln(1-z)\left[\frac{3}{2}z - (5+11z)\ln z\right] \\
&\quad + 2(1-3z)\mathrm{Li}_2(1-z) \\
&+ (1-z)\Bigg[\frac{4}{3}\pi^2 + \frac{7}{2}\ln^2 2 - 2\ln^2(1-z) + \ln 2\big[4\ln(1-z) - 6\big] + \ln^2 z \\
&\quad + \mathrm{Li}_2(1-z)\Bigg] + (1+z)\Bigg[-\frac{\pi^2}{3}\ln z - \frac{7}{4}\ln^2 2\ln z - 2\ln 2\ln(1-z)\ln z \\
&\quad + 4\ln^2(1-z)\ln z - \frac{\ln^3 z}{3} + \big[4\ln(1-z) - 2\ln 2\big]\mathrm{Li}_2(1-z)\Bigg] \\
&\quad + \left[\frac{1+z^2}{1-z}\right]\ln(1-z)\big[3\mathrm{Li}_2(1-z) - 2\ln^2 z\big] - \frac{5-3z^2}{1-z}\mathrm{Li}_3(1-z) \\
&\quad + \frac{\ln z}{(1-z)}\left[12\ln(1-z) - \frac{3-5z^2}{2}\ln^2(1-z) - \frac{7+z^2}{2}\ln 2\ln z\right]\Bigg] \\
&\quad + C_A C_F \Bigg[-\frac{22}{3}\bar{\mathcal{D}}_2(z) + \left(\frac{134}{9} - \frac{2}{3}\pi^2\right)\bar{\mathcal{D}}_1(z) + \Bigg[-\frac{802}{27} + \frac{11}{18}\pi^2 \\
&\quad + (2\pi^2 - 1)\frac{\ln 2}{3} + 11\ln^2 2 + 16\zeta_3\Bigg]\bar{\mathcal{D}}_0(z) + \frac{37-28z}{9} + \frac{1-4z}{3}\ln 2 \\
&\quad - \left(\frac{61}{9} + \frac{161}{18}z\right)\ln(1-z) + (1+z)\ln(1-z)\left[\frac{\pi^2}{3} - \frac{22}{3}\ln 2\right] \\
&\quad - (1-z)\left[\frac{\pi^2}{6} + \mathrm{Li}_2(1-z)\right] - \frac{2+11z^2}{3(1-z)}\ln 2\ln z - \frac{1+z^2}{1-z}\mathrm{Li}_2(1-z)\times \\
&\quad \times\big[2\ln 2 + 3\ln(1-z)\big]\Bigg] + R_+^{(c)}\mathcal{D}_0(z) + R^{(c)}(z)\Bigg\}\left\langle \frac{F_{LM}(z\cdot 1,2)}{z}\right\rangle.
\end{aligned}
$$

Proof-of-principle

- Extensively tested in DY production against analytic results [Hamberg, Matsuura, van Neerven '91]:
  - All channels relevant for DY.
  - *NNLO corrections* to cross section agree at < 1 permille.
  - *NNLO corrections* show permille to percent agreement across 5 orders of magnitude in virtuality of vector boson Q.
  - Also in channels which are numerically negligible.

➡ Good control of extreme kinematic regions.





Color singlet decay

- NNLO corrections to $V \to q\bar{q}$ can be calculated with identical strategy.
- Integrated subtraction terms <u>much</u> simpler:

Consider collinear limit of $V \to q(p_1)\bar{q}(p_2)g(p_3)$:

$$C_{31}F_{LM}(1,2,3) = \frac{g_{s,b}^2}{E_1 E_3 \rho_{13}} P_{qq}\left(\frac{E_1}{E_1+E_3}\right) F_{LM}(1+3,2)$$

Integrate over the **full phase space** of all final state particles, so write energy integration as:

$$z = E_1/(E_1+E_3)$$

$$\int [dE_1][dE_3] C_{31} F_{LM}(1,2,3) = \left[\int dz (z(1-z))^{-2\epsilon} P_{qq}(z)\right] \times \left[\int [dE_{13}] E_{13}^{-2\epsilon} F_{LM}(1+3,2)\right]$$
$$= \text{const.} \times \langle F_{LM}(1,2)\rangle.$$

Lower multiplicity terms multiplied by constants rather than splitting functions.

Extension to colored final states

Use DIS as test process (analytic expression known).

- Analytic form for double soft essential (otherwise require numerical integration at each phase space point).
- Combination of corrections to DY production and $H \rightarrow bb$ decay.
- Suggests that extension to **arbitrarily many colored final state particles** conceptually and analytically straightforward.

➡ Compact general analytic formula for NNLO subtraction of arbitrary final state, displaying pole cancellation.

Summary

- **Nested soft-collinear subtraction method** of handling NNLO subtraction, characterized by decoupling of soft and collinear limits.
- Developed iterative subtraction procedure:
  - Manifestly regulated finite term.
  - Integrated subtraction terms: convolutions of splitting function with explicit poles with lower multiplicity processes.
  - Transparent origin of IR poles.
  - Pole cancellation independent of matrix elements.
- Tested in DY and *W* production *for all partonic channels; H → bb* decay
  - Excellent agreement with analytic results in **all partonic channels.**
- Phenomenological application in $VH(\to b\bar{b})$.
- Ongoing work:
  - Remaining channels for color singlet production & color singlet decay.
  - Extension to colored initial-final state.
  - Major obstacle removed: double soft subtraction term known analytically.
  - Analytic expressions for final integrated subtraction term.

THANK YOU!