String Field Theory and its Applications

Ashoke Sen

Harish-Chandra Research Institute, Allahabad, India

Florence, April 2019

What is string field theory?

In the conventional world-sheet approach to string theory, the scattering amplitudes with n external states take the form:

$$\sum_{g\geq 0}(g_s)^{2g}\int_{M_{g,n}} I_{g,n}$$

$M_{g,n}$: Moduli space of genus g Riemann surface with n punctures

$I_{g,n}$: an appropriate correlation function of vertex operators and other operators (ghosts, PCOs) on a genus g Riemann surface.

String field theory is a quantum field theory with infinite number of fields in which perturbative amplitudes are computed by summing over Feynman diagrams.

Each Feynman diagram can be formally represented as an integral over the moduli space of a Riemann surface with

– the correct integrand $I_{g,n}$ (as in world-sheet description)

– but only a limited range of integration.

Sum over all Feynman diagrams reproduces the integration over the whole moduli space $M_{g,n}$.

Why should we study string field theory?

Original motivation: Use string field theory to give a non-perturbative definition of string theory.

In this talk the focus will be to use string field theory to better understand string perturbation theory

– address the ‘infra-red issues’ to make perturbation theory well-defined.

In the rest of the talk we shall focus on closed string field theories.

Review: arXiv:1703.06410

Corinne de Lacroix, Harold Erbin, Sitender Pratap Kashyap, A.S., Mritunjay Verma

General structure of string field theory

Begin with classical closed bosonic string field theory

Saadi, Zwiebach; Kugo, Suehiro; Sonoda, Zwiebach; Zwiebach; $\cdots$

A string field ψ is an element of some vector space $\mathcal{H}$.

$\mathcal{H}$ is a subspace of the full Hilbert space of matter and ghost world-sheet CFT, defined by the constraints:

$$\mathbf{b_0^-}|\psi\rangle = \mathbf{0}, \quad \mathbf{L_0^-}|\psi\rangle = \mathbf{0}, \quad \mathbf{n_g}|\psi\rangle = \mathbf{2}|\psi\rangle$$

$$\mathbf{b_0^\pm} = \mathbf{b_0} \pm \mathbf{\bar{b}_0}, \quad \mathbf{L_0^\pm} = \mathbf{L_0} \pm \mathbf{\bar{L}_0}, \quad \mathbf{c_0^\pm} = \frac{\mathbf{1}}{\mathbf{2}}(\mathbf{c_0} \pm \mathbf{\bar{c}_0})$$

$$\mathbf{n_g} = \text{ghost number}$$

Matter CFT: Any CFT with c=26.

Note: No physical state constraint on $|\psi\rangle$

If $\{|\phi_{\mathbf{r}}\rangle\}$ is a basis in $\mathcal{H}$, then we can expand $|\psi\rangle$ as

$$|\psi\rangle = \sum_{\mathbf{r}} \psi_{\mathbf{r}} |\phi_{\mathbf{r}}\rangle$$

ψ_r are the dynamical degrees of freedom

– path integral $\equiv$ integration over the ψ_r's

$\sum_{\mathbf{r}}$ includes integration over momenta along non-compact directions

$\Rightarrow$ makes $\psi_{\mathbf{r}}$ into fields (in momentum space)

Classical action (setting $g_s = 1$):

$$S = \frac{1}{2}\langle\psi|c_0^- Q_B|\psi\rangle + \sum_n \frac{1}{n!}\{\psi^n\}$$

Q_B: BRST charge

For $|A_i\rangle \in \mathcal{H}$, $\{A_1 \cdots A_n\}$ is constructed from correlation functions of the vertex operators A_i on the sphere, integrated over a subspace $\mathcal{S}$ of the moduli space $M_{0,n}$.

1. Since A_i's are off-shell, the correlation function depends on the choice of world-sheet metric, or equivalently the choice of local coordinate system z in which the metric = $|dz|^2$ locally.

2. The subspace $\mathcal{S}$ avoids all degenerations, and its choice is correlated with the choice of local coordinates in step 1.

Different choices (z, $\mathcal{S}$) give equivalent string field theories related by field redefinition

$$\mathbf{S} = \frac{\mathbf{1}}{\mathbf{2}}\langle\psi|\mathbf{c_0^-}\mathbf{Q_B}|\psi\rangle + \sum_{\mathbf{n}}\frac{\mathbf{1}}{\mathbf{n}!}\{\psi^{\mathbf{n}}\}$$

This action has infinite parameter gauge invariance of the form

$$\delta|\psi\rangle = \mathbf{Q_B}|\lambda\rangle + \cdots$$

$|\lambda\rangle$ **represents gauge transformation parameter.**

This theory can be quantized using Batalin-Vilkovisky (BV) formalism

– introduces ghosts and anti-fields

Net result: Relax the constraint on the ghost number of $|\psi\rangle$.

The action has similar structure:

$$\mathbf{S_{BV}} = \frac{1}{2}\langle\psi|\mathbf{c_0^- Q_B}|\psi\rangle + \sum_{\mathbf{n}} \frac{1}{\mathbf{n}!}\{\psi^{\mathbf{n}}\}$$

But now $\{\mathbf{A_1} \cdots \mathbf{A_n}\}$ contains contribution from integrals over subspaces of $\mathbf{M_{g,n}}$ for all g

The higher genus contributions are needed to cancel gauge non-invariance of the path integral measure.

Note: We shall continue to use the symbols

$\mathcal{H}$ for this extended Hilbert space carrying arbitrary $\mathbf{n_g}$

$|\psi\rangle$ for the extended string field $\in \mathcal{H}$

$\{\mathbf{A_1} \cdots \mathbf{A_n}\}$ for the new, quantum corrected product.

In Siegel gauge $b_0^+|\psi\rangle = 0$, the action takes the form:

$$S_{gf} = \frac{1}{2}\langle\psi|c_0^- c_0^+ L_0^+|\psi\rangle + \sum_n \frac{1}{n!}\{\psi^n\}$$

Propagator:

$$b_0^+\, b_0^-\, \frac{1}{L_0^+}\delta_{L_0^-} = b_0^+\, b_0^-\, \frac{1}{2\pi}\int_0^\infty ds\, e^{-sL_0^+}\int_0^{2\pi} d\theta e^{-i\theta\, L_0^-}$$

Second step is valid only for $L_0^+ > 0$.

Once we have the propagator we can compute amplitudes using Feynman diagrams.

Each Feynman diagram has vertices and propagators.

We have some integrals from the vertices (integration over subspaces of $M_{g',n'}$).

g', n' refer to individual vertices

We also have two integrals from each propagator (s, θ)

Together the total set of integrals can be interpreted as integral over a subspace of $M_{g,n}$ with the correct integrand

(g,n) refer to the full amplitude

Sum over all Feynman diagrams generate integration over the full moduli space $M_{g,n}$ with the correct integrand

Instead of summing over all Feynman diagrams, one could sum over only one particle irreducible (1PI) diagrams

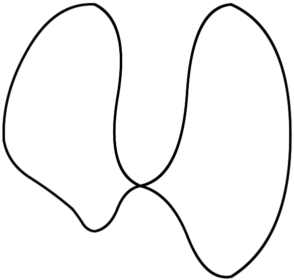
– gives 1PI effective action

$$S_{1PI} = \frac{1}{2}\langle\psi|c_0^- Q_B|\psi\rangle + \sum_n \frac{1}{n!}\{\psi^n\}_{1PI}$$

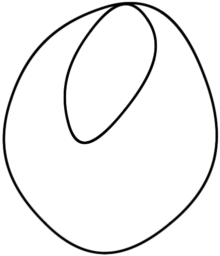
The definition of $\{A_1 \cdots A_n\}_{1PI}$ remains similar to that of $\{A_1 \cdots A_n\}$, except that the subspace of $M_{g,n}$ that we integrate over is larger

– includes boundaries of the moduli space that are non-separating type

(degenerating cycle that does not split the Riemann surface into two disconnected parts.)



Separating



Non-separating

Note: For bosonic string theory, the 1PI effective action is a formal object due to tachyons propagating in the loop.

But there will be no such problem in heterotic and type II theories.

Heterotic string theory:

World-sheet theory contains β, γ ghosts and associated ξ, η, ϕ system after bosonization

$$\beta = \partial\xi\, \mathbf{e}^{-\phi}, \qquad \gamma = \eta\, \mathbf{e}^{\phi}$$

Hilbert space $\mathcal{H}$ splits into direct sum $\oplus_{\mathbf{n}} \mathcal{H}_{\mathbf{n}}$

n: picture number

– integer for NS sector, integer + 1/2 for R sector

Picture changing operator (PCO) Friedan, Martinec, Shenker; Knizhnik

$$\mathcal{X}(\mathbf{z}) = \{\mathbf{Q_B}, \xi(\mathbf{z})\}$$

Heterotic string field theory: A.S.

Introduce a pair of string fields

$$|\psi\rangle \in \mathcal{H}_{-1} + \mathcal{H}_{-1/2}, \quad |\phi\rangle \in \mathcal{H}_{-1} + \mathcal{H}_{-3/2}$$

Action

$$S = \langle\phi|c_0^- Q_B|\psi\rangle - \frac{1}{2}\langle\phi|c_0^- Q_B G|\phi\rangle + \sum_n \frac{1}{n!}\{\psi^n\}$$

G=Identity in NS sector, $G = \mathcal{X}_0 \equiv \oint dz\, z^{-1}\, \mathcal{X}(z)$ **in R sector**

$$S = \langle\phi|c_0^- Q_B|\psi\rangle - \frac{1}{2}\langle\phi|c_0^- Q_B G|\phi\rangle + \sum_n \frac{1}{n!}\{\psi^n\}$$

$\{A_1 \cdots A_n\}$ is defined as in bosonic string theory, with the extra ingredient that we have to insert certain number of PCO's to conserve picture number

Total picture no: (2g-2) on a genus g Riemann surface

Different string field theory actions, associated with different choices of PCO locations, are related by field redefinition.

$$\langle\phi|c_0^- Q_B|\psi\rangle - \frac{1}{2}\langle\phi|c_0^- Q_B G|\phi\rangle + \sum_n \frac{1}{n!}\{\psi^n\}$$

Note: We have doubled the number of degrees of freedom ($|\phi\rangle$ and $|\psi\rangle$)

However since $|\phi\rangle$ enters the action at most quadratically, it describes free field degrees of freedom

– completely decouples from the interacting part of the theory described by $|\psi\rangle$

– has no observable effects.

Quantization of this theory proceeds in the same way as in bosonic string theory.

For type II string theory the structure of the theory is similar.

$$|\psi\rangle \in \mathcal{H}_{-1,-1} \oplus \mathcal{H}_{-1,-1/2} \oplus \mathcal{H}_{-1/2,-1} \oplus \mathcal{H}_{-1/2,-1/2}$$

$$|\phi\rangle \in \mathcal{H}_{-1,-1} \oplus \mathcal{H}_{-1,-3/2} \oplus \mathcal{H}_{-3/2,-1} \oplus \mathcal{H}_{-3/2,-3/2}$$

$$S = \langle\phi|c_0^- Q_B|\psi\rangle - \frac{1}{2}\langle\phi|c_0^- Q_B G|\phi\rangle + \sum_n \frac{1}{n!}\{\psi^n\}$$

G: identity in NSNS sector, $\mathcal{X}_0$ in NSR sector,

$\bar{\mathcal{X}}_0$ in RNS sector, $\mathcal{X}_0\bar{\mathcal{X}}_0$ in RR sector

The tree level ψ-ψ propagator has standard form in the 'Siegel gauge'

$$(L_0 + \bar{L}_0)^{-1}\, b_0^+\, b_0^-\, G\, \delta_{L_0,\bar{L}_0}$$

We could (formally) represent this as

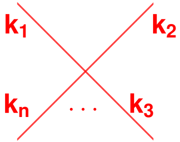
$$b_0^+\, b_0^-\, G\, \frac{1}{2\pi} \int_0^\infty ds\, e^{-sL_0^+} \int_0^{2\pi} d\theta\, e^{-i\theta\, L_0^-}$$

and (formally) recover the usual representation of amplitudes as integrals over $M_{g,n}$.

But we could also regard string field theory as a field theory with infinite number of fields and momentum space propagator

$$(k^2 + M^2)^{-1} \times \text{polynomial in momentum}$$

The polynomial comes from matrix element of $b_0^+\, b_0^-\, G$.



Vertices are accompanied by a suppression factor of

$$\exp\left[-\frac{A}{2}\sum_i (k_i^2 + m_i^2)\right]$$

A: a positive constant whose precise value depends on the choice of coordinate system used to define the off-shell vertex.

Hata, Zwiebach

This makes

– momentum integrals UV finite (almost)

– sum over intermediate states converge

Momentum dependence of vertex includes

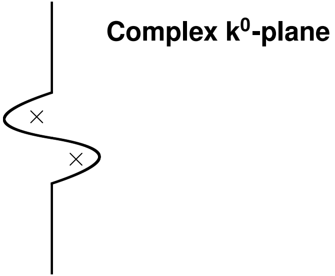
$$\exp\left[-\frac{A}{2}\sum_i (k_i^2 + m_i^2)\right] = \exp\left[-\frac{A}{2}\sum_i (\vec{k_i}^{\,2} + m_i^2) + \frac{A}{2}(k_i^0)^2\right]$$

Integration over $\vec{k_i}$ converges for large $|\vec{k_i}|$, but integration over k_i^0 diverges at large $|k_i^0|$.

The spatial components of loop momenta can be integrated along the real axis, but we have to treat integration over loop energies more carefully.

Resolution: Need to have the ends of loop energy integrals approach $\pm i\infty$.

In the interior the contour may have to be deformed away from the imaginary axis to avoid poles from the propagators.



We shall now describe how to choose the loop energy integration contour.

General procedure:

Pius, A.S.

1. Begin with a configuration of off-shell external momenta where all energies are imaginary and all spatial momenta are real.

2. In this case we can take all loop energy contours to lie along the imaginary axis without encountering any singularity.

3. Now deform the energies to real values (Wick rotation)

4. If some pole of a propagator approaches the loop energy integration contours, deform the contours away from the pole, keeping their ends at $\pm i\infty$.

Result: Such deformations are always possible

– the loop energy contours do not get pinched by poles from two sides during this deformation.

Applications

Momentum space integrals vs integrals over $M_{g,n}$:

Key link:

$$\frac{1}{L_0^+}\delta_{L_0^-} = \frac{1}{2\pi}\int_0^\infty ds\, e^{-sL_0^+}\int_0^{2\pi} d\theta\, e^{-i\theta\, L_0^-}$$

1. For $L_0^+ < 0$ the left hand side is finite but the right hand side is divergent (as $s \to \infty$)

2. For $L_0^+ = 0$ both sides are divergent.

<u>All</u> divergences appearing in the world-sheet description have their origin in one of these two cases.

– divergences appearing at the boundary of $M_{g,n}$ where the Riemann surface degenerates

$$\frac{1}{L_0^+}\delta_{L_0^-} = \frac{1}{2\pi}\int_0^\infty ds\, e^{-sL_0^+}\int_0^{2\pi} d\theta\, e^{-i\theta L_0^-}$$

For $L_0^+ < 0$ the left hand side is finite

$\Rightarrow$ string field theory gives perfectly finite result even though integral over $M_{g,n}$ diverges

For $L_0^+ = 0$ both sides are divergent

But in string field theory these are associated with some internal propagator going on-shell

$\Rightarrow$ field theory intuition tells us how to handle them

Applications of string field theory:

1. Non-essential cases: String field theory is helpful but not essential ($L_0^+ < 0$ cases)

2. Possibly non-essential cases: String field theory is the only one that is successful at present, but world-sheet methods may work ($L_0^+ \leq 0$ cases).

3. Essential cases: String field theory is necessary ($L_0^+ = 0$ cases)

Note: All calculations in string field theory eventually are expressed in terms of integrals over (subspaces of) $M_{g,n}$

The role of string field theory is to tell us what to do near degeneration when we encounter divergences.

Example of a non-essential case:

Consider N tachyon amplitude in bosonic string theory

$$\propto \int \prod_{i=4}^{N} d^2 z_i \prod_{i<j} |z_i - z_j|^{p_i . p_j}$$

This integral diverges if $p_i . p_j < -2$ for any pair (i,j).

– needs to be defined via analytic continuation.

Not very useful for numerical evaluation.

Witten's $i\epsilon$ prescription treats $\mathrm{Re}(\ln z_i)$ and $\mathrm{Im}(\ln z_i)$ as complex variables and turns the $\mathrm{Re}(\ln z_i)$ contours into complex plane.

Makes divergent integrals into <u>oscillatory</u> integrals, but we still need to apply some numerical trick to evaluate the integrals.

In string field theory these divergences can be associated with $L_0^+ < 0$ states propagating in the internal line.

We need to represent the propagator by $1/L_0^+$ instead of using Schwinger parametrization.

This suggests a specific procedure.

1. Remove all regions where 2 or more z_i's approach each other

– gives a finite integral.

2. The missing regions are compensated for by adding boundary terms.

Also need additional boundary terms where two boundaries intersect etc.

The boundary terms correspond to Feynman diagrams with one or more internal propagators.

The bulk term corresponds to the elementary N-string vertex.

Examples of possibly non-essential cases:

1. Proof of unitarity

2. Finding the domain of analyticity of the S-matrix

Unitarity: Mercus; Amano, Tsuchiya; Sundborg; Berera; D'Hoker, Phong; Iengo, Russo; Witten

String theory amplitudes, written as integrals over $M_{g,n}$, are always formally hermitian ($T - T^\dagger = 0$)

– apparently violates unitarity

However one finds by examining the integrals over $M_{g,n}$ that the integrals diverge whenever the external momenta go above the threshold of production of intermediate states

– related to the fact that some particle propagating in the loop has $L_0^+ < 0$.

Therefore the apparent reality is only formal, and one has to define the integral by analytic continuation.

Witten's iϵ prescription gives a way to define these integrals in a way that makes them manifestly finite but complex.

But to construct a direct proof that the amplitudes defined this way satisfy the Cutkosky rules is hard.

String field theory can be used to solve this problem. Pius, A.S.

Since string field theory defines the loop amplitudes as momentum integrals in complex momentum space, it automatically comes with some 'iϵ prescription'.

1. One can give a direct proof of Cutkosky rules using Feynman diagram analysis.

2. One can show that this agrees with Witten's iϵ prescription.

Domain of analyticity:

Consider a general amplitude characterized by a set of Mandelstam variables:

$$s_{ij} = -(p_i + p_j)^2$$

In which domain in the complex s_{ij} plane is the S-matrix analytic?

– related to the question of crossing symmetry (ability to analytically continue from one physical channel to another)

– believed to be related to the question of locality

In principle this question can be addressed without going off-shell but at present this seems to be hard.

The generalization of Witten's iϵ prescription for complex external momenta is not known yet.

String field theory allows us to use an old approach in (axiomatic) quantum field theory.

Jost, Lehmann; Dyson; Bros, Messiah, Stora; Bros, Epstein, Glaser

1. Use the locality of the position space Green's function to prove analyticity of the off-shell momentum space Green's functions in certain primitive domain.

2. Then extend the domain using general properties of functions of many complex variables.

3. Study the intersection of this domain with the mass-shell.

This has been generalized to string field theory. de Lacroix, Erbin, A.S.

Non-trivial step is step 1 since we do not have analog of position space Green's function on which we impose locality.

Instead we analyze momentum space Feynman diagrams directly to find the primitive domain.

Examples of essential cases:

Pius, Rudra, A.S.; A.S.

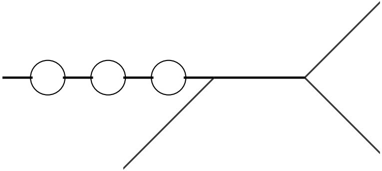
1. Vacuum shift

2. Mass renormalization

For a given amplitude, the usual world-sheet description of string perturbation theory gives one term at every loop order

– usually considered an advantage, but this may not always be the case

e.g. in a quantum field theory, self energy insertions on external legs need special treatment.



Steps required:

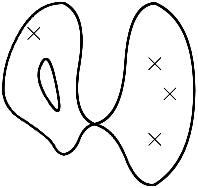
1. Separate graphs with self-energy insertions on external lines

2. Resum to compute off-shell 2-point function

3. Look for pole positions

In the usual world-sheet approach we do not do any of this.

Result: integration over $M_{g,n}$ diverges from the separating type degeneration.



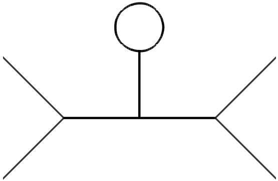
Vacuum shift:

Suppose we have massless ϕ^3 theory in which one loop correction generates a term linear in ϕ:

$$V = A\, g_s^{-2}\, \phi^3 - B\, \phi$$

A,B: constants, **g_s: coupling constant**

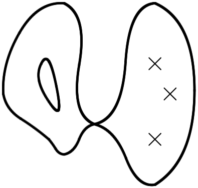
Naive perturbation theory diverges.



Correct procedure: Expand the action around the minimum at $\phi = g_s \sqrt{B/3A}$ and derive new Feynman rules.

Not possible in usual string perturbation theory since we do not have separate tadpole graphs.

Result: Tadpole divergence in integration over $M_{g,n}$.



In contrast, in string field theory we can deal with mass renormalization and vacuum shift by following the standard procedure in quantum field theory.

Vacuum shift and mass renormalization in string field theory

Step 1. Construct the 1PI effective action

$\{A_1 \cdots A_N\}_{1PI}$: 1PI amplitude of states $A_1, \cdots, A_N$ computed from the string field theory

$$\{A_1 \cdots A_N\}_{1PI} = \sum_g (g_S)^{2g} \int_{\mathcal{R}_{g,N}} \langle \cdots A_1 \cdots A_N \rangle_{g,N}$$

$\mathcal{R}_{g,N}$: a subspace of $M_{g,N}$ corresponding to sum of 1PI diagrams

$$S_{1PI} = \frac{1}{g_S^2} \left[-\frac{1}{2} \langle \phi | c_0^- Q_B \, G | \phi \rangle + \langle \phi | c_0^- Q_B | \psi \rangle + \sum_{N=1}^{\infty} \frac{1}{N!} \{\psi^N\}_{1PI} \right]$$

Step 2: Derive equations of motion from S_{1PI}

$$Q_B(G|\phi\rangle - |\psi\rangle) = 0$$

$$Q_B|\phi\rangle + \sum_{N=0}^{\infty} \frac{1}{N!}[\psi^N] = 0$$

$[A_1 \cdots A_N] \in \mathcal{H}$ is defined via:

$$\langle A|c_0^-[A_1 \cdots A_N]\rangle = \{A\, A_1 \cdots A_N\}_{1PI}$$

Combine two equations to get the interacting field equation:

$$Q_B|\psi\rangle + \sum_{N=0}^{\infty} \frac{1}{N!}\, G\, [\psi^N] = 0$$

Systematically construct the vacuum solution in the zero momentum sector in power series in g_s:

$$Q_B|\psi_v\rangle + \sum_{N=0}^{\infty} \frac{1}{N!}\, G\, [\psi_v^N] = 0$$

Step 3. Define

$$|\chi\rangle = |\psi\rangle - |\psi_{\mathbf{v}}\rangle$$

and expand the action in powers of $|\chi\rangle$ to derive new Feynman rules.

For mass renormalization, analyze linearized equations of motion of $|\chi\rangle$ in the background $|\psi_{\mathbf{v}}\rangle$.

Result:

$$\mathbf{Q_B}|\chi\rangle + \sum_{\mathbf{N}=0}^{\infty} \frac{\mathbf{1}}{\mathbf{N}!} \mathbf{G}\,[\psi_{\mathbf{v}}^{\mathbf{N}}\chi] = \mathbf{0}$$

Solve this equation systematically in a power series in g_s

If the solution exists at $p^2 + M^2 = 0$ then M is the renormalized mass.

Solutions which exist for all p are pure gauge

Details of the iterative construction of the vacuum solution:

Suppose $|\psi_k\rangle$ is the solution to order ${g_s}^k$. $(|\psi_0\rangle = 0)$

P: projection operator to $L_0^+ \equiv L_0 + \bar{L}_0 = 0$ states.

Then

$$|\psi_{k+1}\rangle = -\frac{b_0^+}{L_0^+} \sum_{N=0}^{k+1} \frac{1}{N!} (1 - P)\, G\, [\psi_k^N] + |\xi_{k+1}\rangle\,,$$

$|\xi_{k+1}\rangle$ is an $L_0^+ = 0$ state satisfying

$$Q_B |\xi_{k+1}\rangle = -\sum_{N=0}^{k+1} \frac{1}{N!} P\, G\, [\psi_k^N] + \mathcal{O}({g_s}^{k+2})\,.$$

$$|\psi_{k+1}\rangle = -\frac{b_0^+}{L_0^+}\sum_{N=0}^{k+1}\frac{1}{N!}(1-P)\,G\,[\psi_k^N] + |\xi_{k+1}\rangle\,,$$

$$Q_B|\xi_{k+1}\rangle = -\sum_{N=0}^{k+1}\frac{1}{N!}P\,G\,[\psi_k^N] + \mathcal{O}({g_s}^{k+2})\,.$$

Possible obstruction to solving these arise from the second equation.

rhs could contain a component along a non-trivial element of BRST cohomology.

– reflects the existence of zero momentum massless tadpoles in perturbation theory.

Unless this equation can be solved we have to declare the vacuum inconsistent.

$$|\psi_{k+1}\rangle = -\frac{b_0^+}{L_0^+}\sum_{N=0}^{k+1}\frac{1}{N!}(1-P)\,G\,[\psi_k^N] + |\xi_{k+1}\rangle\,,$$

$$Q_B|\xi_{k+1}\rangle = -\sum_{N=0}^{k+1}\frac{1}{N!}P\,G\,[\psi_k^N] + \mathcal{O}({g_s}^{k+2})\,.$$

Once these equations have been solved, we do not encounter any further tadpole divergence in perturbation theory.

Note: The full solution $|\psi_v\rangle$ is $|\psi_\infty\rangle$, but in practice we shall stop at some fixed order in g_s.

Similar iterative procedure can be used to solve linearized equations for fluctuations around the vacuum and hence the renormalized masses.

This also allows us to deal with the cases involving multiple solutions, e.g. when a scalar field χ in low energy theory has potential

$$c\, g_s^{-2}\, (\chi^2 - K\, g_s{}^2)^2\,.$$

At order g_s we have three solutions $\chi = 0, \pm g_s\sqrt{K}$.

In 1PI effective string field theory this will be reflected in the existence of multiple solutions for $|\psi_1\rangle$.

The solution corresponding to $\chi = 0$ will have non-zero dilaton one point function at higher order

$\Rightarrow$ an obstruction to extending the corresponding 1PI effective field theory solution to higher order.

The solutions corresponding to $\chi = \pm g_s\sqrt{K}$ will not encounter such obstructions.

This situation arises in SO(32) heterotic string compactification on Calabi-Yau manifolds.

String field theory analysis reproduces all the results expected from supersymmetry.

1. Existence of multiple solutions at low order.

2. Correct one loop renormalized scalar and fermion masses, both at the maximum and the minimum of the potential.

3. Correct value of two loop dilaton tadpole at the maximum.

4. Vanishing dilaton tadpole at the minimum.

Similar iterative approach can also be used to construct solutions in string field theory corresponding to marginal deformations by NSNS or RR fields.

Cho, Collier, Yin

Practical difficulties in pushing this to higher order:

Construction of 1PI effective action requires choosing local coordinates on the world-sheet and PCO locations to define the 1PI vertices.

It is hard to find an explicit systematic procedure for making these choices.

Final result is known to be independent of these choices but we have to make these choices to do computation

Once a systematic procedure for making these choices is found then one could make this into an automated procedure.

So far there are two concrete proposals based on minimal area metric and constant negative curvature metric.

Zwiebach; Moosavian, Pius

Requires more effort to make them into useful computational tools.

Alternative approach: Try to develop a formalism where the independence of physical quantities on the choice of local coordinates and PCO locations is manifest all through the computation.

Conclusion

String field theory was originally designed to study non-perturbative aspects of string theory.

However it is also a useful tool for making perturbation theory well defined.

When the world-sheet approach works fine, we do not need string field theory.

When in doubt, we can invoke string field theory.