

# Pre-BBN Cosmology: Primordial Gravitational Waves and Dark Matter

Fazlollah Hajkarim

Institute for Theoretical Physics,  
Goethe University Frankfurt

October 8, 2019, GGI, Florence, Italy

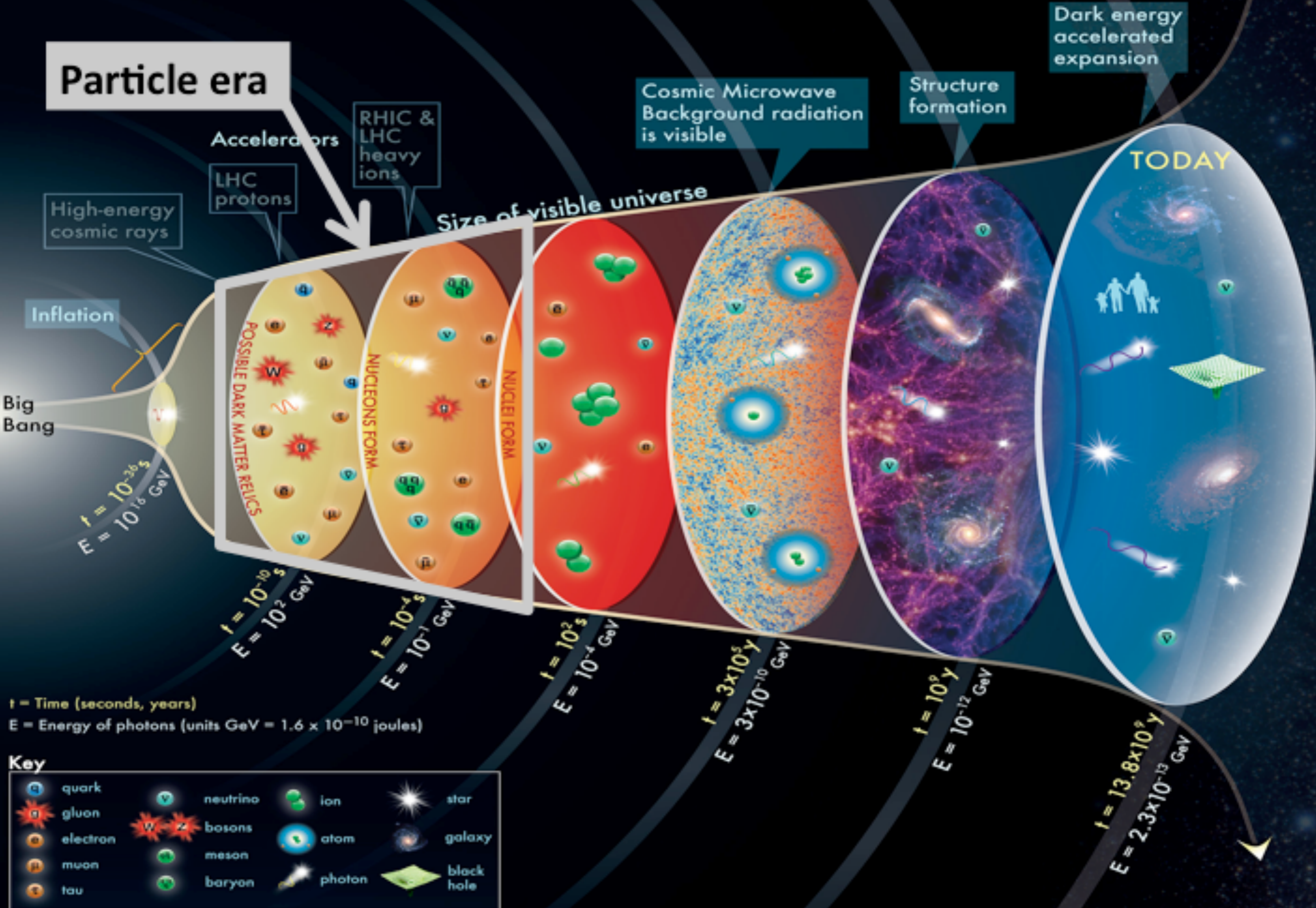


---

## Overview (1st)

- Effects of the QCD equations of state and lepton asymmetry on primordial gravitational waves  
F. H., J. Schaffner-Bielich, S. Wystub, M. M. Wygas, arXiv:1904.01046 (PRD)
- Primordial gravitational waves in nonstandard cosmologies  
N. Bernal, F. H., arXiv:1905.10410 (PRD)
- Thermal history of the early universe and the induced PGW  
F. H., arXiv:1910.XXXXXX

# HISTORY OF THE UNIVERSE



The concept for the above figure originated in a 1986 paper by Michael Turner.

Big Bang

# Primordial Gravitational Waves

Big Bang plus  $10^{-43}$  seconds

quantum-gravity era

Big Bang plus  $10^{-35}$  seconds?

inflation

cosmic microwave background

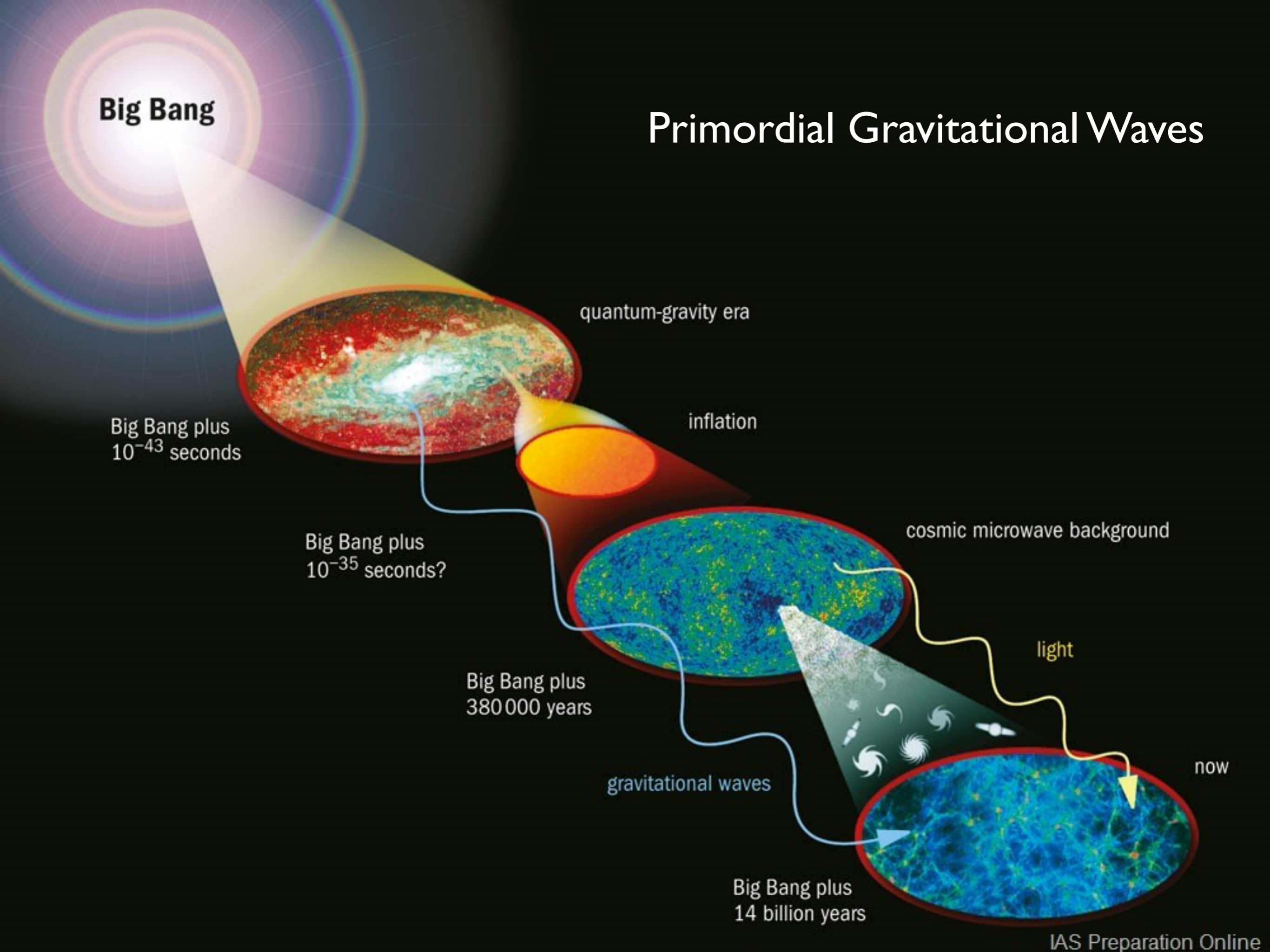
Big Bang plus 380 000 years

light

gravitational waves

now

Big Bang plus 14 billion years



---

# Cosmological (Phase) Transitions and the Standard Model

- QCD transition is a smooth crossover transition.
- EW transition in the SM with the measured Higgs mass is also crossover.
- Cosmological transitions only assuming the SM are not first order phase transitions and do not produce any GW.
- They can leave an imprint on the PGW produced after the Big Bang from the inflationary scenario.

# Equations of Tensor Perturbation and Gravitational Waves Relic Density

Evolution equation for gravitational wave amplitude “h”:

$$h''(k, \eta) + 2\mathcal{H}(\eta)h'(k, \eta) + k^2h(k, \eta) = 0, \quad \mathcal{H} = a'/a = aH$$

Wave number

Conformal time

Hubble rate:

$$H^2 = \frac{8\pi G}{3}\rho_{\text{tot}}$$

Tensor perturbation polarisation modes:

$$h_\lambda(k, \eta) = h_\lambda^{\text{prim}}(k)Y(\eta, k) = \frac{v(k, \eta)}{a(\eta)}$$

Transfer function

Tensor perturbation in the comoving frame:

$$v''(k, \eta) + \left(k^2 - \frac{a''}{a}\right)v(k, \eta) = 0$$

---

PGW energy density:

$$\rho_{\text{GW}}(\eta) = \frac{M_{\text{Pl}}^2}{32\pi a(\eta)^2} \langle h'_{ij}(k, \mathbf{x}) h^{ij'}(k, \mathbf{x}) \rangle$$

$$\langle h'_{ij}(k, \mathbf{x}) h^{ij'}(k, \mathbf{x}) \rangle = \int \frac{dk}{k} \mathcal{P}_T(k, \eta)$$

Tensor power spectrum:

$$\mathcal{P}_T(k, \eta) = \frac{k^3}{\pi^2} \sum_{\lambda} \langle |h_{\lambda}(k, \eta)|^2 \rangle = \mathcal{P}_T^{\text{prim}}(k) [Y(k, \eta)]^2$$

PGW relic density:

$$\Omega_{\text{GW}}(k, \eta) = \frac{\mathcal{P}_T^{\text{prim}}(k)}{12a(\eta)^2 H(\eta)^2} [Y'(k, \eta)]^2$$

At horizon crossing:

$$[Y'(k, \eta)]^2 = k^2 [Y(k, \eta)]^2$$

---

In standard cosmology:

Assuming no phase transition or modified cosmology  $\longrightarrow$  Entropy conservation

Friedmann equation:

$$\frac{a''}{a} = \frac{4\pi G}{3} a^2 (\rho_{tot} - 3p_{tot})$$

Trace anomaly:

$$\frac{I(T)}{T^4} = \frac{\rho_{tot} - 3p_{tot}}{T^4} = T \frac{\partial}{\partial T} \left( \frac{p_{tot}}{T^4} \right)_{\mu/T}$$

Horizon scale:

$$k = a(\eta_{hc}) H(\eta_{hc})$$

We do not consider the effect of neutrinos and photons damping on the PGW, since we are interested in temperatures above 10 MeV. So there is no source term for GW.



---

PGW relic density:

$$\Omega_{\text{GW}}(k, \eta_0) \propto \Omega_{\text{GW}}(k, \eta_{\text{hc}}) \propto k^5 |v(k, \eta_{\text{hc}})|^2$$

It gives roughly a flat spectrum

PGW relic density and the SM equation of state:

$$\Omega_{\text{GW}}(k, \eta_0) \propto \rho_{\text{tot}}(T_{\text{hc}}) s_{\text{tot}}(T_{\text{hc}})^{-4/3}$$

F. H., J. Schaffner-Bielich, S. Wüsteb, M. M. Wygas, arXiv:1904.01046 (PRD)

QCD affects the GW background in the frequency range of pulsar timing arrays, e.g. EPTA, SKA, etc.

# Thermodynamics of the Standard Model

## Noninteracting part in ideal gas limit

Energy density: 
$$\rho_{\text{tot}}(T, \mu) = \sum_i \frac{g_i}{2\pi^2} \int_{m_i}^{\infty} dE \times E^2 \sqrt{E^2 - m_i^2} \left( \frac{1}{e^{\frac{E-\mu_i}{T}} \pm 1} \right)$$

Pressure density: 
$$p_{\text{tot}}(T, \mu) = \sum_i \frac{g_i}{6\pi^2} \int_{m_i}^{\infty} dE \times (E^2 - m_i^2)^{3/2} \left( \frac{1}{e^{\frac{E-\mu_i}{T}} \pm 1} \right)$$

Entropy density: 
$$T s_{\text{tot}}(T, \mu) = \rho_{\text{tot}}(T, \mu) + p_{\text{tot}}(T, \mu) - \sum_i \mu_i n_i(T, \mu_i)$$

Number density of each particle:

$$n_i(T, \mu_i) = \frac{g_i}{2\pi^2} \int_{m_i}^{\infty} dE \times E \sqrt{E^2 - m_i^2} \left( \frac{1}{e^{\frac{E-\mu_i}{T}} \pm 1} - \frac{1}{e^{\frac{E+\mu_i}{T}} \pm 1} \right)$$

## Interacting part from lattice QCD

$$\chi_{ab} = \frac{\partial^2 p^{QCD}(T, \mu)}{\partial \mu_a \partial \mu_b} = \chi_{ba} \quad p^{QCD}(T, \mu) = p^{QCD}(T, 0) + \frac{1}{2} \mu_a \chi_{ab}(T) \mu_b$$

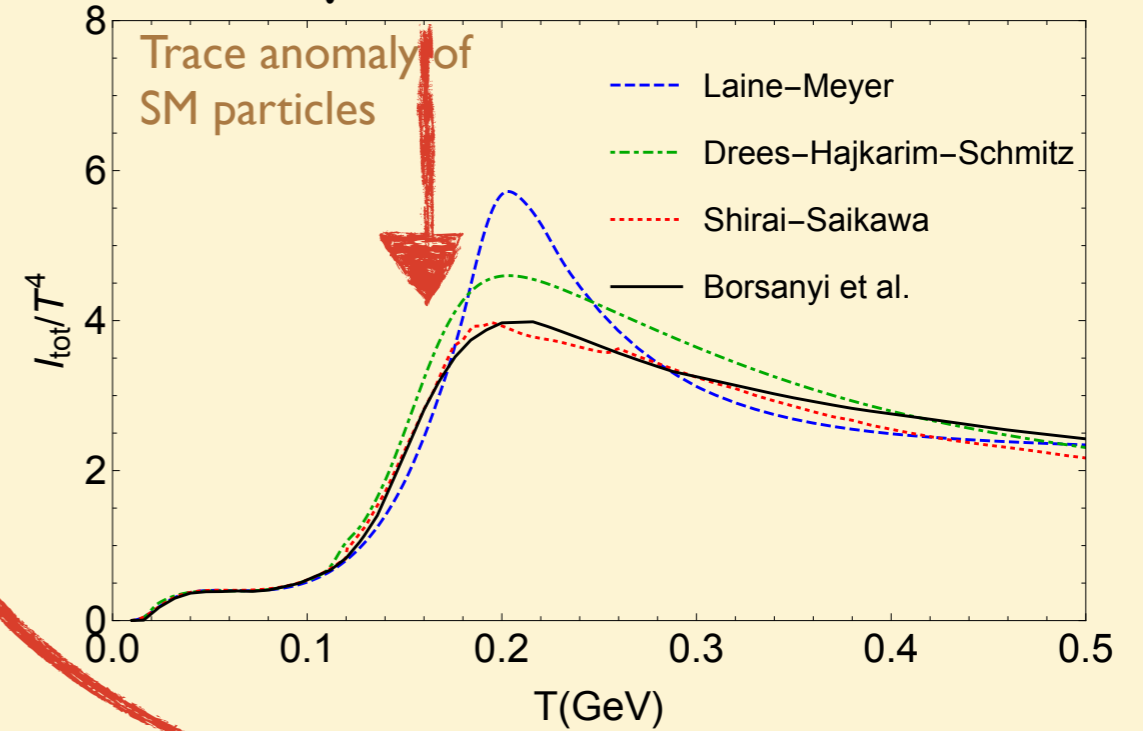
↙ **Susceptibility**
↘ **Conserved charges: Q, B**

# Effects of the QCD Equation of State on the Primordial Gravitational Waves

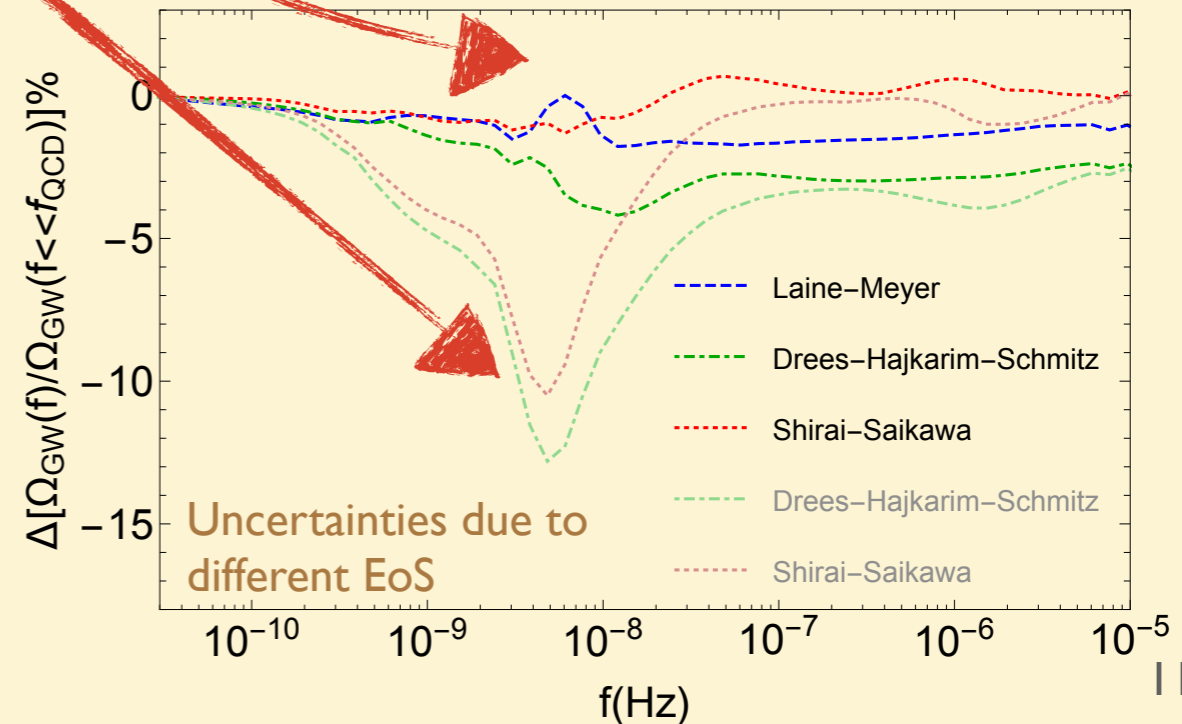
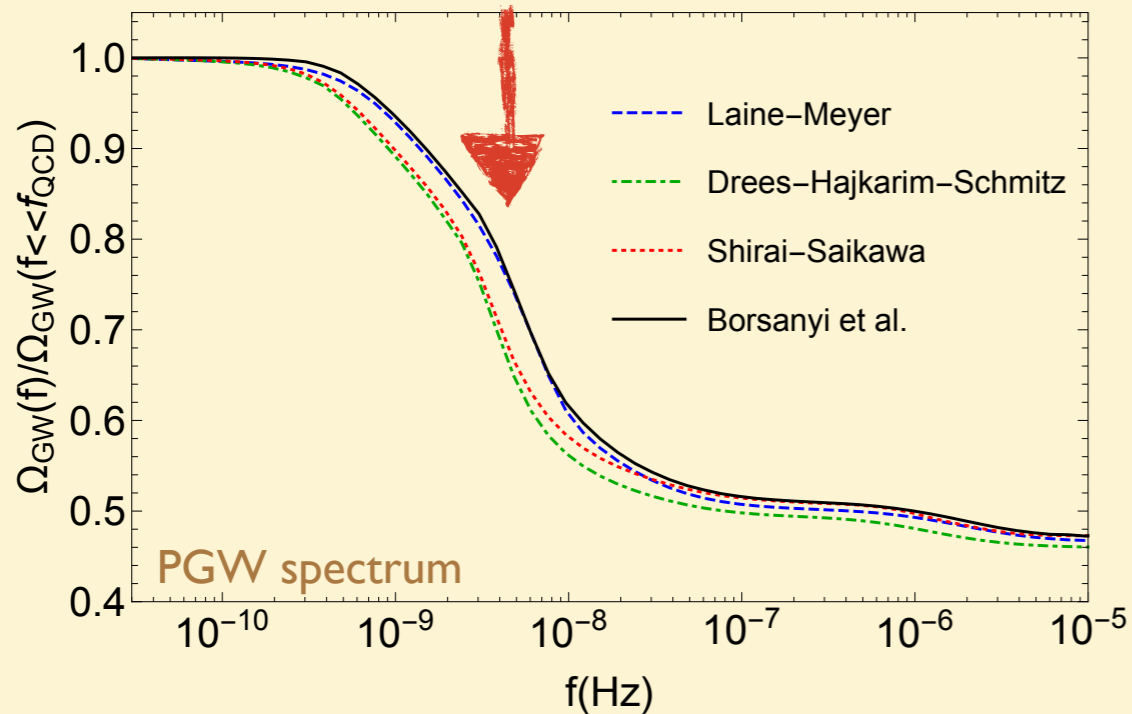
Deviation in the PGW relic due to different QCD equations of state is up to 3.5 %.

Neutrino decoupling and electron annihilation lead to 10% effects in the predicted relic density of the PGW (frequency shift).

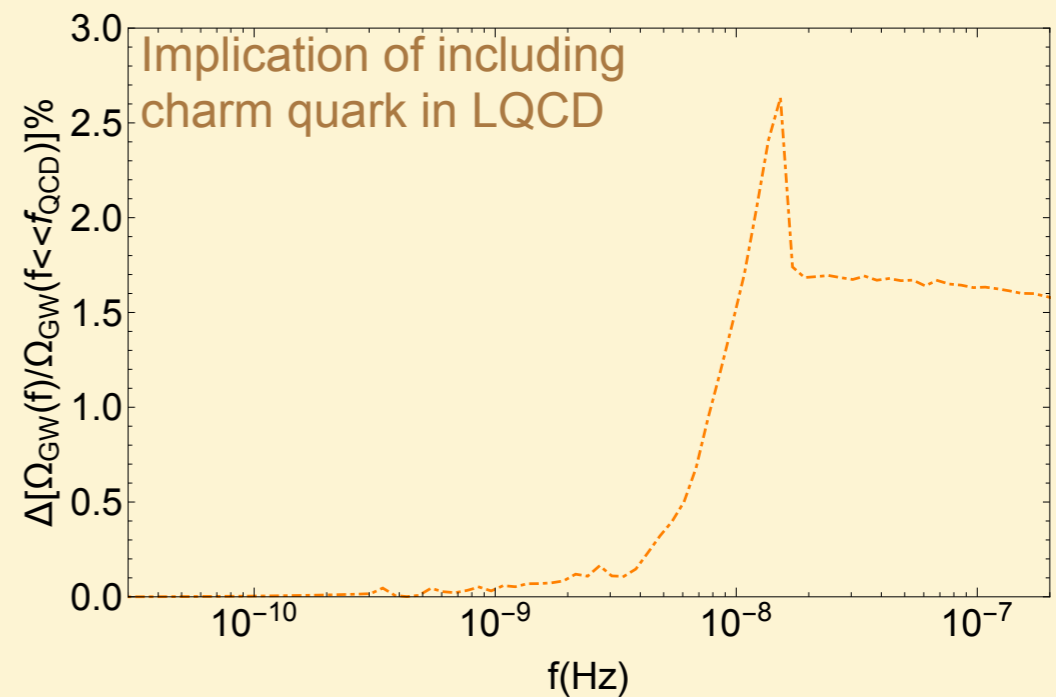
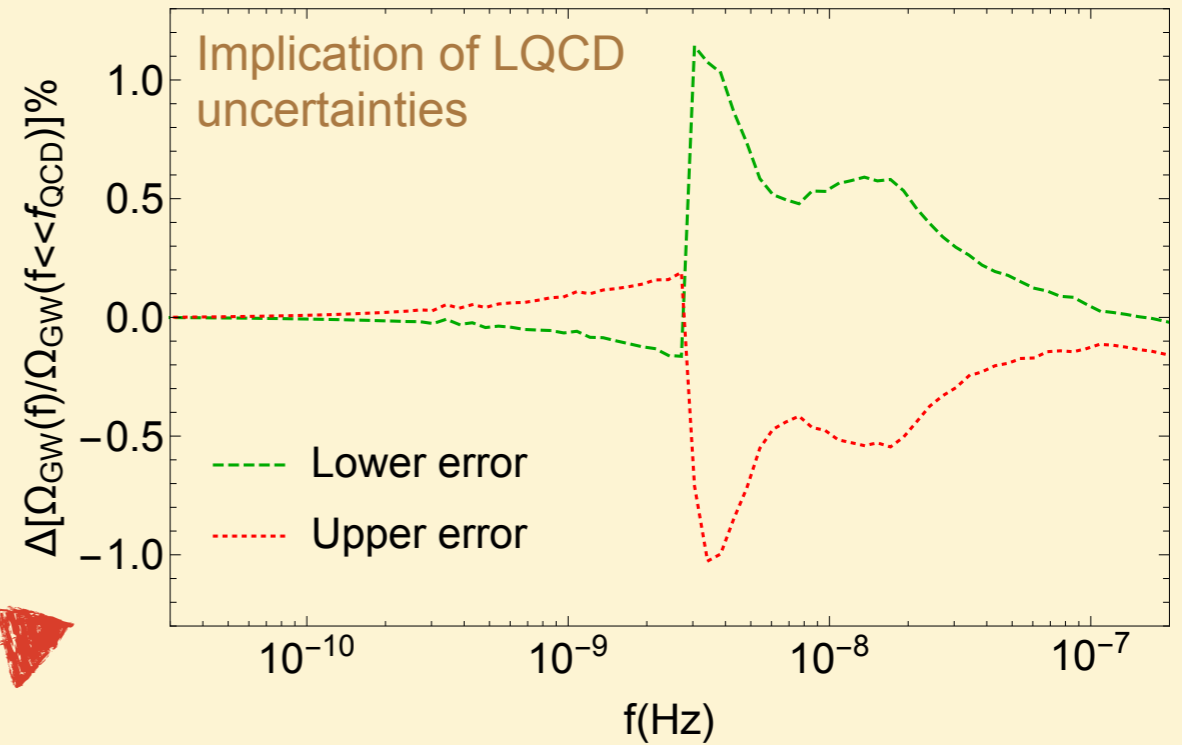
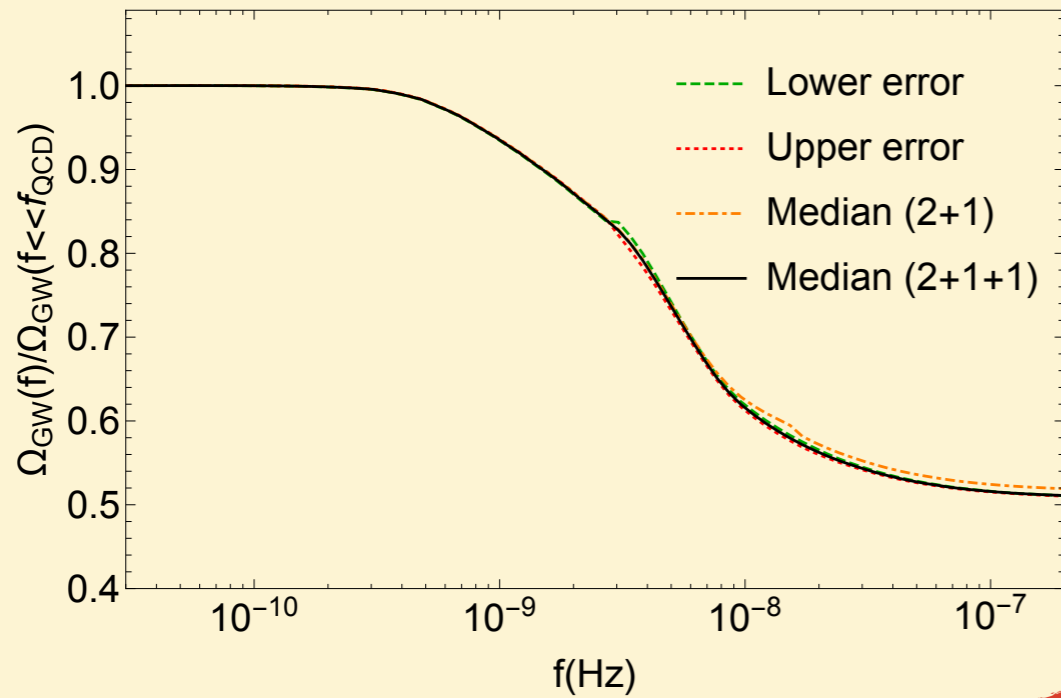
$$T_{\text{QCD}} \sim 150 \text{ MeV}$$



$$f_{\text{QCD}} \approx 3 \times 10^{-9} \text{ Hz}$$



# Charm Quark and Lattice Uncertainties Imprints on the PGW



Effect of lattice QCD error bars on the PGW energy spectrum is up to  $\sim 1\%$ .

Effect of including lattice QCD data with charm will be up to  $\sim 2.5\%$ .

# Vanishing and Nonvanishing Lepton Asymmetry & PGW

Assuming nonvanishing lepton asymmetry leads to nonzero lepton chemical potentials in cosmology. Using lattice QCD equation of state for nonvanishing chemical potentials the evolution of SM chemical potentials are possible.

Constraint equations for finding the evolution of SM chemical potentials:

I) Constraint on baryon asymmetry:

$$b = \frac{n_B}{s} \approx 8 \times 10^{-11}$$

$$bs(T, \mu) = \sum_i b_i n_i(T, \mu_i)$$

II) Constraint on lepton asymmetry:

$$l = \frac{n_L}{s} \lesssim 0.012$$

from CMB and number of effective neutrinos

$$l_f s(T, \mu) = n_f(T, \mu_f) + n_{\nu_f}(T, \mu_{\nu_f}), f = e, \mu, \tau$$

III) Charge neutrality of the universe:

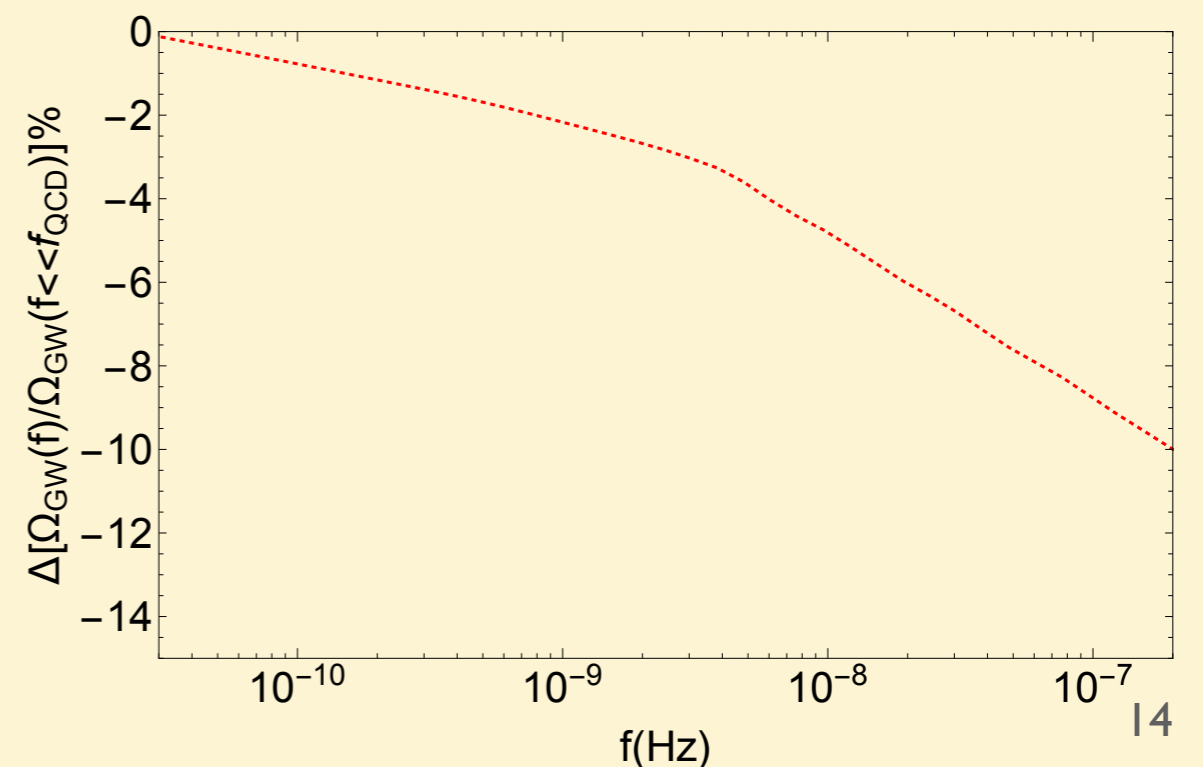
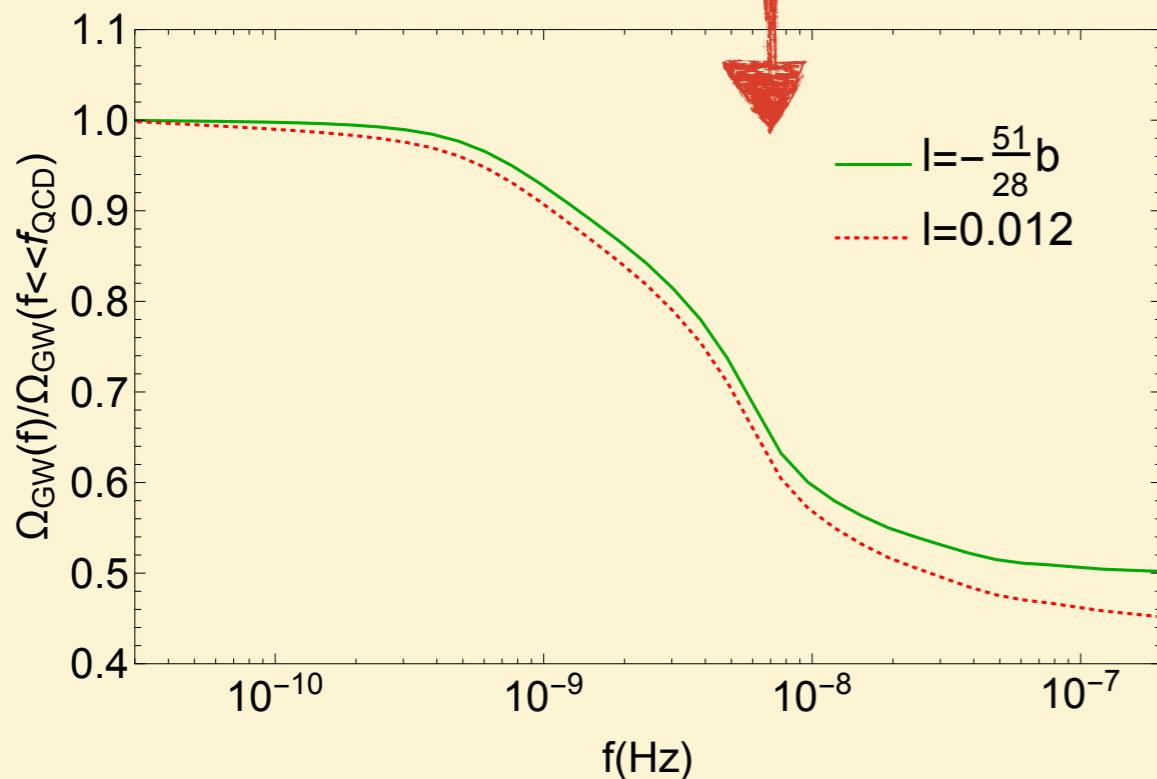
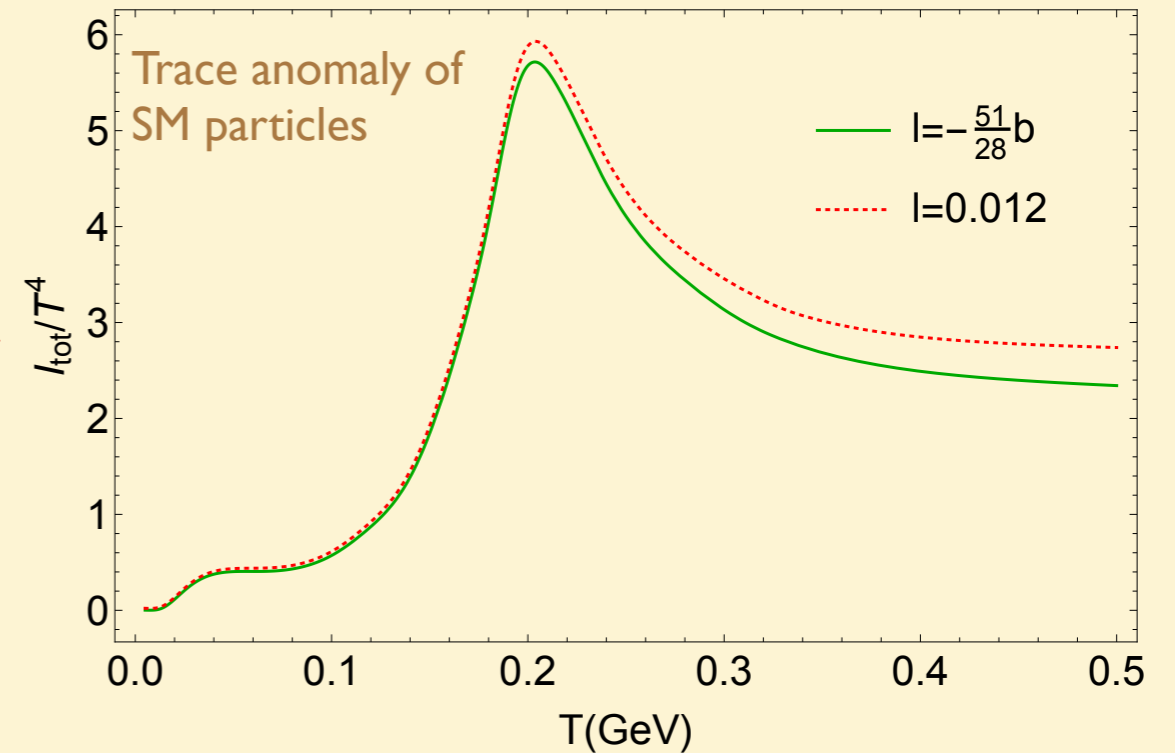
$$0 = \sum_i q_i n_i(T, \mu_i)$$

# Vanishing and Nonvanishing Lepton Asymmetry & PGW

Non-vanishing lepton asymmetry leads to non-vanishing lepton chemical potentials, which affect the cosmic trajectory, equation of state and the PGW.

Wygas et al. 2018

Large lepton asymmetry can affect the PGW spectrum up to ~10%. Effects might be larger allowing for lepton flavour asymmetries.



---

## Nonstandard cosmologies and the PGW

- The history of universe before big bang nucleosynthesis is unknown.
- UV completion theories predict nonstandard cosmologies (by new scalar fields) beyond the standard radiation dominated era before BBN and after inflationary epoch.
- The production mechanism of dark matter in the early universe is unknown. No hints for DM produced from the standard radiation dominated scenario!

N. Bernal, F. H., [arXiv:1905.10410](#) (PRD)

Friedmann equations in nonstandard cosmology:

Equation of state parameter for the dominant component

$$\frac{d\rho_\phi}{dt} + 3(1 + \omega_\phi)H\rho_\phi = -\Gamma_\phi\rho_\phi$$

$$\frac{ds_R}{dt} + 3Hs_R = +\frac{\Gamma_\phi\rho_\phi}{T}$$

$$H^2 = \frac{\rho_\phi + \rho_R + \rho_m + \rho_\Lambda}{3M_{Pl}^2}$$

Temperature at which  $\phi$  decays:

$$T_{\text{dec}}^4 = \frac{90}{\pi^2 g_\star(T_{\text{dec}})} M_{Pl}^2 \Gamma_\phi^2$$

SM degrees of freedom

Decay width

Ratio of initial densities:

$$\xi \equiv \left. \frac{\rho_\phi}{\rho_R} \right|_{T=T_{\text{max}}}$$

Initial value for temperature in the numerical calculation (not physical only to cover all possible cases that cross the experimental constraints):

$$T_{\text{max}} = 10^{14} \text{ GeV}$$

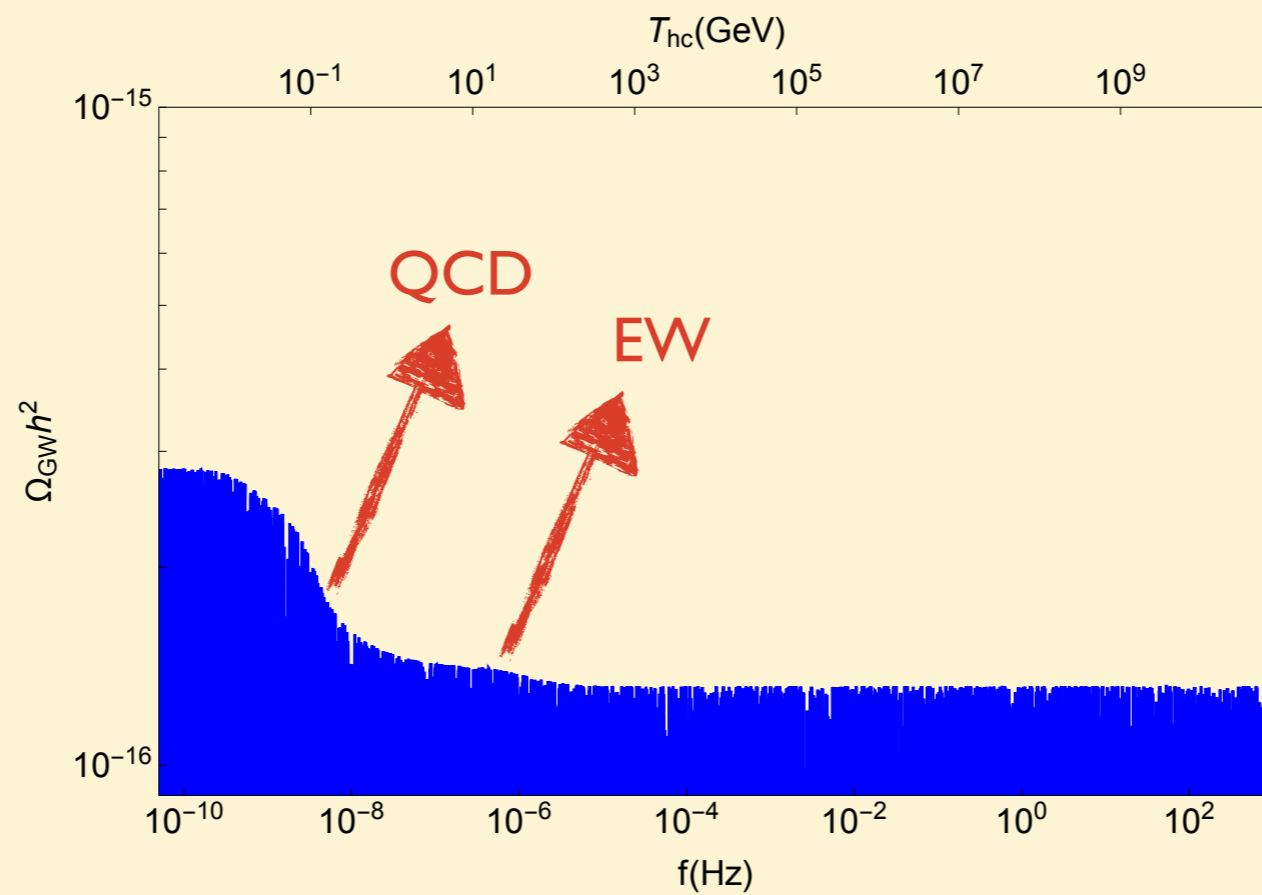
Organizer





Scale independent tensor power spectrum and the scale of inflation:

$$\mathcal{P}_T(k) = \frac{2}{3\pi^2} \frac{V_{\text{inf}}}{M_{Pl}^4}, \quad V_{\text{inf}}^{1/4} = 1.5 \times 10^{16} \text{ GeV}$$



# Scale Dependent Power Spectrum and Constraints

Tensor power spectrum and its scale dependence:

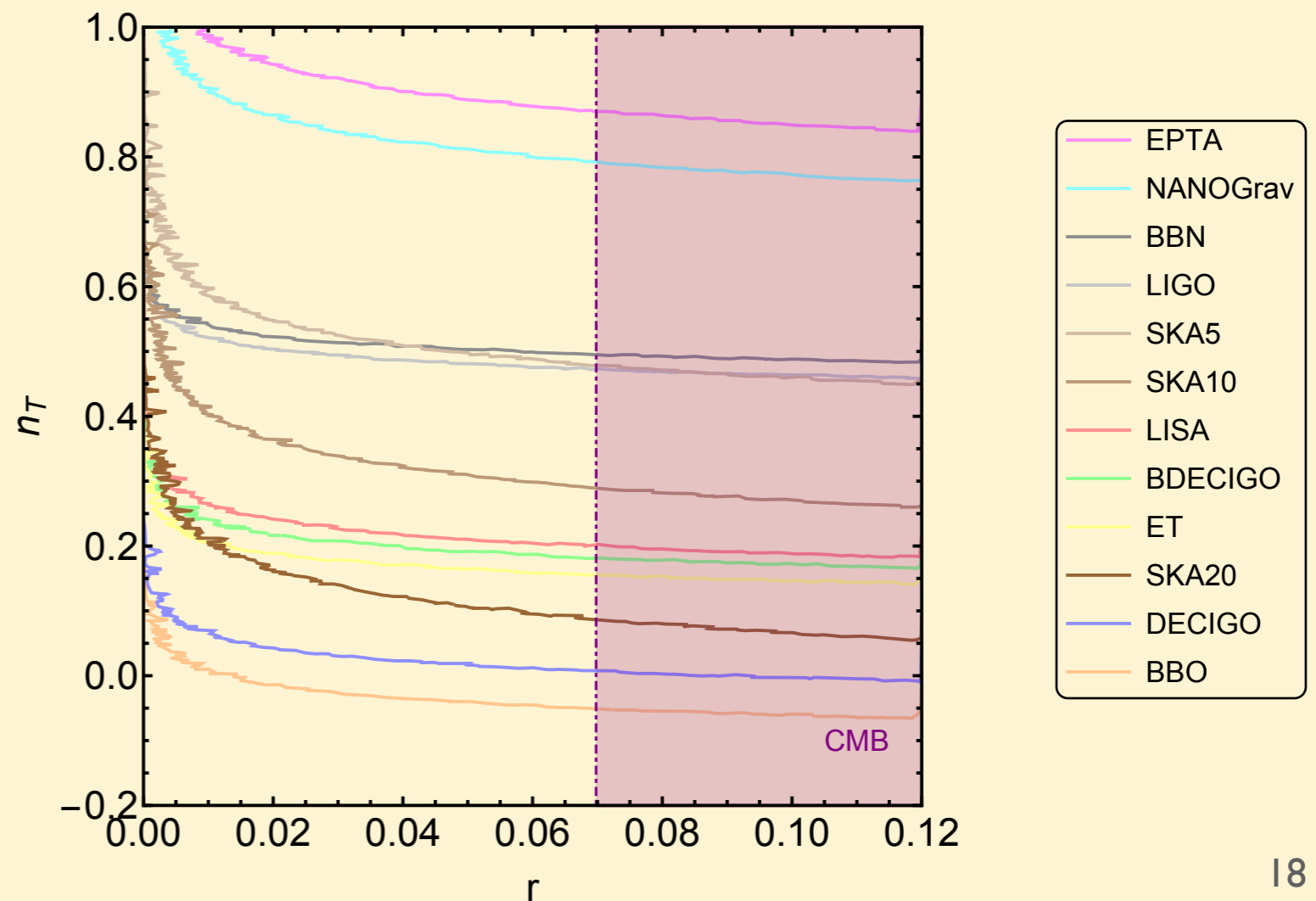
$$\mathcal{P}_T(k) = A_T \left( \frac{k}{\tilde{k}} \right)^{n_T} \longrightarrow \text{Tensor tilt}$$

Pivot scale  $\longleftarrow$

Tensor to scalar perturbation ratio:

$$r \equiv \frac{A_T}{A_S}$$

By fixing the scalar perturbation amplitude from Planck data:



---

Hubble rate during radiation domination:

$$H_R \propto a^{-2}$$

Hubble rate in  $\phi$  domination:

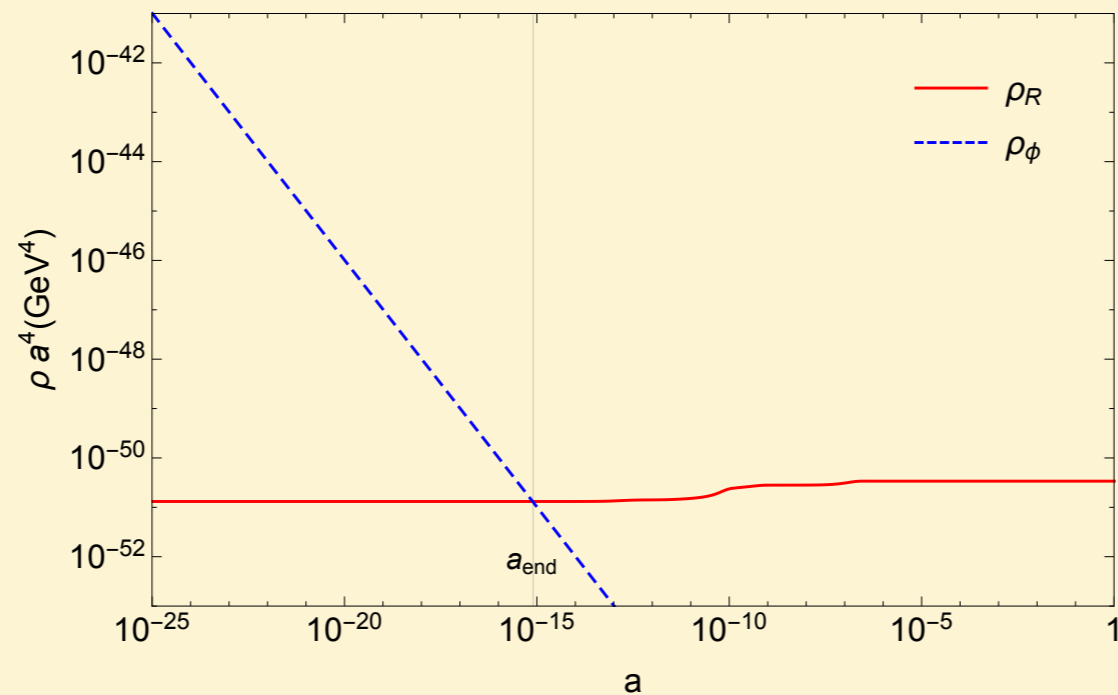
$$H_\phi \propto a^{-\frac{3}{2}(1+\omega_\phi)}$$

The PGW relic for modes come inside the horizon during  $\phi$  domination:  $\Omega_{GW} \propto k^{-2\frac{1-3\omega_\phi}{1+3\omega_\phi}}$

The minimum value of ratios to have a modified radiation density due to nonstandard cosmological scenario ( $\rho_R \propto a^{-4} \rightarrow a^{-\frac{3}{2}(1+\omega_\phi)}$ ):

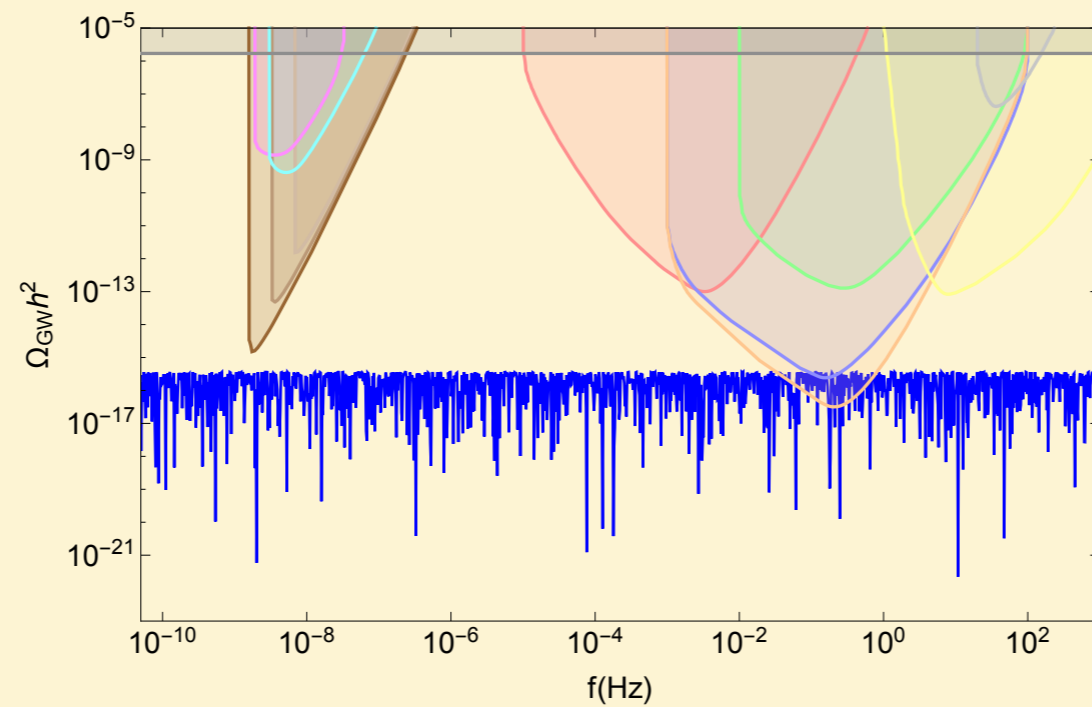
$$\xi_{\min} \approx \left[ \left( \frac{g_\star(T_{\max})}{g_\star(T_{\text{dec}})} \right)^{\frac{1}{4}} \frac{T_{\max}}{T_{\text{dec}}} \right]^{3\omega_\phi - 1}$$

$$\omega_\phi = 1/3, \xi = 10^{25}$$



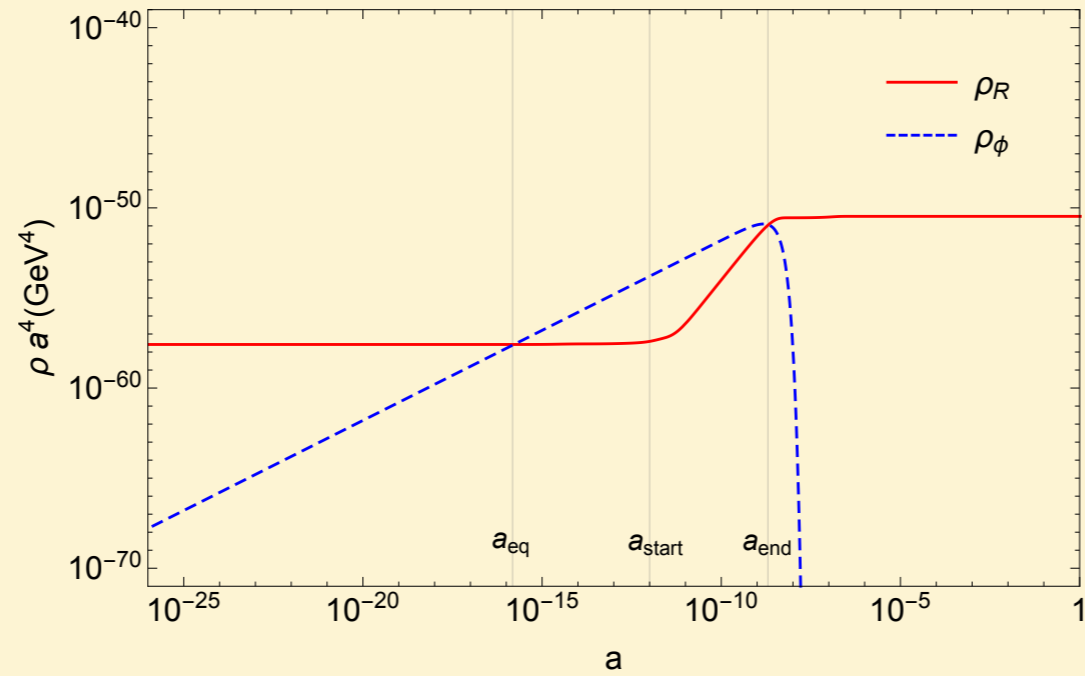
Radiation  
domination like  
scenario

$$T_{\text{dec}} = 10 \text{ MeV}$$



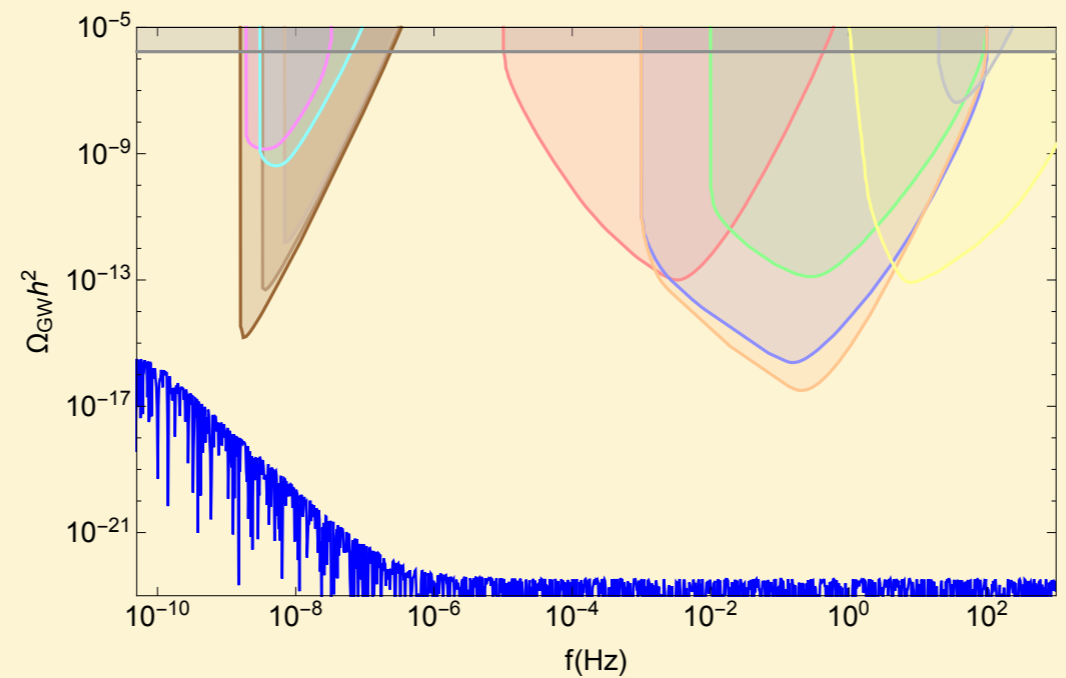
- SKA5
- SKA10
- SKA20
- EPTA
- NANOGrav
- LISA
- BDECIGO
- DECIGO
- BBO
- ET
- LIGO
- BBN

$$\omega_\phi = 0, \xi = 10^{-11}$$



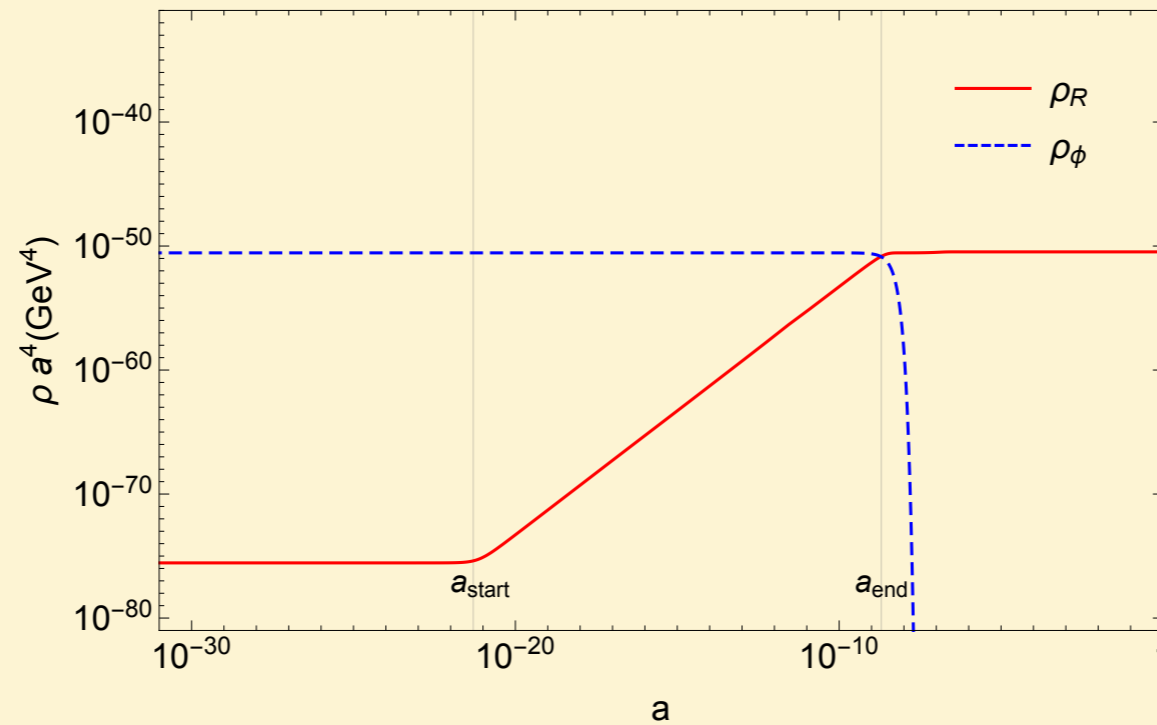
Modulus or matter  
domination like  
scenario

$$T_{\text{dec}} = 10 \text{ MeV}$$



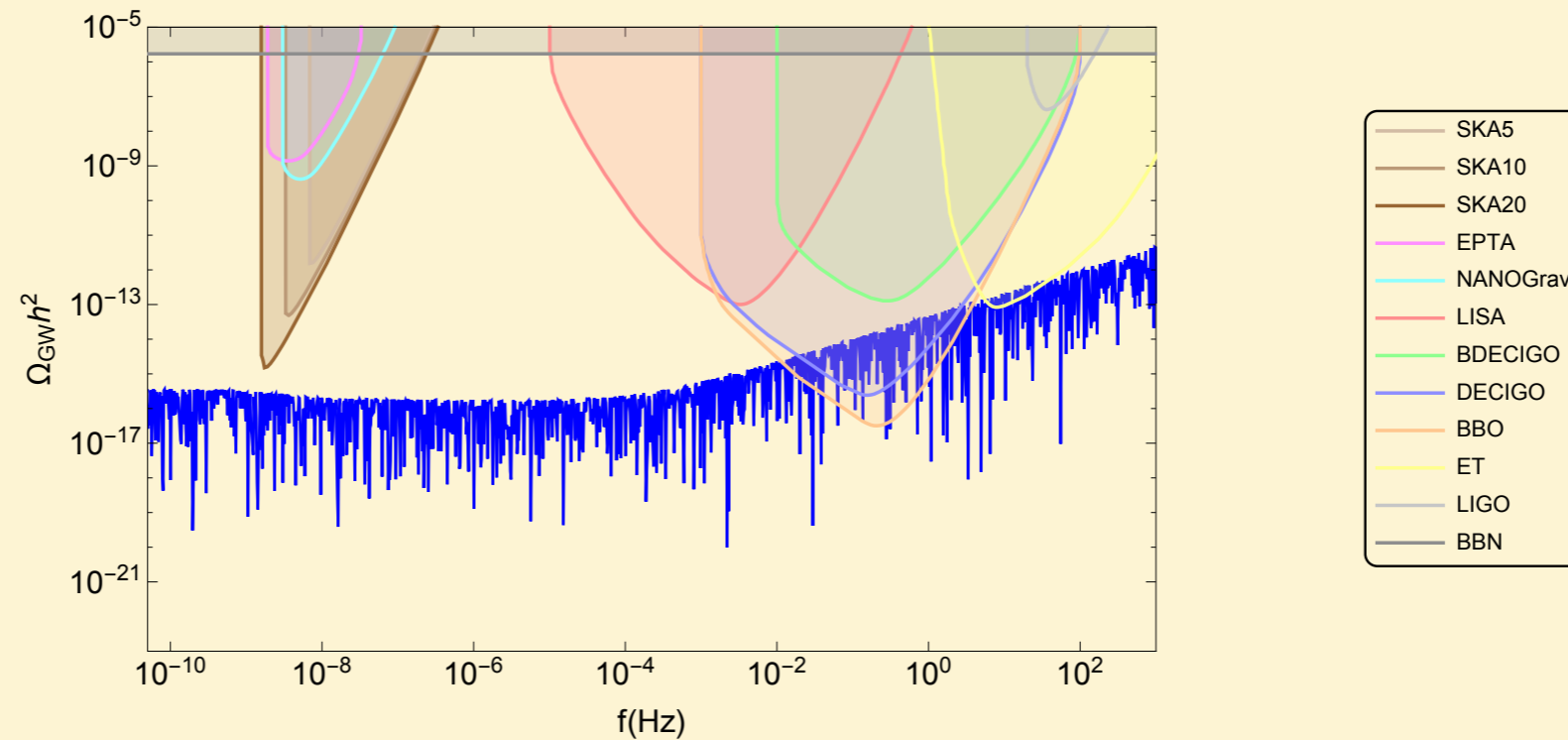
- SKA5
- SKA10
- SKA20
- EPTA
- NANOGrav
- LISA
- BDECIGO
- DECIGO
- BBO
- ET
- LIGO
- BBN

$$\omega_\phi = 2/3, \xi = 10^{10}$$



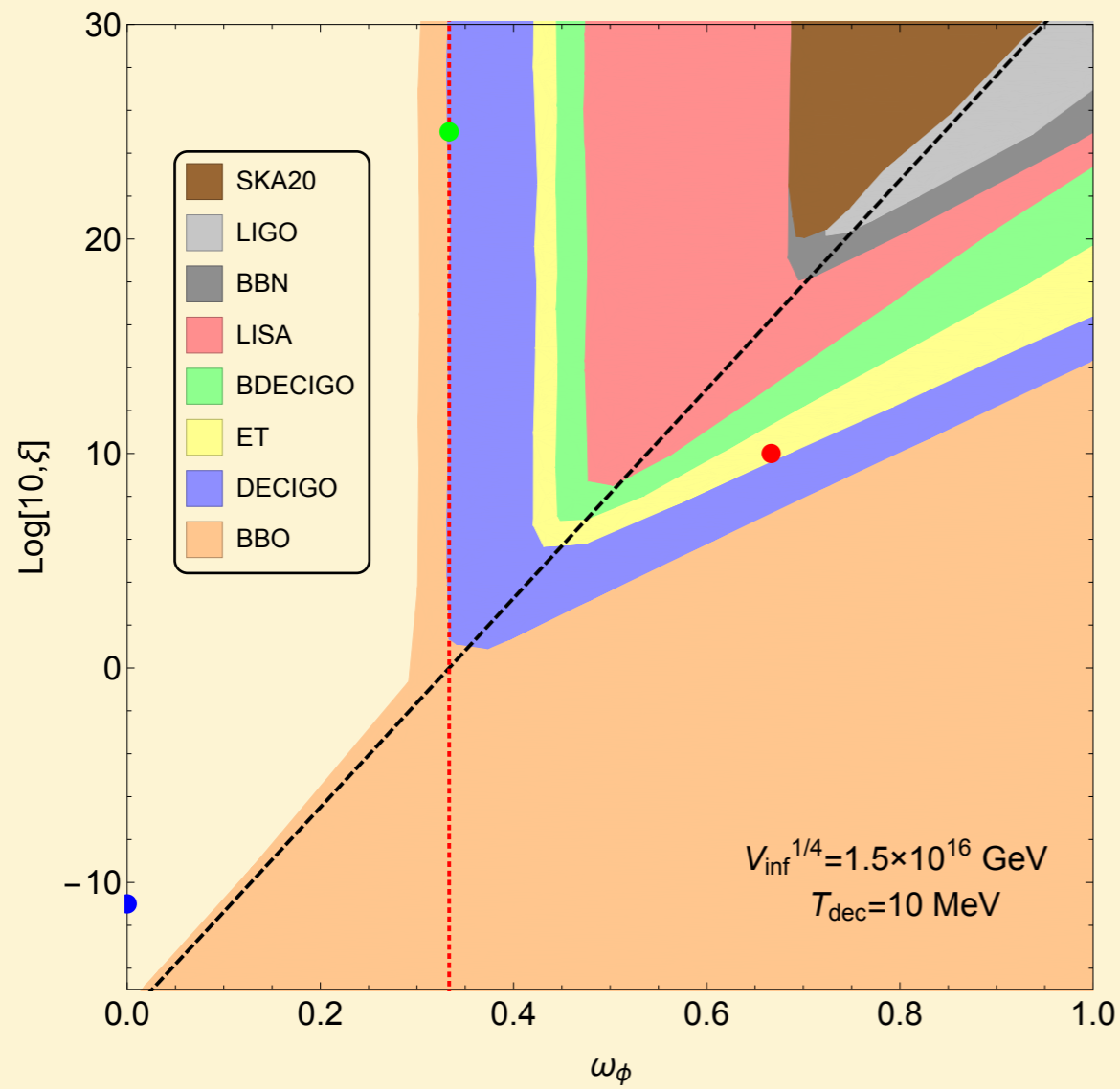
Kination domination  
like scenario

$$T_{\text{dec}} = 10 \text{ MeV}$$

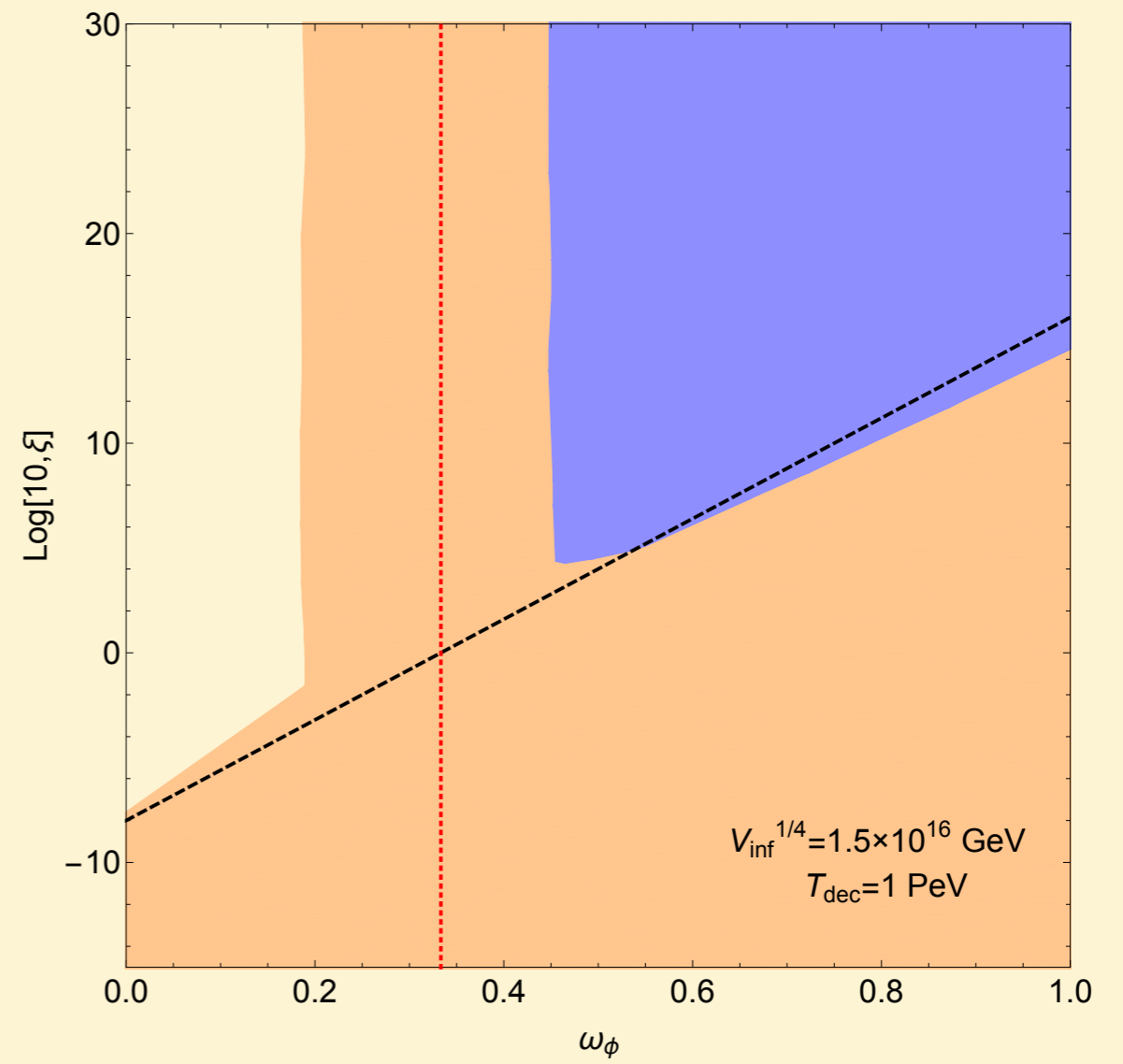


Scanning over the parameter space of  $[\omega_\phi, \xi]$ :

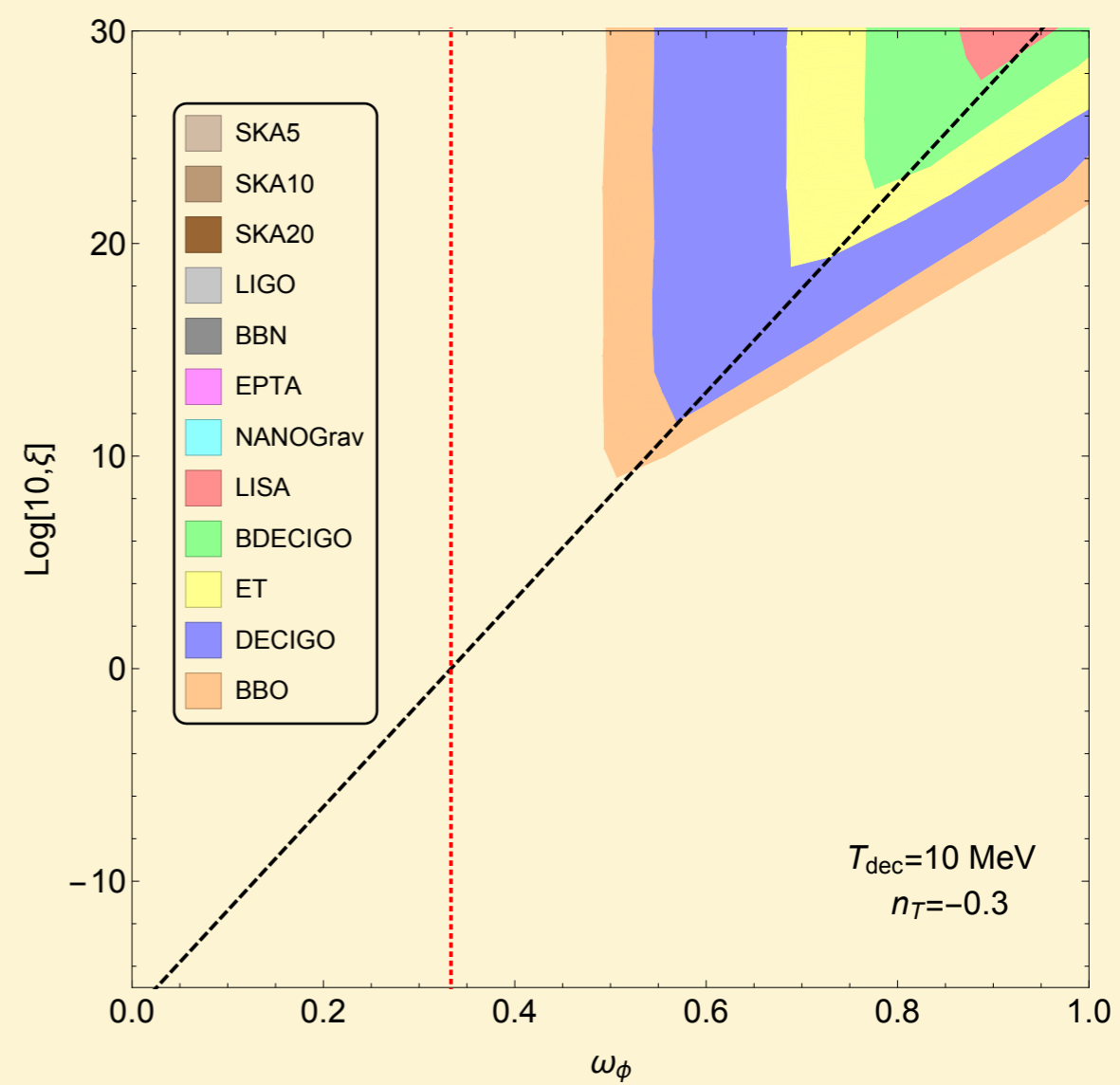
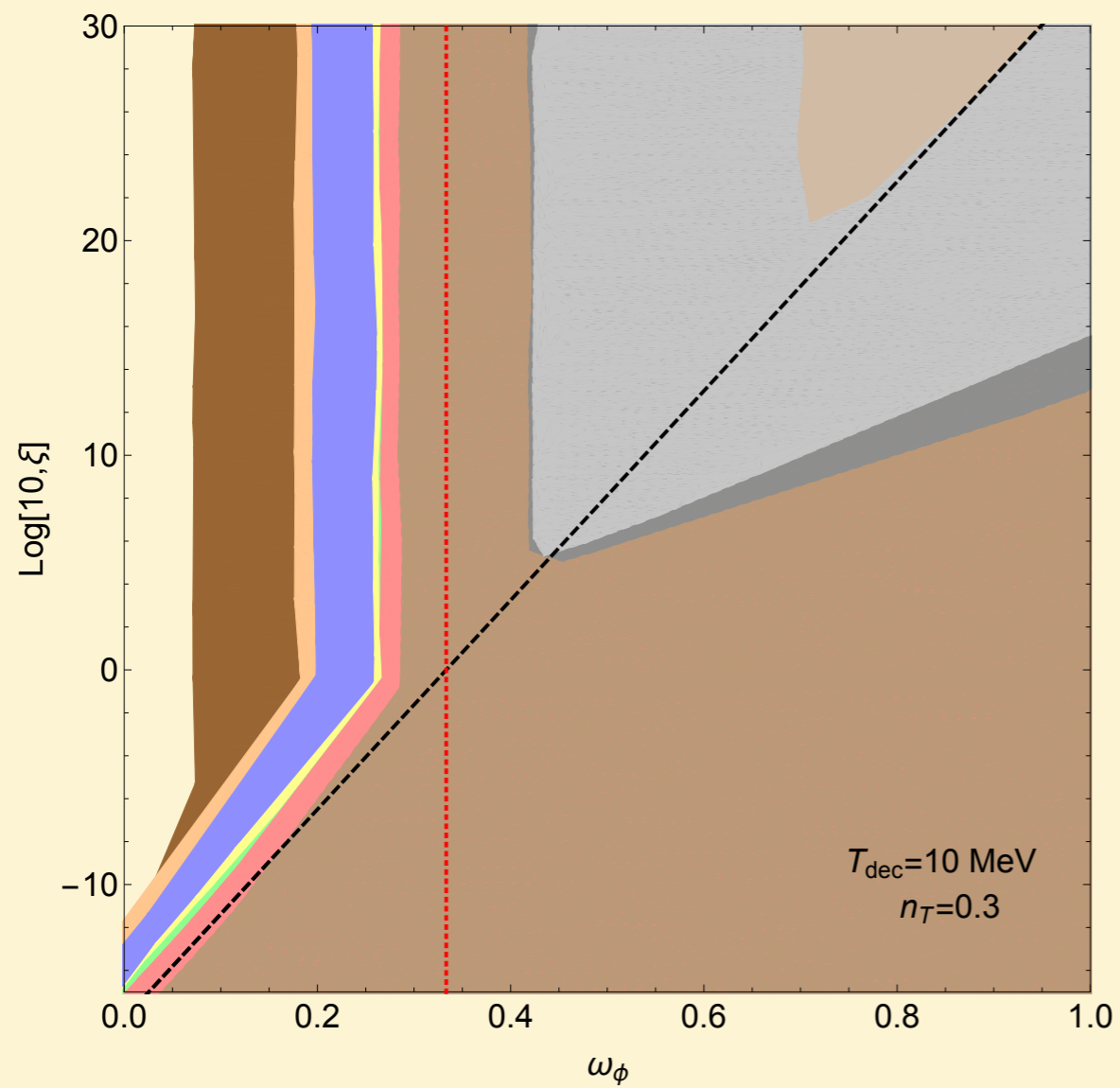
$T_{\text{dec}} = 10 \text{ MeV}$



$T_{\text{dec}} = 1 \text{ PeV}$



Scanning over the parameter space of  $[\omega_\phi, \xi]$ :





---

## Induced (2nd order) PGW from Scalar Perturbations

- In case the tensor-to-scalar ratio is very small then the first order PGW will be negligible and might not be in the range accessible by future experiments.
- There is not any lower limit on “r” at first order.
- The second order tensor perturbations induced from scalar perturbation will be the lower limit on the GW background.

$$h''_{\mathbf{k}}(\eta) + 2\mathcal{H}h'_{\mathbf{k}}(\eta) + k^2 h_{\mathbf{k}}(\eta) = 4S_{\mathbf{k}}(\eta)$$

## 2nd Order Tensor Perturbation Sourced by Scalar Perturbation

Tensor perturbation equation at second order:

$$h''_{\mathbf{k}}(\eta) + 2\mathcal{H}h'_{\mathbf{k}}(\eta) + k^2 h_{\mathbf{k}}(\eta) = 4S_{\mathbf{k}}(\eta)$$

Source from second order scalar perturbation ( $\omega = \frac{p}{\rho}$ ):

$$S_{\mathbf{k}} = \int \frac{d^3q}{(2\pi)^{3/2}} e_{ij}(\mathbf{k}) q_i q_j \left( 2\Phi_{\mathbf{q}}\Phi_{\mathbf{k}-\mathbf{q}} + \frac{4}{3(1+w)} (\mathcal{H}^{-1}\Phi'_{\mathbf{q}} + \Phi_{\mathbf{q}}) (\mathcal{H}^{-1}\Phi'_{\mathbf{k}-\mathbf{q}} + \Phi_{\mathbf{k}-\mathbf{q}}) \right)$$

Second order tensor perturbation in comoving frame:

$$a(\eta)h_{\mathbf{k}}(\eta) = 4 \int^{\eta} d\bar{\eta} \underset{\substack{\downarrow \\ \text{Green's function}}}{G_{\mathbf{k}}(\eta, \bar{\eta})} a(\bar{\eta}) S_{\mathbf{k}}(\bar{\eta})$$

## 2nd Order Tensor Perturbation Sourced by Scalar Perturbation

Evolution of scalar perturbation ( $c_s^2 = \frac{dp}{d\rho}$ ):

$$\Phi_{\mathbf{k}}'' + 3\mathcal{H}(1 + c_s^2)\Phi_{\mathbf{k}}' + (2\mathcal{H}' + (1 + 3c_s^2)\mathcal{H}^2 + c_s^2 k^2)\Phi_{\mathbf{k}} = 0$$

Second order tensor power spectrum:

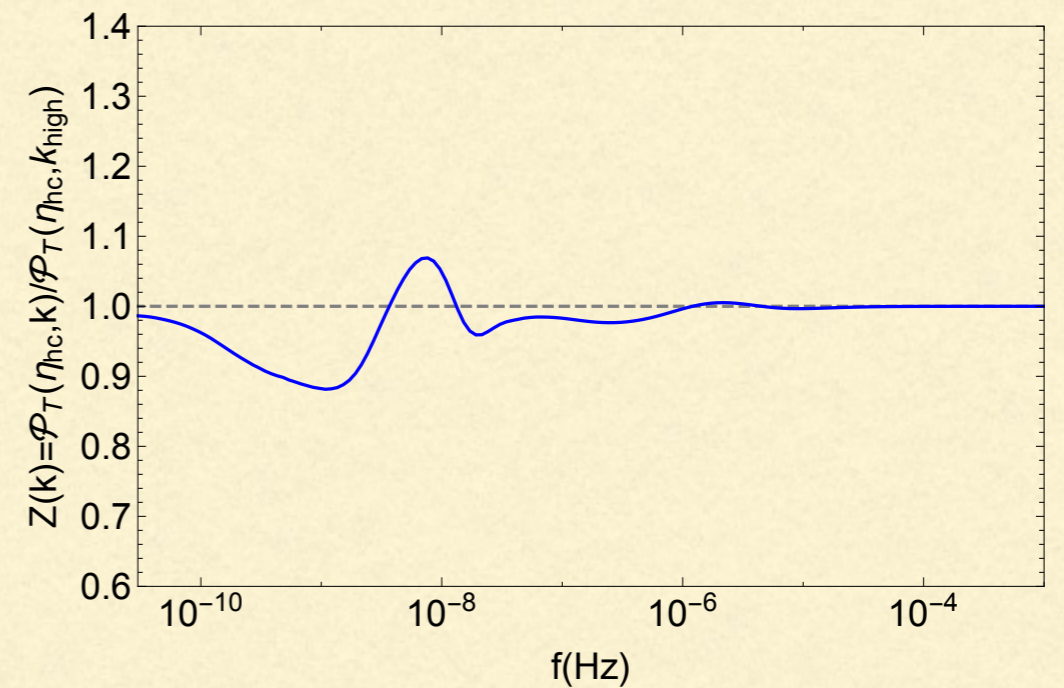
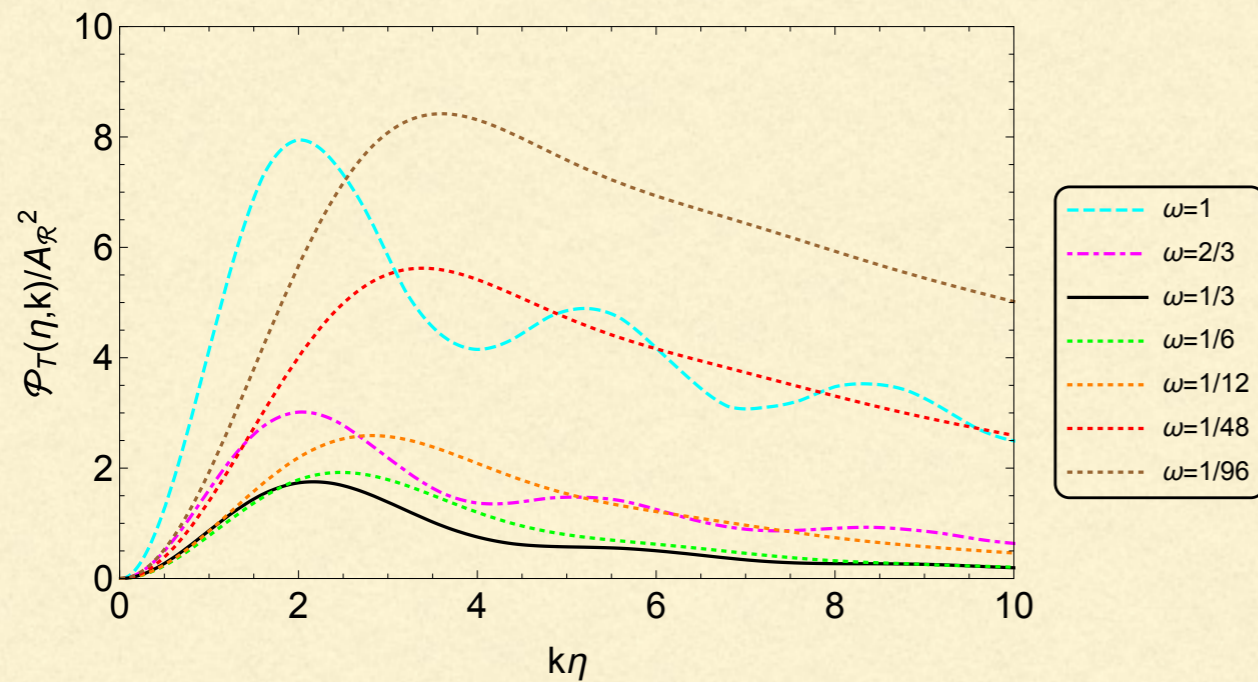
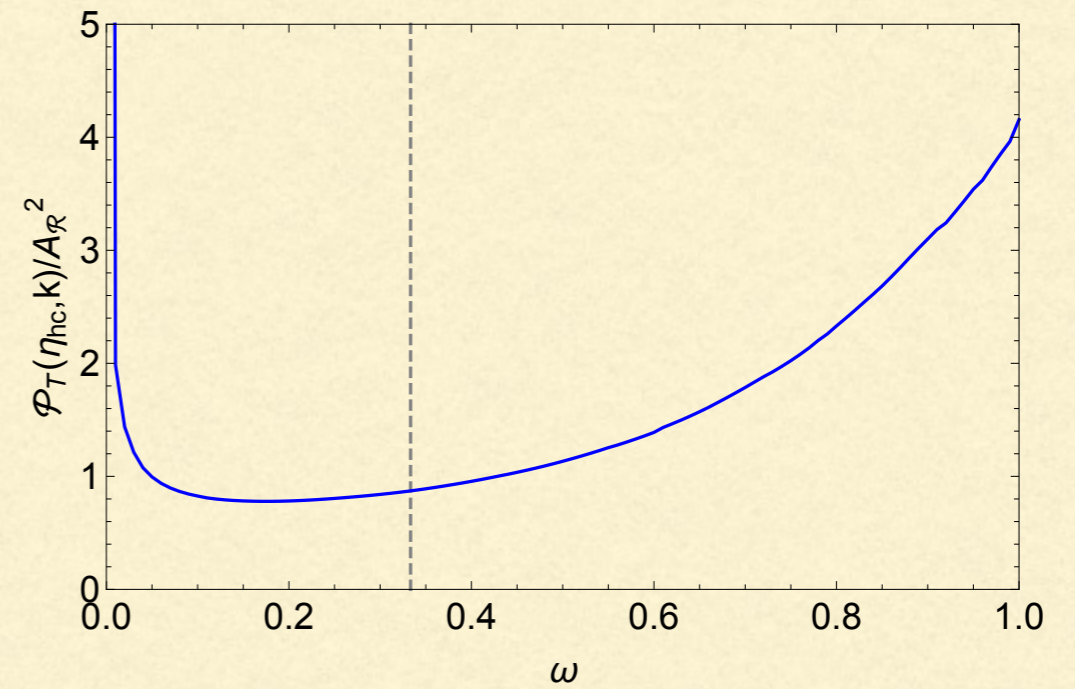
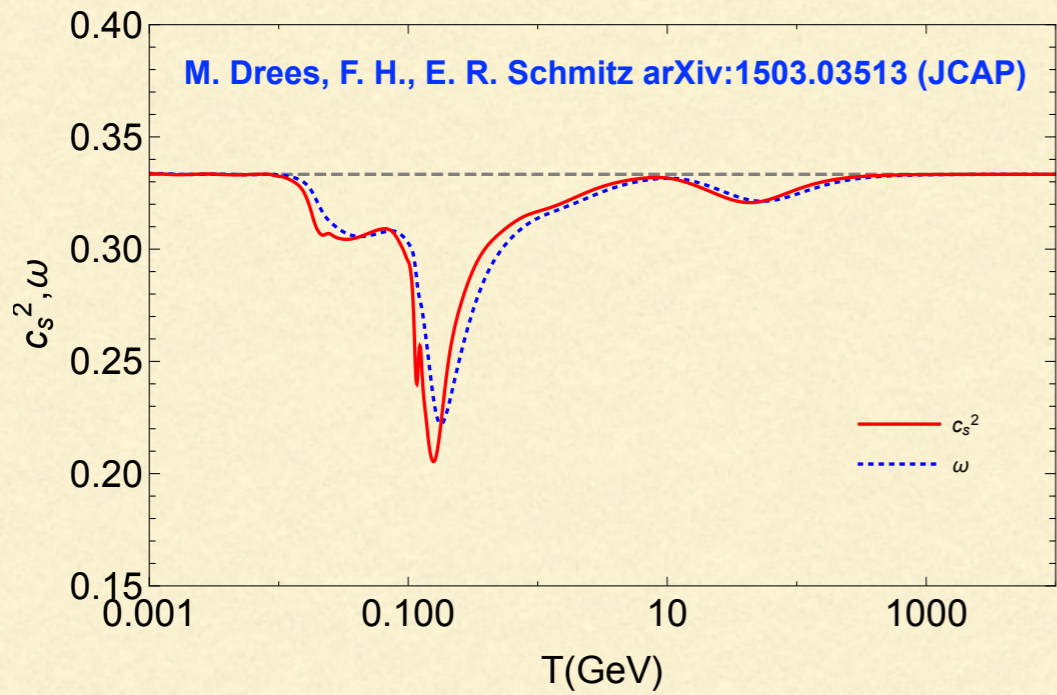
$$\mathcal{P}_T(\eta, k) = 4 \int_0^\infty \int_{|1-v|}^{1+v} dv du \left[ \frac{4v^2 - (1 + v^2 - u^2)^2}{4vu} \right]^2 I^2(v, u, x) \mathcal{P}_{\mathcal{R}}(kv) \mathcal{P}_{\mathcal{R}}(ku)$$

$$I(v, u, x) = \int_0^x d\bar{x} \frac{a(\bar{\eta})}{a(\eta)} k G_k(\eta, \bar{\eta}) f(v, u, \bar{x})$$

$$f(v, u, \bar{x}) = \frac{6(w+1)}{3w+5} \Phi(v\bar{x})\Phi(u\bar{x}) + \frac{12(w+1)}{(3w+5)^2 \mathcal{H}} (\partial_{\bar{\eta}} \Phi(v\bar{x})\Phi(u\bar{x}) + \partial_{\bar{\eta}} \Phi(u\bar{x})\Phi(v\bar{x})) + \frac{12(1+w)}{(3w+5)^2 \mathcal{H}^2} \partial_{\bar{\eta}} \Phi(v\bar{x}) \partial_{\bar{\eta}} \Phi(u\bar{x})$$

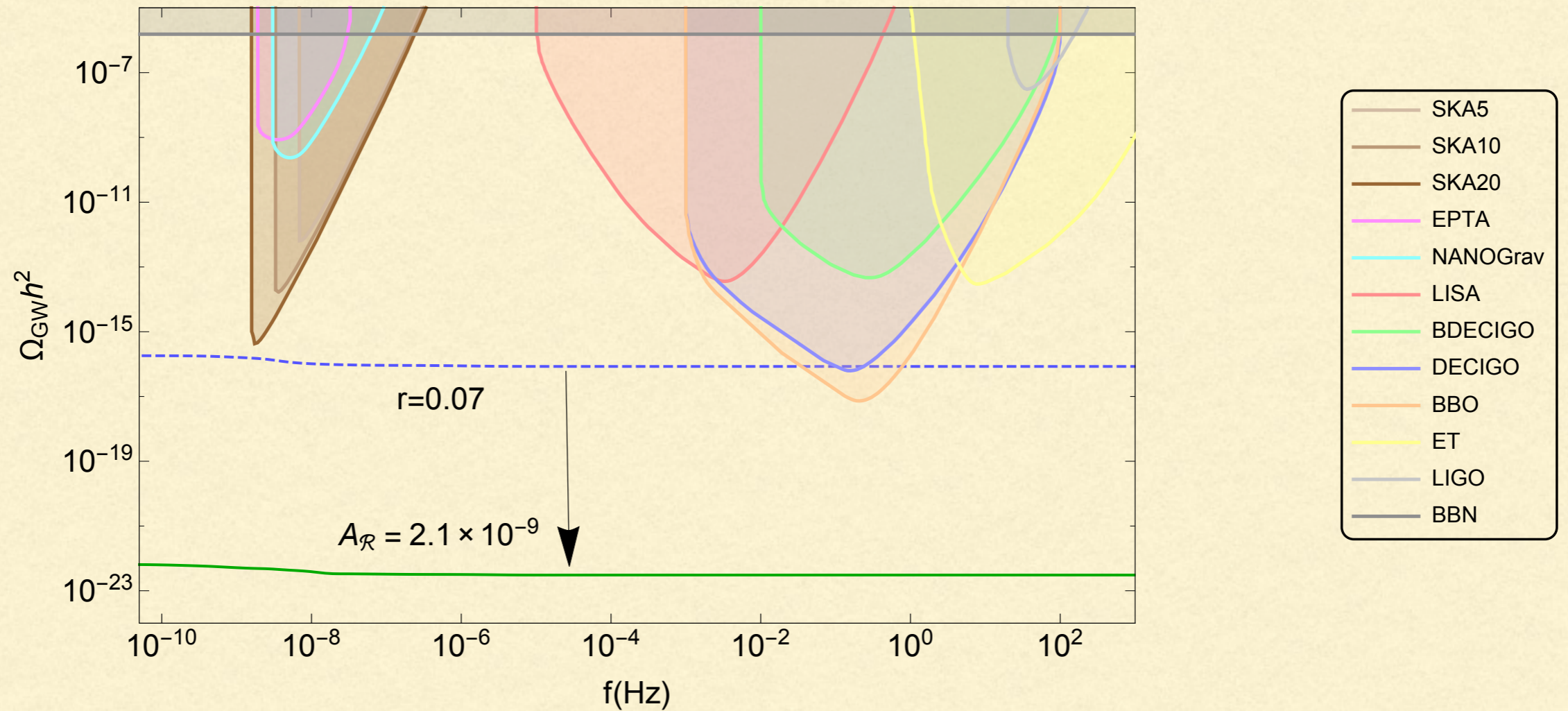
Assuming scale invariant curvature power spectrum:

$$\mathcal{P}_{\mathcal{R}} = A_{\mathcal{R}}$$



$$\Omega_{GW}(k, \eta_0) \propto Z(T_{hc}) \rho_{tot}(T_{hc}) s_{tot}(T_{hc})^{-\frac{4}{3}}$$

↓  
a correction factor from scalar perturbation and SM DoF



Nongaussianity effect can enhance the spectrum as some peaks with large relic density accessible by experiments.

---

## Overview (2nd)

---

- *Dark matter production in an early matter dominated epoch*  
M. Drees, F. H. [arXiv:1711.05007 \(JCAP\)](#)
- *Neutralino dark matter in scenarios with early matter domination*  
M. Drees, F. H. [arXiv:1808.05706 \(JHEP\)](#)

---

# Dark Matter Production in an Early Matter Dominated Epoch

---

## Motivations:

- UV-completion theories like string theory predict a period of matter domination by a heavy scalar after inflation and before radiation domination.
- Lack of experimental evidence in direct, indirect detection and collider experiments for thermal WIMP dark matter (DM).
- It is possible to suppress the over/under-production of WIMP DM candidates (e.g. Bino, Higgsino, and Wino like neutralinos in SUSY) in non-thermal scenario.
- The history of universe between big bang nucleosynthesis (BBN) and inflationary era is unknown.

# Friedmann-Boltzmann Equations in Early Matter Domination

DM in non-thermal cosmology can be produced by a heavy scalar with Planck suppressed decay rate during the time it dominates the universe before radiation domination era and after inflation. To understand the production of DM and radiation we require the following Friedmann - Boltzmann equations (S≠0 : entropy production):

Modulus energy density

Modulus decay rate

$$\dot{\rho}_\phi + 3H\rho_\phi = -\Gamma_\phi\rho_\phi$$

Assuming DoF of energy and entropy densities are equal.

$$\dot{\rho}_R + 4H\rho_R = (1 - \bar{B})\Gamma_\phi\rho_\phi + 2E_{X'}\langle\sigma v\rangle'(n_{X'}^2 - n_{X',EQ}^2)$$

DM energy

$$\bar{B} = \frac{E_{X'}B_{X'}}{M_\phi}$$

$$\dot{n}_{X'} + 3Hn_{X'} = \frac{B_{X'}}{M_\phi}\Gamma_\phi\rho_\phi - \langle\sigma v\rangle'(n_{X'}^2 - n_{X',EQ}^2)$$

$$H^2 = \frac{8\pi}{3M_{Pl}^2}(\rho_\phi + \rho_R + \rho_{X'})$$

M. Drees, F. H. arXiv:1711.05007 (JCAP)



Radiation energy and entropy densities:

$$\rho_R(T) = \frac{\pi^2}{30} g_{\text{eff}}(T) T^4 \quad s_R(T) = \frac{2\pi^2}{45} h_{\text{eff}}(T) T^3$$

Equilibrium number density of DM particles:

$$n_{X',\text{EQ}} \equiv \frac{g_{X'} T M_{X'}^2}{2\pi^2} K_2 \left( \frac{M_{X'}}{T} \right) \rightarrow \begin{cases} \frac{\tilde{g}_{X'} \zeta(3) T^3}{\pi^2} & \text{if } T \gg M_{X'} \\ g_{X'} \left( \frac{M_{X'} T}{2\pi} \right)^{\frac{3}{2}} \exp(-M_{X'}/T) & \text{if } T \ll M_{X'} \end{cases}$$

Reheating temperature and moduli decay width:

$$H_{RH} = \Gamma_\phi \quad \Gamma_\phi = \alpha \frac{M_\phi^3}{M_{\text{Pl}}^2}, \quad \alpha = \frac{C}{8\pi} = \text{constant} \quad T_{RH} = \sqrt{\Gamma_\phi} M_{\text{Pl}} \left( \frac{45}{4\pi^3 g_{\text{eff}}(T_{RH})} \right)^{1/4}$$

Using comoving densities to do the numerical calculation:

$$\Phi \equiv \frac{\rho_\phi A^3}{T_{RH}^4}, \quad R \equiv \rho_R \frac{A^4}{T_{RH}^4}, \quad X' \equiv n_{X'} \frac{A^3}{T_{RH}^3}$$

Initial conditions: modulus domination + vanishing radiation and DM initial densities

$$A = 1, \quad \Phi_I = \frac{3H_I^2 M_{\text{pl}}^2}{8\pi T_{RH}^4}, \quad R_I = X'_I = 0 \quad H_I = \gamma \Gamma_\phi$$

To consider the evolution of DoF we require the entropy density:

$$\frac{d s_R}{dt} + 3H s_R = \frac{1}{T} \left[ (1 - \bar{B}) \Gamma_\phi \rho_\phi + 2E_{X'} \langle \sigma v \rangle' (n_{X'}^2 - n_{X',EQ}^2) \right]$$

Relic density in an early matter dominated era:

current CMB temperature

↓

$$\Omega_{DM} h^2 = \frac{\rho_{X'}(T_{\text{now}})}{\rho_\gamma(T_{\text{now}})} \Omega_\gamma h^2 = M_{X'} \frac{X'(T_F)}{R(T_F)} \frac{A_F T_F g_{\text{eff}}(T_F) h_{\text{eff}}(T_{\text{now}})}{2T_{\text{now}} T_{\text{RH}} h_{\text{eff}}(T_F)} \Omega_\gamma h^2$$

↑

a temperature before matter-radiation equality and after neutrino decoupling

By considering the domination of a heavy long-lived scalar after inflation and before radiation domination era the following **DM production mechanisms** are possible (Giudice et al. [hep-ph/0005123]; Kane et al. [arXiv:1502.05406]; Gelmini and Gondolo, [arXiv:hep-ph/0602230]):

### Efficient annihilation:

- Quasi-static equilibrium (QSE<sub>nr</sub>); when  $\frac{B_{X'}}{M_\phi} \Gamma_{\phi\rho\phi} - \langle\sigma v\rangle' (n_{X'}^2 - n_{X',\text{EQ}}^2) \approx 0$

$$\Omega h^2[\text{QSE}_{\text{nr}}] \propto \frac{M_{X'}}{g_{\text{eff}}(T_{\text{RH}})^{1/6} M_{\text{Pl}} \langle\sigma v\rangle' T_{\text{RH}}}$$

- Freeze-out in radiation dominated era(FO<sub>rad</sub><sup>nr</sup>)  $H_{\text{rad}} = n_{X',\text{eq}} \langle\sigma v\rangle'$

$$\Omega h^2[\text{FO}_{\text{nr}}^{\text{rad}}] \propto \frac{1}{g_{\text{eff}}(\hat{T}_{\text{FO}})^{1/2}} \frac{\hat{x}'_{\text{FO}}}{M_{\text{Pl}} \langle\sigma v\rangle'}, \quad \text{with } \hat{x}'_{\text{FO}} = \frac{M_{X'}}{\hat{T}_{\text{FO}}}$$

## Inefficient annihilation:

- Dark matter production from direct decay of modulus ( $\Phi$ -decay).

$$\Omega_{\text{decay}} h^2 \propto B_{X'} \frac{T_{\text{RH}} M_{X'}}{M_\phi}$$

- Freeze-out in modulus domination ( $\text{FO}_{\text{mod}}^{\text{nr}}$ )  $H_{\text{mod}} = n_{X',eq} \langle \sigma v \rangle'$ ;

$$\Omega_{\text{ann}} h^2 [\text{FO}_{\text{nr}}^{\text{mod}}] \propto \frac{g_{\text{eff}}(T_{\text{RH}})^{1/2}}{g_{\text{eff}}(T_{\text{FO}})} \frac{T_{\text{RH}}^3 x'_{\text{FO}}{}^4}{M_{X'}^3 M_{\text{Pl}} \langle \sigma v \rangle'}, \text{ with } x'_{\text{FO}} = \frac{M_{X'}}{T_{\text{FO}}}$$

- Inverse annihilation during modulus domination happens when  $n_{X',I} \approx 0$ , before the second reheating completion, the annihilation of SM particles to DM particles is dominant. The final abundance will depend on  $n_{X',EQ}$ .

- Relativistic inverse annihilation ( $\text{IA}_r$ );

$$\Omega_{\text{ann}} h^2 [\text{IA}_r] \propto \frac{T_{\text{RH}} M_{X'} M_{\text{Pl}} \langle \sigma v \rangle'}{g_{\text{eff}}(T_{\text{RH}})^{3/2}}$$

- Non-relativistic inverse annihilation ( $\text{IA}_{\text{nr}}$ ).

$$\Omega_{\text{ann}} h^2 [\text{IA}_{\text{nr}}] \propto \frac{g_{\text{eff}}(T_{\text{RH}})^{3/2} T_{\text{RH}}^7 M_{\text{Pl}} \langle \sigma v \rangle'}{g_{\text{eff}}(T_*)^3 M_{X'}^5}$$

DoF constant  
(at reheating)

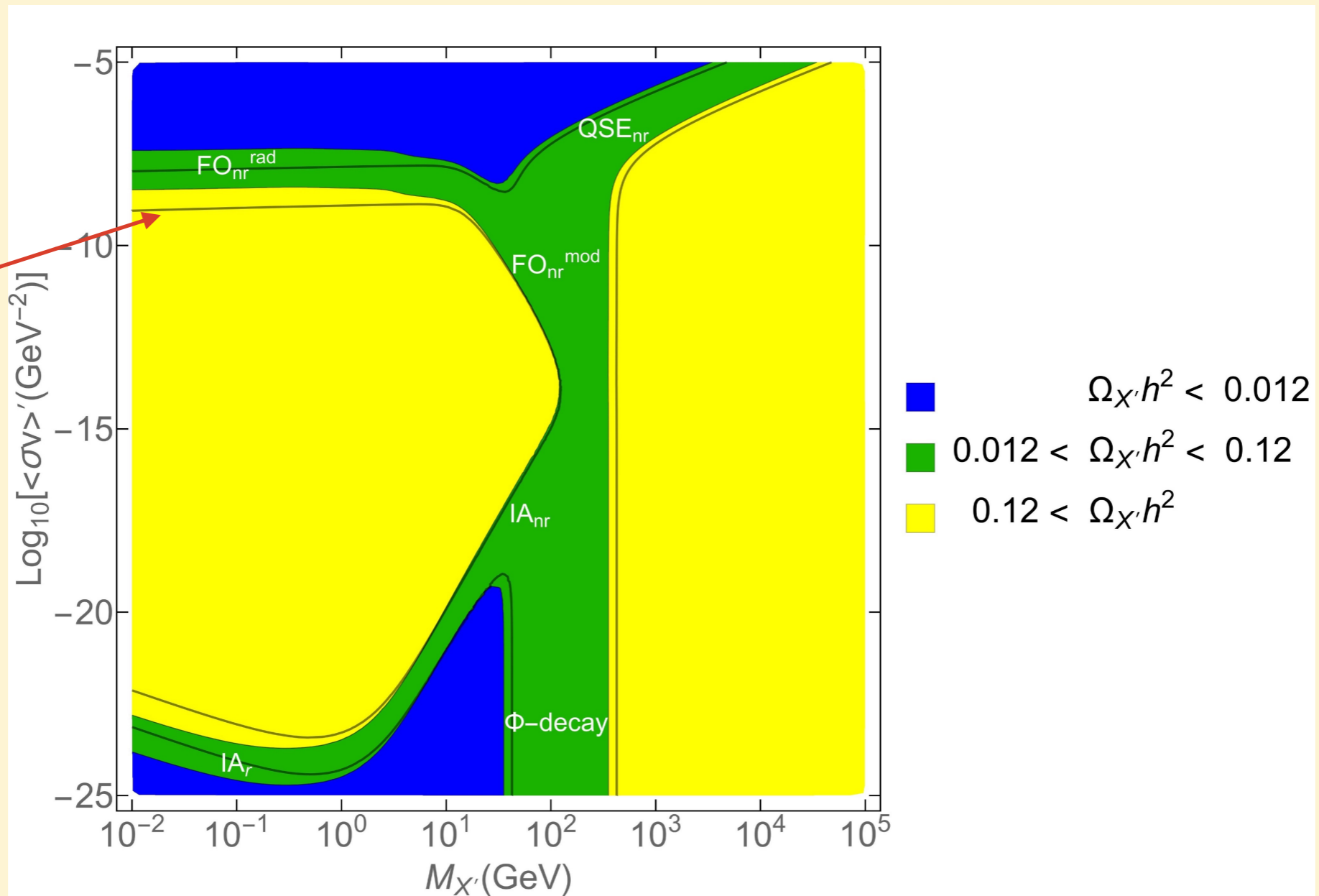


Figure: Thermally averaged cross section of DM versus its mass for non-thermal scenario with  $M_\phi = 5 \times 10^6 \text{ GeV}$  and  $B_{X'} = 10^{-5}$

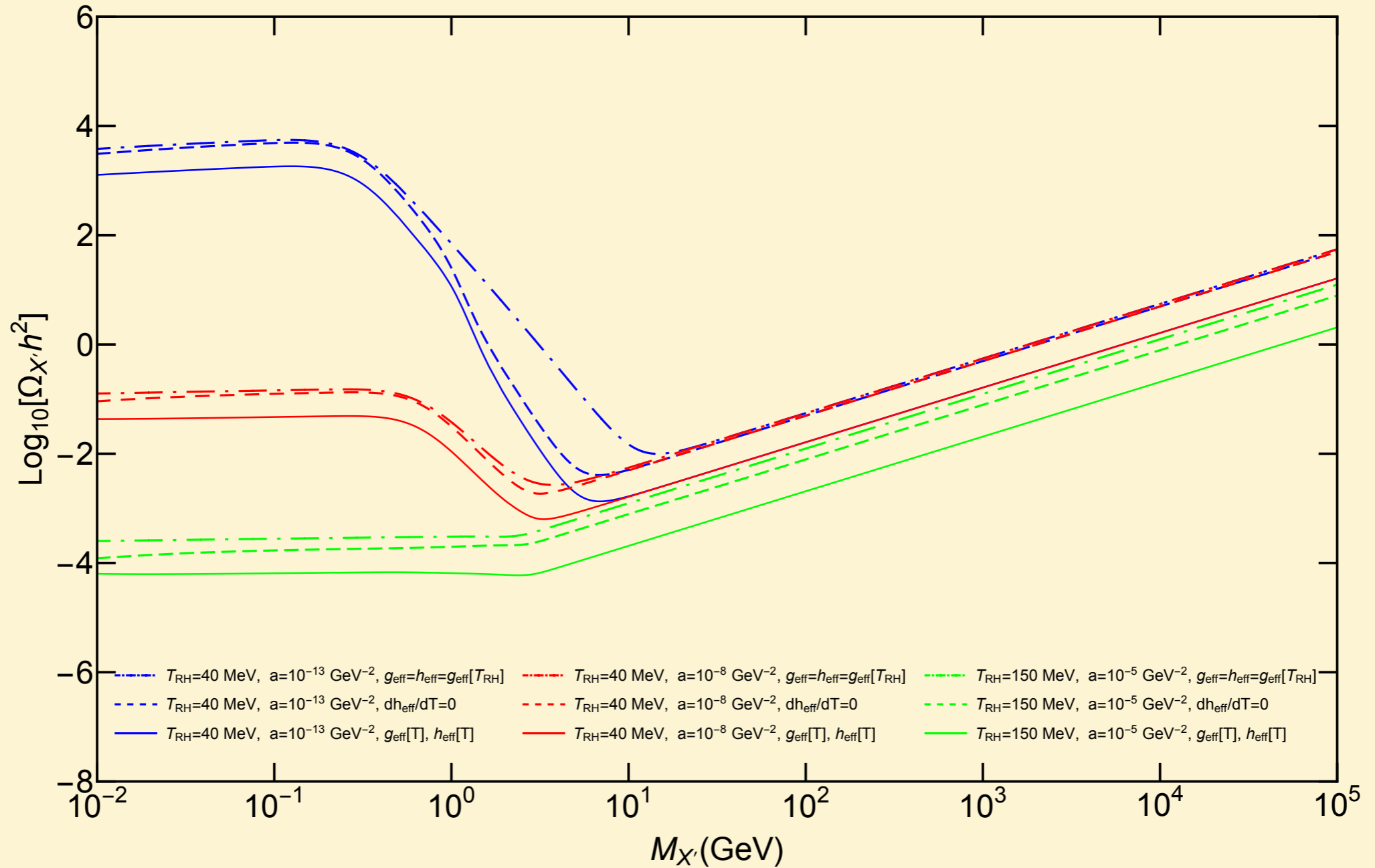
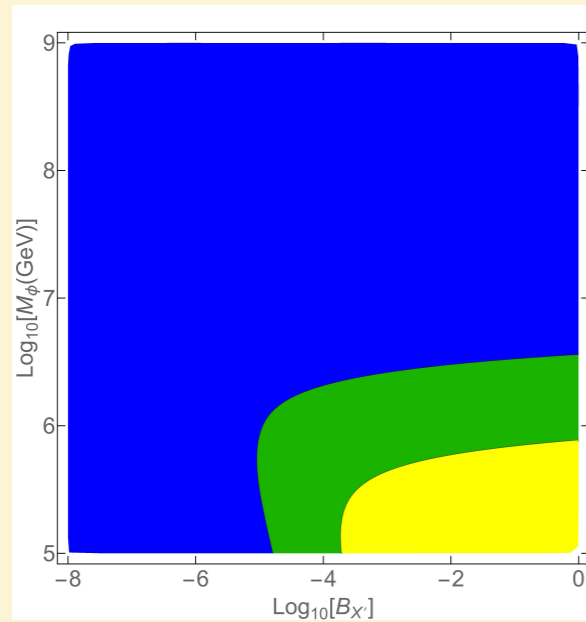


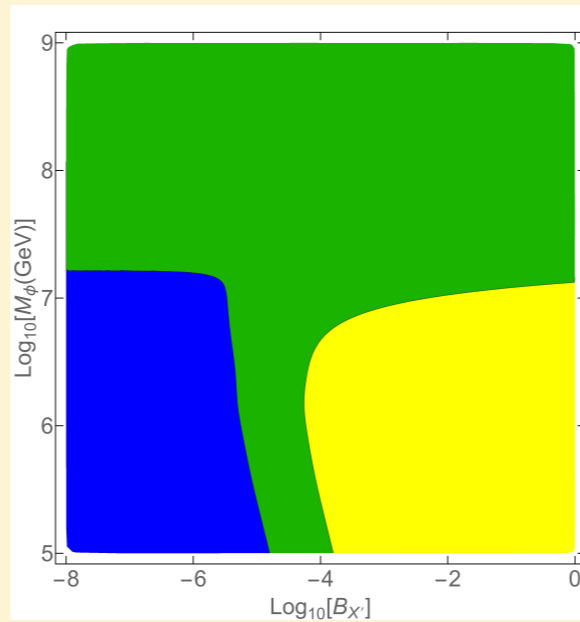
Figure: Relic density of DM versus its mass for non-thermal scenario for different treatments of DoF

For cross sections lower than WIMP and  $B_{X'} \lesssim 10^{-4} \left( \frac{M_{X'}}{100 \text{ GeV}} \right)$ ,  $M_\phi \lesssim 10^7 \text{ GeV} \left( \frac{M_{X'}}{100 \text{ GeV}} \right)^{2/3}$

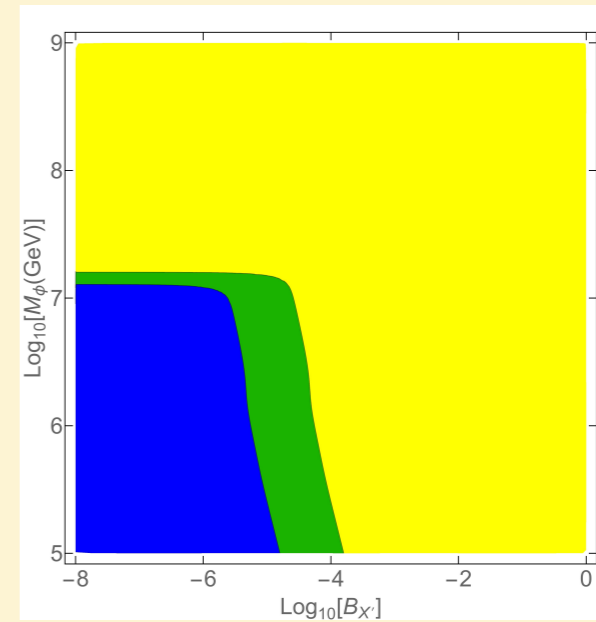
The relic abundance is smaller than the observed value:  $\Omega_{X'} h^2 \approx 0.12$



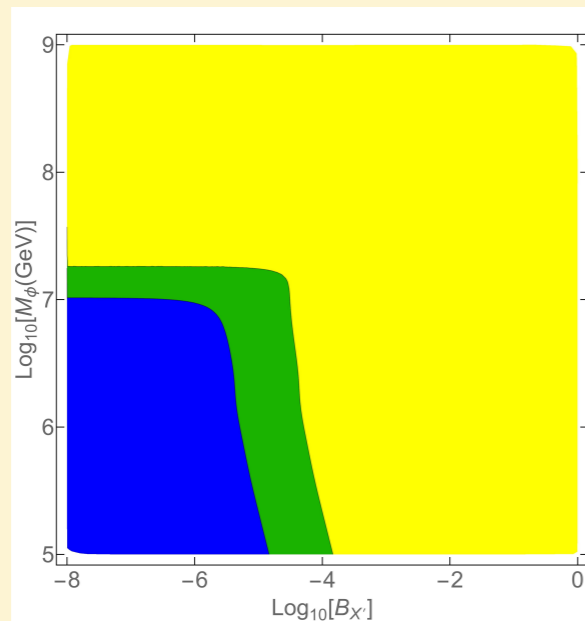
$$\langle\sigma v\rangle' = 10^{-6} \text{ GeV}^{-2}$$



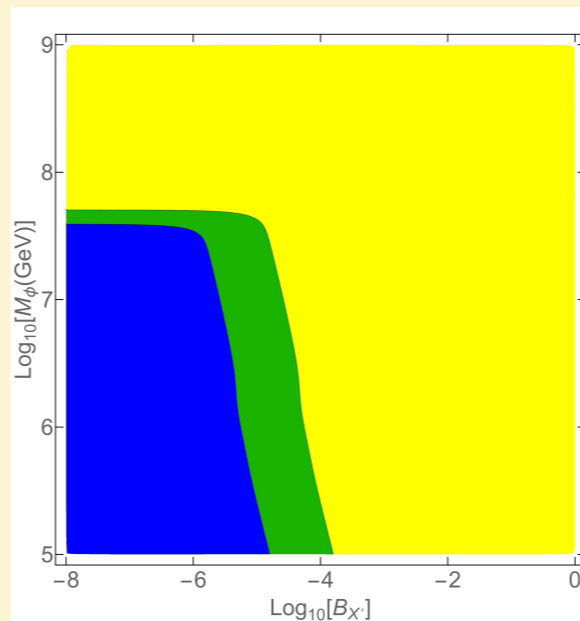
$$\langle\sigma v\rangle' = 10^{-8} \text{ GeV}^{-2}$$



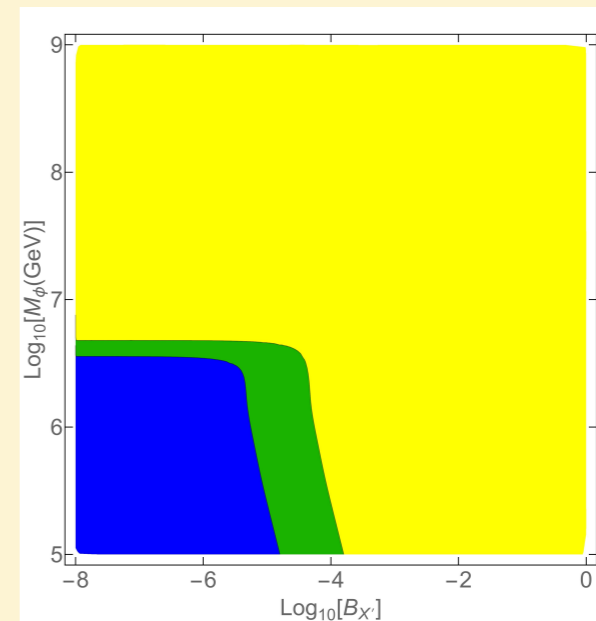
$$\langle\sigma v\rangle' = 10^{-9} \text{ GeV}^{-2}$$



$$\langle\sigma v\rangle' = 10^{-14} \text{ GeV}^{-2}$$



$$\langle\sigma v\rangle' = 10^{-20} \text{ GeV}^{-2}$$



$$\langle\sigma v\rangle' = 10^{-25} \text{ GeV}^{-2}$$

Figure: Regions of valid relic density for different moduli masses versus branching ratios for

$$M_{X'} = 100 \text{ GeV}$$

Non-vanishing radiation and DM densities:

$$\mu = \frac{\rho_{R,I}}{\rho_{\phi,I}}$$

$$\Phi_I = \frac{3M_{\text{Pl}}^2 H_I^2}{8\pi T_{\text{RH}}^4 (1 + \mu)}, R_I = \mu \Phi_I$$

If this inequality holds, there will not be any difference between vanishing and non-vanishing initial radiation and DM abundances:

$$X'_I \ll X'_F$$

$$\frac{\mu^{3/4}(1 + \mu)^{1/4}}{\gamma^{1/2}} \ll \kappa_{\phi\text{-decay}} B_{X'} \left( \frac{\alpha M_\phi}{M_{\text{Pl}}} \right)^{1/2}$$

$$\frac{\mu^{3/4}(1 + \mu)^{1/4}}{\gamma^{1/2}} \ll \kappa_{IA_{nr}} \frac{\alpha^{1/2} M_\phi^{3/2} M_{\text{Pl}}^{1/2} T_{\text{RH}}^6 \langle \sigma v \rangle'}{M_{X'}^6}$$

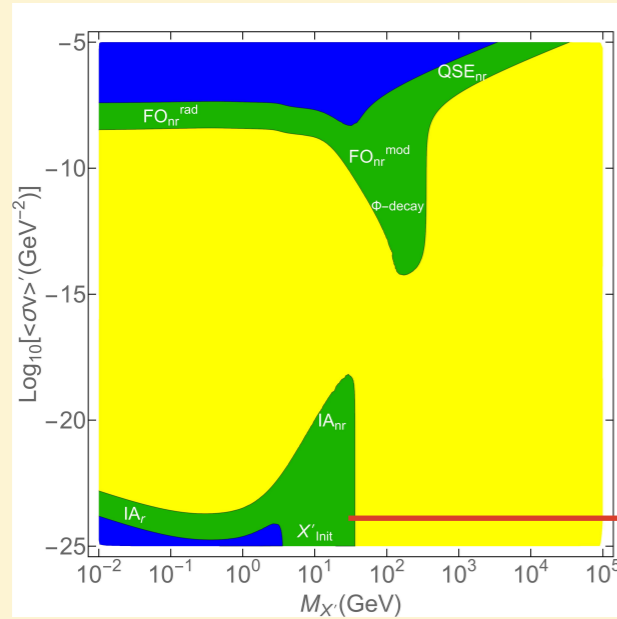
$$\frac{\mu^{3/4}(1 + \mu)^{1/4}}{\gamma^{1/2}} \ll \kappa_{IA_r} \alpha^{1/2} M_\phi^{3/2} M_{\text{Pl}}^{1/2} \langle \sigma v \rangle'$$

Some constants  
depending on the  
production scenario

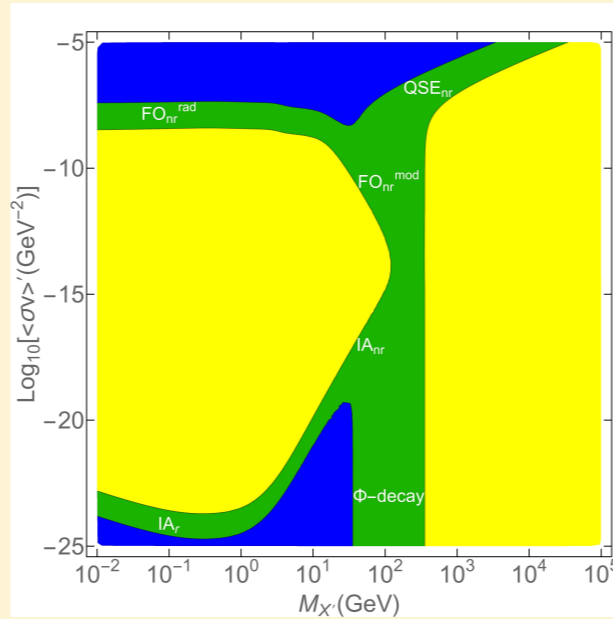
Otherwise the final DM abundance will be equal to its initial abundance.



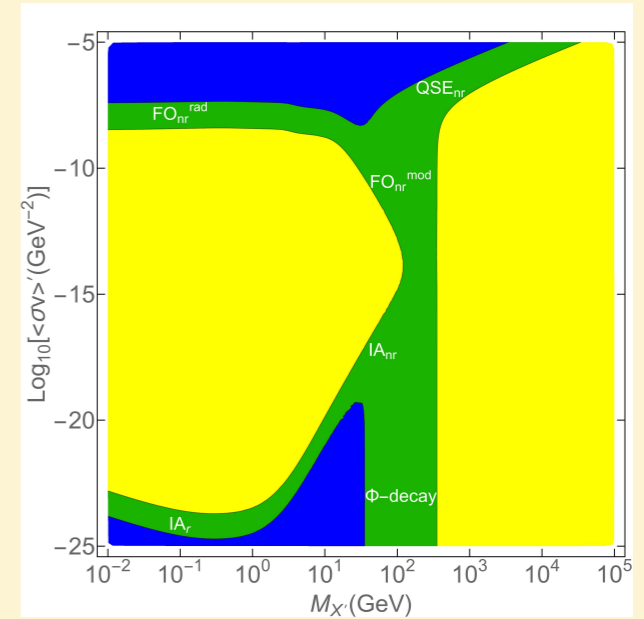
$$\gamma > 10^{20} \mu^{3/2} \sqrt{1 + \mu} \left( \frac{10^{-5}}{B_{X'}} \right)^2 \left( \frac{10^7 \text{ GeV}}{M_\phi} \right) \longrightarrow \text{goes back to initially vanishing density limit}$$



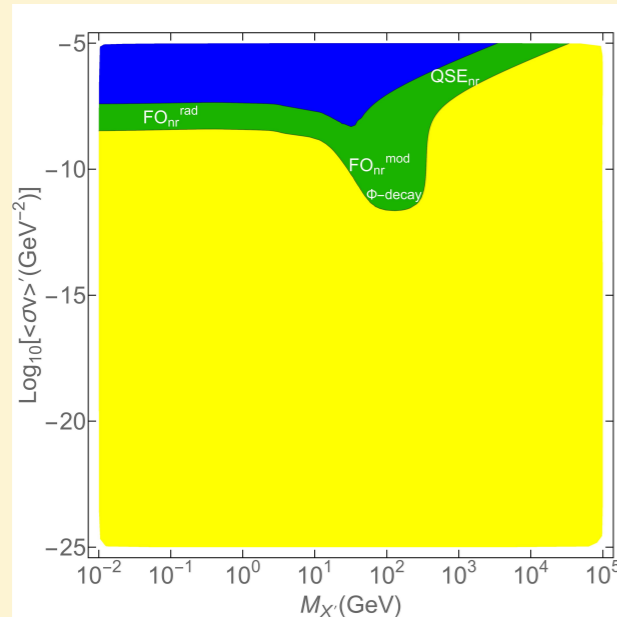
$$\mu = 0, \gamma = 10^{10}$$



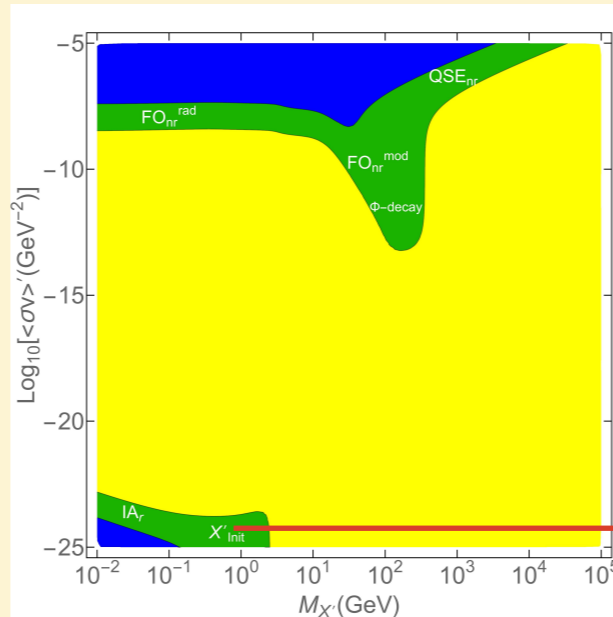
$$\mu = 0, \gamma = 10^{15}$$



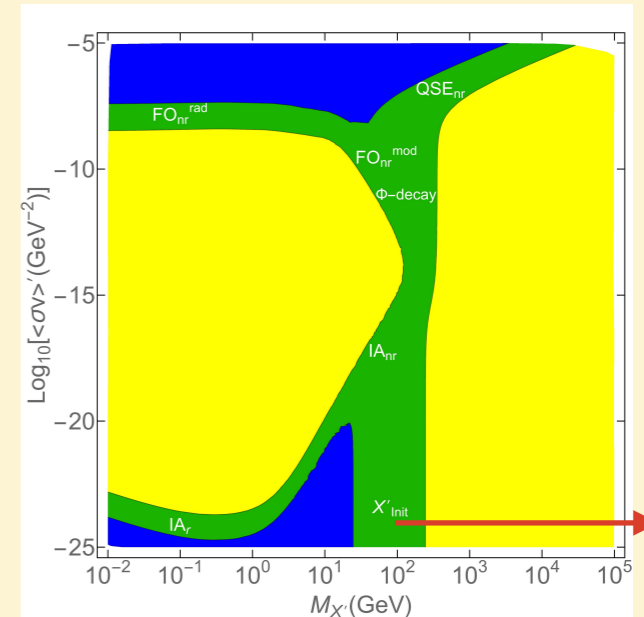
$$\mu = 0, \gamma = 10^{20}$$



$$\mu = 1, \gamma = 10^{10}$$



$$\mu = 1, \gamma = 10^{15}$$



$$\mu = 1, \gamma = 10^{20}$$

Figure: Regions of valid relic density for thermally averaged cross section versus DM mass for different initial conditions

---

# Neutralino Dark Matter in Scenarios with Early Matter Domination

---

- Supersymmetry can solve some problems in particle physics and cosmology like hierarchy problem, gauge unification, dark matter, etc. by assigning a fermionic (bosonic) super partner to each boson (fermion) in SM.
- Neutralino can play the role of the lightest supersymmetric particle in minimal supersymmetric version of SM with R-parity conservation.
- Neutralinos are the mass eigenstates of gauginos (in flavour basis).

- In thermal production scenario:

Bino-like neutralino  $\longrightarrow$  overabundance

Higgsino- and wino-like neutralino  $\longrightarrow$  under abundance

(Drees and Nojiri [hep-ph/9207234 ], Allahverdi et al. arXiv:1307.5086)

Bino

Higgsino

Wino

$$\langle \sigma_{\tilde{B}} v \rangle \propto \frac{M_1^2}{m_{\tilde{t}}^4}$$

$$\langle \sigma_{\tilde{H}} v \rangle \propto \frac{1}{\mu^2}$$

$$\langle \sigma_{\tilde{W}} v \rangle \propto \frac{1}{M_2^2}$$

$$\text{Thermal relic density} \longrightarrow \Omega_\chi h^2 \propto \frac{1}{\langle \sigma v \rangle}$$

To consider neutralino in an early matter dominated era:

$$X' \rightarrow \chi, M_{X'} \rightarrow M_\chi, \langle \sigma v \rangle' \rightarrow \langle \sigma v \rangle, \text{etc.}$$

To do the scan we used *micrOMEGAs* (relic density, cross section for direct and indirect searches), *SUSPECT* (RGE running), and *T3PS* (parallelized scan). The range of variables come from pMSSM parameter space.

The only constraint we considered:  $122 \text{ GeV} < m_h < 128 \text{ GeV}$ .

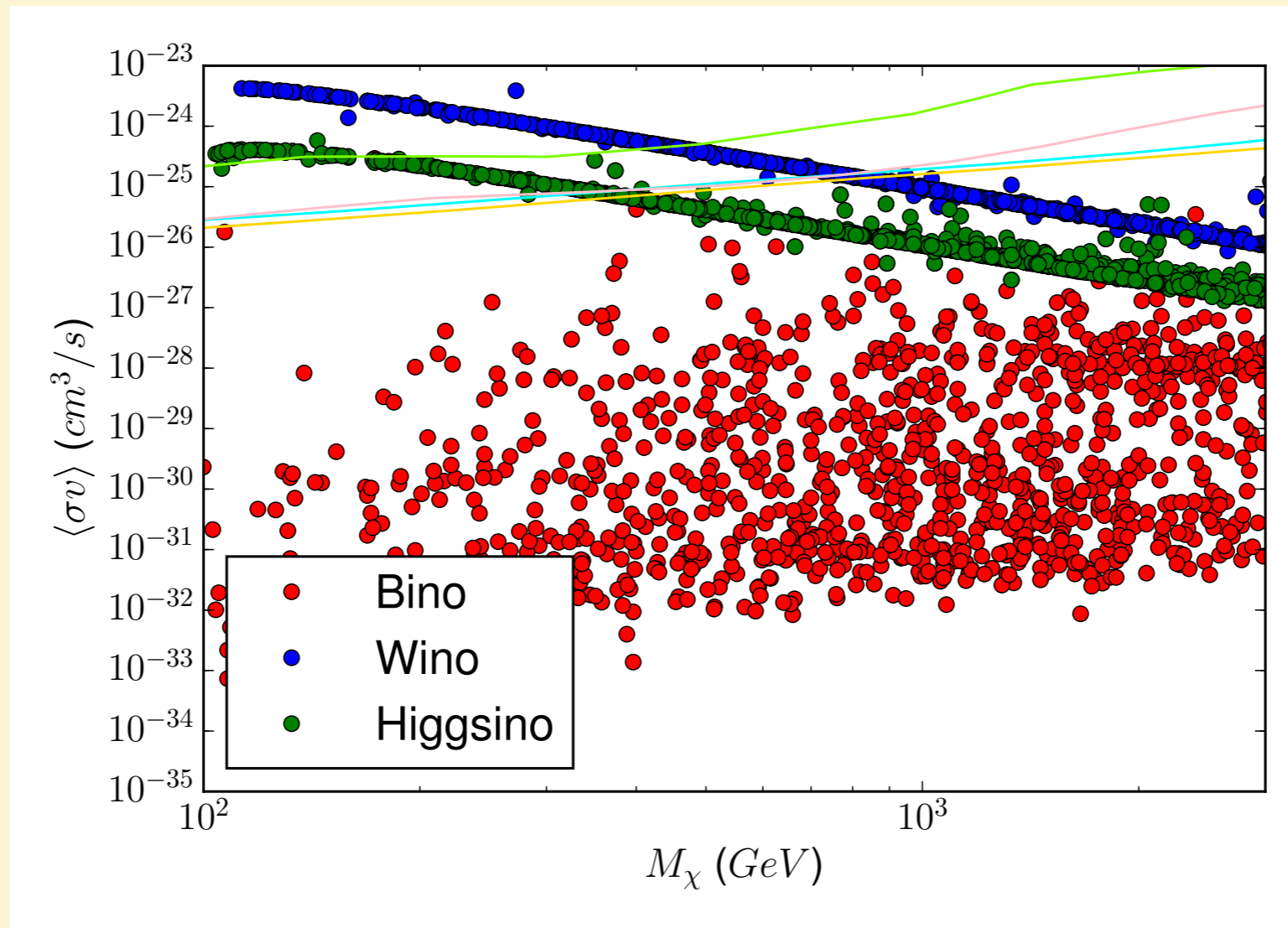


Figure: Thermally averaged cross section of neutralino versus its mass in zero velocity limit. Constraints are from the combination of MAGIC and Fermi-LAT experiments. Cyan, yellow, pink, and light green lines are for  $\chi\chi \rightarrow W^+W^-, b^+b^-, \tau^+\tau^-, \mu^+\mu^-$ .

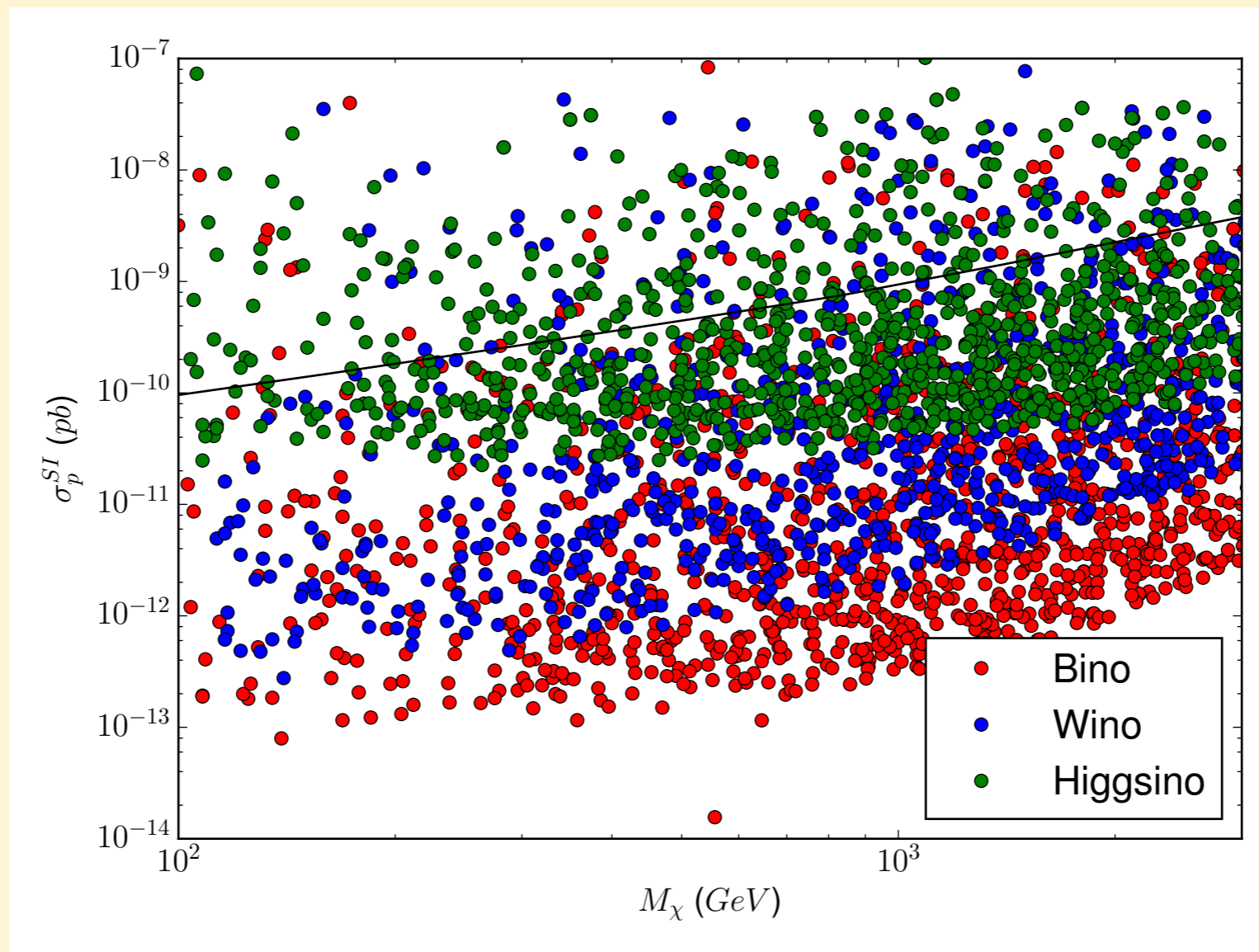


Figure: Spin independent cross section of neutralino-nucleon versus neutralino mass  
(black: Xenon-1T-2018)

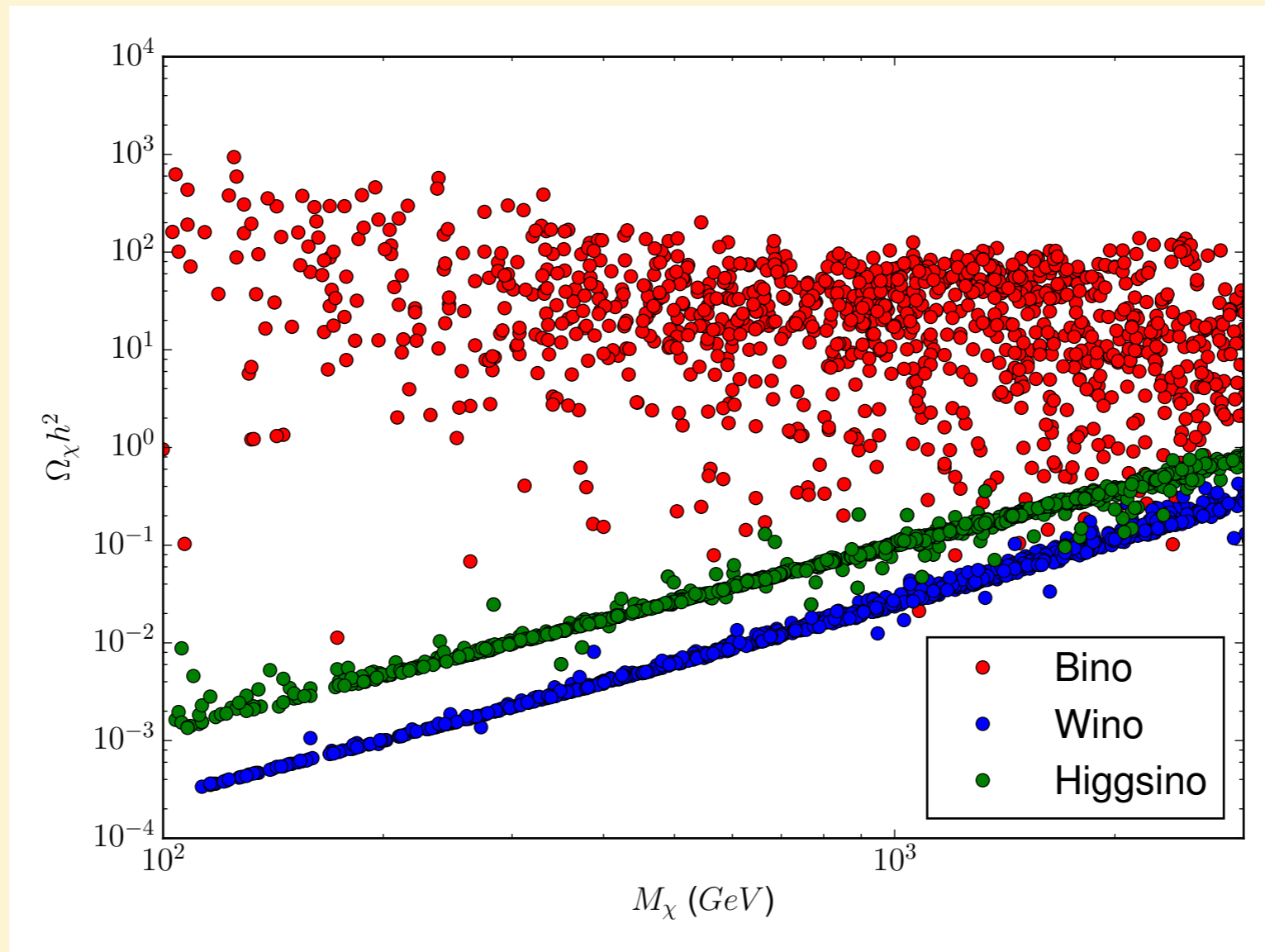
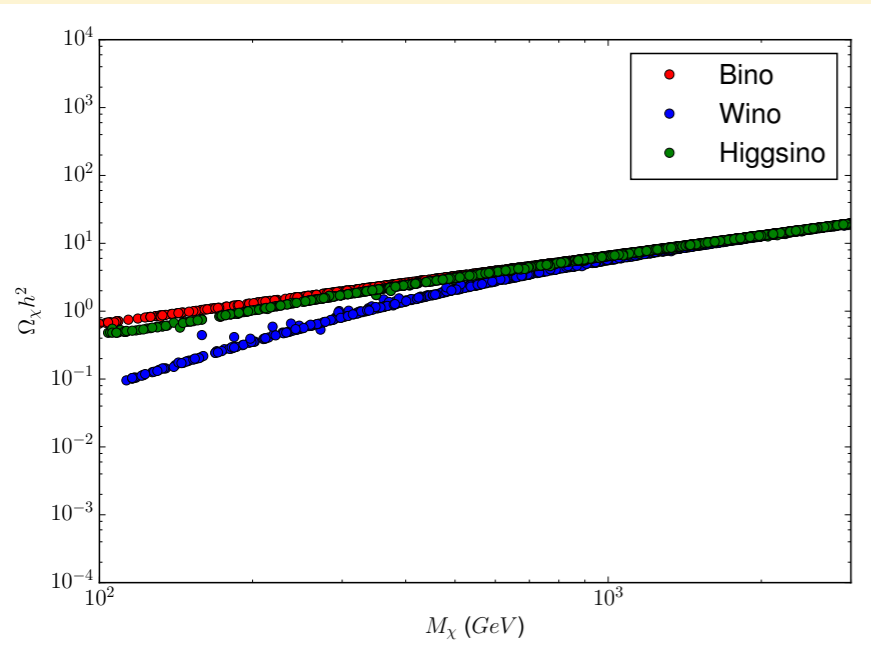
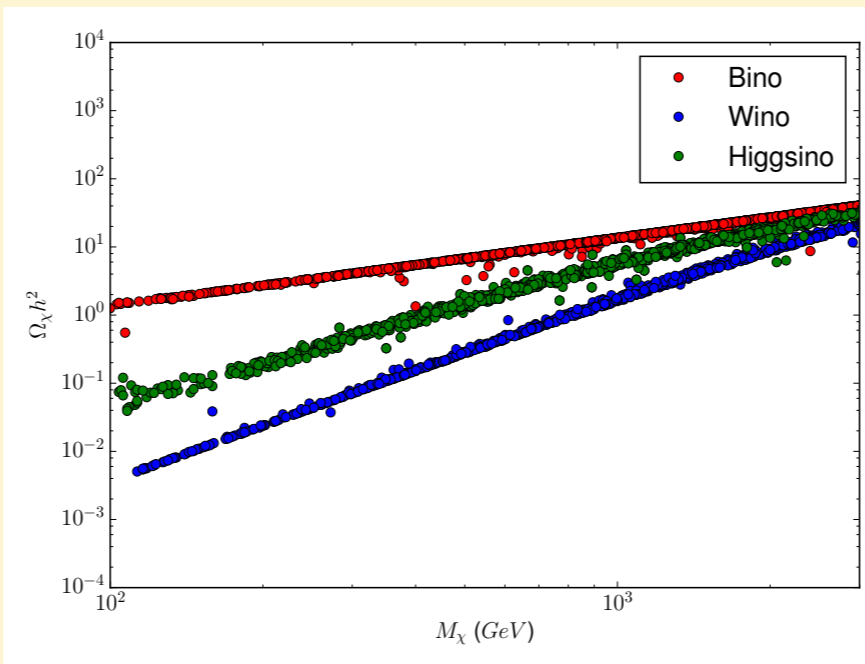


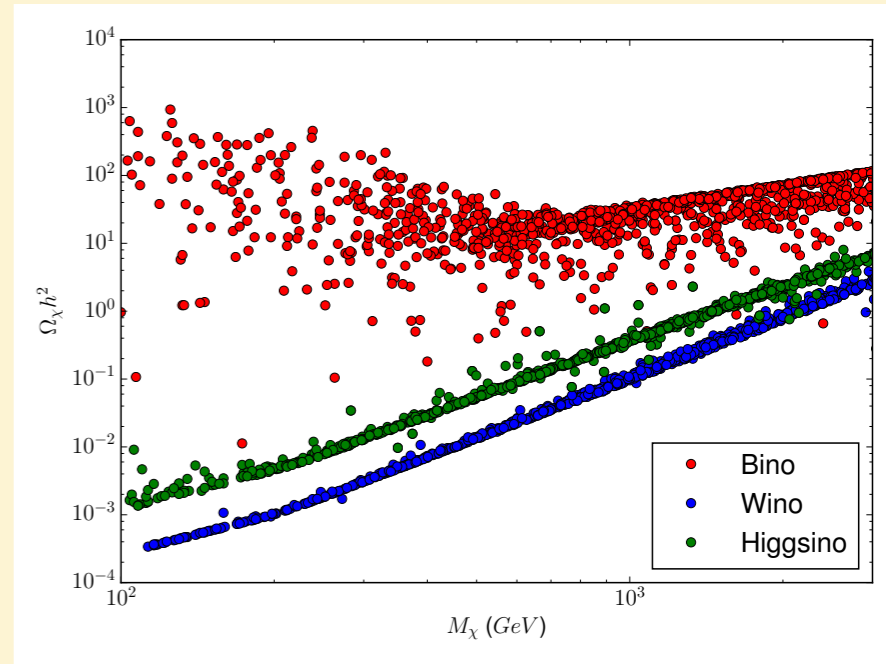
Figure: Thermal relic density of neutralino versus its mass



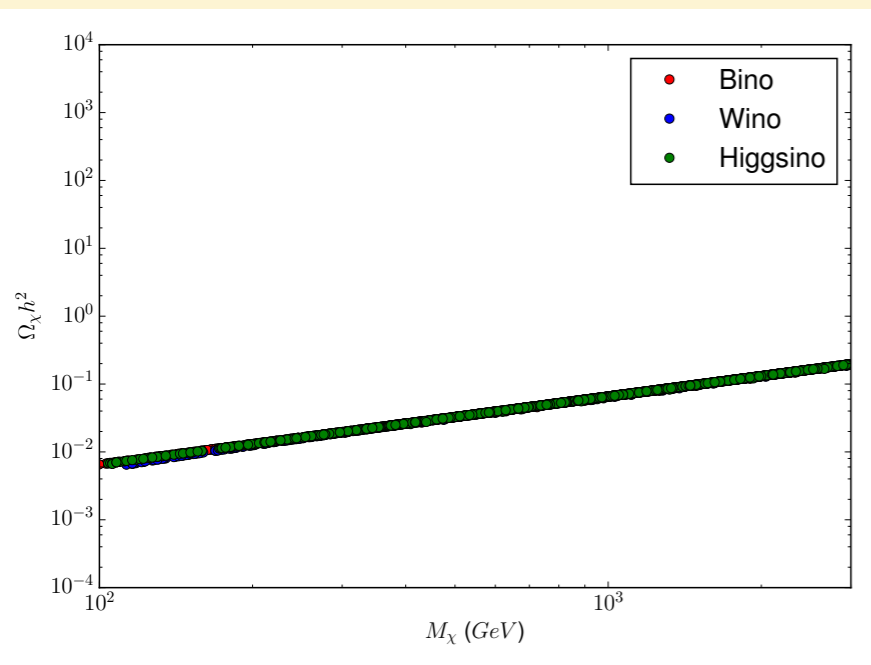
$$B_\chi = 10^{-3}, M_\phi = 5 \times 10^5 \text{ GeV}$$



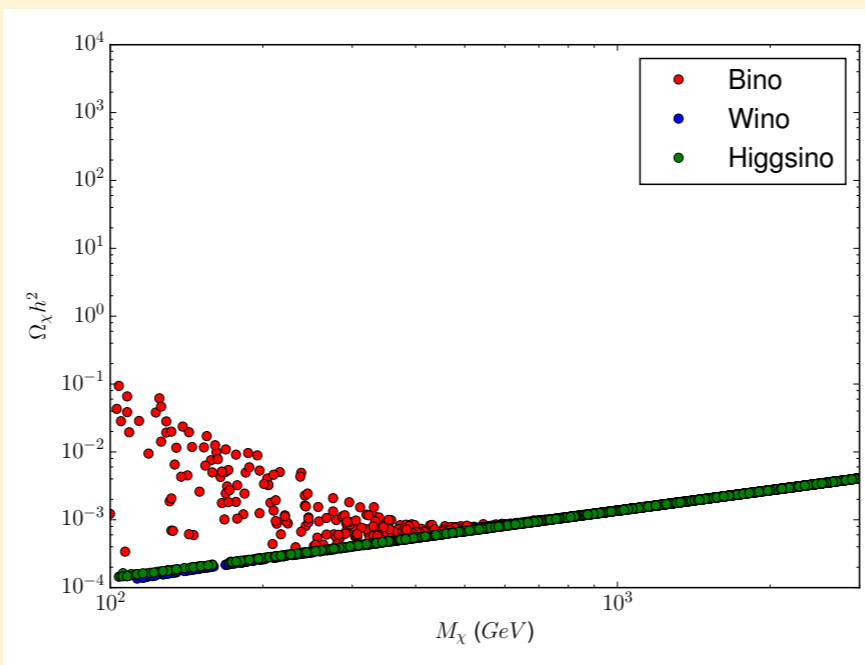
$$B_\chi = 10^{-3}, M_\phi = 5 \times 10^6 \text{ GeV}$$



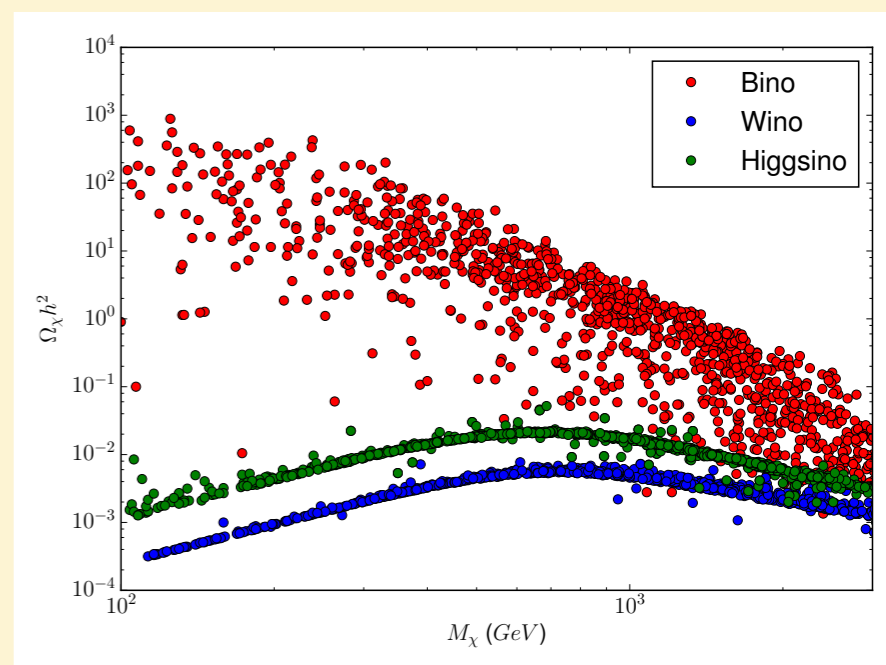
$$B_\chi = 10^{-3}, M_\phi = 5 \times 10^7 \text{ GeV}$$



$$B_\chi = 10^{-5}, M_\phi = 5 \times 10^5 \text{ GeV}$$



$$B_\chi = 10^{-7}, M_\phi = 5 \times 10^6 \text{ GeV}$$



$$B_\chi = 10^{-9}, M_\phi = 5 \times 10^7 \text{ GeV}$$

Figure: The relic density of lightest neutralino versus its mass in different non-thermal scenarios

---

# Summary

- QCD equations of state affect the PGW up to a few percent. Pulsar Timing Arrays can observe such effect in near future if the PGW relic is around  $\Omega_{GW}h^2 \approx 10^{-16}$
  - Nonvanishing lepton asymmetry can leave an imprint on the PGW up to 10% in comparison to the Vanishing case which might be an indirect signature of lepton asymmetry in the early universe.
  - GW experiments can constrain the parameter space of nonstandard cosmology regimes dominated before BBN.
  - They can indirectly measure the regimes might lead to nonthermal production of dark matter which can help us to understand the DM production in the early universe and give an indirect hint for DM mass and cross section.
  - Thermal history of the SM can affect the induced PGW differently from the first order one.
-



---

# Summary

- Considering the precise evolution of DoF in the early universe is important for both thermal and non-thermal cosmology.
  - A period of early matter domination can predict new valid regions and mechanisms for DM masses and thermally averaged cross sections beyond the assumption of thermal WIMP production. This can describe the lack of detection of thermal WIMPs.
  - There is an upper bound on the mass of modulus (long-lived) particle if the DM annihilation cross section is below that corresponding to thermal WIMP DM in standard cosmology.
  - Assuming non-vanishing initial DM and radiation densities before early matter domination can affect the final abundance of DM in specific regions of parameter space for non-thermal production. It will also depend on the initial Hubble rate through its effect on the initial conditions of Boltzmann equations.
  - Non-thermally produced neutralino DM in R-parity conserved MSSM can still be a valid DM candidate even with considering current experimental constraints.
-



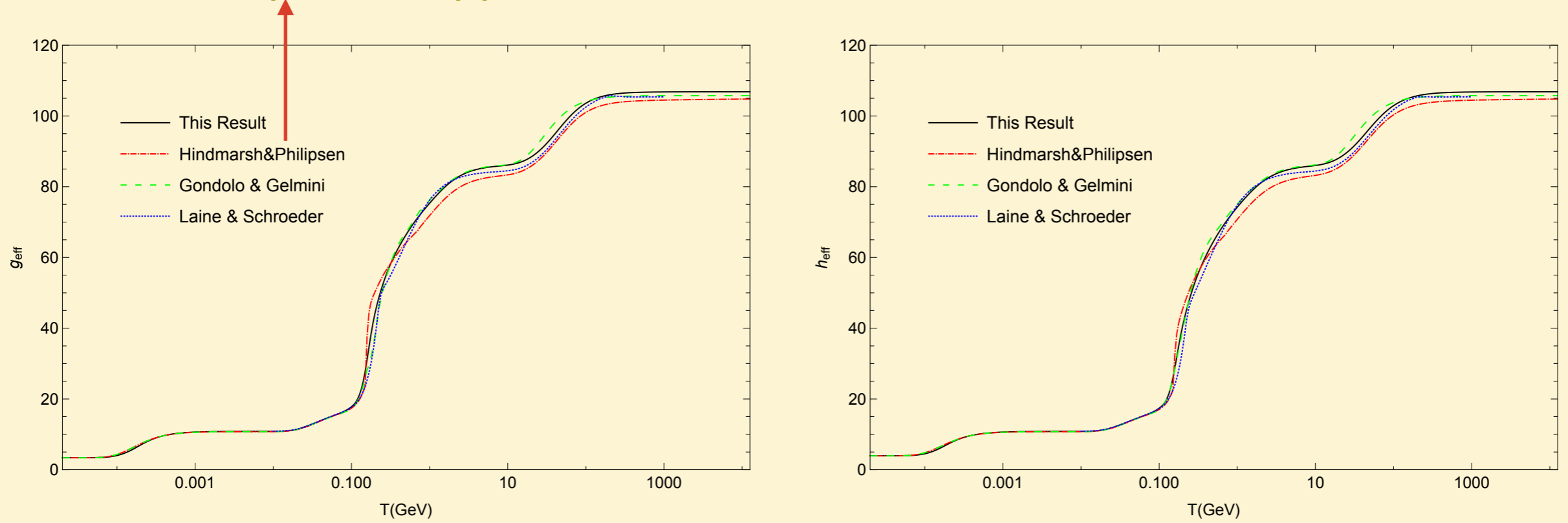
*Thanks for your attention!*

---

*The organisation was  
Good?! or Bad?!*



M. Hindmarsh and O. Philipsen arXiv:hep-ph/0501232



P. Gondolo, G. Gelmini, 1990

M. Laine and Y. Schroeder hep-ph/0603048

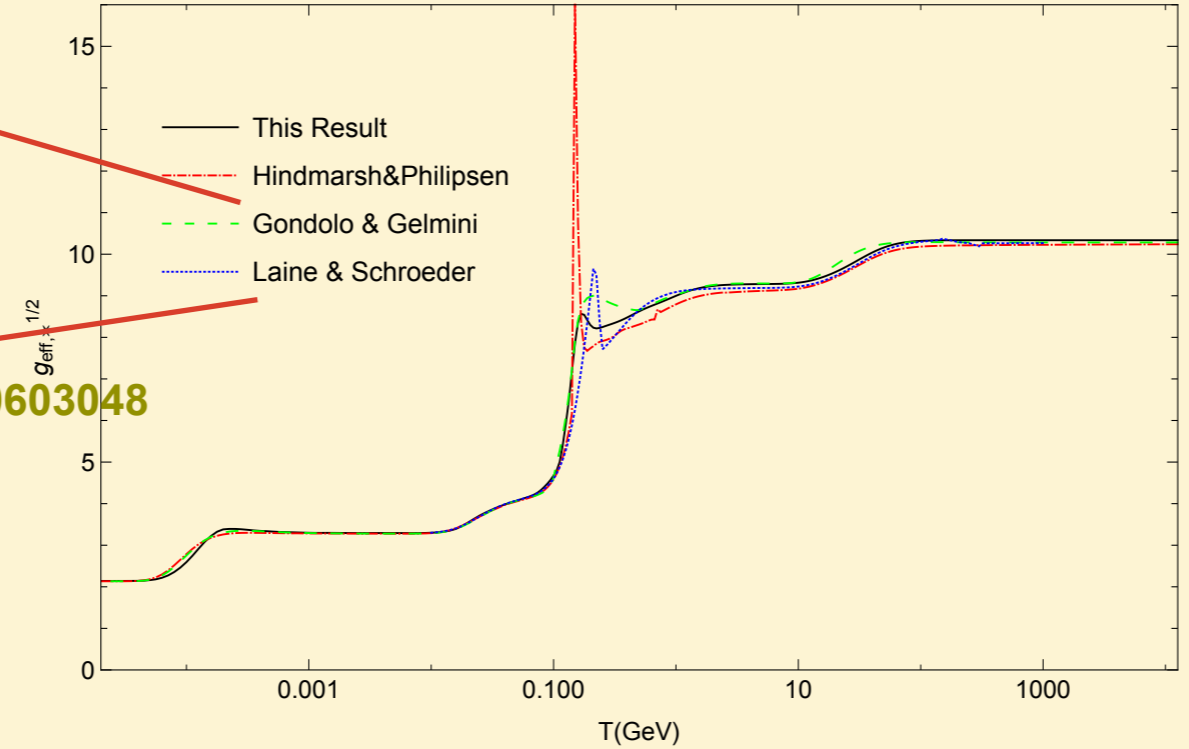


Figure: Degrees of freedom  $h_{\text{eff}}$ ,  $g_{\text{eff}}$ , and  $g_{\text{eff}}^{1/2}$ .

M. Drees, F. H., E. R. Schmitz arXiv:1503.03513

# The relative difference between various DoF treatments

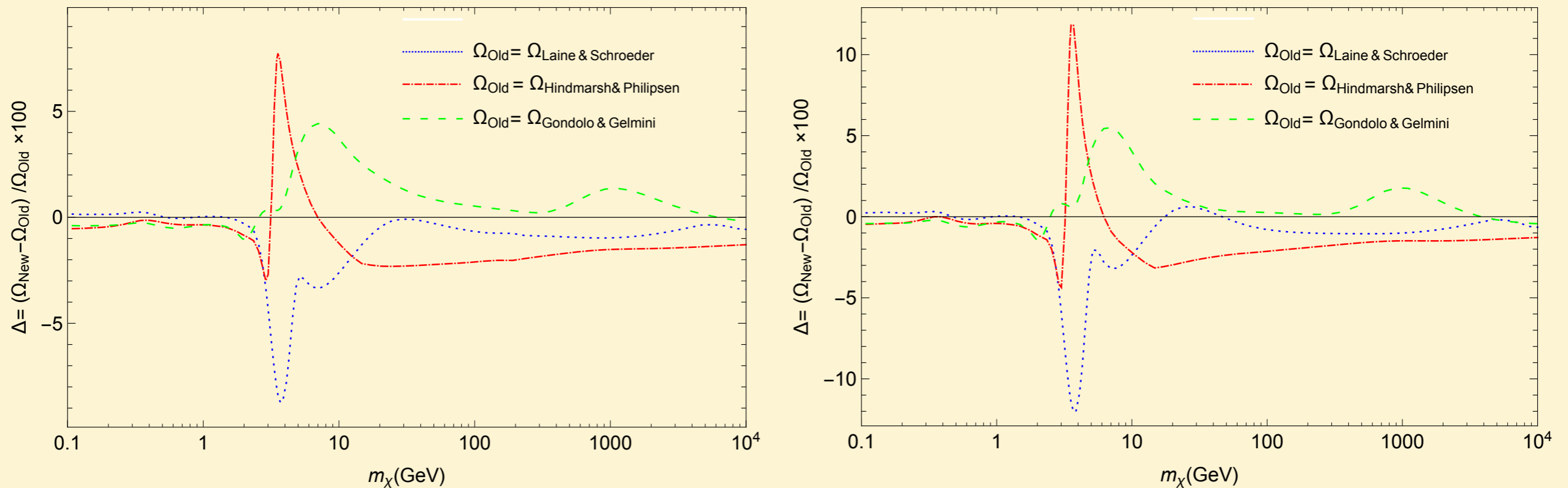


Figure: The relative difference between the predicted relic density of a Majorana WIMP between our calculation and a calculation using older results for the functions  $h_{\text{eff}}$ ,  $g_{\text{eff}}$ , and  $g_{\text{eff}}^{1/2,*}$  for S-wave and P-wave annihilation cross sections.

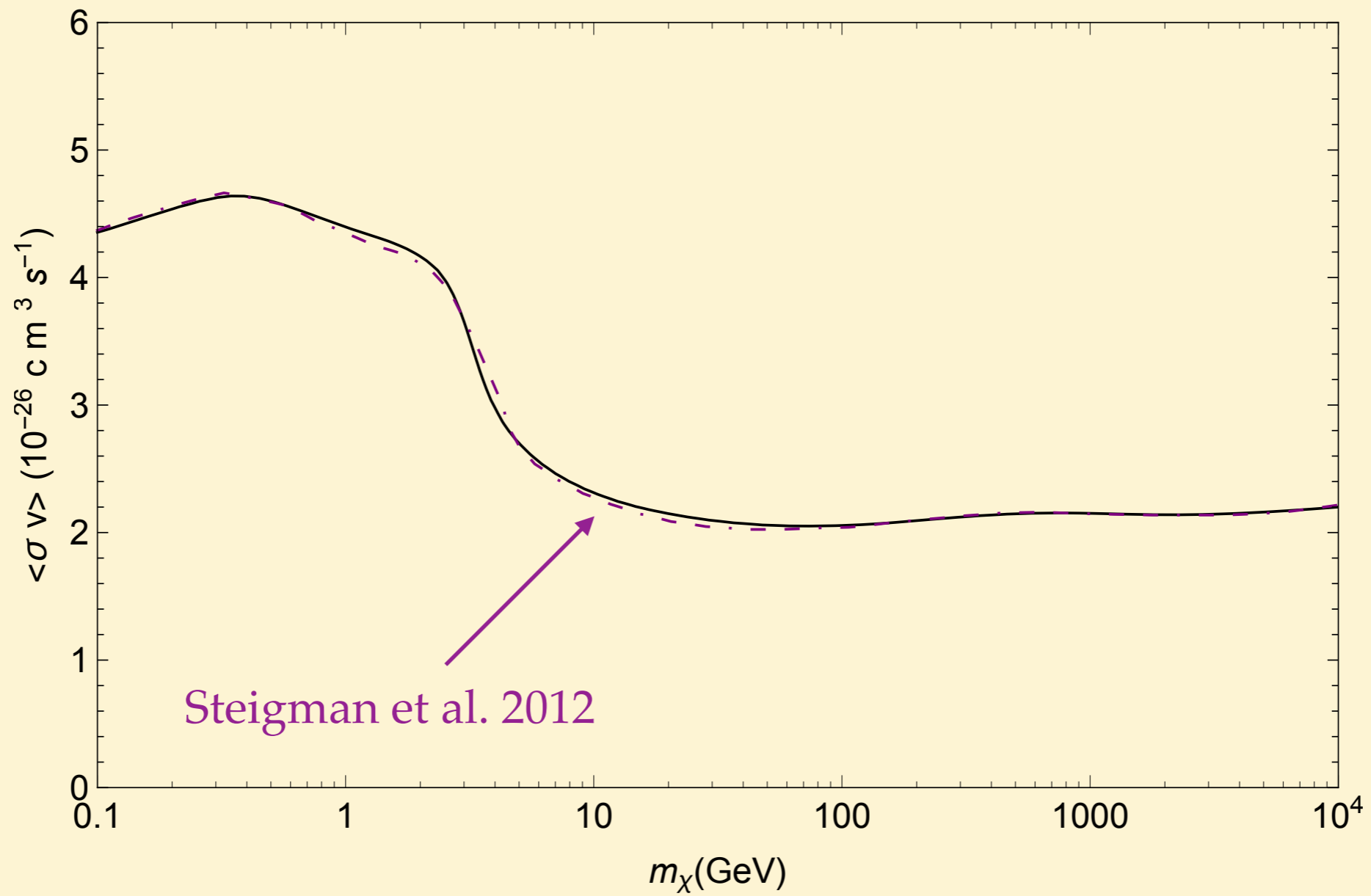


Figure: Thermally averaged cross section (S-wave) of WIMP DM versus its mass for the observed relic density  $\rightarrow 0.12$