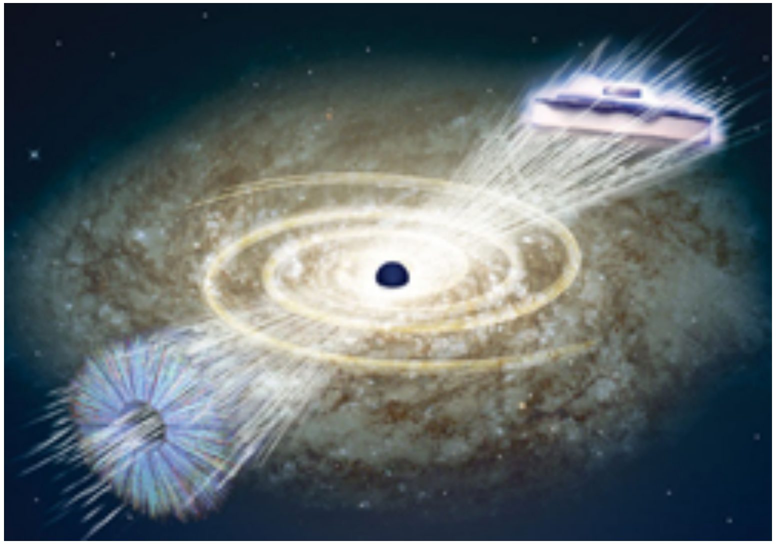
Beyond the Standard Model: Historical-Critical Perspectives, GGI, Oct. 21 2019.

The Holographic Correspondence



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Plan

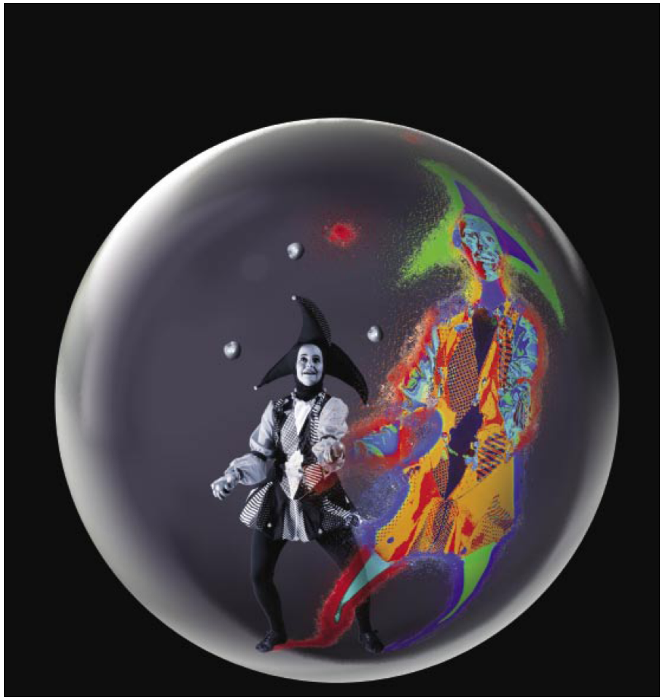
- What is it ?
- *Historical interlude*
- How does it work ?
- What is it for ?

Plan

- What is it ?
- *Historical interlude*
- How does it work ?
- What is it for ?

The statement

Quantum Gravity in D+1 dimensions	=	Quantum Field Theory in D dimensions



The statement

Quantum Gravity in D+1 dimensions	=	Quantum Field Theory in D dimensions

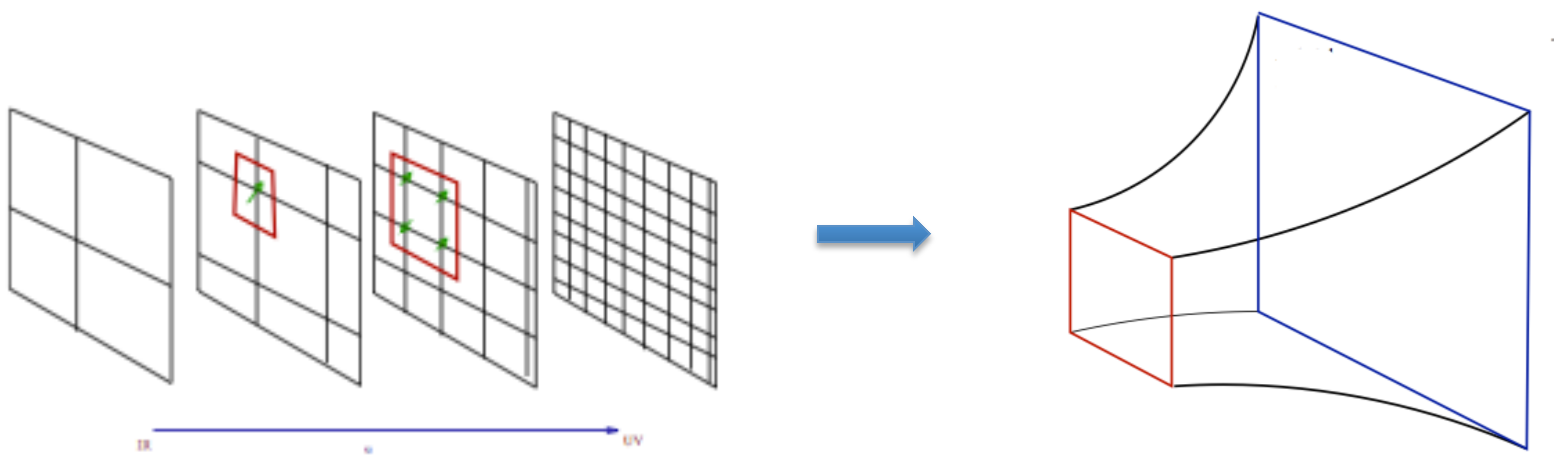
Is this reasonable?
It is not, but…

Heuristic Hint 1: RG flow

- Renormalization Group equations in QFT are local in the energy scale u

$$u\frac{dg}{du} = \beta(g)$$

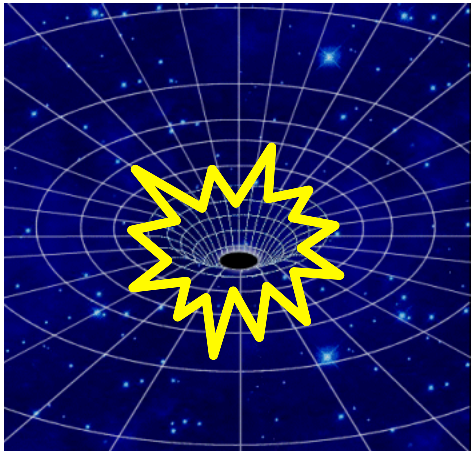
- Idea: RG flow of a D-dim QFT as “foliation” in D+1 dims .
- RG scale u = Extra dimension



Heuristic Hint 2: black holes

- Model in D+1 must have same number of d.o.f. as the QFT in D-dims
- Gravity is a good candidate: it is "holographic"
- See black hole physics

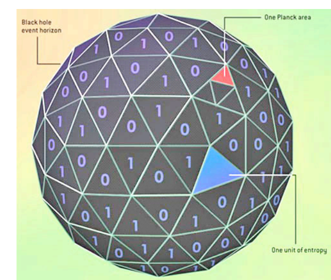
Black holes… are not so black



- Quantum effects: emit thermal radiation.
- Obey laws of thermodynamics
- Entropy scales like horizon area
  [Bekenstein, Hawking 1974]

$$S_{BH} = \frac{k_B c^3}{\hbar} \frac{A_H}{4G}$$

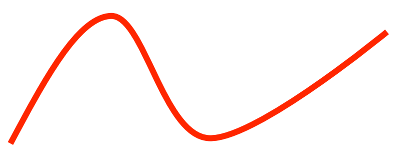
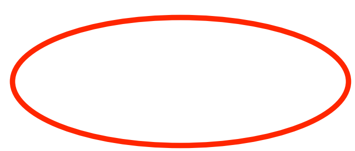
- The holographic "principle" ['t Hooft, Susskind 1994]
- *Quantum gravity, whatever it is, is holographic.*



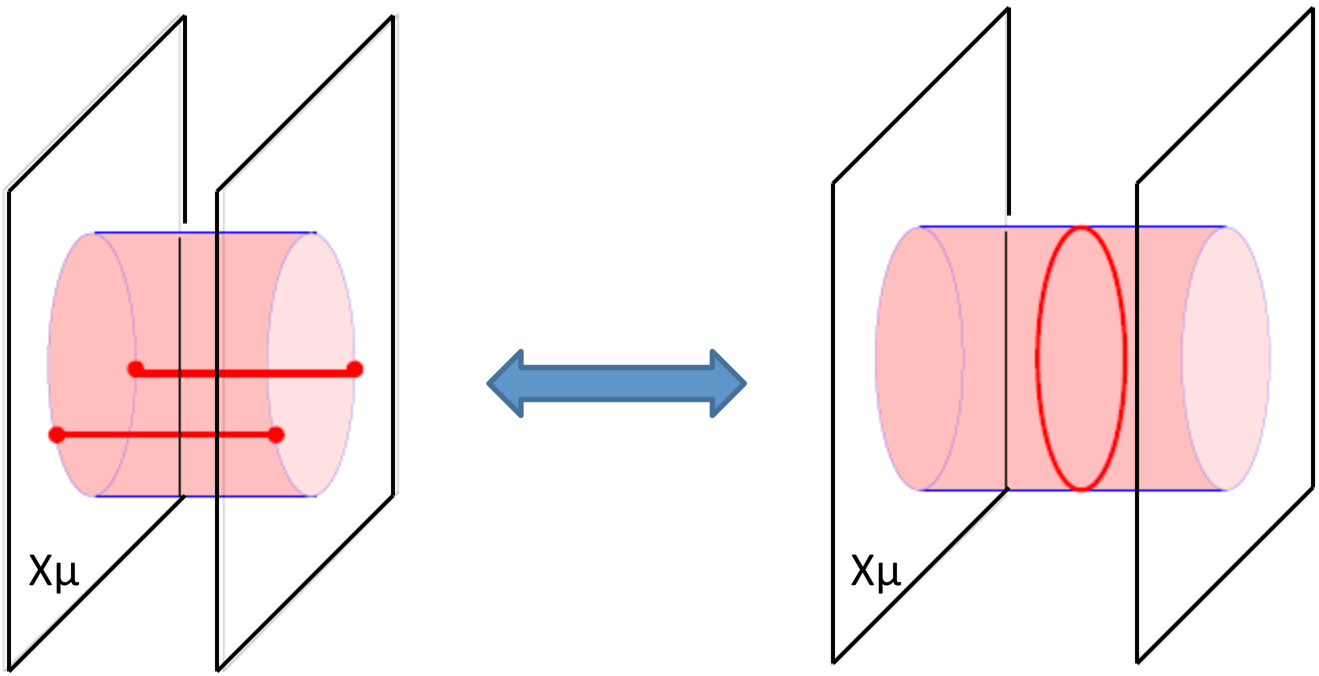
Degrees of freedom of QG in D+1 dim. spacetime volume	=	Degrees of freedom of QFT in D dim. boundary

Heuristic Hint 3: String theory

- Assumption: fundamental constituents are string-like
- Point particles are different modes of a vibrating string

String		Particle
Open		photon (gluon) +…
Closed		graviton +…..

- Open/closed string duality (or: 2 ways of drawing a cylinder)

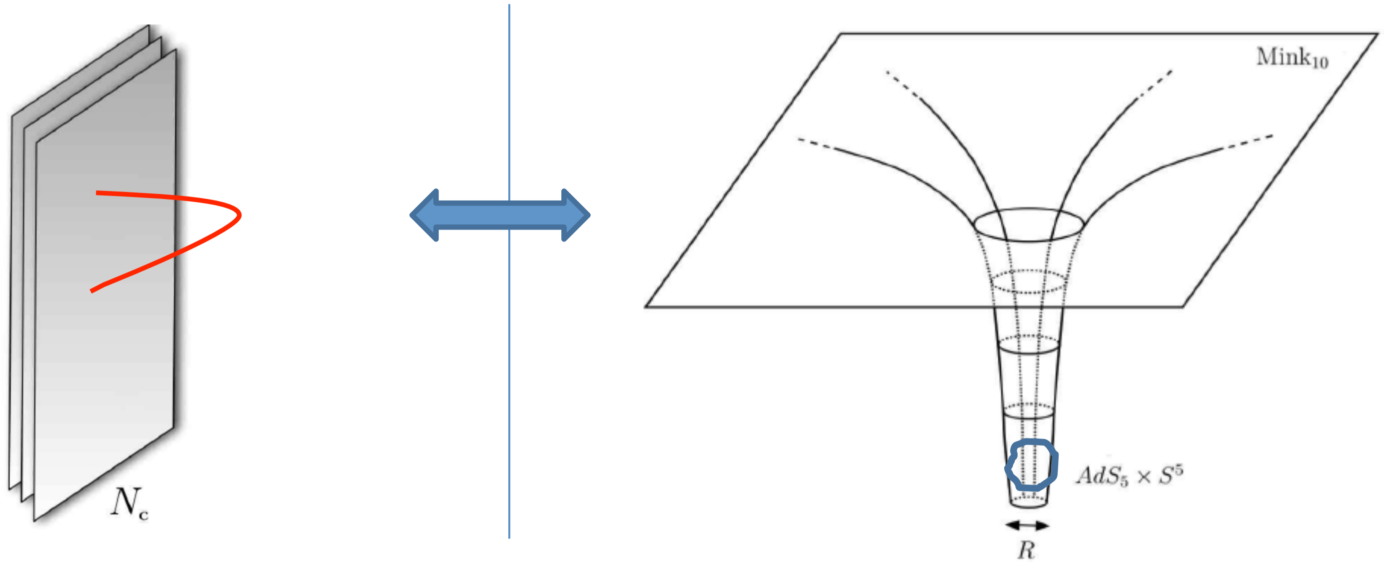


Open string loop (quantum)
Quantum Field Theory
Xμ, u (RG scale)

⟺

Closed string propagation (classical)
Theory of gravity
Xμ, r (extra dimension)

- The dual nature of Dp-branes [Polchinski, 95]



- Taking low energy limit on both sides, two interacting theories [J.M. Maldacena, 97]:

- Left: 4d SU(N_c) susy Yang-Mills. Low energy limit of open strings on Nc D3-branes
- Right: closed strings (gravity) on Anti-de-Sitter 5d background (times S^5)

- String theory provided the first explicit realization of the holographic correspondence [Maldacena 1997] a.k.a. AdS/CFT

3+1 dim $\mathcal{N}$=4 SU(N) Yang-Mills = IIB string on $AdS_5 \times S_5$

(Conformal Field Theory) (Quantum gravity on Anti de Sitter)

- …and a very detailed map between observables of the corresponding theories [Witten; Gubser, Klebanov, Polyakov, 1998]

- This has produced both extensions and an enormous amount of quantitative validity checks of the correspondence…and concrete applications.

- Holography is changing our way of understanding gravity and quantum field theories. It does not come out from nowhere: the connections between strings and gauge theories (like QCD) have a long history.

Plan

- What is it ?
- *Historical interlude (oversimplified)*
- How does it work ?
- What is it for ?

From Gabriele to Juan Martin (and back again)

1968

- Gabriele Veneziano was born and grew up in Florence.
- He got his M.Sc. in Physics right in this place, in 1965.
- In 1968 he is 26 years hold. His paper containing the famous "Veneziano Amplitude" will contribute to the birth of string theory, see e.g. [Cappelli, Castellani, Colomo, Di Vecchia, 2012].

- Juan Martin Maldacena, italian-argentinian nationality, was born in Buenos Aires

- In the ’50s everything looked clear: electron, proton, neutron and few other particles (muon, pion, positron)
- In the ’60s however, a pletora of other hadrons: kaons, rho mesons, Delta and Omega baryons, Lambda, Sigma, Eta, Nu, Upsilon…
- Difficult to believe they were all elementary.
- 1964. Murray Gell-Mann, Zweig: quarks as building blocks.

  - Neutron, proton and all other baryons: three quarks
  - Pion, rho and other mesons: quark-antiquark pairs.

- The new hadrons posed new problems for theoretical physics. If a strong interaction happens through an exchange of some of them, the scattering amplitude increases as the energy increases.

- 1968. Veneziano: if an infinite number of particles is exchanged, the scattering amplitude does not diverge with energy anymore.
- This goes with the name of Veneziano amplitude.

- 1970. Nambu, Nielsen e Susskind: Veneziano amplitude can be interpreted in terms of a theory of strings.

- Regge trajectories $M^2 \sim J$: mesons as rotating strings with quark endpoints?

- However string theory gave other predictions which turned out to be in conflict with experimental data.

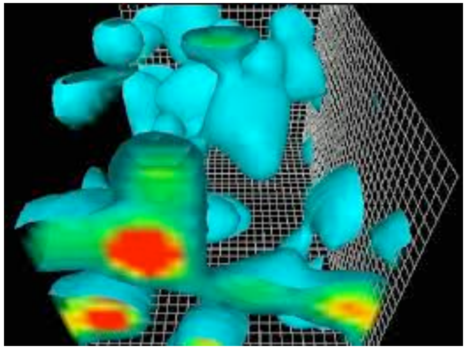
- 1973. Gross, Wilczek, Politzer: Quantum Chromodynamics (QCD).
- A SU(3) Yang-Mills gauge theory coupled to quarks.
- QCD revealed to be the correct theory of strong interactions.

- String theory almost abandoned
- In the first ’70s only very few scientists, among which Green and Schwartz, kept working on it.

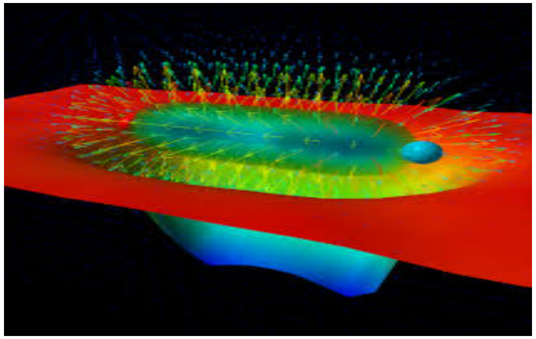
- By the way: QCD is hard.

- Asymptotic freedom: gauge coupling is small at high energies
- Low energy limit (hadron spectra, confinement, chiral symmetry breaking…) cannot be studied using perturbation theory.

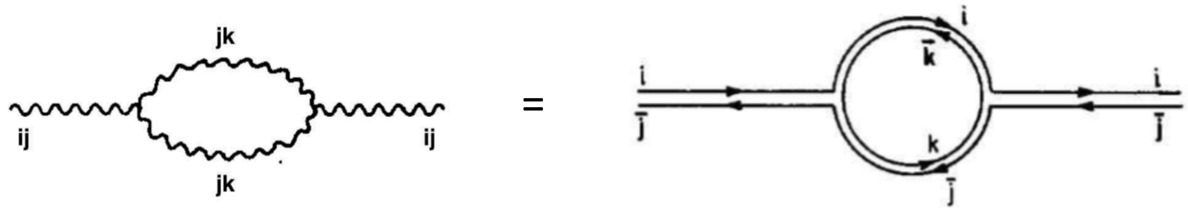
- Even understanding QCD vacuum is really very challenging.
- Can get a lot of info studying QCD on a Euclidean Lattice [Wilson 1974]
- and using Monte Carlo numerical simulations



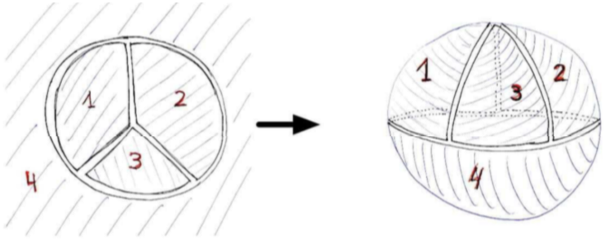
- Moreover: are we so sure that strings do not enter into the game at all?
- In SU(3) Yang-Mills the flux tubes are confined.
- Potential energy between external quarks scales linearly with the distance.



- ‘t Hooft [1974]: consider SU(N).
- Take large N limit with $\lambda = g^2_{YM} N$ fixed
- Double line notation



- At large N planar diagrams are leading, non-planar diagrams subleading
- Perturbative expansion = sum over topologies
- Planar diagrams = spheres; non planar: higher genus surfaces (torus, etc)



- Same as perturbative expansion of string amplitudes with $g_s \sim 1/N$

- 1974. Scherk and Schwarz realize that string theory contains a massless spin 2 particle which corresponds to the graviton.
- String theory = quantum gravity?
- Not so many people interested, since the theory had various inconsistencies (anomalies)

- 1984. Green and Schwarz discover that actually these inconsistencies are not there.
- Since then, the interest in string theory grew up enormously

- 1995. Witten suggests that the 5 different consistent string theories are just different manifestations of a mother theory, M-theory

- 1995. Polchinski: D-branes and their double nature.
- 1996. Strominger,Vafa: black hole entropy from D-branes.

- 1997. Maldacena: holographic conjecture
- String theories (and thus quantum gravity) can be equivalent to quantum field theories (like QCD) with no gravity, in at least one dimensions less.

- This raises the hope to find a string theory model which is equivalent to QCD, coming back to the origins in a sense.

- (It is fair to say that we do not have found a string dual to QCD, yet)

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“ It works in a very subtle way,
as a strong/weak coupling duality.”

" Certain regimes where QFT is strongly interacting, mapped into classical (i.e. weakly interacting) gravity! "

(and the other way around)

- For the master example [Maldacena 1997]:

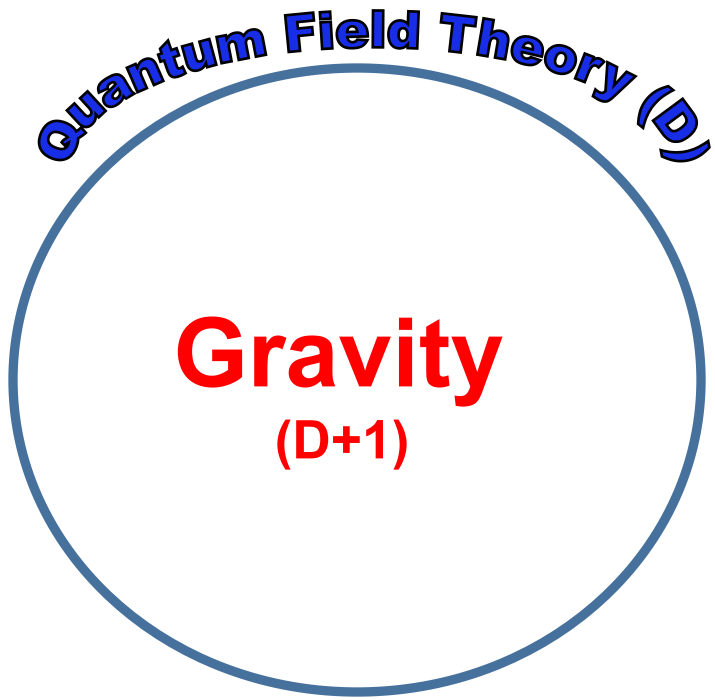
  $\mathcal{N} = 4$ $SU(N_c)$ SYM in $D = 4$ dual to gravity on $AdS_5 \times S^5$.
  Classical gravity regime: $N_c \gg 1, \quad \lambda = g_{YM}^2 N_c \gg 1$.

- Large number of d.o.f. (large N)
- Strong coupling

- Non-perturbative QFT problems can be solved by classical gravity!
- Quantum gravity from a dual perturbative QFT!

How to compute?

QFT vacuum ➔ Gravity background

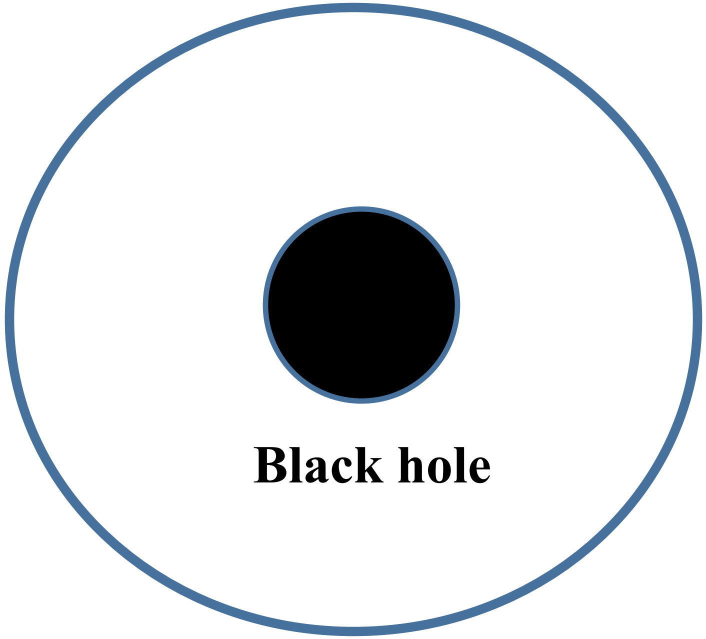


$$Z_{QFT} = Z_{QG(String)} \approx e^{-S_{\rm gravity}(\rm on-shell)}$$

[Gubser, Klebanov, Polyakov; Witten, 1998]

QFT vacuum ➔ Gravity background

QFT at finite temperature $Z = Tre^{-\frac{H}{T}}$

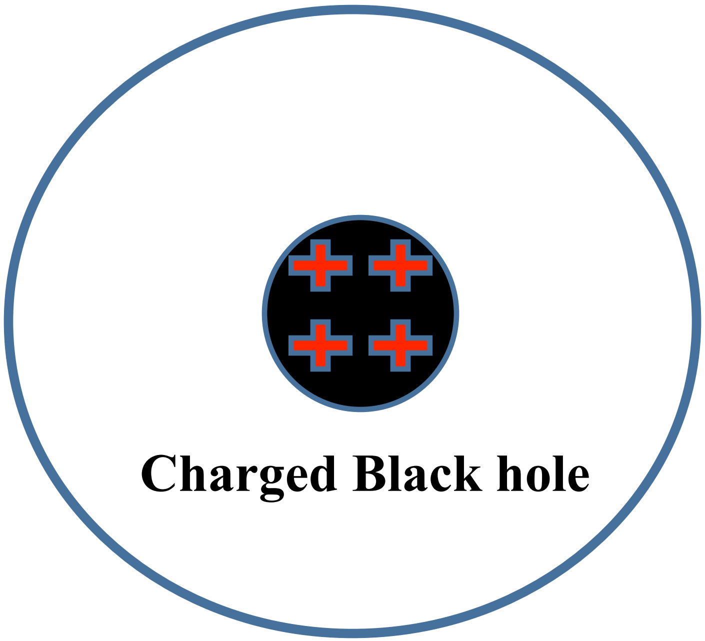


[Witten, 98]

Log Z ≈ - S[gravity on shell]

QFT vacuum ➔ Gravity background

QFT at finite temperature and density $Z = Tr e^{-\frac{H-\mu N}{T}}$



QFT charge density = electric flux on the boundary

Log Z ≈ - S[gravity on shell]

CFT d

???

$$x_\mu \to \lambda x_\mu$$
$$E \to \lambda^{-1} E$$

$\Longleftrightarrow$

$$ds^2 = \quad dx_\mu dx^\mu + \quad dr^2$$

E ≈ r

CFT d

AdS d+1

$$x_\mu \to \lambda x_\mu$$
$$E \to \lambda^{-1} E$$

⟺

$$ds^2 = \frac{r^2}{R^2} dx_\mu dx^\mu + \frac{R^2}{r^2} dr^2$$

E ≈ r

(R=AdS radius)

CFT d

$$x_\mu \to \lambda x_\mu$$
$$E \to \lambda^{-1} E$$

⟺

E ≈ r

AdS d+1

$$ds^2 = \frac{r^2}{R^2} dx_\mu dx^\mu + \frac{R^2}{r^2} dr^2$$

(R=AdS radius)

CFT at finite T

$$Z = Tr e^{-\frac{H}{T}}$$

⟺

AdS black hole

$$ds^2 = \frac{r^2}{R^2}\left[-b[r]dt^2 + dx_i dx_i\right] + \frac{R^2}{r^2}\frac{dr^2}{b[r]}$$

$$b[r] = 1 - \frac{r_h^d}{r^d}$$

$$T_{CFT} = T_{BH} = \frac{r_h\, d}{4\pi R^2}; \qquad S_{CFT} = S_{BH} = \frac{A_h}{4G_N} \sim V_{d-1} T^{d-1}$$

CFT at finite T and μ

$$Z = Tr e^{-\frac{H - \mu N}{T}}$$

⟺

Charged RN-AdS black hole

$$A_t \sim \mu - \frac{\rho}{r^{d-2}}$$

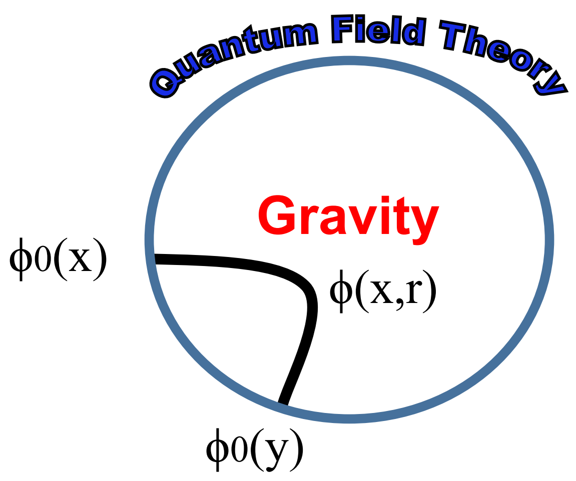
Correlators

- In a D dim. QFT we compute the generating functional

$$Z_{QFT}[\phi_0] = \int D\Psi \exp\left(i[S_{QFT} + \int d^D x\, \phi_0(x)\mathcal{O}[\Psi](x)]\right)$$

- N-point correlators of O[x]: from n-th derivarives of Log Z w.r.t. external source ϕ_0(x).

- Holography: treat external source ϕ_0(x) as boundary value of a gravity field ϕ(x,r) which is "dual" to the operator O[x]



Correlators

- Then: [Gubser, Klebanov, Polyakov; Witten; 1998]

$$Z_{QFT}[\phi_0] = Z_{QG/String} \approx e^{iS_{gravity}[\phi_0]}|_{``\phi(x,r)\to\phi_0(x)"}$$

- So that for example: $\langle \mathcal{O}(x)\mathcal{O}(y)\rangle \sim \frac{\delta^2 S_{grav}[\phi_0]}{\delta\phi_0(x)\delta\phi_0(y)}|_{\phi_0=0}$

- Can compute correlators, just solving equation of motion for ϕ(x,r) !
- Retarded correlators at finite temperature: incoming b. c. at horizon

Operator O(x) ➔ Gravity field ϕ(x,r)

- Example 1(stress tensor): $T^{\mu\nu}(x) \to g_{\mu\nu}(x,r)$

- Example 2 (conserved current): $J^{\mu}(x) \to A_{\mu}(x,r)$

- Example 3 (scalar operator): $TrF^2(x) \to \Phi(x,r)$

- Global symmetries in QFT are mapped into local symmetries in the gravity side

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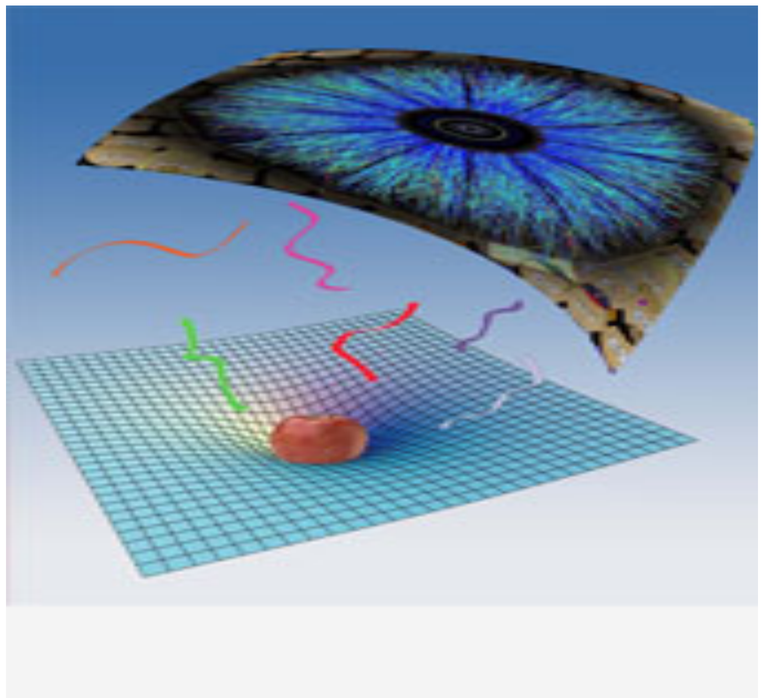
Strongly correlated QFTs arise in many places:

- QCD: confinement, mass gap, quark-gluon plasma phase, neutron star core…
- Quantum critical regions in condensed matter: high Tc superconductors, strange metals …

- Often non-equilibrium, finite density challenging: need novel tools .
- Holography is emerging as a promising one.

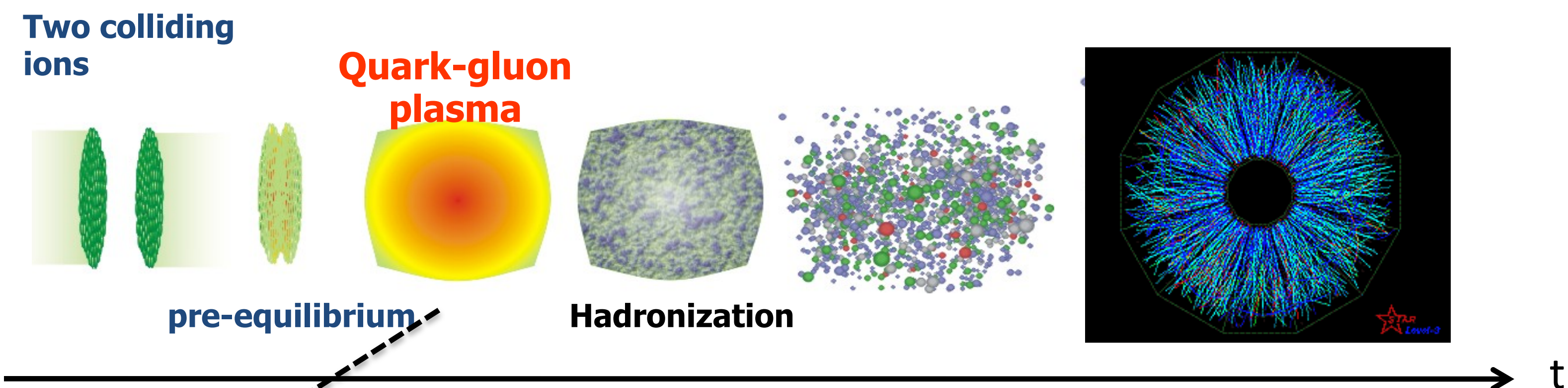
- Often analytic control on the models. Novel intuitions.
- Can deal with both static and real-time dynamical properties.
- Still limited to toy models.
- Provide benchmarks and info on universal behavior at strong coupling.

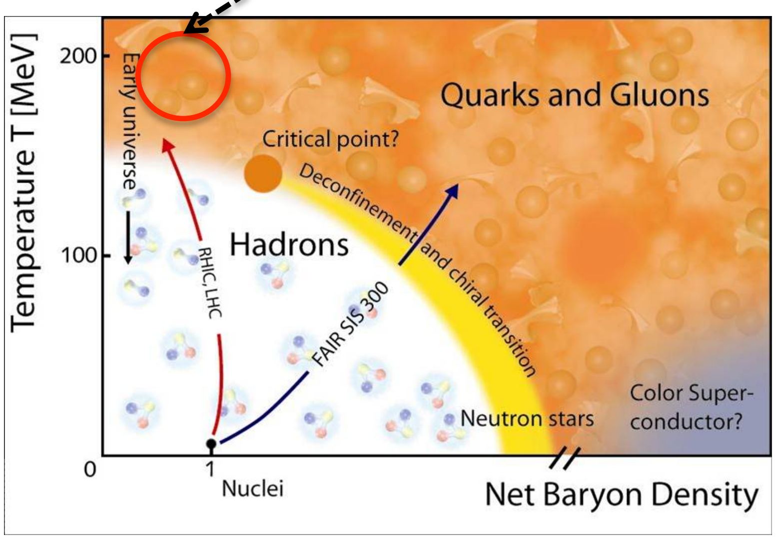
Example 1:
Holography, black holes and the Quark-Gluon Plasma



The Quark-Gluon-Plasma

Au+Au collisions at STAR (RHIC). 200 GeV/nucleon pair.
Pb + Pb collisions at ALICE (LHC). 3 TeV/nucleon pair





A novel state of matter where QCD is deconfined

The Quark-Gluon-Plasma

- Heavy ion collisions at RHIC and LHC indicate that QGP:

- behaves like liquid
- Very small shear viscosity over entropy density ratio
- Hence: strongly coupled
- Nearly scale invariant
- Very opaque (large jet quenching)

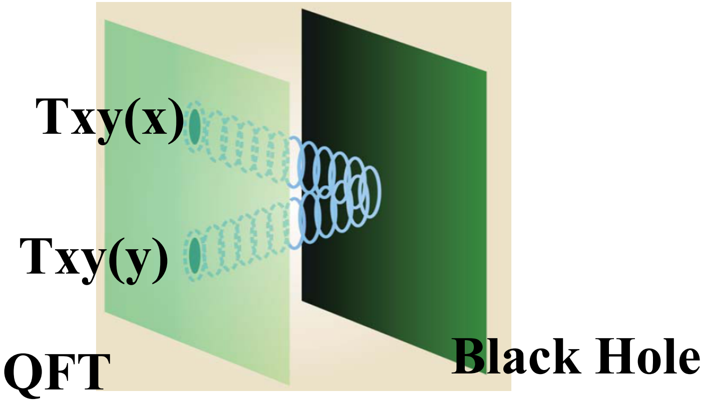
- Challenging, dynamical system. Try with holography
- Strongly coupled thermal QFT ➔ Black Hole in higher dim.
- QFT Thermodynamics ➔Black Hole thermodynamics.
- Hydrodynamics ➔ Fluctuations around black hole background

Shear viscosity from holography

$$\eta = -\mathrm{Lim}_{\omega \to 0} \frac{1}{\omega} \mathrm{Im} G^R_{xy,xy}(\omega, \mathbf{0})$$

$$G^R_{xy,xy}(\omega, \mathbf{0}) = \int dt\, d\mathbf{x}\, e^{i\omega t} \theta(t) \langle [T_{xy}(t, \mathbf{x}), T_{xy}(0, \mathbf{0})] \rangle$$

- Compute using holography: $T_{xy}(x)$ dual to $g_{xy}(x,r)$



QFT correlator = classical scattering of gravitons from black hole

$$\frac{\eta}{s} = \frac{1}{4\pi} \frac{\hbar}{K_B}$$

[T. Damour, Ph.D. Thesis 1979; Policastro, Son, Starinets, 2001; Kovtun, Son, Starinets 2004]

- Universal: for any isotropic fluid with classical gravity dual
- Surprisingly close to the measured value for QGP

Second order hydrodynamics from holography

[Romatschke 2009; F.B., Cotrone, Tarrio; F.B. , Cotrone, 2010]

Model: conformality broken by marginally relevant operator

$$\Delta \equiv (1 - 3c_s^2)$$

$$\frac{\eta}{s} = \frac{1}{4\pi} \qquad T\tau_\pi = \frac{2-\log 2}{2\pi} + \frac{3(16-\pi^2)}{64\pi}\Delta \qquad \frac{T\kappa}{s} = \frac{1}{4\pi^2}\left(1 - \frac{3}{4}\Delta\right)$$

$$\frac{T\lambda_1}{s} = \frac{1}{8\pi^2}\left(1 + \frac{3}{4}\Delta\right) \qquad \frac{T\lambda_2}{s} = -\frac{1}{4\pi^2}\left(\log 2 + \frac{3\pi^2}{32}\Delta\right) \qquad \frac{T\lambda_3}{s} = 0$$

$$\frac{T\lambda_4}{s} = 0 \qquad \frac{T\kappa^*}{s} = -\frac{3}{8\pi^2}\Delta \qquad T\tau_\pi^* = -\frac{2-\log 2}{2\pi}\Delta$$

$$\frac{\zeta}{\eta} = \frac{2}{3}\Delta \qquad T\tau_\Pi = \frac{2-\log 2}{2\pi} \qquad \frac{T\xi_1}{s} = \frac{1}{24\pi^2}\Delta$$

$$\frac{T\xi_2}{s} = \frac{2-\log 2}{36\pi^2}\Delta \qquad \frac{T\xi_3}{s} = \frac{T\xi_4}{s} = 0 \qquad \frac{T\xi_5}{s} = \frac{1}{12\pi^2}\Delta \qquad \frac{T\xi_6}{s} = \frac{1}{4\pi^2}\Delta$$

Note: QCD fireball nearly conformal and strongly coupled for $1.5T_c \lesssim T \lesssim 4T_c$ (RHIC, LHC).

Jet quenching parameter

- Transport coefficient characterizing probe parton energy loss
- Evaluated holographically in *N=4 SYM* [Liu, Rajagopal, Wiedemann 06]
- Adding Nf dynamical flavors [Bigazzi, Cotrone, Mas, Paredes, Ramallo, Tarrio 2009]

$$\hat{q} = \frac{\pi^{3/2}\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}\sqrt{\lambda}\, T^3 \left[1 + \frac{2+\pi}{64\pi^2}\lambda\frac{N_f}{N_c} + \ldots\right]$$

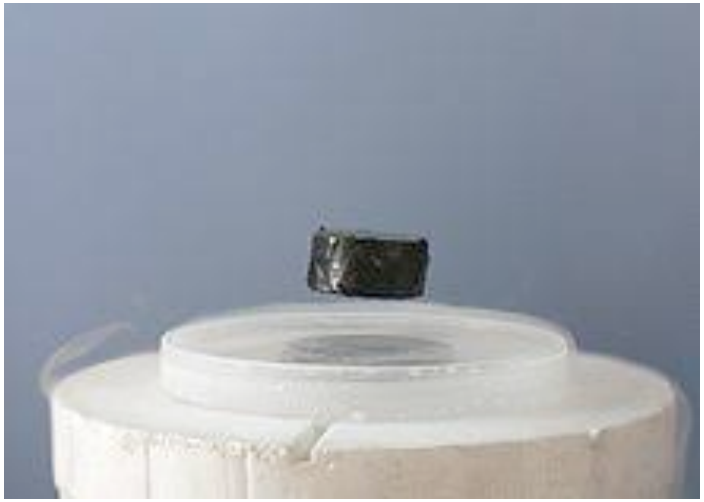
Quarks enhance jet quenching

Extrapolating to QGP: Nc=Nf=3, λ=6π, T=300 MeV, get

q≈ 4÷5 GeV^2/fm

right in the ballpark of data

Example 2:
Holography and condensed matter



- Strongly correlated electrons (Condensed Matter) (often layered, 2+1)

"High-T_c" Cuprates

$YBa_2Cu_3O_{7-x}$ (YBCO)

$La_{2-x}Sr_xCuO_4$ (LSCO)

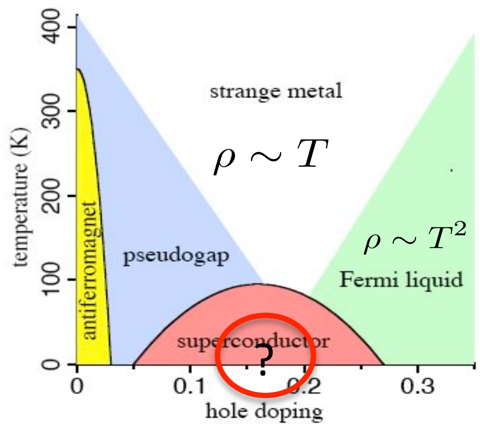
$Bi_2Sr_2CaCu_2O_{8+x}$ (BSCCO)

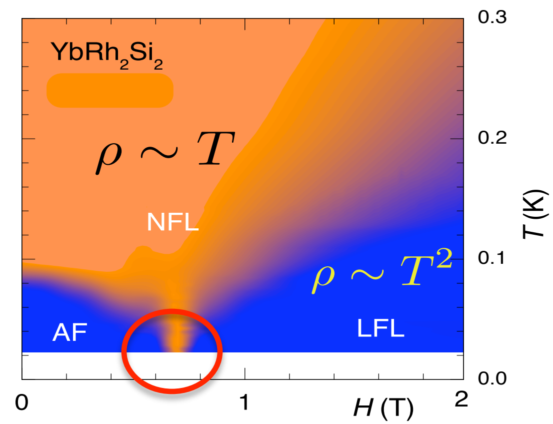
Heavy Fermions

$CeCu_{6-x}Au_x$

$CeCoIn_5$

$YbRh_2Si_2$

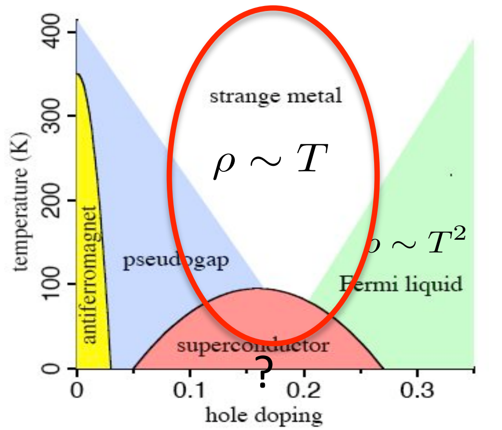


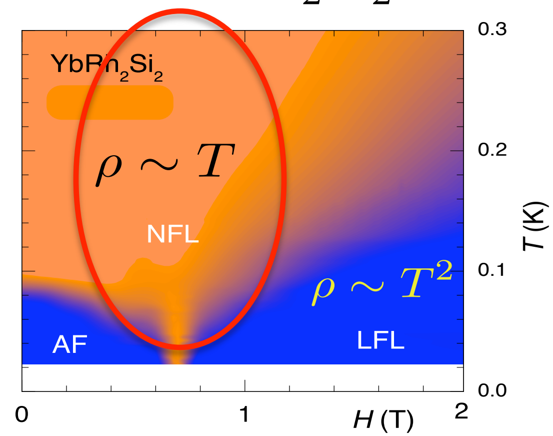


- Quantum phase transitions (T=0). Scale invariant QFT. Very large correlations

2. Strongly correlated electrons (Condensed Matter) (often layered, 2+1)

"High-T_c" Cuprates		Heavy Fermions
$YBa_2Cu_3O_{7-x}$	(YBCO)	$CeCu_{6-x}Au_x$
$La_{2-x}Sr_xCuO_4$	(LSCO)	$CeCoIn_5$
$Bi_2Sr_2CaCu_2O_{8+x}$	(BSCCO)	$YbRh_2Si_2$





- Quantum phase transitions (T=0). Scale invariant QFT. Very large correlations
- Quantum critical region. Affected by quantum critical point at T=0 [Sachdev]
- Exhibit phases which escape standard paradigms based on quasi-particle description

- E.g. strange metallic phase with linear (in T) resistivity
- No satisfactory theoretical understanding

- 2. Strongly correlated electrons (Condensed Matter)

"High-T$_c$" Cuprates		Heavy Fermions
$YBa_2Cu_3O_{7-x}$	(YBCO)	$CeCu_{6-x}Au_x$
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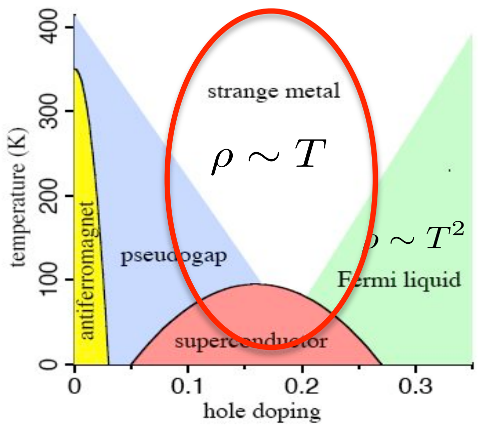
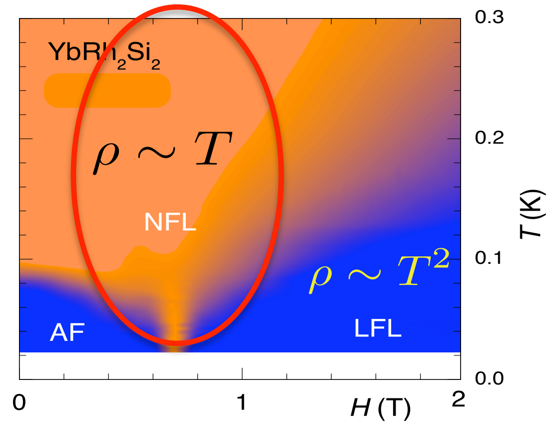




- Example of challenging observable at strong coupling: (optical) conductivity
- Ohm's law: J = σ E.   σ: retarded correlator of U(1) current J;   (ρ=1/Re[σ(0)])

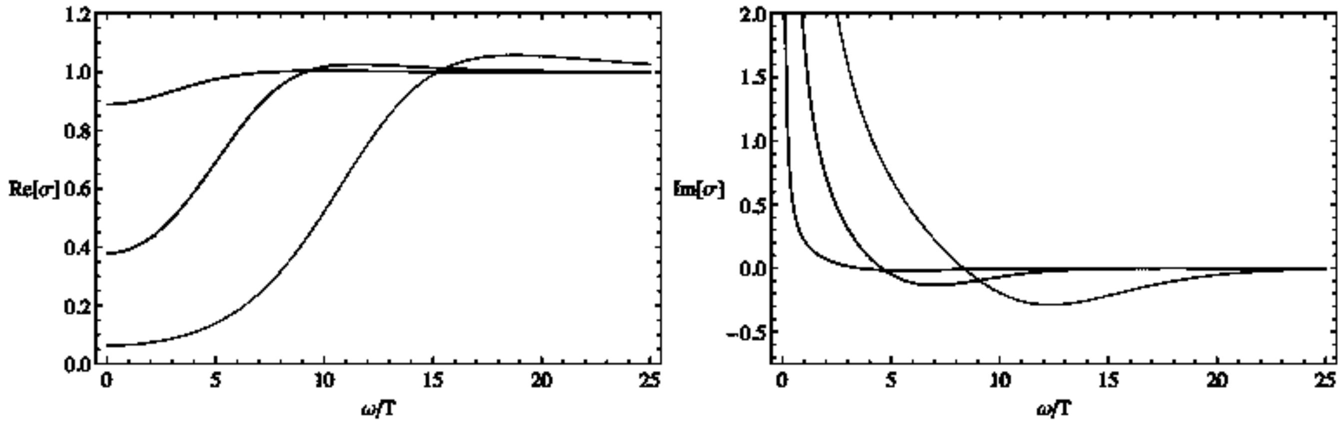
$$\sigma(\omega) = \mathrm{Lim}_{k\to 0}\frac{G^R_{JJ}(\omega,k)}{i\omega}$$

$$G^R_{JJ}(\omega,k) = -i\int d^{d-1}x\,dt\,e^{i\omega t - ikx}\theta(t)\langle[J(t,x),J(0,0)]\rangle$$

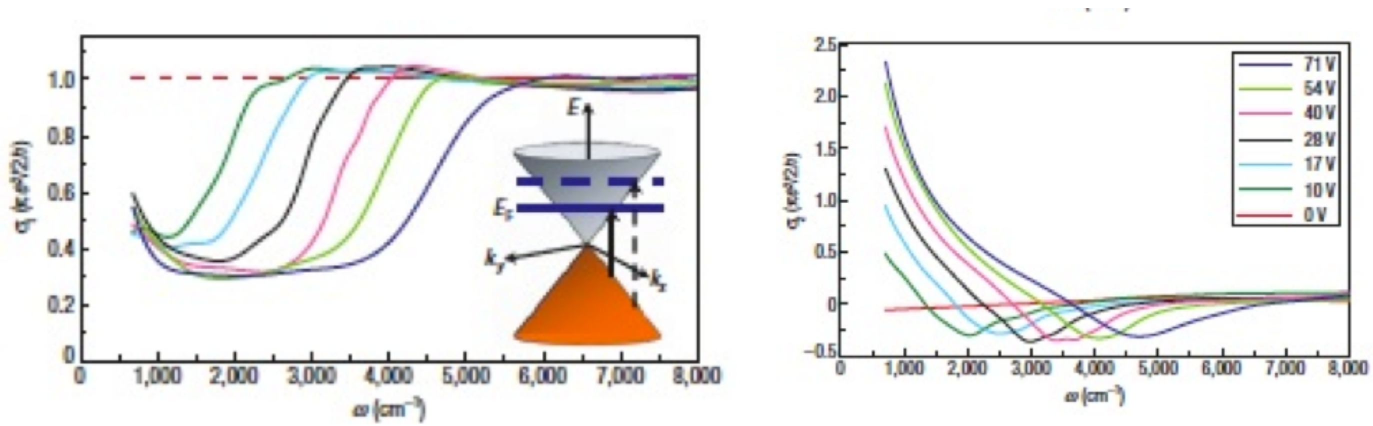
Optical conductivity in d=2+1 from holography

From fluctuations around dual charged black hole [Herzog, Kovtun, Sachdev, Son, 07]

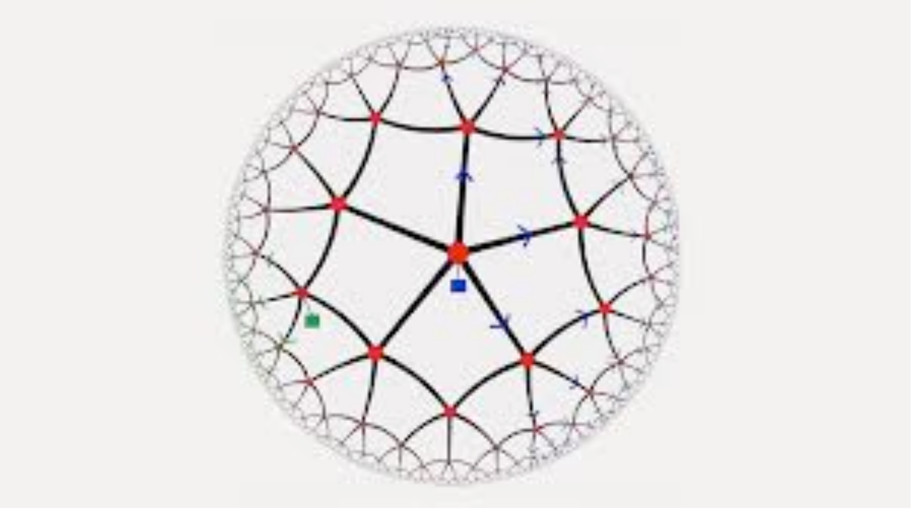
- Jx = σ Ex = - iω σ Ax



- Cfr with graphene (at low energy a relativistic theory in 2+1) [Li et al 2008]



Example 3:
Holography, black holes and quantum information

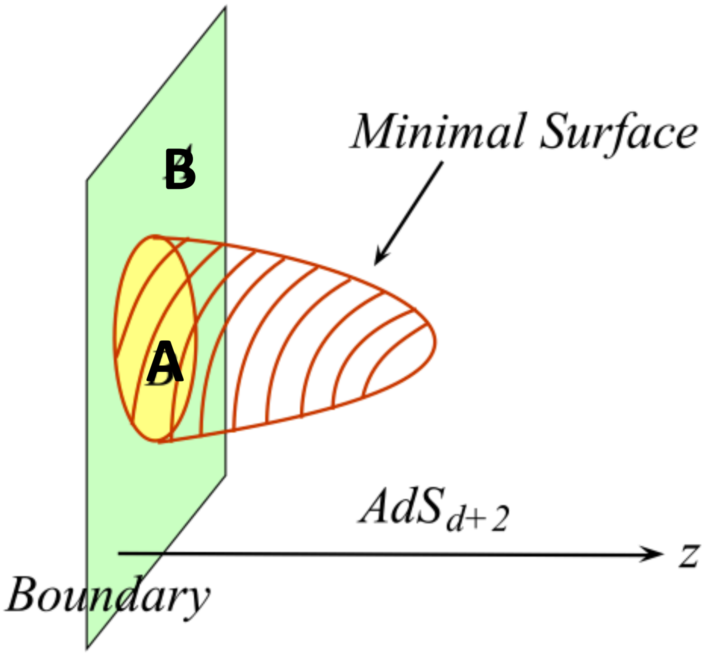


Holography and Entanglement Entropy

- Consider region A in a space A+B
- If ignorant on B, define reduced density matrix $\rho_A = tr_B \rho$

- Entanglement entropy $S_A = -tr_A \rho_A \log \rho_A$

- Holographically [Ryu, Takayanagi 2006]



$$S_A = \text{extremum}_{\partial M = \partial A} \frac{\text{area}(M)}{4G_N}$$

- For AdS_3 this gives,

$$S_A = \frac{c}{3} \log \frac{l}{a}$$

- Same as in 2d CFT [Wilczek; Cardy, Calabrese]

Concluding comments

- Holography: theory of strings (QG) = lower dim. QFT without gravity.
- In a sense this could provide a background independent definition of QG.

- An enormous amount of quantitative checks.

- Many applications to strongly coupled QFT, from hep to cond-mat

- In recent years many efforts for going in the other direction:

  - Gravity as an emergent quantum many-body phenomenon?

  - What role do quantum information concepts such as entanglement and circuit complexity play in this connection ?

- A lot of fun!

Thank you for your time