Classicalization, Scrambling and Thermalization in QCD at high energies

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Outline of lectures

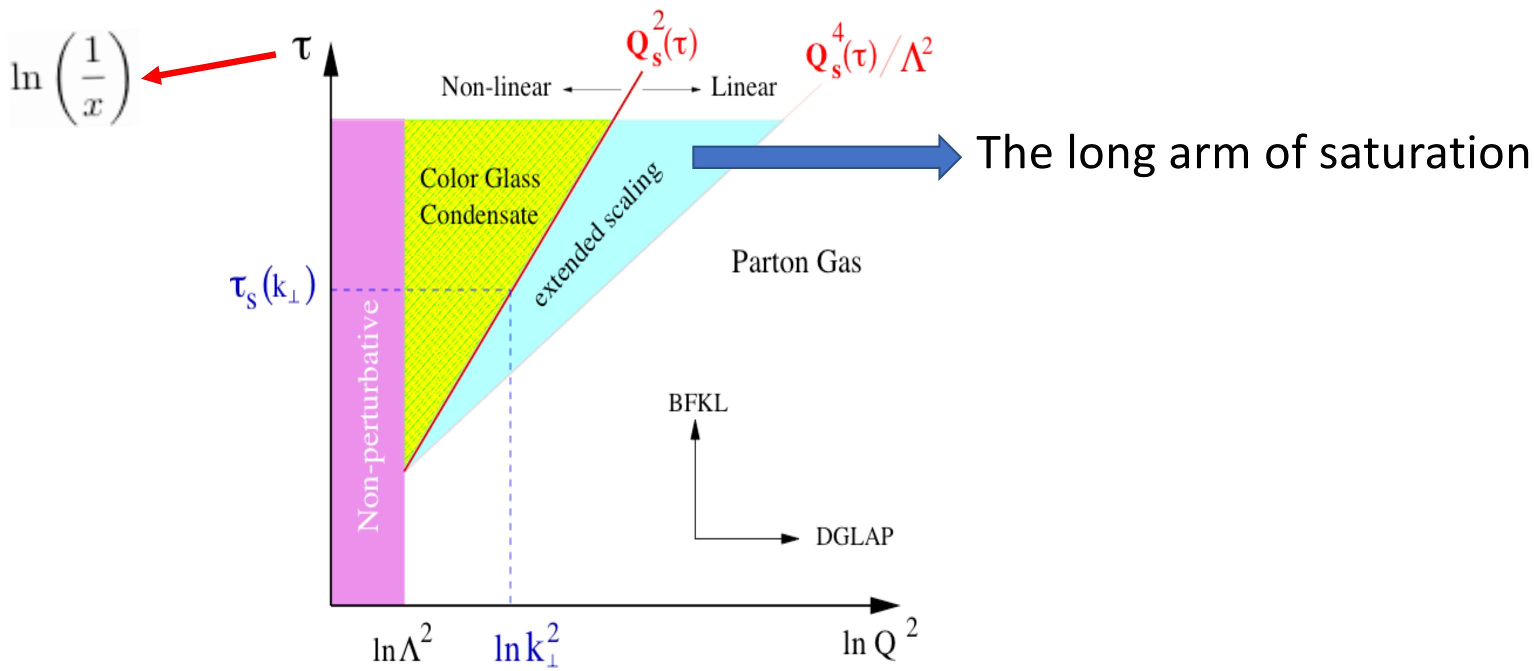
Lecture I: Classicalization: The hadron wavefunction at high energies as a Color Glass Condensate

Lecture II: The CGC Effective Field Theory

Lecture III: From CGC to the Glasma, key features of the Glasma

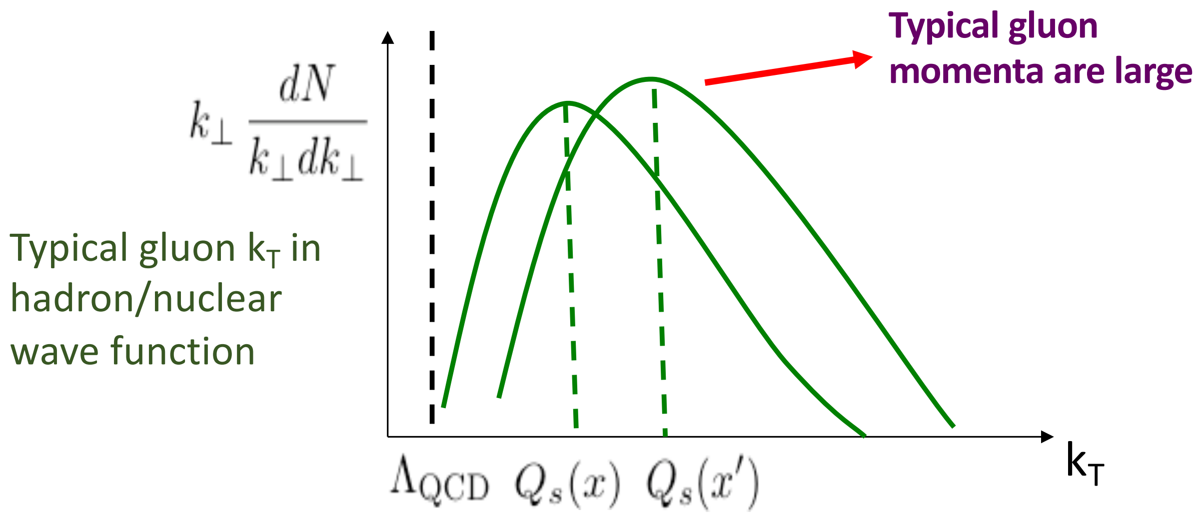
Lecture IV: Thermalization and interdisciplinary connections

The hadron wave-function at high energies



- Description of gluon saturation, geometrical scaling, and matching to pQCD in the CGC EFT
- Powerful tools to compute n-body correlators and their energy evolution: MV, BK, JIMWLK
- Precision computations: state of the art for a number of processes in NLO+NLLx

Basic idea: emergent saturation scale grows with energy



Bulk of high energy cross-sections:

a) obey dynamics of novel non-linear QCD regime
b) Can be computed systematically in weak coupling in QCD

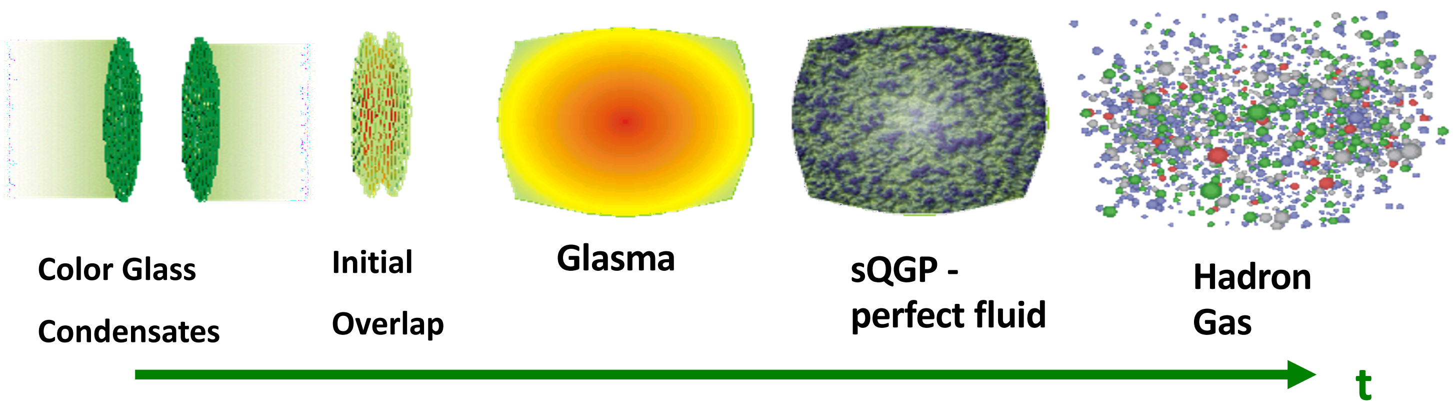
Traditional picture of heavy-ion collisions

***@$#! on *@$#!**

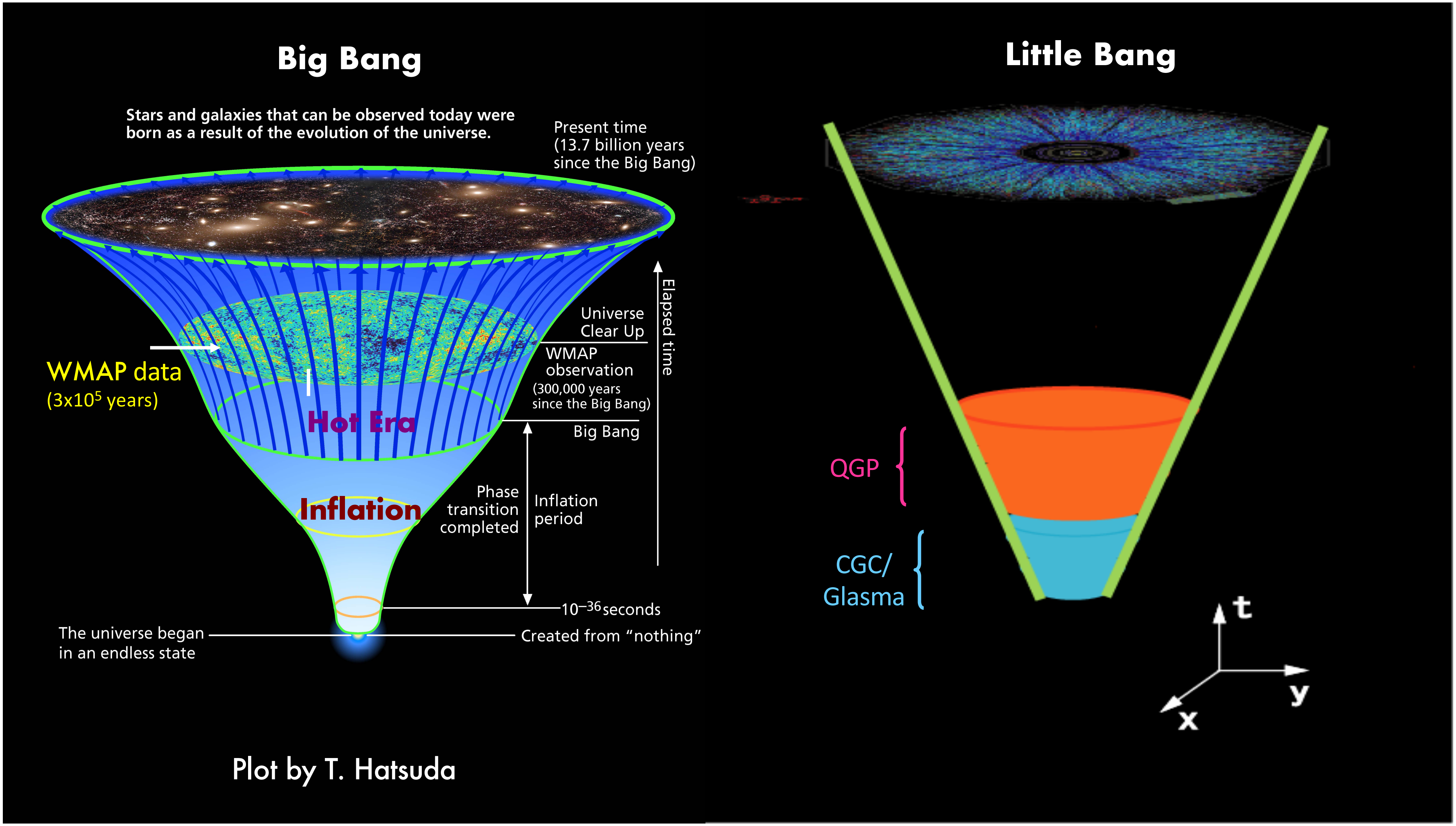
Well known physicist (circa early 1980s)

Standard model of Heavy-Ion Collisions

RV, Plenary Talk, ICHEP (2010), arXiv:1012.4699

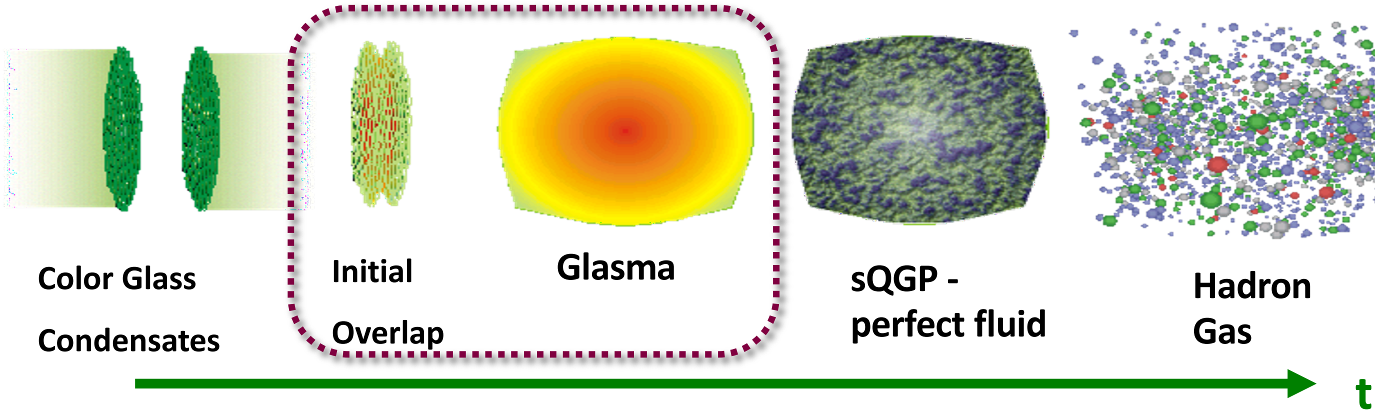


We will argue that key features of this space-time evolution can be described from first principles in the Regge limit of QCD


Big Bang
Stars and galaxies that can be observed today were born as a result of the evolution of the universe.
Present time (13.7 billion years since the Big Bang)
Elapsed time
Universe Clear Up
WMAP observation (300,000 years since the Big Bang)
WMAP data (3x10^5 years)
Hot Era
Big Bang
Inflation
Phase transition completed
Inflation period
10^-36 seconds
The universe began in an endless state
Created from "nothing"
Plot by T. Hatsuda
Little Bang
QGP
CGC/ Glasma
t
y
x

Standard model of Heavy-Ion Collisions

RV, Plenary Talk, ICHEP (2010), arXiv:1012.4699



Glasma (\Glahs-maa\): *Noun:* non-equilibrium matter between Color Glass Condensate (CGC) & Quark Gluon Plasma (QGP)

Big Bang vs. Little Bang

Decaying Inflaton
with occupation # $1/g^2$

↔

Decaying Glasma
with occupation # $1/g^2$

Explosive amplification
of low momentum small
fluctuations (preheating)

↔

Explosive amplification
of low momentum small fluctuations
(Weibel instabilities)

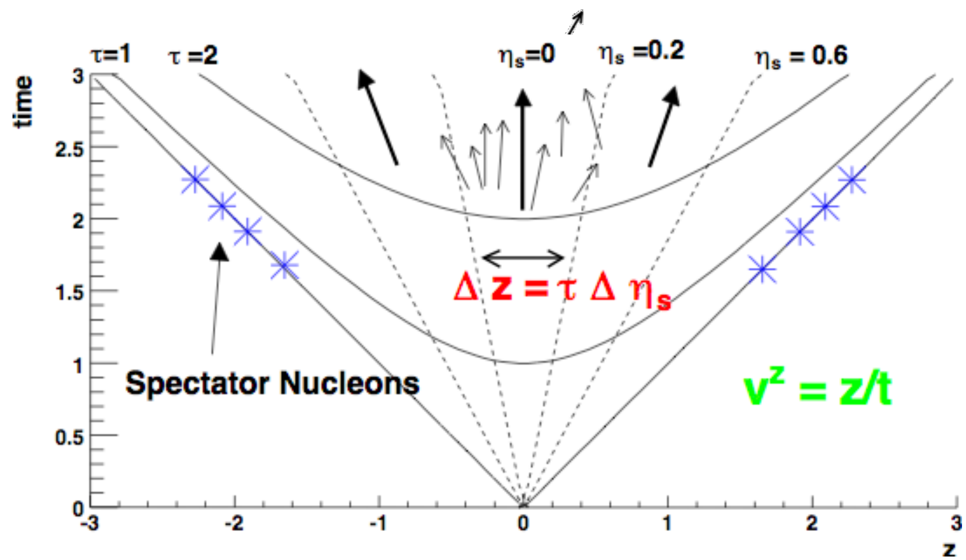
Interaction of fluctutations/inflaton
-> thermalization?

↔

Interaction of fluctutations/Glasma
-> thermalization?

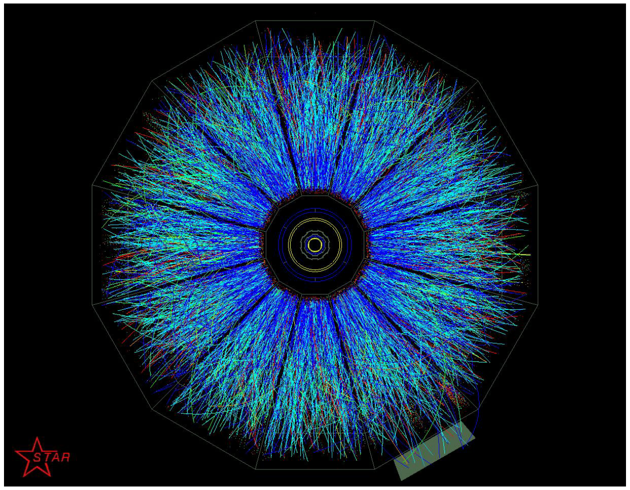
Other common features: topological defects, turbulence,…

Forming a Glasma in the little Bang



Problem: Compute particle production in QCD with *strong time dependent* sources

THE LITTLE BANG



How can we compute multiparticle production *ab initio* in HI collisions ?

~~*perturbative VS non-perturbative,*~~

strong coupling VS weak coupling

Always non-perturbative for questions of interest in this talk!

Similar to computations of pair production in strong E&M fields (Schwinger mechanism) and Hawking radiation in the vicinity of a Black Hole

Approaches to multi-particle production in QCD

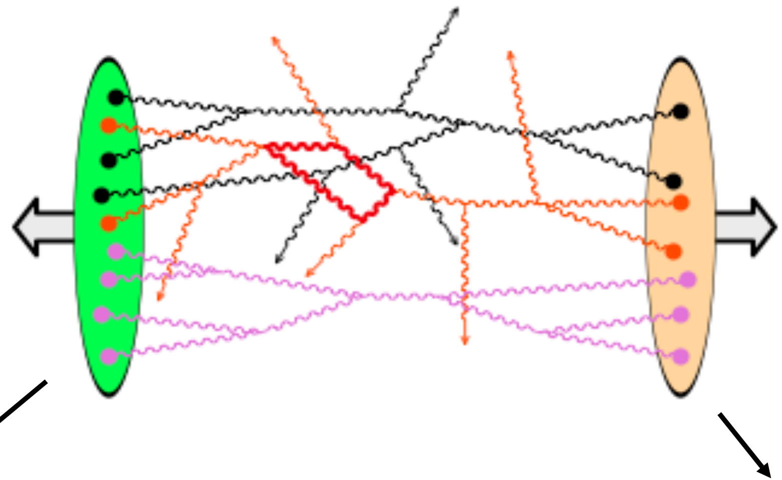
Two “clean” theoretical limits:

- Holographic thermalization (based on duality of strongly coupled ($g^2 N_c \to \infty$; $N_c \to \infty$) N=4 SUSY YM to classical gravity in $AdS_5 \times S_5$)

- Highly occupied QCD at weak coupling ($g^2 \to 0$; $g^2 f \sim 1$)

Our focus: non-equilibrium strongly correlated gluodynamics at weak coupling

Particle production in presence of strong time-dependent sources



$$\rho_A \sim \frac{1}{g} >> 1 \qquad \rho_B \sim \frac{1}{g} >> 1$$

P_n obtained from cut vacuum graphs in field theories with strong time dependent sources

Probability of producing n particles in theory with sources

Lehmann-Symanzik-Zimmerman (LSZ)

$$\langle p_1\cdots p_{n\,\mathrm{out}}|0_{\mathrm{in}}\rangle = \frac{1}{Z^{n/2}}\int\left[\prod_{i=1}^{n} d^4x_i\, e^{ip_i\cdot x_i}\,(\partial^2_{x_i}+m^2)\frac{\delta}{i\delta\rho(x_i)}\right]e^{i\mathcal{V}[\rho]}$$

n-particle probability $$P_n=\frac{1}{n!}\prod_{i=1}^{n}\int\frac{d^3p_i}{2E_{p_i}}|\langle p_1\cdots p_n\mathrm{out}|0_{\mathrm{in}}\rangle|^2$$

$$P_n=\frac{1}{n!}\mathcal{D}^n[j_+,j_-]\,\exp\left(iV[j_+]-iV^\star[j_-]\right)|_{j_+=j_-=j}$$

Schwinger-Keldysh contour

$$D[j_+,j_-]\equiv\frac{1}{Z}\int_{x,y}G^0_{+-}(x,y)(\partial_x^2+m^2)(\partial_y^2+m^2)\frac{\delta}{\delta j_+(x)}\,\frac{\delta}{\delta j_-(y)}$$

$$\int\frac{d^3\vec{p}}{(2\pi)^3 2E_p}\,e^{ip\cdot(x-y)}$$

$$G^0_{++}=\frac{i}{p^2-m^2+i\epsilon}$$

$$G^0_{--}=\frac{-i}{p^2-m^2-i\epsilon}$$

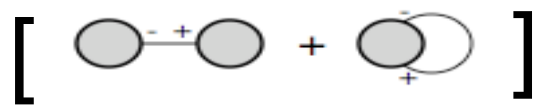
$$G^0_{+-}=2\pi\theta(-p^0)\delta(p^2-m^2)$$

N-particle distributions: inclusive multiplicity

$$\langle n \rangle = \sum_n n\, P_n \equiv D\left[\underbrace{e^{D} e^{iV} e^{-iV}}_{e^{iV_{\rm SK}}}\right]$$

Gelis, RV ; NPA776 (2006)135
NPA 779 (2006)177

$$\langle n \rangle = \int_{x,y} Z\, G^0_{+-}(x,y)\left[\Gamma_+(x)\,\Gamma_-(y) + \Gamma_{+-}(x,y)\right]$$



$$\Gamma_\pm(x) = \frac{\partial_x^2 + m^2}{Z}\,\frac{\delta i V_{\rm SK}}{\delta j_\pm(x)}\Big|_{j_+=j_-=j}$$

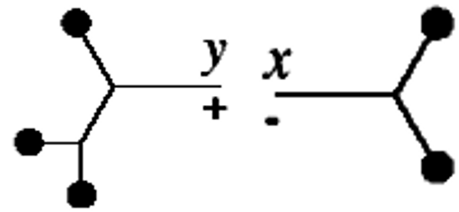
One-point function in the background field

$$\Gamma_{+-}(x,y) = \frac{(\partial_x^2 + m^2)(\partial_y^2 + m^2)}{Z}\,\frac{\delta^2 i V_{\rm SK}}{\delta j_\pm(x)\delta j_-(y)}\Big|_{j_+=j_-=j}$$

Two-point function
in the background field

Inclusive multiplicity to LO in strong fields: O $(1/g^2)$

$$\langle n \rangle_{\rm LO} =$$



$\mathrm{but}\ (g\rho)^\infty$ means arbitrary number of insertions of sources ρ

In the Schwinger Keldysh formalism, each node of a tree includes a sum over $\pm$

$$G^0_{i+}(x,y) - G^0_{i-}(x,y) = G^0_R(x,y)\ ;\ i = \pm$$

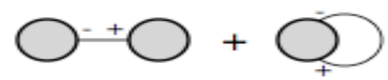
Recursive use of this identity shows that sum of all tree diagrams is
is the *retarded* solution of classical equations of motion with

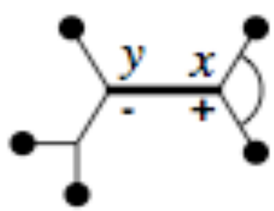
$$\lim_{x^0 \to -\infty} \phi_c(x) = 0$$

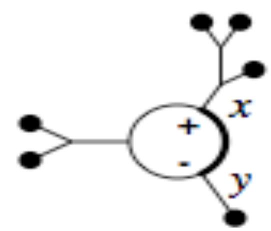
Hence, leading order result for the inclusive multiplicity
in the strong fields in a heavy-ion collision
is given by solutions of the QCD Yang-Mills equations !

Inclusive multiplicity at NLO in strong fields: O (g^0)

$$\langle n \rangle = \int_{x,y} Z\, G^0_{+-}(x,y)\, [\Gamma_+(x)\, \Gamma_-(y) + \Gamma_{+-}(x,y)]$$

[]

$\langle n \rangle_{\mathrm{NLO}} =$ 

$+$ 

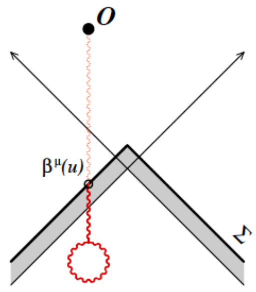
$$\phi_c^{(0)}\, \phi_c^{(1)} \qquad \langle \delta\phi_{\mathrm{quant.}}\, \delta\phi_{\mathrm{quant.}} \rangle$$

Product of classical field and
1-loop correction to classical field

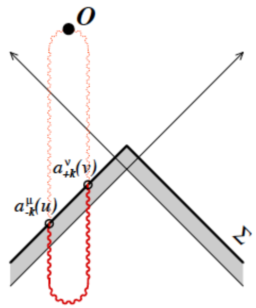
Small fluctuation propagator
in classical background field

QCD factorization of wee gluon distributions of the nuclei

Gelis,Lappi,RV (2008)



$\beta(\vec{u})$



$\mathcal{G}(\vec{u},\vec{v})$

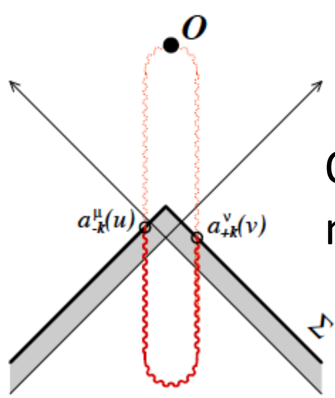
$$\mathcal{O}_{\rm NLO} = \left[\frac{1}{2}\int_{\vec{u},\vec{v}} \mathcal{G}(\vec{u},\vec{v})\; \mathcal{T}_u \mathcal{T}_v + \int_{\vec{u}} \beta(\vec{u})\; \mathcal{T}_u\right] \mathcal{O}_{\rm LO}$$

$$\mathcal{T}_u = \frac{\delta}{\delta A(\vec{u})}$$

linear operator of source on initial "Cauchy" surface

These 1-and 2-point have logs - are resummed to all orders by the JIMWLK Hamiltonian

$$\mathcal{O}_{\rm NLO} = \left[\ln\left(\frac{\Lambda^+}{p^+}\right)\mathcal{H}_1 + \ln\left(\frac{\Lambda^-}{p^-}\right)\mathcal{H}_2\right]\mathcal{O}_{\rm LO}$$



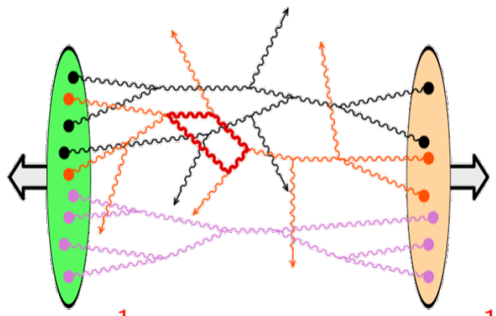
Quantum fluctuations that cross-talk between nuclei before the collision are suppressed $\Rightarrow$ QCD factorization !

Factorization + temporal evolution in the Glasma

$$O\left(\frac{Q_S^4}{g^2}\right)$$

$$T^{\mu\nu}_{\rm LO} = \frac{1}{4} g^{\mu\nu} F^{\lambda\delta} F_{\lambda\delta} - F^{\mu\lambda} F^{\nu}_{\lambda}$$

ε=20-40 GeV/fm^3 for τ=0.3 fm @ RHIC
Scale set by Q_S in the nuclei



$$\langle T^{\mu\nu}(\tau, \eta, x_\perp)\rangle_{\rm LLog} = \int [D\rho_1 d\rho_2]\, W_{Y_1}[\rho_1]\, W_{Y_2}[\rho_2]\, T^{\mu\nu}_{\rm LO}(\tau, x_\perp)$$

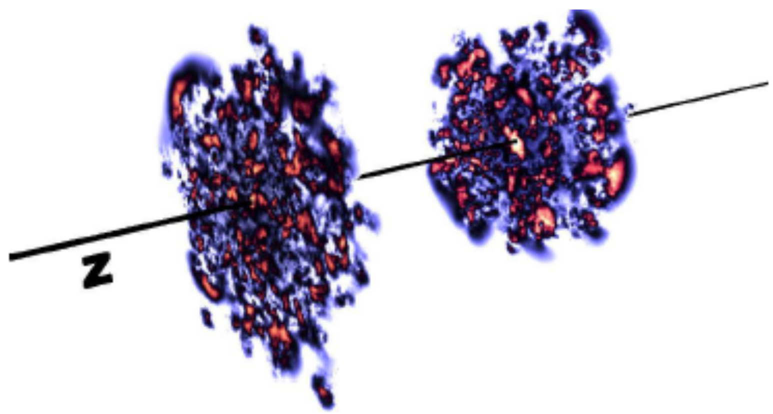
$$Y_1 = Y_{\rm beam} - \eta\,;\; Y_2 = Y_{\rm beam} + \eta$$

Glasma factorization => universal "density matrices W" ⊗ "matrix element"

The lumpy Glasma at LO: Yang-Mills equations

Collisions of lumpy gluon ``shock” waves

Non-equil. computations on lattice:
Krasnitz, RV (1998)
Krasnitz, Nara, RV (2001)
Lappi (2003)



Leading order solution: Solution of QCD Yang-Mills eqns

$$D_\mu F^{\mu\nu,a} = \delta^{\nu+}\rho_A^a(x_\perp)\delta(x^-) + \delta^{\nu-}\rho_B^a(x_\perp)\delta(x^+)$$

$$x^\pm = t \pm z$$

$$F^{\mu\nu,a} = \partial_\mu A^{\nu,a} - \partial_\nu A^{\nu,a} + g f^{abc} A^{\mu,b} A^{\nu,c}$$

The lumpy Glasma at LO: Yang-Mills equations

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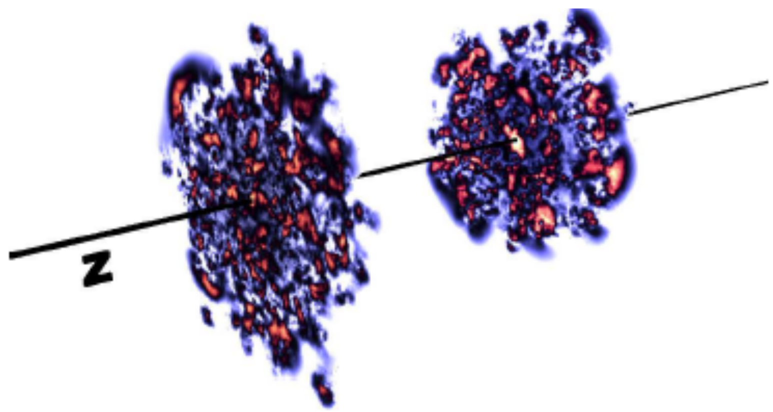


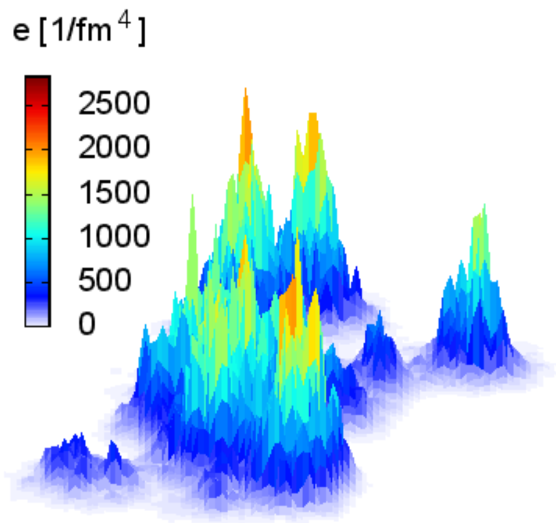
Leading order solution: Solution of QCD Yang-Mills eqns

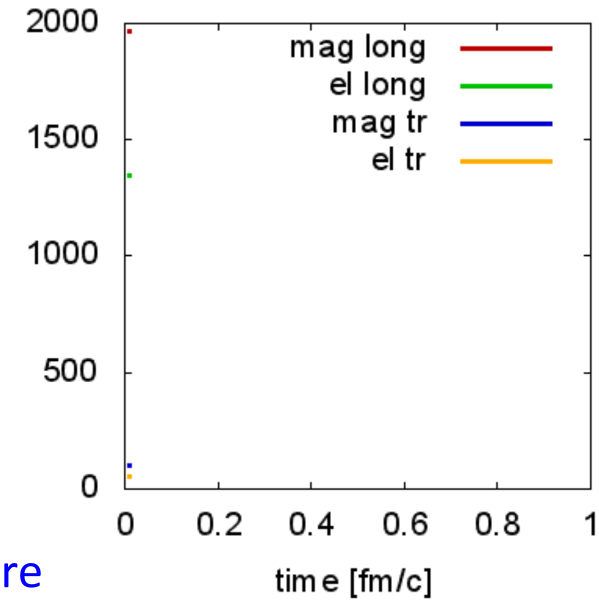
$$D_\mu F^{\mu\nu,a} = \delta^{\nu+}\rho^a_A(x_\perp)\delta(x^-) + \delta^{\nu-}\rho^a_B(x_\perp)\delta(x^+)$$

$$\langle \rho^a_{A(B)}(x_\perp)\rho^a_{A(B)}(y_\perp)\rangle = Q^2_{S,A(B)}\delta^{(2)}(x_\perp - y_\perp)$$

Q_S (x,b_T) determined from saturation model fits to HERA inclusive and diffractive DIS data

The lumpy Glasma at LO: Yang-Mills solutions





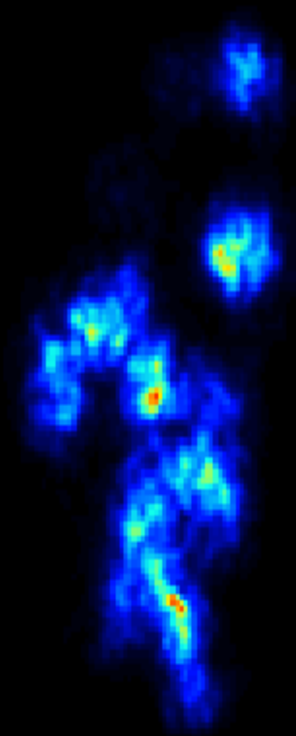
Glasma energy density and pressure

$$T_{\mu\nu}(\tau=0) = \frac{1}{2}(B_z^2 + E_z^2) \times \text{diag}(1,1,1,-1)$$

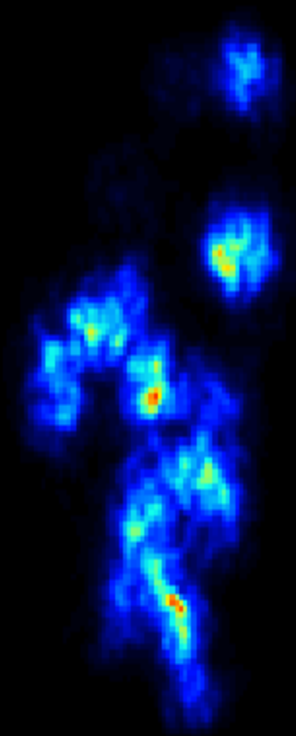
Initial longitudinal pressure is negative:

Goes to $P_L = 0$ from below with time evolution

The Glasma: colliding gluon shock waves



Glasma color fields



Glasma color fields matched to viscous hydrodynamics

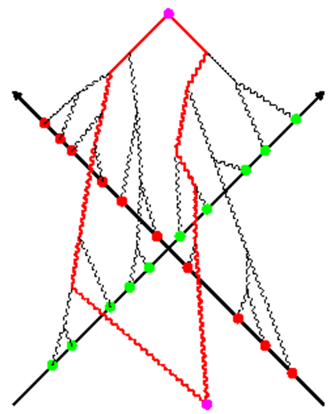
Krasnitz,Venugopalan, Nucl.Phys.B557 (1999)
Lappi, Phys.Rev. C67 (2003)
Schenke,Tribedy,Venugopalan,PRL108 (2012)

t = 0.0 fm/c

Note: 1 fm/c = $3*10^{-24}$ seconds!

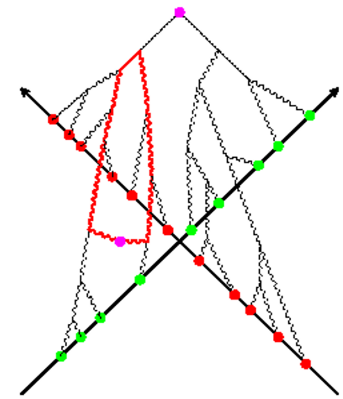
The Glasma at NLO: explosive growth of time-dependent fluctuations

$$\langle n \rangle_{\rm NLO} =$$



Gluon pair production contribution

$+$



One loop corrections to classical field

In our previous discussion, we proved the factorization of only static modes in each nucleus (for which the initial Cauchy surface was the backward "Milne" wedge in spacetime)

– these correspond to p_η=0

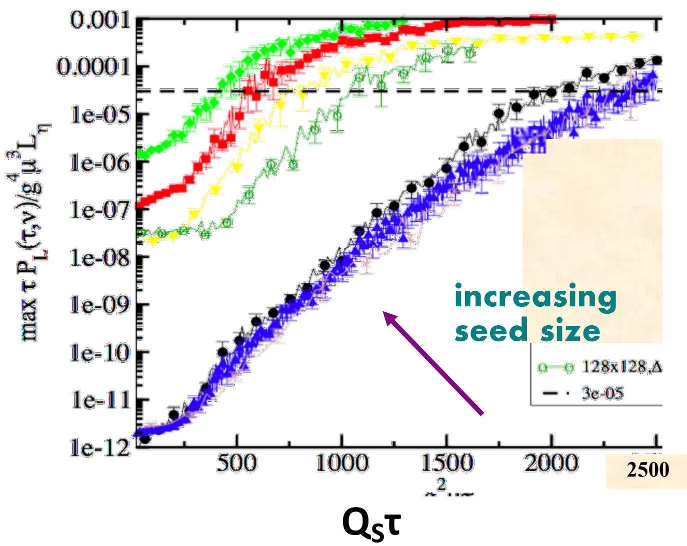
But for $p_\eta \neq 0$ modes, $\mathbf{T}_u \mathcal{A}(x) \sim \dfrac{\delta \mathcal{A}(x)}{\delta \mathcal{A}(0,y)} \sim \exp\left(\sqrt{Q_s \tau}\right)$

The Glasma at NLO: plasma instabilities

Romatschke,Venugopalan (2005)
Dusling,Gelis,Venugopalan (2011)
Gelis, Epelbaum (2013)

At LO: boost invariant gauge fields $A_{cl}^{\mu,a}(x_T,\tau) \sim 1/g$

NLO: $A^{\mu,a}(x_T,\tau,\eta) = A_{cl}^{\mu,a}(x_T,\tau) + a^{\mu,a}(\eta)$



$a^{\mu,a}(\eta) = O(1)$

- Small fluctuations grow exponentially as

$$e^{\sqrt{Q_S \tau}}$$

- Same order of classical field at

$$\tau = \frac{1}{Q_S} \ln^2 \frac{1}{\alpha_S}$$

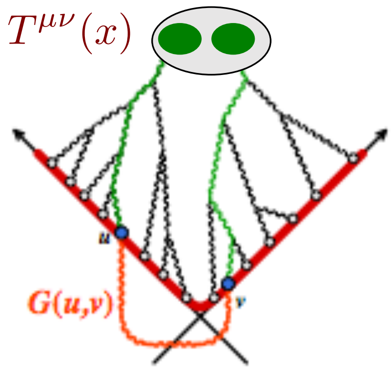
- Resum such contributions to all orders

$$(g\ e^{\sqrt{Q_S \tau}})^n$$

$$T^{\mu\nu}_{\text{resum}} = \int_{\tau=0^+} [da]\ F_{\text{init.}}[a]\ T_{\text{LO}}[A_{\text{cl}} + a]$$

Spectrum of initial fluctuations in the little bang

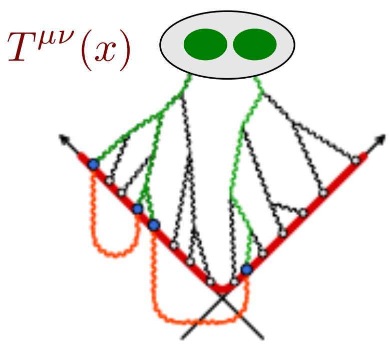
Dusling,Gelis,RV (2011)
Gelis,Epelbaum (2013)



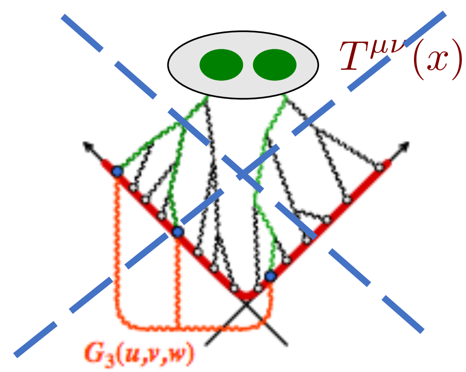
$$\mathcal{G}^{\mu\nu} = \int \frac{d^3k}{(2\pi)^3 2E_k} a^{\mu}_{-k}(\vec{u})\; a^{\nu}_{+k}(\vec{v})$$

$$\left[\frac{\delta^2 S_{\rm YM}}{\delta A^{\mu} A^{\nu}}\right]_{\rm A=A_{cl}} a^{\nu}_{\pm k} = 0 \qquad \lim_{x^0\to-\infty} a^{\mu}_{\pm k,\lambda a}(x) = \epsilon^{\mu}(k)\; T^a\; e^{\pm i k\cdot x}$$

Higher orders:

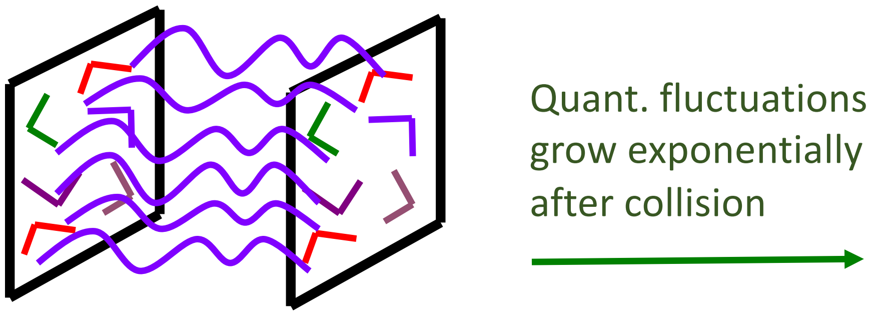


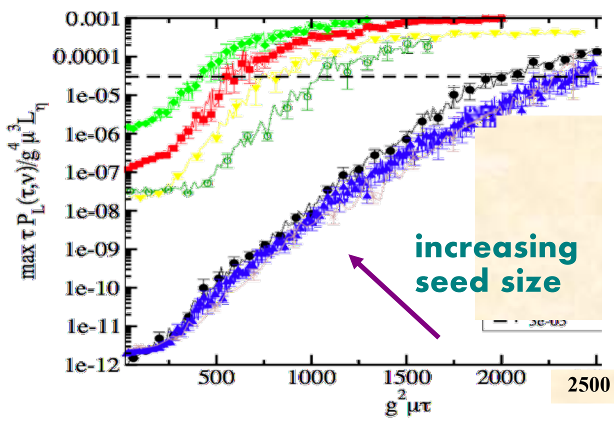
$$(g\,\exp(\sqrt{Q_S\tau}))^4 \sim O(1)$$



$$g(g\,\exp(\sqrt{Q_S\tau}))^3 \sim O(g)$$

From Glasma to Plasma





$$\langle\langle T^{\mu\nu}\rangle\rangle_{\mathrm{LLx+Linst.}} = \int [D\rho_1][D\rho_2]\, W_{\mathrm{Y_{beam}-Y}}[\rho_1]\, W_{\mathrm{Y_{beam}+Y}}[\rho_2] \times \int [da(u)]\; F_{\mathrm{init}}[a]\; T^{\mu\nu}_{\mathrm{LO}}[A_{\mathrm{cl}}(\rho_1,\rho_2)+a]$$

Path integral over multiple initializations of classical trajectories in one event can lead to quasi-ergodic "eigenstate thermalization" Berry; Srednicki; Rigol et al.; …

This *scrambling of information* is seen in many systems in nature and can be understood To lead to decoherence of the primordial classical fields

Scrambling and pre-thermalization from quantum fluctuations

Dusling,Epelbaum,Gelis,RV (2011)

"Toy" example: scalar Φ^4 theory

Gaussian random variable

$$\langle c_{\nu k} c_{\mu l} \rangle = 0$$

$$\langle c_{\nu k} c^*_{\mu l} \rangle = 2\pi\delta(\nu - \mu)\delta_{kl}$$

$$\phi(\tau, \eta, x_\perp) = \phi_{\rm cl.}(\tau, x_\perp) + \frac{1}{2}\int \frac{d\nu}{2\pi}\, d\mu_k\; c_{\nu k}\; e^{i\nu\eta} \chi_k(x_\perp) H_{i\nu}(\lambda_k \tau) + c.c$$

satisfies the small fluctuation equation

$$[-\partial_\perp^2 + V''(\phi_0)]\chi_k = \lambda_k^2\, \chi_k$$

These quantum modes satisfy a "Berry conjecture":

1) The high-lying energy quantum eigenmodes of a classically chaotic system are random Gaussian functions
2) With the dynamics controlled by the corresponding two-point Wigner function of these modes.

Further conjecture: isolated quantum systems that obey Berry's conjecture display "eigenstate thermalization"

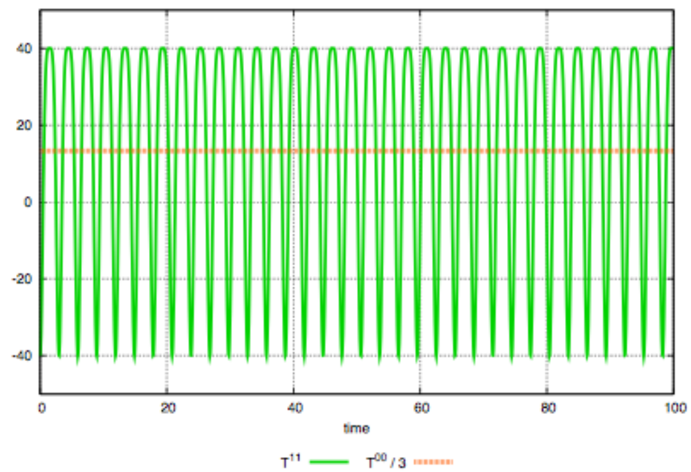
Srednicki (1994)

EOS in a toy mode from quantum fluctuations

Dusling,Epelbaum,Gelis,RV (2011)

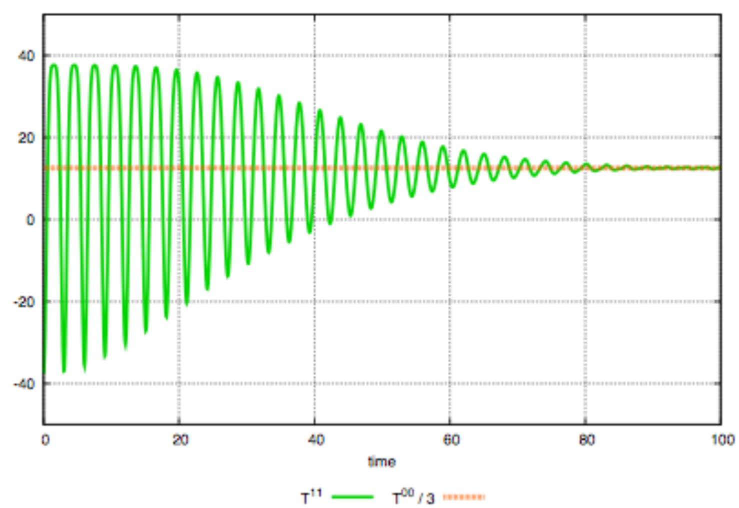
Consider scalar Φ^4 theory:

Energy density and pressure
without averaging over fluctuations



Energy density and pressure
after averaging over fluctuations

Converges to single valued "EOS"



"Pre-thermalization"

Berges,Borsanyi,Wetterich PRL (2005)

From scrambling to universality

- Scrambling and pre-thermalization are suggestive that it may be sufficient (from the point of view of thermalization) to consider Gaussian random variables
-- "details" of the spectrum of fluctuations may not matter

- We will see that this conjecture is confirmed and leads to a novel, universal turbulent attractor in the Glasma

Initial conditions in the overpopulated Glasma

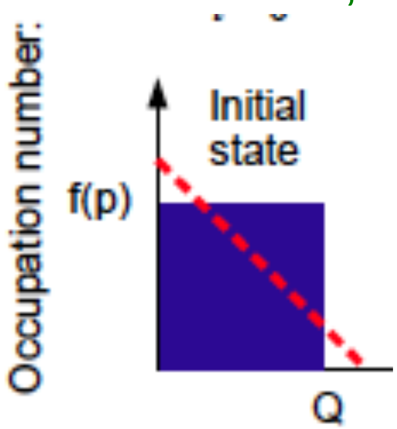
Choose for the Gaussian random gauge fields for the initial conditions

$$A_\mu^a(\tau_0, \mathbf{x}_\perp, \eta) = \sum_{\lambda=1,2} \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} \frac{d\nu}{2\pi} \sqrt{f(\mathbf{k}_\perp, \nu, \tau_0)} \times \left[c_{\lambda,a}^{\mathbf{k}_\perp \nu} \xi_\mu^{(\lambda)\mathbf{k}_\perp \nu +}(\tau_0)\, e^{i\mathbf{k}_\perp \mathbf{x}_\perp}\, e^{i\nu\eta} + c.c. \right]$$

Stochastic random variables

$$\begin{aligned}
\langle c^{(\lambda)\boldsymbol{k}_\perp \nu} c^{(\lambda')\boldsymbol{k}'_\perp \nu'} \rangle &= 0\,, \\
\langle c^{(\lambda)\boldsymbol{k}_\perp \nu} c^{*(\lambda')\boldsymbol{k}'_\perp \nu'} \rangle &= (2\pi)^3 \delta^{\lambda\lambda'} \delta(\boldsymbol{k} - \boldsymbol{k}') \delta(\nu - \nu') \\
\langle c^{*(\lambda)\boldsymbol{k}_\perp \nu} c^{*(\lambda')\boldsymbol{k}'_\perp \nu'} \rangle &= 0.
\end{aligned}$$

Polarization vectors ξ expressed in terms of Hankel functions in Fock-Schwinger gauge $A^\tau = 0$



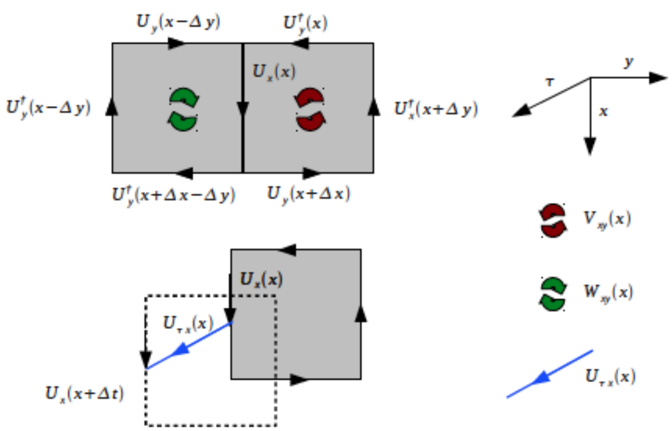
$$f(p_\perp, p_z, t_0) = \frac{n_0}{\alpha_S} \Theta\left(Q - \sqrt{p_\perp^2 + (\xi_0 p_z)^2} \right)$$

Controls “prolateness” or “oblateness” of initial momentum distribution

Temporal evolution in the overpopulated QGP

Solve Hamilton's equation for 3+1-D SU(2) gauge theory in Fock-Schwinger gauge

Berges,Boguslavski,Schlichting,Venugopalan
arXiv: 1303.5650, 1311.3005



Fix residual gauge freedom imposing Coloumb gauge at each readout time

$$\partial_i A_i + t^{-2}\partial_\eta A_\eta = 0$$

Largest classical-statistical numerical simulations of expanding Yang-Mills to date:
$256^2 \times 4096$ lattices