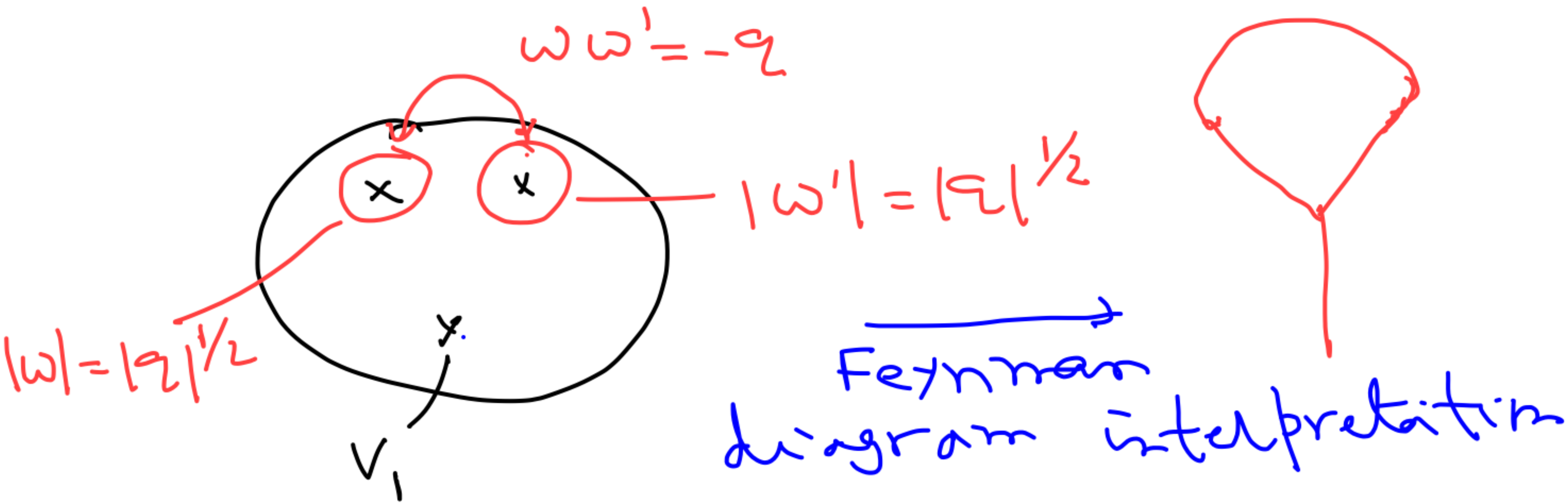
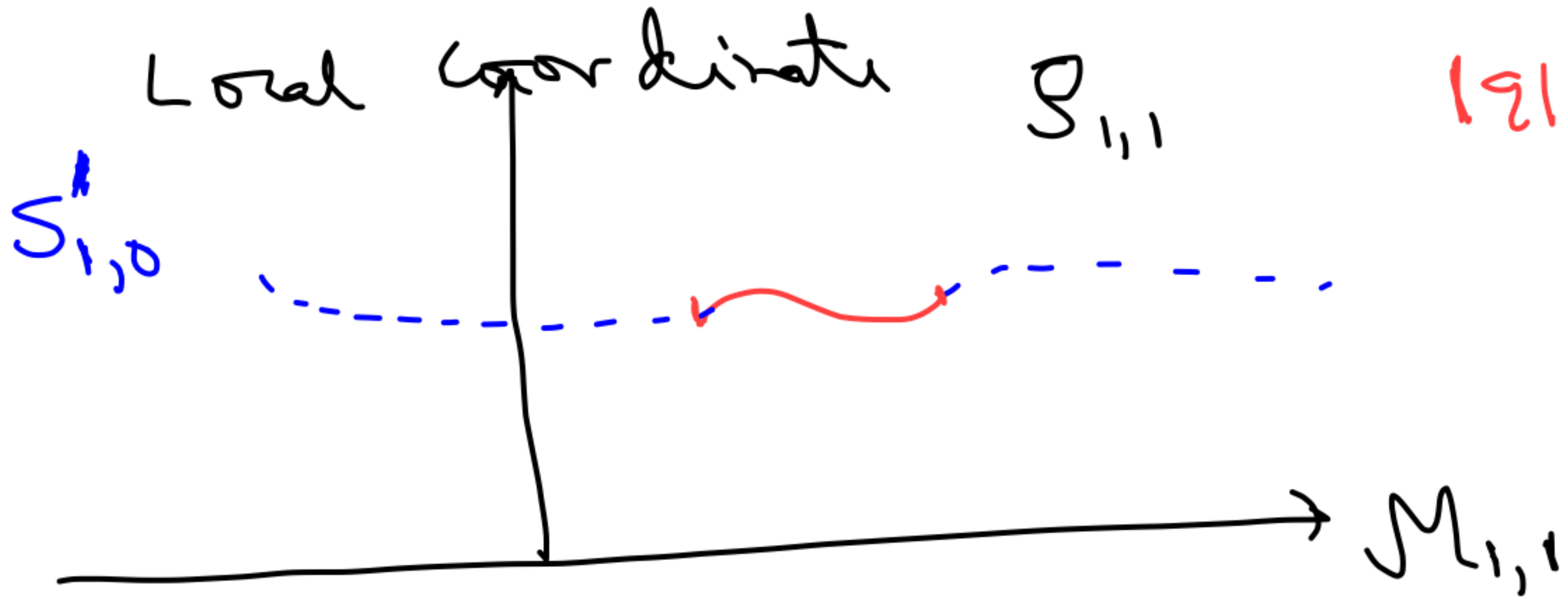
Sphere 3 & 4 pt. fn. (off-shell)

→ sum of Feynman diagrams.

$$\langle f_1 \circ V_1(0)\; f_2 \circ V_2(0)\; f_3 \circ V_3(0) \rangle$$



$|q| \leq 1$



$$g_s \int_{S'_{1,1}} \Omega^{(1,1)}_{3g-3+n \; 2}(V_1) \longrightarrow \ast$$

One point vertex.

$\Downarrow$

Finite
One loop 'counterterm' $\rightarrow$ not to cancel divergences but for gauge invariance...

Generalize this to higher order

Iterative in $6g-6+2n$

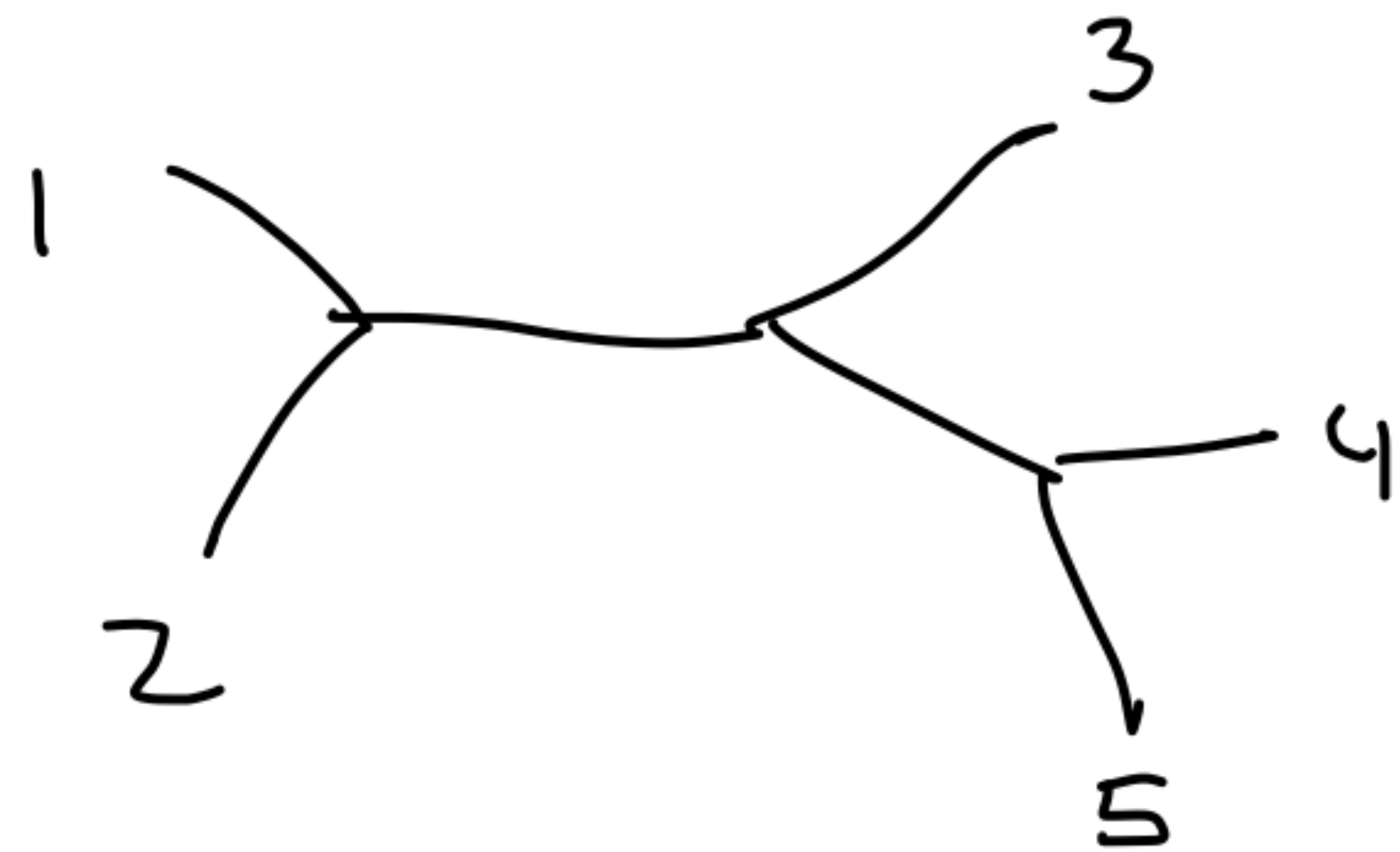
Plumbing fixture $\to$ adds two extra variable to integrate over $q, \bar{q}$

(a) First construct all the section segments obtained by plumbing fixture of lower order vertices.

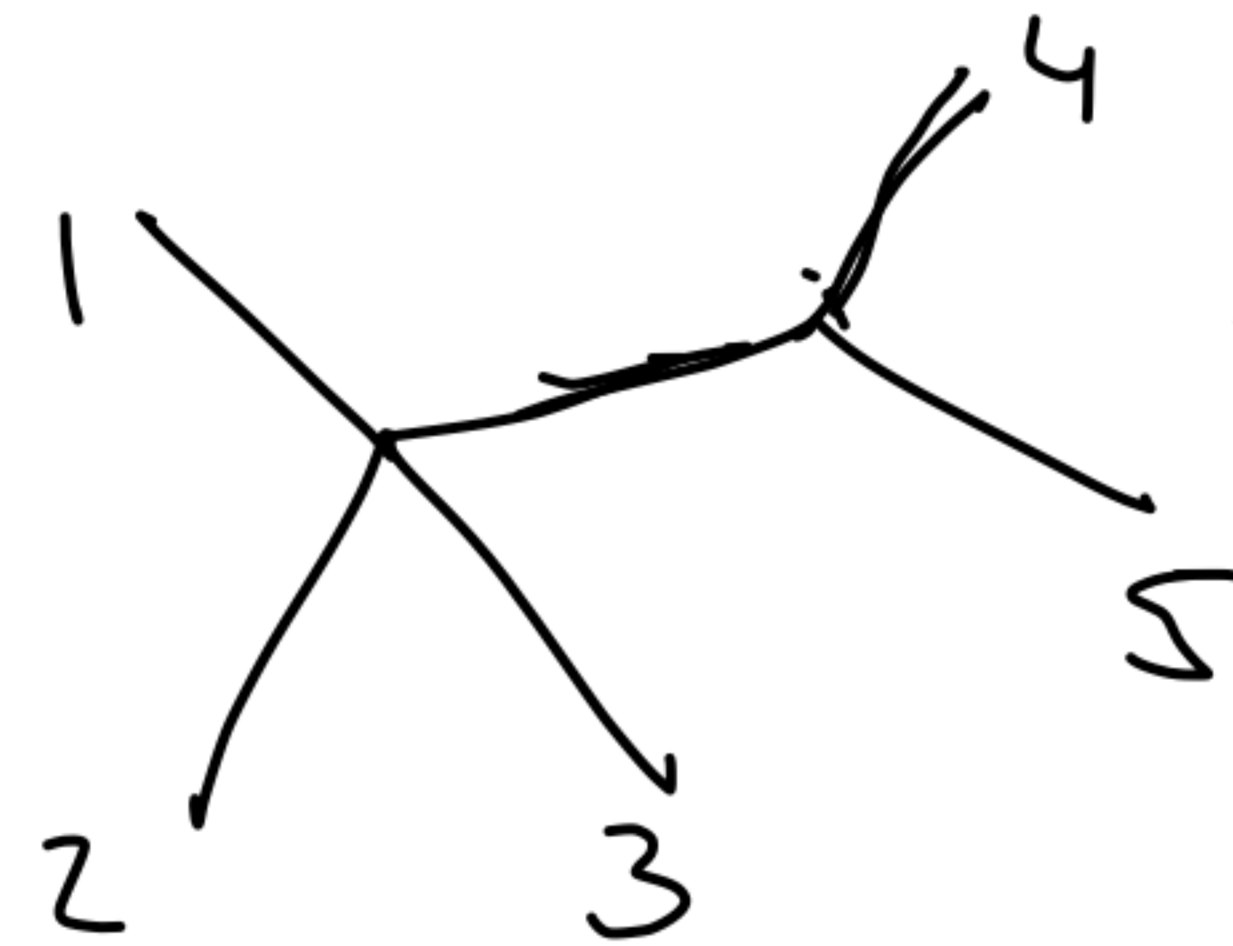
(b) Fill the gap smoothly by $S'_{g,n}$.

$$g_s^{3g-3+n} \int_{S'_{g,n}} \Omega^{(g,n)}_{6g-6+2n}(V_1, \cdots V_n) \to g \text{ loop, } n \text{ point counterterm}$$

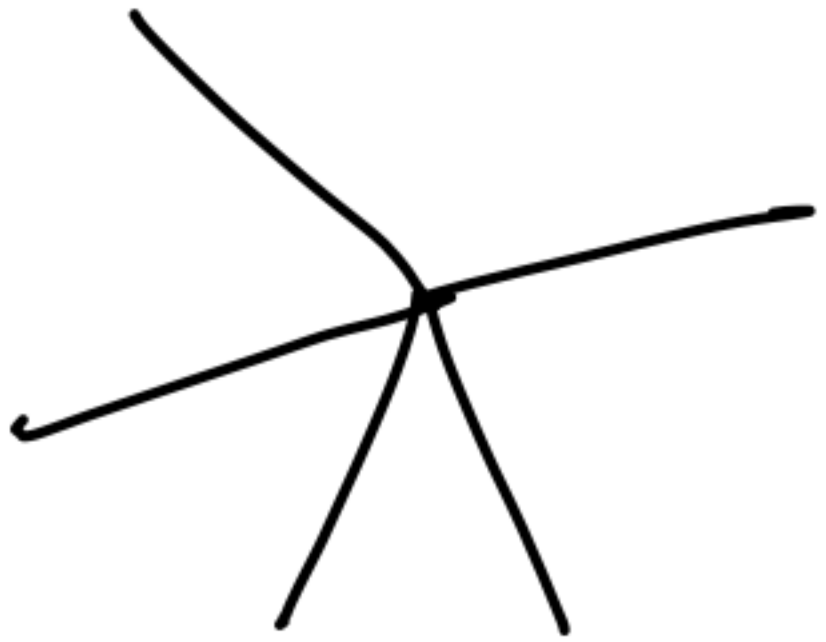
$g=0, \quad n=5$



$+ \cdots\cdots + $



$+ \cdots$



$: \; g_s^2 \int_{S'_{0,5}} \Omega_4^{(0,5)}(V_1, \dots V_5)$

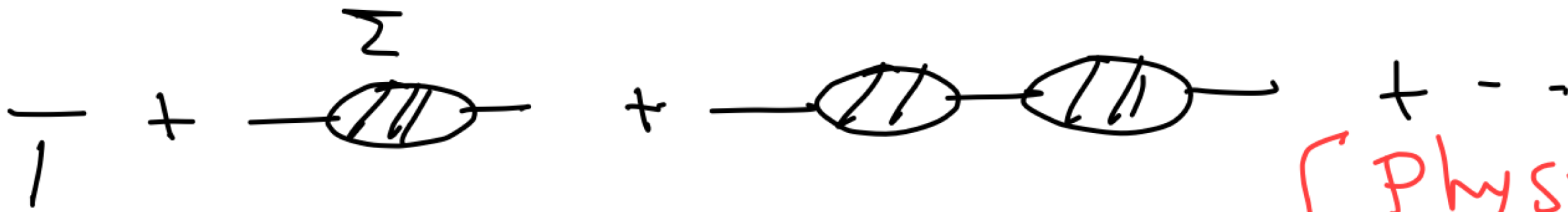
This procedure is practical, but not necessarily "elegant"

$\Downarrow$

choose section segments in a "natural" way (minimal area metric, hyperbolic metric, ...)

Once the amplitudes are arranged as sum over Feynman diagrams, we can use QFT insight (including LSZ) to deal with the problem of on-shell internal propagator.

Example: self energy diagram.



$$\frac{1}{k^2+m^2} = \frac{1}{k^2+m^2-\Sigma(k)}$$

$\Rightarrow$ Physical outcome like pole position, S-matrix are indep. of the choice of $S'_{g,r}$.

Σ: sum of 1PI diagrams.

Need to know what are 1PI diagrams!

This split depends on the choice of $S'_{g,r}$.

$\rightarrow$ ambiguous

The Feynman rules can be obtained by gauge fixing a gauge invariant string field theory. $\Rightarrow$ will not be discussed.

$\Rightarrow$ Decoupling of pure gauge states, independence of physical quantities of the choice of $S_{g,n}'$ etc. follow from the usual Ward identities of gauge theory.

Issues in bosonic string theory:

① Tachyon → makes the theory inconsistent.

→ will be resolved in superstring theory.

② $D=26$ but we live in $D=4$.

→ can be resolved by taking 22 of the space dimensions to be small and compact e.g. $X^i \equiv X^i + 2\pi R^i$ for $i=4, \cdots 25$

As long as R^i are small enough these dimensions are not observable.

In actual analysis:

$$k^i = \frac{n^i}{R^i}, \quad n^i \text{ integers.}$$

We also have winding modes:

$$X^i(\sigma + 2\pi) = X^i(\sigma) + 2\pi R^i w^i,$$

w^i: integer.

→ winding modes.

The spectrum changes & the correlation fns change, but we still have CFT with $c=26$ & all the general discussions still hold.

More generally, we can use any $c=26$ CFT to construct string theory.

$\rightsquigarrow$ landscape.

$\rightsquigarrow$ different classical solutions of string theory.

Superstrings.

World-sheet theory has diffeomorphism and local SUSY (all in two dim.)

Gauge fixing $\rightarrow$ ghosts $b, c, \bar{b}, \bar{c}, \underbrace{\beta, \gamma, \bar{\beta}, \bar{\gamma}}$

grassmann even.

$\bar{\partial}\beta = 0$, $\bar{\partial}\gamma = 0$, $\partial\bar{\beta} = 0$, $\partial\bar{\gamma} = 0$

Matter sector: X^μ, ψ^μ, $\bar{\psi}^\mu$. $\bar{\partial}\psi^\mu = 0$ $\partial\bar{\psi}^\mu = 0$.

β, γ system can be "bosonized"

$\beta = \partial\xi e^{-\phi}$, $\gamma = \eta e^{\phi}$, ϕ boson, ξ, η: fermions

$\bar{\partial}\xi = 0$, $\bar{\partial}\eta = 0$, $\bar{\partial}\phi = 0$.

$e^{q\phi}$ has grassmann parity $(-1)^q$ though ϕ is a boson.

<u>operator product expansion</u>

$$\psi^\mu(z)\,\psi^\nu(w) \simeq -\frac{\eta^{\mu\nu}}{2(z-w)}\,, \qquad \xi(z)\,\eta(w) \simeq \frac{1}{z-w}$$

$$\partial\phi(z)\,\partial\phi(w) \simeq -(z-w)^{-2}\,,$$

$$e^{q\phi}(z)\,e^{q'\phi}(w) \simeq (z-w)^{-qq'}\, e^{(q+q')\phi}(w) + \cdots$$

$$T_{\beta\gamma} = -\frac{3}{2}\beta\partial\gamma + \frac{1}{2}\gamma\partial\beta\,, \quad T_{\xi\eta} = -\eta\partial\xi$$

$$T_\phi = -\frac{1}{2}(\partial\phi)(\partial\phi) - \partial^2\phi\,, \quad T_\psi = \eta_{\mu\nu}\psi^\mu\partial\psi^\nu.$$

Ex. 1 Find $\beta(z)\gamma(w)$ OPE

2 Conformal weights:

$\xi : (0,0)$, $\eta : (0,1)$, $e^{q\phi} : (0, -\frac{1}{2}q(q+2))$,

$\gamma : (0, -\frac{1}{2})$, $\beta : (0, \frac{3}{2})$, $\psi^\mu : (0, \frac{1}{2})$. $\mu = 0, \cdots, D-1$

3 T_ψ has central charge $\frac{D}{2}$

4 $T_{\beta\gamma}$ has " " 11

Vanishing of the central charge

$D - 26 + \frac{D}{2} + 11 = 0$

$\Rightarrow D = 10$

Some more defs.

Ghost no. $\quad 1$ for $c, \bar{c}, \gamma, \bar{\gamma}, \eta, \bar{\eta}$

$\quad -1$ for $b, \bar{b}, \beta, \bar{\beta}, \xi, \bar{\xi}$

$\quad 0$ for the rest (includes $\partial\phi, e^{q\phi}$)

Picture no. (hol.): $\quad q$ for $e^{q\phi}$, 1 for ξ, -1 for η.

$\quad 0$ for the rest.

β, γ have 0 picture no. $\left(\begin{array}{l} \beta = \partial\xi e^{-\phi} \\ \gamma = \eta e^{\phi} \end{array} \right)$

Similarly define anti-hol. picture no.

One more defn.

$T_F = -\eta_{\mu\nu}\psi^\mu \partial X^\nu$. (superpartner of T^X)

$\bar{T}_F = -\eta_{\mu\nu}\bar{\psi}^\mu \bar{\partial} X^\nu$.

Superconformal matter primary V

$\Downarrow$

made of $X^\mu, \psi^\mu, \bar{\psi}^\mu$

① V must be a primary.

② $T_F(z) V(w,\bar{w}) \simeq \frac{1}{z-w} \tilde{V}(w,\bar{w}) + \cdots$

e.g. ψ^μ is a superconformal primary

but not ∂X^μ.

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More defs:

$\beta = \partial\xi e^{-\phi}$

$\gamma = \eta e^{\phi}$

GSO parity (hol.)

- (-1) for β, γ, ψ^μ, $(-1)^q$ for $e^{q\phi}$,

  1 for the rest. (including ξ, η.)

① Only GSO even operators are kept in the CFT. (Similarly in the anti-hol sector)

② We keep only those operators that involve derivatives of $\xi, \bar{\xi}$ but not $\xi, \bar{\xi}$ themselves

$\partial\xi\, \partial X^\mu$ ✓

$\xi\, \partial X^\mu$ ✗

$\Rightarrow$ Small Hilbert space.

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$\int \delta'(k)\, e^{ik\cdot x}\, dk \sim i x$

Ex: On the sphere only operators with total picture no. $(-2,-2)$ and equal no. of (ξ, η) and equal no of $(\bar{\xi}, \bar{\eta})$ have non-zero correlation fn.

$\times \quad \langle \xi(z)\eta(w)\rangle = 0 \qquad h_\xi = (0,0), \quad h_\eta = (0,1)$

$\langle \partial\xi(z)\, \eta(w)\rangle \simeq \dfrac{1}{z-w} \quad \Big| \quad \langle \partial\xi(z)\, \eta(w)$ $\neq 0$

$c(z_1)\, c(z_2)\, c(z_3)$

$\bar{c}(\bar{z}_1')\, \bar{c}(\bar{z}_2')\, \bar{c}(\bar{z}_3')$

$e^{-2\phi}(y_1)\, e^{-2\bar{\phi}}(\bar{y}_1)\rangle$

Ex. Using plumbing fixture show that on genus g surface we need picture no. $(2g-2, 2g-2)$, equal no. of ξ, η & equal no. of $\bar{\xi}, \bar{\eta}$ to get non-zero correlator.

Normalization:

$$\langle k| c_{-1}\bar{c}_{-1} c_0 \bar{c}_0 c_1 \bar{c}_1 \, e^{-2\phi}(0)\, e^{-2\bar{\phi}}(0)|k'\rangle$$

$$= (2\pi)^{10} \cdot \delta^{(10)}(k+k').$$

$$\Downarrow$$

$$e^{ik'\cdot X}(0)|0\rangle$$

BRST current:

$$j_B(z) = c(T^X + T_{\beta\gamma}) + \gamma T_F + bc\partial c - \frac{1}{4}\gamma^2 b$$

$$\gamma(z)^2 = \lim_{w\to z} \eta e^{\phi}(w)\, \eta e^{\phi}(z) \overset{\text{Ex.}}{=} -\partial\eta\,\eta\, e^{2\phi}(z)$$

Similarly $\bar{j}_B(\bar{z})$.

$$Q_B = \oint j_B(z)\,dz + \oint \bar{j}_B(\bar{z})\,d\bar{z}$$

Ex. $Q_B^2 = 0$ for $D=10$.

More defn:

Picture changing operator:

$$X(z) = \{Q_B, \xi(z)\} = \oint^{z} dw\, j_B(w)\, \xi(z)$$

$$= c\partial\xi + e^{\phi} T_F - \frac{1}{4}\partial\eta e^{2\phi} b - \frac{1}{4}\partial(\eta e^{2\phi} b)$$

Note: ξ is not in the small Hilbert space but X is in the small Hilbert space.

Similarly $\bar{X}$

Ex. check that $X, \bar{X}$ are dimension $(0,0)$ primaries.

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As in bosonic string theory, we can construct local operators from products of (derivatives of) ∂X^μ, ψ^μ, $\partial\xi$, η, $\partial\phi$, anti-hol. counterparts and $e^{q\phi}$, $e^{\bar{q}\phi'}$ for $q, \bar{q} \in \mathbb{Z}$, $e^{ik\cdot X}$.

$$\Downarrow$$

subject them to GSO projection on the hol. and anti-hol. side. to one of four sectors of theory (NS NS)

Neveu-Schwarz

We also have RNS, NSR and RR sector

Ramond.

Construction of R-Sector:

ψ^μ l.c. of $e^{i\phi_k}$ and $e^{-i\phi_k}$

① Bosonize the ψ^μ's.

$$\psi^1 + i\psi^2 \sim e^{i\phi_1}, \quad \psi^3 + i\psi^4 \sim e^{i\phi_2}, \cdots$$

Construct operators $e^{\frac{i}{2}(\pm\phi_1 \pm \phi_2 \pm \cdots \pm \phi_5)}$

$\Rightarrow$ 32 operators transforming in the spinor representation of $SO(10)$

$(SO(9,1))$

Even no. of $-$signs : S_α : Chiral spinor

Odd no. of $-$signs : S^α of $SO(9,1)$

$\rightarrow$ anti-chiral spinors

Take product $e^{-\phi}\psi^\mu(z)$ with $e^{-\frac{\phi}{2}}S_\alpha$

$\alpha = 1, \cdots, 16$.

(NSR)-sector operators are obtained by taking products of $e^{-\phi/2}S_\alpha$ and GSO even NSNS operators

Ex. $e^{-\frac{3\phi}{2}}S^\alpha$ is GSO even, $e^{-\frac{\phi}{2}}S^\alpha$ is GSO odd.

Anti-hol. sector:

Type IIB string theory:

GSO even RNS operators are obtained by taking the product of $e^{-\frac{\bar{\phi}}{2}}\bar{S}_\alpha$ with GSO even NSNS operators

Type IIA string theory:

GSO even RNS operators are obtained by taking the product of $e^{-\frac{\bar{\phi}}{2}}\bar{S}^\alpha$ with GSO even NSNS operators.

NSNS $\rightarrow$ integer spin of $SO(9,1)$

RNS, NSR $\Rightarrow$ half integer spin

$\Rightarrow$ physical space-time fermion states

RR $\rightarrow$ integer spin $\Rightarrow$ space-time bosons.

IIA and IIB have space-time SUSY.

Next time we'll see gravitinos in the spectrum $\longrightarrow$ supergravity.

$N=2$ supergravity in $D=10$.