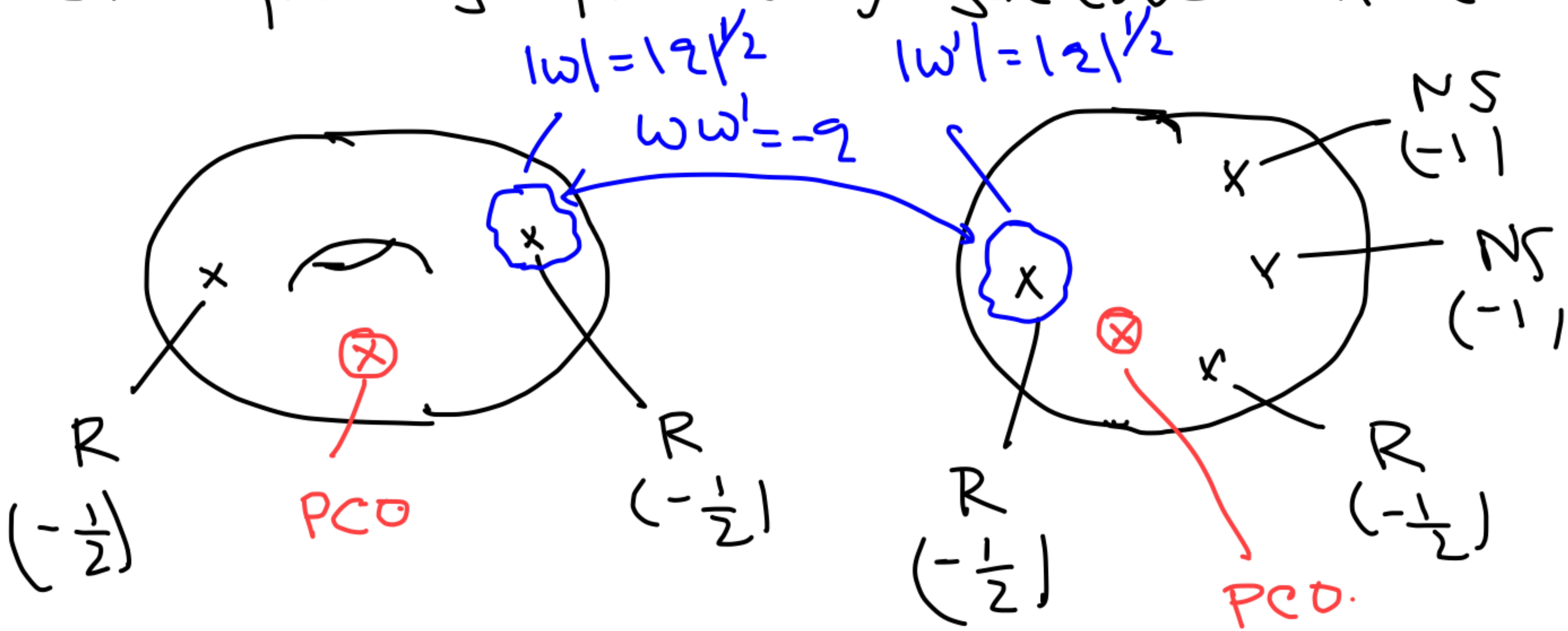
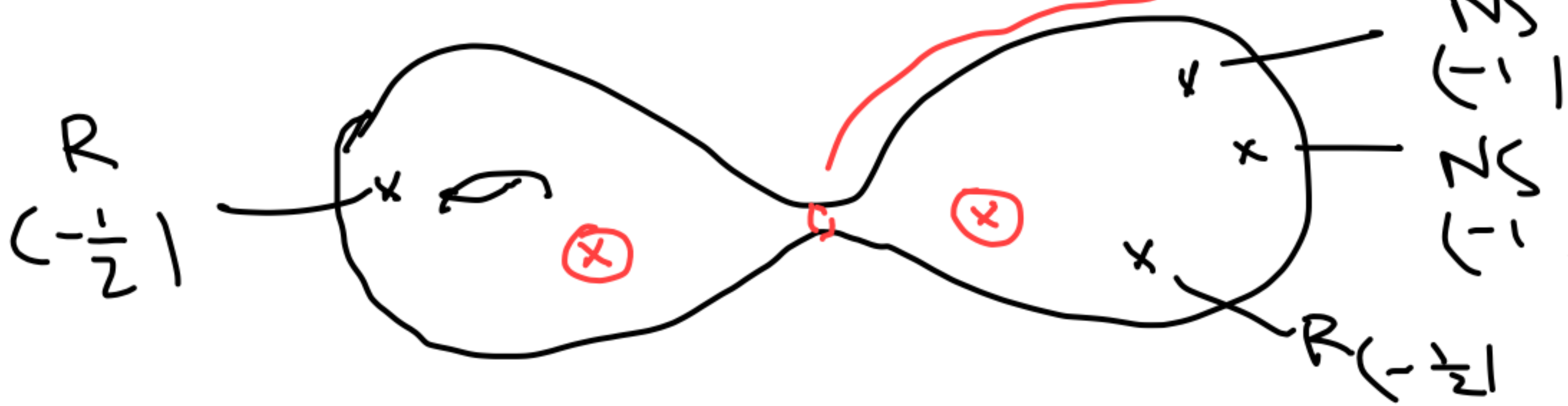
Example of plumbing fixture in the R-sector:



# of PCO: $2g-2+m+\frac{n}{2}$



$$X_0 = \oint \frac{d\omega}{\omega} X(\omega)$$

$g=1,\ m=2,\ n=2$

$$2g-2+m+\frac{n}{2} = 2+\frac{2}{2} = 3$$

Plumbing fixture with R:

$$\phi_s(0)\ \phi_r(0) \int_{|z|\le 1} \langle \phi_s^c | b_0 \bar{b}_0 X_0 | \phi_r^c \rangle\, q^{h_s} \bar{q}^{\bar{h}_s}\, dq \wedge d\bar{q}$$

ϕ $-1/2$ $\quad -1/2 \quad -3/2 \quad -3/2$

Ex. $\beta_0 |-\frac{3}{2}, k\rangle \neq 0$, $\gamma_0 |-\frac{1}{2}, k\rangle \neq 0$.

$\beta_0^N |-\frac{3}{2}, k\rangle$ is degenerate with $|-\frac{3}{2}, k\rangle$ for any N.

$\langle -\frac{3}{2}, k | \beta_0^M$ " " " $\langle -\frac{3}{2}, k |$

ghost no. $-N$ $\quad$ ghost no. $-M$. $\Rightarrow$ matrix element vanishes unless compensated by +ve gh. no. ops.

γ_{-n} not useful.

c_1, c_0, c_{-1}, c_{-2}

$$\phi_r(0) \quad \phi_s(0) \quad \langle \phi_r^c | b_0 \bar{b}_0 | \phi_s^c \rangle \, q^{h_r} \bar{q}^{h_r}$$

$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$

$$-\tfrac{1}{2} \qquad -\tfrac{3}{2} \qquad -\tfrac{3}{2} \qquad -1/2$$

$$\text{Ex.} \; \gamma_0 |-\tfrac{1}{2}, k\rangle \neq 0$$

$$\langle -\tfrac{3}{2}, k | \beta_0^M \qquad (\gamma_0)^M |-\tfrac{1}{2}, k\rangle$$

$\Rightarrow$ Divergent sum.

In the R sector the propagator

$$\langle \phi_r^c | b_0 \bar{b}_0 \, \delta_{L_0, \bar{L}_0} \frac{2}{L_0 + \bar{L}_0} X_0 | \phi_s^c \rangle$$

plays the role of ~~term~~ in the num. of Dirac propagator.

Spurious poles

In bosonic string theory, all divergences come from $q \to 0$ limit of some plumbing fixture variable.

→ can be treated by regarding the amplitude as sum of Feynman diagrams.

In superstring/heterotic string the integrand has additional poles. ⇒ spurious poles (arise from breakdown of gauge choice for local world-sheet susy) ⇒ cannot be dealt with using Feynman diagrams.

Example :

① $y_\alpha \to y_\beta$ since $X(y)X(z) \to \infty$ as $y \to z$ (Ex. check this).

② $y_\alpha \to 0$ in w_r coordinate.

③ On $g \geq 1$ surfaces, ξ, η, ϕ correlation fr. have poles at points where no operators collide.

Verlinde & Verlinde, PLB 192, p 95

$$f(\{y_\alpha\}, \{m_i\}) = 0$$

PCO locations

parameters of $\mathcal{M}_{g,m,n}$.

In $\mathcal{P}_{g,m,n}$ there are codimension 2 subspaces along which the spurious pole loci lie.

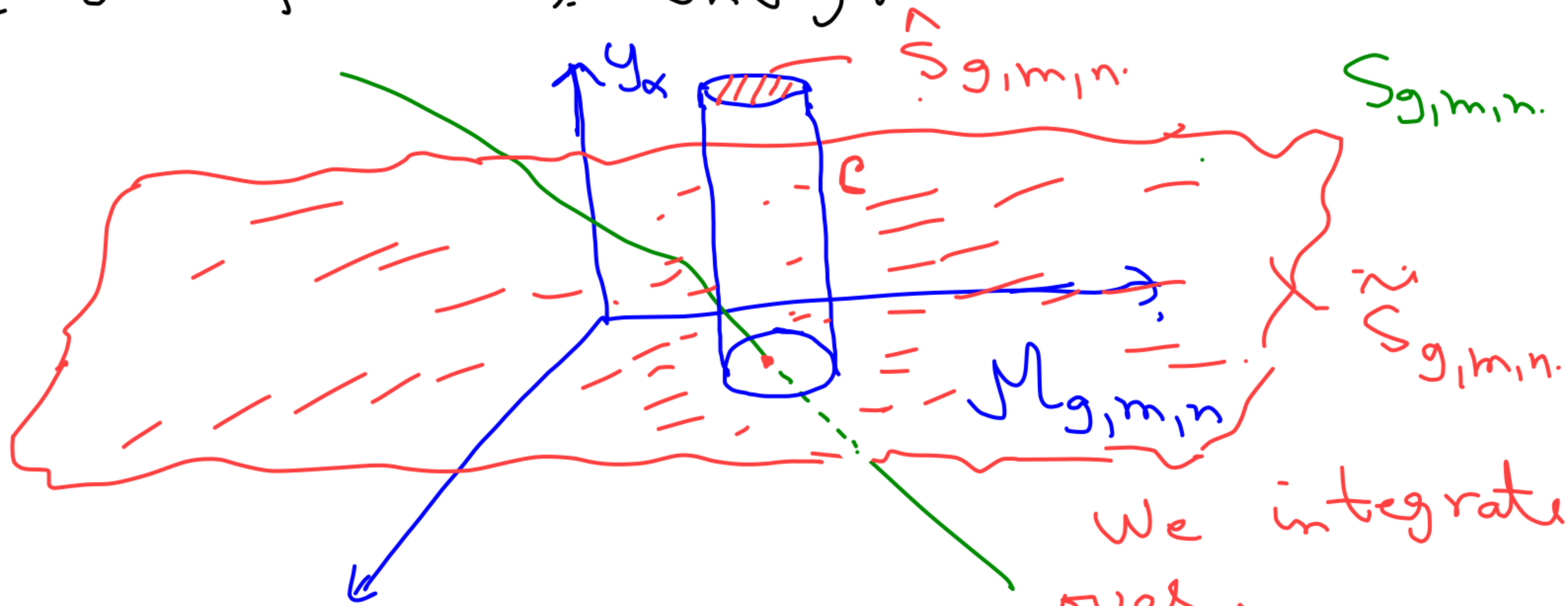
$\{t^m\}$: coordinates of $\mathcal{P}_{g,m,n}$.

① $y_\alpha(\vec{t}) - y_\beta(\vec{t}) = 0 \Rightarrow$ 2 conditions
Real, Imaginary.

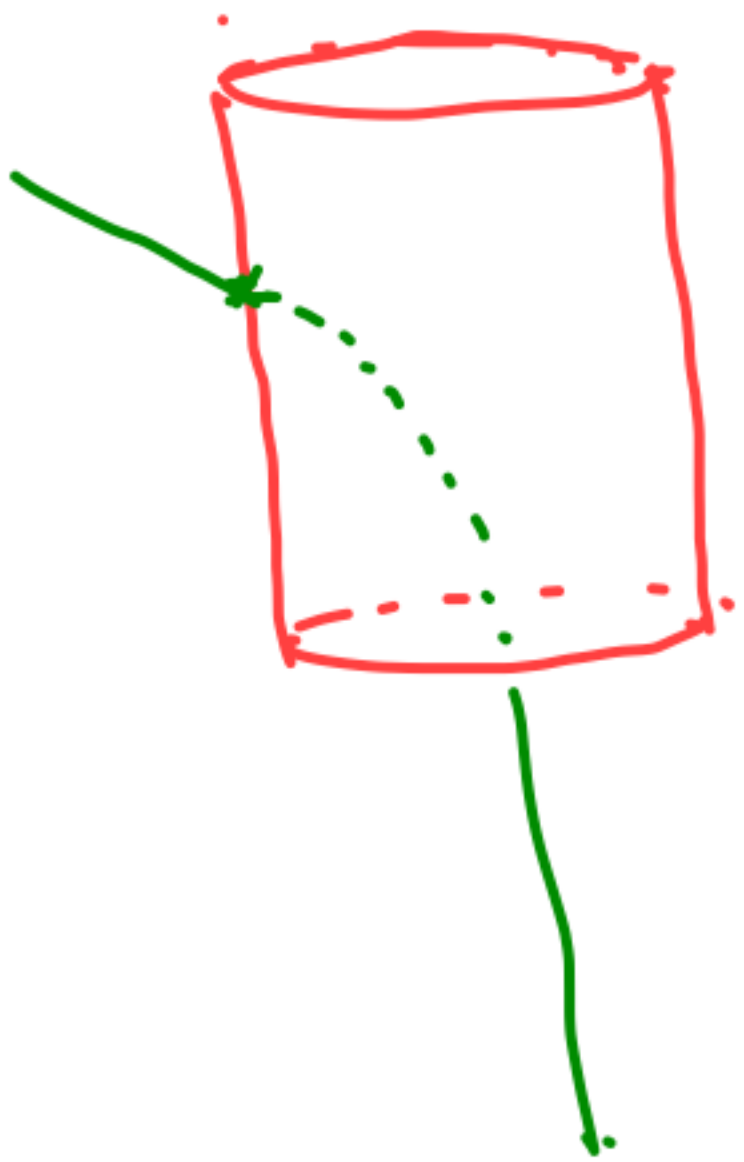
② $y_\alpha(\vec{t}) = 0 \rightarrow$ 2 conditions.

③ $f\left(\{y_\alpha(\vec{t})\}, \{m_i(\vec{t})\}\right) = 0 \Rightarrow$ 2 conditions
from real,
imaginary.

A section $S_{g,m,n}$ will intersect the spurious pole loci on codimension 2 subspaces $\Rightarrow$ integrand becomes singular.



We integrate over $\tilde{S}_{g,m,n} \oplus \hat{S}_{g,m,n} \oplus C$.



Along the vertical direction only the coordinate y_α of a PCO varies:

$F_s(\tau_s)$, y_β for $\beta \neq \alpha$ remain fixed.

t: The coordinate of $\xi_{g,min}$ along the vertical direction.

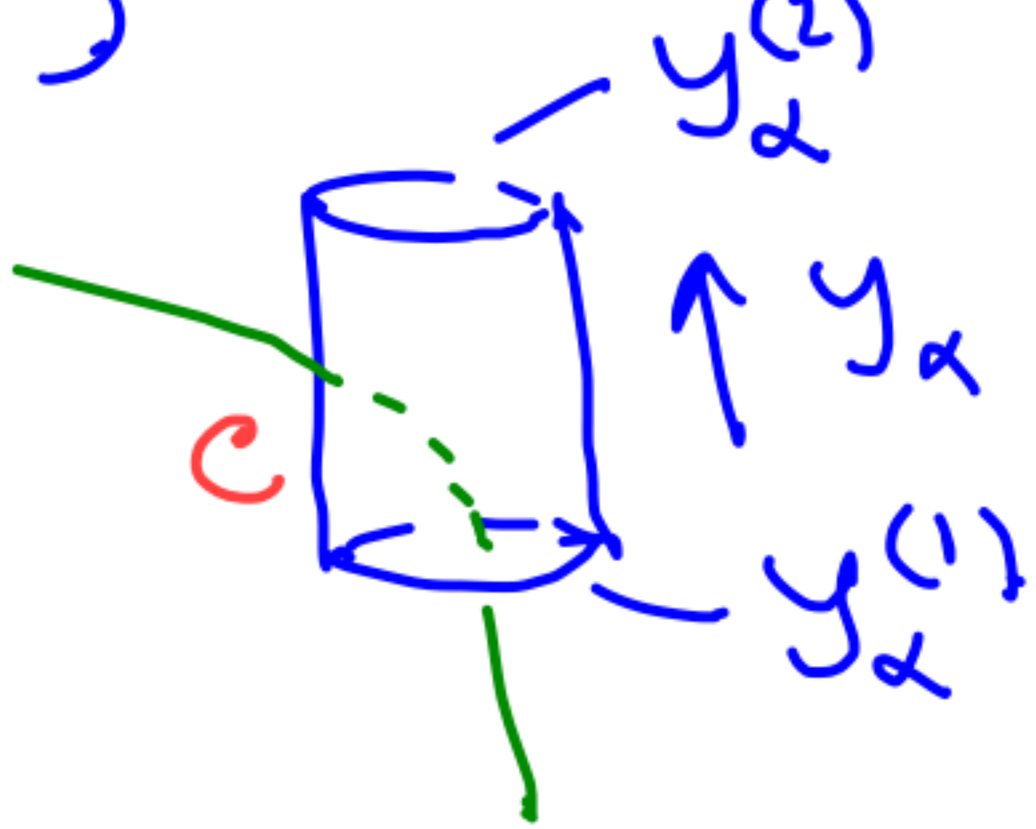
$$B_t = \sum_s \oint_{C_s} \frac{\partial F_s}{\partial t}\, b(\sigma_s)\, d\sigma_s + \sum_s \oint_{C_s} \frac{\partial \bar{F}_s}{\partial t}\, \bar{b}(\bar{\sigma}_s)\, d\bar{\sigma}_s$$

$$- \sum_\beta \frac{1}{X(y_\beta)} \frac{\partial y_\beta}{\partial t}\, \partial \xi(y_\beta) \longrightarrow - \frac{1}{X(y_\alpha)} \frac{\partial y_\beta}{\partial t}\, \partial \xi(y_\beta)$$

$$\int dt \langle B_t \cdots \rangle$$

$$= \int dt \left\langle (-1) \cancel{\frac{1}{X(y_\alpha)}} \frac{\partial y_\alpha}{\partial t} \partial \xi(y_\alpha) \prod B_{t_i} \prod_{\beta \neq \alpha} X(y_\beta) \prod V_i \prod W_i \right\rangle$$

$$= \int \left\langle \left(\xi(y_\alpha^{(1)}) - \xi(y_\alpha^{(2)}) \right) \prod B_{t_i} \prod_{\beta \neq \alpha} X(y_\beta) \prod V_i \prod W_i \right\rangle dt_1 \wedge \cdots$$



$\rightarrow$ free from spurious poles.

Suppose that we want to move 2 PCO's

$\left(y_\alpha^{(1)}, y_\beta^{(1)}\right) \to \left(y_\alpha^{(2)}, y_\beta^{(2)}\right)$

bottom of C

top of C

<u>2- choices</u>: $\left(y_\alpha^{(1)}, y_\beta^{(1)}\right) \to \left(y_\alpha^{(2)}, y_\beta^{(1)}\right) \to \left(y_\alpha^{(2)}, y_\beta^{(2)}\right)$

or $\left(y_\alpha^{(1)}, y_\beta^{(1)}\right) \to \left(y_\alpha^{(1)}, y_\beta^{(2)}\right) \to \left(y_\alpha^{(2)}, y_\beta^{(2)}\right)$.

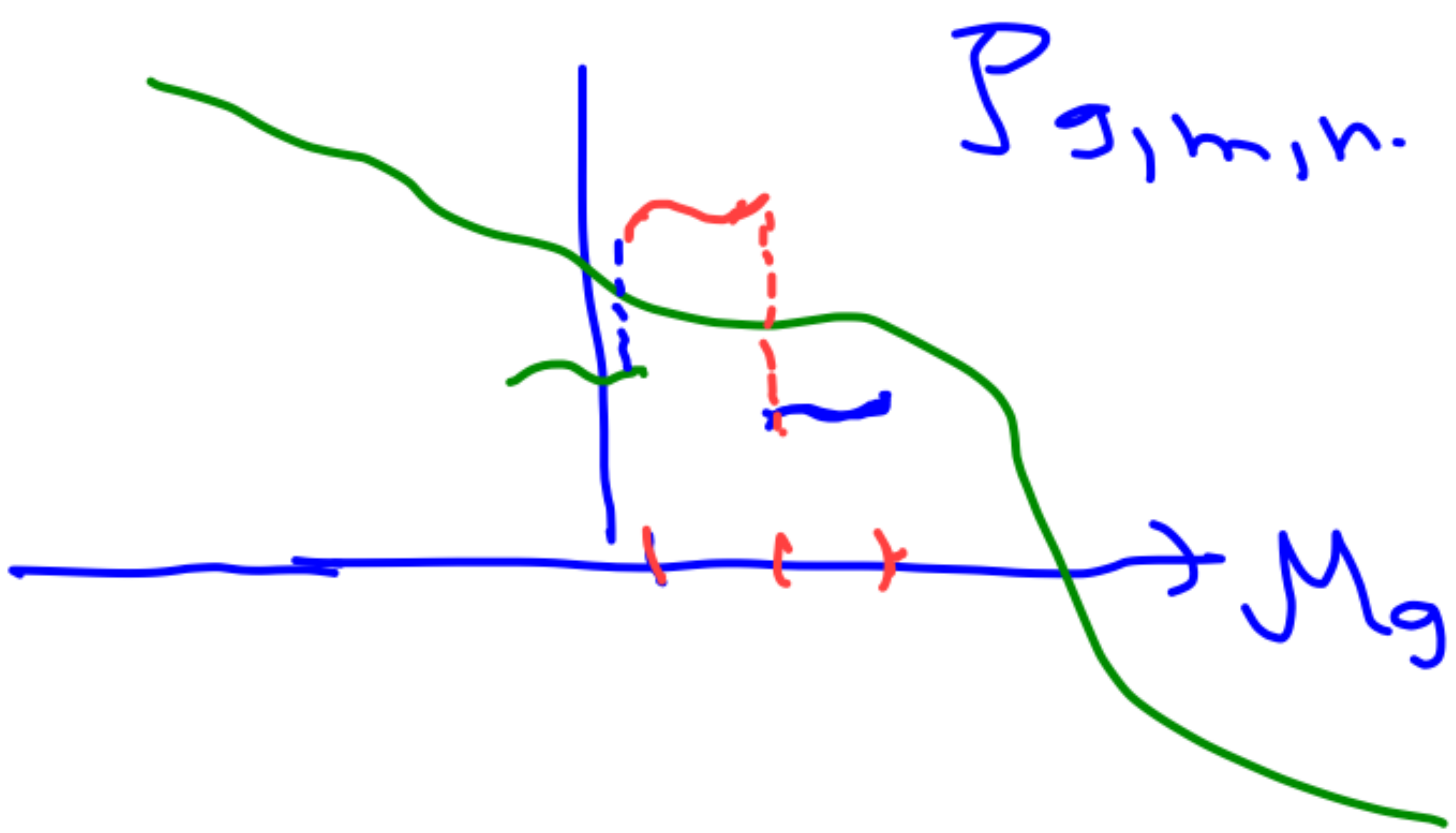
→ should be regarded as different choice of "section" which differ only in their vertical segments

gives same result for physical state amplitudes

General procedure for integrating over $\mathcal{M}_{g,m,n}$ with multiple PCO insertions.

① Divide $\mathcal{M}_{g,m,n}$ into small chambers.

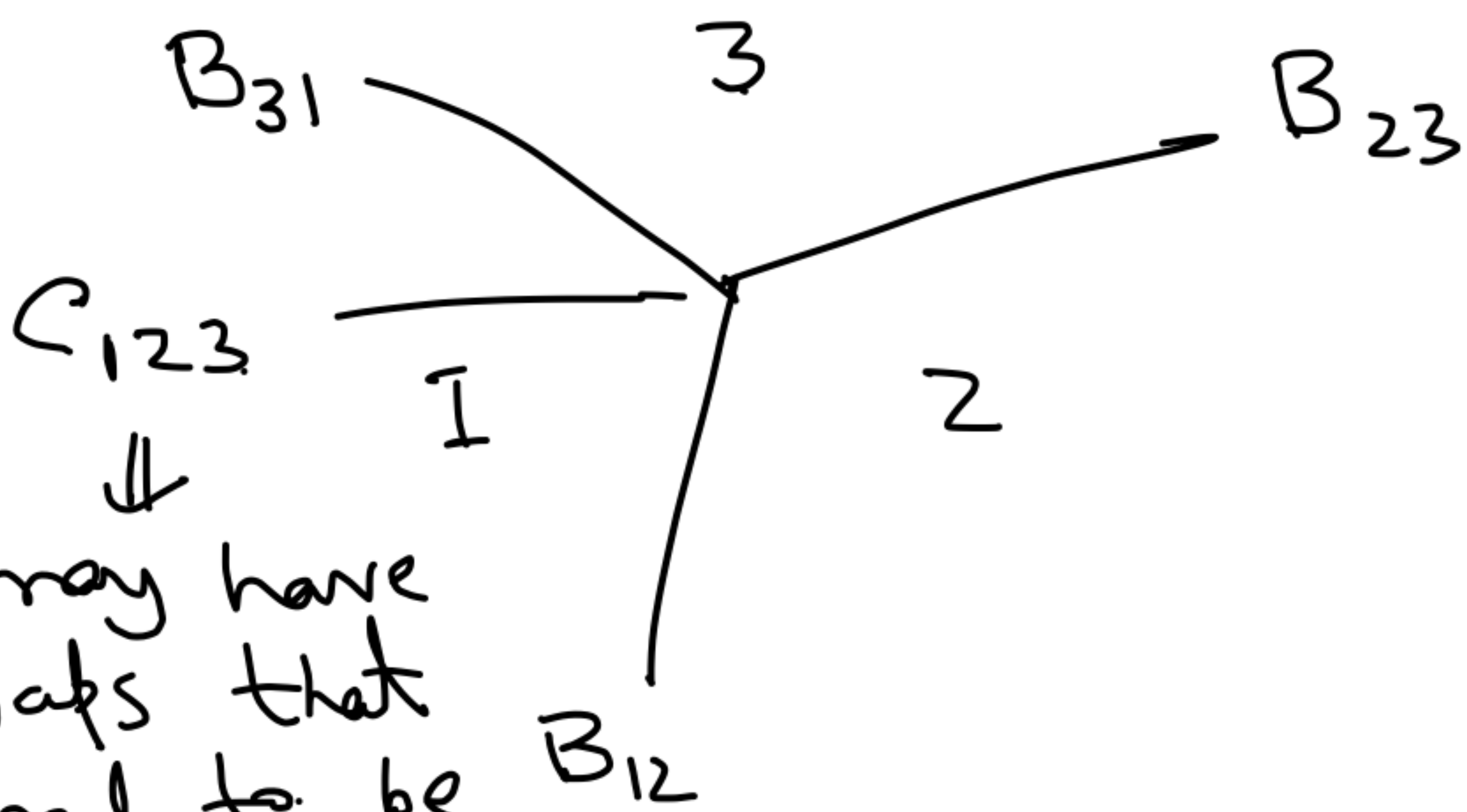
② On each chamber find a section segment avoiding spurious poles.



③ Across the boundary separating two chambers PCO's jump.

Fill the gap using vertical segment.

There may be further mismatch when these codimension 1 boundaries between the chambers meet on a codimension 2 subspace.



$\Downarrow$ may have gaps that need to be filled.

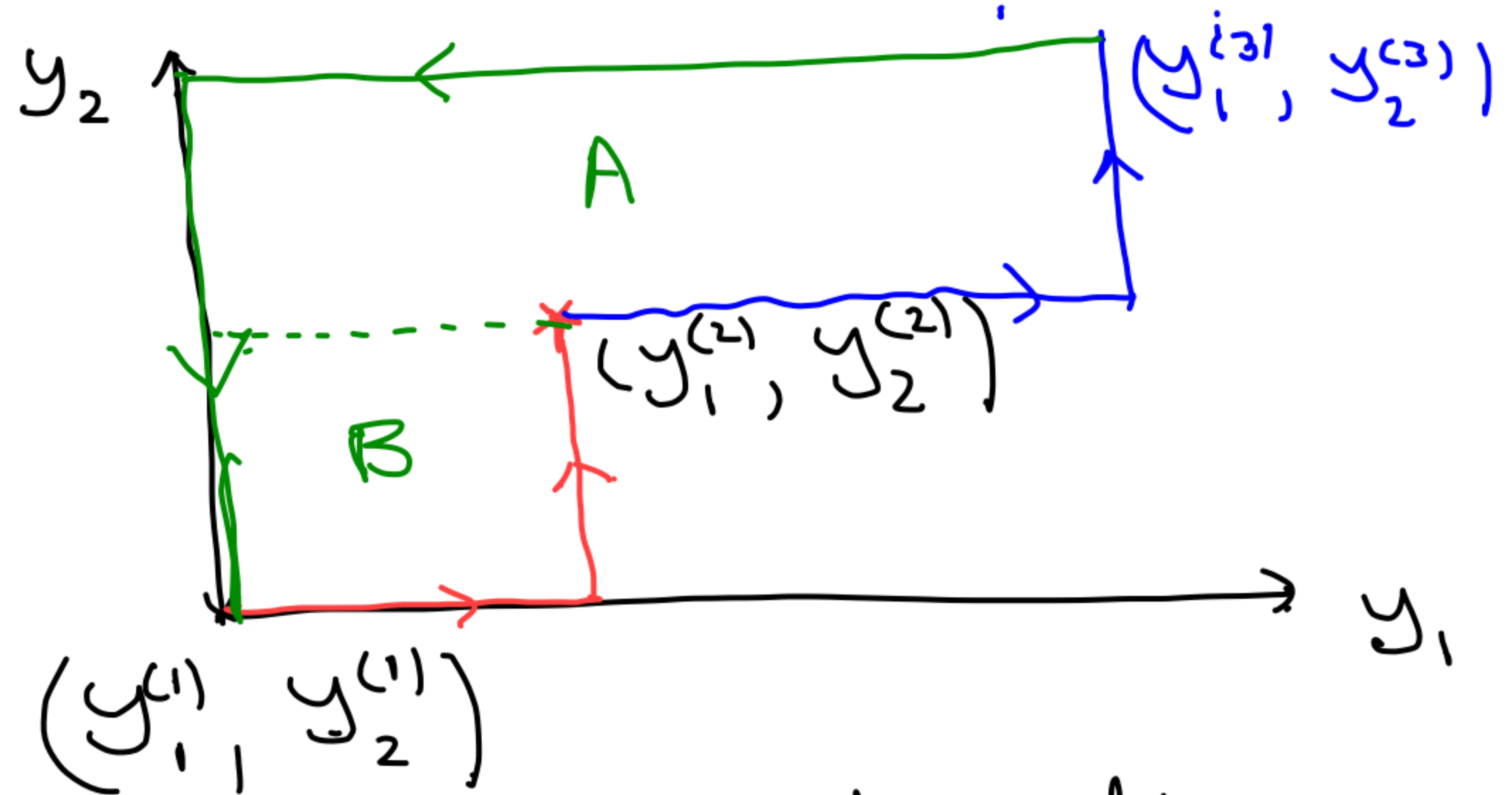
Take two PCO's y_1, y_2

On B_{12}: $(y_1^{(0)}, y_2^{(1)})$ $\to (y_1^{(2)}, y_2^{(1)}) \to (y_1^{(2)}, y_2^{(2)})$

On B_{23}: $(y_1^{(2)}, y_2^{(2)})$ $\to (y_1^{(3)}, y_2^{(2)}) \to (y_1^{(3)}, y_2^{(3)})$

On B_{13}: $(y_1^{(3)}, y_2^{(3)}) \to (y_1^{(1)}, y_2^{(3)}) \to (y_1^{(1)}, y_2^{(1)})$

whiteboardApp.org



A: $\int dt_1 \wedge dt_2 \, \frac{dy_1}{dt_1} \frac{dy_2}{dt_2} \, \partial \xi(y_1) \, \partial \xi(y_2)$

$= \left(\xi(y_1^{(3)}) - \xi(y_1^{(1)})\right)\left(\xi(y_2^{(3)}) - \xi(y_2^{(2)})\right)$

B: $\left(\xi(y_1^{(2)}) - \xi(y_1^{(1)})\right)\left(\xi(y_2^{(2)}) - \xi(y_2^{(1)})\right)$

This procedure can be continued.

→ Fully consistent rules for avoiding spurious poles.

Only left-over divergences come from $q \to 0$ limit of some plumbing fixture variable?

→ Can be treating using Feynman diagram viewpoint.

The procedure is complicated but is well defined and finite.

→ can in principle be coded in a computer program.

More elegant formalisms may be possible by clever choice of PCO locations and/or local coordinates, supermoduli integration etc.

→ Work for the future.

Freedom we have is in the choice of $S'_{g,m,n}$

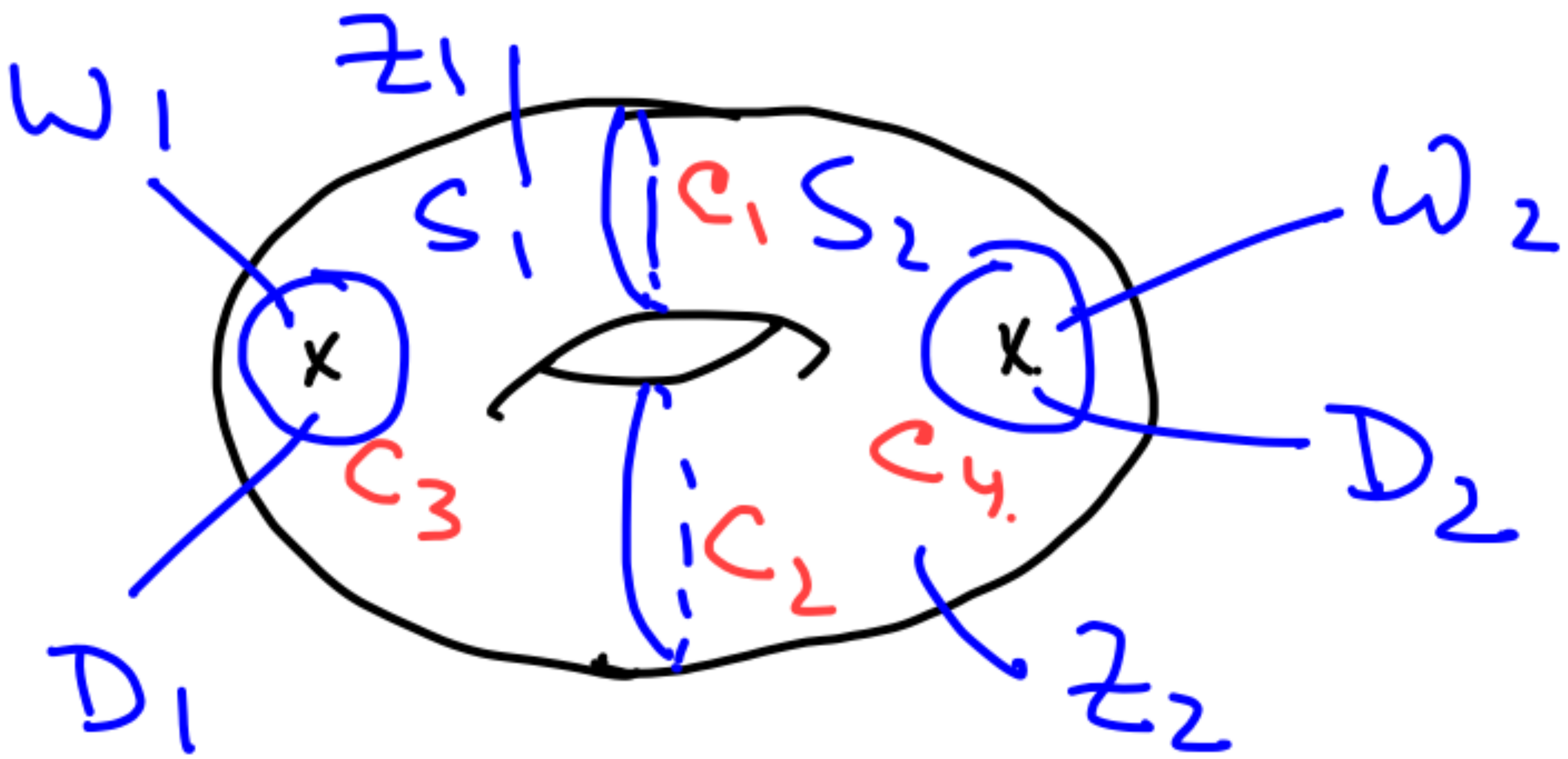
One can show that if we use local coordinates with large stubs, then plumbing fixture of $S'_{g,m,n}$ avoids spurious poles as long as $S'_{g,m,n}$ avoids them.

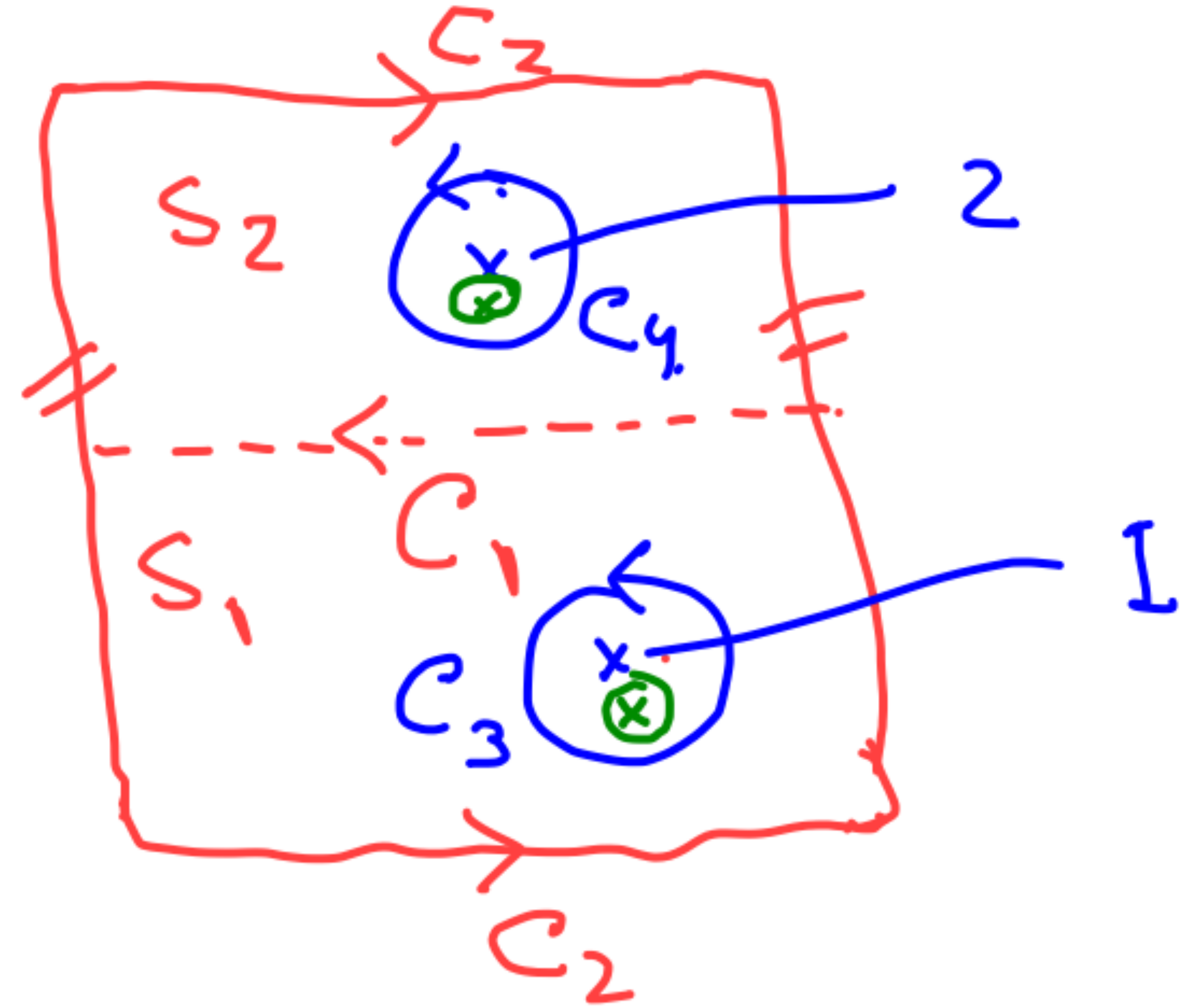
$\phi_r(0)\, \phi_s(0)\, \langle \phi_r^c | b_0 \bar{b}_0 \underset{1}{x_0} | \phi_s^c \rangle\, q^{h_s} \bar{q}^{\bar{h}_s}$

Genus 1, 2 pt. fn. of NS states

# of PCO's : $2g-2+m+\frac{n}{2} = 2$

# of moduli: $6g-6+2(m+n) = 4.$





On c_3: $z_1 = w_1$

on c_1: $z_1 = z_2$

on c_4: $z_2 = w_2 + \sigma$

on c_2: $z_1 = z_2 - \tau$

(σ, τ) label $M_{1,2,0}$

PCO locations
on D_1 at y_1
on D_2 at y_2
indep of σ, τ by choice.

ex. Compute B_τ, $B_{\bar{\tau}}$,
B_σ, $B_{\bar{\sigma}}$

Ex. $B_\tau = \oint_{C_2} (-1)\, b(z_1)\, dz_1$, $B_{\bar\tau} = \oint_{C_2} (-1)\, \bar b(\bar z_1)\, d\bar z_1$

$B_\varsigma = \oint_{C_4} (-1)\, b(\omega_2)\, d\omega_2$, $B_{\bar\varsigma} = \oint_{C_4} (-1)\, \bar b(\bar\omega_2)\, d\bar\omega_2$

$$A = \sum_{\substack{\text{spin} \\ \text{structure}}} \int d\tau \wedge d\bar\tau \wedge d\varsigma \wedge d\bar\varsigma\, X(y_1)\, X(y_2) \left\langle B_\tau B_{\bar\tau} B_\varsigma B_{\bar\varsigma}\, V_1(\omega_1 = 0)\ V_2(\omega_2 = 0) \right\rangle$$

Choose $V_i = c\bar c\, e^{-\phi}\, W_i \xrightarrow{} $ dim $(1, \frac{1}{2})$ matter superconformal primary.

Simplify by ~~taking~~ $y_i \to 0$ | Ex. $\lim_{y \to 0} X(y) V_i(0) = \text{finite} = V_i^{(0)}$.

$B_\varsigma, B_{\bar\varsigma}$ can be simplified using residue thm.

There is a subtlety:

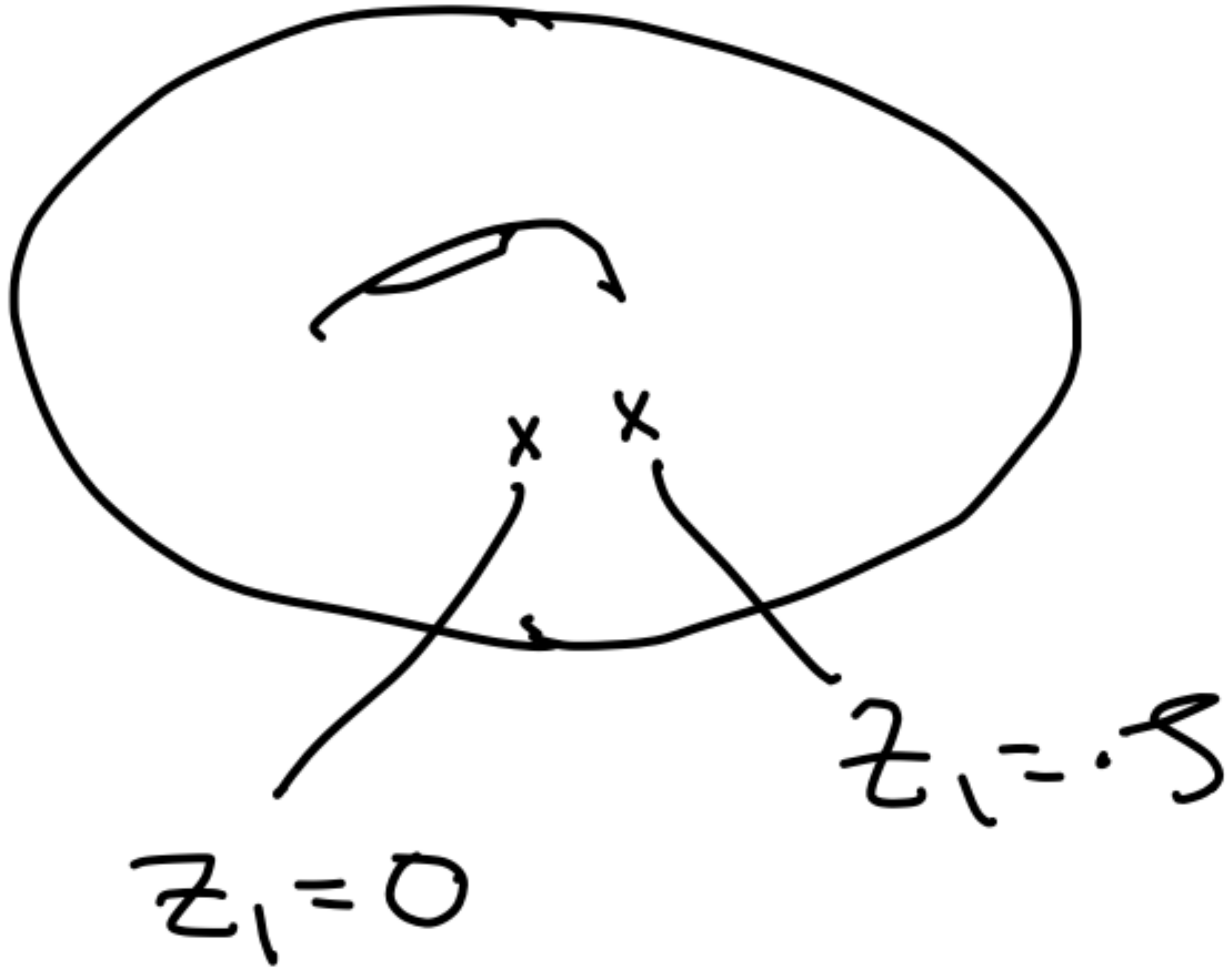
$$z_1 = w_1 = 0$$

$$w_2 = 0$$

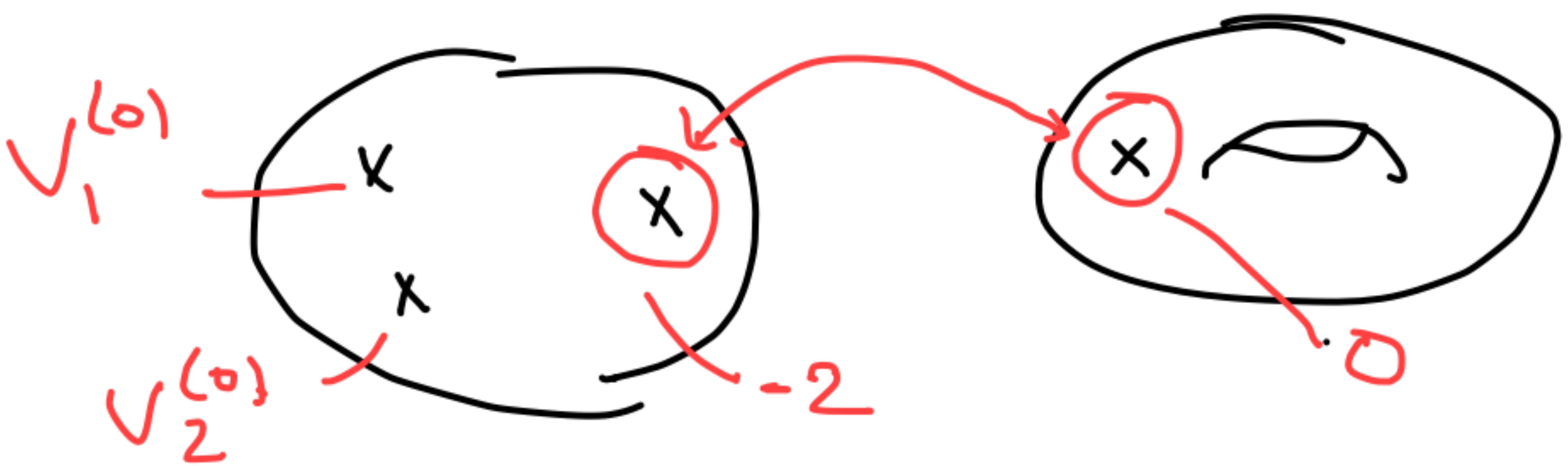
$$z_2 = w_2 + \varsigma = \varsigma$$

$$z_1 = z_2 = \varsigma$$

$\varsigma \to 0$ limit.



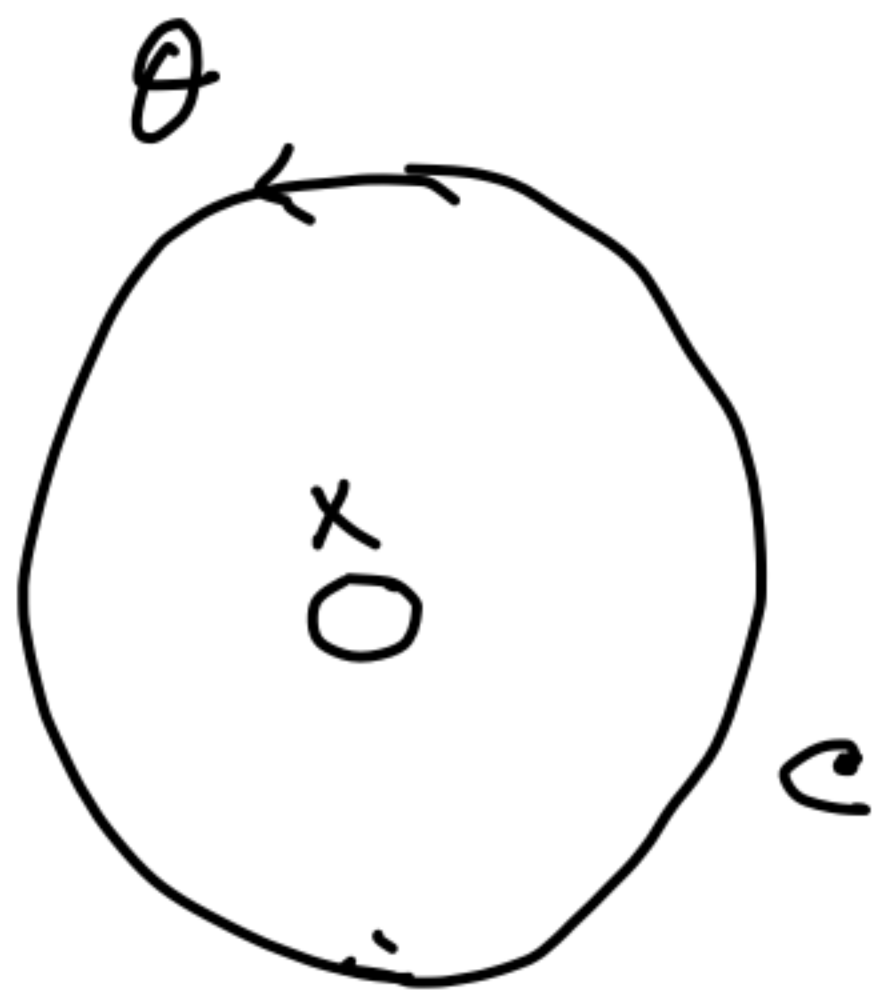
$\varsigma \to 0$ limit:



$\to$ Wrong!

We should have -1 picture on each side.

Soln: Look at the S-plane.



For S outside C, we can use $V_1^{(0)}$ and $V_2^{(0)}$

On C: use a vertical segment & move the PCO from S to S_0 (far way)

On C we have a contribution

$$\propto \int \langle V_1^{(0)}(0)\left(\xi(S) - \xi(S_0)\right) V_2(S)\, \pi B \rangle \; d\tau \wedge d\bar{\tau} \wedge d\theta$$

Inside c:

$$\int \langle (\Pi B| \; V_1^{(a)}(0) \; V_2(s) \; X(s_0) \rangle$$

$$\int \partial \xi(z) \; (\eta(z_0))$$

$$\sim \frac{1}{(z-z_0)^2}$$