Electromagnetic Couplings of Axions

Andreas Ringwald
Galileo Galilei Institute for Theoretical Physics Workshop
Axions across Boundaries between Particle Physics, Astrophysics, Cosmology and Forefront Detection Technologies
Florence, Italy
April 26 – June 9, 2023

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[Anton Sokolov, AR, 2104.02574; 2109.08503; 2205.02605; 2303.10170]

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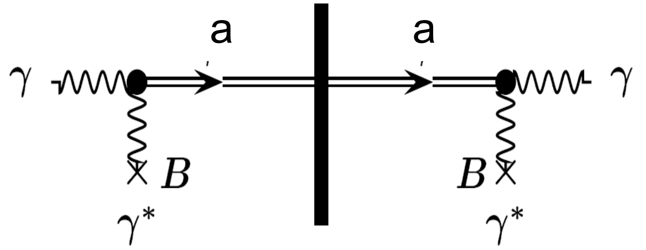
$$\mathcal{L} \supset \frac{g_{a\gamma\gamma}}{4}\, a\, F_{\mu\nu}\tilde{F}^{\mu\nu} \equiv g_{a\gamma\gamma}\, a\, \mathbf{E}\cdot\mathbf{B}$$

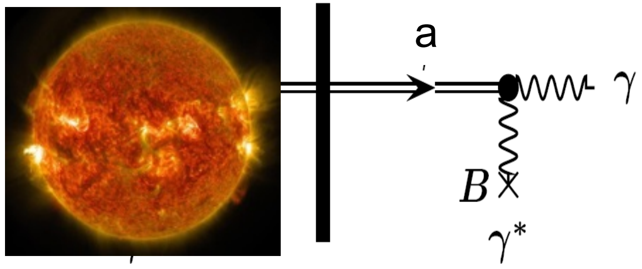
Light shining through a wall

[Anselm 85; van Bibber 87]



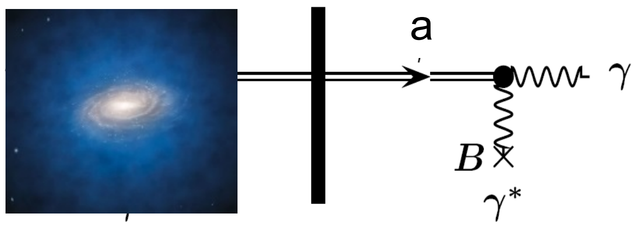
Helioscope

[Sikivie 83]



Haloscope

[Sikivie 83]



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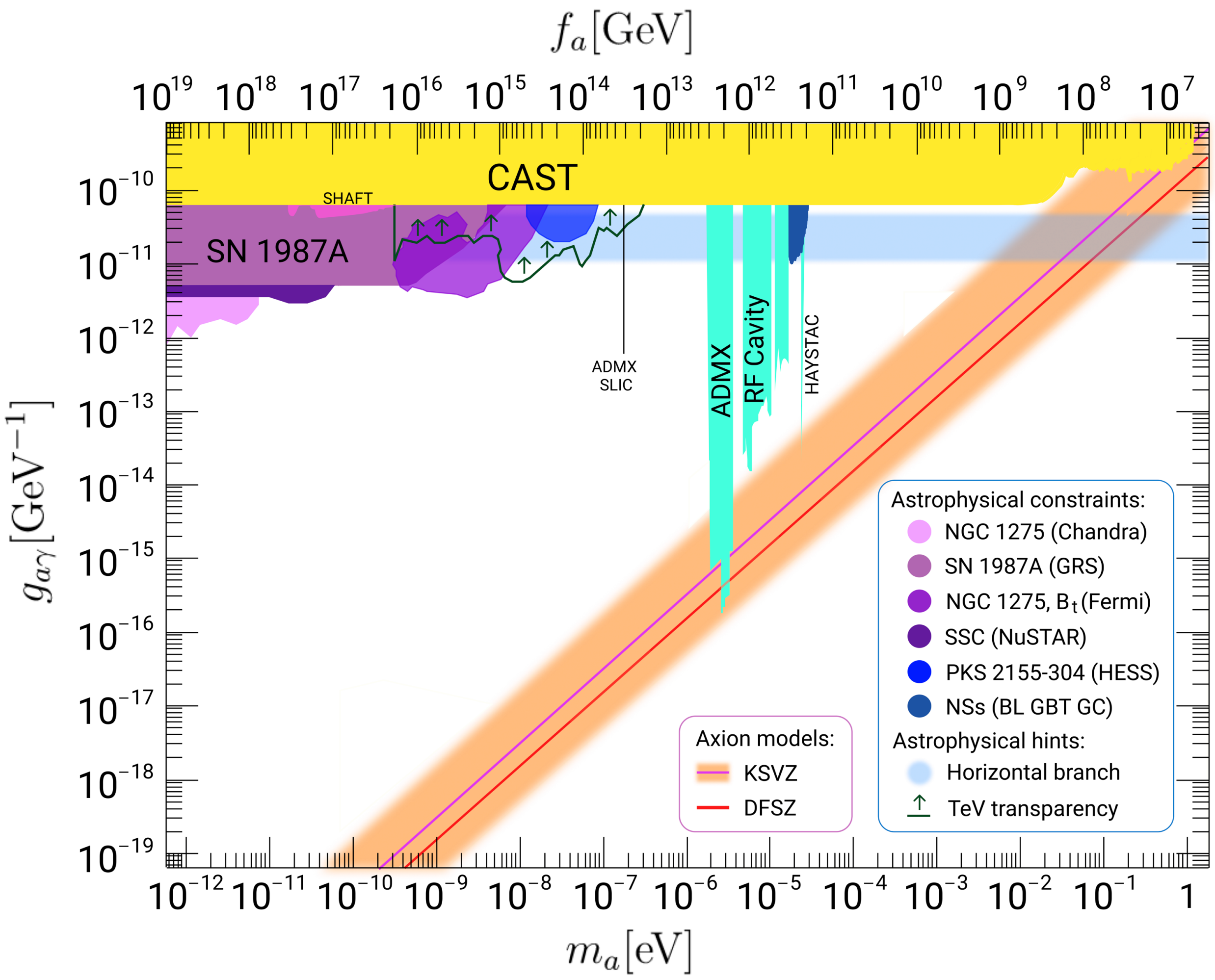
$$g_{a\gamma\gamma} \equiv \frac{\alpha}{2\pi f_a} C_{a\gamma} \simeq \frac{\alpha}{2\pi f_\pi}\frac{m_a}{m_\pi}\frac{1+z}{\sqrt{z}}\left(\frac{E}{N} - \frac{2}{3}\frac{4+z}{1+z}\right)$$



adapted from [Sokolov,AR, 2104.02574]

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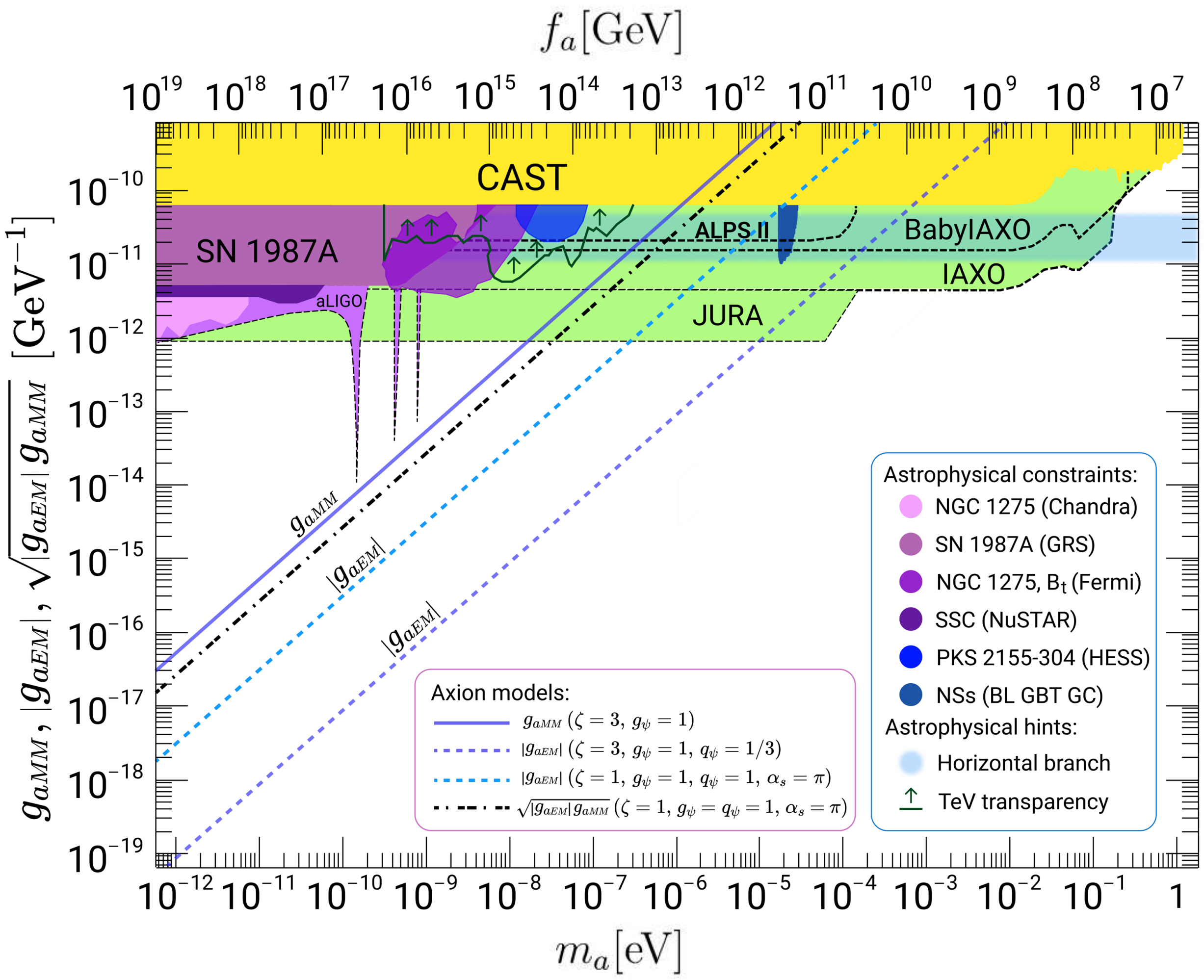
- **Report here on the construction of variants of the KSVZ model with enhanced electromagnetic couplings which can be probed very soon**

[Sokolov,AR, 2104.02574, 2109.08503, 2205.02605, 2303.10170]



adapted from [Sokolov,AR 2205.02605]

KSVZ Axion Model

[Kim 79;Shifman,Vainshtein,Zakharov 80]

Recap

- Add to SM a singlet complex scalar field σ, featuring a spontaneously broken global $U(1)_{\rm PQ}$ symmetry, and a vector-like fermion $\mathcal{Q} = \mathcal{Q}_L + \mathcal{Q}_R$ in the fundamental of colour, singlet under $SU(2)_L$ and neutral under hypercharge.

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- Assuming that under $U(1)_{\rm PQ}$ the fields transform as

$$\sigma \to e^{i\alpha}\sigma\,, \qquad \mathcal{Q}_L \to e^{i\alpha/2}\mathcal{Q}_L\,, \qquad \mathcal{Q}_R \to e^{-i\alpha/2}\mathcal{Q}_R$$

  the most general renormalizable Lagrangian can be written as

$$\mathcal{L}_{\rm KSVZ} = |\partial_\mu\sigma|^2 - \lambda_\sigma\left(|\sigma|^2 - \frac{v_\sigma^2}{2}\right)^2 + \overline{\mathcal{Q}}\,i\gamma_\mu D^\mu\mathcal{Q} - \left(y_{\mathcal{Q}}\overline{\mathcal{Q}}_L\mathcal{Q}_R\sigma + \text{h.c.}\right)$$

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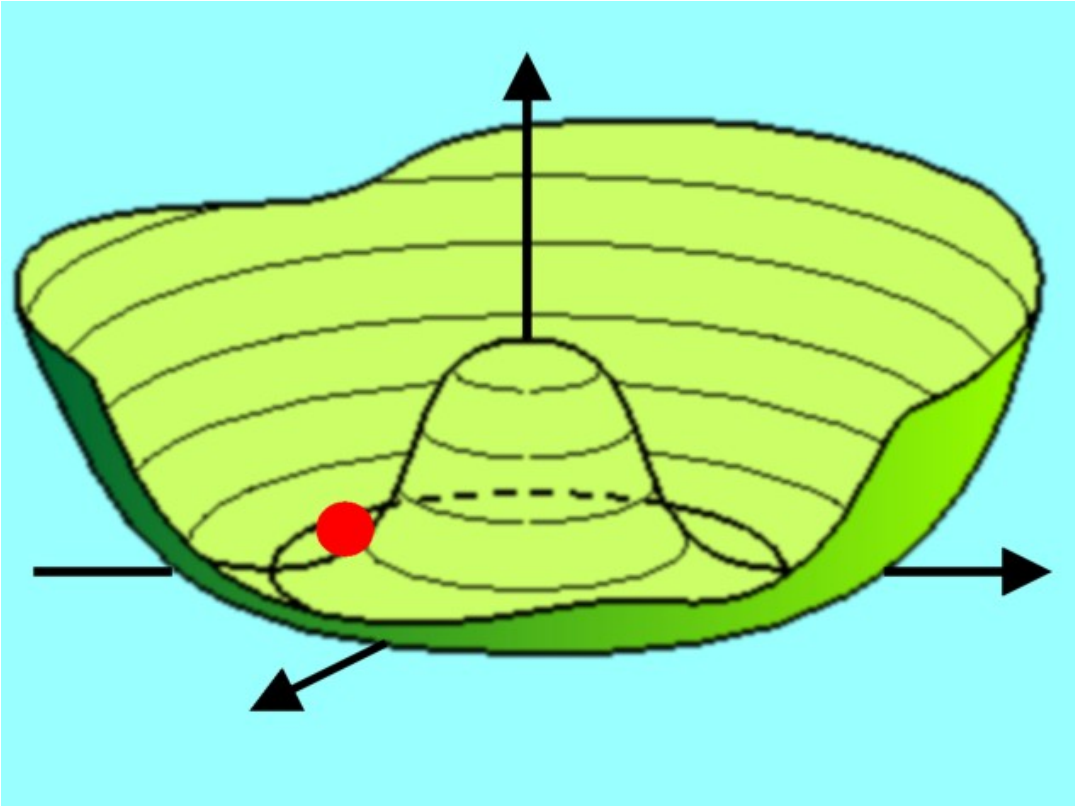
- Decomposing the scalar field in polar coordinates,

$$\sigma(x) = \frac{1}{\sqrt{2}} \left(v_\sigma + \rho(x) \right) \mathrm{e}^{i a(x)/v_\sigma}$$

  we see that this model features three BSM particles:

1. Excitation of Goldstone field $a(x)$: massless at tree level
2. Excitation of radial field $\rho(x)$: $m_\rho = \sqrt{2\lambda_\sigma} v_\sigma$
3. New fermion: $m_\mathcal{Q} = \frac{y_\mathcal{Q}}{\sqrt{2}} v_\sigma$



[Raffelt]

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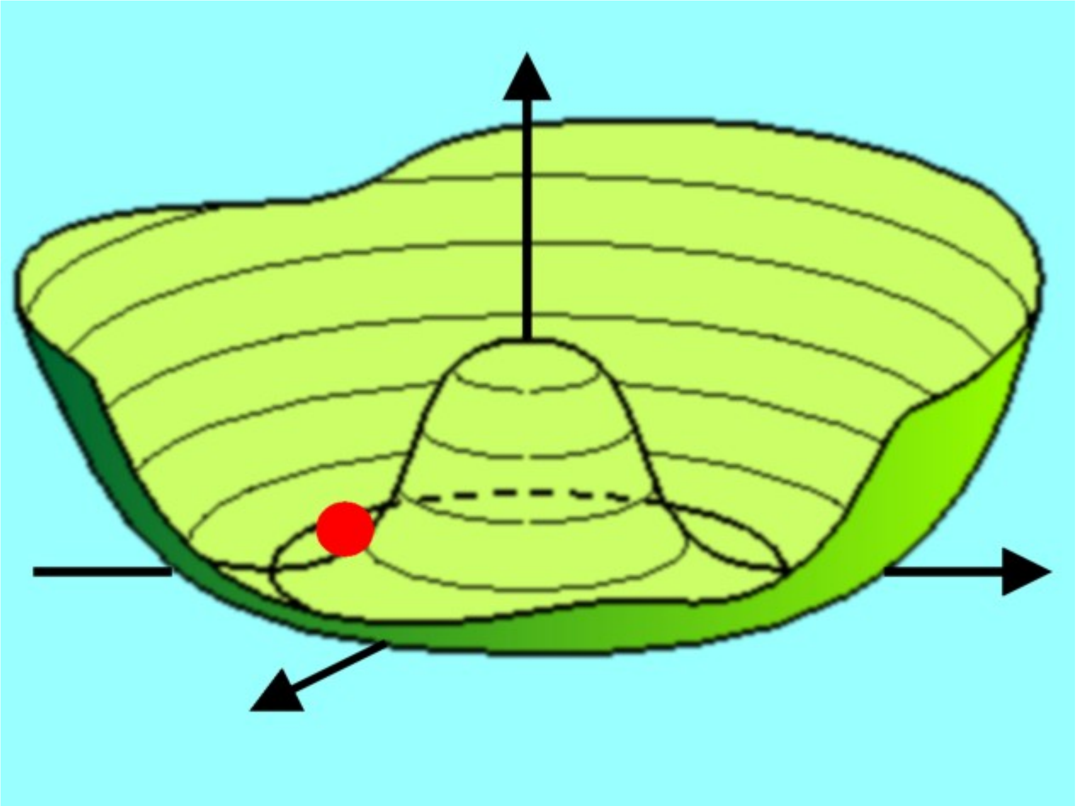
- For large PQ breaking scale v_σ, the latter two are heavy and may be integrated out, if we are only interested at the effective theory at energies much less than the breaking scale



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- Integrate out $\rho(x)$:

$$\mathcal{L}_{\rm KSVZ} \simeq \frac{1}{2}\partial_\mu a \partial^\mu a + \overline{\mathcal{Q}}\, i\gamma_\mu D^\mu \mathcal{Q} - \left(m_{\mathcal{Q}} \overline{\mathcal{Q}}_L \mathcal{Q}_R e^{ia/v_\sigma} + \text{h.c.} \right)$$

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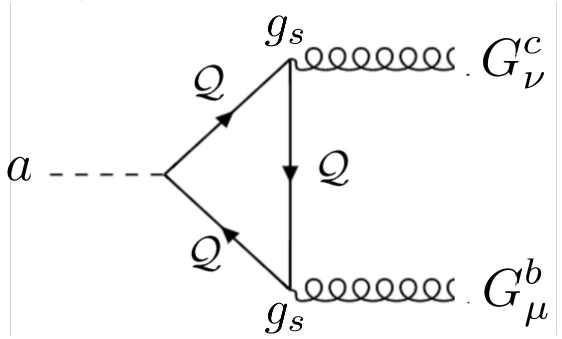

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- Now we can also safely integrate out the heavy exotic quark:

$$\mathcal{L}_{\rm KSVZ} \simeq \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{g_s^2}{32\pi^2}\frac{a}{v_\sigma} G\tilde{G}$$

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- Now we can also safely integrate out the heavy exotic quark: $\theta = a/v_\sigma$ is a dynamical $\overline{\theta}$-parameter!

$$\mathcal{L}_{\rm KSVZ} \simeq \frac{f_a^2}{2}\partial_\mu \theta\, \partial^\mu \theta + \theta \frac{\alpha_s}{8\pi} G\tilde{G} \qquad f_a = v_\sigma$$

KSVZ Axion Model

Allowing for electric charge of exotic quark

- Allowing for electric charge of the exotic quark, that is charged under $U(1)_E$, generalized KSVZ axion described by

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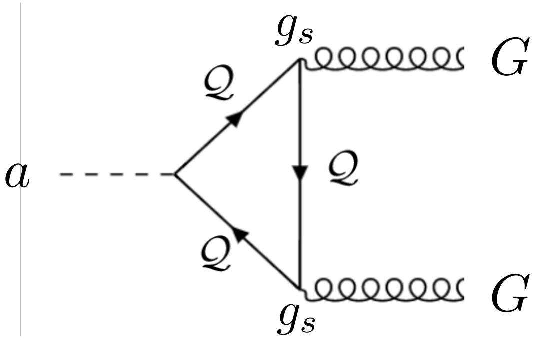
  - Anomaly coefficients N and E:

$U(1)_{PQ} \times SU(3)_c \times SU(3)_c$



$N = 1/2$

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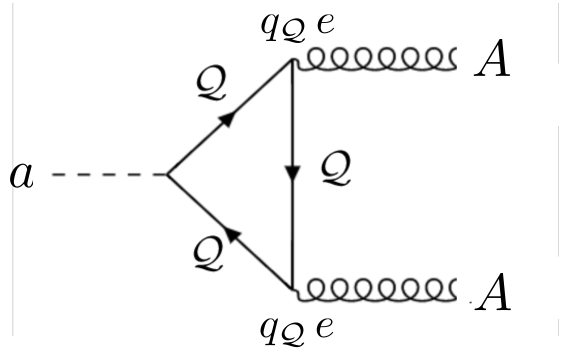

$E = 3\, q_\mathcal{Q}^2$

KSVZ Axion Model

Effective field theory below QCD scale

$$\mathcal{L}_{\rm KSVZ} \supset \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2} m_a^2 a^2 + \frac{\alpha}{2\pi}\,\frac{C_{a\gamma}}{f_a}\,\frac{a}{4}\,F_{\mu\nu}\tilde{F}^{\mu\nu}$$

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[Gorghetto et al. 18]

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[Kaplan 85; Srednicki `85]

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$$C_{a\gamma} = \frac{E}{N} - 1.92(4)$$

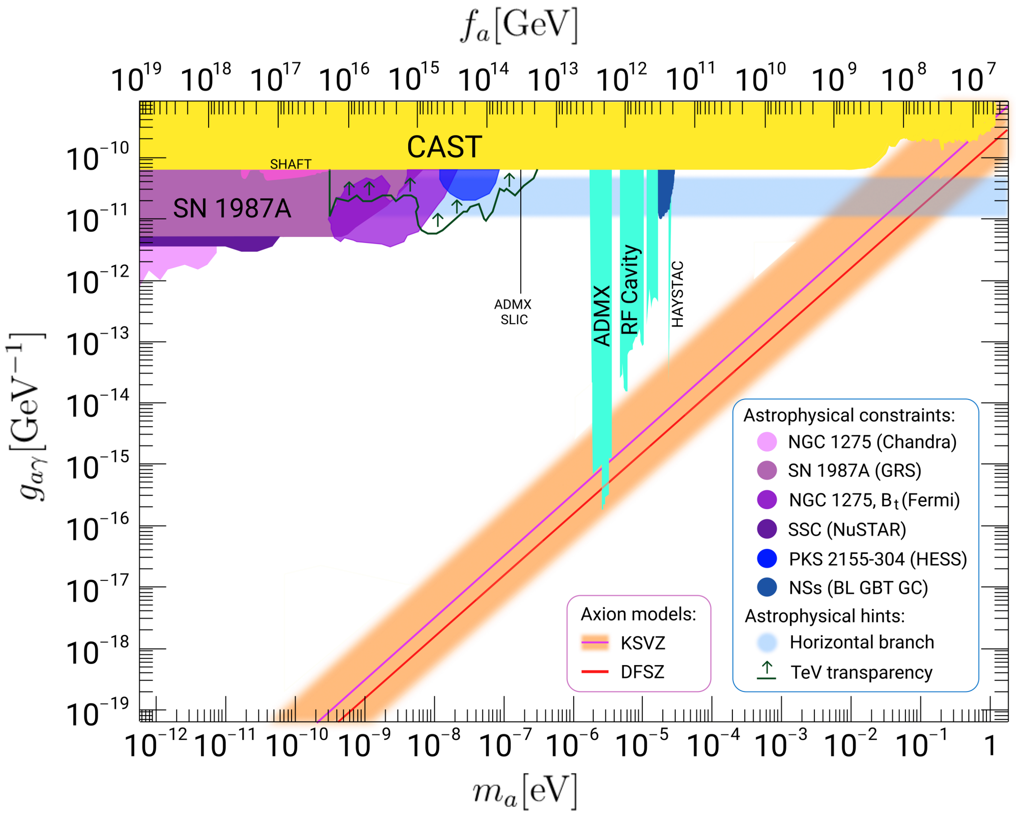
[Grilli di Cortona et al. `16]

KSVZ Axion Model

Electromagnetic coupling versus mass

$$g_{a\gamma\gamma} \equiv \frac{\alpha}{2\pi f_a} C_{a\gamma} \simeq \frac{\alpha}{2\pi f_\pi} \frac{m_a}{m_\pi} \frac{1+z}{\sqrt{z}} \left(\frac{E}{N} - \frac{2}{3} \frac{4+z}{1+z} \right)$$

- “Band” of predictions for electromagnetic coupling



adapted from [Sokolov,AR, 2104.02574]

see also [Di Luzio,Mescia,Nardi, `17]

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- What if exotic quark carries a magnetic charge?

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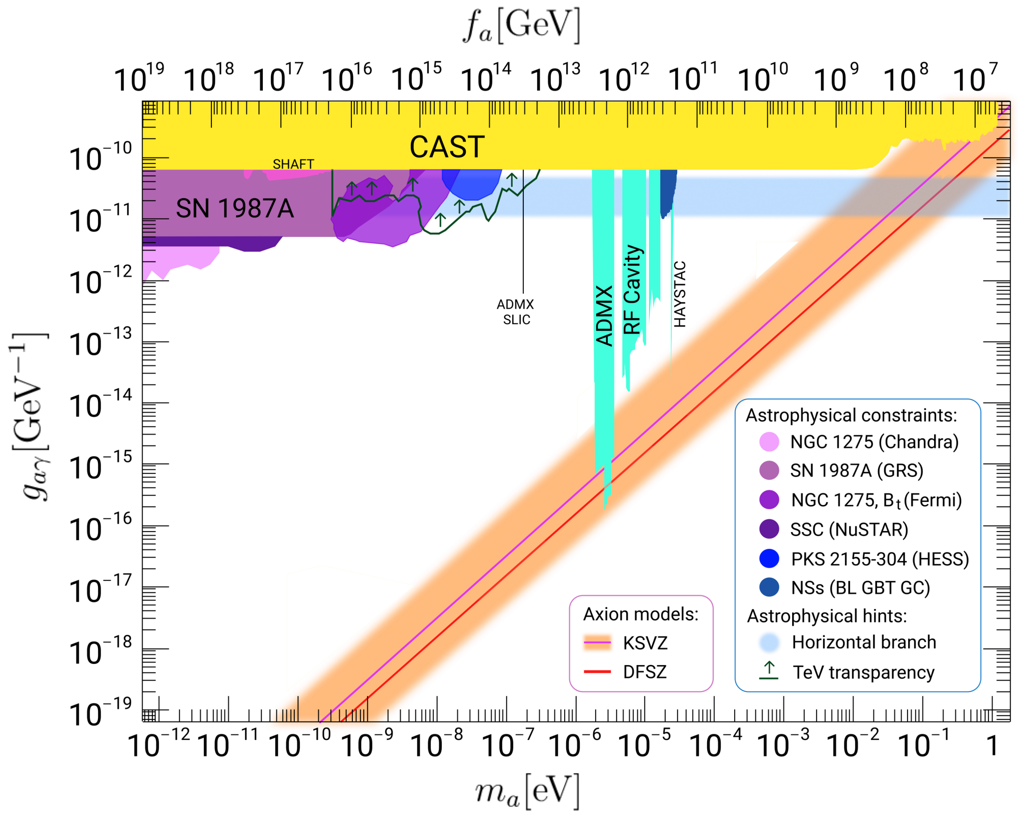

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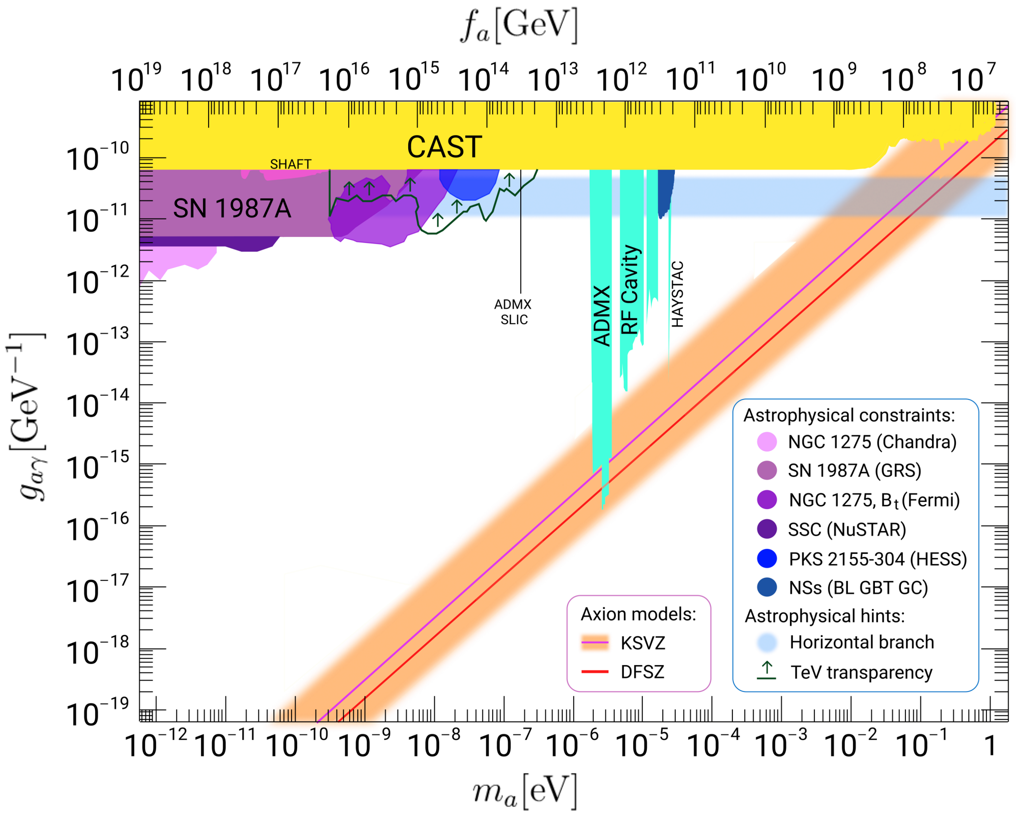

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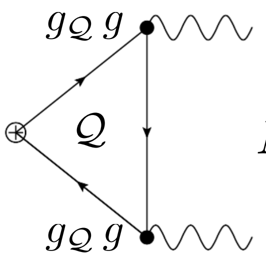
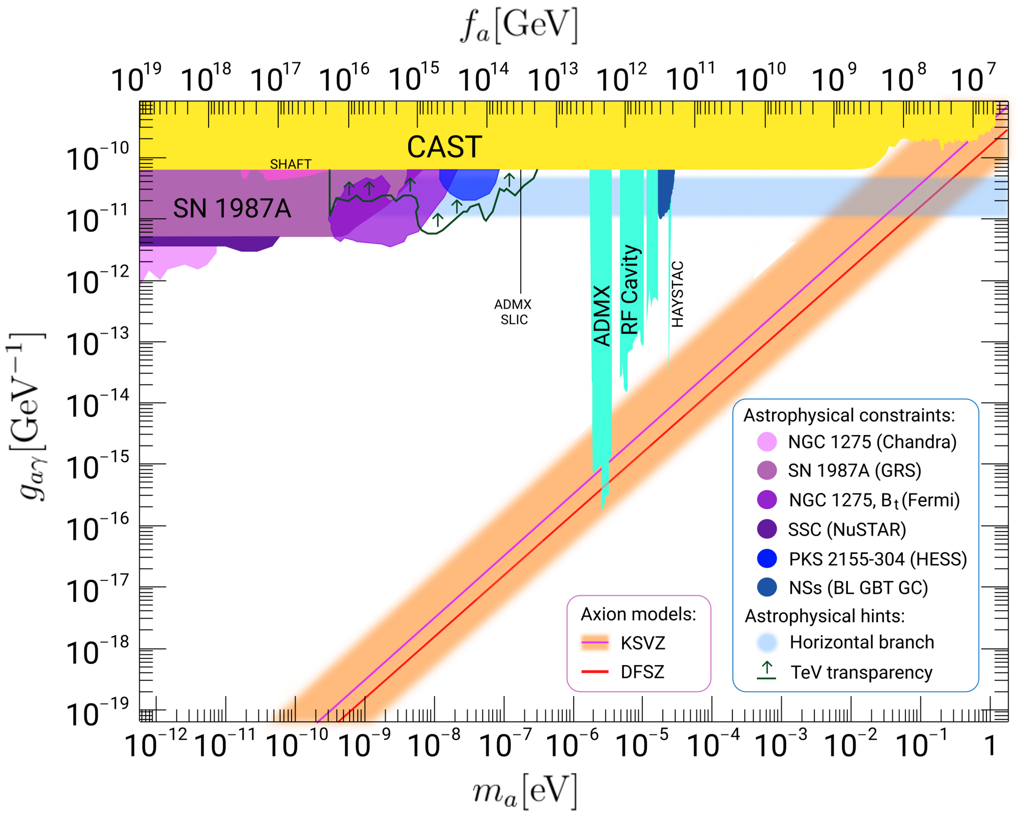


$$M = 3\, g_Q^2$$



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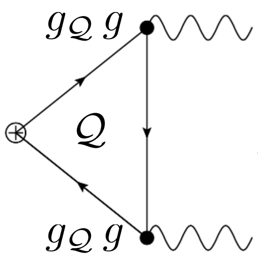

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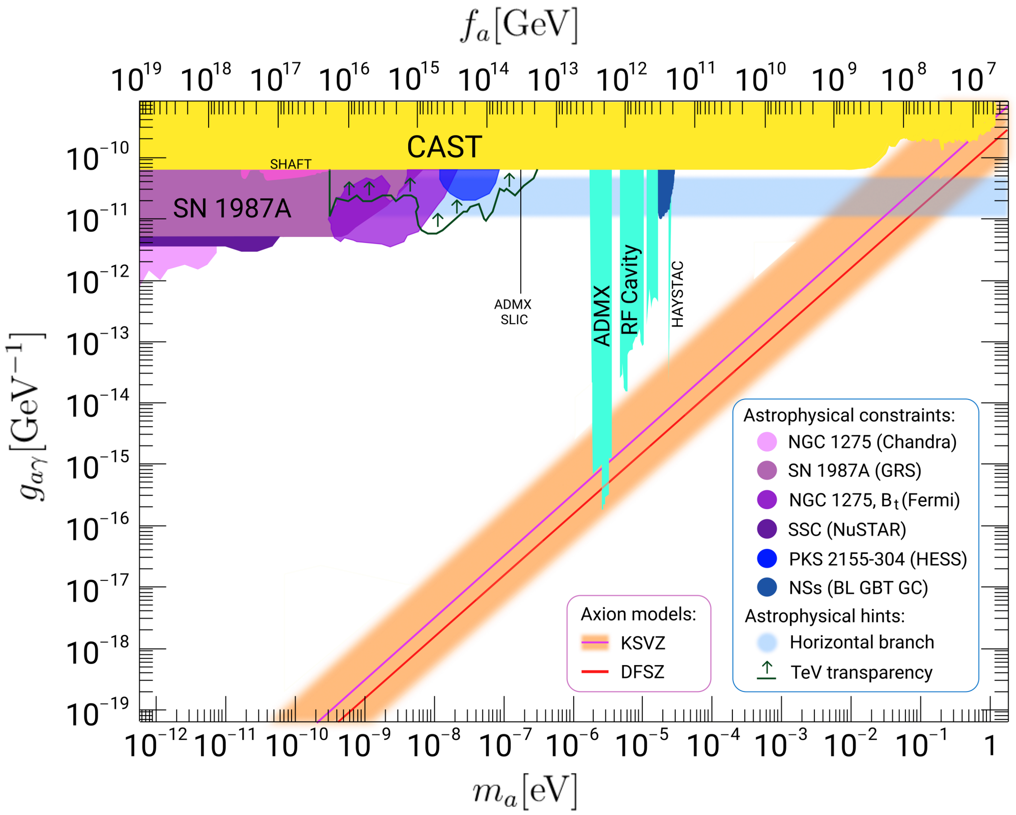
  - Parametrically enhanced due to charge quantisation:

$$\alpha_{\rm M} = \frac{g^2}{4\pi} \sim \frac{\pi^2/e^2}{4\pi} \sim \alpha^{-1} \quad \Rightarrow \frac{g_{aMM}}{g_{a\gamma\gamma}} \sim \alpha^{-2} \sim 10^4$$



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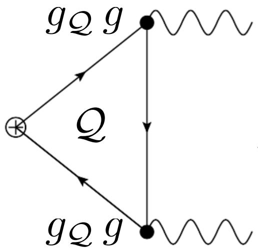

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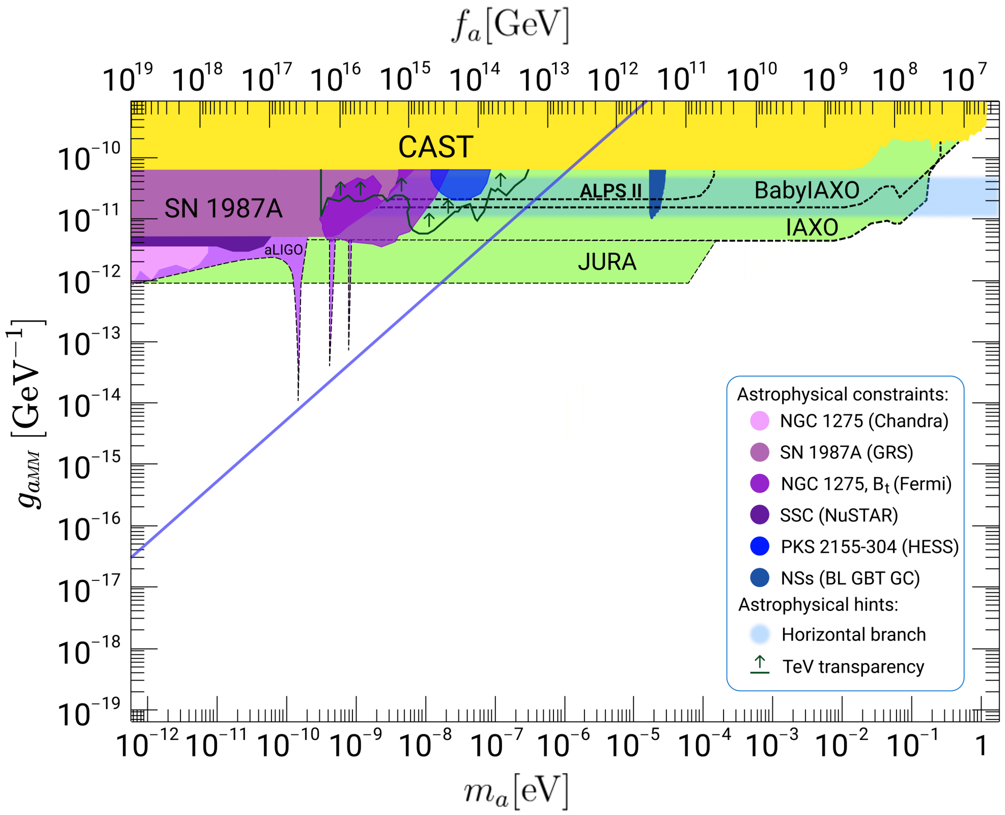

adapted from [Sokolov,AR 2205.02605]

Monopole-Philic Axion Model

What if the exotic quark carries also a magnetic charge?

- Want to integrate out a heavy, magnetically charged quark

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    - Lorentz invariance of physical observables, provided that charge quantization condition is satisfied, can be shown by path integral techniques [Brandt, Neri, Zwanziger `78]

Monopole-Philic Axion Model

Formulation of generalized Maxwell's equations in terms of two gauge fields

- Zwanziger's local Lagrangian based on the fact that the general solutions of the electromagnetic field strength tensor and its dual appearing in Maxwell's equations,

$$\partial_\mu F^{\mu\nu} = j_e^{\,\nu}\,, \quad \partial_\mu F^{d\,\mu\nu} = j_m^{\,\nu}$$

in the presence of conserved electric and magnetic currents,

$$\partial_\mu j_e^\mu = \partial_\mu j_m^\mu = 0$$

can be expressed in terms of two gauge fields, A_μ and C_μ,

$$F = \partial \wedge A - (n\cdot\partial)^{-1}(n \wedge j_m)^d\,, \qquad F^d = \partial \wedge C + (n\cdot\partial)^{-1}(n \wedge j_e)^d\,,$$

where n is an arbitrary fixed four-vector and $(n\cdot\partial)^{-1}(x-y)$ an integral operator satisfying

$$n \cdot \partial\,(n\cdot\partial)^{-1}(x) = \delta^4(x)$$

  - Notation: $a\cdot b = a_\mu b^\mu, (a\wedge b)^{\mu\nu} = a^\mu b^\nu - a^\nu b^\mu,\ (a\cdot G)^\nu = a_\mu G^{\mu\nu},\ F^{d\,\mu\nu} \equiv \epsilon^{\mu\nu\kappa\lambda}F_{\kappa\lambda}/2$

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- The gauge fields depend on the choice of n, the choice of $(n\cdot\partial)^{-1}$, and the choice of gauge

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- Exploiting the identity $X = \frac{1}{n^2}\left\{[n \wedge (n\cdot X)] \; - \; [n \wedge (n\cdot X^d)]^d\right\}$, which holds for any antisymmetric tensor, and using that $n\cdot F = n\cdot(\partial \wedge A), \quad n\cdot F^d = n\cdot(\partial \wedge C)$, one finds that the field strength and its dual can be expressed locally in terms of the two potentials alone:

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\begin{aligned}
F &= \frac{1}{n^2}\left\{[n \wedge (n\cdot(\partial \wedge A))] \; - \; [n \wedge (n\cdot(\partial \wedge C))]^d\right\} \\
F^d &= \frac{1}{n^2}\left\{[n \wedge (n\cdot(\partial \wedge A))]^d \; + \; [n \wedge (n\cdot(\partial \wedge C))]\right\}
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- To any solution F of Maxwell's equations there exist potentials A and C related locally to F by (1) and satisfying (2).

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$$

$$
\frac{1}{n^2}\left(n\cdot\partial\, n\cdot\partial C^\mu - n\cdot\partial\, \partial^\mu n\cdot C - n^\mu\, n\cdot\partial\, \partial\cdot C + n^\mu\, \partial^2\, n\cdot C - n\cdot\partial\, \epsilon^\mu{}_{\nu\rho\sigma} n^\nu \partial^\rho A^\sigma\right) = j_m^\mu .
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- To any solution F of Maxwell's equations there exist potentials A and C related locally to F by (1) and satisfying (2). Conversely, every pair of potentials satisfying (2) define a unique solution F of Maxwell's equations. A and C are highly non-unique.

Monopole-Philic Axion Model

Local Lagrangian for gauge fields in QEMD

[Zwanziger `71]

- Generalized Maxwell equation in terms of gauge fields:

$$\begin{aligned} \frac{1}{n^2}\left(n\cdot\partial\, n\cdot\partial A^\mu - n\cdot\partial\,\partial^\mu n\cdot A - n^\mu\, n\cdot\partial\,\partial\cdot A + n^\mu\,\partial^2\, n\cdot A - n\cdot\partial\,\epsilon^\mu{}_{\nu\rho\sigma} n^\nu \partial^\rho C^\sigma\right) &= j_e^\mu\,, \\ \frac{1}{n^2}\left(n\cdot\partial\, n\cdot\partial C^\mu - n\cdot\partial\,\partial^\mu n\cdot C - n^\mu\, n\cdot\partial\,\partial\cdot C + n^\mu\,\partial^2\, n\cdot C - n\cdot\partial\,\epsilon^\mu{}_{\nu\rho\sigma} n^\nu \partial^\rho A^\sigma\right) &= j_m^\mu\,. \end{aligned} \quad (2)$$

- Local Lagrangian $\mathcal{L} = \mathcal{L}_Z + \mathcal{L}_I$ whose Euler-Lagrange equations coincide with (2):

$$\mathcal{L}_Z = \frac{1}{2n^2}\left\{-\,[n\cdot(\partial\wedge A)]\cdot[n\cdot(\partial\wedge C)^d] + [n\cdot(\partial\wedge C)]\cdot[n\cdot(\partial\wedge A)^d] - [n\cdot(\partial\wedge A)]^2 - [n\cdot(\partial\wedge C)]^2\right\}$$

$$\mathcal{L}_I = -\,j_e\cdot A - j_m\cdot C$$

Monopole-Philic Axion Model

Relativistic classical dynamics of electrically and magnetically charged point particles

[Zwanziger `71]

- Consider a realization of the electric and magnetic currents by point particles with trajectories $x_i(\tau_i)$,

$$j_e^\nu(x) = \sum_i e_i \int \delta^4(x - x_i(\tau_i))\, dx_i^\nu \,, \qquad j_m^\nu(x) = \sum_i g_i \int \delta^4(x - x_i(\tau_i))\, dx_i^\nu$$

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- Their classical equations of motion plus the equations of motion of the gauge potentials can be obtained from the requirement that the action $S = S_Z + S_p + S_I$ be an extremum with respect to variation of the particle trajectories and the gauge fields, where

$$S_Z = \int \mathcal{L}_Z(x) d^4x \qquad S_p = -\sum_i \int m_i (u_i^2)^{1/2} d\tau_i \qquad S_I \; = \; = \; - \sum_i \int [e_i A(x_i) \; + \; g_i C(x_i)] \cdot u_i d\tau_i$$

$$u_i^\mu = dx_i^\mu / d\tau_i$$

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- Generalized Maxwell equations in terms of field strength fine, but Lorentz force

$$\frac{d}{d\tau_i}\left(\frac{m_i u_i}{(u_i^2)^{1/2}}\right) = \left(e_i\, [\partial \wedge A(x_i)] + g_i\, [\partial \wedge C(x_i)]\,\right)\cdot u_i \,,$$

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$$S_Z = \int \mathcal{L}_Z(x) d^4x \qquad S_p = -\sum_i \int m_i (u_i^2)^{1/2} d\tau_i \qquad S_I = -\sum_i \int [e_i A(x_i) + g_i C(x_i)]\cdot u_i d\tau_i$$

$$u_i^\mu = dx_i^\mu / d\tau_i$$

- Generalized Maxwell equations in terms of field strength fine, but Lorentz force

$$\frac{d}{d\tau_i}\left(\frac{m_i u_i}{(u_i^2)^{1/2}}\right) = \left(e_i\,[\partial \wedge A(x_i)] + g_i\,[\partial \wedge C(x_i)]\,\right)\cdot u_i\,,$$
$$= \left[e_i F(x_i) + g_i F^d(x_i)\,\right]\cdot u_i - \sum_j (e_i g_j - g_i e_j)\, n\cdot\int (n\cdot\partial)^{-1}(x_i - x_j)\ (u_i \wedge u_j)^d\, d\tau_j$$

made up of familiar local term plus interparticle action at a distance depending on $(n\cdot\partial)^{-1}(x-y)$

Monopole-Philic Axion Model

Relativistic classical dynamics of electrically and magnetically charged point particles

[Zwanziger `71]

$$\frac{d}{d\tau_i}\left(\frac{m_i u_i}{(u_i^2)^{1/2}}\right) = \left[e_i F(x_i) + g_i F^d(x_i)\right]\cdot u_i - \sum_j (e_i g_j - g_i e_j)\, n\cdot\int (n\cdot\partial)^{-1}(x_i - x_j)\ (u_i \wedge u_j)^d\, d\tau_j$$

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- Last term, which seems to spoil Lorentz-invariance, does not contribute to dynamics
  - Representing the kernel in the form ,

$$(n\cdot\partial)^{-1}(x) = \int_0^\infty \Big(a_s\, \delta^4(x - ns) - (1 - a_s)\, \delta^4(x + ns)\Big)\, ds$$

one finds that the support of $(n\cdot\partial)^{-1}\,(x_i(\tau_i) - y_j(\tau_j))$ is restricted to the “string”

$$x_i^\mu(\tau_i) - y_j^\mu(\tau_j) = n^\mu s\,,\;\; -\infty < s, \tau_i, \tau_j < +\infty$$

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  - This condition will not be satisfied anywhere along a trajectory unless it is exceptional, because there are four equations, but only three free parameters

Monopole-Philic Axion Model

Quantum field theory of electrically and magnetically charged particles

- Quantum Electromagnetodynamics (QEMD):

[Zwanziger `71]

$$\mathcal{L}_{\mathrm{QEMD}} = \mathcal{L}_Z + \mathcal{L}_G + \mathcal{L}_D$$

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- Can be shown by path integral techniques that time-ordered Green's functions of gauge-invariant local operators are independent of n^μ if the **Dirac-Schwinger-Zwanziger charge quantization condition** holds,

$$e_i g_j - e_j g_i = 2\pi n_{ij}, \quad n_{ij}\in\mathbb{Z}$$

[Brandt, Neri, Zwanziger `78]

Monopole-Philic Axion Model

What if the exotic quark carries also a magnetic charge?

- Integrate out $\rho(x)$:

$$\mathcal{L}_{\rm KSVZ} \simeq \frac{1}{2}\partial_\mu a \partial^\mu a + \overline{\mathcal{Q}}\, i\gamma_\mu D^\mu \mathcal{Q} - \left(m_{\mathcal{Q}} \overline{\mathcal{Q}}_L \mathcal{Q}_R e^{ia/v_\sigma} + \text{h.c.} \right)$$

- Last term can be brought to the form of a standard mass term by performing the field-dependent axial transformation, $\mathcal{Q} \to e^{-\frac{i}{2}\gamma_5 \frac{a}{v_\sigma}} \mathcal{Q}$, that is

$$\mathcal{Q}_L \to e^{\frac{i}{2}\frac{a}{v_\sigma}} \mathcal{Q}_L\,, \qquad \mathcal{Q}_R \to e^{-\frac{i}{2}\frac{a}{v_\sigma}} \mathcal{Q}_R$$

- However, fermionic measure in path integral is not invariant under axial transformations, cf.

$$\mathcal{D}\mathcal{Q}\mathcal{D}\bar{\mathcal{Q}} \to \mathcal{D}\mathcal{Q}\mathcal{D}\bar{\mathcal{Q}}\; e^{i\int d^4x \mathcal{L}_F(x)} \quad \text{where} \quad \mathcal{L}_{\rm F} = \frac{\alpha_s}{8\pi}\frac{a}{v_\sigma} G\tilde{G} + \mathcal{L}_{\rm F}^{\rm QEMD}$$

[Anton Sokolov, AR, arXiv:2205.02605]

$$\mathcal{L}_{\rm F}^{\rm QEMD} = \frac{a}{v_\sigma} \cdot \lim_{\substack{\Lambda\to\infty \\ x\to y}} \text{tr}\left\{\gamma_5 \exp\left(\not{\mathcal{D}}^2/\Lambda^2\right) \delta^4(x-y)\right\} \quad \text{with} \quad \mathcal{D}_\mu = \partial_\mu - ie\,q_{\mathcal{Q}} A_\mu - ig_0\, g_{\mathcal{Q}} C_\mu$$

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$$\mathcal{L}_{\rm F}^{\rm QEMD} = \frac{a}{v_\sigma}\left(\frac{\alpha}{8\pi} E\,{\rm tr}\left\{(\partial\wedge A)\,(\partial\wedge A)^d\right\} + \frac{\alpha_M}{8\pi} M\,{\rm tr}\left\{(\partial\wedge C)\,(\partial\wedge C)^d\right\} + \frac{\sqrt{\alpha\alpha_M}}{4\pi} D\,{\rm tr}\left\{(\partial\wedge A)\,(\partial\wedge C)^d\right\}\right)$$

- Coefficients: $E = 3\, q_{\mathcal{Q}}^2\,,\quad M = 3\, g_{\mathcal{Q}}^2\,,\; D = 3\, q_{\mathcal{Q}} g_{\mathcal{Q}}$

Monopole-Philic Axion Model

Generalized axion Maxwell equations

[Anton Sokolov, AR, arXiv:2205.02605]

- Resulting generalized axion Maxwell equations for experiment and phenomenology:

$$\nabla \times \mathbf{B}_a - \dot{\mathbf{E}}_a = g_{a\mathrm{EE}} \left(\mathbf{E}_0 \times \nabla a - \dot{a} \mathbf{B}_0 \right) + g_{a\mathrm{EM}} \left(\mathbf{B}_0 \times \nabla a + \dot{a} \mathbf{E}_0 \right) ,$$

$$\nabla \times \mathbf{E}_a + \dot{\mathbf{B}}_a = -g_{a\mathrm{MM}} \left(\mathbf{B}_0 \times \nabla a + \dot{a} \mathbf{E}_0 \right) - g_{a\mathrm{EM}} \left(\mathbf{E}_0 \times \nabla a - \dot{a} \mathbf{B}_0 \right) ,$$

$$\nabla \cdot \mathbf{B}_a = -g_{a\mathrm{MM}} \, \mathbf{E}_0 \cdot \nabla a + g_{a\mathrm{EM}} \, \mathbf{B}_0 \cdot \nabla a \, ,$$

$$\nabla \cdot \mathbf{E}_a = g_{a\mathrm{EE}} \, \mathbf{B}_0 \cdot \nabla a - g_{a\mathrm{EM}} \, \mathbf{E}_0 \cdot \nabla a \, ,$$

$$\left(\partial^2 + m_a^2 \right) a = - \left(g_{a\mathrm{EE}} - g_{a\mathrm{MM}} \right) \mathbf{E}_0 \cdot \mathbf{B}_0 + g_{a\mathrm{EM}} \left(\mathbf{E}_0^2 - \mathbf{B}_0^2 \right)$$

  - Here, $\mathbf{E}_a$ and $\mathbf{B}_a$ are axion-induced electric and magnetic fields, while $\mathbf{E}_0$ and $\mathbf{B}_0$ are background electric and magnetic fields created in experiments of astrophysical environments

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  - Electromagnetic couplings:

$$g_{a\mathrm{MM}} = \frac{\alpha_M}{2\pi f_a} \frac{M}{N} \qquad g_{a\mathrm{EM}} = \frac{\sqrt{\alpha \alpha_M}}{2\pi f_a} \frac{D}{N} \qquad g_{a\mathrm{EE}} = \frac{\alpha}{2\pi f_a} \left(\frac{E}{N} - 1.92 \right)$$

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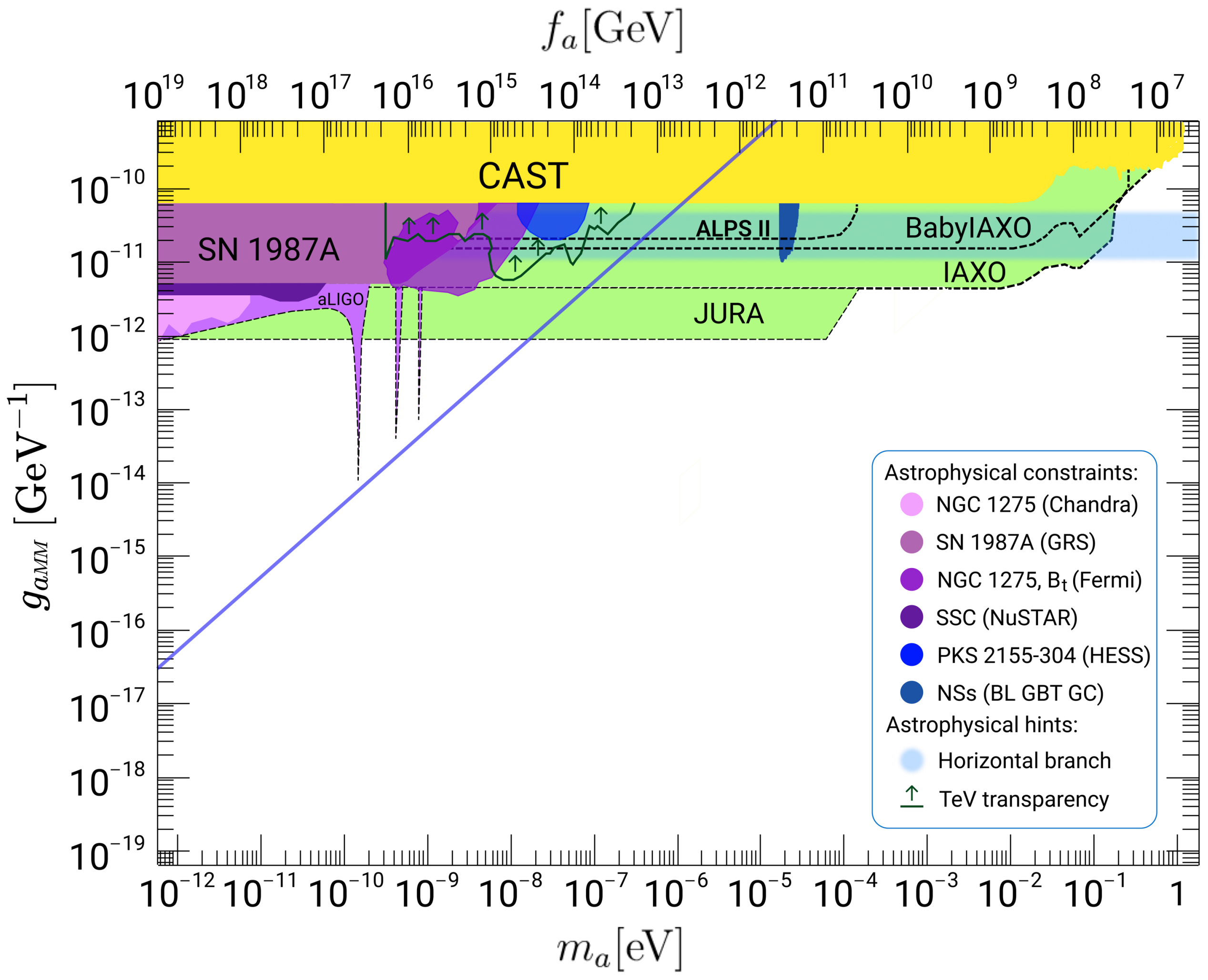
  - Huge hierarchy because of DSZ charge quantization condition:

$$\alpha \equiv e^2/4\pi \approx 1/137, \qquad \alpha_M \equiv g_0^2/4\pi = 9\pi/\alpha \approx 3.87\times 10^3$$

Monopole-Philic Axion Model

Phenomenological implications

- Conversion of relativistic axions into photons and back in transverse magnetic fields and solar axion emission through Primakoff effect or photon coalescence are all given by conventional expressions, but with $g_{a\gamma\gamma} \to g_{aMM}$



adapted from [Sokolov,AR 2205.02605]

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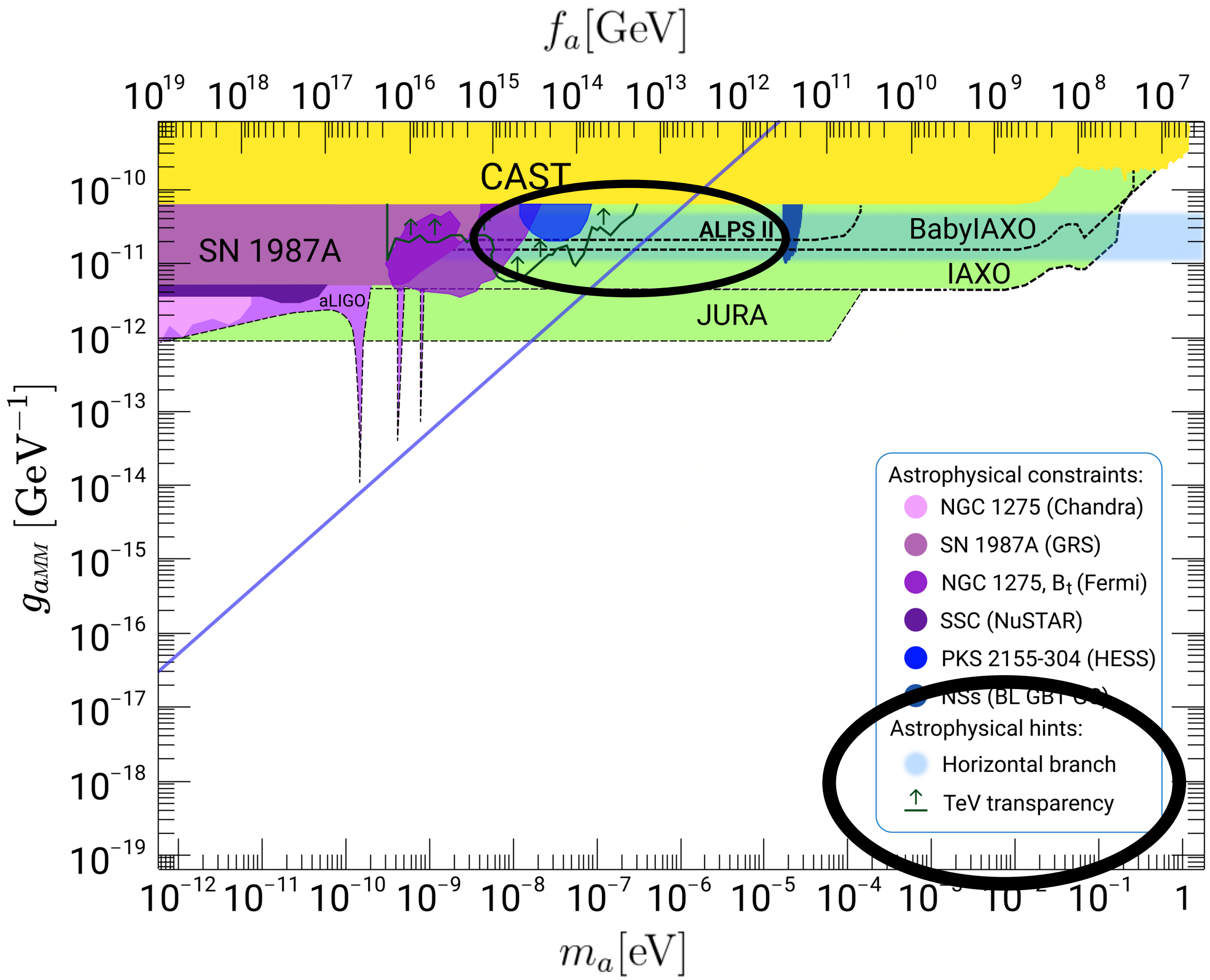
  - Excessive energy losses of horizontal branch stars in globular clusters
  - Transparency of universe for TeV gamma rays



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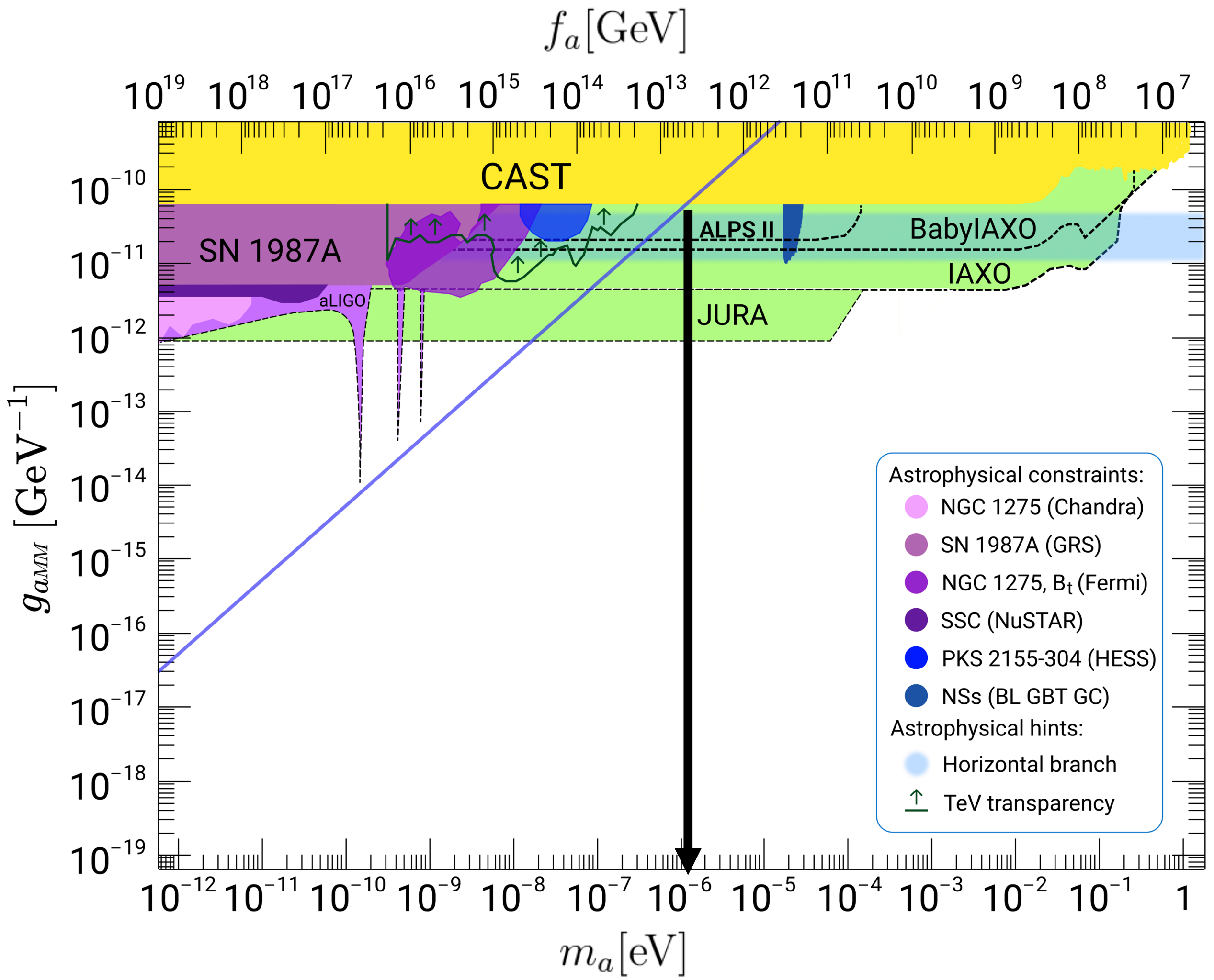

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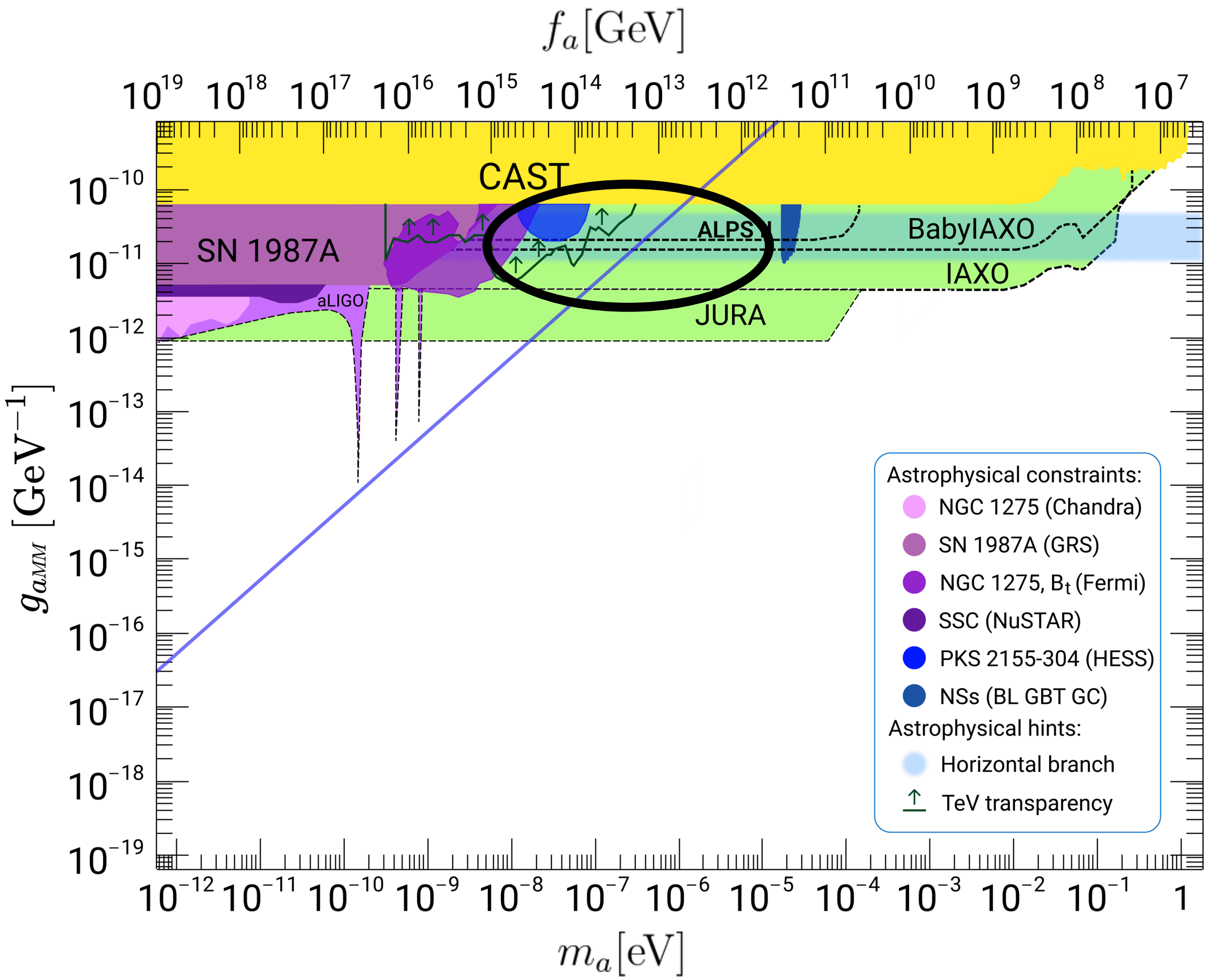
- ALPS II and (Baby)IAXO can probe this axion down to a mass of around 10^{-7} eV



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Monopole-Philic Axion Model

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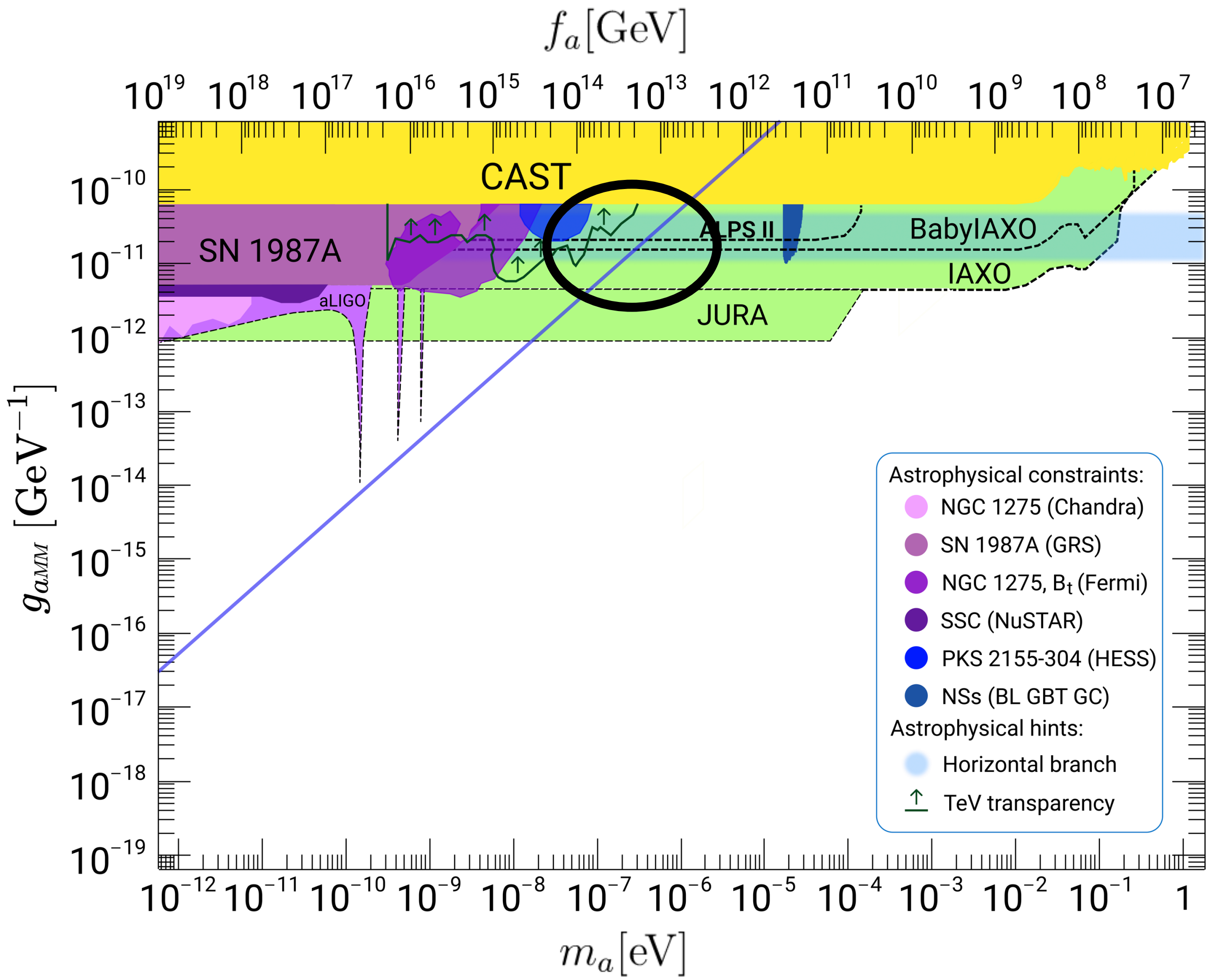
- An axion in the parameter range by ALPS II and (Baby)IAXO naturally 100% DM, cf.

$$\frac{\Omega_{\rm axion\ DM}}{\Omega_{\rm DM}} \approx \left(\frac{6\ \mu{\rm eV}}{m_a}\right)^{1.165} \theta^2_{\rm initial}$$



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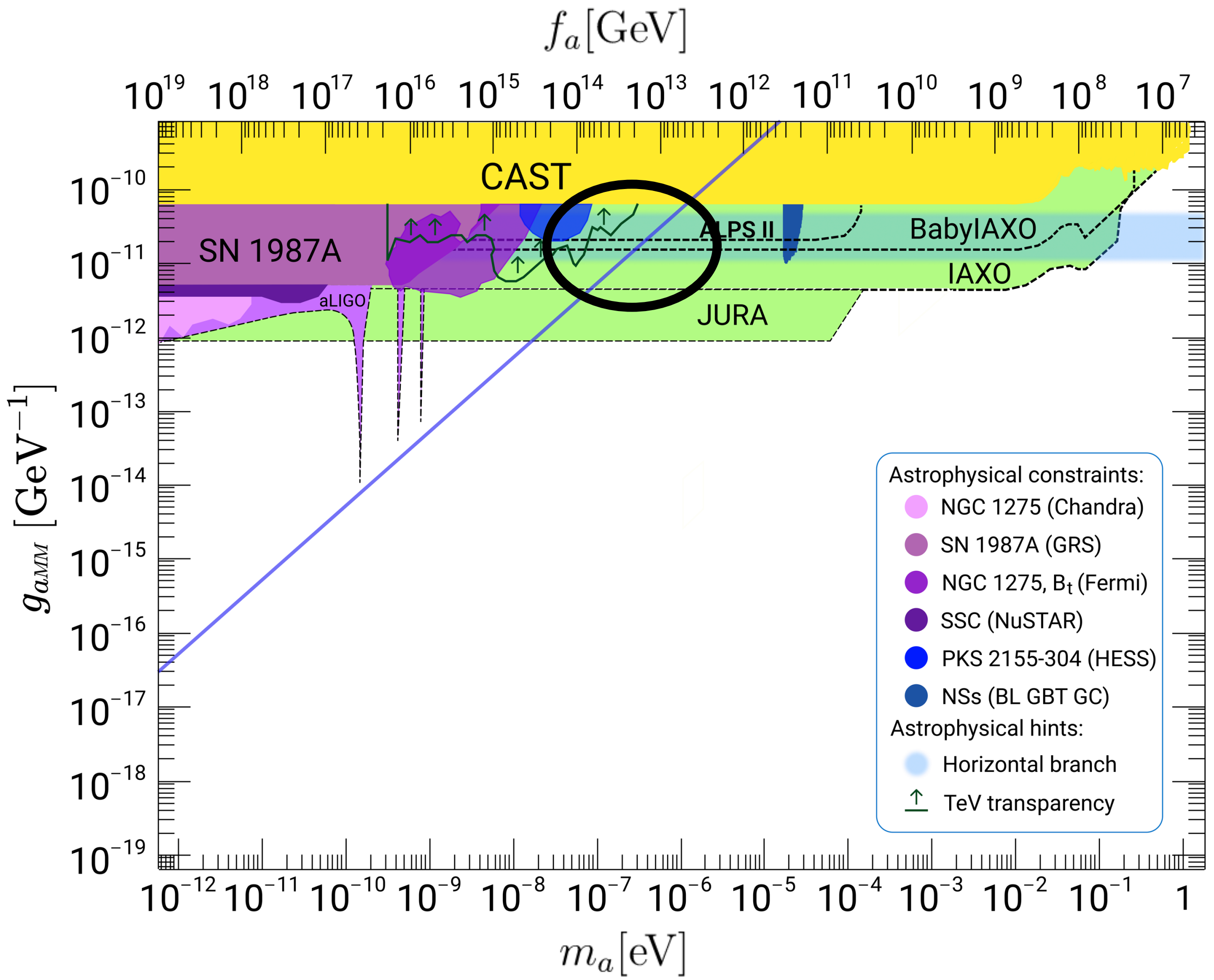

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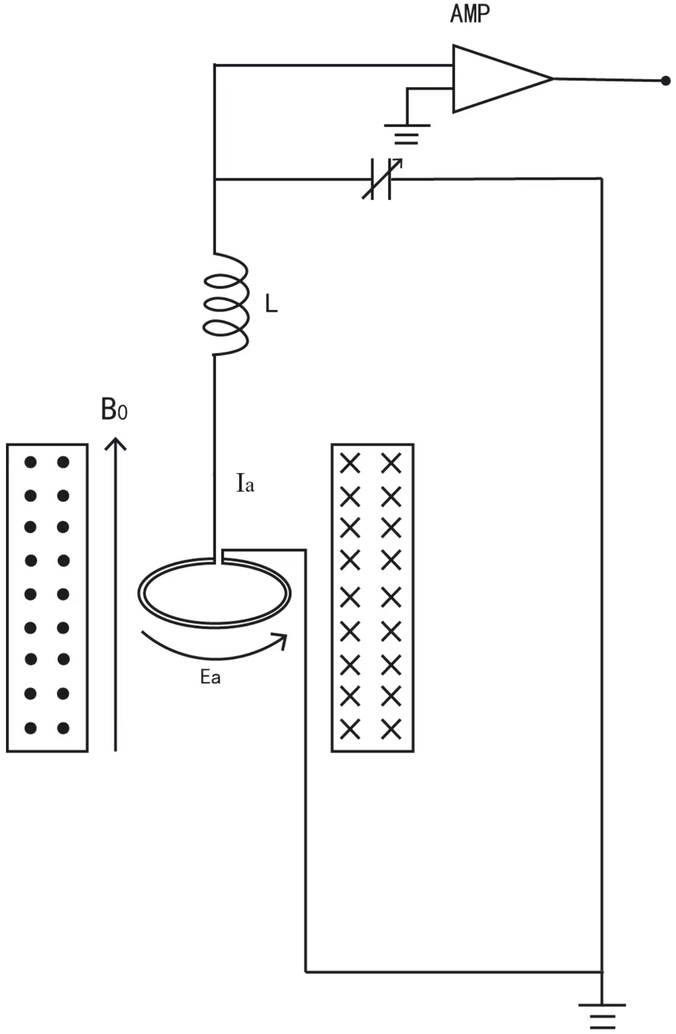

[Li,Zhang,Dai, `22]

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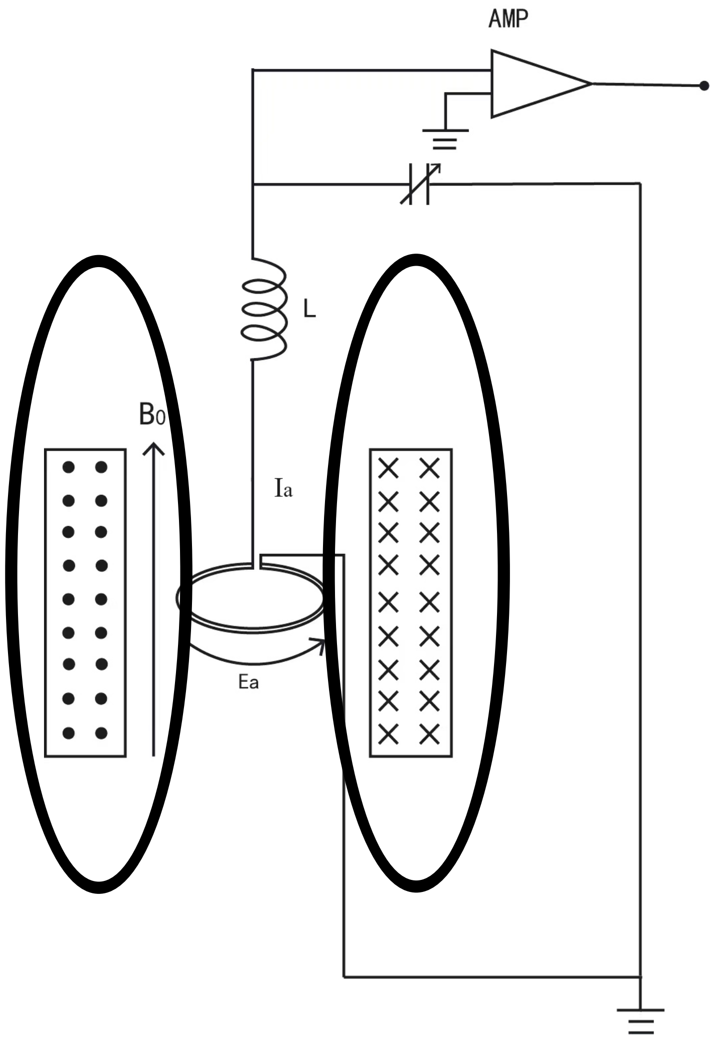

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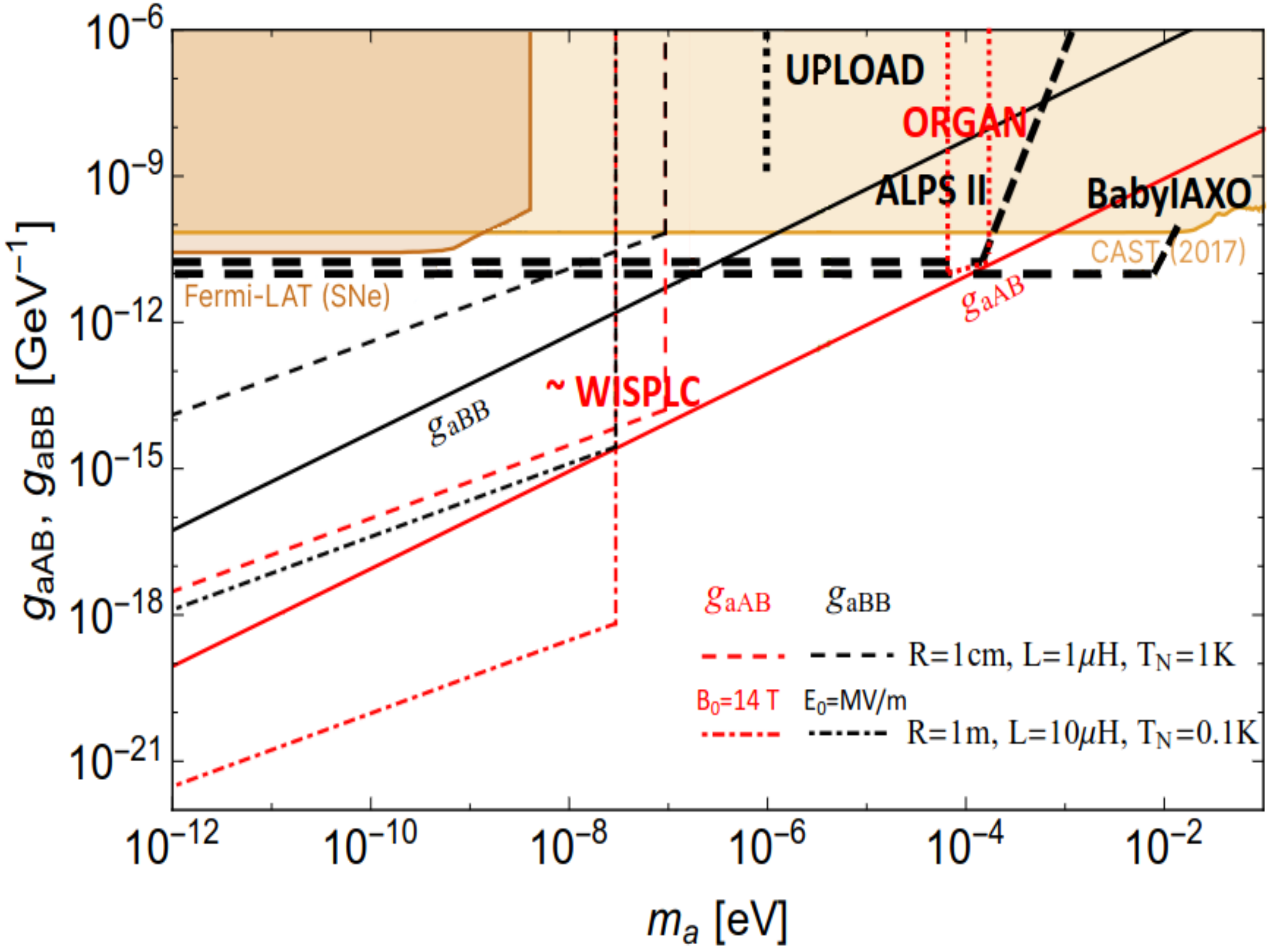


adapted from [Li,Zhang,Dai, `22]

Conclusions

- Proposed a new family of KSVZ-type axion models where the exotic quark carries magnetic charge
- These models have parametrically enhanced electromagnetic couplings
- These models can explain various “hints” with one stroke
  - Strong CP conservation
  - Quantisation of charge
  - Observed dark matter abundance
  - Anomalous TeV-transparency of the Universe
  - Cooling of horizontal branch stars in globular clusters
- For masses above 10^{-7} eV, these models can be probed decisively with ALPS II and (Baby)IAXO
- Lumped element search based low-mass haloscopes have to be re-designed to be sensitive to the enhanced couplings

Conclusions

- Proposed a new family of KSVZ-type axion models where the exotic quark carries magnetic charge
- These models have parametrically enhanced electromagnetic couplings
- These models can explain various “hints” with one stroke
  - Strong CP conservation
  - Quantisation of charge
  - Observed dark matter abundance
  - Anomalous TeV-transparency of the Universe
  - Cooling of horizontal branch stars in globular clusters
- For masses above 10^{-7} eV, these models can be probed decisively with ALPS II and (Baby)IAXO
- Lumped element search based low-mass haloscopes have to be re-designed to be sensitive to the enhanced couplings

Anton Sokolov will provide more details on the phenomenology of the monopole-philic axion later in this workshop!

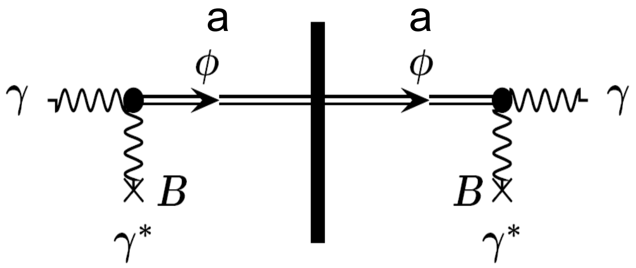
Monopole-Philic Axion Model

Phenomenological implications

- Axion-photon conversion in external field described by

$$\left(\partial^2 + m_a^2\right) a \;=\; -\left(g_{a\mathrm{EE}} - g_{a\mathrm{MM}}\right) \mathbf{E}\cdot\mathbf{B} + g_{a\mathrm{EM}} \left(\mathbf{E}^2 - \mathbf{B}^2\right)$$

- Constraints from axion-photon conversion stay approximately the same, with the identification $g_{a\gamma\gamma} \to g_{aMM}$
- LSW:



$$P(\gamma_{\|} \to a \to \gamma) \simeq 16\, \frac{(g_{a\mathrm{MM}}\omega B_0)^4}{m_a^8}\, \sin^4\left(\frac{m_a^2\, L_{B_0}}{4\omega}\right),$$

$$P(\gamma_{\perp} \to a \to \gamma) \simeq 16\, \frac{(g_{a\mathrm{EM}}\omega B_0)^2 (g_{a\mathrm{MM}}\omega B_0)^2}{m_a^8}\, \sin^4\left(\frac{m_a^2\, L_{B_0}}{4\omega}\right)$$

  - If signal detected in both channels, one can compare the theoretically derived ratio of CP-violating and CP-conserving couplings in a given model with the experiment

Monopole-Philic Axion Model

Quantum field theory of electrically and magnetically charged particles

- Idea of proof of Lorentz invariance of QEMD

[Brandt, Neri, Zwanziger `78]

  - For the proof of n-indendence (Lorentz invariance) of the point-particle theory it was essential to use the point structure of particle charge
  - The quantum field theoretic proof of n-indepence (and Lorentz invariance) is based on the functional integral formulation of QEMD
    - Generating functional of conserved-current Green's functions of gauge-invariant in terms of a functional integral over fermion and gauge fields

$$\mathcal{Z}[\tilde{a},\tilde{c}] \;=\; \mathcal{N}\int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}A\mathcal{D}C\,\exp\left\{i\left(S_Z[A+\tilde{a},C+\tilde{c}]+S_G[A+\tilde{a},C+\tilde{c}]+S_D[A+\tilde{a},C+\tilde{c},\psi,\bar{\psi}]\right)\right\}$$

    - Functional integral over fermions can be converted into series of integrals over closed point-particle trajectories
    - Remaining Gaussian integration over gauge fields can be performed

[Feynman `48; Schwinger 51]

    - n-dependent term in the effective action functional is proportional to $e_i g_j - e_j g_i$ and the number of times the trajectory of one particle intersects some oriented n_μ-dependent three-surface associated to the trajectory of another particle, which is simply an integer but for some exceptional trajectories that form a measure zero subset and can therefore be omitted in the integral over all trajectories
    - Requiring DSZ quantization condition $e_i g_j - e_j g_i = 2\pi m,\ m\in\mathbb{Z}$ for of all possible (i,j) pairs ensures that the n-dependent contribution to the action is always equal to $2\pi k,\ k\in\mathbb{Z}$, ensuring that $\mathcal{Z}[\tilde{a},\tilde{c}]$ is independent of n