Model-independent bubble wall velocities in local thermal equilibrium.

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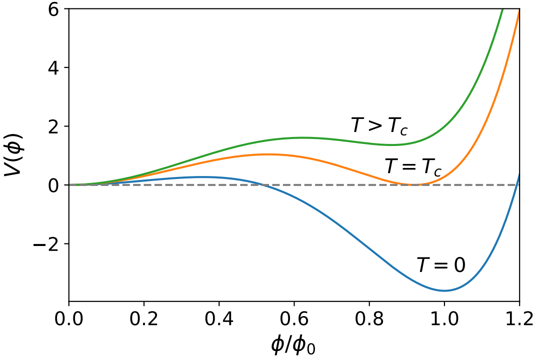
Outline

- First-order phase transitions: A brief review
- Wall velocity: General framework
- LTE approximation and model-independent fluid equations
- Results and discussion

First-order phase transitions: A brief review

First-order phase transitions

- FOPTs can happen when a potential has several local minima separated by a **barrier**,
- If the relative depth between the minima changes with T, the true vacuum state can change (at $T = T_c$) and cause a phase transition where some fields acquire a VEV,
- Because of the potential barrier, the phase transition does not happen at T_c, but at the nucleation temperature $T_n < T_c$.



Bubble nucleation

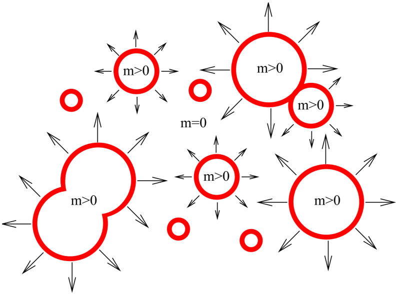
The tunneling rate is

$$\Gamma = Ae^{-S_3/T},$$

where S_3 is the O(3)-symmetric bounce action

$$S_3 = 4\pi \int dr\, r^2 \left[\frac{1}{2} \left(\frac{d\phi}{dr} \right)^2 + V_T \right]$$

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = \frac{\partial V_T}{\partial \phi}$$

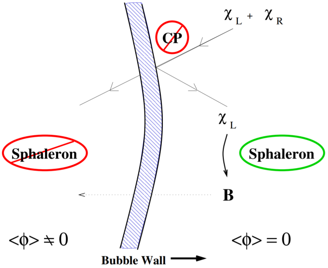


Cline (hep-ph/0609145)

Why are they interesting?

Baryogenesis:

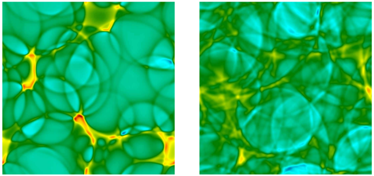
- FOPTs offer a mechanism to explain the observed baryon asymmetry of the Universe (e.g. electroweak baryogenesis),
- Baryogenesis requires a departure from equilibrium, which can be provided by the bubble wall.



Morrissey and Ramsey-Musolf (1206.2942).

Gravitational waves:

- When bubbles collide, spherical symmetry is broken and GWs can be generated,
- In the main mechanism, the energy in the wall is converted into sound waves which are then dissipated as GWs,
- These GWs could be detected by future space-based experiments such as LISA.

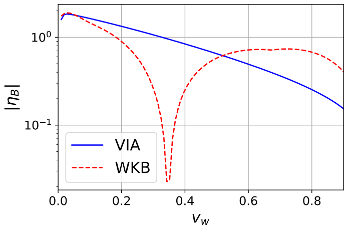


Hindmarsh, Huber, Rummukainen and Weir (1304.2433)

Wall velocity: General framework

Why is ξ_w important?

Baryon asymmetry:



Cline and Laurent (2108.04249)

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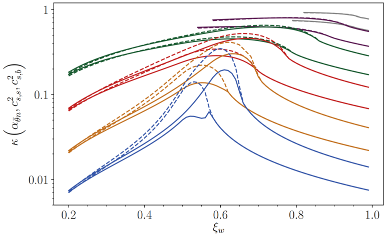


Giese *et al.* (2010.09744)

Fluid equations

The evolution of the bubble wall can be derived from the conservation of the energy-momentum tensor

$$0 = \partial_\mu T^{\mu\nu} = \partial_\mu (T^{\mu\nu}_\phi + T^{\mu\nu}_{\text{plasma}}),$$

where

$$\begin{aligned} T^{\mu\nu}_\phi &= \partial^\mu \phi \, \partial^\nu \phi - \eta^{\mu\nu} \left[\frac{1}{2} \partial_\alpha \phi \, \partial^\alpha \phi - V_0(\phi) \right], \\ T^{\mu\nu}_{\text{plasma}} &= \sum_i \int \frac{d^3 p}{(2\pi)^3 E_i} p^\mu p^\nu f_i(\mathbf{x}, \mathbf{p}). \end{aligned}$$

The distribution functions are expanded in equilibrium and out-of-equilibrium parts:

$$f_i = f_{\text{eq}} + \delta f_i,$$

$$f_{\text{eq}} = \left[\exp\left(\frac{1}{T(\mathbf{x})} p_\mu u^\mu_{\text{pl}}(\mathbf{x}) \right) \pm 1 \right]^{-1}.$$

$T^{\mu\nu}_{\text{plasma}}$ then becomes

$$T^{\mu\nu}_{\text{plasma}} = w u^\mu_{\text{pl}} u^\nu_{\text{pl}} - p_T \eta^{\mu\nu} + T^{\mu\nu}_{\text{out}}(\delta f),$$

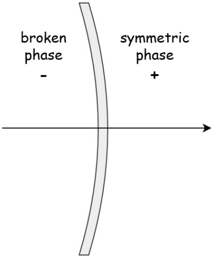
Matching equations

Assuming a thin wall compared to the bubble radius, the fluid equations can be directly integrated:

$$\Delta\left(w\gamma_{\rm pl}^2 v_{\rm pl}\right) = 0,$$
$$\Delta\left(w\gamma_{\rm pl}^2 v_{\rm pl}^2 + p\right) = 0.$$

where

$$\begin{aligned}
\Delta X &= X_+ - X_-, \\
p &= p_T - V_0 \\
&= \pm T \sum_i \int \frac{d^3p}{(2\pi)^3} \log[1 \pm \exp(-E_i/T)] - V_0, \\
w &= T\frac{\partial p}{\partial T}.
\end{aligned}$$



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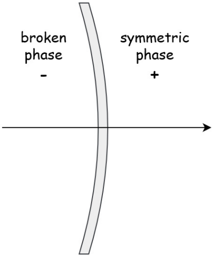

In the simplest case, we know v_+ and T_+, but v_-, ξ_w and T_- are unknown.

We need one more equation!

3 types of solutions

Depending on the wall velocity and the strength of the phase transition, 3 types of solutions are possible:

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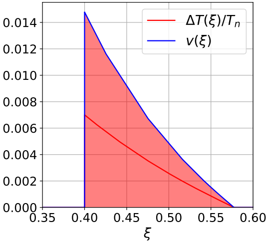
Deflagration:

- Subsonic: $\xi_w < c_s$
- $v_- = \xi_w$ and $T_+^{\text{sw}} = T_n$
- Shock wave



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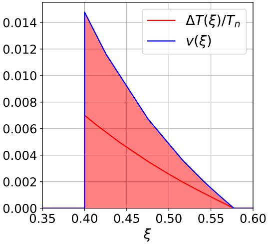

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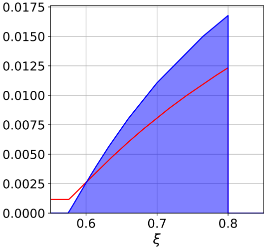
Detonation:

- Supersonic: $\xi_w \geq \xi_J > c_s$
- $v_+ = \xi_w$ and $T_+ = T_n$
- Rarefaction wave



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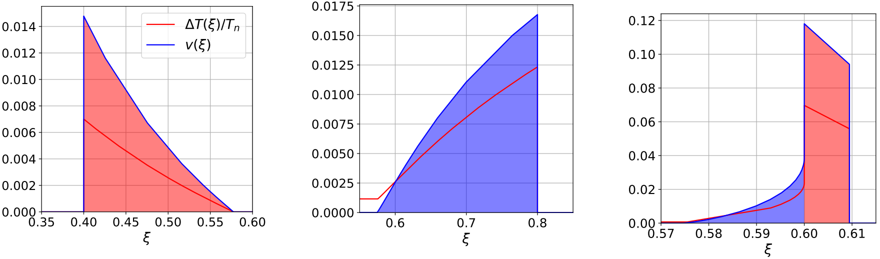
Hybrid:

- $c_s < \xi_w < \xi_J$
- $v_- = c_s$ and $T_+^{\rm sw} = T_n$
- Rarefaction and shock waves



$\xi_w = 0.4$ ($\xi = r/t$)

$\xi_w = 0.8$

$\xi_w = 0.6$

Equation of motion:

$$\partial^2\phi + \frac{\partial V_T(\phi)}{\partial\phi} + \frac{1}{2}\sum_i \frac{\partial(m_i^2)}{\partial\phi}\int\frac{d^3p}{(2\pi)^3E_i}\delta f_i(\mathbf{x},\mathbf{p}) = 0$$

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Boltzmann equation:

$$\left(p_z\partial_z - \frac{1}{2}\partial_z(m_i^2)\partial_{p_z}\right)f_i(z,\mathbf{p}) = -\mathcal{C}_i[f]$$

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Collision integral:

$$\mathcal{C}_i[f] = \sum_{j,m,n}\frac{1}{2N_i}\int\frac{d^3p_jd^3p_md^3p_n}{(2\pi)^5 8E_jE_mE_n}\left|\mathcal{M}_{ij\to mn}\right|^2\delta^{(4)}(p_i+p_j-p_m-p_n)\mathcal{P}[f],$$

$$\mathcal{P}[f] = f_if_j(1\pm f_m)(1\pm f_n) - (i,j\leftrightarrow m,n).$$

LTE approximation and model-independent fluid equations

Entropy conservation in LTE

Let's assume $\delta f_i = 0$.
We can contract the divergence of $T^{\mu\nu}$ with u^ν to get

$$0 = u_\nu \partial_\mu T^{\mu\nu} = u^\nu \partial_\nu \phi \left[\partial^2 \phi + \frac{\partial V_T}{\partial \phi} \right] + T \partial_\nu S^\nu,$$

with the entropy flux

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We now have 3 matching equations

$$\Delta(w\gamma^2 v) = \Delta(w\gamma^2 v^2 + p) = \Delta(\gamma T) = 0.$$

for the 3 unknown quantities v_-, T_- and ξ_w.

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- Set an upper bound on the true ξ_w,
- There are large uncertainties on $\mathcal{C}[f]$,
- $\mathcal{C}[f]$ must be linearized.

Template model

It is common to approximate the equation of state (EOS) with the bag EOS, where one assumes $c_s^2 = 1/3$ and $p \sim T^4$.

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It is straightforward to generalize this EOS to get any constant speed of sound. One then obtains the so-called **template model**:

$$p_+(T) = \frac{1}{3}a_+T^\mu - \epsilon, \quad p_-(T) = \frac{1}{3}a_-T^\nu,$$

with

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Any particle physics model can be fitted to the template model, as long as the plasma has approximately uniform speeds of sound:

$$\frac{dc_+(r)}{dr} \approx \frac{dc_-(r)}{dr} \approx 0.$$

Model-independent fluid equations

We define

$$\bar{\theta} = e - \frac{p}{c_-^2}, \quad \alpha(T) = \frac{\bar{\theta}_+(T) - \bar{\theta}_-(T)}{3w_+(T)}, \quad \Psi(T) = \frac{w_-(T)}{w_+(T)}.$$

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One can eliminate T_- from the matching equations:

$$\frac{v_+}{v_-} = \frac{v_+v_-(\nu-1) - 1 + 3\alpha(T_+)}{v_+v_-(\nu-1) - 1 + 3v_+v_-\alpha(T_+)},$$

$$v_+v_- = \frac{1 - 3\alpha(T_+) - \left(\frac{\gamma_+}{\gamma_-}\right)^\nu \Psi(T_+)}{3\nu\alpha(T_+)}[1 - (\nu-1)v_+v_-].$$

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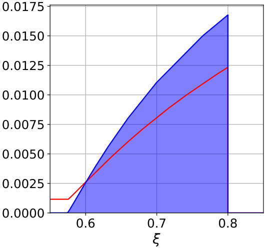
The system can be described in terms of **4 parameters** evaluated at T_n:

$$\boxed{\alpha(T_n), \quad \Psi(T_n), \quad \mu, \quad \nu.}$$

Results and discussion

Detonations

$$v_+ = \xi_w > c_- \text{ and } T_+ = T_n$$



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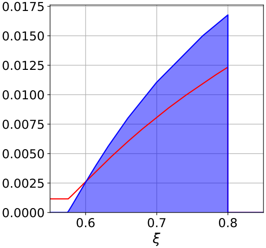
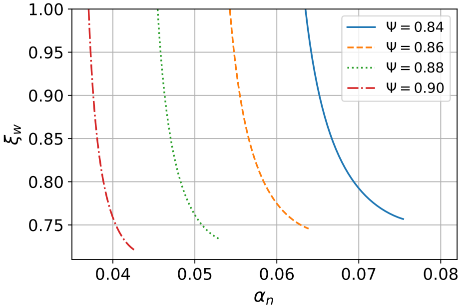
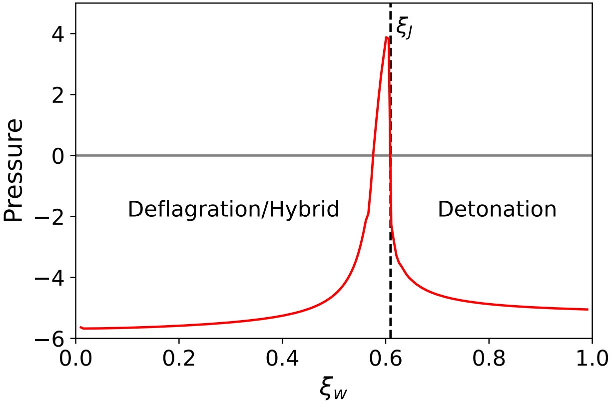




Are LTE detonations physical?



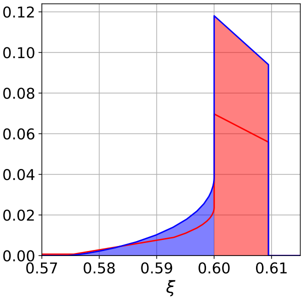
$v_- = \min(\xi_w, c_-)$ and $T_+^{\rm sh} = T_n$

One must integrate the fluid equations through the shock wave:

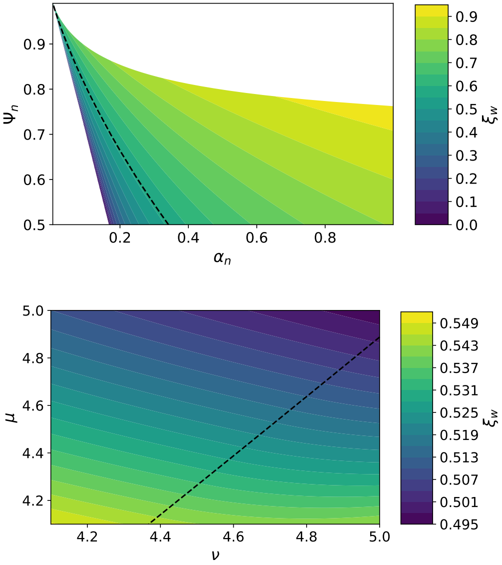
$$\partial_\xi v = \frac{2v}{\xi\gamma^2(1-v\xi)}\left[\frac{\mu^2(\xi,v)}{c^2}-1\right]^{-1}$$

$$\partial_\xi w = w\left(1+\frac{1}{c^2}\right)\gamma^2\mu(\xi,v)\partial_\xi v$$

with $\xi = r/t$ and $\mu(\xi,v) = \frac{\xi-v}{1-\xi v}$.



Deflagrations/Hybrids



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One can show that

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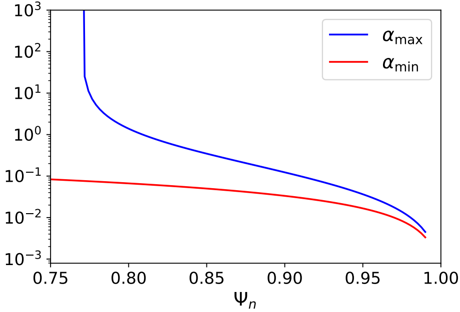
No bubble nucleation can happen below that bound.

The upper bound $\alpha_{\rm max}$ is reached when $\xi_w = \xi_{\rm sh} = \xi_J$.



For $\mu = \nu = 4$, $\alpha_{\rm max} \to \infty$ when $\Psi_n \to \frac{4}{3\sqrt{3}} \approx 0.77$.

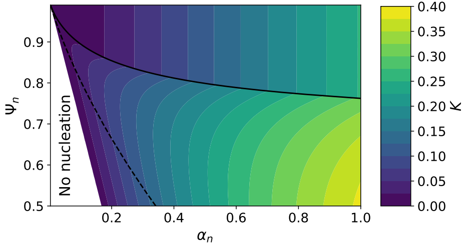
Kinetic energy fraction

Let's define

$$K = \frac{\rho_{\rm fl}}{e_n}, \quad \rho_{\rm fl} = \frac{3}{\xi_w^3}\int d\xi\, \xi^2 v^2 \gamma^2 w$$

This quantity is important to determine the GW spectrum:

$$\Omega_{\rm gw} \sim K^{3/2} \text{ or } K^2.$$



Summary

- The LTE approximation can significantly simplify the fluid equations used to compute ξ_w,
- Even in the case where LTE is badly violated, it can be useful as it offers an upper bound on ξ_w,
- Using the template model, the fluid equations can be written in a model-independent way which only depends on 4 free parameters,
- For $\Psi_n \lesssim 0.75$, there is always a deflagration/hybrid solution, no matter how strong the phase transition is,
- For these models, baryogenesis and GW production could be highly efficient.

Thank you!