Axion Mass from Magnetic Monopole Loops

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Based on:

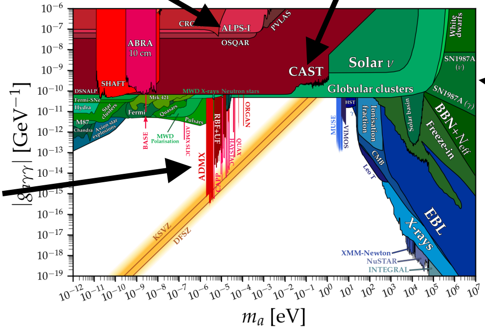
[arXiv: 2105.09950, PRL 127 (2021) 13, 131602 with J. Fan, M. Reece, J. Stout]

Searching for Axions

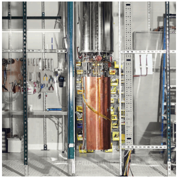
Many experiments are searching for axions, which depend on properties like mass and different couplings. The axion-photon coupling is often easiest to probe.

[ALPS-I: DESY]

[CAST: CERN]



[SN1987a: ESO]



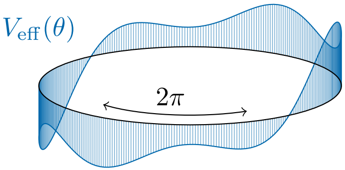
[ADMX: Scientific American]

[https://github.com/cajohare/AxionLimits]

The Typical Story: How the Axion gets its mass

- The QCD axion gets a potential from the $\theta G\tilde{G}$ coupling. It can be computed with:
  - Instantons at high temp
  - Chiral lagrangian at low temp

- These effects don't exist for axions that only couple to abelian gauge fields. Can these axions be massless? If not, what is their mass?



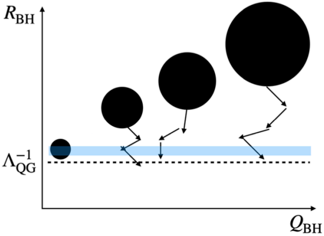
What quantum gravity tells us

Quantum Gravity tells us axions should not be massless. Massless axions would have a continuous shift symmetry, and we do not expect global symmetries in QG.

[Banks and Seiberg: 1011.5120]

But QG also gives us something else: that we expect monopole states to exist ("completeness hypothesis")

[Polchinski: hep-th/0304042]



[Image: Matt Reece TASI Notes]

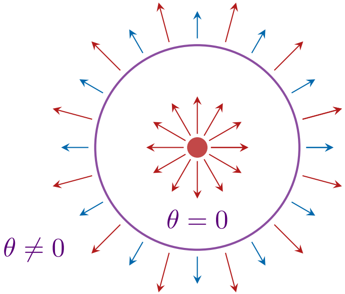
The Short Version

When coupled to abelian gauge fields, the axion gets a potential from loops of magnetic monopoles.

1. In the presence of a θ angle, monopoles acquire electric charge and become dyons.
2. The electric charge changes the dyon masses in a θ-dependent way
3. Axions get a potential when we integrate out the dyons.

The Witten Effect

Witten Effect: Monopoles in the presence of a non-zero θ acquire electric charge



$$\nabla \cdot E = -\frac{\alpha}{\pi}(\nabla\theta \cdot B)$$

$$q_E = -\frac{\alpha}{\pi} q_M \theta$$

- Put a magnetic monopole inside a $\theta = 0$ region, surrounded by $\theta \neq 0$
- Monopole magnetic field induces an electric field when θ changes
- Take size $\to 0$ limit because electric charge is independent of the inner region size

[Wilczek: PRL 58 (1987) 1799]
[P. Sikivie, Phys. Lett. B137 (1984) 353]
[Review: David Tong gauge theory lectures]

θ-dependent Dyon Masses

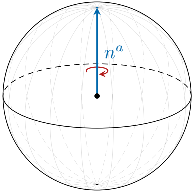
The electric charge changes the dyon masses in a θ-dependent way

This is a general statement, but it is easiest to see in an example: the 't Hooft Polyakov monopole in the Georgi-Glashow model.

Field Content:

- SU(2) Gauge Field A_μ^a
- Scalar triplet ϕ^a
- $V(\phi) = \lambda(\phi^a\phi^a - v^2)^2$

Admits a Monopole Soln for ϕ^a, A_i^a
because $\pi_2(SU(2)/U(1)) = \mathbb{Z}$

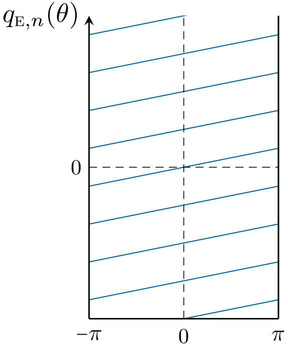


$$E = \int d^3x \Big[\frac{1}{2}\Big(\frac{1}{g}B_i^a - \mathscr{D}_i\phi^a\Big)\Big(\frac{1}{g}B_i^a - \mathscr{D}_i\phi^a\Big) + \underbrace{\frac{1}{g}B_i^a\mathscr{D}_i\phi^a}_{M_M = vQ_M}\Big]$$

[See Shifman Book for review]

θ-dependent Dyon Masses

- The solution can be quantized by studying its collective coordinates, with one for each broken symmetry
- There are four collective coordinates: three translations $\vec{x}_0$ and $\sigma \sim \sigma + 2\pi$ from the unbroken U(1).



$$S = \int dt \left[\frac{1}{2} m_M \dot{x}^2 + \frac{1}{2} \frac{m_M}{m_W^2} \dot{\sigma}^2 + \frac{\theta}{2\pi} \dot{\sigma} \right]$$

Quantize $\downarrow$

$$q_{E,n}(\theta) = g\left(n - \frac{\theta}{2\pi}\right)$$

$$M_M = v Q_M$$

$$E_{D,n} = \frac{1}{2} \frac{m_W^2}{m_M} \left(n - \frac{\theta}{2\pi}\right)^2 \sim q_{E,n}^2(\theta)/r_M$$

[See Shifman Book for review]

The Effective Potential

In the worldline formalism, the effective potential is

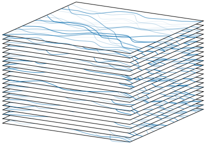
$$V_{\text{eff}}(\theta) = -\lim_{V\to\infty} \frac{1}{V} \log Z(\theta)$$

Path Integral:

$$Z(\theta) = \sum_{\text{worldlines}} \int D(\text{fields})\, e^{-S_E[fields, worldlines, \theta]}$$

Spacetime Volume

where we sum over all periodic monopole world lines.



For weakly interacting monopoles, we restrict to disconnected paths

$$V_{\text{eff}}(\theta) = -\lim_{V\to\infty} \frac{1}{V} Z_{S_1}(\theta)$$

transition amplitude for trajectories that return to the same point they started at

$$Z(\theta) = \sum_{n=0}^{\infty} \frac{1}{n!} (Z_{S_1})^n$$

The Dyon Picture

The easiest way to get $Z_{S_1}(\theta)$: Integrate out the tower of heavy dyons.

In the worldline picture, the free particle propagator is

$$\langle x'|x\rangle_\tau = \frac{1}{2(2\pi\tau)^2}\exp\left(\frac{-1}{2\tau}(x-x')^2 - \frac{m^2\tau}{2}\right)$$

Invariant Length τ
(Schwinger proper time)

Sum over circular paths:

Dyon Mass

$$m_{D,n}^2 = m_M^2 + m_\Delta^2\left(n - \frac{\theta}{2\pi}\right)^2$$

$$Z_{S_1} = \sum_{n\in\mathbb{Z}}\int_0^\infty \frac{d\tau}{2\tau}\frac{1}{2(2\pi\tau)^2}\exp\left(-\frac{m_{D,n}^2\tau}{2}\right)$$

Plugging this in and Poisson resumming gives $V_{\text{eff}}(\theta)$

$$\sum_{n\in\mathbb{Z}} e^{-\frac{1}{2}m_\Delta^2\tau\left(n-\frac{\theta}{2\pi}\right)^2} = \sum_{\ell\in\mathbb{Z}}\sqrt{\frac{2\pi}{m_\Delta^2\tau}}\exp\left(-\frac{2\pi^2\ell^2}{m_\Delta^2\tau} + i\,\ell\,\theta\right)$$

The Calculation: Final Result

For a general monopole,

$S = \frac{2\pi m_M}{m_\Delta}$

$$V_{eff}(\theta) = -\sum_{\ell=1}^{\infty} \frac{m_\Delta^4}{128\pi^6} e^{-\ell S} \cos(\ell\theta) \left(\frac{3}{\ell^5} + \frac{3S}{\ell^4} + \frac{S^2}{\ell^5} \right)$$

In the ’t Hooft Polyakov case, $m_M = 4\pi v/g$ and $m_\Delta = g v$, so

$$S = \frac{8\pi^2}{g^2}$$

This is the same as the BPST Yang Mills instanton action, obtained with only low energy information!

The Winding Picture

There is another way to get $Z_{S_1}(\theta)$: sum over winding around the coordinate σ, which acts like an extra compact spatial dimension.

$$S = m_M \int d\lambda \sqrt{\dot{x}_\mu \dot{x}^\mu + m_\Delta^{-2} \dot{\sigma}^2} + \frac{\theta}{2\pi} \int d\lambda \, \dot{\sigma} + \ldots$$

Then the free particle propagator sums over windings

$$\langle x, \sigma \,|\, x', \sigma' \rangle_\tau = \sum_{\ell \in \mathbb{Z}} e^{i\ell\theta} \langle x, \sigma \,|\, s', \sigma' + 2\pi\ell \rangle_{\tau,nl}$$

$$\langle x, \sigma \,|\, 0,0 \rangle_{\tau,nl} = \frac{1}{2(2\pi\tau)^{5/2}} \exp\left(-\frac{1}{2\tau}[x^2 + m_\Delta^{-2}\sigma^2] - \frac{1}{2} m_M^2 \tau \right)$$

Sum over connected paths:

$$V_{\text{eff}}(\theta) = -\frac{1}{V} \int_0^\infty \frac{d\tau}{2\tau} \int d^4x \, d\sigma \langle x, \sigma \,|\, x, \sigma \rangle_\tau$$

Recap: The Big Picture

When coupled to abelian gauge fields, the axion gets a potential from magnetic monopole loops.

Monopole states exist (from QG)

↓

In the presence of a θ angle, monopoles acquire electric charge and become dyons.

↗

The electric charge changes the dyon masses in a θ -dependent way

↓

Axions get a potential when we integrate out the dyons.

A Distinction from Previous Work

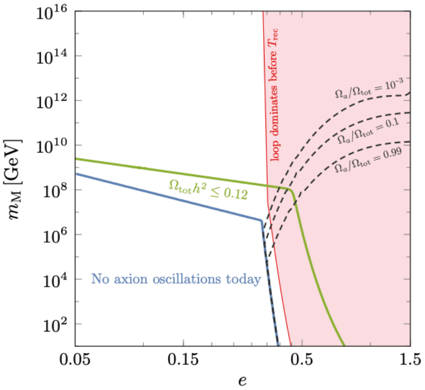
This is ***different*** from the potential that is generated by the monopole and anti-monopole plasma.

[Fischler & Preskill `83, Kawasaki et al: 1511.05030 & 1708.06047, Nomura et al: 1511.06347]

In that case, physical monopoles are generated in the early universe, for example through the Kibble Zurek mechanism.

Our potential is from ***virtual*** monopole/dyon loops.

$U(1)_{dark}$ Hidden Sector Model



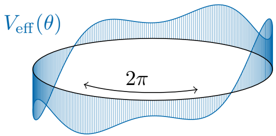
red region = region of phase space where our contribution dominates

Consider a model with a hidden gauged $U(1)_{dark}$ and a spontaneously broken $U(1)_{PQ}$. Then $m_a(T) = m_a^{loop} + m_a^{plasma}(T)$:

- m_a^{loop} is the contribution from virtual monopole loops (this talk)
- $m_a^{plasma}(T)$ is the temperature dependent contribution from the monopole/anti-monopole plasma (previously known)

Summary

The axion's compact field space constrains its coupling to topological defects and can create qualitatively new effects.



Here, we show this leads to a new contribution to the axion potential for axions coupled to abelian gauge fields, which comes from heavy dyons generated by the Witten effect.

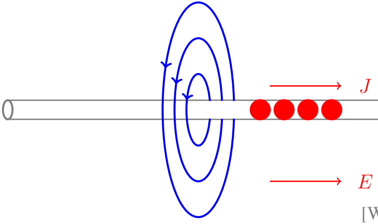
Next Steps:

- Adding light fermions?
- Is the potential actually dominated by single monopole loops? What about long-range coulomb interactions?
- Can this be formally connected to the instanton potential in the UV?

Preview: Axion String Zero Modes

Another example of how the axion's compact field space leads to new effects when coupling to topological defects is superconducting axion strings.

In the background of an axion string, both bosons and fermions can have trapped massless modes on the string, which can lead to superconductivity (a current that grows with imposed electric current).

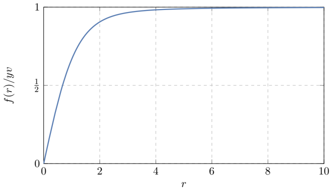


[Witten, Nuclear Physics B 1985]
[Fukuda et al, 2010.02763]
[Callan & Harvey, Nuclear Physics B, 1985]

Preview: Axion String Zero Modes

It is possible to show massless modes explicitly.

However, a common intuitive explanation is that bulk particles become massless at the core of the string.



We show that this is not necessary by studying a massive Dirac fermion. There can be zero modes even when bulk particles remain massive everywhere if the explicit PQ breaking isn't too large.

This could be useful for DFSZ axion strings.

[2306.XXXXX with J. Stout, S. Homiller, H. Bagherian]

Back Up Slides

The Witten Effect: Another Derivation

In QED without an axion and away from charged objects and currents, the magnetic dual gauge field is defined by

$$dA_M \sim \star\, dA$$

This works because $d(\star dA) = 0$ away from charges and currents.

Once you add an axion, this fails because $d(\star dA) \sim d\theta \wedge dA$. Instead, define

$$dA_M = \star\, dA - \theta dA$$

since $d(\star dA - \theta dA) = 0$. Now when we shift $\theta \to \theta + 2\pi$, A_M shifts by A. This is the Witten effect.

[Reece, Heidenreich, Mcnamara, In Progress]

Higher Order Monopole Corrections

- The details of the model came into the calculation only through the mass

Coefficient of $\int d_A\sigma \wedge {}^*d_A\sigma$

$$m_{D,n}^2 = m_M^2 + m_\Delta^2\left(n - \frac{\theta}{2\pi}\right)^2, \; m_\Delta^2 = m_M/l_\sigma$$

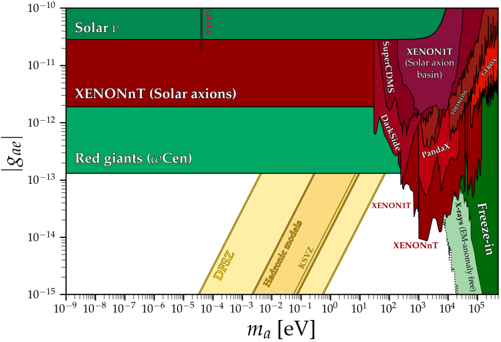
- Operators such as $(F_{\mu\nu}F^{\mu\nu})^2$,$(F_{\mu\nu}\tilde{F}^{\mu\nu})^2$, and higher powers of $(\partial_\mu\sigma + kA_\mu)$ can generate B^4, $(E\cdot B)^2$, B^2E^2, E^4 terms, which correct the dyon mass.

- The leading corrections are from the terms $c_{2j}E^{2j}/\Lambda^{4(j-1)}$:

$$m_{D,n}^2 = m_M^2 + m_\Delta^2\sum_{j=1}^{\infty} c_{2j}[e^2k^2/(16\pi^2(r_*\Lambda)^4)]^{j-1}\left(n - \frac{\theta}{2\pi}\right)^{2j}$$

These are small for $j > 1$.

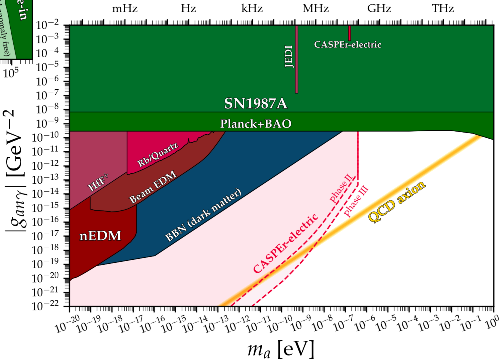
Limits on Other Axion Couplings



[https://github.com/cajohare/AxionLimits]

Axion Electron Coupling

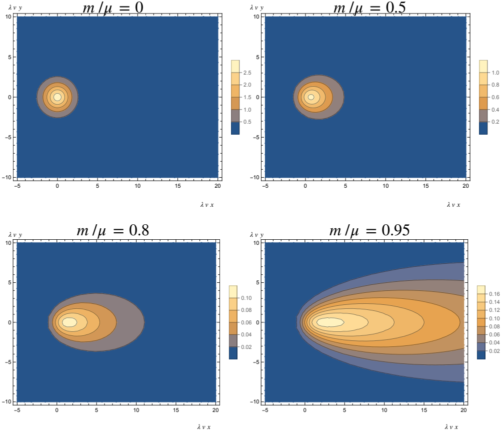
Neutron EDM Coupling



Axion Strings: Numerical Solutions

We can study a massive Dirac fermion in an axion string background by solving the Dirac equation.

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(i\not{D} - m)\psi + |\partial_\mu \Phi|^2 + y\overline{\psi}(\Phi_1 + i\gamma^5\Phi_2)\psi - \lambda(|\Phi|^2 - v^2)^2$$



- As we turn on a Dirac mass, the zero mode stretches out away from the string, until the critical point $m = \mu$.
- In the critical case ($m = \mu$), the zero modes live on a wedge in the x-y plane about $\varphi = 0$.
- Above the critical point, the zero modes vanish.

Anomaly Inflow: The Big Picture

In the presence of the axion string, the U(1) gauge theory is anomalous unless additional degrees of freedom localized to the string can cancel the anomaly. This is anomaly inflow.

To determine when we have anomaly inflow, we will

1. Integrate out the heavy fermions to get an EFT for the gauge field and axion
2. Do a gauge transformation in the EFT to find the anomaly.

We need zero modes when the path integral in the EFT is not gauge invariant.

[Callan & Harvey, Nuclear Physics B, 1985]

When is there an anomaly?

There is an anomaly when the path integral is not gauge invariant. Since

$$Z_\psi(\varphi) \sim \exp\left[\frac{i}{8\pi^2}\int \delta(\rho,\varphi) F\wedge F\right]$$

gauge invariance is determined by whether $\delta(\varphi) = \arg(m - \mu e^{i\varphi})$ is single or multivalued

If $\delta(\varphi)$ is single valued:

- $\frac{1}{8\pi^2}\int \delta(\varphi) F\wedge F$ is well-defined and gauge invariant

If $\delta(\varphi)$ is multi-valued:

- Need to integrate by parts for a well-defined answer
- Under the U(1) transformation $\delta_\Lambda \psi = ie\Lambda(x)\psi$, $\delta_\Lambda A_\mu = \partial_\mu \Lambda(x)$, the integral is not gauge invariant

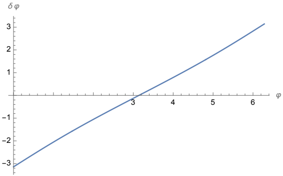
$$-\frac{1}{8\pi^2}\int d\delta\wedge A\wedge F \to \frac{1}{8\pi^2}\int d^2\delta(\varphi)\wedge \Lambda F \neq 0$$

since $d^2\delta(\varphi)\neq 0$ in the presence of an axion string.

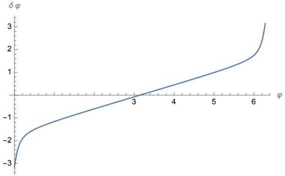
When is there an anomaly?

We see $\delta(\varphi) = \arg(m - \mu e^{i\varphi})$ is multi-valued for $m/\mu \leq 1$ and single valued for $m/\mu > 1$. Therefore, we get zero modes only for $m/\mu \leq 1$, but not for $m/\mu > 1$.

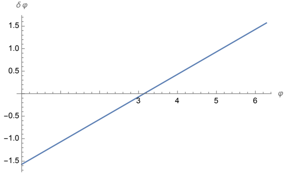
$m/\mu = 0.1$



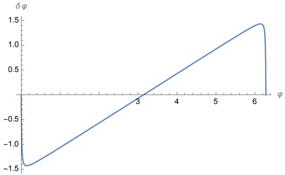
$m/\mu = 0.91$



$m/\mu = 1$



$m/\mu = 1.01$



$m/\mu = 2$