Amplitude evolution

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At the GGI Workshop
Precision LHC Physics
Firenze | 21 September 2023

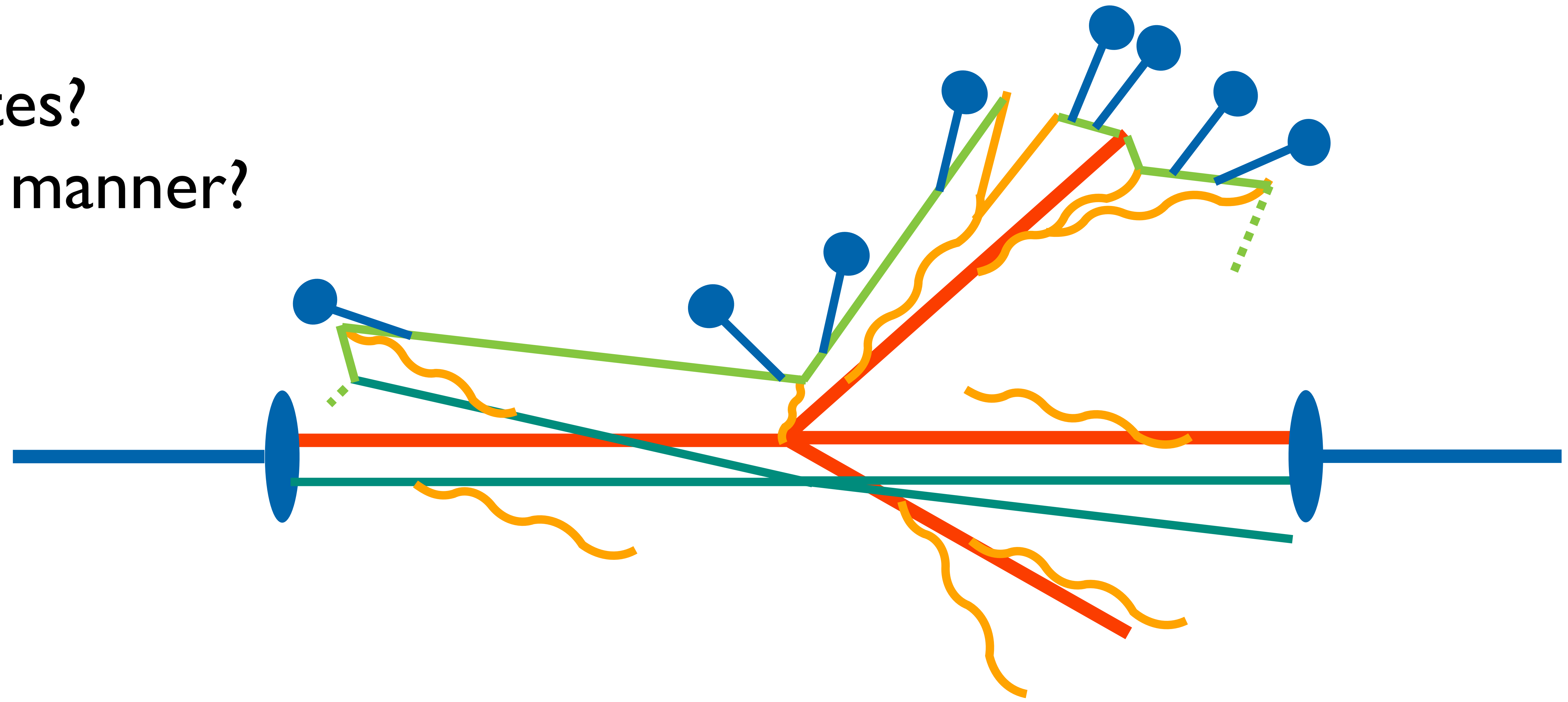
Current challenges

How do we accurately describe details of final states?
How do we quantify precision in a comprehensive manner?

Matching beyond NLO QCD?
Solve shower bottlenecks first?

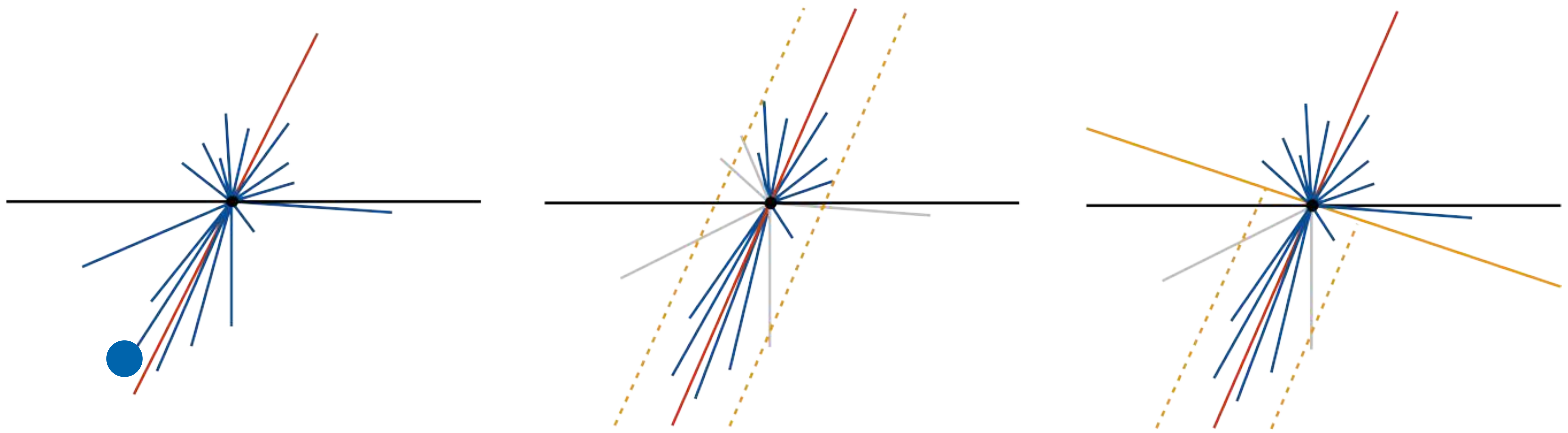
How to benchmark precision of QCD algorithms?
How to accurately include EW and QED?

How to constrain hadronization models?
What is their response to perturbative variations?



$$\mathrm{d}\sigma \sim \mathrm{L} \times \mathrm{d}\sigma_H(Q) \times \mathrm{PS}(Q \to \mu) \times \mathrm{MPI} \times \mathrm{Had}(\mu \to \Lambda) \times \ldots$$

Accuracy of Parton Showers



Fragmentation is fine if we get
collinear physics right.

Coherent branching parton showers

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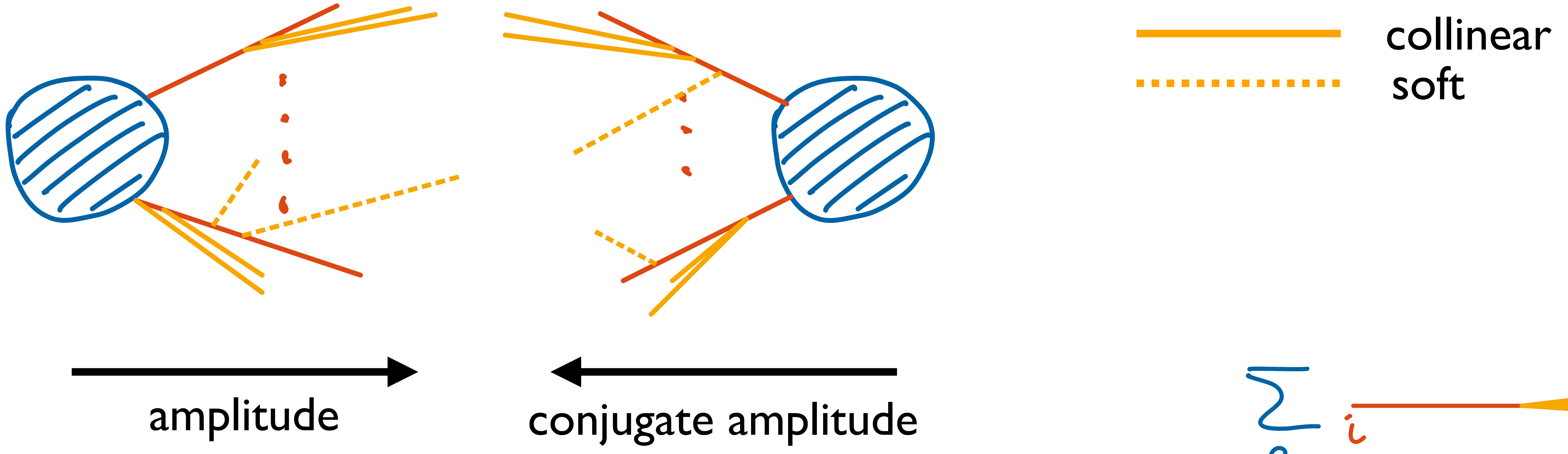
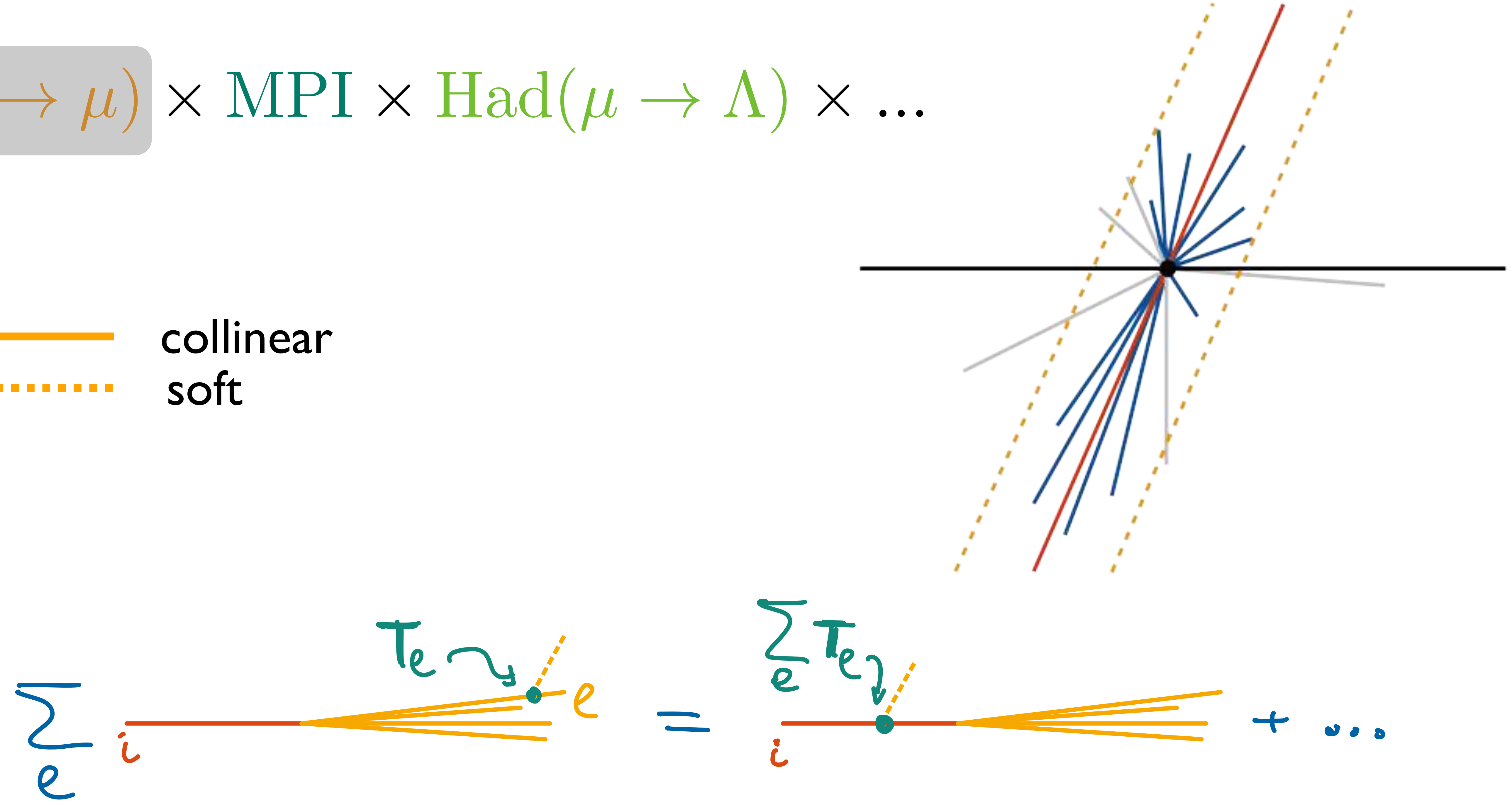
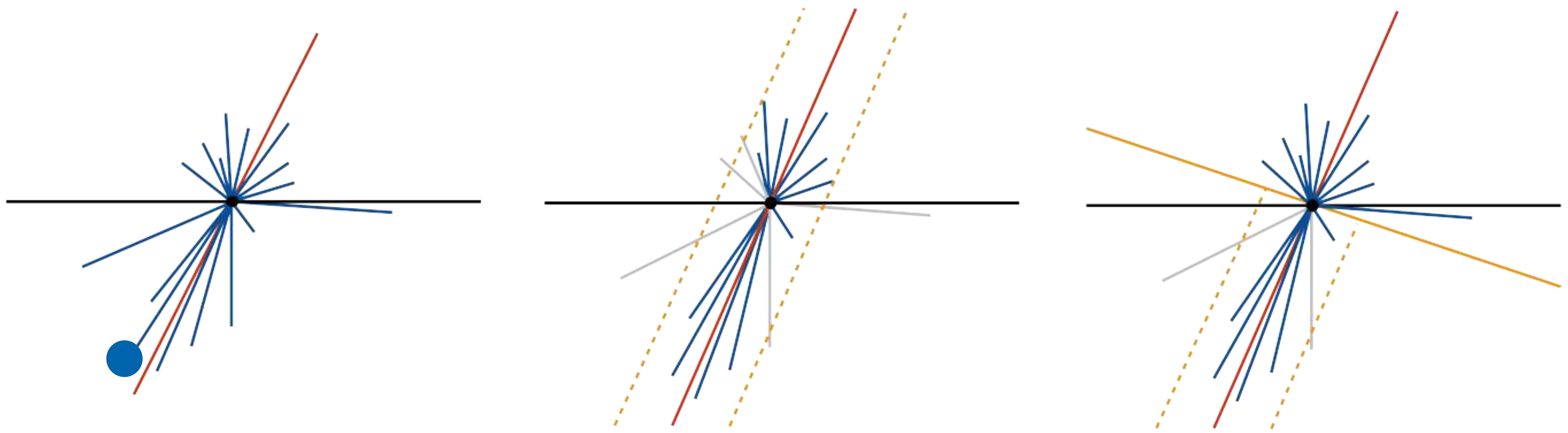




Move soft colour charges towards hard process and use angular ordering for azimuthal average around jet axes:

Accuracy of Parton Showers

[Catani, Trentadue, Webber, Marchesini ….]



Fragmentation is fine if we get collinear physics right.

Global event shapes from coherent branching — for two jets.

$$H(\alpha_s) \times \exp\left(Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots\right)$$

LL — qualitative

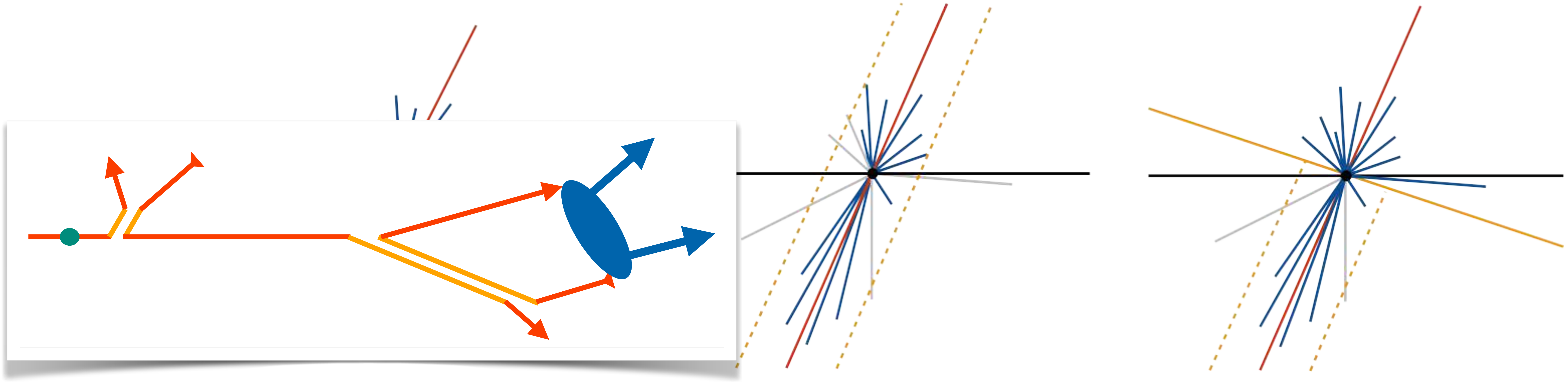
NLL — quantitative

NNLL — precision

$$\alpha_s L \sim 1$$

Accuracy of Parton Showers

[Catani, Trentadue, Webber, Marchesini ….]



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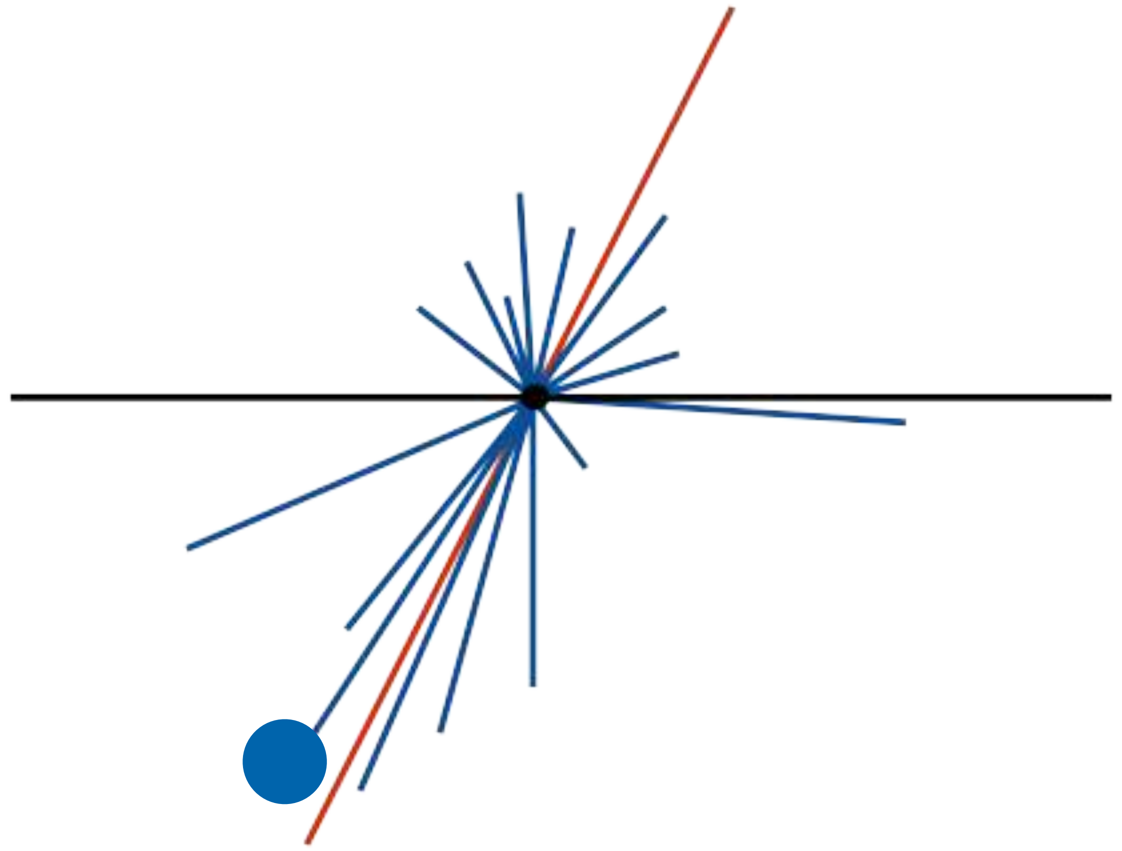

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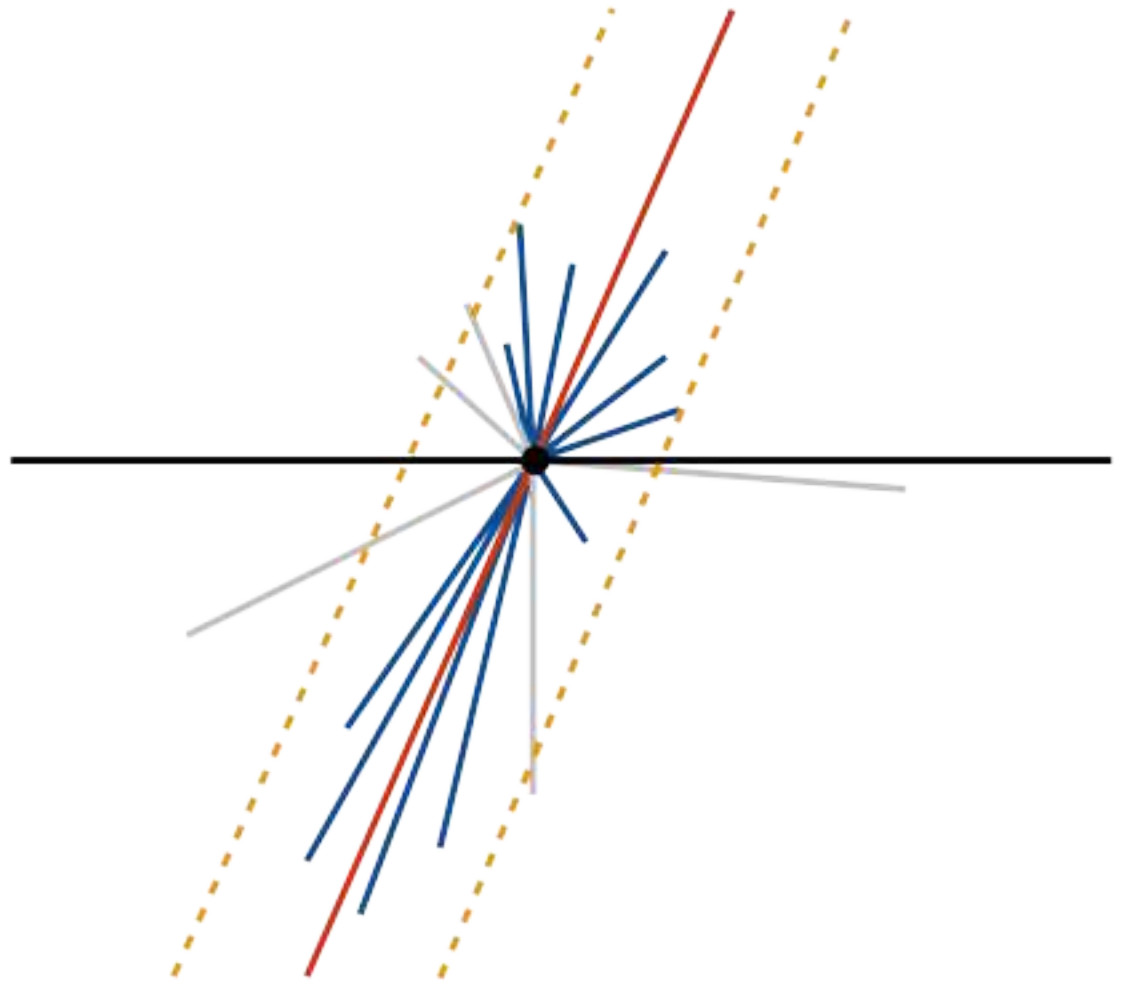

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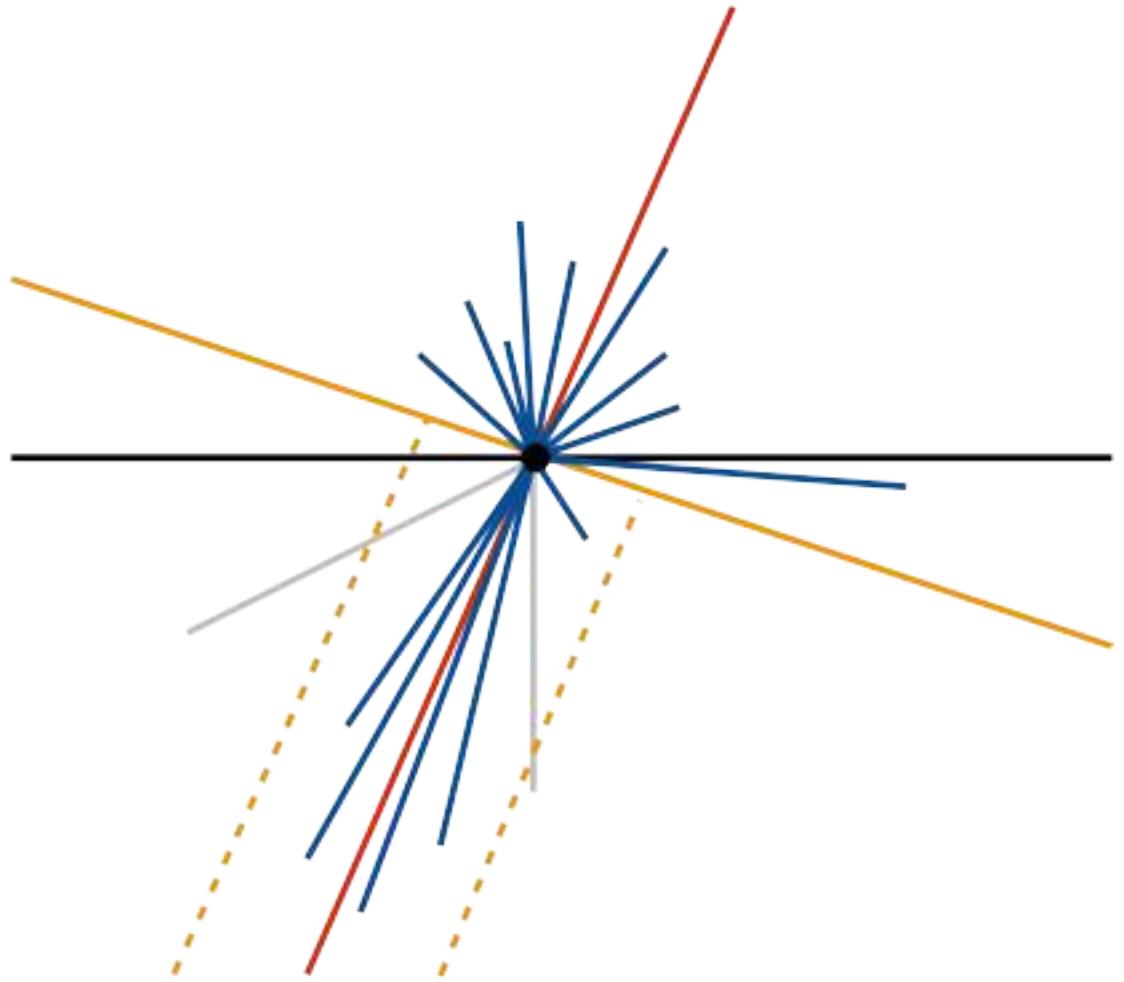

Coherence breaks down for non-global observables.

$T_k T_l T_i \cdot T_j T_m T_n$

large-N limit ↓

$$\frac{\partial G_{ab}(t)}{\partial t} = -\int_{\text{in}} \frac{\mathrm{d}\Omega_k}{4\pi} \omega_{ab}(k) G_{ab}(t) + \int_{\text{out}} \frac{\mathrm{d}\Omega_k}{4\pi} \omega_{ab}(k) \left[G_{ak}(t) G_{kb}(t) - G_{ab}(t) \right]$$

[Banfi, Marchesini, Smye ’02]

Colour

Colour reconnection and hadronization is about subleading-N.
So are shower accuracy and interference terms.

Colour factor algorithms

Coherent, NLL-accurate dipole showers

[Gustafson] [PanScales '21]
[Forshaw, Holguin, Plätzer '21]

Colour ME corrections

Colour-exact real emissions as far as possible

[Plätzer, Sjödahl '12, '18]
[Höche, Reichelt '20]

Full amplitude evolution

Colour-exact real and virtual corrections

[Forshaw, Plätzer + … '13 …]
[Nagy, Soper '12 …]

$$\mathrm{d}\sigma \sim \mathrm{L} \times \mathrm{d}\sigma_H(Q) \times \mathrm{PS}(Q \to \mu) \times \mathrm{MPI} \times \mathrm{Had}(\mu \to \Lambda) \times \ldots$$

Amplitude evolution

Colour reconnection and hadronization is about subleading-N.
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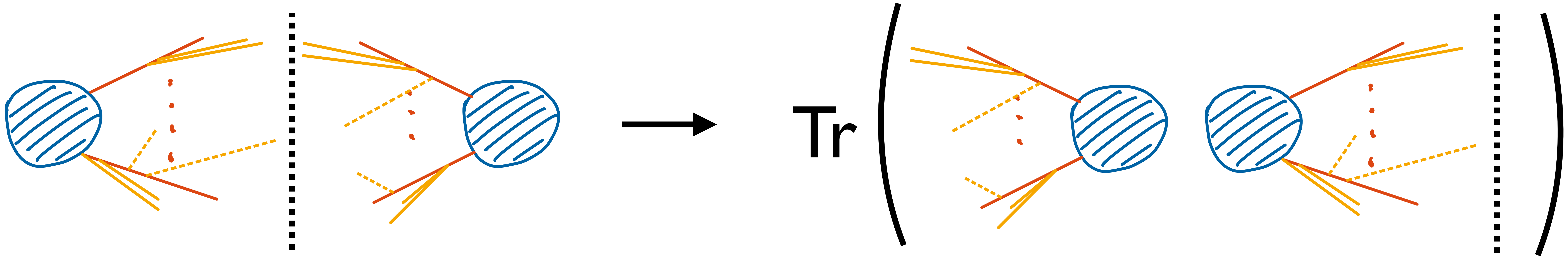
Colour factor algorithms	Colour ME corrections	Full amplitude evolution
Coherent, NLL-accurate dipole showers	Colour-exact real emissions as far as possible	Colour-exact real and virtual corrections
[Gustafson] [PanScales '21] [Forshaw, Holguin, Plätzer '21]	[Plätzer, Sjödahl '12, '18] [Höche, Reichelt '20]	[Forshaw, Plätzer + … '13 …] [Nagy, Soper '12 …]

$$d\sigma \sim \mathrm{Tr}\left[\mathbf{PS}(Q \to \mu)\mathrm{d}\mathbf{H}(Q)\mathbf{PS}^\dagger(Q \to \mu)\mathbf{Had}(\mu \to \Lambda)\right]$$

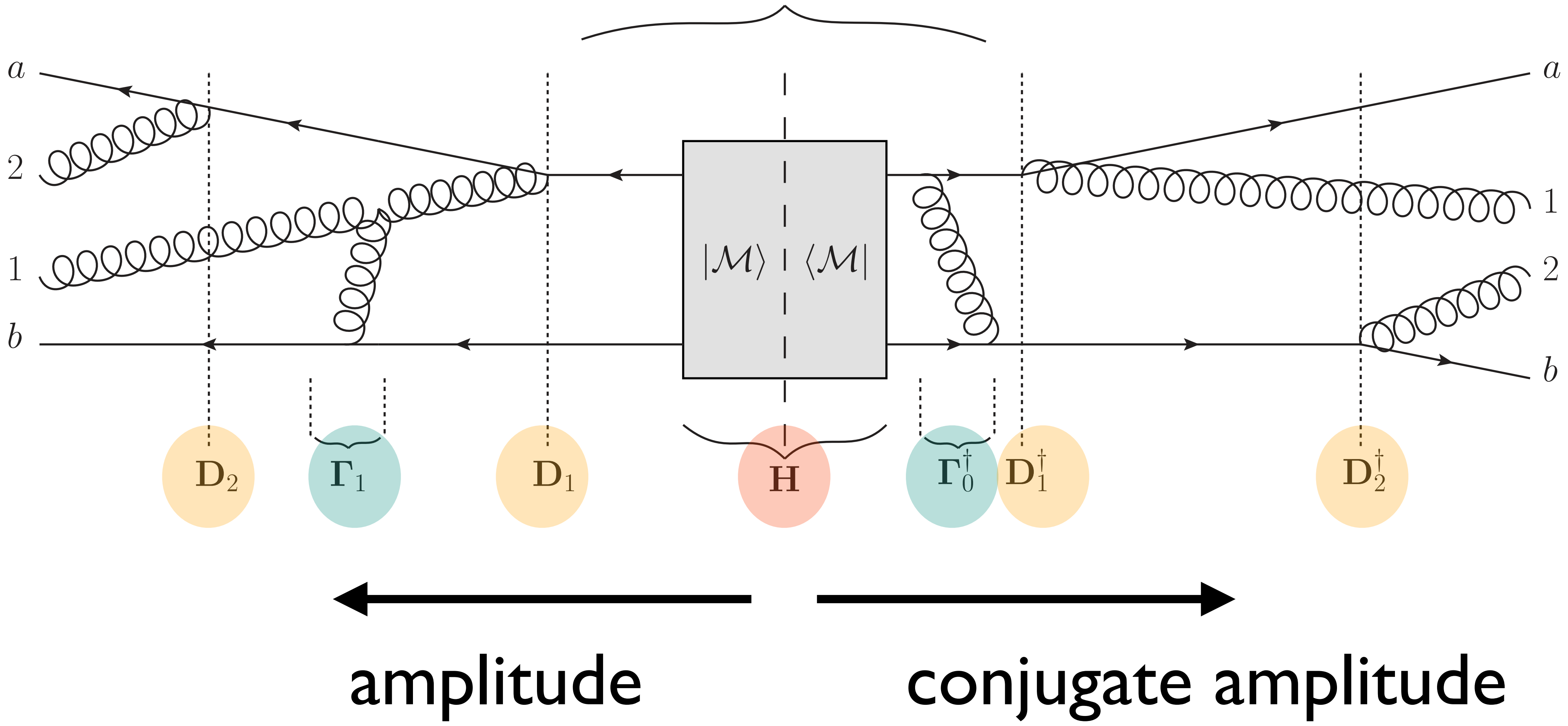
Amplitude evolution: CVolver



$$\mathbf{A}_n(q) = \int_q^Q \frac{\mathrm{d}k}{k}\ \mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\boldsymbol{\Gamma}(k')}\ \mathbf{D}_n(k)\ \mathbf{A}_{n-1}(k)\ \mathbf{D}_n^\dagger(k)\ \overline{\mathrm{P}}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\boldsymbol{\Gamma}^\dagger(k')}$$

Markovian algorithm at the amplitude level:
Iterate gluon exchanges and emission.

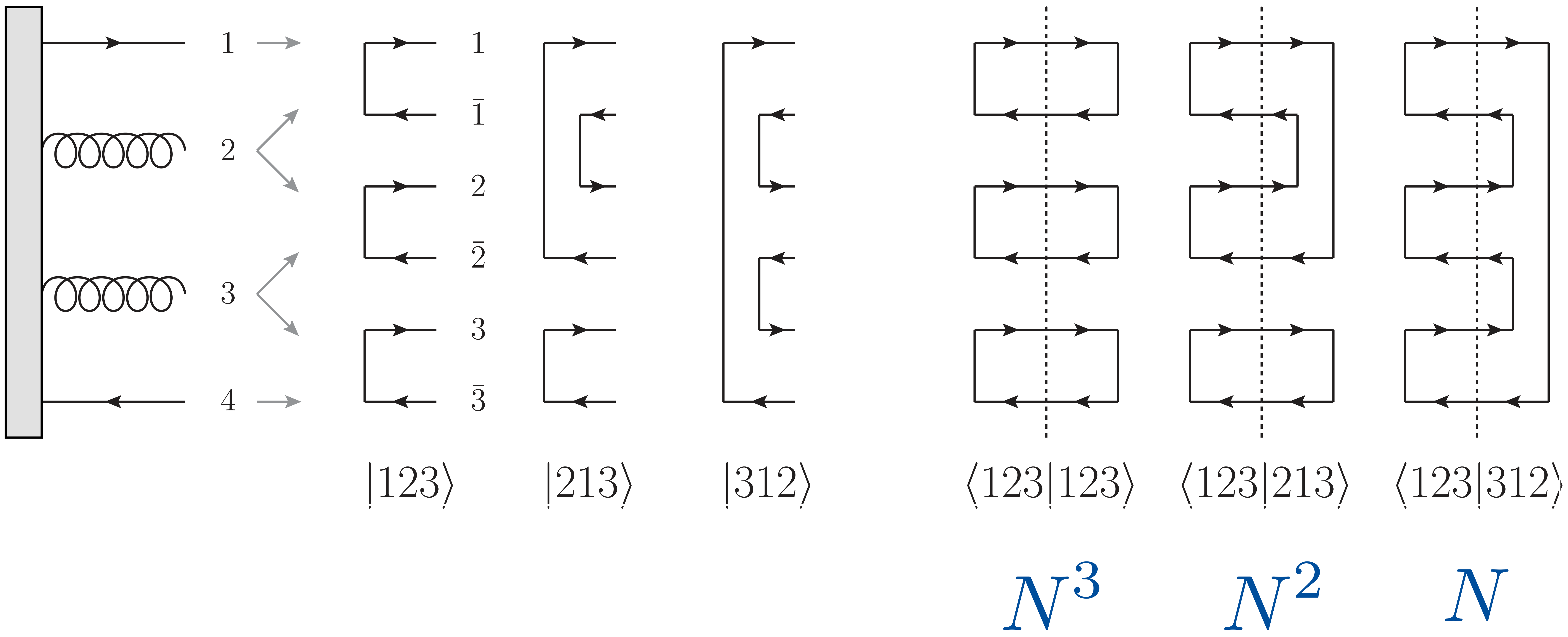
Different histories in amplitude and conjugate amplitude needed to include interference.



[Angeles, De Angelis, Forshaw, Plätzer, Seymour – '18]
[Forshaw, Holguin, Plätzer – '19]

Tracking colour

Decompose amplitudes in flow of colour charge. $(t^a)^i{}_k (t^a)^j{}_l = T_R \left(\delta^i_l \delta^j_k - \frac{1}{N} \delta^i_k \delta^j_l \right)$



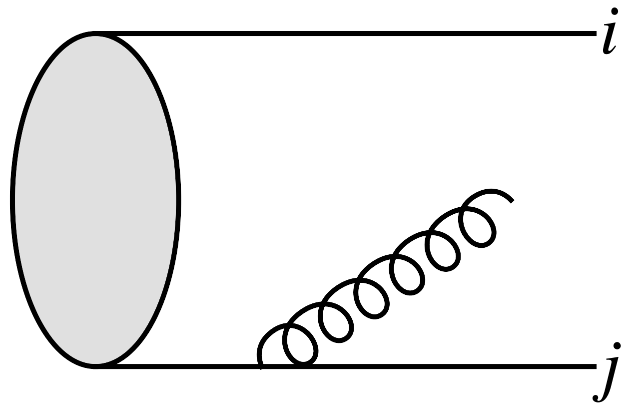
Suppression of interferences outside of colour connected dipoles.

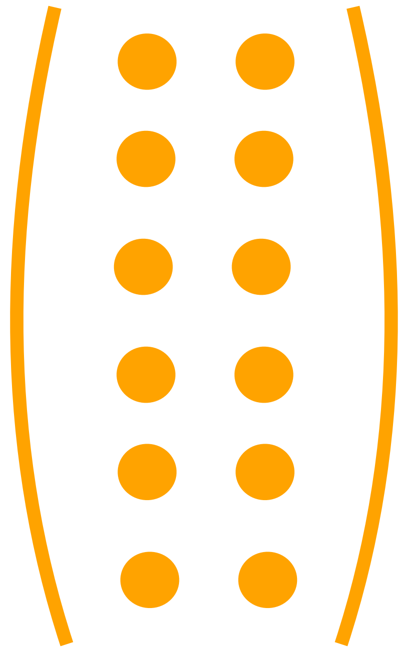
[Plätzer '13]
[Angeles, De Angelis, Forshaw, Plätzer, Seymour '18]

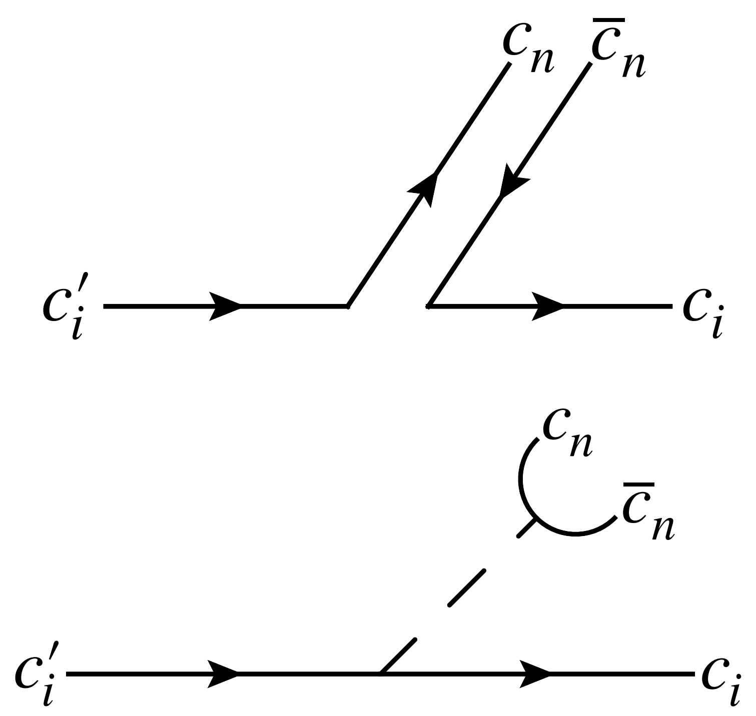
Tracking colour

Gluon emission

$$\mathbf{D}_n(k)$$



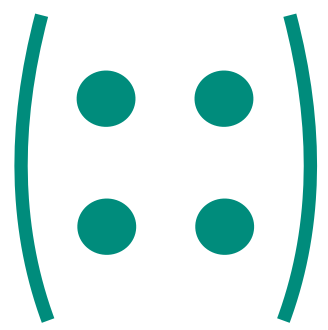


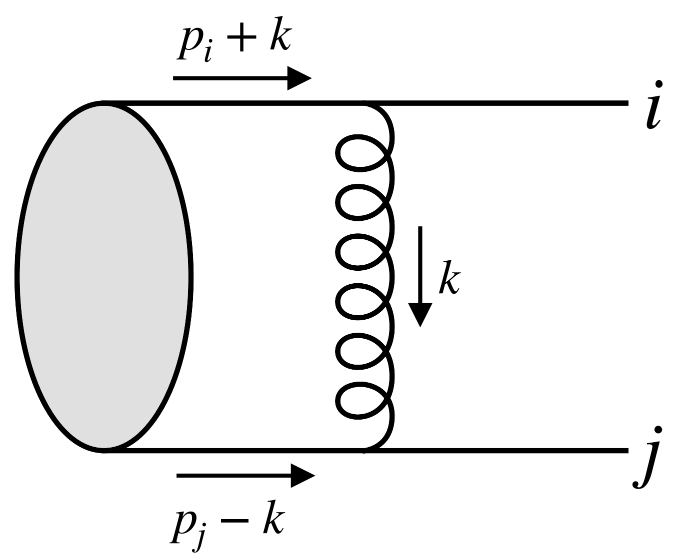


Explicit suppression in 1/N

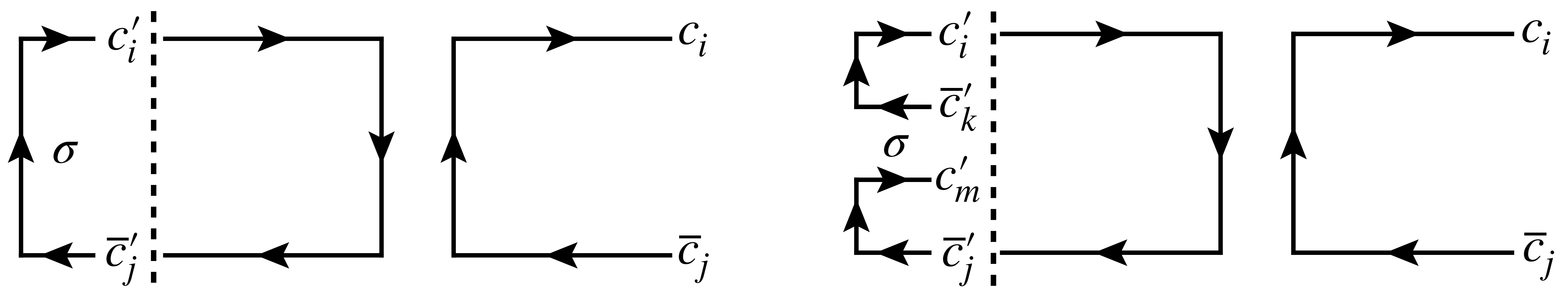
Gluon exchange

$$\mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}(k')}$$





$$[\tau|\mathbf{\Gamma}|\sigma\rangle = (\alpha_s N)[\tau|\mathbf{\Gamma}^{(1)}|\sigma\rangle + (\alpha_s N)^2[\tau|\mathbf{\Gamma}^{(2)}|\sigma\rangle + \ldots$$



$$[\tau|\mathbf{\Gamma}^{(1)}|\sigma\rangle = \left(\Gamma_\sigma^{(1)} + \frac{1}{N^2}\rho^{(1)}\right)\delta_{\sigma\tau} + \frac{1}{N}\Sigma_{\sigma\tau}^{(1)}$$

dipole flips — implicit suppression in 1/N

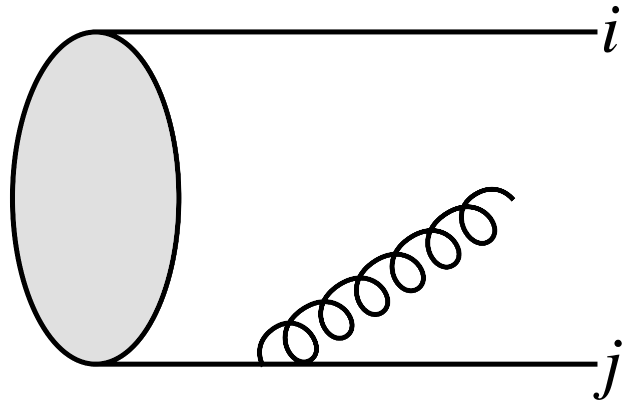
Systematically expand around large-N limit summing towers of terms enhanced by $\alpha_S N$

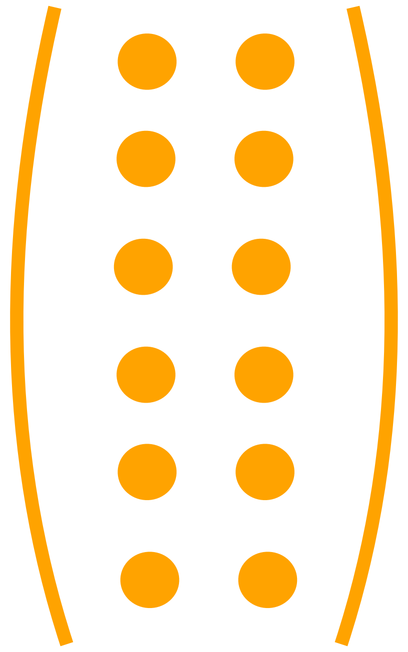
[Plätzer – '13] — diagrams from [Ruffa, MSc thesis 2020]

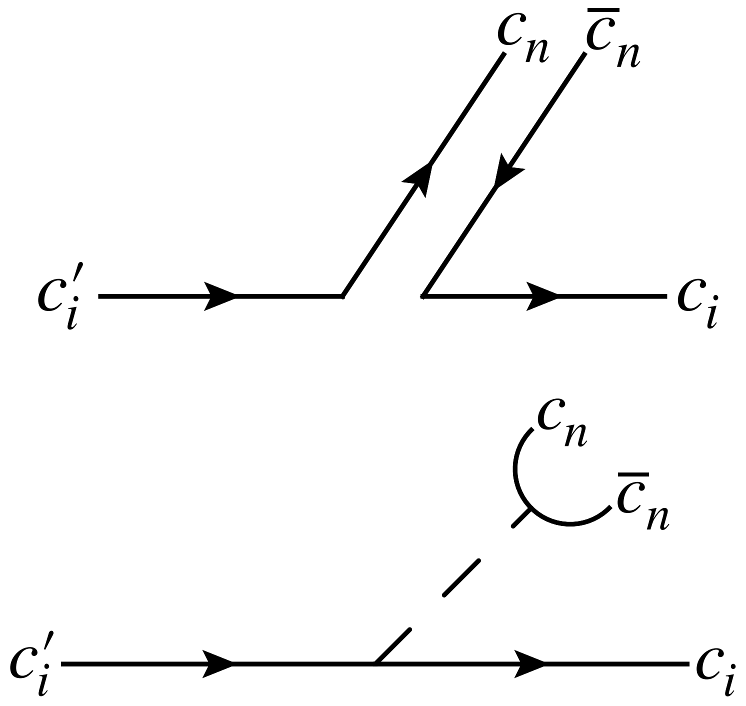
Tracking colour

Gluon emission

$$\mathbf{D}_n(k)$$







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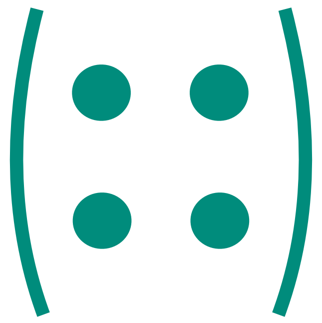

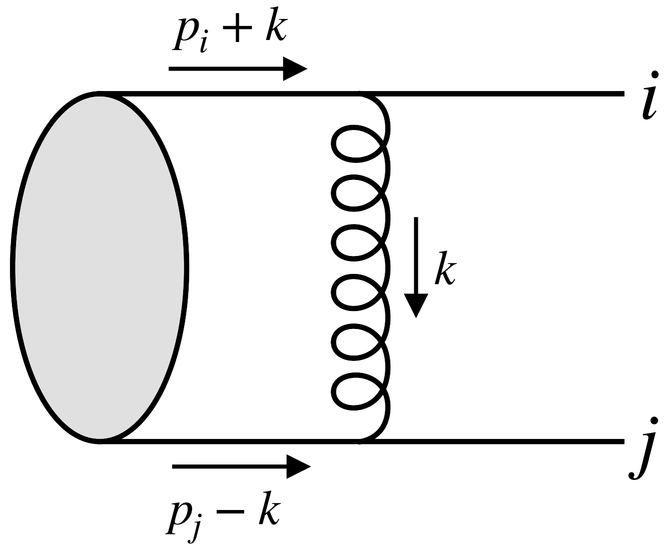

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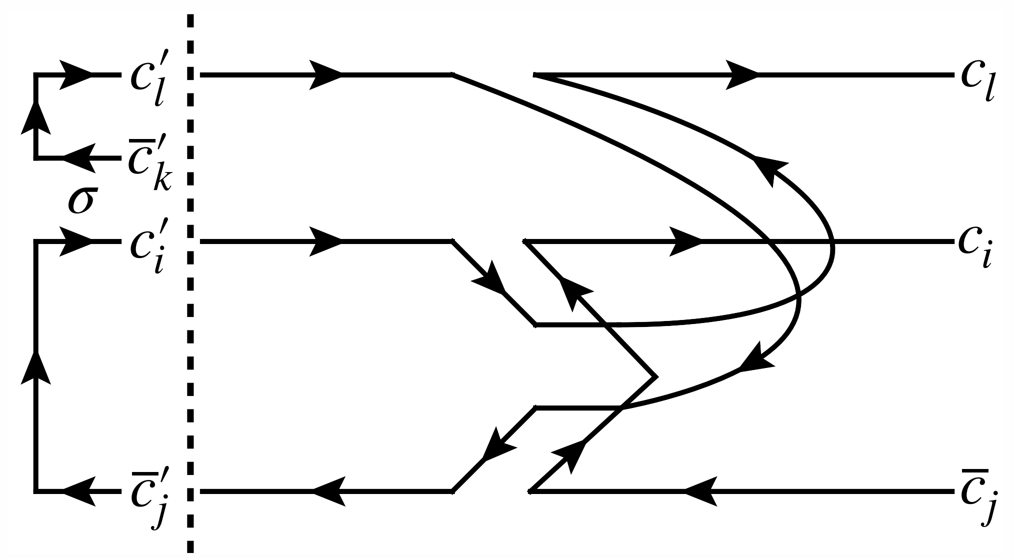

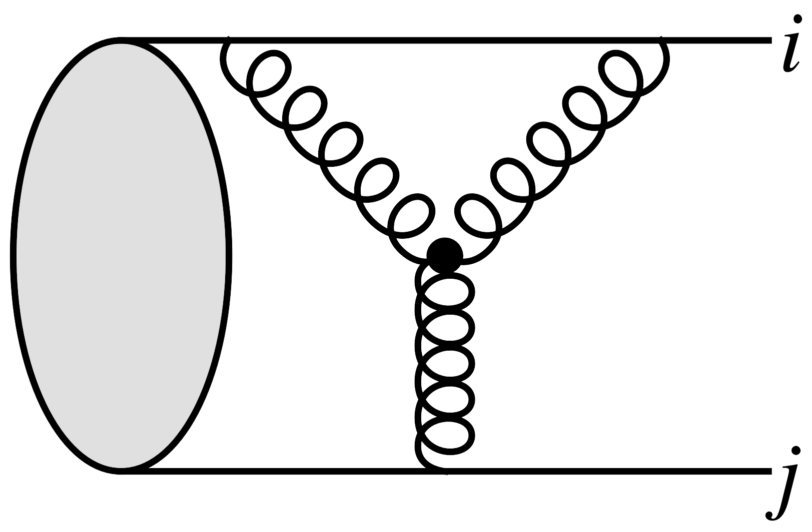

[Plätzer, Ruffa — '21]

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Amplitude evolution — CVolver

CVolver solves evolution equations in colour flow space

[De Angelis, Forshaw, Plätzer '21]
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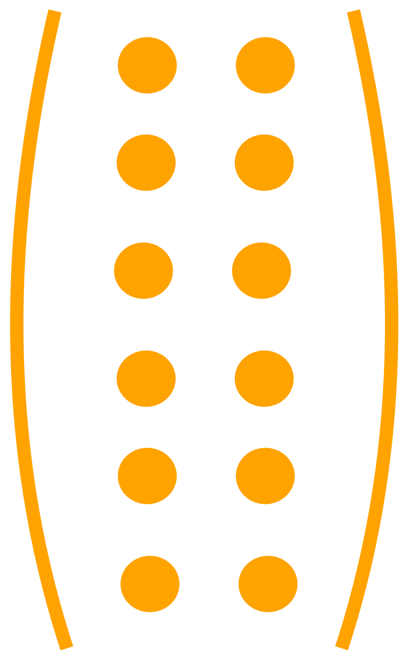

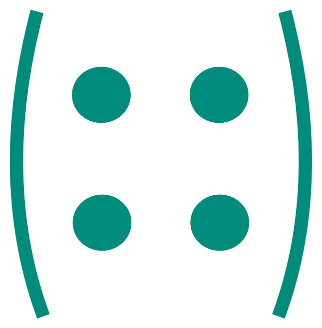

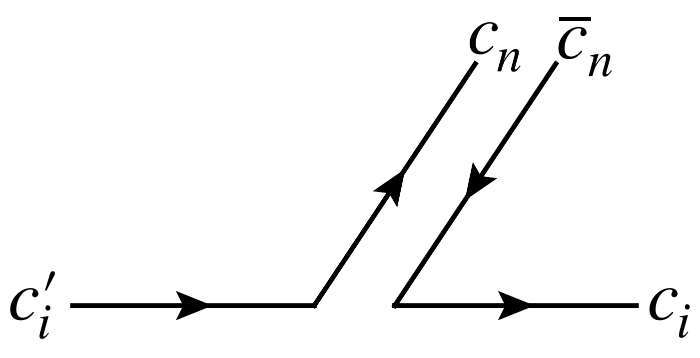

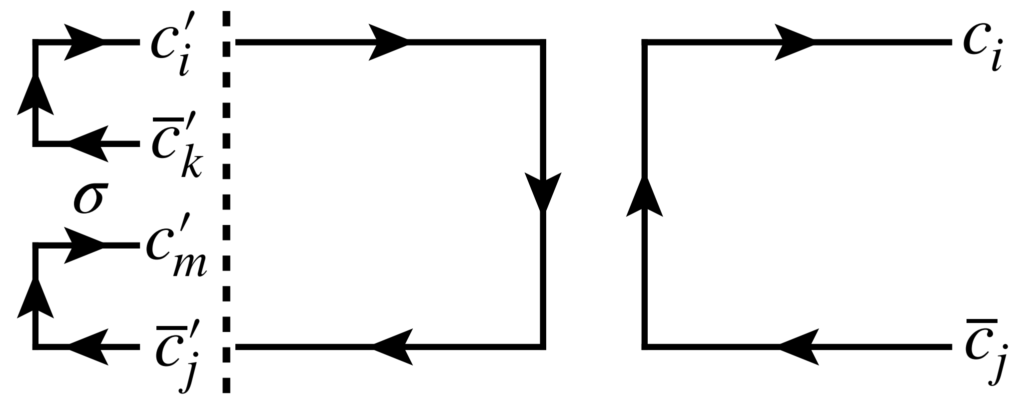

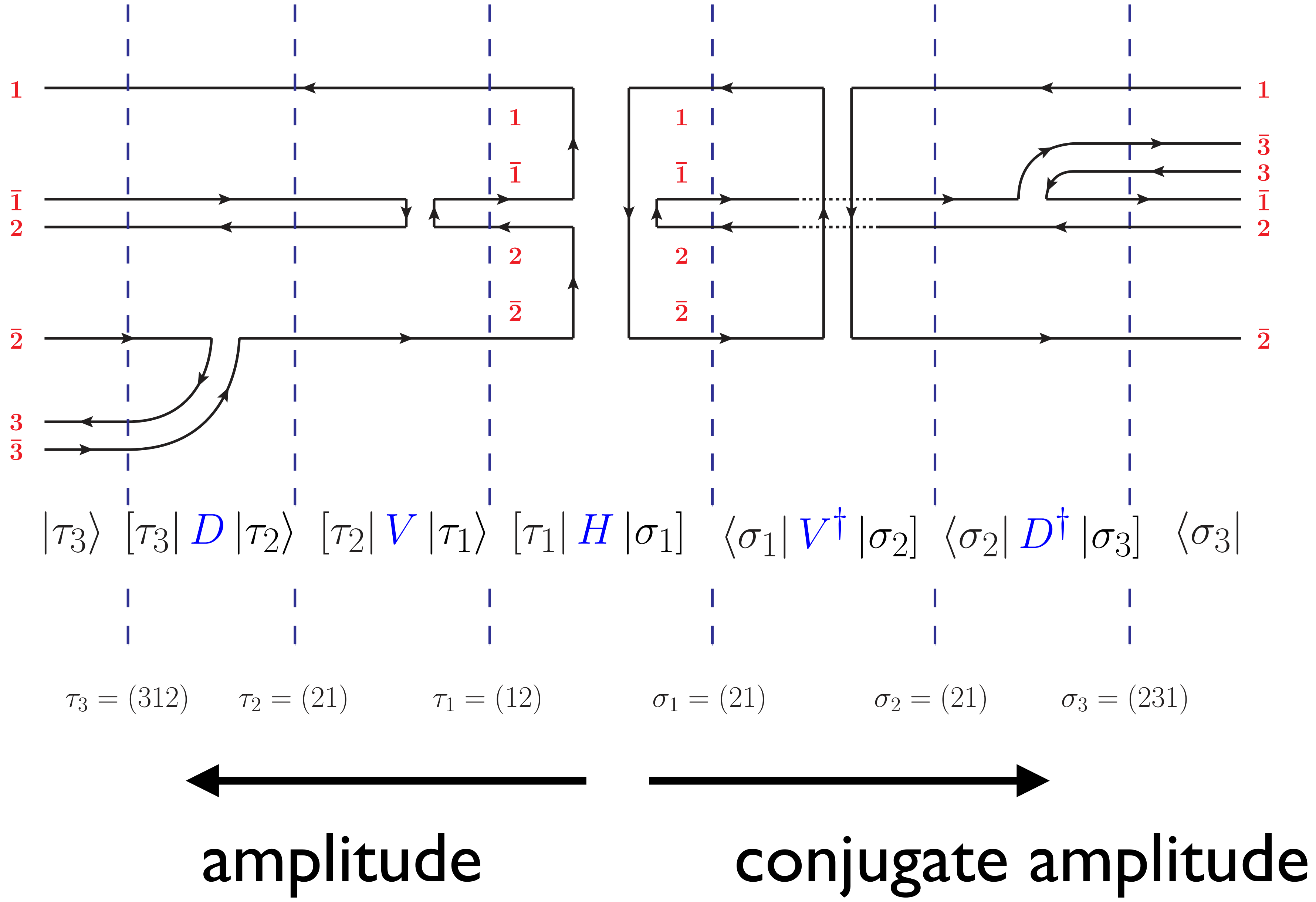

Amplitude evolution: CVolver

CVolver solves evolution equations in colour flow space

[De Angelis, Forshaw, Plätzer '21]
[Plätzer '13]

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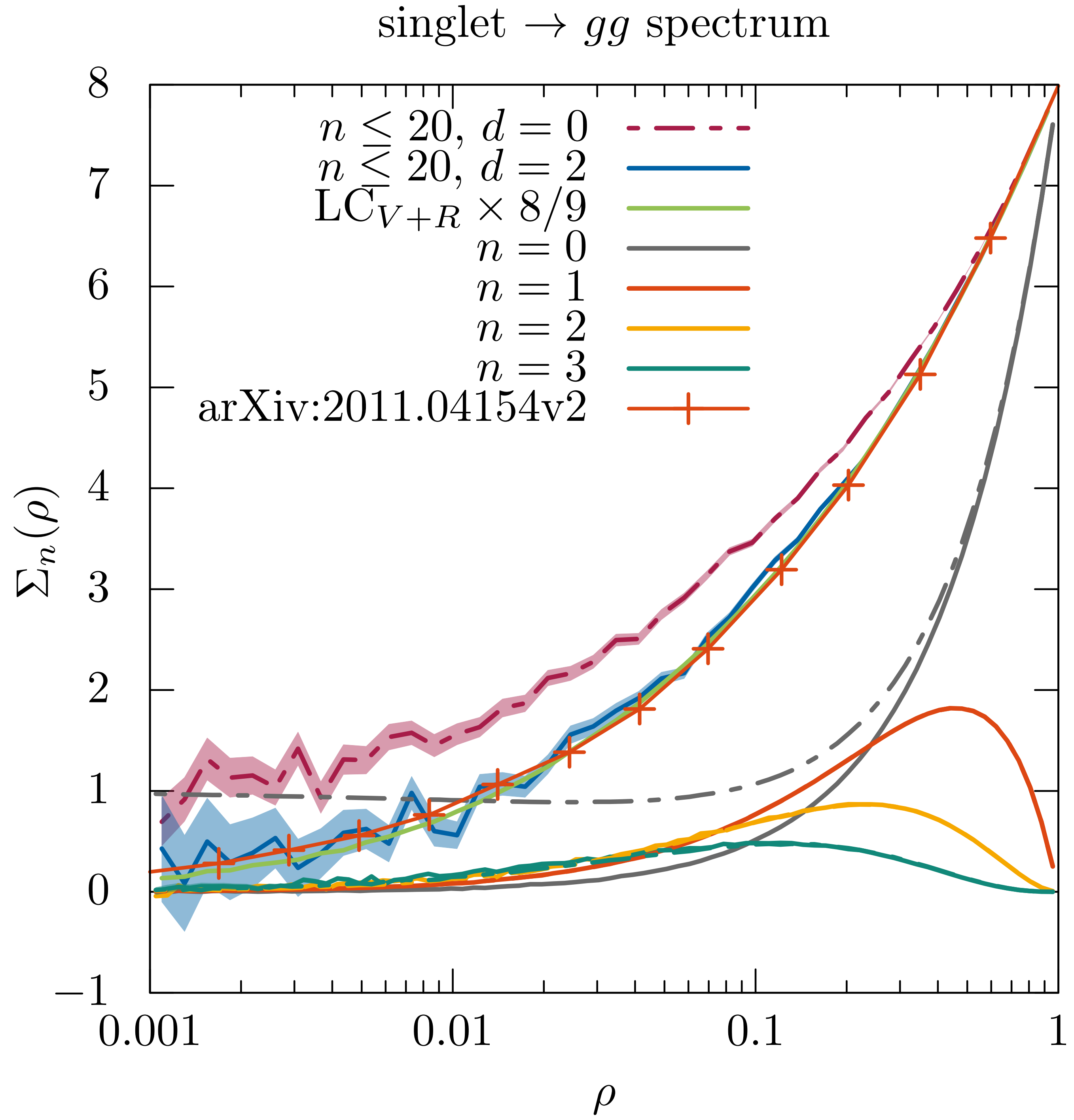
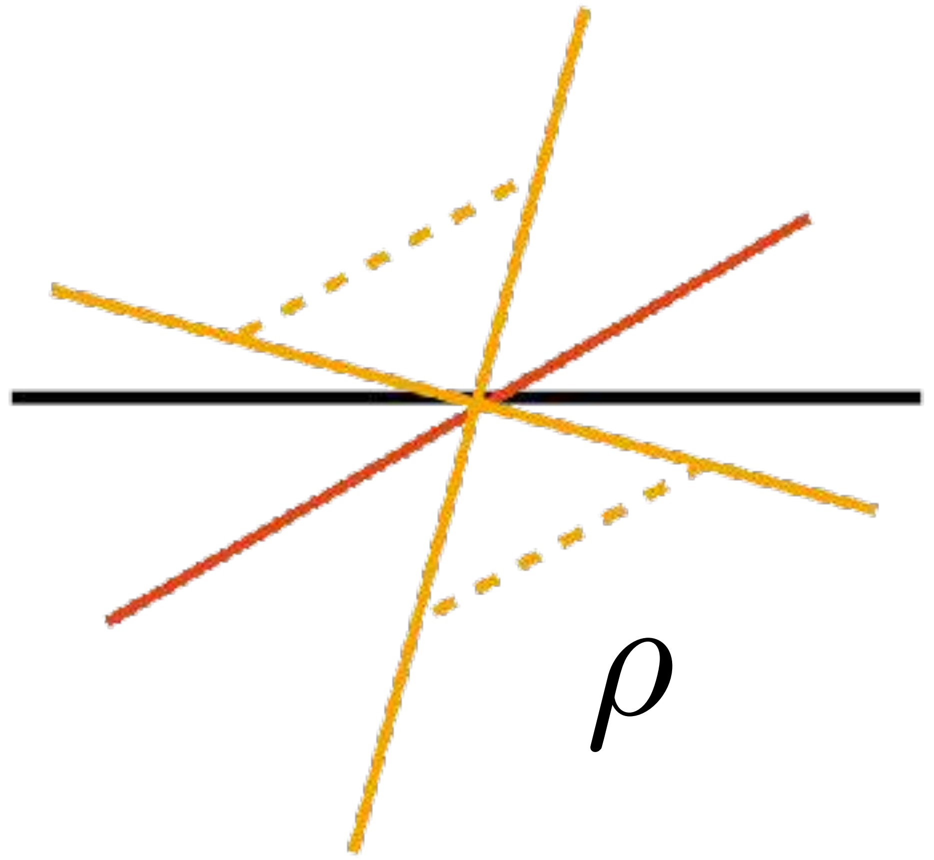
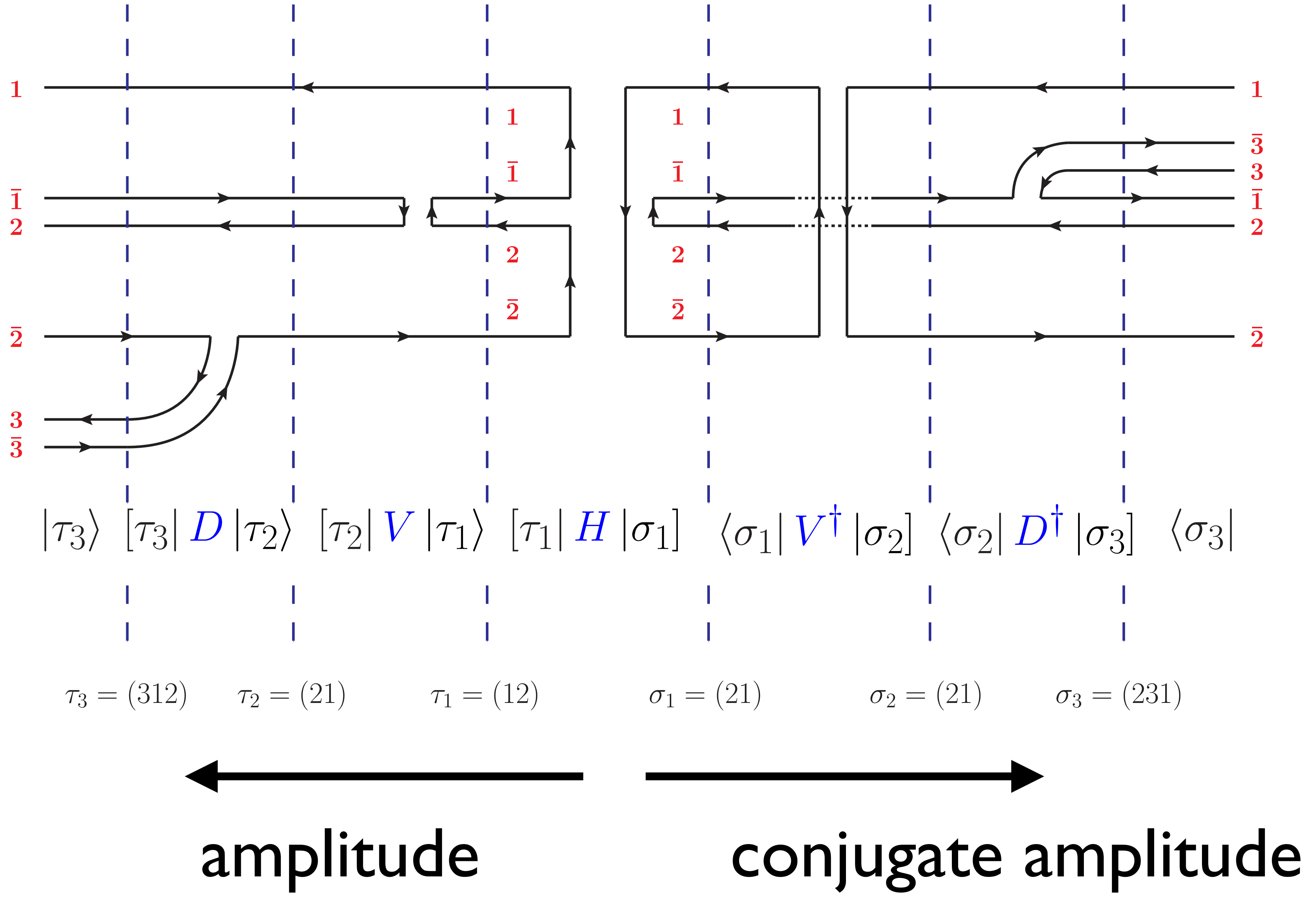




$$\Sigma(\rho) = \sum_n \int \mathrm{d}\sigma(\{p_i\}) \prod_i \theta_{\mathrm{in}}(\rho - E_i)$$



complete agreement with Hatta et al. using equivalent Langevin formulation

Factorisation and evolution

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm} \right] \mathrm{d}\phi_m u(\phi_m)$$

[Plätzer – ’22]

Factorisation and evolution

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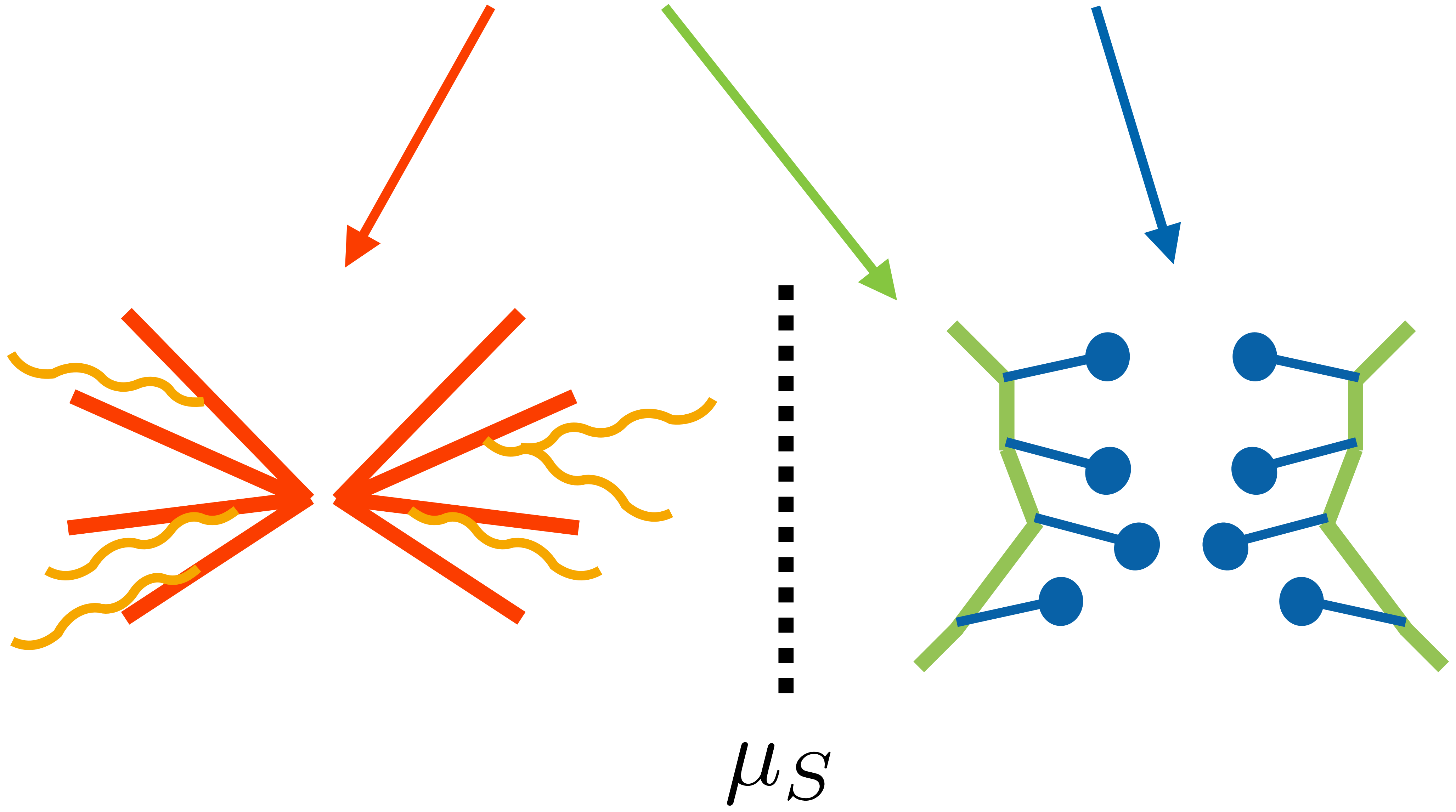
$$0 = \frac{\mathrm{d}}{\mathrm{d}\mu_S}$$



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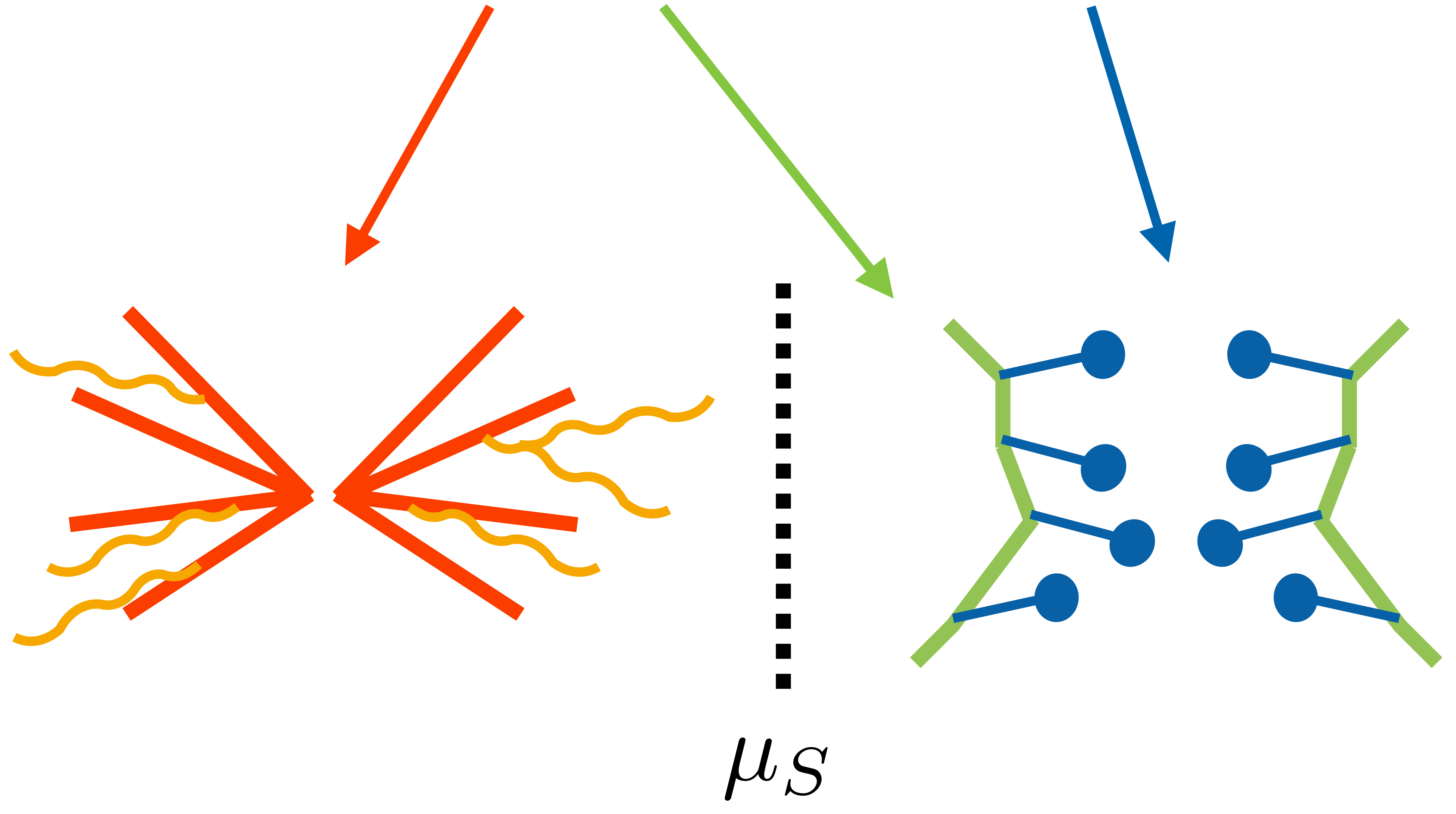

derive evolution

calculate building blocks

construct model response

constrain by data

[Plätzer – ’22]

Factorisation and evolution

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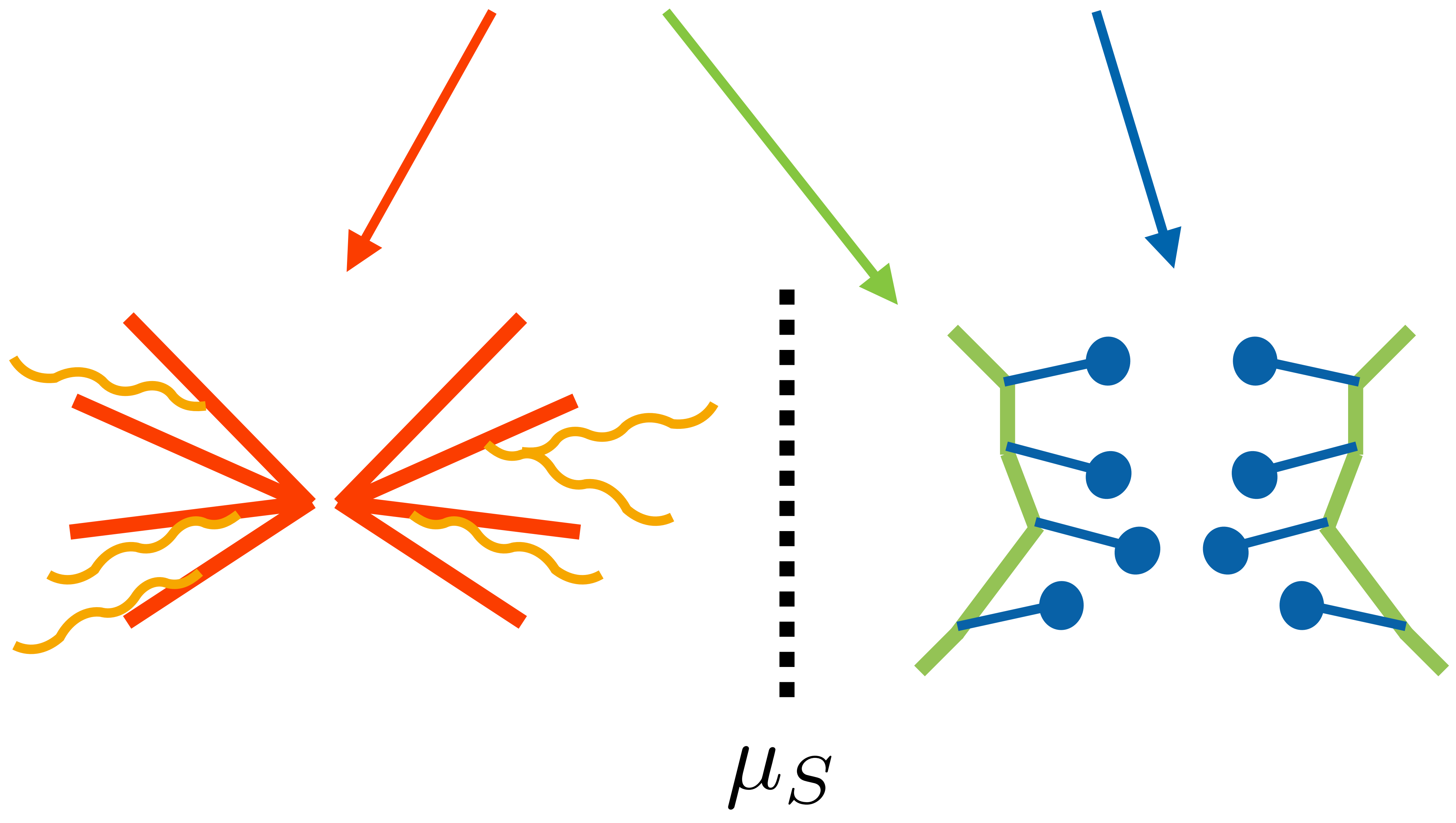


Not limited to a hadronization model — can also re-arrange partonic observables in this way.

[e.g. resummation of NGL in SCET — Becher, Neubert et al.]

[Plätzer – '22]

Redefinitions of "bare" operators

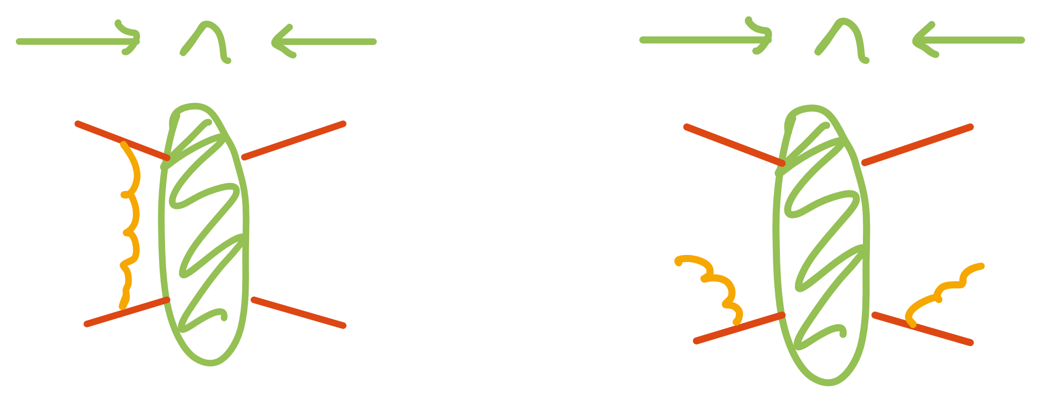
How do we consistently hadronize in light of (improved) shower algorithms?

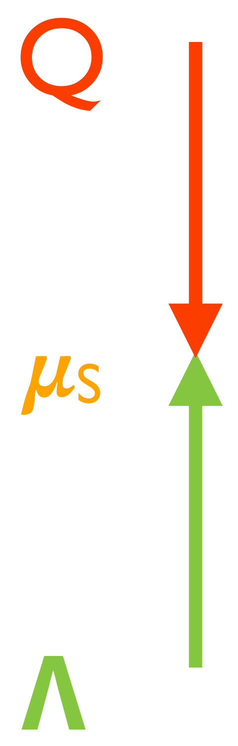
How to do this at subleading N and higher order shower evolution?

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

Subtract IR divergencies in unresolved regions

$$\mathbf{U}_n = \mathcal{X}_n \left[\mathbf{S}(\mu_S), \mu_S\right] = \mathbf{X}_n^\dagger \mathbf{S}_n \mathbf{X}_n - \sum_{s=1}^{\infty} \alpha_S^s \int \mathbf{F}_{n+s}^{(s)\dagger} \mathbf{S}_{n+s} \mathbf{F}_{n+s}^{(s)} \prod_{i=n+1}^{n+s} \mu_R^{2\epsilon} [\mathrm{d}p_i] \tilde{\delta}(p_i)$$





Re-arrange to resum IR enhancements

$$\mathbf{M}_n Z_g^n = \mathcal{Z}_n \left[\mathbf{A}(\mu_S), \mu_S\right] = \mathbf{Z}_n \mathbf{A}_n \mathbf{Z}_n^\dagger + \sum_{s=1}^{n} \alpha_S^s \mathbf{E}_n^{(s)} \mathbf{A}_{n-s} \mathbf{E}_n^{(s)\dagger}$$

$$\mathbf{M}_n = \sum_{l=0}^{\infty} \alpha_0^l \, \mathbf{M}_n^{(l)}$$

Cross section is RG invariant

$$\sigma = \sum_n \alpha_S^n \int \mathrm{Tr}\left[\mathbf{A}_n(\mu_S) \mathbf{S}_n(\mu_S)\right] \mathrm{d}\phi_n$$

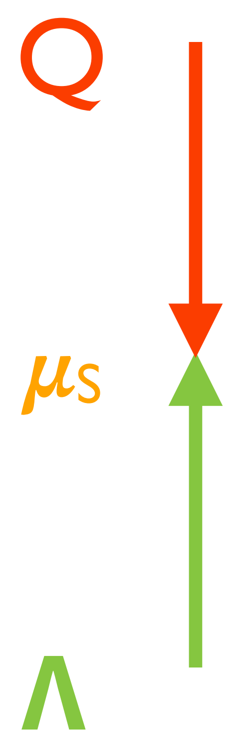
Redefinitions of "bare" operators

How do we consistently hadronize in light of (improved) shower algorithms?

How to do this at subleading N and higher order shower evolution?

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

Redefinitions of hard and soft factor **inverse** to each other:



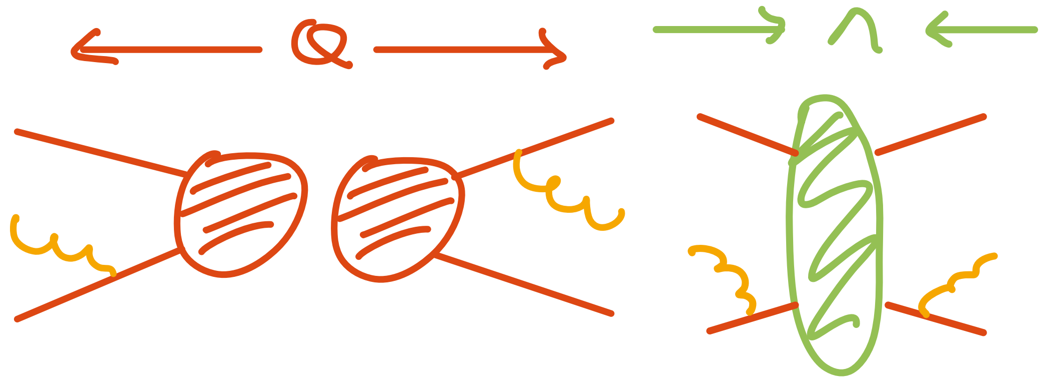
$$\mathbf{Z}_n = \mathbf{X}_n^{-1} \qquad \mathbf{X}_n \mathbf{E}_n^{(s)} \circ \mathbf{E}_n^{(s)\dagger} \mathbf{X}_n^\dagger - \mathbf{F}_n^{(s)} \mathbf{Z}_{n-s} \circ \mathbf{Z}_{n-s}^\dagger \mathbf{F}_n^{(s)\dagger} - \sum_{t=1}^{s-1} \mathbf{F}_n^{(t)} \mathbf{E}_{n-t}^{(s-t)} \circ \mathbf{E}_{n-t}^{(s-t)\dagger} \mathbf{F}_n^{(t)\dagger} = 0$$

dressing of hard process ~ parton shower

soft evolution ~ measurement and hadronization model

$$\sum_n \int \alpha_S^n \, \mathrm{Tr}\left[(\mathbf{A}_n + \boldsymbol{\Delta}_n)\mathbf{S}_n\right] \mathrm{d}\phi(Q) \prod_{i=1}^n \mu_R^{2\epsilon} [\mathrm{d}p_i] \tilde{\delta}(p_i)$$

α_s corrections to tower of logarithms in A —
truncation error of relation of Z factors

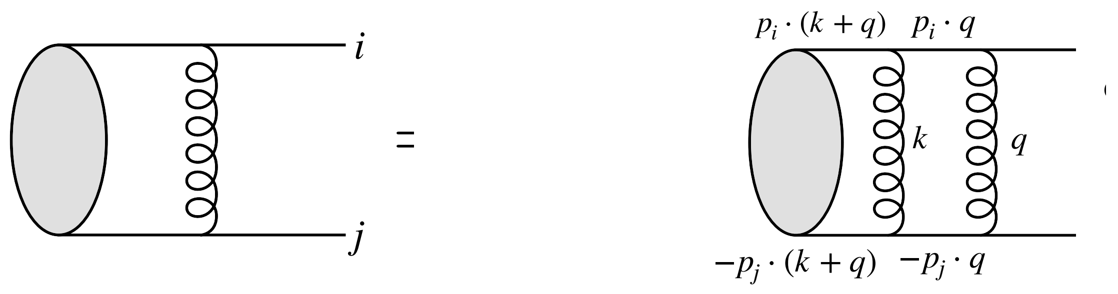


(Soft) factorisation of amplitudes

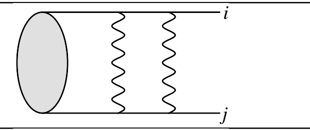
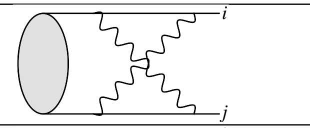
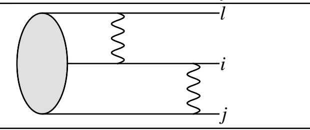
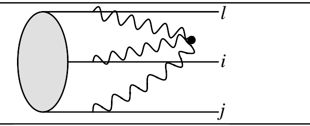
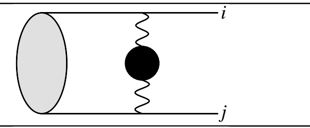
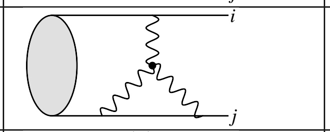
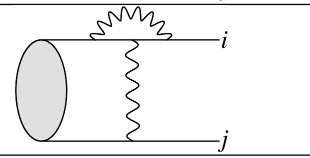
$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm} \right] \mathrm{d}\phi_m u(\phi_m)$$

Factorisation of virtual contributions

$$\mathbf{M}_n^{(l)} = \mathbf{V}^{(1)} \mathbf{M}_n^{(l-1)} + \mathbf{M}_n^{(l-1)} \mathbf{V}^{(1)\dagger}$$
$$+ \mathbf{V}^{(1)} \mathbf{M}_n^{(l-2)} \mathbf{V}^{(1)\dagger} + \mathbf{V}^{(2)} \mathbf{M}_n^{(l-2)} + \mathbf{M}_n^{(l-2)} \mathbf{V}^{(2)\dagger} + \ldots$$

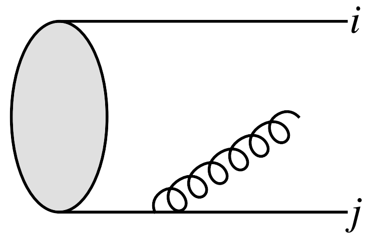


Handle virtual as phase-space type integrals to remove divergencies with subtractions.

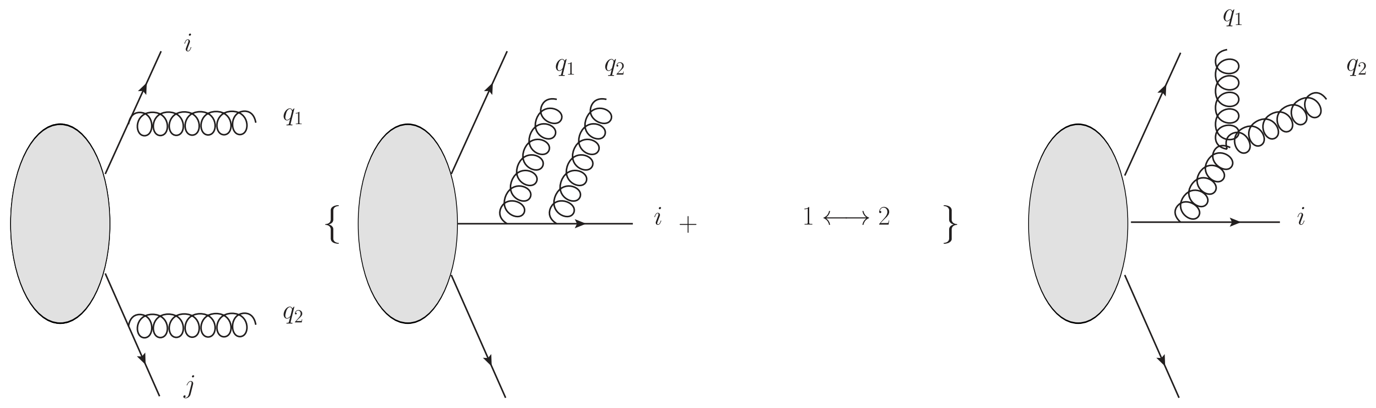
Coefficient	Diagram	Colour-factor
$\Omega_{ij}^{(2)}$		$(\mathbf{T}_i \cdot \mathbf{T}_j)(\mathbf{T}_i \cdot \mathbf{T}_j)$
$\tilde{\Omega}_{ij}^{(2)}$		$\mathbf{T}_i^a \mathbf{T}_i^b \mathbf{T}_j^b \mathbf{T}_j^a$
$\Omega_{ijl}^{(2)}$		$(\mathbf{T}_i \cdot \mathbf{T}_l)(\mathbf{T}_i \cdot \mathbf{T}_j)$
$\hat{\Omega}_{ijl}^{(2)}$		$i f^{abc} \mathbf{T}_i^a \mathbf{T}_j^b \mathbf{T}_l^c$
$\Omega_{ij,\mathrm{self-en.}}^{(2)}$		$T_R (\mathbf{T}_i \cdot \mathbf{T}_j)$
$\Omega_{ij,\mathrm{vertex-corr.}}^{(2)}$		$T_R (\mathbf{T}_i \cdot \mathbf{T}_j)$
$\hat{\Omega}_{ij}^{(2)}$		$\mathbf{T}_i^b \mathbf{T}_i^a \mathbf{T}_i^b \mathbf{T}_j^a$

[Plätzer, Ruffa — '21]

Factorisation of real contributions



$$\mathbf{M}_n^{(l)} = \mathbf{D}_n^{(1,0)} \mathbf{M}_{n-1}^{(l)} \mathbf{D}_n^{(1,0)\dagger}$$
$$+ \mathbf{D}_n^{(1,1)} \mathbf{M}_{n-1}^{(l-1)} \mathbf{D}_n^{(1,0)\dagger} + \mathbf{D}_n^{(1,0)} \mathbf{M}_{n-1}^{(l-1)} \mathbf{D}_n^{(1,1)\dagger}$$
$$+ \mathbf{D}_n^{(2,0)} \mathbf{M}_{n-2}^{(l)} \mathbf{D}_n^{(2,0)\dagger} + \ldots$$

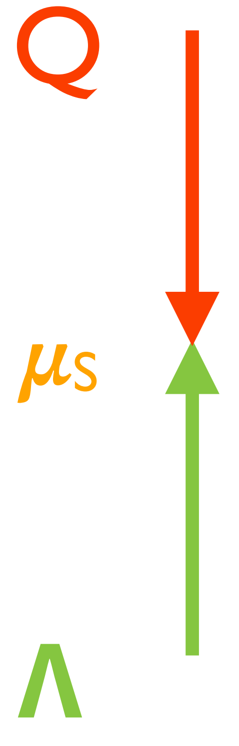


$$\sum_{(a,b),(c,d)} \sum_{i,j,k,l=1}^{n} \omega_{ijkl}^{abcd} \, \mathrm{T}_i^{(a)} \mathrm{T}_j^{(b)} \circ \mathrm{T}_k^{(c),\dagger} \mathrm{T}_l^{(d),\dagger}$$

[Majcen — M.Sc. thesis 2022]
based on Catani & Grazzini

Infrared subtractions

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm} \right] \mathrm{d}\phi_m u(\phi_m)$$



Subtractions necessitate a resolution:
what is it we call ‘unresolved’?
Encompass all singular regions!

$$\mathbf{U}_n = \mathbf{X}_n^\dagger \mathbf{S}_n \mathbf{X}_n - \sum_{s=1}^{\infty} \alpha_S^s \int \mathbf{F}_{n+s}^{(s)\dagger} \mathbf{S}_{n+s} \mathbf{F}_{n+s}^{(s)} \prod_{i=n+1}^{n+s} \mu_R^{2\epsilon} [\mathrm{d}p_i] \tilde{\delta}(p_i)$$

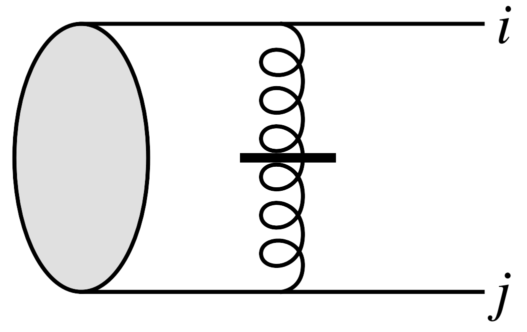
resolution function for (cut) loop momenta

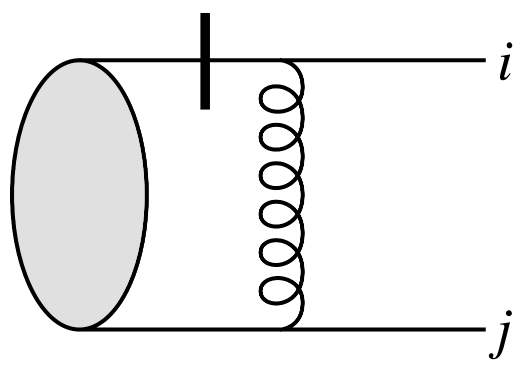
$$\mathbf{X}_n^{(1)} = \hat{\mathbf{V}}_n^{(1)} [\Xi_{n,1}]$$

$$\hat{\mathbf{V}}_n^{(l)} [\Xi_{n,l}] = \sum_\alpha \int \mathcal{I}_{n,\alpha}^{(l)}(p_1, \ldots, p_n; k_1, \ldots, k_l) \; \Xi_{n,l}^{(\alpha)} \prod_{i=1}^{l} \mu_R^{2\epsilon} [\mathrm{d}k_i]$$

$$\mathbf{F}_n^{(1,0)} \circ \mathbf{F}_n^{(1,0)\dagger} = \mathbf{D}_n^{(1,0)} \circ \mathbf{D}_n^{(1,0)\dagger} \Theta_{n,1}$$

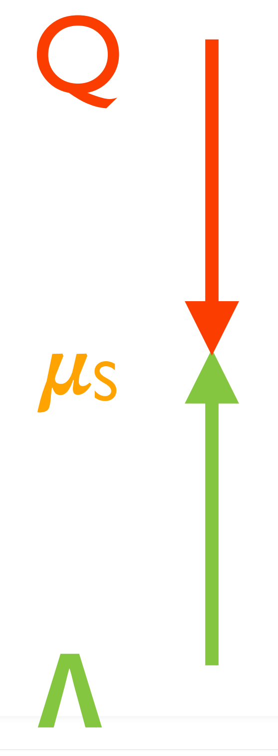
resolution function for real emission





Infrared subtractions

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$



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resolution function for (cut) loop mome

$$\mathbf{X}_n^{(1)} = \hat{\mathbf{V}}_n^{(1)}[\Xi_{n,1}]$$

$$\mathbf{F}_n^{(1,0)} \circ \mathbf{F}_n^{(1,0)\dagger} = \mathbf{D}_n^{(1,0)} \circ \mathbf{D}_n^{(1,0)\dagger} \Theta_{n,1}$$

resolution function for real emission

Continues to higher orders …

$$\mathbf{X}_n^{(2)} = \hat{\mathbf{V}}_n^{(2)}[\Xi_{n,2}] - \hat{\mathbf{V}}_n^{(1)}[\Xi_{n,1}]\hat{\mathbf{V}}_n^{(1)}$$

$$\begin{aligned}\mathbf{F}_n^{(1,1)} \circ \mathbf{F}_n^{(1,0)\dagger} = &\ \mathbf{D}_n^{(1,1)}\left[\Xi_{n-1,1}\right] \circ \mathbf{D}_n^{(1,0)\dagger}\Theta_{n,1} \\ &+ \mathbf{D}_n^{(1,1)}\left[1 - \Xi_{n-1,1}\right] \circ \mathbf{D}_n^{(1,0)\dagger}\Theta_{n,1} + \mathbf{D}_n^{(1,1)}\left[\Xi_{n-1,1}\right] \circ \mathbf{D}_n^{(1,0)\dagger}(1 - \Theta_{n,1}) \\ &- \hat{\mathbf{V}}_n^{(1)}\left[\Xi_{n-1,1}\right]\mathbf{D}_n^{(1,0)} \circ \mathbf{D}_n^{(1,0)\dagger} + \mathbf{D}_n^{(1,0)}\hat{\mathbf{V}}_{n-1}^{(1)} \circ \mathbf{D}_n^{(1,0)\dagger}\Theta_{n,1}\end{aligned}$$

$$\mathbf{F}_n^{(2,0)} \circ \mathbf{F}_n^{(2,0)\dagger} = \mathbf{D}_n^{(2,0)} \circ \mathbf{D}_n^{(2,0)\dagger}\Theta_{n,2} - \mathbf{D}_n^{(1,0)}\mathbf{D}_{n-1}^{(1,0)} \circ \mathbf{D}_{n-1}^{(1,0)\dagger}\mathbf{D}_n^{(1,0)\dagger}\Theta_{n,1}$$

Infrared subtractions

$$\sigma = \sum_{n,m} \int\int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

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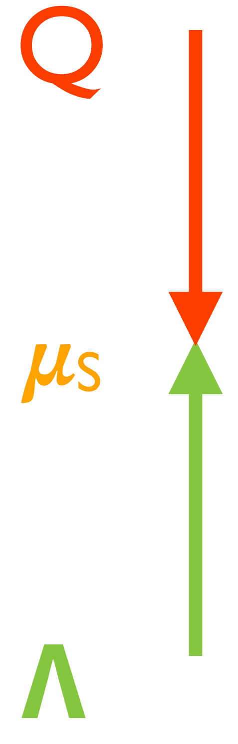

resolution function for (cut) loop momenta

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$$\mathbf{F}_n^{(1,0)} \circ \mathbf{F}_n^{(1,0)\dagger} = \mathbf{D}_n^{(1,0)} \circ \mathbf{D}_n^{(1,0)\dagger} \Theta_{n,1}$$

resolution function for real emission

Resolution functions introduce cutoff dependence, e.g. energy ordering:

$$\Theta_{n,1} = 1 - \hat{\Theta}_{n,1} \theta(E_n - \mu_S)$$

"soft or collinear"

$$\Theta_{n,2} = 1 - \hat{\Theta}_{n,2} \theta(E_{n-1} - \mu_S) \theta(E_n - \mu_S)$$

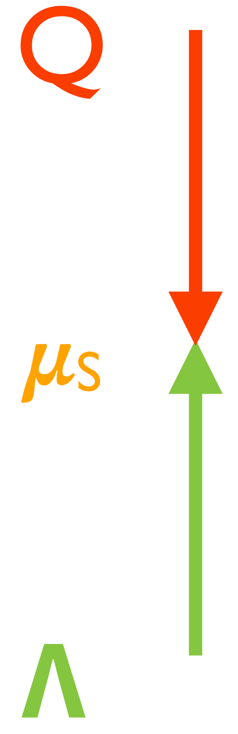
Infrared subtractions

How do we consistently hadronize in light of (improved) shower algorithms?

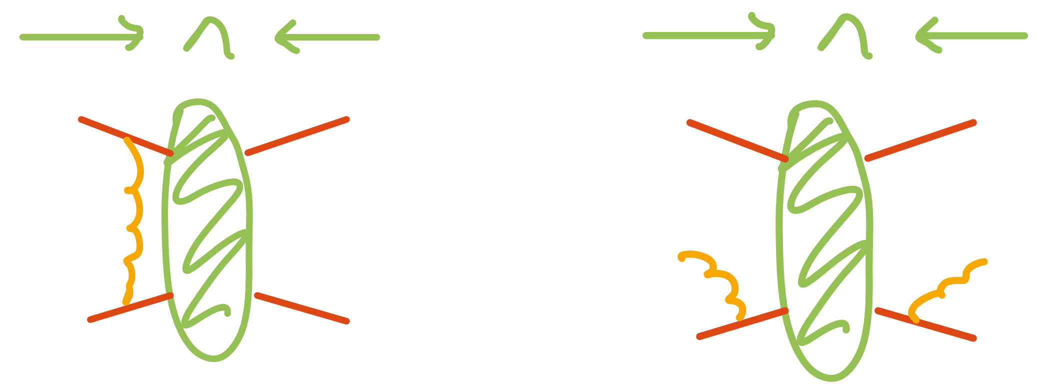
How to do this at subleading N and higher order shower evolution?

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

Hard factor reproduces CVolver algorithm and predicts key features of second order evolution.



$$\partial_S \mathbf{S}_n = -\tilde{\mathbf{\Gamma}}^{\dagger}_{S,n}\mathbf{S}_n - \mathbf{S}_n\tilde{\mathbf{\Gamma}}_{S,n} + \sum_{s\geq 1} \alpha_S^s \int \tilde{\mathbf{R}}^{(s)\dagger}_{S,n+s}\mathbf{S}_{n+s}\tilde{\mathbf{R}}^{(s)}_{S,n+s} \prod_{i=n+1}^{n+s} [\mathrm{d}p_i]\tilde{\delta}(p_i)$$



$$\partial_S \mathbf{A}_n = \mathbf{\Gamma}_{n,S}\mathbf{A}_n + \mathbf{A}_n\mathbf{\Gamma}^{\dagger}_{n,S} - \sum_{s\geq 1} \alpha_S^s \mathbf{R}^{(s)}_{S,n}\mathbf{A}_{n-s}\mathbf{R}^{(s)\dagger}_{S,n}$$

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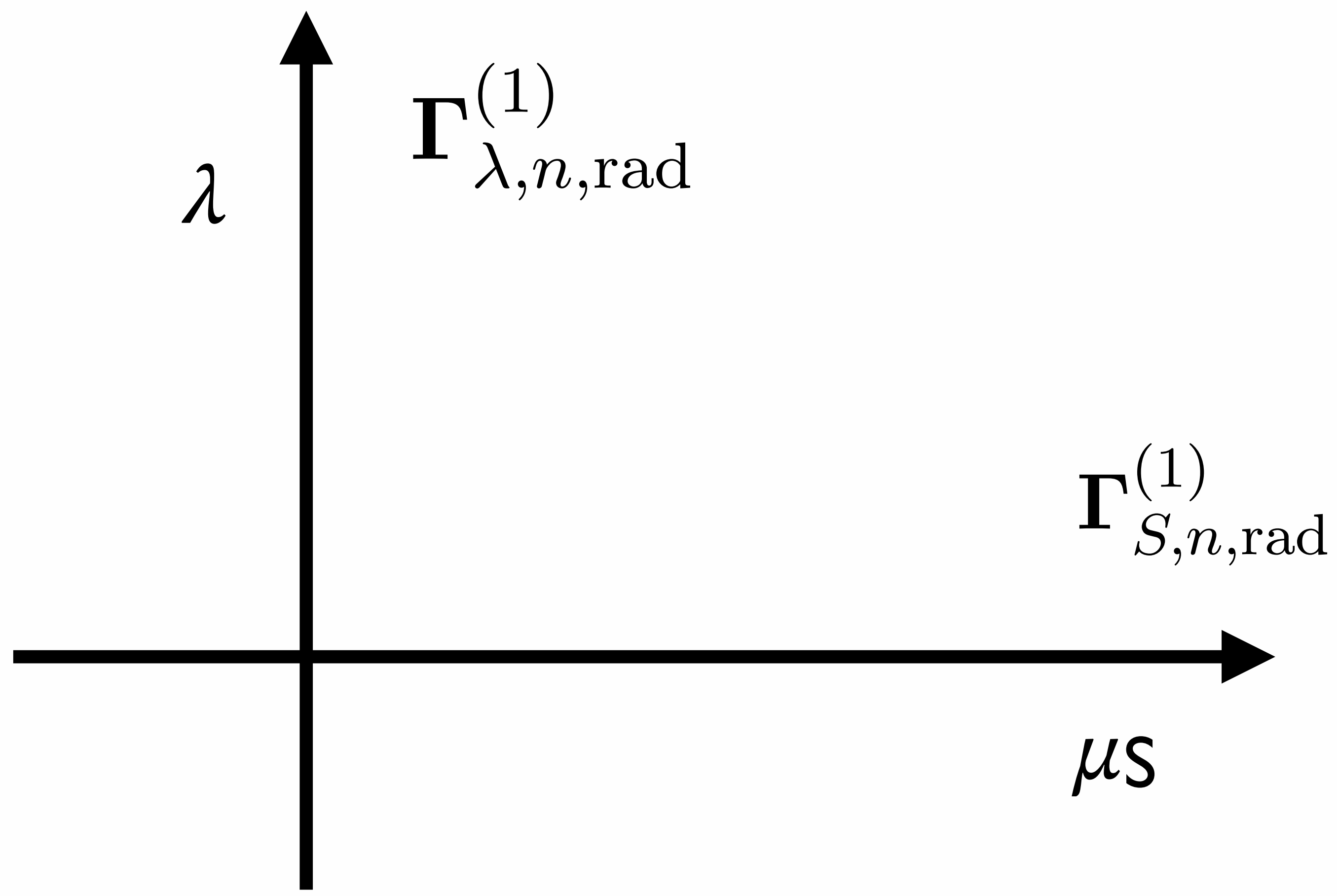
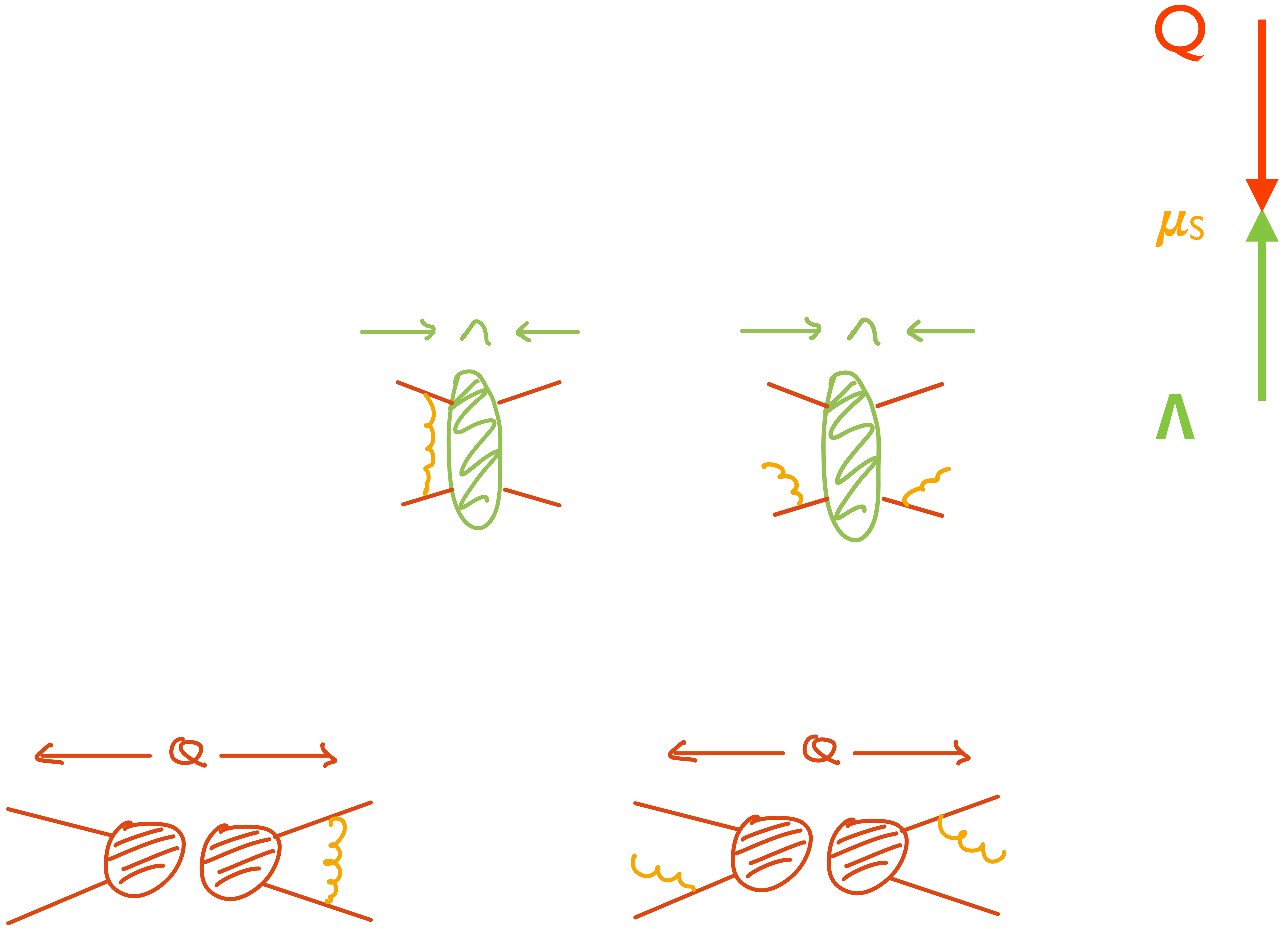
Hard factor reproduces CVolver algorithm and predicts key features of second order evolution.

$\partial_S \mathbf{S}_{,}$ (p_i)



$\partial_S \mathbf{A}$

$$\Theta_{n,1} = 1 - \hat{\Theta}_{n,1}\theta(E_n - \mu_S)$$



Infrared subtractions

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Subtractions necessitate a resolution:
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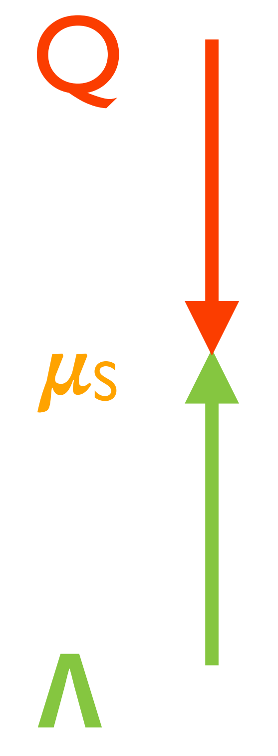
$$\mathbf{U}_n = \mathbf{X}_n^\dagger \mathbf{S}_n \mathbf{X}_n - \sum_{s=1}^{\infty} \alpha_S^s \int \mathbf{F}_{n+s}^{(s)\dagger} \mathbf{S}_{n+s} \mathbf{F}_{n+s}^{(s)} \prod_{i=n+1}^{n+s} \mu_R^{2\epsilon} [\mathrm{d}p_i] \tilde{\delta}(p_i)$$



Enters the re-definition of observables, e.g. demanding a jet cross section

use unitarity and pick equal resolutions $\int \mathbf{D}_{n+1}^{(1,0)\dagger} \mathbf{D}_{n+1}^{(1,0)} \Theta_{n,1} \mu_R^{2\epsilon} [\mathrm{d}p_{n+1}] \tilde{\delta}(p_{n+1}) = -\frac{1}{2} \hat{\mathbf{V}}_n^{(1)} \left[\Theta_{n,1}\right]$

$$\mathbf{U}_n = \mathbf{1}_n u(p_1, \ldots, p_n) - \alpha_s \int \mu_R^{2\epsilon} [\mathrm{d}p_{n+1}] \tilde{\delta}(p_{n+1}) \hat{\mathbf{D}}_{n+1}^{(1,0)\dagger} \hat{\mathbf{D}}_{n+1}^{(1,0)} \Theta_{n,1} \left[u(p_1, \ldots, p_n, p_{n+1}) - u(p_1, \ldots, p_n)\right] + \mathcal{O}(\alpha_s^2)$$

Proof this to vanish or to generate a power correction.

Constructing evolution algorithms

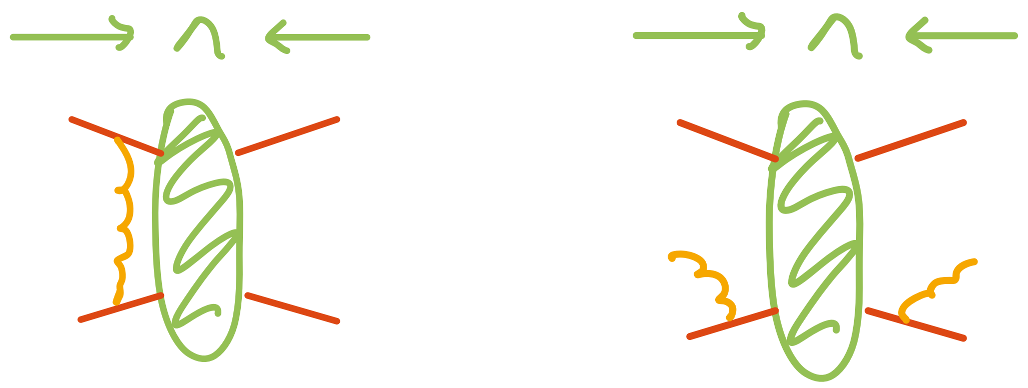
How do we consistently hadronize in light of (improved) shower algorithms?
How to do this at subleading N and higher order shower evolution?

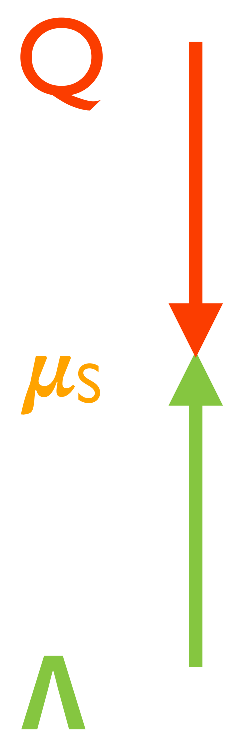
$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

Hard factor reproduces CVolver algorithm and predicts key features of second order evolution.

Subtract iterated contribution in ordered phase space.

$$\mathbf{R}_n^{(2,0)} \circ \mathbf{R}_n^{(2,0)\dagger} = \left(\hat{\mathbf{D}}_n^{(0,2)} \circ \hat{\mathbf{D}}_n^{(0,2)\dagger}\hat{\Theta}_{n,2} - \hat{\mathbf{D}}_n^{(0,1)}\hat{\mathbf{D}}_{n-1}^{(0,1)} \circ \hat{\mathbf{D}}_{n-1}^{(0,1)\dagger}\hat{\mathbf{D}}_n^{(0,1)\dagger}\hat{\Theta}_{n-1,1}\hat{\Theta}_{n,1}\right) \times\theta(E_{n-1}-\mu_S)\delta(E_n-\mu_s)$$
$$+ \hat{\mathbf{D}}_n^{(0,2)} \circ \hat{\mathbf{D}}_n^{(0,2)\dagger}\hat{\Theta}_{n,2}\theta(E_n-\mu_S)\delta(E_{n-1}-\mu_S)$$





Use full double gluon matrix element outside.

Similar consequences for virtual corrections.

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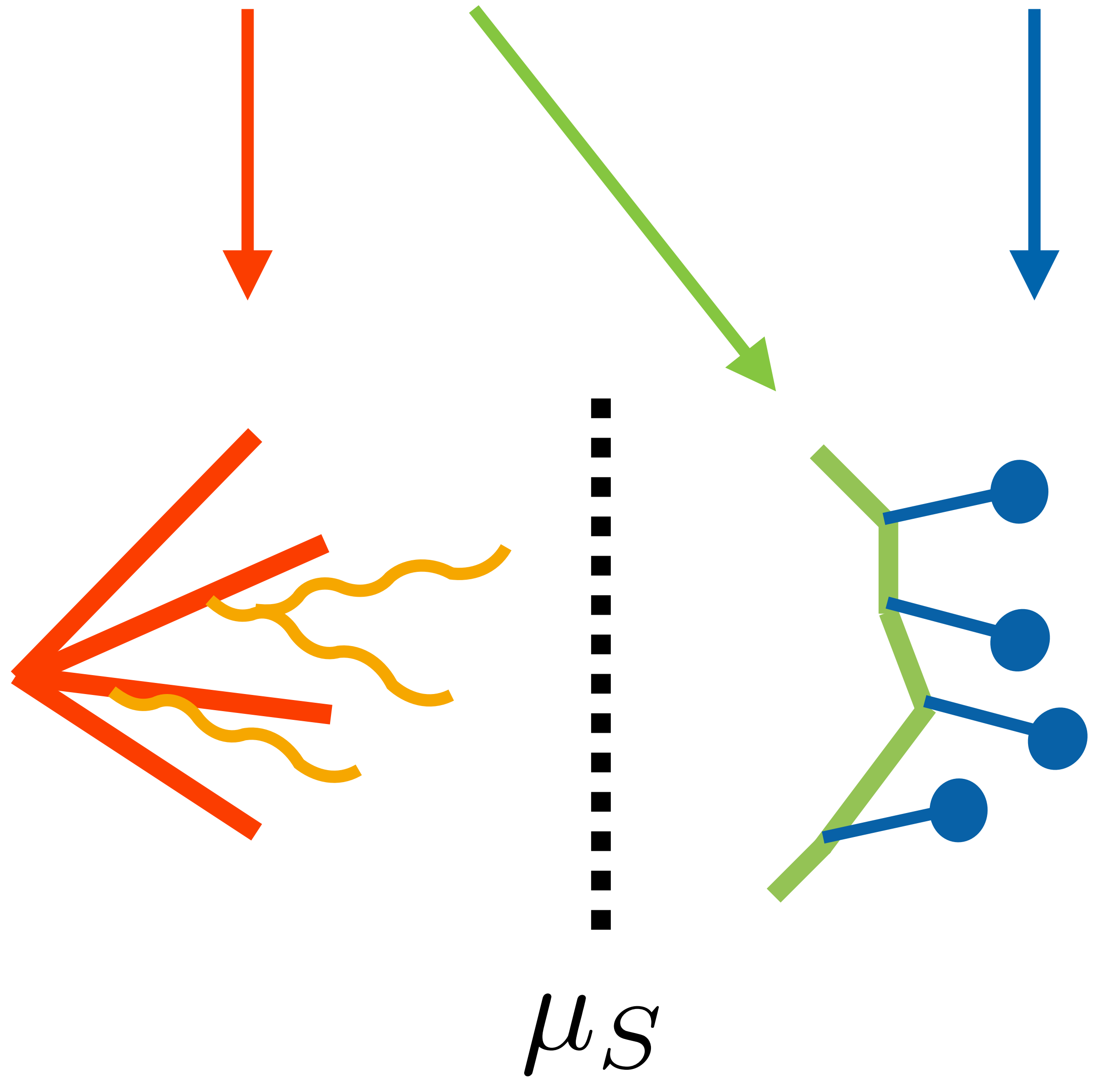
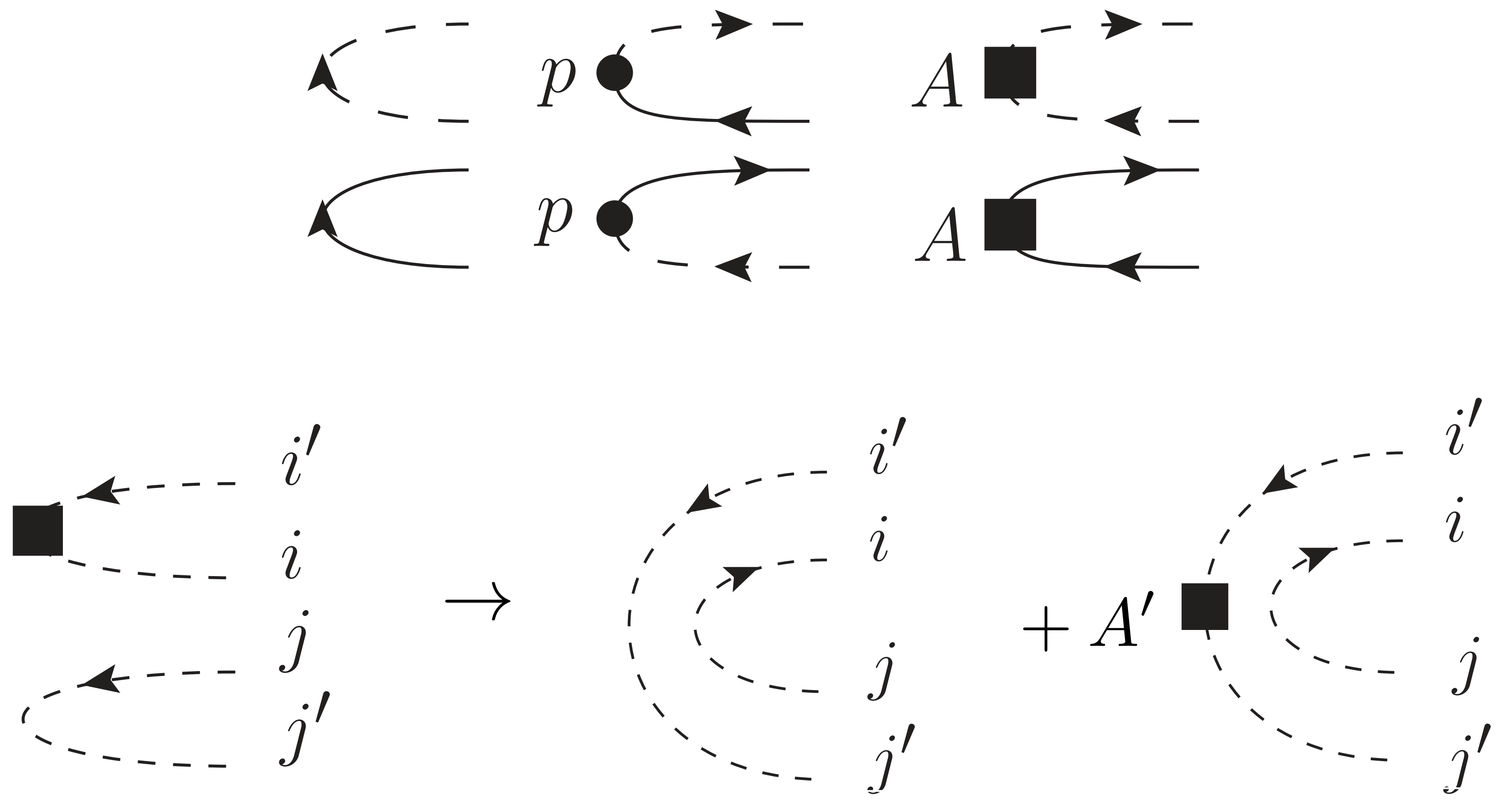
$$0 = \frac{\mathrm{d}}{\mathrm{d}\mu_S}$$



Construct electroweak evolution.
Measurement projection is ubiquitous.



Basis and mixing of chirality structures.

[Plätzer, Sjödahl — '21]

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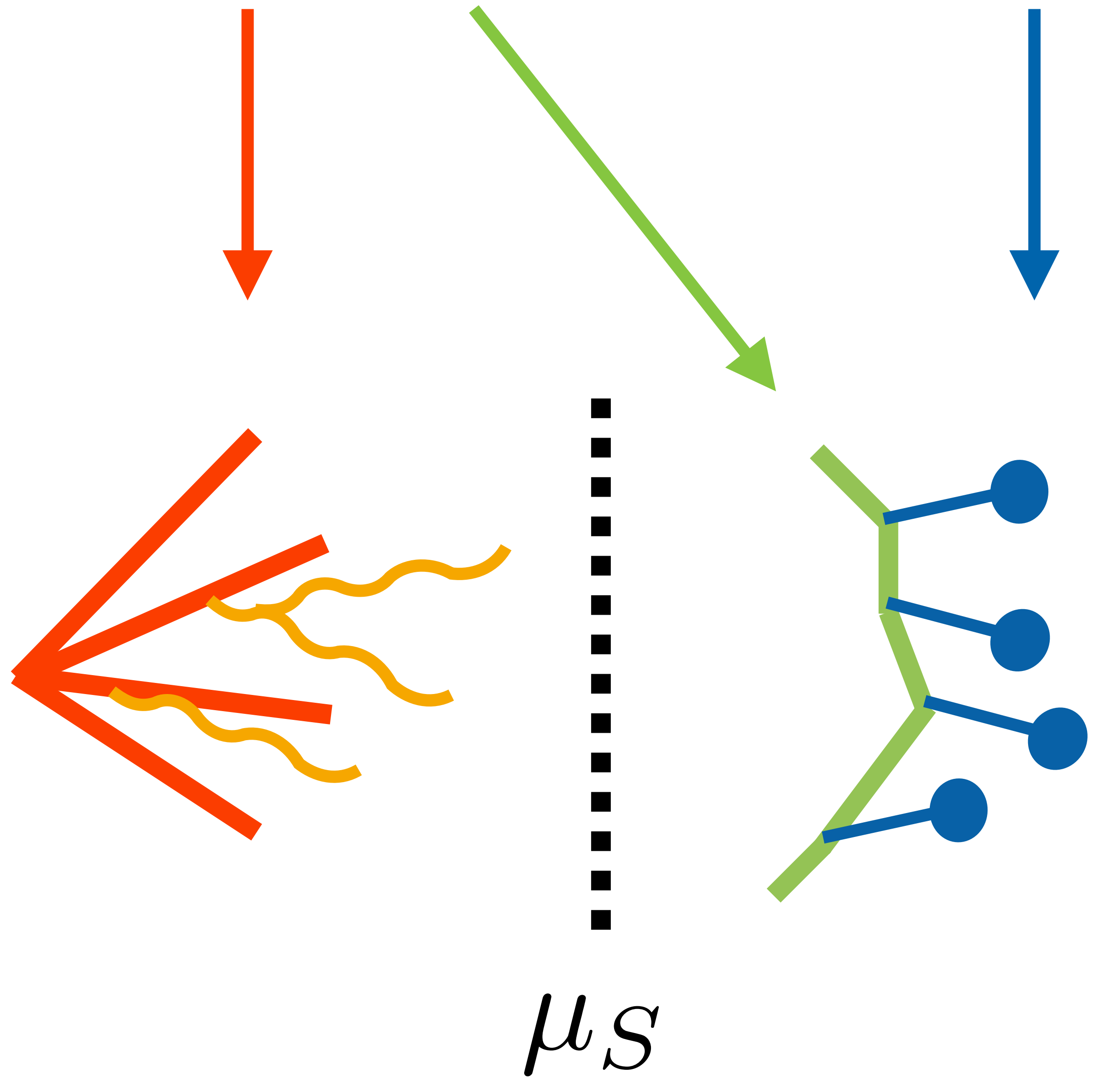

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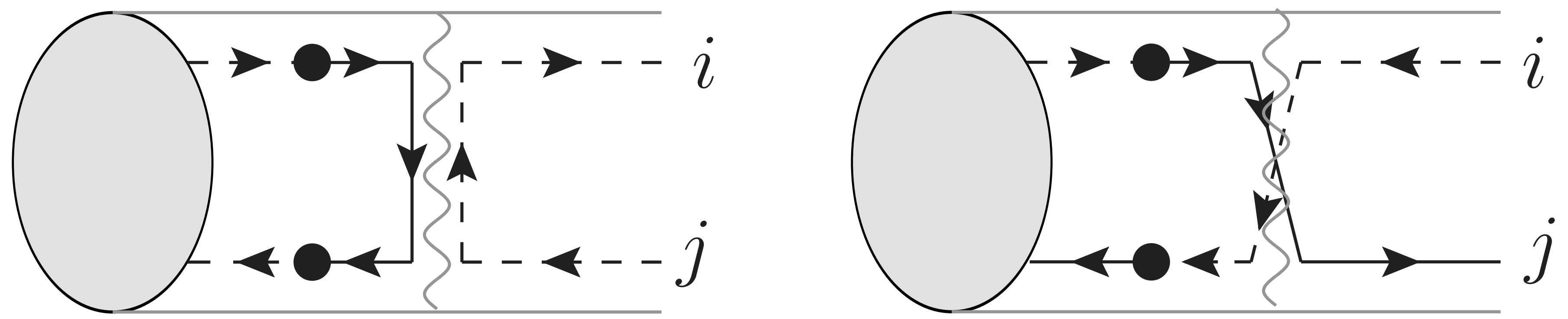

Factorisation and kinematics.

[Plätzer, Sjödahl — '21]

Constructing evolution algorithms

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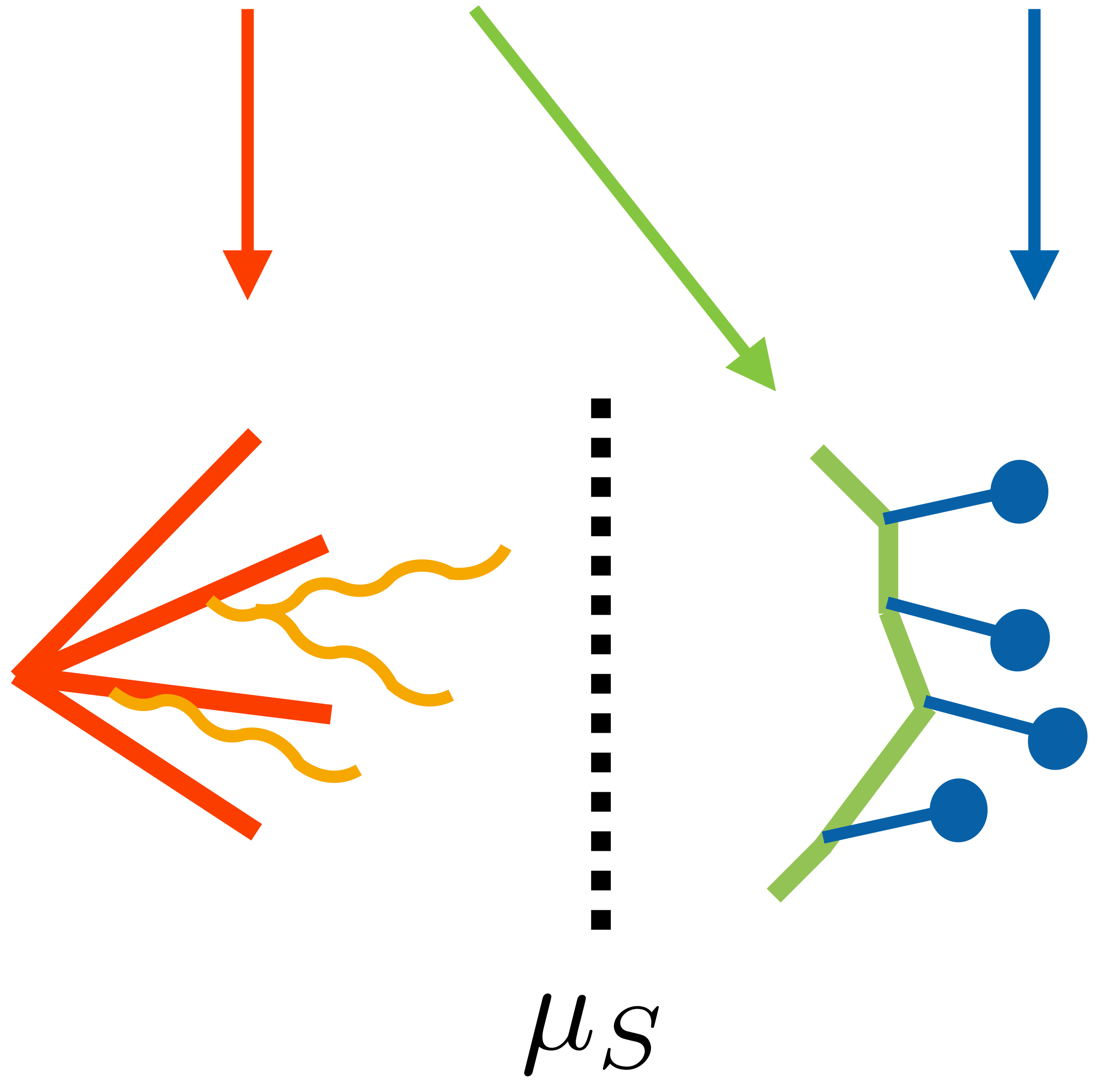


Construct electroweak evolution.

Cutting indicates that subtraction terms refer to different final states — unitarity?

$$\frac{1}{k^2 - m^2 - im\Gamma\ \mathrm{sign}(T\cdot k)} =$$
$$\frac{1}{k^2 - m^2 + im\Gamma} + 2i \frac{m\Gamma}{(k^2-m^2)^2 + m^2\Gamma^2}\theta(T\cdot k)$$

Factorisation and kinematics.

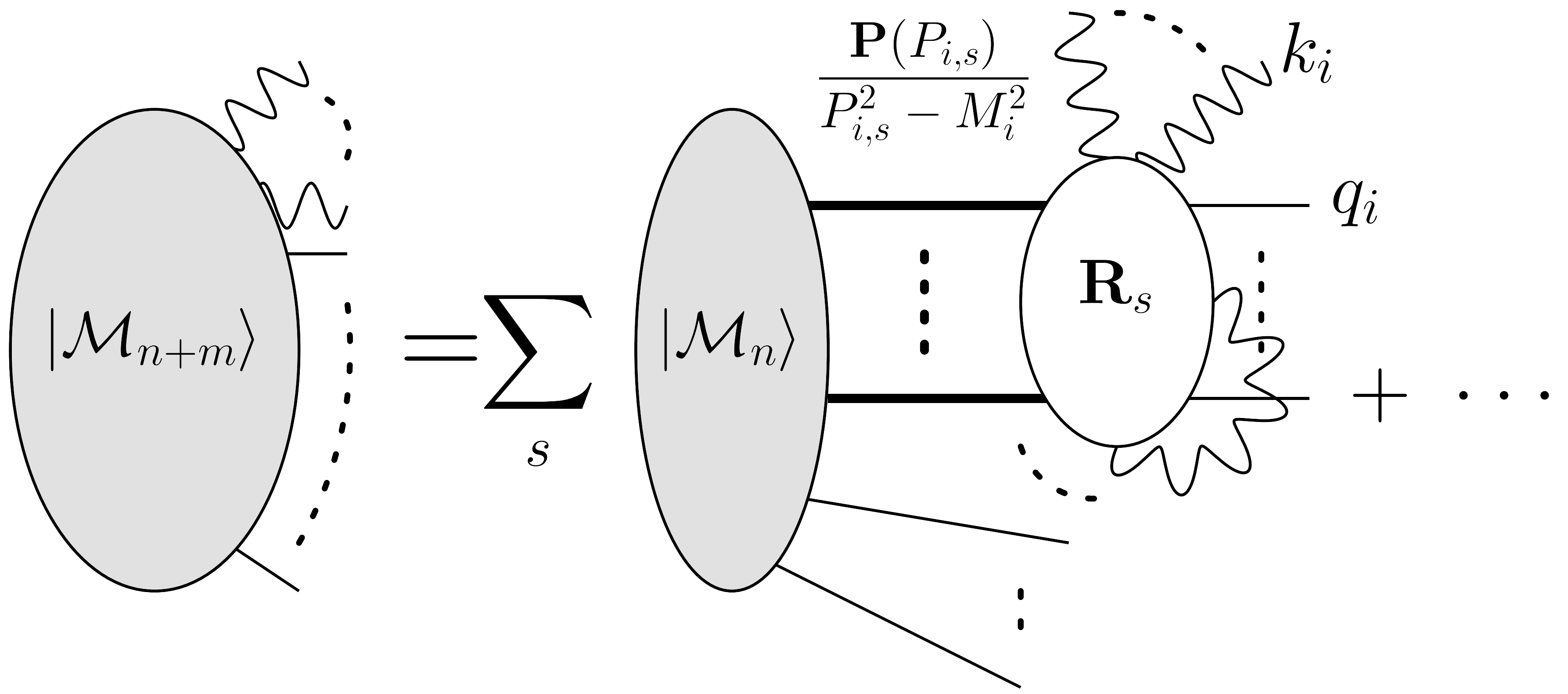
[Plätzer, Sjödahl — '21]

Electroweak evolution

Momentum remapping closely tie in with how factorisation is performed.
Eikonal approximation needs to separate true soft degrees of freedom.

$$(q_i + K_{i,s})^2 - M_i^2 = 2p_i \cdot Q_{i,s}$$

$$p_i \cdot Q_{i,s} \ll p_i \cdot n_{i,s} \equiv S_{i,s}$$



$$K_{i,s}^\mu = \Lambda^\mu{}_\nu \left(Q_{i,s}^\nu + \delta_{i,s}\, n_{i,s}^\nu\right)$$

$$q_i^\mu = \Lambda^\mu{}_\nu \left(\alpha p_i^\nu + \frac{(1-\alpha^2)M_i^2 + p_i \cdot Q_{i,s}}{2\alpha\; n_{i,s} \cdot p_i} n_{i,s}^\nu\right) - K_{i,s}^\mu$$

$$\sum_{n=0}^{\infty} \left(\frac{\mathbf{P}(q_i + K_{i,s}, M_i)}{(q_i + K_{i,s})^2 - \tilde{M}_{R,i}^2} \mathbf{\Sigma}(q_i + K_{i,s}) \right)^n \frac{\mathbf{P}(q_i + K_{i,s}, M_i)}{(q_i + K_{i,s})^2 - \tilde{M}_{R,i}^2} = \frac{1}{2p_i \cdot Q_{i,s}} \frac{\Psi(\Lambda p_i, M_i)\bar{\Psi}(\Lambda p_i, M_i)}{1 - \Sigma'(M_i^2)} + \mathcal{O}(\lambda) \;,$$

[Plätzer, Sjödahl — '21]

Constructing evolution algorithms

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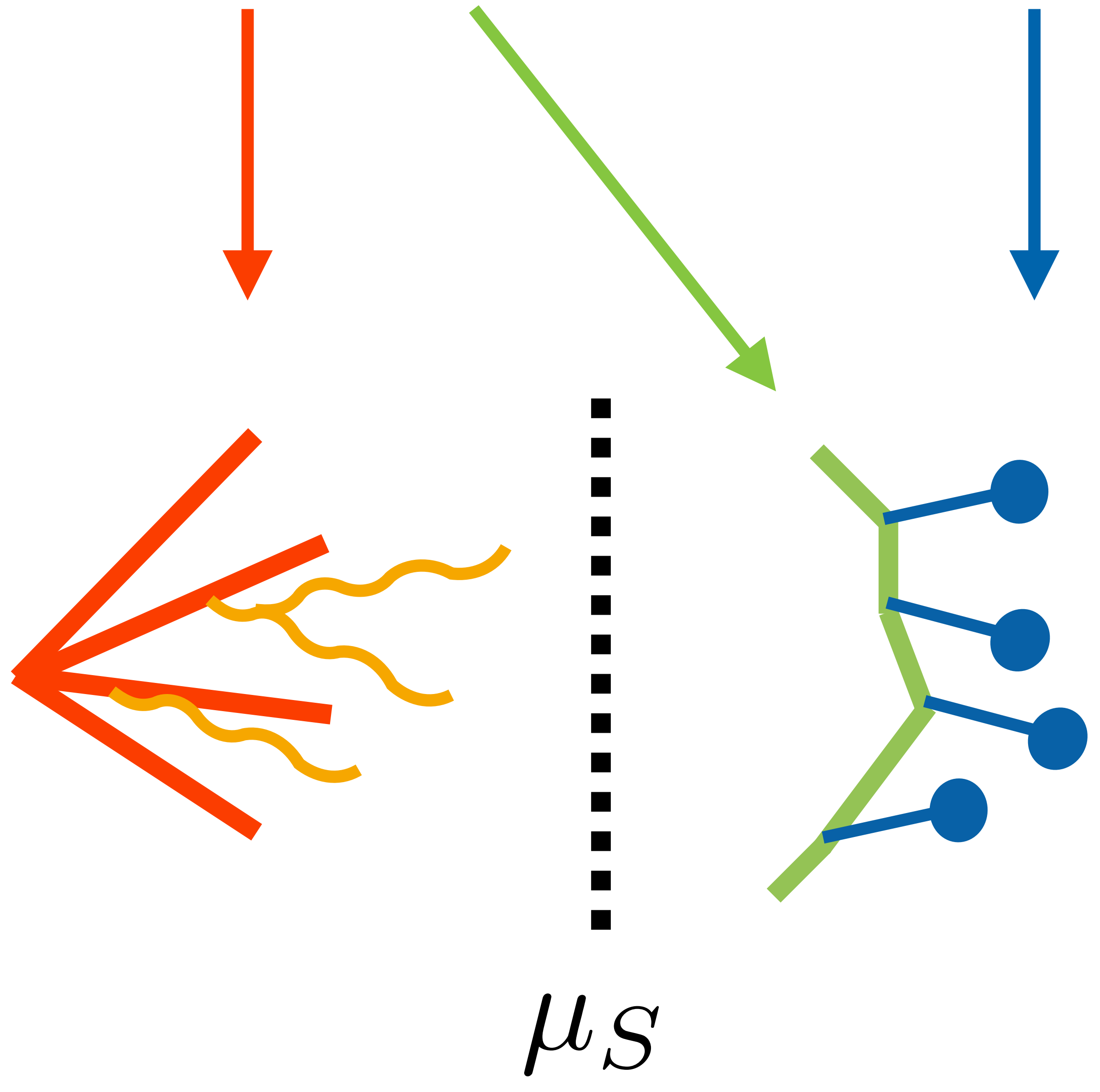
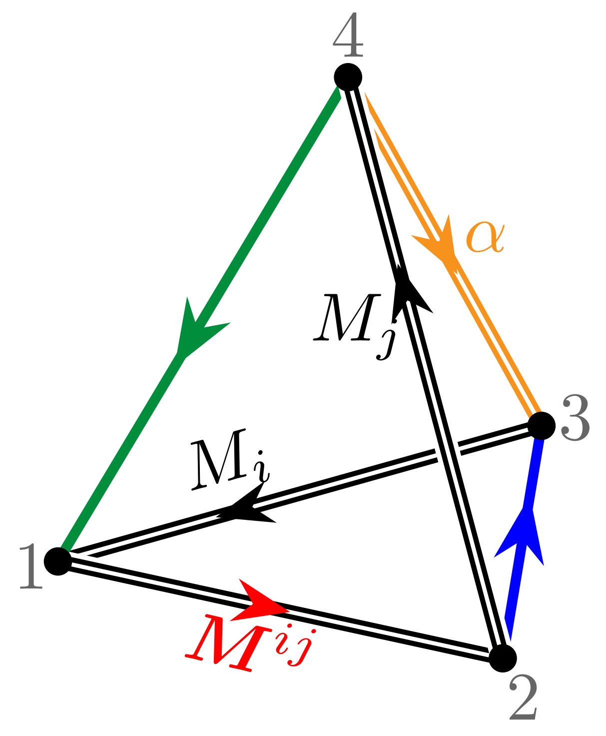
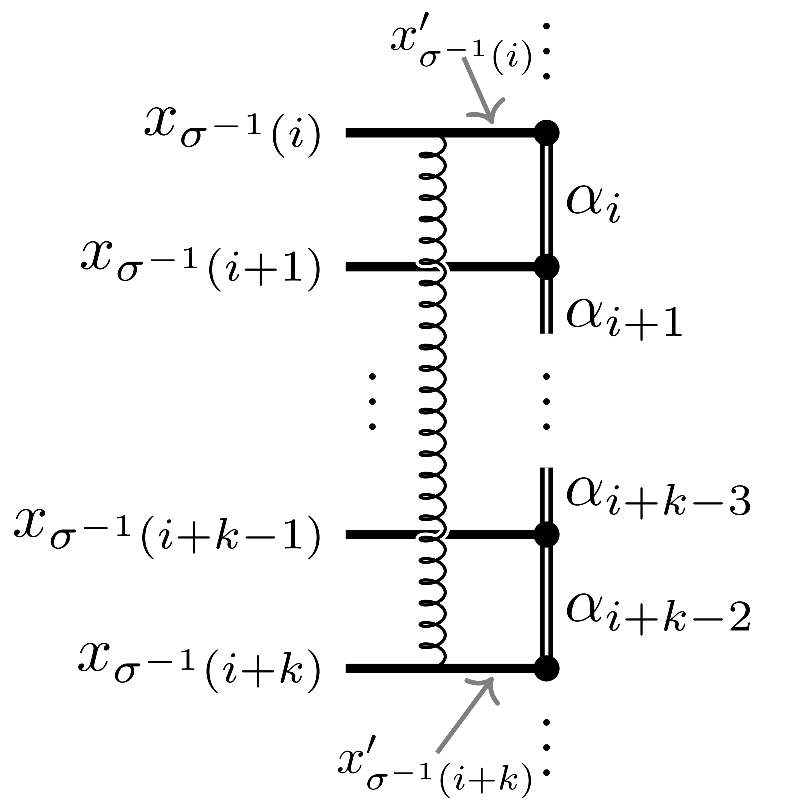
$$0 = \frac{\mathrm{d}}{\mathrm{d}\mu_S}$$



Understand colour multiplets for many legs.





[Alcock-Zeilinger, Keppeler, Plätzer, Sjödahl – ’22 & in progress]

Constructing evolution algorithms

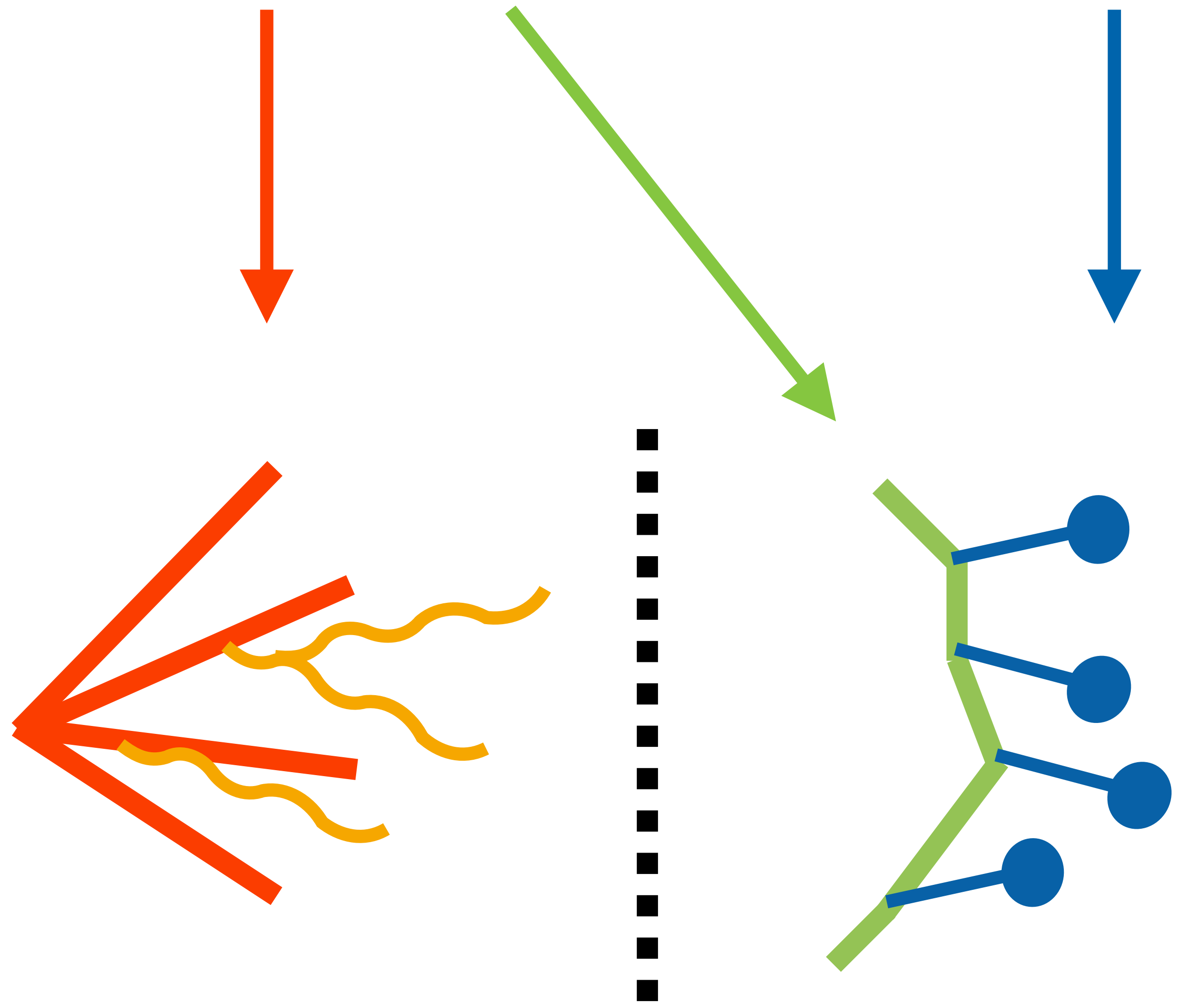
How do we consistently hadronize in light of (improved) shower algorithms?

How to do this at subleading N and higher order shower evolution?

[Plätzer – '22]

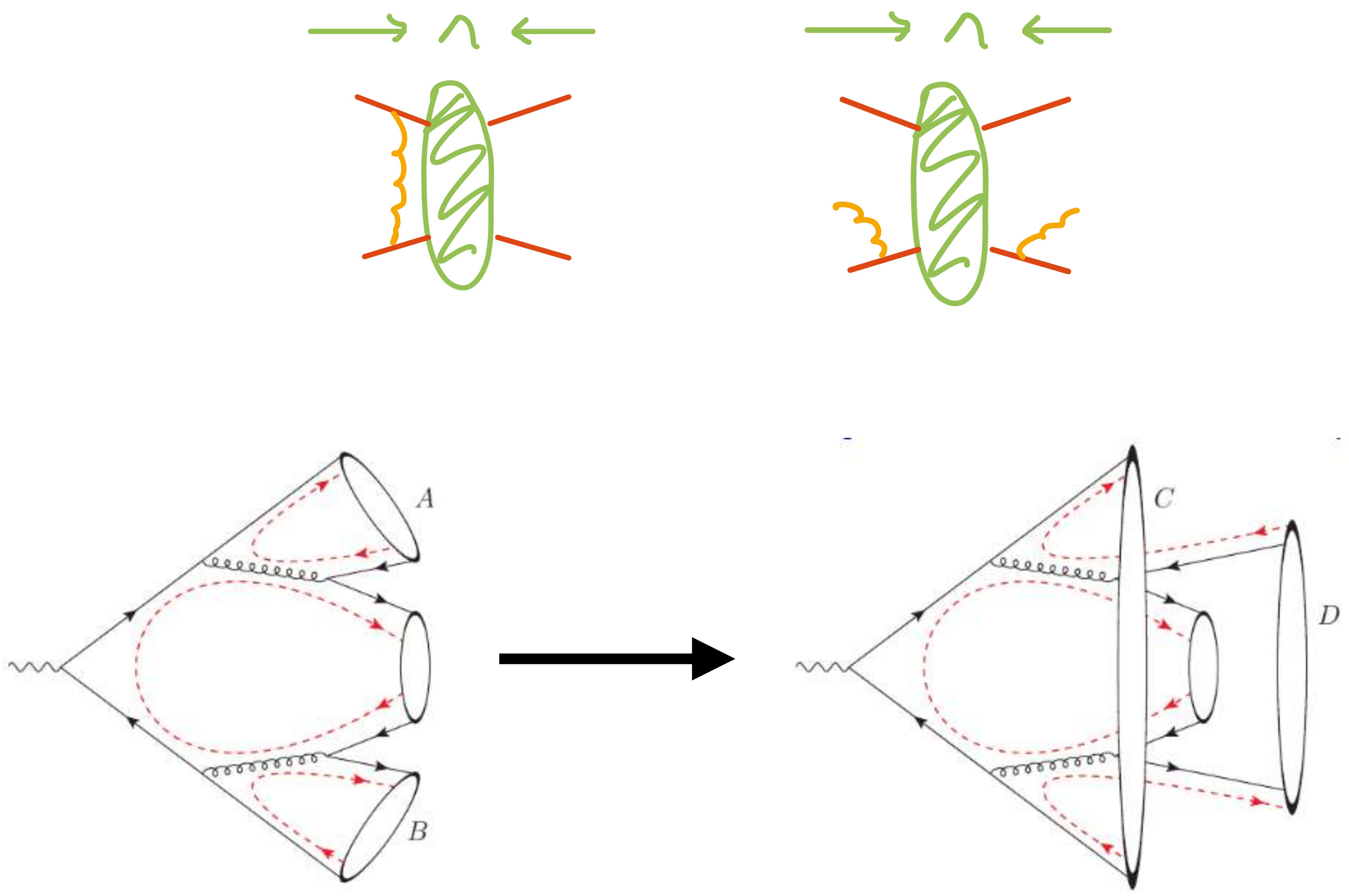
$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

$$0 = \frac{\mathrm{d}}{\mathrm{d}\mu_S}$$



μ_S

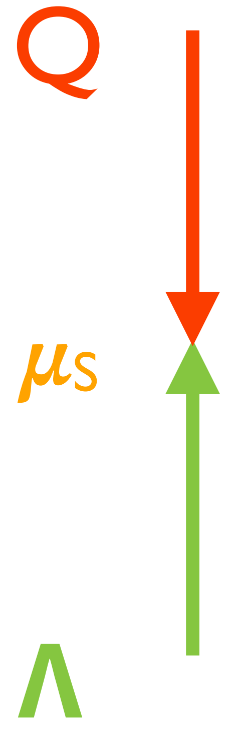
Construct perturbative end of hadronization.



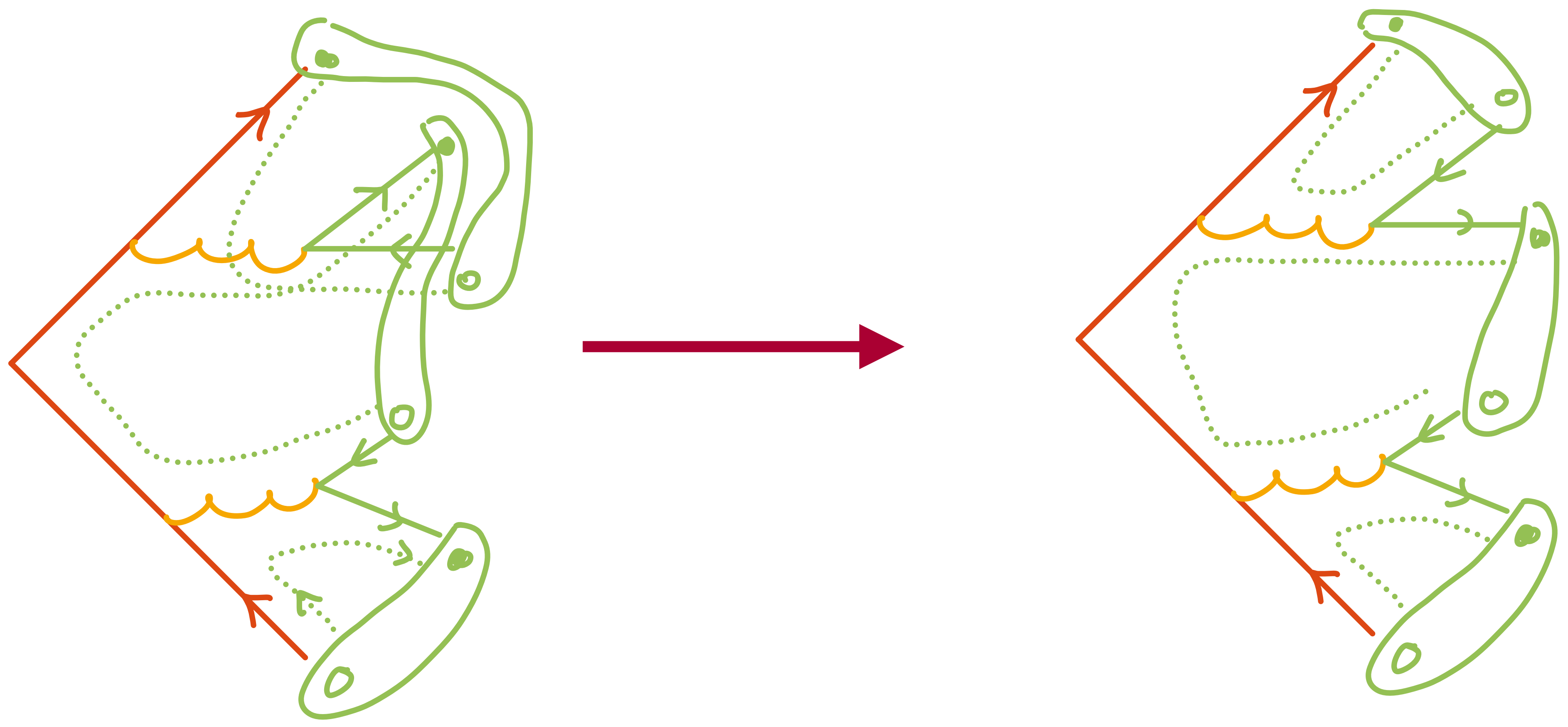
e.g. colour reconnection *implied* just as observed in [Gieseke, Kirchgaesser, Plätzer – '18 …]

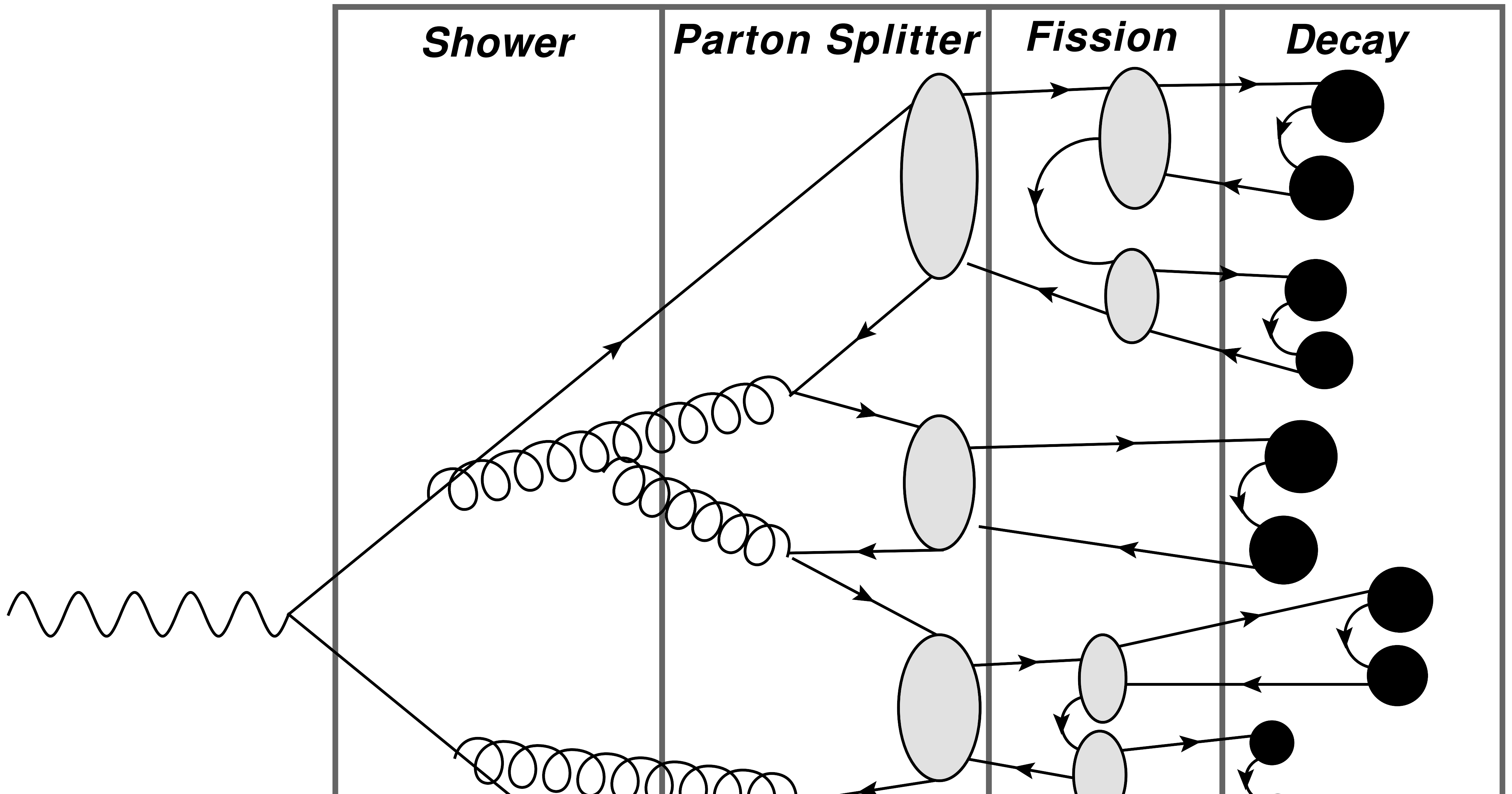
Hadronization models

$$\mathrm{d}\sigma \sim \mathrm{L} \times \mathrm{d}\sigma_H(Q) \times \mathrm{PS}(Q \to \mu) \times \mathrm{MPI} \times \mathrm{Had}(\mu \to \Lambda) \times \ldots$$



$$\partial_S \mathbf{S}_n = -\tilde{\mathbf{\Gamma}}^{\dagger}_{S,n}\mathbf{S}_n - \mathbf{S}_n\tilde{\mathbf{\Gamma}}_{S,n} + \sum_{s\geq 1}\alpha_S^s \int \tilde{\mathbf{R}}^{(s)\dagger}_{S,n+s}\mathbf{S}_{n+s}\tilde{\mathbf{R}}^{(s)}_{S,n+s}\prod_{i=n+1}^{n+s}[\mathrm{d}p_i]\tilde{\delta}(p_i)$$

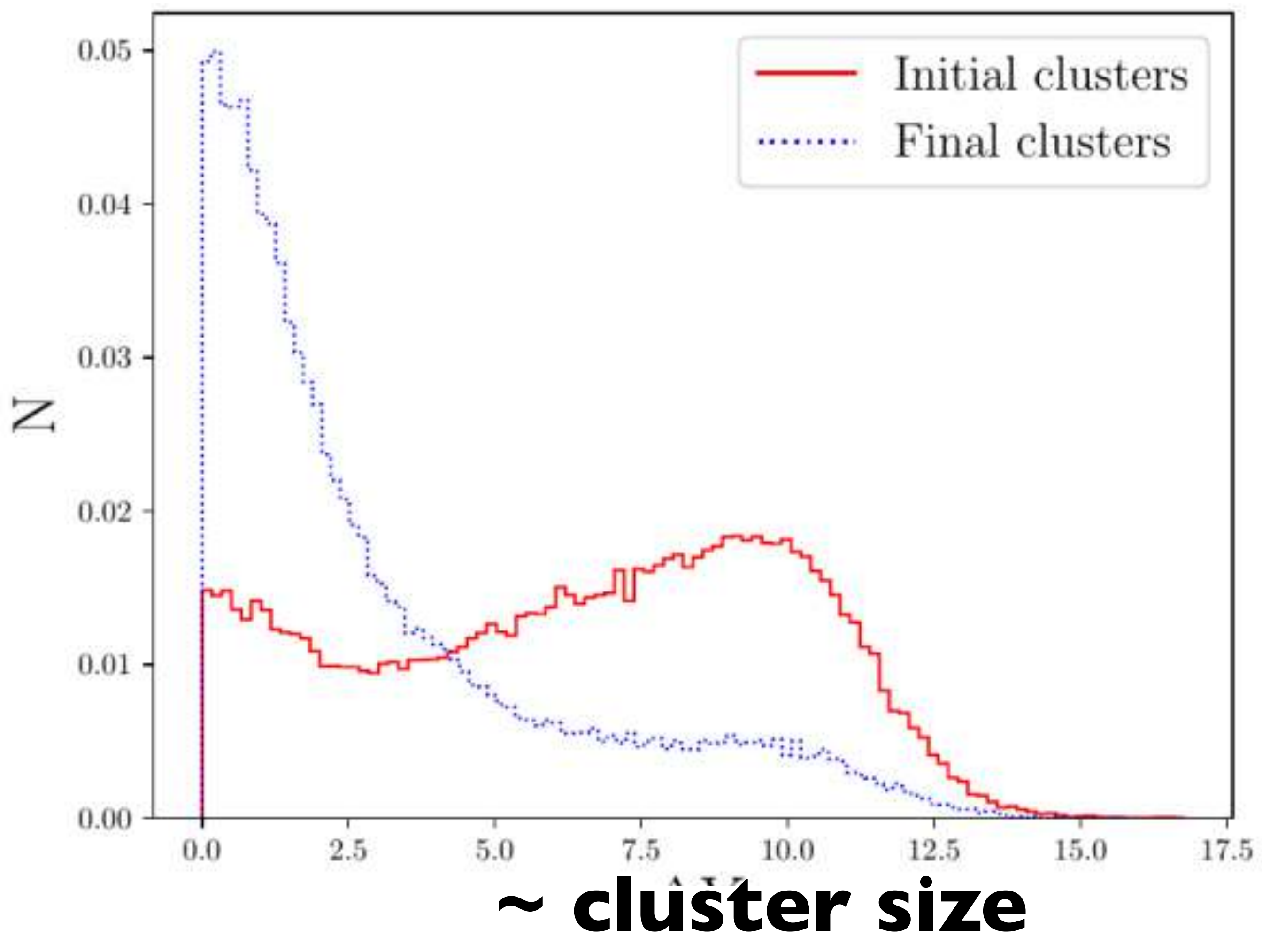




Hadronization models

$$d\sigma \sim \mathrm{L} \times d\sigma_H(Q) \times \mathrm{PS}(Q \to \mu) \times \mathrm{MPI} \times \mathrm{Had}(\mu \to \Lambda) \times \ldots$$

amplitude evolution



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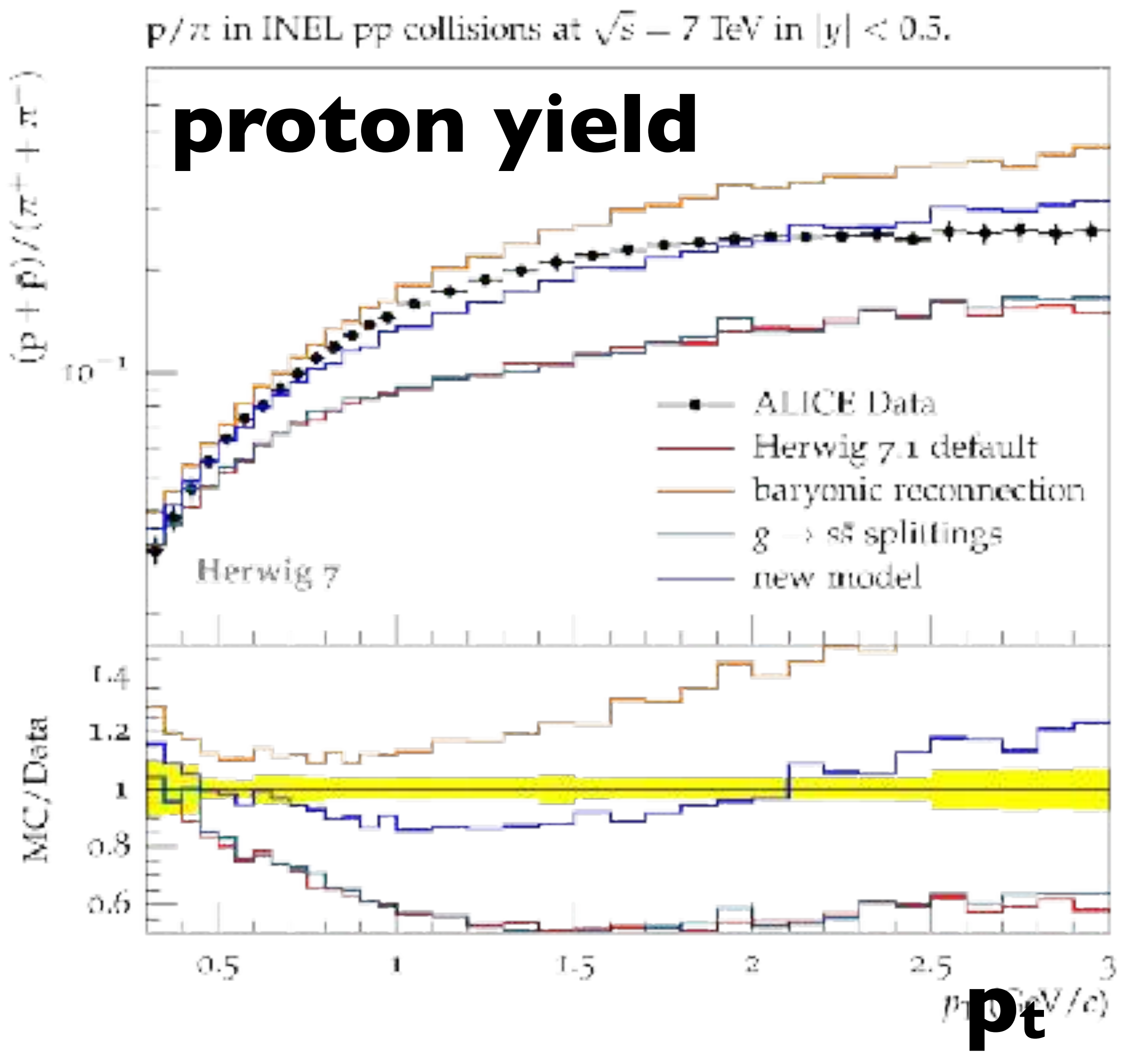
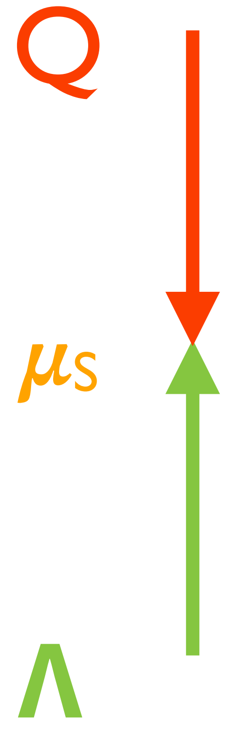
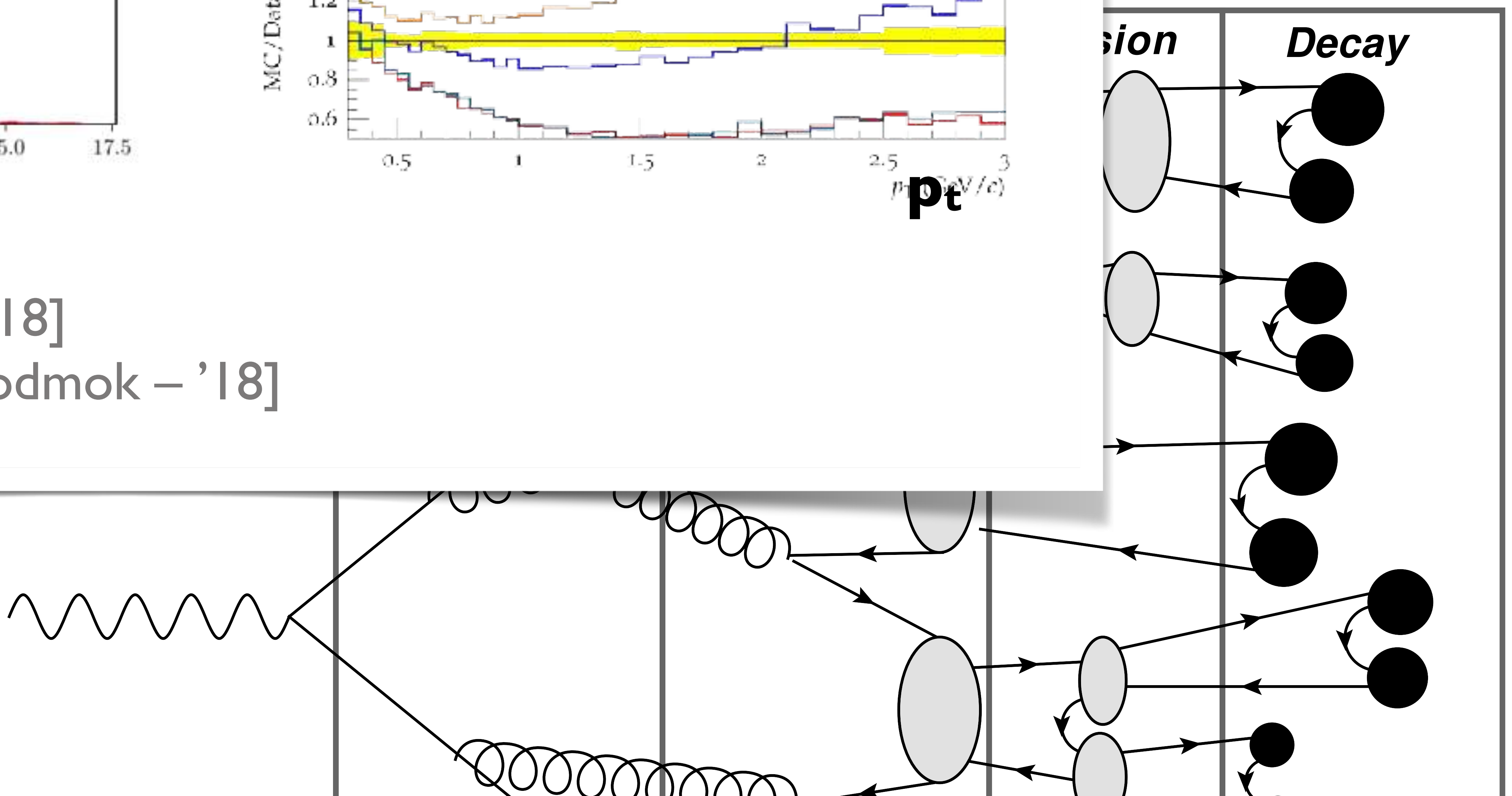


[Gieseke, Kirchgaesser, Plätzer – '18]
[Gieseke, Kirchgaesser, Plätzer, Siodmok – '18]





Summary

Amplitude evolution is much more than just studying subleading-N effects.
We use to as a theoretical tool and algorithm in its own right.

We can address the structure of evolution algorithms, at leading and higher orders.
Systematic break down in large-N allows us to solidify structure of new algorithms.

[in progress for second order]

Infrared cutoff is the factorisation scale to hadronization models and allows us to construct their high-energy end from perturbative considerations, including colour reconnection.

explored for non-globals and in Herwig, see Andrzej's talk
[Hoang, Plätzer, Samitz — in progress]
[Gieseke, Kiebacher, Plätzer, Priedigkeit — in progress]

If we want to thoroughly understand electroweak evolution beyond the quasi-collinear limit nothing allows us to bypass this framework.

Thank you.