A gauge choice for infrared singularities

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Interpolating gauge

- I describe *interpolating gauge*, invented by Doust (1987) and by Baulieu and Zwanziger (1999).
- Our interest is in simplifying the description of the soft and collinear singularities of QCD.
- This may be useful for defining subtractions that remove the soft and collinear singularities in perturbative QCD calculations.
- Our particular interest is in defining the splitting functions for a parton shower at order α_s^2.

- Interpolating gauge interpolates between Feynman gauge (or Lorenz gauge) and Coulomb gauge.

- Doust and Baulieu and Zwanziger were interested in providing a better definition of Coulomb gauge.

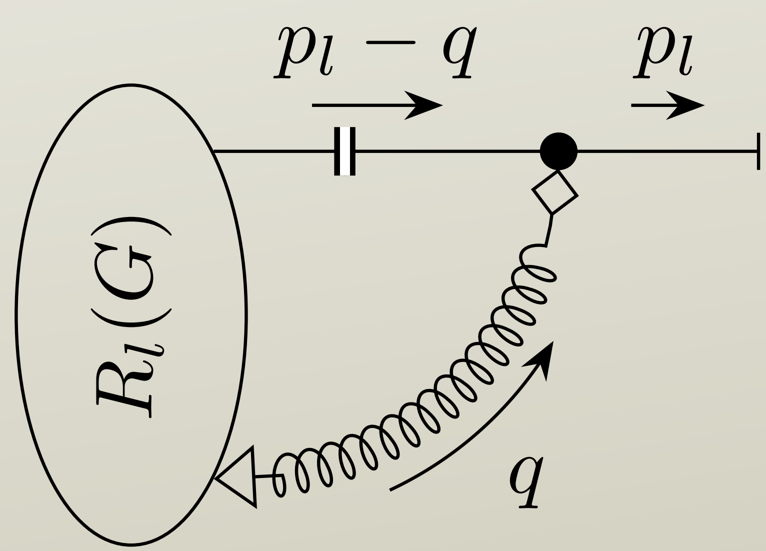
- With our different goal, we adopt a different notation and emphasize different features of the gauge.

- We also explore technical issues in some detail.

Why not Feynman gauge?

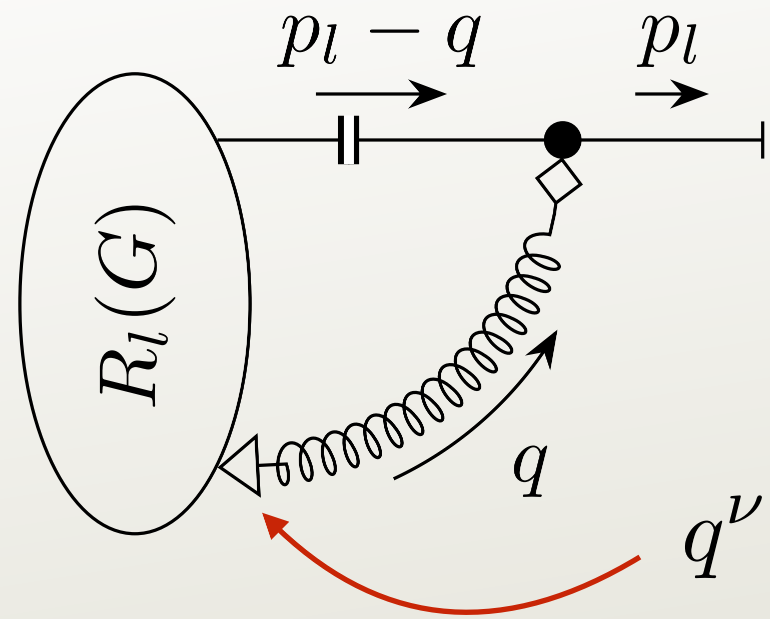
- The gluon propagator in Feynman gauge is very simple.
- But consider a virtual gluon with momentum q that couples to an external line with momentum p_l.

$$\sum_l \int \frac{d^d q}{(2\pi)^d} \sum_G$$



- There are collinear singularities that give a logarithmic divergences from $q \to x p_l$.

$$\sum_l \int \frac{d^d q}{(2\pi)^d} \sum_G$$

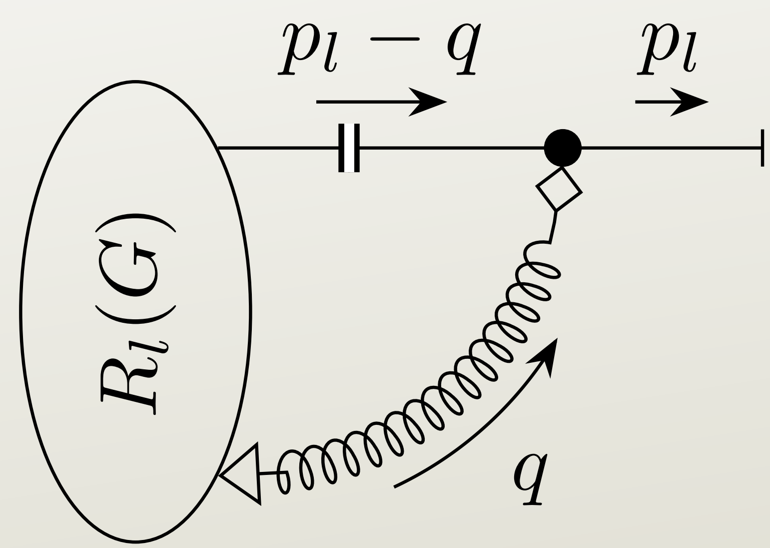


- The collinear divergences appear even when the gluon couples to an off-shell internal line in the graph.
- The gluon has an unphysical polarization:

$$J_\mu D^{\mu\nu}(q) \propto q^\nu$$

- We can use Ward identities to get rid of these.
- But this is more complicated when there are multiple gluons collinear to different external partons.
- *Cf.* C. Anastasiou and G. Sterman, *Locally finite two-loop QCD amplitudes from IR universality for electroweak production*, JHEP 05 (2023) 242.

$$\sum_{l} \int \frac{d^d q}{(2\pi)^d} \sum_{G}$$



- It might be better if these unphysical collinear singularities did not occur.

Definition of interpolating gauge

- Use a special reference frame defined by a vector n, with $n^2 = 1$.
- Define a tensor $h^{\mu\nu}$ with components in the $\vec{n} = 0$ frame

$$h^{\mu\nu} = \begin{pmatrix} 1/v^2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- For any vector q we define an associated vector $\tilde{q}$ by

$$\tilde{q}^\mu = h^\mu_\nu \, q^\nu$$

$$\tilde{q} = \left(\frac{q^0}{v^2}, \vec{q} \right)$$

- Use gauge fixing condition $G[A] = 0$ with

$$G[A]_c(x) = \tilde{\partial}_\mu A_c^\mu(x) - \omega_c(x)$$

- Compare this to

$$G'[A]_c(x) = \partial_\mu A_c^\mu(x) - \omega_c(x)$$

for a covariant gauge.

- Thus we replace g_ν^μ by the modified metric tensor h_ν^μ in the gauge fixing.

$$\partial_\mu A_c^\mu(x) = \partial_\mu g_\nu^\mu A_c^\nu(x) \rightarrow \tilde{\partial}_\mu A_c^\mu(x) = \partial_\mu h_\nu^\mu A_c^\nu(x)$$

- The gauge fixing Lagrangian is

$$\mathcal{L}_{\mathrm{GF}}(x) = -\frac{v^2}{2\xi}\,(\tilde{\partial}_\mu A_a^\mu(x))(\tilde{\partial}_\nu A_a^\nu(x))$$

- Parameters ξ and v (and n) determine the gauge choice.

- The tree-level gluon propagator is then

$$D^{\mu\nu}(q) = \frac{1}{q^2 + \mathrm{i}0}\left[-\, g^{\mu\nu} + \frac{q^\mu\, \tilde{q}^\nu + \tilde{q}^\mu\, q^\nu}{q\cdot\tilde{q} + \mathrm{i}0} - \left(1 + \frac{1}{v^2}\right)\frac{q^\mu\, q^\nu}{q\cdot\tilde{q} + \mathrm{i}0}\right] - \frac{\xi - 1}{v^2}\,\frac{q^\mu\, q^\nu}{(q\cdot\tilde{q} + \mathrm{i}0)^2}$$

- Usually we choose $\xi = 1$.

The gluon propagator

- We divide the tree-level propagator into two parts:

$$D^{\mu\nu}(q) = D_{\mathrm{T}}^{\mu\nu}(q) + D_{\mathrm{L}}^{\mu\nu}(q)$$

- Choose $\xi = 1$. Then

$$D_{\mathrm{T}}^{\mu\nu}(q) = \frac{1}{q^2 + \mathrm{i}0} \sum_{s=1,2} \varepsilon^{\mu}(q,s)\,\varepsilon^{\nu}(q,s)$$

- Here $\varepsilon(q,s) \cdot \varepsilon(q,s') = -\delta_{s,s'}$ and

$$\varepsilon(q,s) \cdot n = 0$$

$$\varepsilon(q,s) \cdot q = 0$$

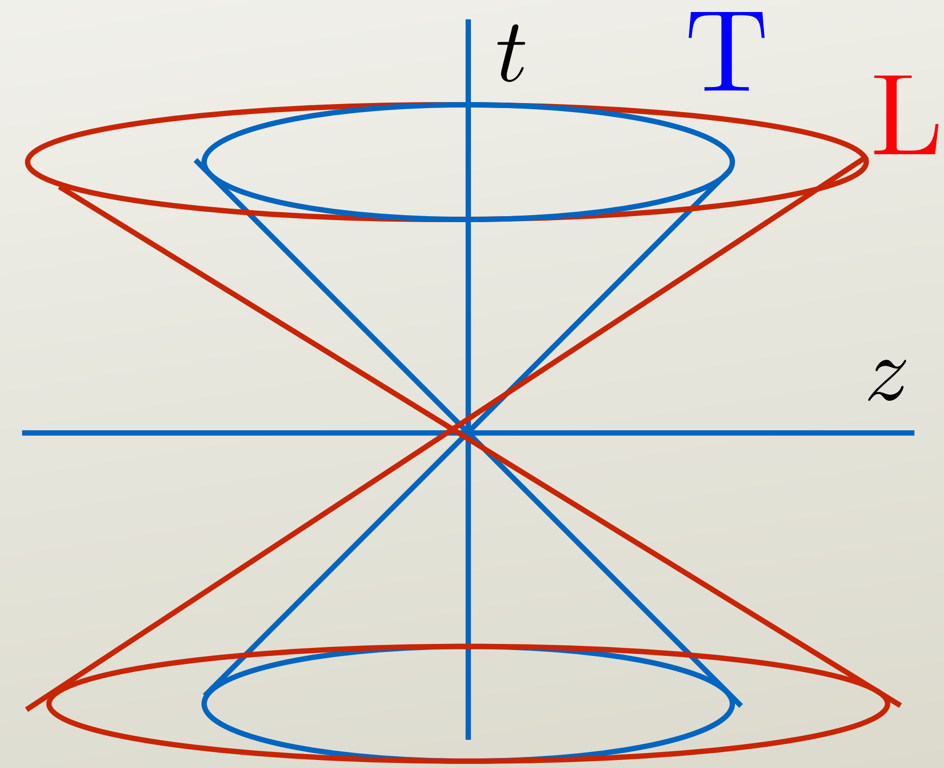
- This describes the propagation of transversely polarized gluons.

- The tree-level propagator for the L-gluons is

$$D_{\rm L}^{00}(q) = -\frac{1}{q\cdot\tilde{q} + {\rm i}0}\ ,$$

$$D_{\rm L}^{0i}(q) = D_{\rm L}^{i0}(q) = 0\ ,$$

$$D_{\rm L}^{ij}(k) = \frac{1}{v^2}\,\frac{1}{q\cdot\tilde{q} + {\rm i}0}\,\frac{q^i q^j}{\vec{q}^{\,2}}$$



- Note the denominators

$$\frac{1}{q\cdot\tilde{q} + {\rm i}0}$$

- $q\cdot\tilde{q} = (q^0)^2/v^2 - \vec{q}^{\,2}$, the condition for on-shell propagation is

$$q^0 = \pm v|\vec{q}| \qquad\qquad |\vec{x}| = vt$$

- The L-gluons propagate with speed v in the $\vec{n} = 0$ frame.

- If we take $v = 1$, we get Feynman gauge for $\xi = 1$ and Lorenz gauge for $\xi = 0$.

- If we take $v \to \infty$, we get Coulomb gauge.

$$\frac{1}{q \cdot \tilde{q} + \mathrm{i}0} \to -\frac{1}{|\vec{q}\,|^2}$$

- The L-gluons give the Coulomb force, which propagates with infinite speed in the $\vec{n} = 0$ frame.

- Now Coulomb gauge is defined as a limit.

- We do not need $v \to \infty$: $v = 2$ is fine.

The full gluon propagator

- The full propagator obeys

$$G^{\mu}_{\nu}(p) = D^{\mu}_{\nu}(p) + G^{\mu}_{\alpha}(p)\Pi^{\alpha}_{\beta}(p)D^{\beta}_{\nu}(p)$$

$$D^{\mu\nu}(p) = D^{\mu\nu}_{\mathrm{T}}(p) + D^{\mu\nu}_{\mathrm{L}}(p)$$

$$D^{\mu\nu}_{\mathrm{T}}(p) = \frac{P^{\mu\nu}_{\mathrm{T}}(p)}{p^2 + \mathrm{i}0} \qquad D^{\mu\nu}_{\mathrm{L}}(p) = \frac{P^{\mu\nu}_{\mathrm{L}}(p)}{p \cdot \tilde{p} + \mathrm{i}0}$$

- We can also decompose $\Pi^{\mu\nu}$:

$$\Pi^{\mu\nu}(p) = \Pi_{\rm T}^{\mu\nu}(p) + \Pi_{\rm L}^{\mu\nu}(p)$$

where

$$\Pi_{\rm T}^{\mu\nu}(p) = P_{\rm T}^{\mu\nu}(p)\, \pi_{\rm T}(p)$$

$$\Pi_{\rm L}^{\mu\nu}(p) = \alpha\, p^\mu p^\nu + \beta\, n^\mu n^\nu + \gamma\, (p^\mu n^\nu + n^\mu p^\nu)$$

- Also

$$D_{\rm L}^{\mu\nu}(p) = \alpha'\, p^\mu p^\nu + \beta'\, n^\mu n^\nu + \gamma'\, (p^\mu n^\nu + n^\mu p^\nu)$$

- $P_{\rm T}$ has the properties

$$p_\nu P_{\rm T}^{\nu\mu}(p) = 0 \qquad\qquad n_\nu P_{\rm T}^{\nu\mu}(p) = 0$$

- This gives us

$$D_{\mathrm{T}} \cdot \Pi_{\mathrm{L}} = \Pi_{\mathrm{L}} \cdot D_{\mathrm{T}} = 0$$

$$\Pi_{\mathrm{T}} \cdot D_{\mathrm{L}} = D_{\mathrm{L}} \cdot \Pi_{\mathrm{T}} = 0$$

- Then

$$G^{\mu\nu}(p) = G^{\mu\nu}_{\mathrm{T}}(p) + G^{\mu\nu}_{\mathrm{L}}(p)$$

$$G_{\mathrm{T}} = D_{\mathrm{T}} + D_{\mathrm{T}} \cdot \Pi_{\mathrm{T}} \cdot D_{\mathrm{T}} + D_{\mathrm{T}} \cdot \Pi_{\mathrm{T}} \cdot D_{\mathrm{T}} \cdot \Pi_{\mathrm{T}} \cdot D_{\mathrm{T}} + \cdots$$

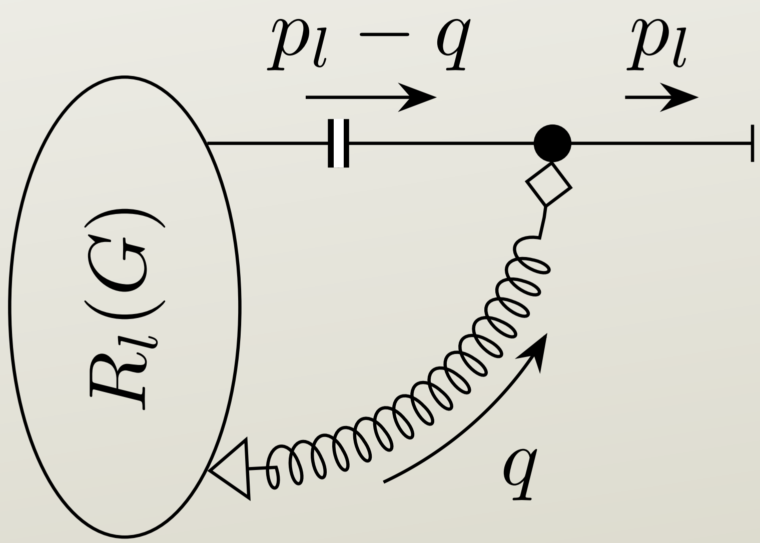
$$G_{\mathrm{L}} = D_{\mathrm{L}} + D_{\mathrm{L}} \cdot \Pi_{\mathrm{L}} \cdot D_{\mathrm{L}} + D_{L} \cdot \Pi_{\mathrm{L}} \cdot D_{\mathrm{L}} \cdot \Pi_{\mathrm{L}} \cdot D_{\mathrm{L}} + \cdots$$

- The propagator $G^{\mu\nu}_{\mathrm{T}}(p)$ for T-gluons has poles at $p^2 = 0$ but no poles at $p \cdot \tilde{p} = 0$.

- The propagator $G^{\mu\nu}_{\mathrm{L}}(p)$ for L-gluons has poles at $p \cdot \tilde{p} = 0$ but no poles at $p^2 = 0$.

Why might interpolating gauge be useful?

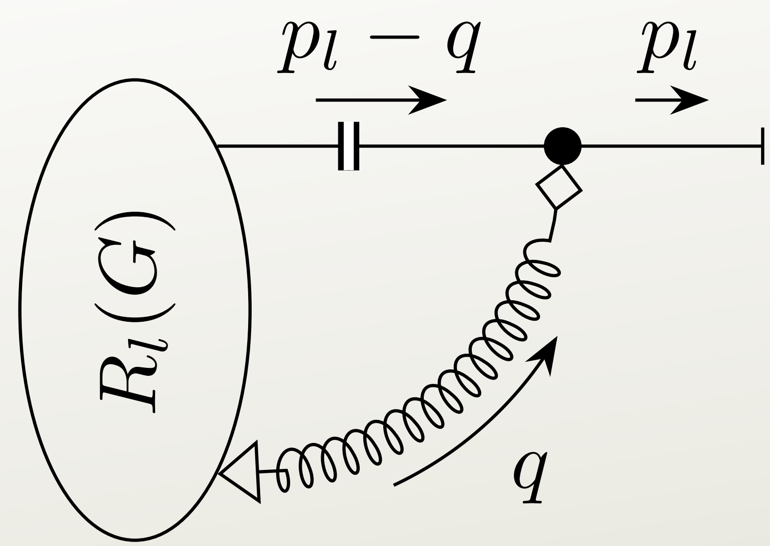
$$\sum_l \int \frac{d^dq}{(2\pi)^d} \sum_G$$



- T-gluons do not give collinear divergences except for self-energy insertions on an external leg.

- That is because $q \cdot \varepsilon(q, s) = 0$.

$$\sum_l \int \frac{d^d q}{(2\pi)^d} \sum_G$$



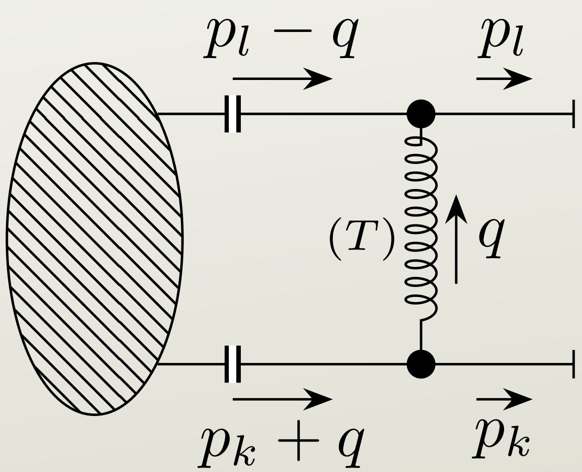
- L-gluons do not give collinear divergences.

- That is because if $p_l^2 = 0$ and $q = x p_l$ then $(p_l - q)^2 = 0$ but $q \cdot \tilde{q} \neq 0$.

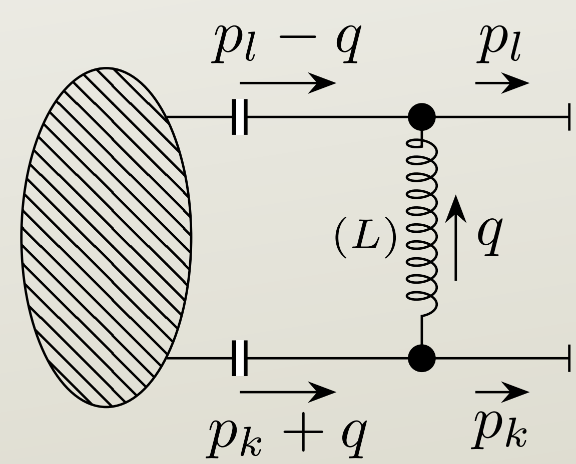
- Thus interpolating gauge is like a physical gauge with respect to collinear divergences.

- Both T-gluons and L-gluons create soft ($q \to 0$) divergences when they couple to two external legs.

$$\sum_{l<k} \int \frac{d^d q}{(2\pi)^d}$$



$$\sum_{l<k} \int \frac{d^d q}{(2\pi)^d}$$



- These are soft divergences, but without collinear divergences.

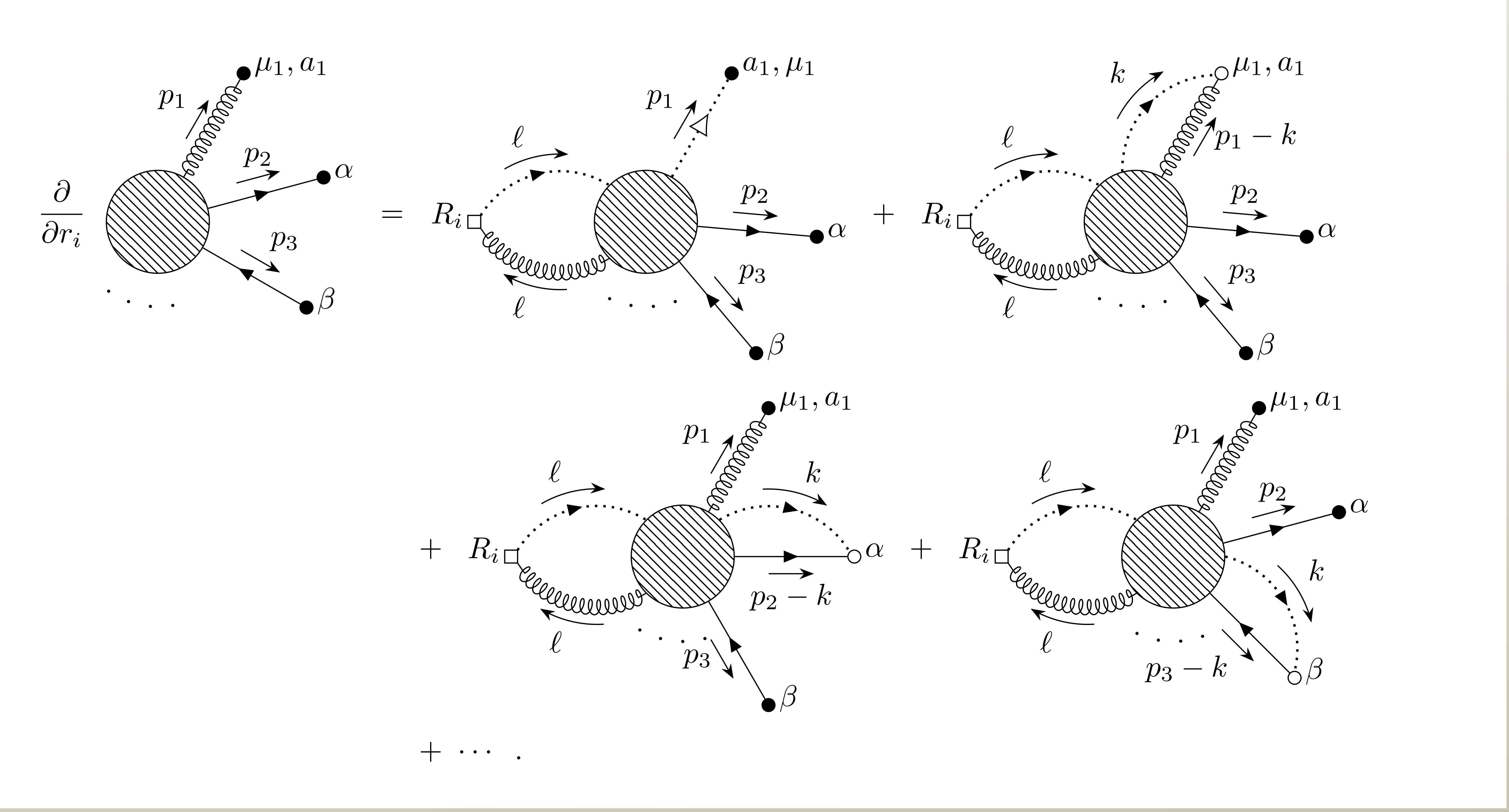
Technical issues

- Renormalization works. We calculate $[Z_A^{1/2}]^\mu_\nu$, $Z_\psi^{1/2}$, Z_η, Z_g, Z_v, and Z_ξ at order $\alpha_{\rm s}$.

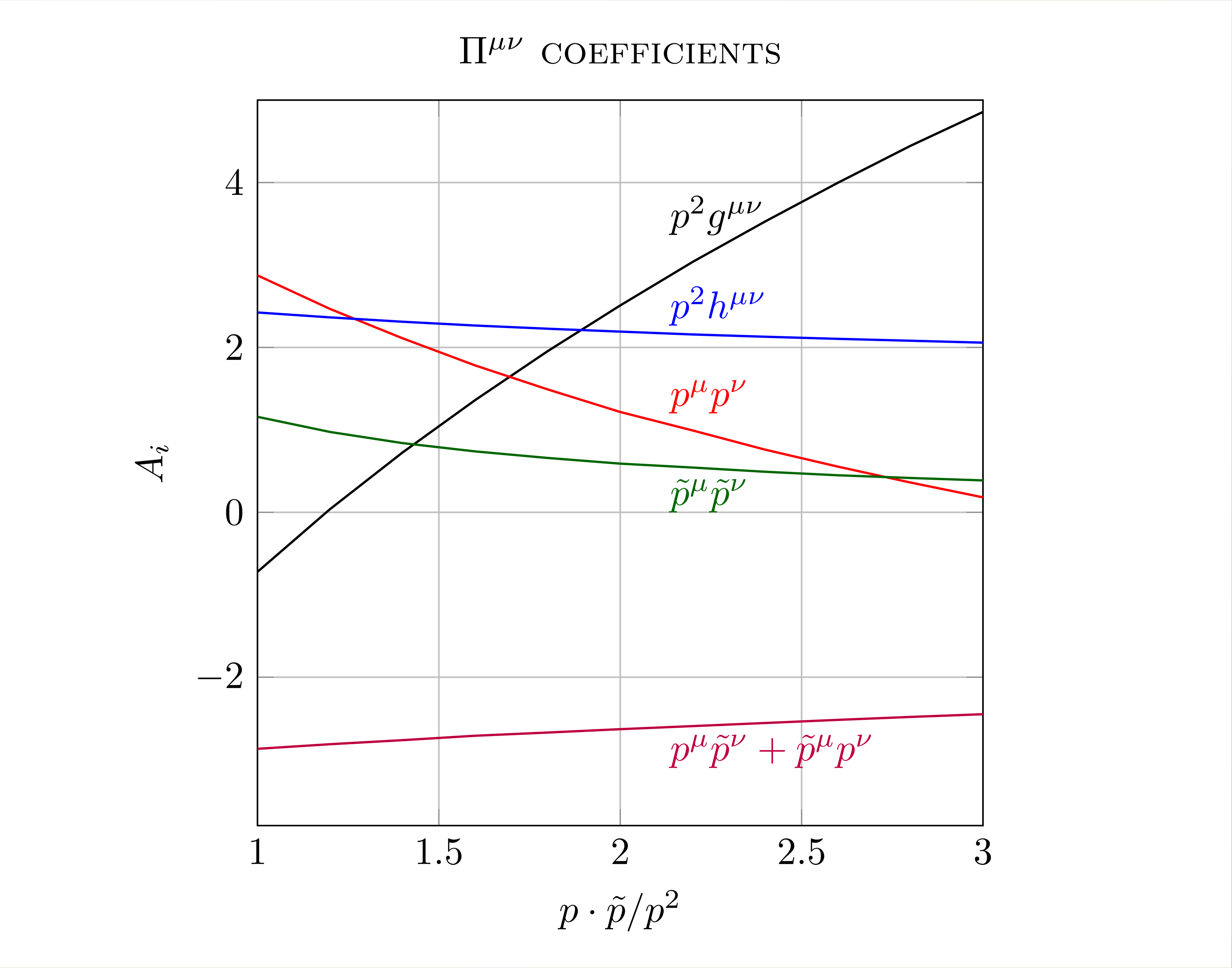
- BRST invariance shows that the S-matrix is independent of v, ξ, and n.

- BRST identity for the variation of Green functions as we vary the gauge parameters r_i:

$$\frac{\partial \mathcal{L}(x)}{\partial r_i} = \delta_{\text{brst}} \mathcal{R}_i(x)$$



- One can calculate loop integrals with the help of Feynman parameterization and then numerical integration:



Conclusion

- Interpolating gauge may be useful for calculations that aim to isolate soft and collinear singularities of QCD.