Progress towards full Next-to-Leading Logarithmic Accuracy to Scattering at High Energy

Jeppe R. Andersen
and collaborators

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Introduction

Outline of talk:

1. Amplitudes in the High Energy Limit
2. From amplitudes to cross sections
3. Progress towards full next-to-leading logarithmic accuracy

High Energy Jets:

- **Factorisation of matrix elements** using **currents** retains analytic properties such as **crossing symmetries**
- systematic **power expansion of QCD amplitudes** for real emissions
- all-order **leading and sub-leading logarithmic corrections** Focus of this talk
- **matching**, **results**. . . Results reported at earlier talk by J. Smillie

Regge theory

Regge theory describes scattering from a **central potential** in terms of the projections on Legendre polynomial and states of **definite orbital angular momentum** (partial wave analysis)

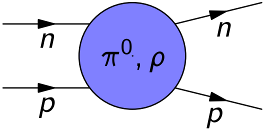
The analysis of **analytic scattering amplitudes** in terms of Regge Theory: Regge (1959)

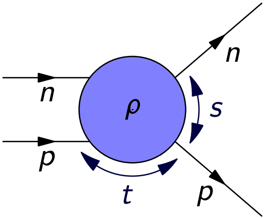
$$\mathcal{M} = \sum_i \Gamma_i(t)\,(s)^{j_i}$$

At **large energies** s, the contribution from particle of **highest spin j dominates**

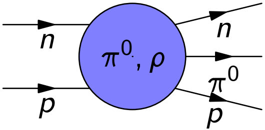
$$\mathcal{M} \to \Gamma(t)\,(s)^j$$

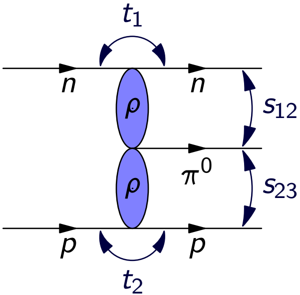
Regge limit: $s \gg -t$ or $s \gg p_t^2$





Large *s* of course leads to the possibility of **multi-particle production**





Multi-Regge limit: Brower, DeTAR, Weis (1974)

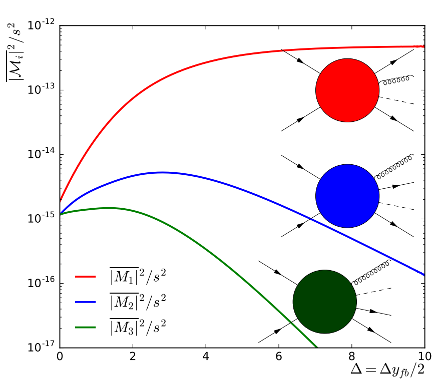
$s_{12}, s_{23} \gg p_{t_i}^2, |t_i|, \quad |t_i| \sim |t_j|, \quad |p_{t_i}| \sim |p_{t_j}|$

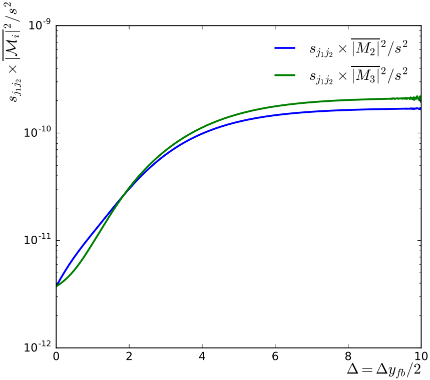
$$\mathcal{M} = s_{12}^j s_{23}^j \, \Gamma(t_1, t_2, s/(s_{12}s_{23}))$$

No underlying theory for strong interactions; derives constraints on the high energy behaviour based on the constraints from an **analytic scattering amplitude**.

Scaling of QCD Amplitudes

The scaling extends to all QCD processes involving also Higgs bosons, W, Z and photon production.





The **scaling** for different kinematic evaluations of the same amplitude is exactly as predicted by Regge theory applied to the **planar graph** connecting the rapidity-ordered configuration.

M. Heil, A. Maier, J.M. Smillie, JRA, arXiv:1706.01002

Perurbative Corrections in the High Energy Limit

Since the real emision perturbative corrections have $|M|^2/s^2 \to \mathrm{constant}$ for large $\Delta y \sim \log(s/p_t^2)$, it will contribute a correction $\alpha_s \log(s/p_t^2)$ after integration.
The other orderings of momenta (and other processes) contribute sub-leading corrections which can be included at next-to-leading order (see later).
Virtual corrections for $gg \to gg$ at one loop have logarithmic piece:

$$
\begin{aligned}
m_{4:1}(-,-,+,+) &= m_4(-,-,+,+)\ c_\Gamma \\
&\times \left\{ \left(-\frac{\mu^2}{s_{14}}\right)^{\epsilon} \left[N_c \left(-\frac{4}{\epsilon^2} - \frac{11}{3\epsilon} + \frac{2}{\epsilon}\ln\frac{s_{12}}{s_{14}} - \frac{64}{9} - \frac{1}{3} + \pi^2 \right) \right.\right. \\
&\left.\left. + N_f \left(\frac{2}{3\epsilon} + \frac{10}{9} \right) \right] - \frac{\beta_0}{\epsilon} \right\}
\end{aligned}
$$

Logarithmic structure predicted to all orders (BFKL, Regge, VDD,...).
Control perturbative corrections of $\alpha_s^n \log^n(s/p_t^2)$ (leading logarithm).

Regge Theory and QCD

While QCD allows for the calculation of the scattering amplitudes, the amplitudes are still **analytic**, and a Regge analysis can be applied. The amplitude can be reconstructed (to obtain logarithmic accuracy of the cross section) by effective vertices. These **building blocks** can be **calculated in QCD**.

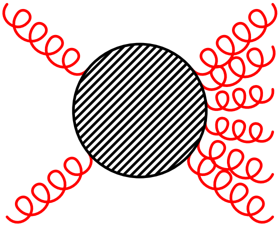
These are the impact factors and kernels in the **BFKL** language. So what is different with High Energy Jets?

The Regge analysis relies on analyticity, e.g. crossing symmetry. However, even the building blocks of BFKL and the standard analysis of the high energy limits do not respect such crossing symmetries. The components in HEJ respects **crossing symmetry**.

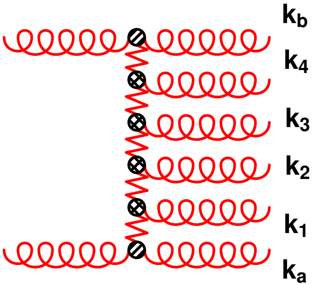
The components also respect **gauge invariance** - everywhere in phase space, not just in asymptotic limits.

More importantly for the **logarithmic accuracy**, phase space restrictions of NLL significance are taken into account (hep-ph/0611011)...

The Power of Reggeisation and BFKL



High Energy Limit
$$\xrightarrow{\qquad}$$
$$|\hat{t}| \quad \text{fixed}, \quad \hat{s} \to \infty$$



Amplitudes approximated - need leading power in s/pt^2 for leading logarithmic accuracy in cross section.
At LL only gluon production; at NLL also quark–anti-quark pairs produced.

Cross sections and the BFKL Equation

Further approximations to phase space integrals applied in BFKL : **Fully inclusive** any-jet **partonic** cross sections **can be calculated analytically**

$$\mathrm{d}\hat{\sigma}(p_a, p_b) = \Gamma_a(\mathbf{p}_a)\, f(\mathbf{p}_a, -\mathbf{p}_b, \Delta)\, \Gamma_b(\mathbf{p}_b)$$

The **evolution of the reggeised gluon** is described by the
BFKL equation

$$\omega f_\omega\left(\mathbf{k}_a, \mathbf{k}_b\right) = \delta^{(2+2\epsilon)}\left(\mathbf{k}_a - \mathbf{k}_b\right) + \int d^{2+2\epsilon}\mathbf{k}' \mathcal{K}_\epsilon\left(\mathbf{k}_a, \mathbf{k}'\right) f_\omega\left(\mathbf{k}', \mathbf{k}_b\right)$$

ω: Mellin conjugated variable to the rapidity y along the evolution.

- The kernel $\mathcal{K}_\epsilon$ consists of the **virtual** corrections of the trajectory and the **real** corrections from the Lipatov vertices.
- dependence on **transverse** degrees of freedom only

Observation (BFKL)

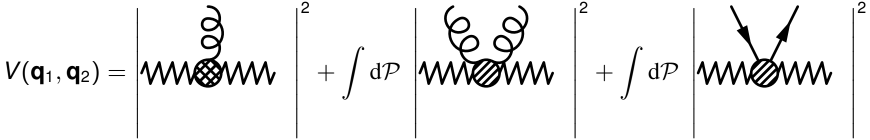
1. Imposing Energy and Momentum conservation (i.e. restricting phase space integral to that accessible at a given energy) is completely unrelated to the NLL corrections to the evolution.
2. Claim: To calculate an observable to full NLL accuracy in $\log(s)$, three ingredients are necessary:
   - NLL Impact Factors
   - NLL Evolution
   - A correct evaluation of s for each of the configurations integrated over

Differences between BFKL and HEJ:

- Momentum configurations constructed explicitly - allows for correct calculation of s
- Impact factors (also at NLL) respect crossing symmetry, gauge invariance etc., matching to fixed order. Important for controlling behaviour away from the strict HE limit

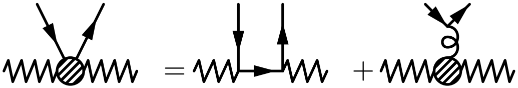
Recalculate a NLL component of the kernel to illustrate these points

The Ingredients of the BFKL NLL Vertex

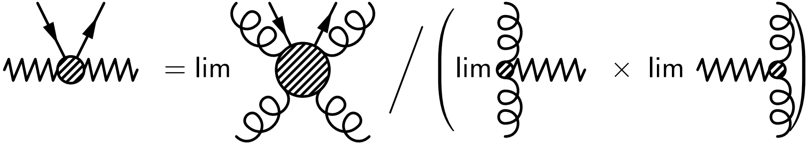


Two methods for obtaining the vertices at NLL:

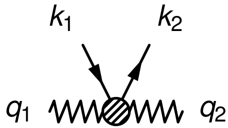
- Fadin & Lipatov:



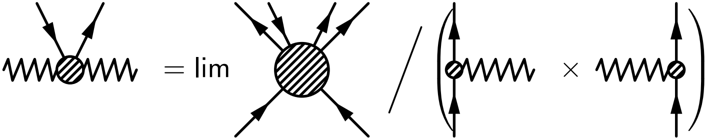
- V. Del Duca:



NLL Vertex

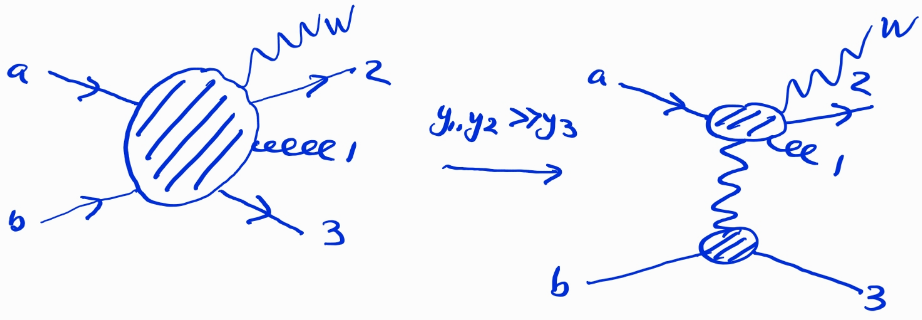


Results from numerical integration: Average rapidity separation for 20GeV partons .5 units of rapidity etc. Clearly not isolated in rapidity, clearly not a single contribution to $\sqrt{s}$. In HEJ we calculate differently - construct approximations to the scattering amplitudes for the processes identified as leading and sub-leading. The calculations apply a smaller set of approximations, ensuring the **analytic properties** of amplitudes.



NLL components for Reggeisation

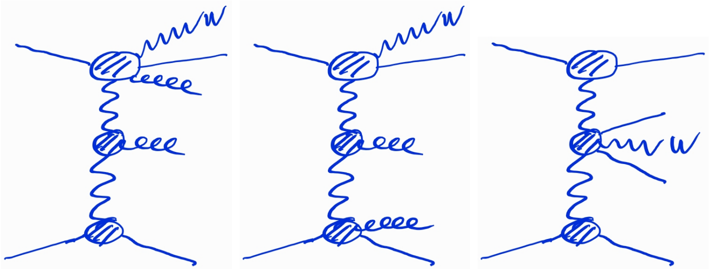
Consider $pp \to W3j$. Next-to-leading logarithmic corrections arise e.g. from the amplitudes in the quasi-multi-Regge-kinematic limit, where the invariant mass between one pair of partons is not large.



Amplitude expressed as $\mathcal{M} = I^{\mu}(a, w, 1, 2)\ J_{\mu}(b, 3)$. Full crossing symmetry, gauge invariance etc. in each component. $I^{\mu}(a, w, 1, 2)$ calculated by projection onto colour octet exchange in the t-channel.

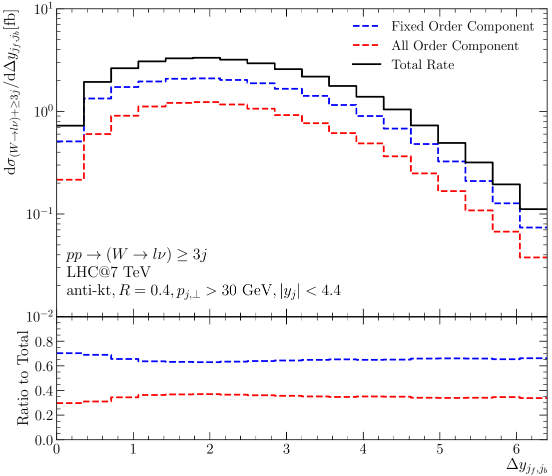
Higher Order Corrections

Can calculate higher order corrections with NLL components by explicit MC integration over the regulated amplitudes, represented by a Reggeised graph

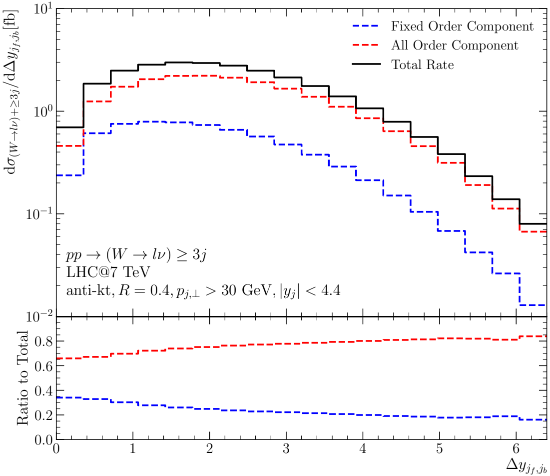


Virtual corrections encoded in the t-channel propagators.
Matching: Sub-processes and phase space points not reached with LL or NLL Reggeised description are treated at fixed order (additive). Resummation points are matched to n-jet matrix elements (multiplicative).

Impact of NLL corrections for W3J

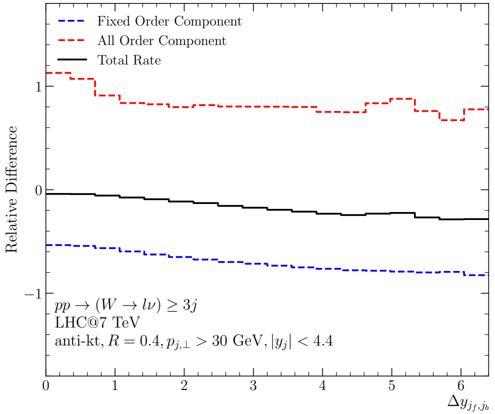


LL channels resummation only



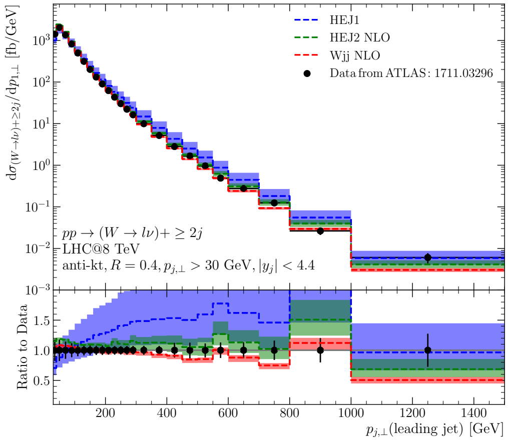
sub-leading channels included

Impact of NLL corrections for W3J



Much less fixed order matching, much bigger resummation component. Final result of the inclusive distribution changes by no more than 25%.

Comparison to Data (WJJ)



The NLL terms included and improvement in matching are sufficient to ensure the predictions agree well with data even in the most difficult regions of phase space.

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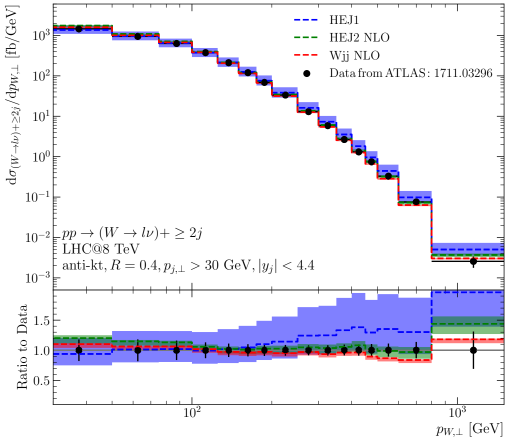

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Ongoing work

The **work to include full NLL accuracy** in the resummation of High Energy Jets is ongoing - need one-loop corrections to the one-particle production components at LL, two-loop component of the reggeised t-channel propagator.
Unsurprisingly, the inclusion of sub-leading logarithms leads to

- small changes in the leading regions of phase space
- a better description in sub-leading regions of phase space

Hall-marks of a well-behaved perturbative expansion.
Further improvements ongoing.