Electroweak precision physics at the LHC: the M_W and $\sin\theta_{\rm eff}^{\rm lept}$ example

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Introduction

EW precision physics = precise determination of some parameter of the EW sector of the Standard Model

- M_W, M_Z, M_H, $\cdots$
- Γ_W, Γ_Z, $\cdots$
- $\sin\theta_W$, $\cdots$

precise measurements inevitably call for precise theoretical predictions (see later)

Theoretical predictions: uncertainties

- truncation of the perturbative expansion/missing higher-order effects
- parametric dependence on input parameters (e.g. EW input parameters, fermion masses, coupling constants, etc)
- uncertainties from non perturbative effects (e.g. hadronic running of α, at hadron colliders PDFs and QCD effects at small p_T)
- technical details of the calculation, like treatment of resonances, combination strategies for QCD and EW effects, beyond fixed-order matching strategies for fixed order predictions matched with parton shower, etc

EW uncertainties and EW input parameter schemes

EW input parameter scheme (EW-IPS)

Choice of the 3 independent parameter (gauge boson masses and/or couplings) to be taken as independent parameters

For a specific order in perturbation theory (say NLO) all EW-IPS are formally equivalent, but some EW-IPS can lead to smaller (EW) theoretical uncertainties

- if the input parameters are determined experimentally with high accuracy $\Rightarrow$ reduced parametric unc.s
- if the NLO EW corrections in the chosen scheme are small $\Rightarrow$ uncertainties from missing $\mathcal{O}(\alpha^2)$ effects should be small ($\sim \delta^2_{\mathrm{NLO}}$)
- some classes of parametric uncertainties might disappear with some scheme choices: e.g. the hadronic contributions to the running of α when using $\alpha(M_Z)$ or G_μ as input

Why EW corrections at the LHC?

- at hadron colliders QCD effects dominate
- $\alpha_S \sim 0.1$, $\alpha \sim 1/137$
- at the cross section level
  - $\delta^{\mathrm{NLO\,EW}} << \delta^{\mathrm{NLO\,QCD}}$
  - $\mathcal{O}(\alpha_S^2) \sim \mathcal{O}(\alpha)$

provided that the event selection is inclusive!

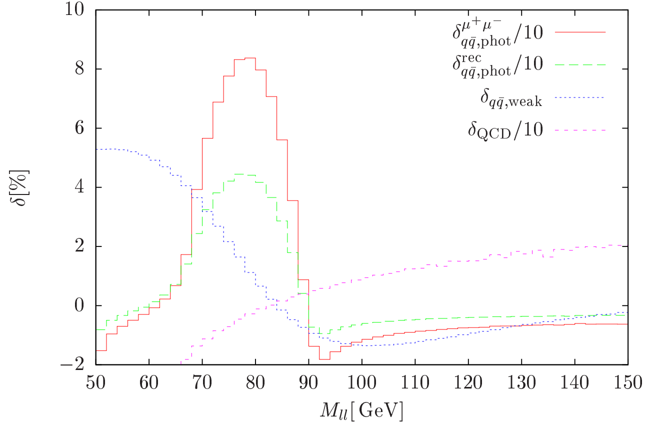
This is NOT always the case, see e.g. VBS scattering:

Order	$\mathcal{O}(\alpha^7)$	$\mathcal{O}(\alpha_S\alpha^6)$	$\mathcal{O}(\alpha_S^2\alpha^5)$	$\mathcal{O}(\alpha_S^3\alpha^4)$
$\delta\sigma_{\mathrm{NLO}}$ [fb]	$-0.2169(3)$	$-0.0568(5)$	$-0.00032(13)$	$-0.0063(4)$
$\delta\sigma_{\mathrm{NLO}}/\sigma_{\mathrm{LO}}$ [%]	-13.2	-3.5	0.0	-0.4

$pp \to W^+W^+jj$ (Denner et al. 1611.02951, 1708.00268, 1906.01863)

remark1: QED collinear FSR

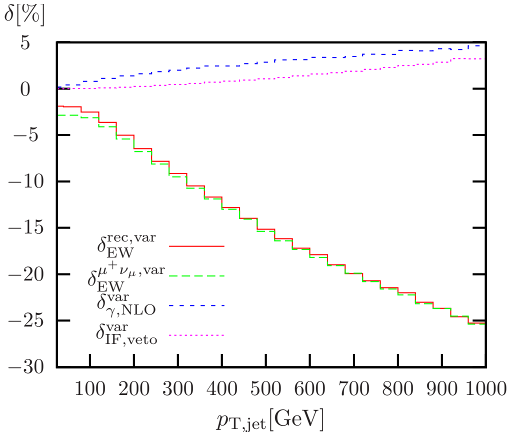
EW corrections can be enhanced by collinear final-state QED radiation $\simeq \frac{\alpha_0}{2\pi} \log \frac{m_f^2}{Q^2}$ leading to non-trivial shape effects



M_{ll} plot in NC DY from 0911.2329

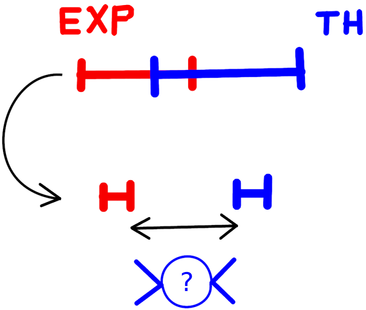
remark2: Weak Sudakov corrections

virtual weak corrections can reach several tens of percent in the Sudakov regime



jet p_T in $V+$ jet from 0906.1656

Precise theoretical predictions, case 1



Measurement that do not depend on theoretical predictions (a little idealized)

Data/theory comparison serves as a precision test of the SM

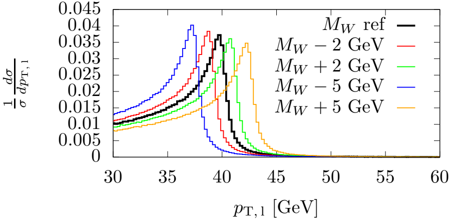
Possible deviations can be interpreted as NP effects (indirect NP searches)

Precise theoretical predictions, case2

Measurement that do not depend on theoretical predictions, like template fits.

Example M_W determination at the LHC:

- generate several Monte Carlo samples with different M_W
- the measured M_W corresponds to the sample that best fits to the data



theory uncertainties = approximation in templates propagate to M_W measurement and become part of the total experimental error budget

Theoretical uncertainties in M_W measurement: strategy

Impact of different EW effects (an theory uncertainties from weak and mixed QCD-EW corrs) on W-mass measurement in: arXiv:1612.02841 (W_ew-BMNNP)

1. pseudodata
   - Monte Carlo samples with a given theoretical accuracy
   - play the role of experimental data
2. templates
   - MC samples at NLO QCD+QCD-PS (or LO) generated for different values of M_W
   - will be fitted to the pseudodata
3. $\Delta M_W = M_W(\text{pseudodata}) - M_W(\text{fit output})$

Theory uncertainties in M_W measurement: tools

Monte Carlo generators

HORACE (Carloni Calame et al. hep-ph/0303102, hep-ph/050626)

- MC event generator for DY
- can generate events at NLO EW+QED-PS, and NLO EW+QED-PS+unresolved l^+l^- radiation
- buit-in QED PS implementation

POWHEG-BOX-V2/W_ew-BMNNP (Barze et al. arXiv:1202.0465)

- MC event generator for charged DY
- can generate events at NLO QCD+QCD-PS and NLO (QCD+EW)+(QCD+QED)-PS
- relies on external shower MC programs (i.e. PYTHIA, PYTHIA+PHOTOS)

Parton Showers

PYTHIA (Sjostrand et al. hep-ph/0603175; arXiv:0710.3820)

- general purpose shower MC generator
- can generate multiple QCD and QED radiation
- used for ISR multiple QCD (and QED) radiation AND non-perturbative QCD effects
- in some runs used for QED FSR (see later)

PHOTOS (Barberio et al. CPC 66 (1991), CPC 79 (1994), Golonka et al. hep-ph/0506026)

- general purpose shower MC generator
- can generate multiple QED radiation off fermions (from W decay)
- in some runs used for QED FSR (see later)

Mixed QCD-EW corrections (1)

$pp \to \mu^+\nu_\mu$, fit to $M_\mathrm{T}(\mu^+\nu_\mu)$

	Templates	Pseudodata	M_W shifts (MeV)
1	LO	POWHEG(QCD) NLO	56.0 $\pm$ 1.0
2	LO	POWHEG(QCD)+PYTHIA(QCD)	74.4 $\pm$ 2.0
3	LO	HORACE(EW) NLO	-94.0 $\pm$ 1.0
4	LO	HORACE (EW,QEDPS)	-88.0 $\pm$ 1.0
5	LO	POWHEG(QCD,EW) NLO	-14.0 $\pm$ 1.0
6	LO	POWHEG(QCD,EW) `two-rad`+PYTHIA(QCD)+PHOTOS	-5.6 $\pm$ 1.0

	samples	M_W shift (MeV)
$\sum_{m=1,n=1}^{\infty} \delta'_{\alpha_s^m \alpha^n} + \sum_{m=2}^{\infty} \delta'_{\alpha_s^m} + \sum_{n=2}^{\infty} \delta'_{\alpha^n}$	[6]-[5]	8.4 ± 1.4 MeV
$\sum_{m=2}^{\infty} \delta'_{\alpha_s^m}$	[2]-[1]	18.4 ± 2.2 MeV
$\sum_{n=2}^{\infty} \delta'_{\alpha^n}$	[4]-[3]	6.0 ± 1.4 MeV

$\sum_{m=1,n=1}^{\infty} \delta'_{\alpha_s^m \alpha^n}$=([6]-[5])-([2]-[1])-([4]-[3])=-16.0$\pm$3.0 MeV

in agreement with the results of Dittmaier et al. 1511.08016 for the full $\mathcal{O}(\alpha\alpha_\mathcal{S})$ corrections in pole approx. (-14 MeV)

non-log QED, weak and mixed EW-QCD contributions (2)

$pp \to W^+$, $\sqrt{s} = 14$ TeV			M_W shifts (MeV)			
Templates accuracy: NLO-QCD+QCD$_{\rm PS}$			$W^+ \to \mu^+\nu$		$W^+ \to e^+\nu$(dres)	
	Pseudodata accuracy	QED FSR	M_T	p_T^ℓ	M_T	p_T^ℓ
1	NLO-QCD+(QCD+QED)$_{\rm PS}$	Pythia	-95.2±0.6	-400±3	-38.0±0.6	-149±2
2	NLO-QCD+(QCD+QED)$_{\rm PS}$	Photos	-88.0±0.6	-368±2	-38.4±0.6	-150±3
3	NLO-(QCD+EW)+(QCD+QED)$_{\rm PS}$`two-rad`	Pythia	-89.0±0.6	-371±3	-38.8±0.6	-157±3
4	NLO-(QCD+EW)+(QCD+QED)$_{\rm PS}$`two-rad`	Photos	-88.6±0.6	-370±3	-39.2±0.6	-159±2

ATLAS collaboration at LHC use as templates
Powheg -QCD+Pythia -QCD+Photos

uncertainties from
non-log QED,
weak,
mixed QCD-EW corr.

		ΔM_W (MeV) bare muons	
	QED FSR model	M_T	p_T^ℓ
LHC	Pythia	+6.2 ± 0.8	+29 ± 4
	Photos	-0.6 ± 0.8	-2 ± 4

non-log QED, weak and mixed EW-QCD contributions

- Full $\mathcal{O}(\alpha\alpha_S)$ (arXiv:2102.12539,arXiv:2201.01754): it would be nice to study their impact on M_W extraction. What is the shift w.r.t. the factorized approximation?
- QED, WEAK, and mixed effects inevitably have an interplay with IS QCD effects (e.g. $\mathrm{PS} \times \overline{B}$)
- in our simulation we only used PYTHIA8 for ISR QED and QCD shower and non-perturbative effects with a default PYTHIA tuning
- how do the shifts change if we use another shower MC, say HERWIG?
- how do the estimates change when changing the PYTHIA tune? (having in mind the ATLAS procedure of tuning PYTHIA to reproduce the Z p_T data)
- how do the shifts change if we use another description of IS effects, say for instance RESBOS like in TEVATRON analyses?

$\sin\theta_{\rm eff}^{\rm lept}$ at the LHC

measured from invariant-mass forward-backward asymmetry (or from the A_4 angular coefficient)

$$A_{FB}(M_{ll}) = \frac{F(M_{ll}) - B(M_{ll})}{F(M_{ll}) + B(M_{ll})}$$

$$F = \int_0^1 \mathrm{d}\cos\theta^* \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta^*}, \qquad B = \int_{-1}^0 \mathrm{d}\cos\theta^* \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta^*}$$

θ^* measured in the Collins-Soper frame

using template fits

- measure $A_{FB}(M_{ll})$
- generate Monte Carlo samples with different values of $\sin\theta_W$
- fit the template to the data

measured $\sin\theta_W$ is the one of the sample that describes best the data

$\sin\theta_{\mathrm{eff}}^{\mathrm{lept}}$ and EW corrections

- calculations are usually done in the on-shell scheme with EW input parameters $(\alpha/G_\mu, M_W, M_Z)$
- in the OS $(\alpha/G_\mu, M_W, M_Z)$ schemes, $\sin\theta$ is constant at all orders

$$\sin\theta_{OS}^2 = 1 - \frac{M_W^2}{M_Z^2}$$

- in the direct determination of $\sin\theta$ we want to extract $\sin\theta$ from the strength of the Zff coupling that is NOT constant at H.O.

$$\sin\theta_{\mathrm{eff}}^2 = \frac{1}{4}\Big(1 - \mathrm{Re}\frac{g_V}{g_A}\Big)$$

- $\sin\theta_{\mathrm{eff}}^2 = \kappa_l \, \sin\theta_{OS}$, ($\kappa_l = 1$ at LO)

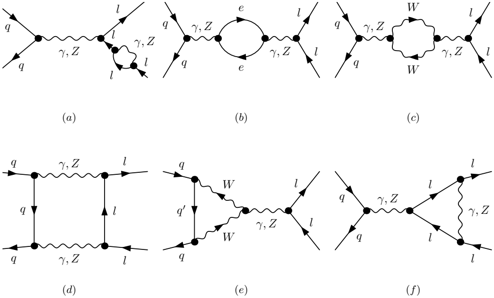
Template fits for $\sin\theta_{\rm eff}^{\rm lept}$ and EW corrections

- Accuracy goal on $\sin^2\theta_W$ is 10^{-4}: EW corrections mandatory
- $\sin\theta_W$ can always be used as input parameter for fits at LO
- The typical input schemes used at the LHC are $(\alpha/G_\mu, M_W, M_Z)$: $\sin\theta_W$ is a derived quantity

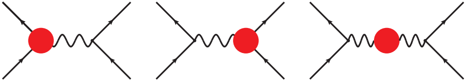
In order to perform a fit at NLO EW and have a clean way to estimate the EW uncertainties, a new input parameter scheme should be used with $\sin\theta_W$ as free parameter [arXiv:1906.11569]

$$(\alpha/G_\mu, \sin\theta, M_Z)$$

NC DY in the $(\alpha/G_\mu, \sin\theta, M_Z)$ scheme



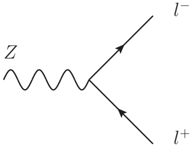
bare diagrams don't change



CT diagrams don't change

w.r.t. the on-shell scheme, different expression for the counterterm functions $\frac{\delta s_W^2}{s_W^2}$ and Δr

Renormalization conditions



$$\frac{ie}{2 s_W c_W} \gamma^\mu \Big[g_V^l - g_A^l \gamma_5 \Big],$$

$$g_V = \frac{g_L + g_R}{2}, \quad g_A = \frac{g_L - g_R}{2}$$

at LO $\sin\theta_{\rm eff}^2 = \frac{1}{4}(1 - {\rm Re}\frac{g_V}{g_A})$

the renormalization condition is

$$\sin\theta_{\rm eff}^2 \Big|_{NLO} = \sin\theta_{\rm eff}^2 \Big|_{LO} \qquad \Rightarrow \qquad \frac{g_V + \delta g_V}{g_A + \delta g_A} = \frac{g_V}{g_A}$$

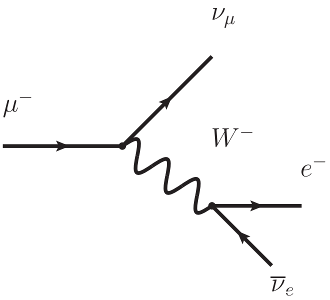
$\delta \sin\theta_{\rm eff}$

$$\frac{\delta \sin\theta^2_{\rm eff}}{\sin\theta^2_{\rm eff}} = {\rm Re}\Big\{ \frac{\cos\theta_{\rm eff}}{\sin\theta_{\rm eff}} \frac{\Sigma_T^{AZ}(M_Z^2)}{M_Z^2} + (1 - \frac{Q_l}{I_3^l}\sin\theta^2_{\rm eff})[\delta V^L - \delta V^R] \Big\}$$

$\delta V^{L/R} =$ bare vertex diagrams+fermion w.f. renorm.

- $\delta^{QED} g_L = \delta^{QED} g_R$: affected only by weak corrections
- no enhancement from logs of fermion masses
- no dependence on $\Delta\rho$ (no m_t^2 enhancement)

Δr, $\Delta\tilde{r}$



computed from the NLO EW corrections to μ-decay after subtracting 1-loop QED corrections in the Fermi model

$$\Delta r = \Delta r(\alpha, M_W, M_Z)$$

$$\Delta\tilde{r} = \Delta r(\alpha, \sin\theta, M_Z)$$

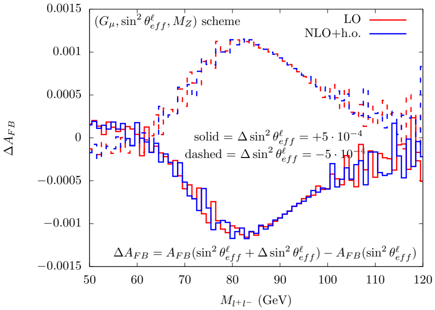
Δr

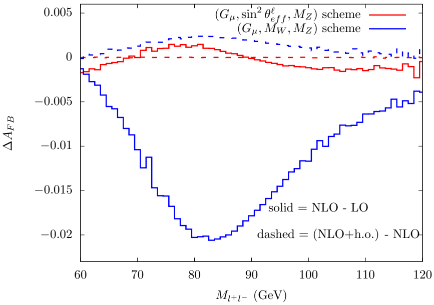
- LO $\simeq \frac{-1}{M_W^2}$
- CT $\simeq \frac{\delta M_W^2}{M_W^4}$
- $\Delta r = \Delta\alpha(s) - \frac{c_W^2}{s_W^2}\Delta\rho + \Delta r_{\mathrm{remn}}$

$\Delta\tilde{r}$

- LO $\simeq \frac{-1}{c_W^2 M_Z^2}$
- CT $\simeq \frac{\delta M_Z^2}{c_W^2 M_Z^4} - \frac{2}{M_Z^2}\frac{s_W^2}{c_W^2}\frac{\delta\tilde{s}_W}{\tilde{s}_W}$
- $\Delta\tilde{r} = \Delta\alpha(s) - \Delta\rho + \Delta\tilde{r}_{\mathrm{remn}}$

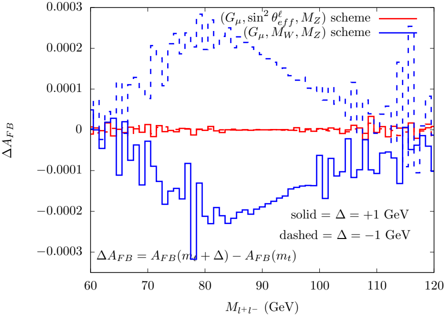
The $(G_\mu, \sin\theta_{\rm eff}, M_Z)$ scheme: numerical results (1)





- sensitivity dominated by LO behaviour
- NLO EW corrections are smaller in the $(G_\mu, \sin\theta_{\rm eff}, M_Z)$ scheme
- H.O. effects smaller in the $(G_\mu, \sin\theta_{\rm eff}, M_Z)$ scheme

The $(G_\mu, \sin\theta_{\rm eff}, M_Z)$ scheme: numerical results (2)



smaller parametric uncertainties from m_t dependence compared to the OS $(\alpha/G_\mu, M_W, M_Z)$ scheme

m_t^2 dependence from $\Delta\rho$

- OS $(\alpha/G_\mu, M_W, M_Z)$ scheme: $\Delta\rho$ enters Δr and δs_W. EW corrections affect γ and Z diagrams in a different way.
- $\sin\theta$ scheme: $\Delta\rho$ enters only Δr. Overall effect, cancels in A_{FB}.

Weak mixing angle: energy dependence

$\sin\theta_{\rm eff}^{\rm lept}$ measures the strength of the $Zf\overline{f}$ coupling at the weak scale

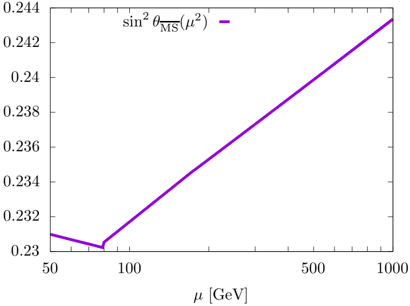
To quantify the energy dependence of the $Zf\overline{f}$ one has to resort to some energy dependent definition of $\sin\theta$

$\Rightarrow \sin\theta_{\overline{\rm MS}}(\mu_R^2)$ (though other choices are possible)

We can use template fits by looking at kinematic distributions like M_{ll} or $A_{\rm fb}$, but

- theoretical ingredients in the templates must match exactly the definitions/conventions used for $\sin\theta_{\overline{\rm MS}}(\mu_R^2)$
- one has to assume the form of the running (e.g. the SM running)
- the fit parameter will be the value of $\sin\theta_{\overline{\rm MS}}(\mu^*)$, taken as the starting value of the evolution starting from a scale μ^* (e.g. midpoint of the invariant mass bins, etc.)

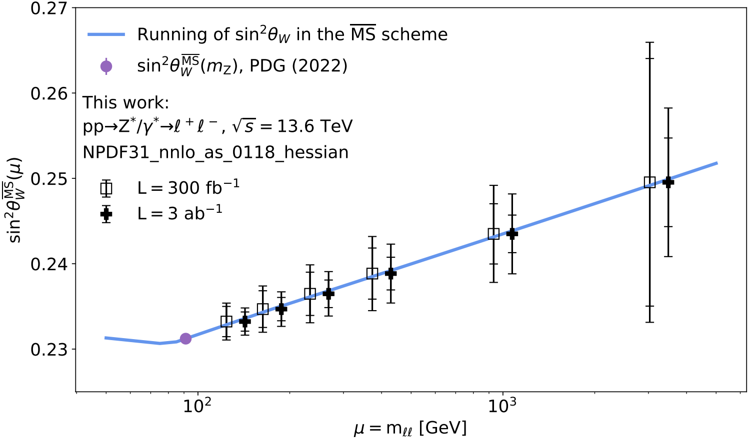
$\overline{\mathrm{MS}}$ weak mixing angle: template preparation



implement

- the $\overline{\mathrm{MS}}$ running of α and $\sin^2\theta$
- the decoupling strategy: M_W, m_{top}
- $\overline{\mathrm{MS}}$ counterterms for α and $\sin^2$

$\overline{\text{MS}}$ weak mixing angle



[arXiv:2302.10782]

Summary

Examples of the role of precision EW calculations in the context of template fits for DY

- M_W determination and estimate of the theory uncertainties
- $\sin\theta_{\rm eff}^{\rm lept}$: subtleties in the parameter extraction at NLO and the input/renormalization scheme with $\sin\theta_{\rm eff}^{\rm lept}$ as input
- possible strategies to measure the running of $\sin\theta$

Backup Slides

non-log QED, weak and mixed EW-QCD contributions (1)

	$pp \to W^+$, $\sqrt{s} = 14$ TeV		M_W shifts (MeV)			
	Templates accuracy: NLO-QCD+QCD$_{\text{PS}}$		$W^+ \to \mu^+\nu$		$W^+ \to e^+\nu$(dres)	
	Pseudodata accuracy	QED FSR	M_T	p_T^ℓ	M_T	p_T^ℓ
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- impact of non-log QED, weak and mixed EW-QCD contributions
- different effects for Photos or Pythia (different non-log QED terms)
- more stable results for M_{T} (less sensitive to mixed EW-QCD corrections)

NLO+PS matching with EW corrections

- NLO EW corrections: $d\sigma = d\sigma_0\,[1 + \delta_\alpha]$

- QED-PS: all order γ radiation in leading log approx.

  $$d\sigma = d\sigma_0\,\left[1 + \sum_{n=1}^{\infty} \delta'_{\alpha^n}\right]$$

- NLO EW+QED-PS: $d\sigma = d\sigma_0\,\left[1 + \delta_\alpha + \sum_{n=2}^{\infty} \delta'_{\alpha^n}\right]$

  matching replaces first PS radiation with NLO real radiation

- HORACE NLO EW+QED-PS: $d\sigma = d\sigma_0\,\left[1 + \delta_\alpha + \sum_{n=2}^{\infty} \delta'_{\alpha^n}\right]$

- POWHEG NLO (QCD+EW)+(QCD+QED)-PS:

$$d\sigma = d\sigma_0 \left[1 \;+\; \delta_{\alpha_s} + \delta_\alpha + \sum_{m=1,n=1}^{\infty} \delta'_{\alpha_s^m \alpha^n} + \sum_{m=2}^{\infty} \delta'_{\alpha_s^m} + \sum_{n=2}^{\infty} \delta'_{\alpha^n}\right]$$

POWHEG-BOX-V2 VS POWHEG-BOX-RES

POWHEG-BOX-V2

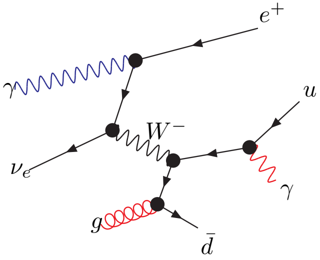
- try to generate one radiation from each $\alpha_{\rm r}$ ($p_{\rm T}^{\alpha_{\rm r}}$)
- find the hardest radiation ($p_{\rm T}^{max}$)
- $p_{\rm T}^{max}$ is the starting scale of the PS

POWHEG-BOX-RES

- try to generate one radiation from each $\alpha_{\rm r}$ ($p_{\rm T}^{\alpha_{\rm r}}$)
- for each resonance r, find the hardest radiation emitted by the resonance ($p_{{\rm T},r}^{max}$)
- $p_{{\rm T},r}^{max}$ is the starting scale of the PS radiation from r

- POHWEG-BOX-RES (like) events contain up to one radiation from each resonance
- PS radiation from each resonance must be vetoed independently
- dedicated interface to PS unavoidable (no LHE accord for multiple scales, `scalup` works for one radiation only)

POWHEG-BOX-RES (like) treatment of resonances

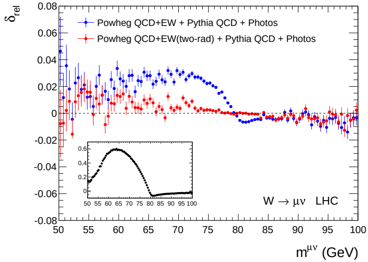


- 3 radiation regions:
  QCD ISR, QED ISR, QED FSR
- 2 resonances: IS, W

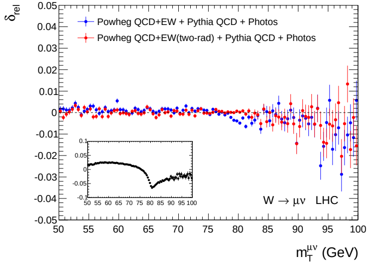
The events contain up to 2 radiations:

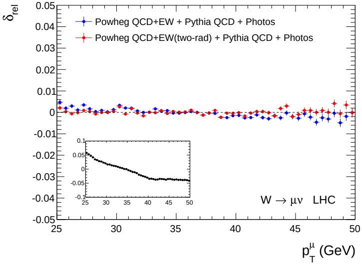
1. one ISR QED or QCD radiation setting the scale of the IS shower
2. one FSR QED radiation setting the scale of the FS shower

POWHEG-BOX-RES (like) treatment of resonances (2)



non negligible effect for observables sensitive to QED FSR corrections but rather insensitive to QCD corrections





HORACE

$$d\sigma^{\infty} = F_{SV}\ \Pi(Q^2,\varepsilon)\sum_{n=0}^{\infty}\frac{1}{n!}\left(\prod_{i=0}^{n}F_{H,i}\right)|\mathcal{M}_{n,LL}|^2 d\Phi_n$$

POWHEG

$$d\sigma = \sum_{f_b}\bar{B}^{f_b}(\mathbf{\Phi}_n)\,d\mathbf{\Phi}_n\Bigg\{\Delta^{f_b}(\mathbf{\Phi}_n, p_T^{min}) + \sum_{\alpha_r\in\{\alpha_r|f_b\}}\frac{\left[d\Phi_{rad}\,\theta(k_T - p_T^{min})\,\Delta^{f_b}(\mathbf{\Phi}_n, k_T)\,R(\mathbf{\Phi}_{n+1})\right]_{\alpha_r}^{\bar{\mathbf{\Phi}}_n^{\alpha_r}=\mathbf{\Phi}_n}}{B^{f_b}(\mathbf{\Phi}_n)}\Bigg\}$$

Universal fermionic corrections (H.O.) (2)

$$\Delta\rho = \frac{\Sigma_T^{ZZ}(0)}{M_Z^2} - \frac{\Sigma_T^{WW}(0)}{M_W^2}$$

$\Delta\rho$ in H.O. calculation:

$$\Delta\rho = 3x_t[1+\rho^{(2)}x_t]\left[1-\frac{2\alpha_S}{9\pi}(\pi^2+3)\right]$$

$$3x_t = \frac{3\sqrt{2}G_\mu m_t^2}{16\pi^2} = \Delta\rho^{(1)}$$

including 2-loop EW and QCD effects

Universal fermionic corrections (H.O.) (1)

- Leading fermionic corrections to DY come from $\Delta\alpha$ and $\Delta\rho$

- They can be included at 2-loop rescaling the relevant parameters in the LO amplitudes (subtracting the terms $\mathcal{O}(\alpha)$ already present at NLO)

- In the OS scheme:

$\alpha_0 \to \frac{\alpha_0}{1-\Delta\alpha(M_Z^2)}$, $s_W^2 \to s_W^2(1+\frac{\delta s_W^2}{s_W^2}) = s_W^2 + \Delta\rho c_W^2$

g_L and g_R diagrams receive different corrections

- In the $\sin\theta$ scheme:

$\alpha_0 \to \frac{\alpha_0}{1-\Delta\alpha(M_Z^2)}$, $G_\mu \to G_\mu(1+\Delta\rho)^2$

overall factor, cancels in A_{FB}