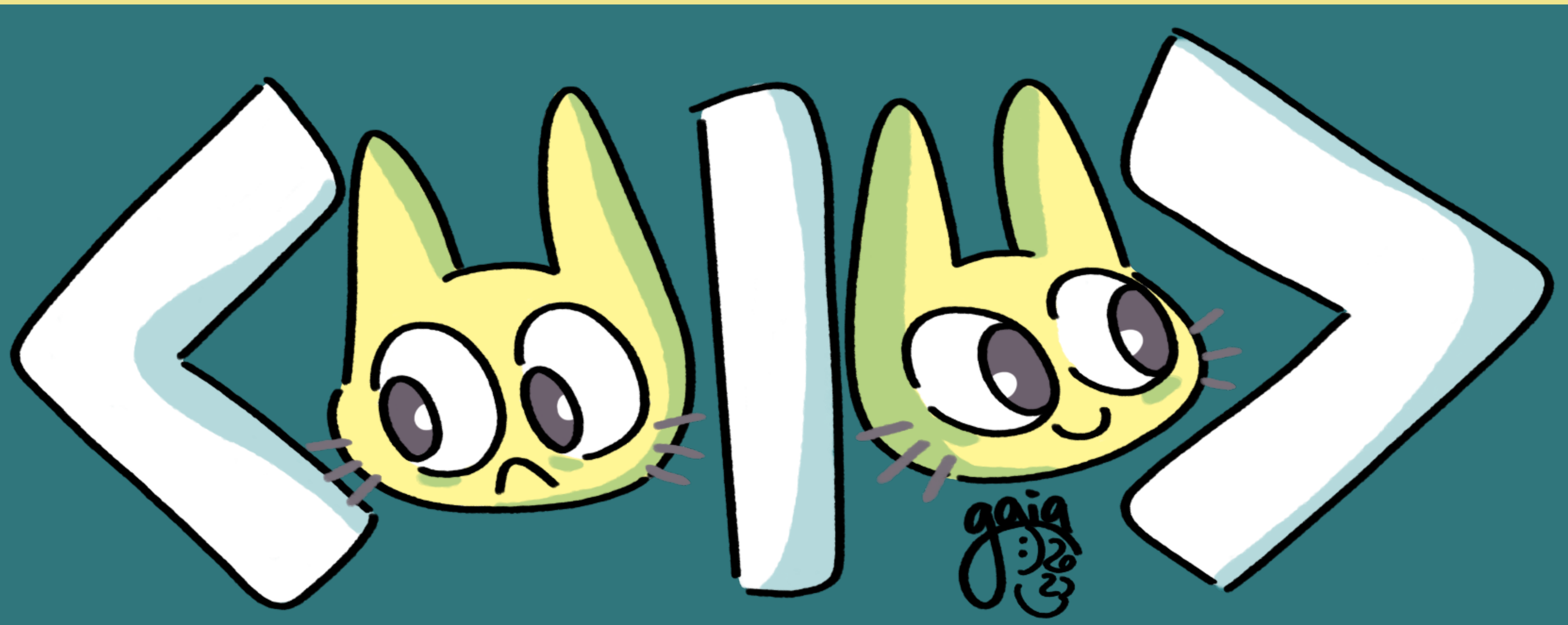
Rational algorithms for the decomposition of Feynman integrals via intersection theory



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Women in theoretical physics

Premio nazionale Milla Baldo Ceolin 2022

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Feynman integrals & intersection numbers

Essential ingredients for theoretical predictions in high energy physics

$$I_j = \int \Big(\prod_{i=1}^{L} \mathrm{d}^d k_i\Big) \frac{1}{D_1^{a_1} \dots D_n^{a_n}}$$

Intersection theory

scalar product between loop integrals:

intersection number

$$\langle I_i | I_j \rangle$$

Project the integrals contributing to a given process

to a basis of independent integrals – master integrals

$$I_i = \sum_j c_{ij}\, G_j \qquad c_j = \sum_k (G^{-1})_{jk} \langle G_k | I \rangle, \qquad G_{jk} \equiv \langle G_j | G_k \rangle$$

Avoid solution of large systems of eq.s:

main bottleneck using traditional approaches

Calculating intersection numbers

Rational scalar products between Feynman integrals

$$\langle I_L | I_R \rangle$$

$$|I_R\rangle = \int d\, z\, \frac{1}{u(z)}\, I_R(z), \quad \langle I_L| = \int d\, z\, u(z)\, I_L(z)$$

Calculated summing over residues of functions locally satisfying a differential equation

$$\langle I_L | I_R \rangle = \sum_{p \in \mathcal{P}_\omega} \mathrm{Res}_{z=p}(\psi\, I_R)$$

$$(\partial_z + \omega)\psi = I_L, \quad \omega \equiv (\partial_z u)/u$$

Solved as an expansion around poles

Non-rational poles:

obstacle at efficient implementation of the algorithm

$p(z)$-adic expansions and residues

Expansion of a rational function in powers of a prime polynomial

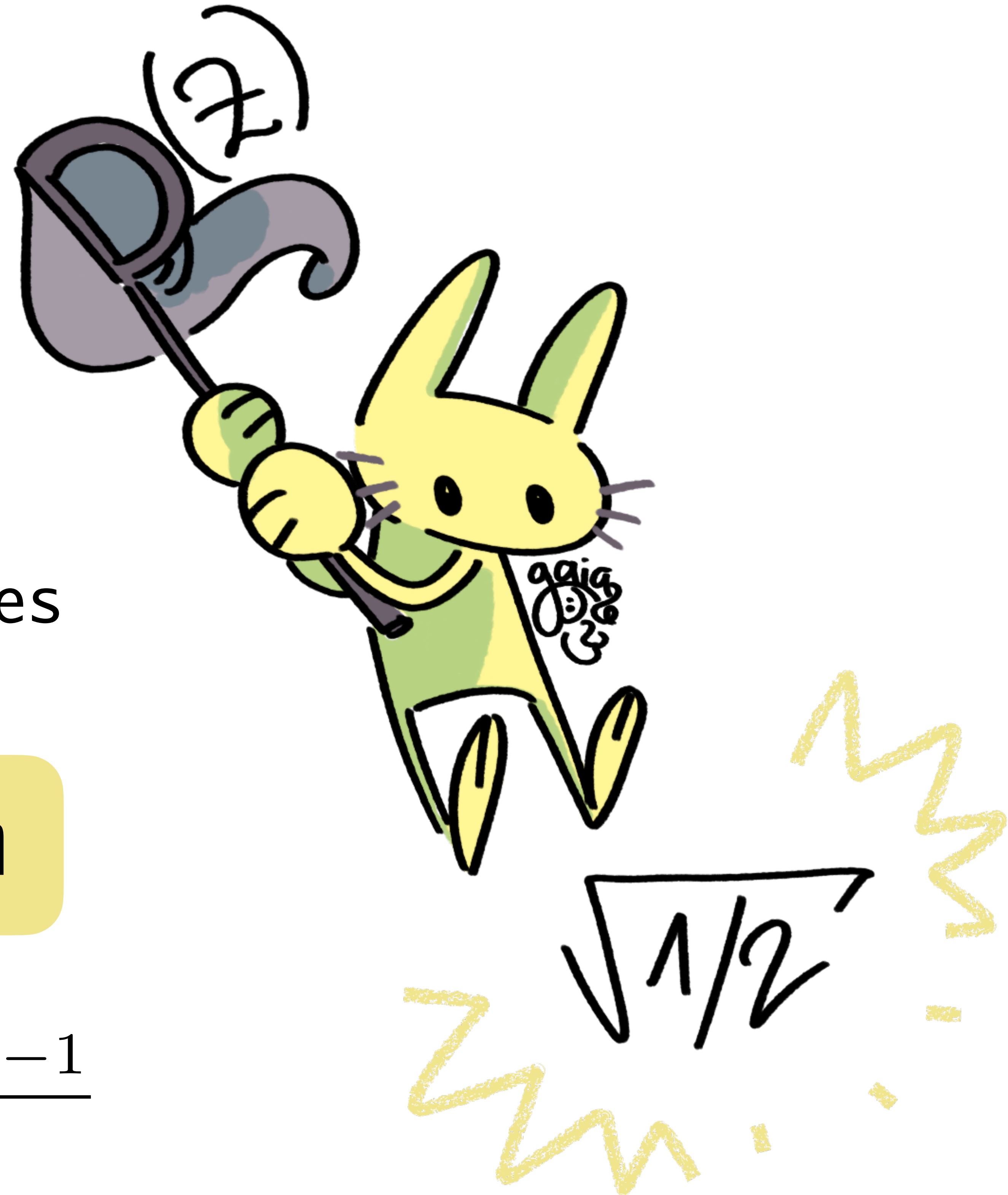
$$f(z) = \sum_{i=min}^{max} c_i(z)\, p^i(z) + \mathcal{O}\left(p(z)^{max+1}\right)$$

prime polynomial

polynomial coefficients $c_i(z)$

$$c_i(z) = \sum_{j=0}^{deg\, p(z)-1} c_{ij}\, z^j$$

- Obtained via repeated polynomial division
- Avoids knowledge of location of irrational poles

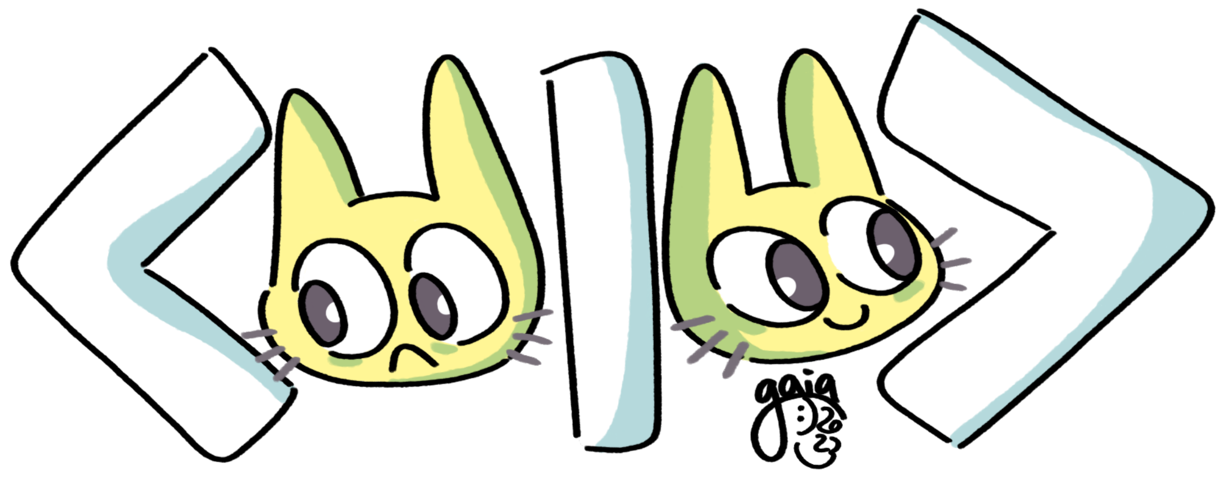


Univariate Global residue theorem

$$\mathrm{Res}_{p(z)}\left(f(z)\right) \equiv \sum_{y\,|\,p(y)=0} \mathrm{Res}_{z=y}\left(f(z)\right) = \frac{c_{-1,deg\ p-1}}{l_c}$$

Conclusions

- **Intersection theory** unveils new mathematical structures in loops
- **$p(z)$-adic expansions :** study rational functions near their roots
  - avoid algebraic extensions
  - no need to know explicit location of roots
  - avoid bottlenecks and enable finite field technologies



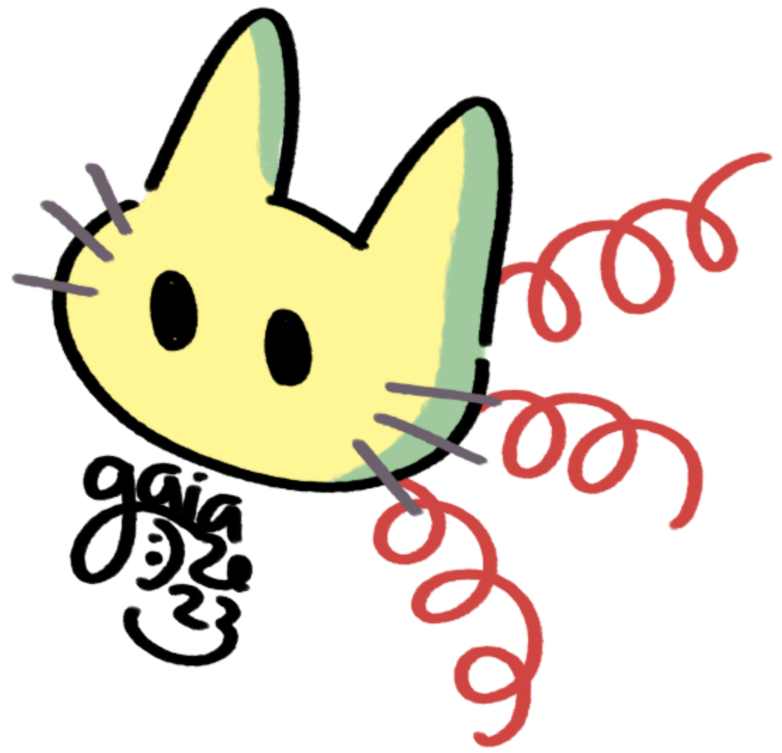
Now, a few words on my PhD project …

with

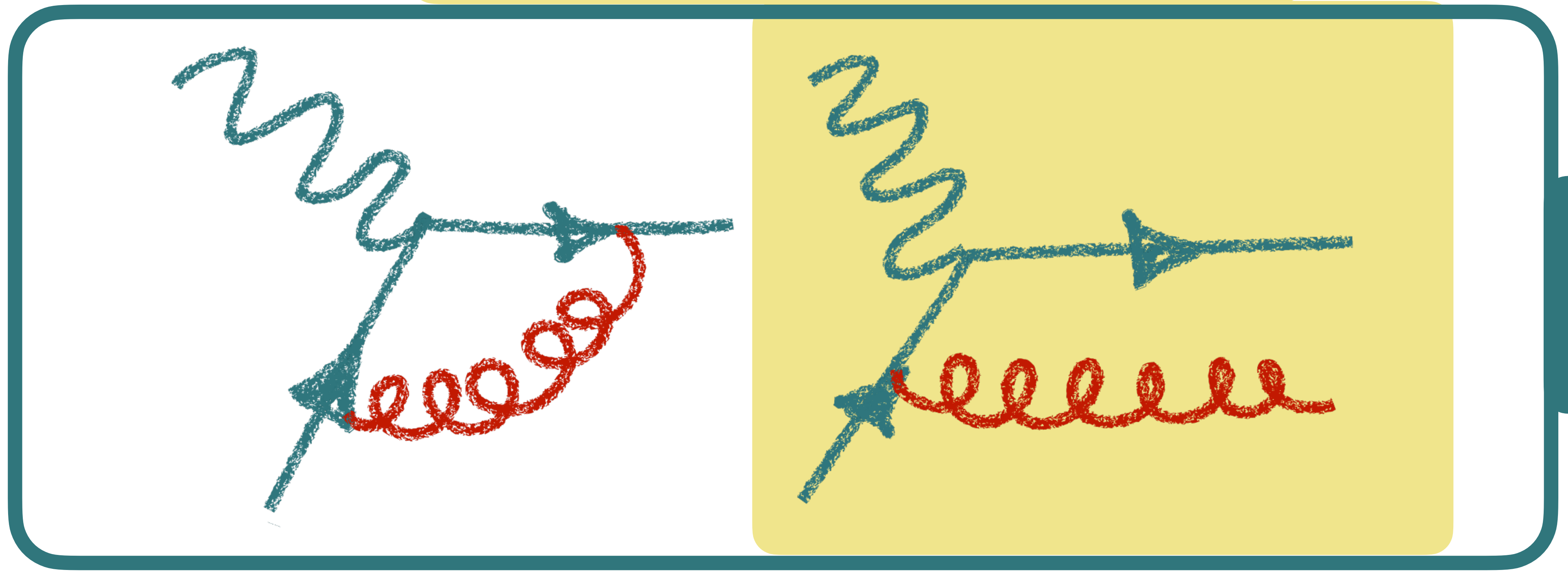
Thomas Gehrmann

Kay Schönwald

To make a theoretical prediction you don't need only loops:

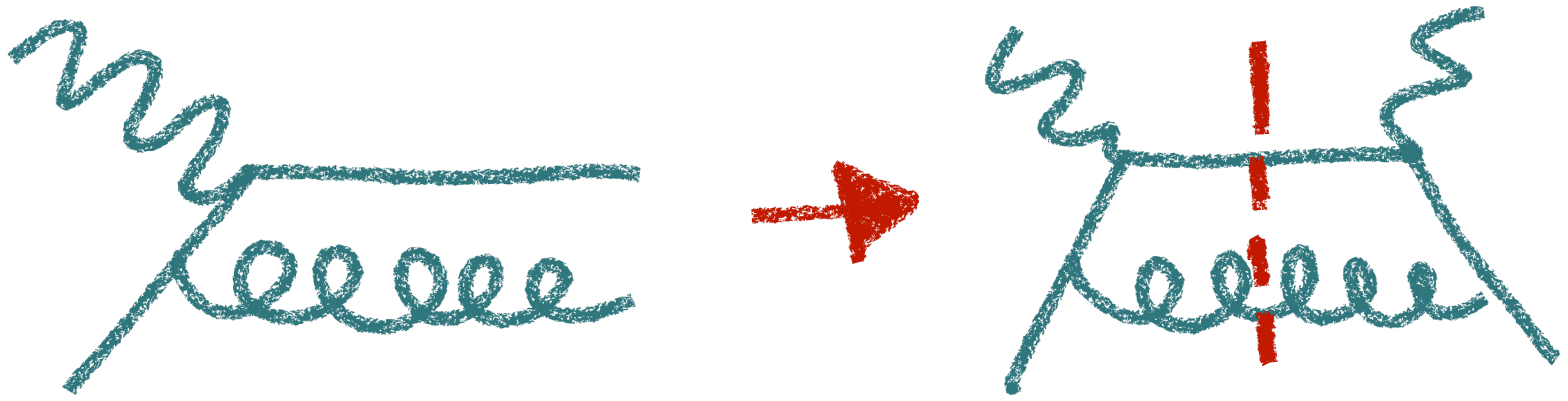


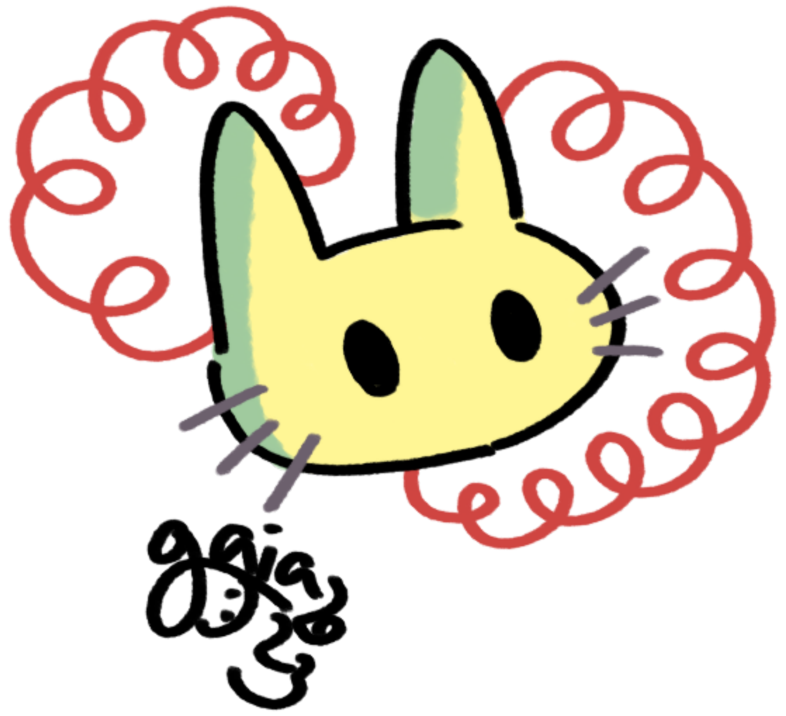
Real corrections



virtual + real = finite

Real corrections calculated via reverse unitarity as cut loops





Subtraction schemes → Antenna subtraction

$$\mathrm{d}\hat{\sigma}_{NLO} = \int_{\mathrm{d}\Phi_m} \mathrm{d}\hat{\sigma}_V^{NLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\hat{\sigma}_S^{NLO} + \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\hat{\sigma}_R^{NLO} - \mathrm{d}\hat{\sigma}_S^{NLO}\right)$$

Thank you for your attention!