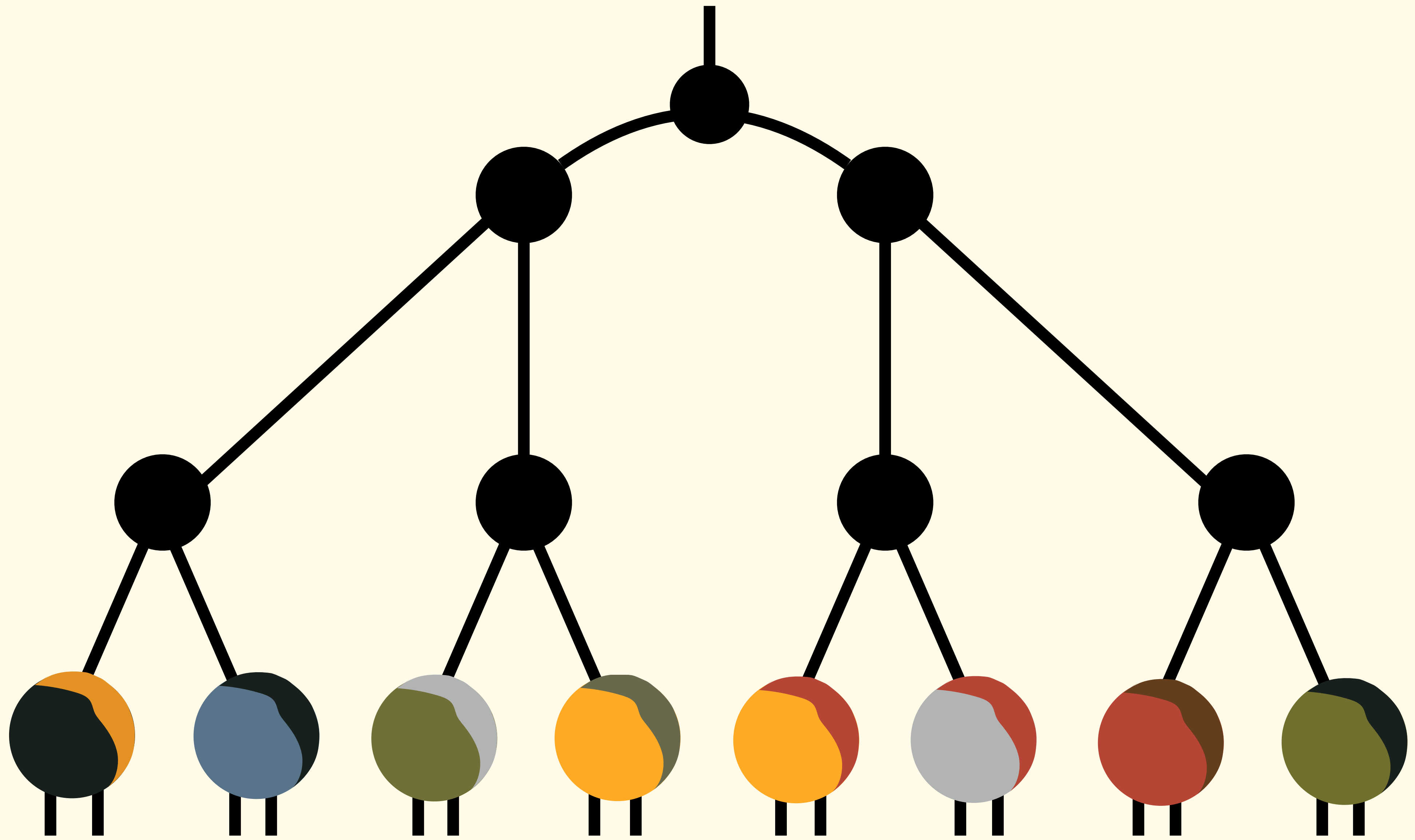
Women in Theoretical Physics - Premio Nazionale “Milla Baldo Ceolin" 2022

# Tree Tensor Networks for Quantum Many-Body Systems at finite temperature


**Nora Reinić**

University of Padova, University of Zagreb

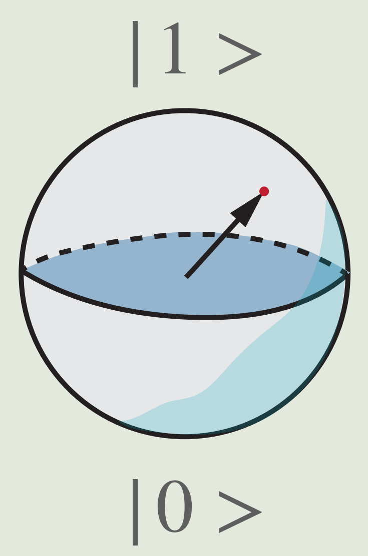



November 2023, Firenze

# Why? Quantum computing

**Bit** → **Qubit**

0 or 1




Quantum **logical gates**

Quantum **algorithms**

**Outperform classical computers** in certain tasks

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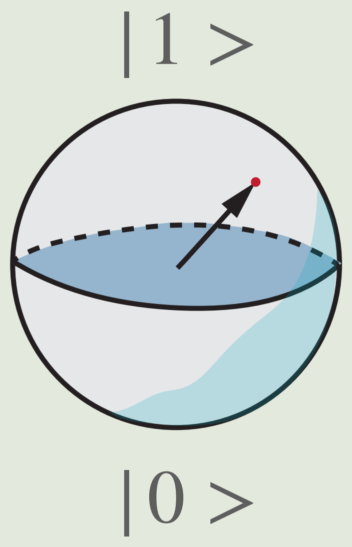
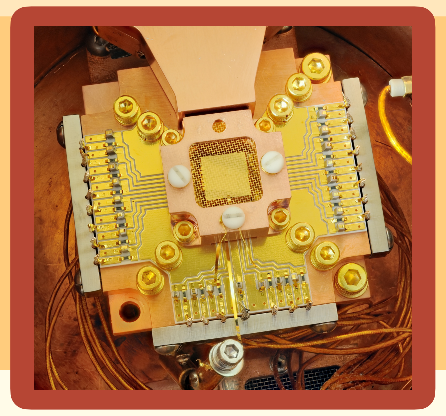



Quantum **logical gates** → Quantum **algorithms** → **Outperform classical computers** in certain tasks

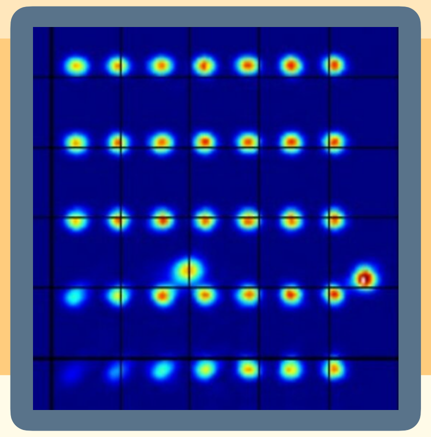
**Experimental quantum computer candidates:**

Superconductors

Trapped ions



**Neutral (Rydberg) atoms**



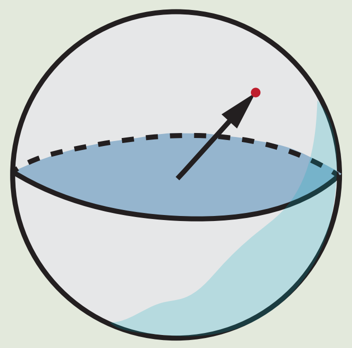
**Why?** 

# Quantum computing

**Bit** ⟶ **Qubit**

0 or 1

$|1>$



$|0>$

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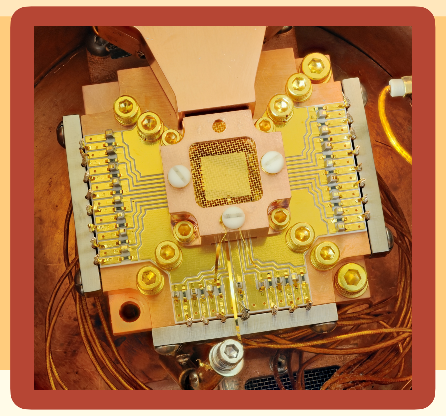

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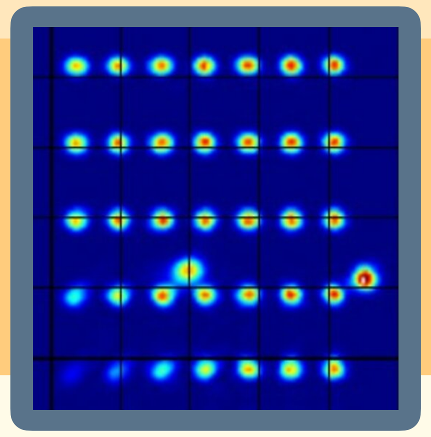

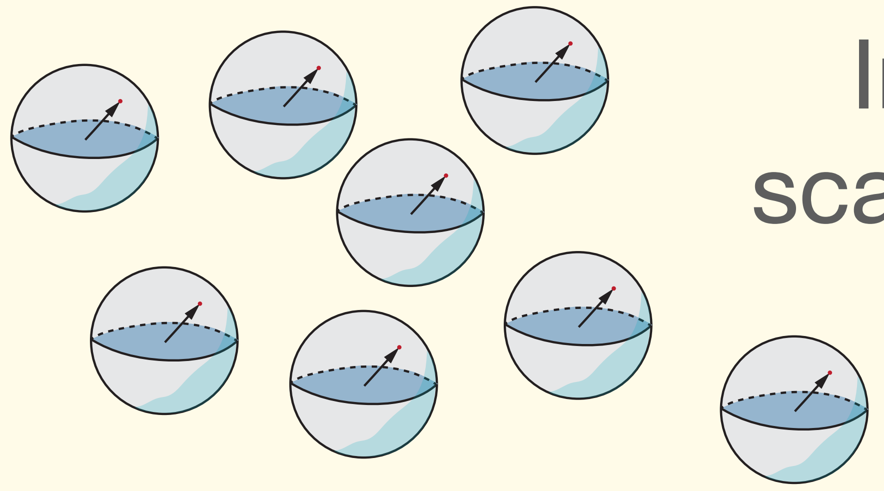

Improve the control over scalable quantum computers $\Longrightarrow$

**What do we do?**

**Examine the quantum many-body physics**

In reality: $T \neq 0$ $\Longrightarrow$ **Finite temperature effects**

**Why is it difficult?**

# Quantum many-body problem

**Hilbert space scales exponentially with the number of particles in the system**

$N$ particles:


$d$ - local Hilbert space dimension

- We need $d^N$ elements to write the state vector

**Storing a Hamiltonian matrix:**

$N = 8 \longrightarrow 1.05$ MB $\quad N = 32 \longrightarrow 10^8$ TB

**Out of reach for any exact diagonalization method!**

Finite temperature

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## Finite temperature

How do we know in which state will the physical system be?

**Statistical physics: at thermal equilibrium, we can assign a classical probability to each state**

$$p_j = \frac{e^{-\beta E_j}}{Z} \qquad Z = \sum_{i=1}^{d^N} e^{-\beta E_i} \qquad \beta = \frac{1}{k_B T}$$

System is described with a density matrix!

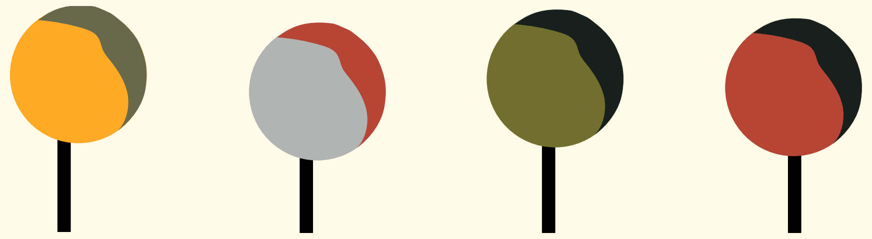
$$\rho = \sum_{j=1}^{d^N} p_j |\psi_j > < \psi_j|$$

How do we do it?

# Tensor Network Methods

**Complexity of quantum many-body state depends on the amount of entanglement in the system**

No entanglement



Easy to simulate

**This scenario is usually not the case**

A lot of entanglement

Exponential number of degrees of freedom

**This scenario is usually not the case**

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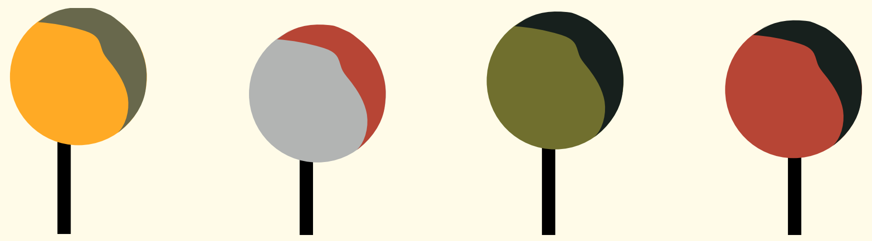

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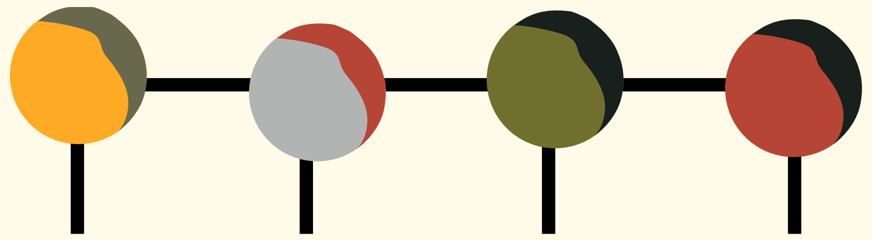
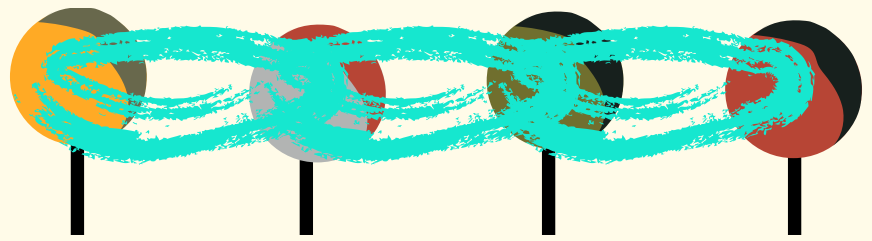
**Many realistic physical systems:**

**the amount of entanglement is sufficiently low**

(area law of entanglement)



A lot of entanglement



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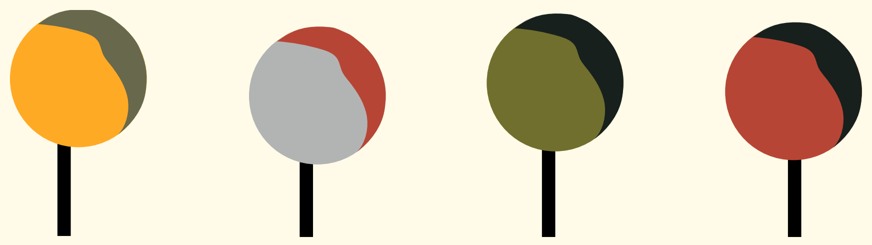

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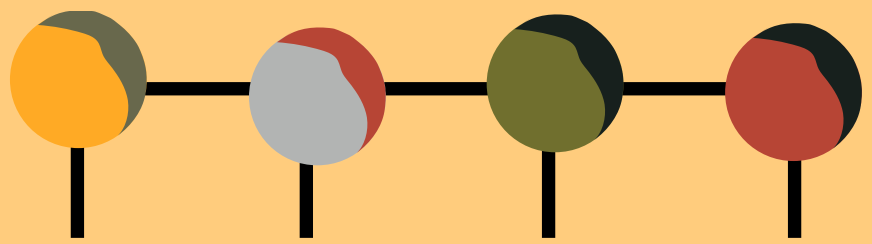
**TENSOR NETWORK METHODS**

State representation written naturally in the language of entanglement



**Keep only certain amount of entanglement in the system and discard the rest**

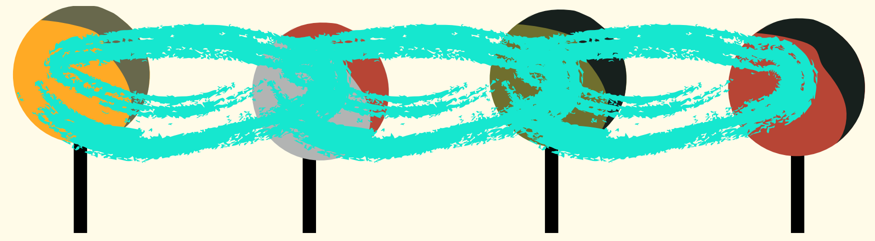
Make quantum many-body simulations possible

Quantum TEA

Quantum GREEN TEA

Quantum TEA LEAVES

A lot of entanglement



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# Master thesis

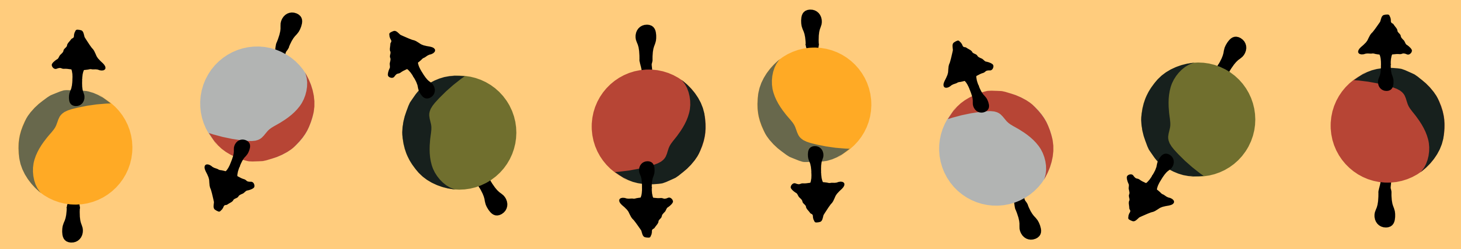
*Tree Tensor Networks for quantum many-body systems at finite temperature*, N.Reinić, (2022), University of Zagreb, Croatia

**Supervisor**: Simone Montangero, University of Padova, INFN Padova

**Developed, implemented, and tested the numerical method for computing the finite-T density matrix**

## Setup

**Quantum many-body spin systems** + **Thermal equilibrium**



**Mixed state!**

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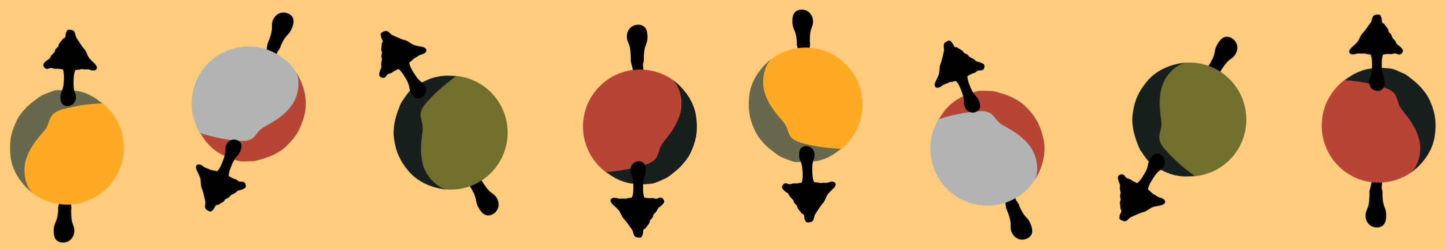
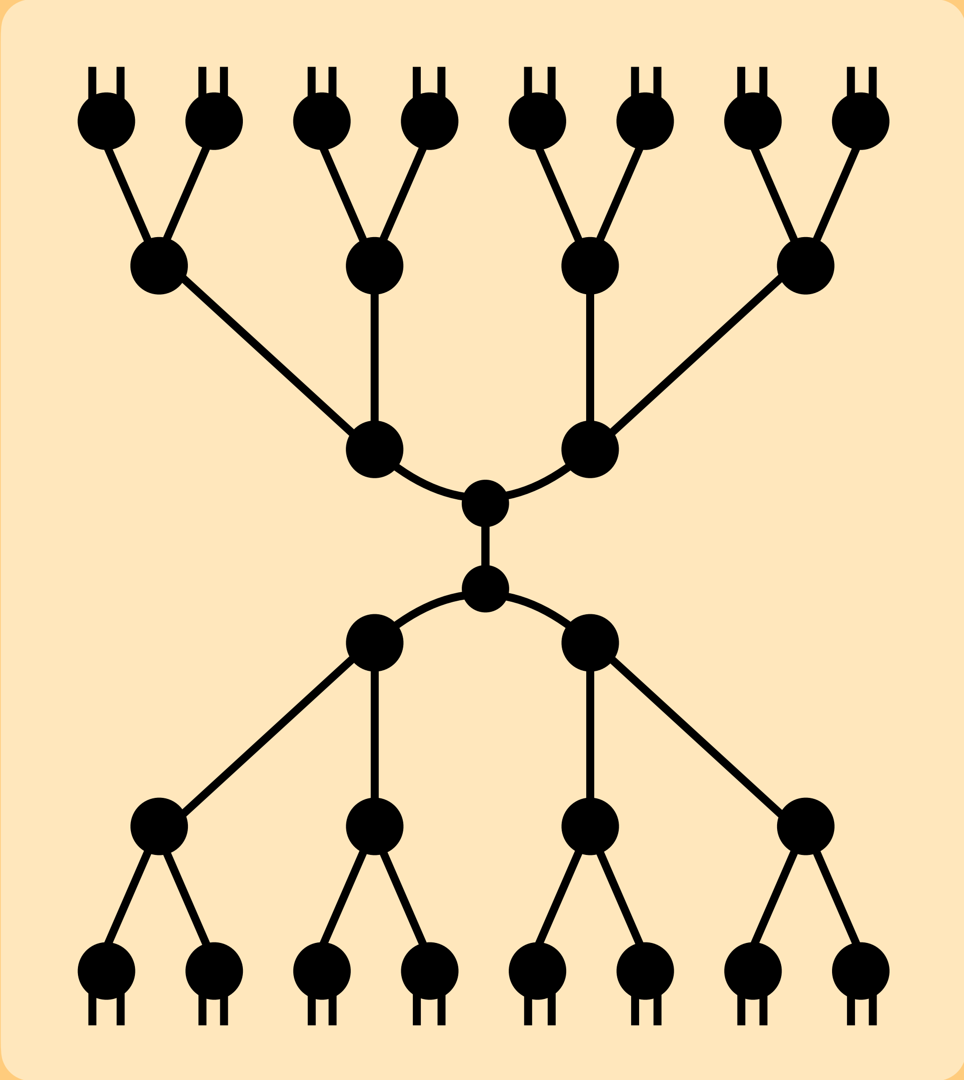


**Mixed state!**

## Numerical method

**Exploiting the Tree Tensor Operator form**

Very suitable for extraction of some of the finite-T properties



We compute:

**Purity**

**Entropy**

**Mixed state entanglement properties**

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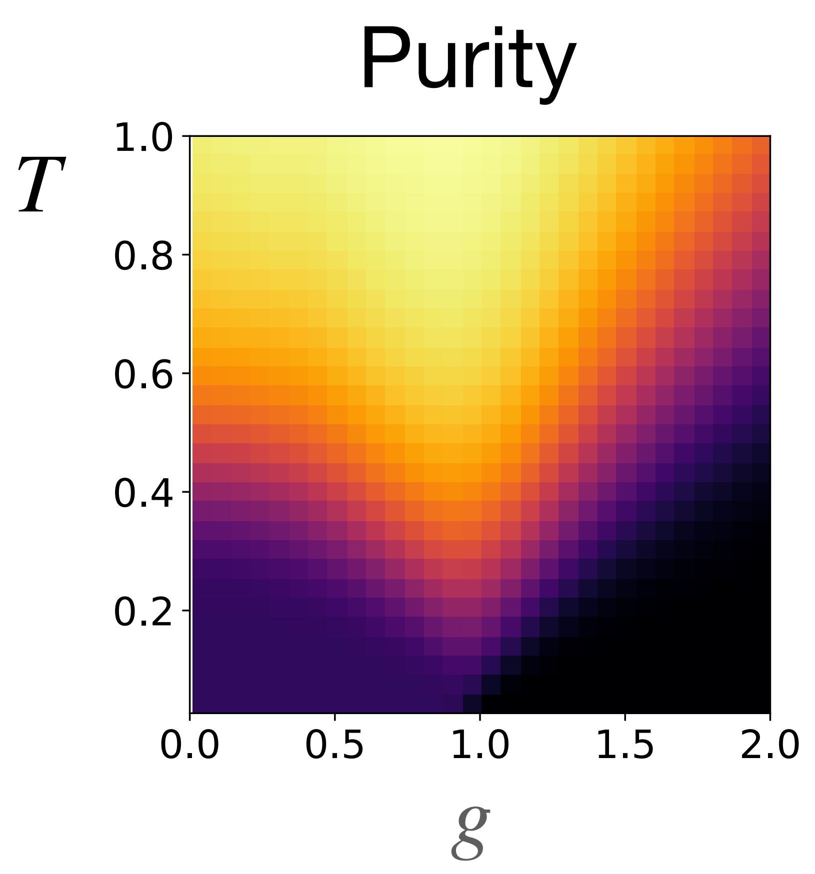
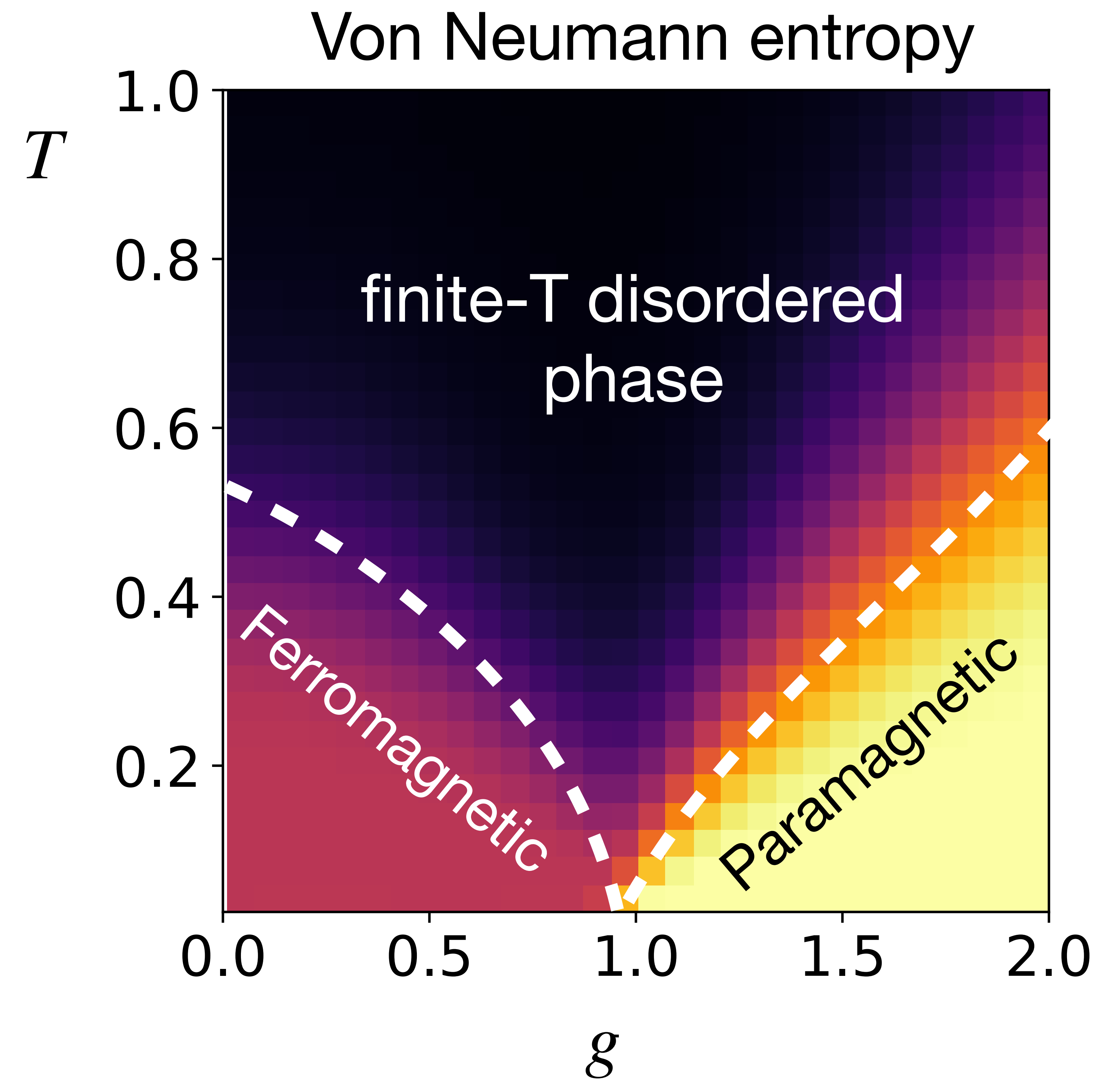
## Result

**Benchmarked and verified the method on the quantum Ising chain for up to $N = 32$ particles**

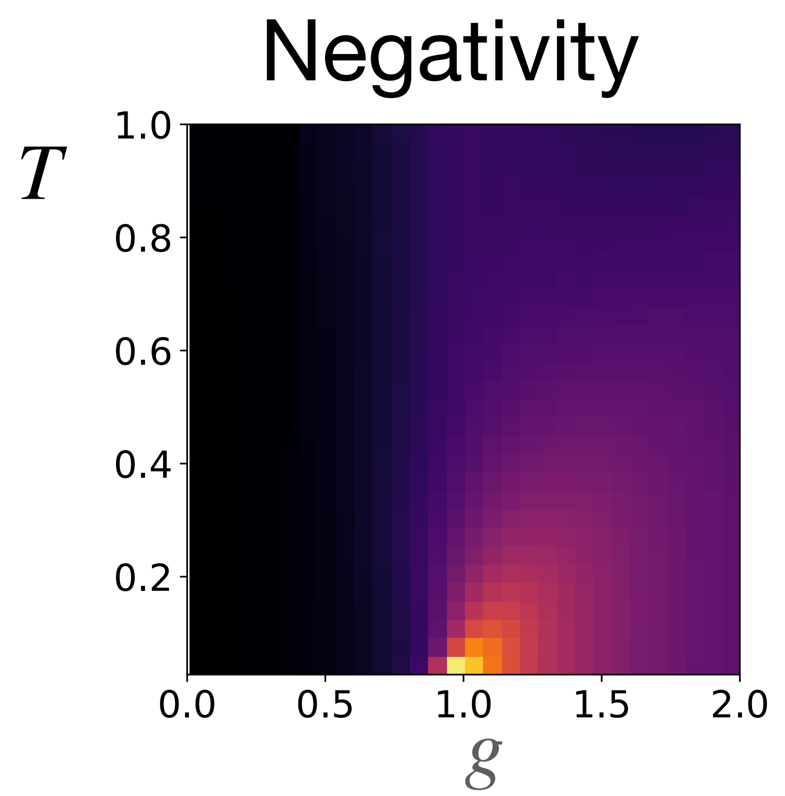
$$\hat{H}_{ising} = -J\Big(\sum_{<ij>} \sigma_x^i \sigma_x^j + g \sum_i \sigma_z^i\Big)$$

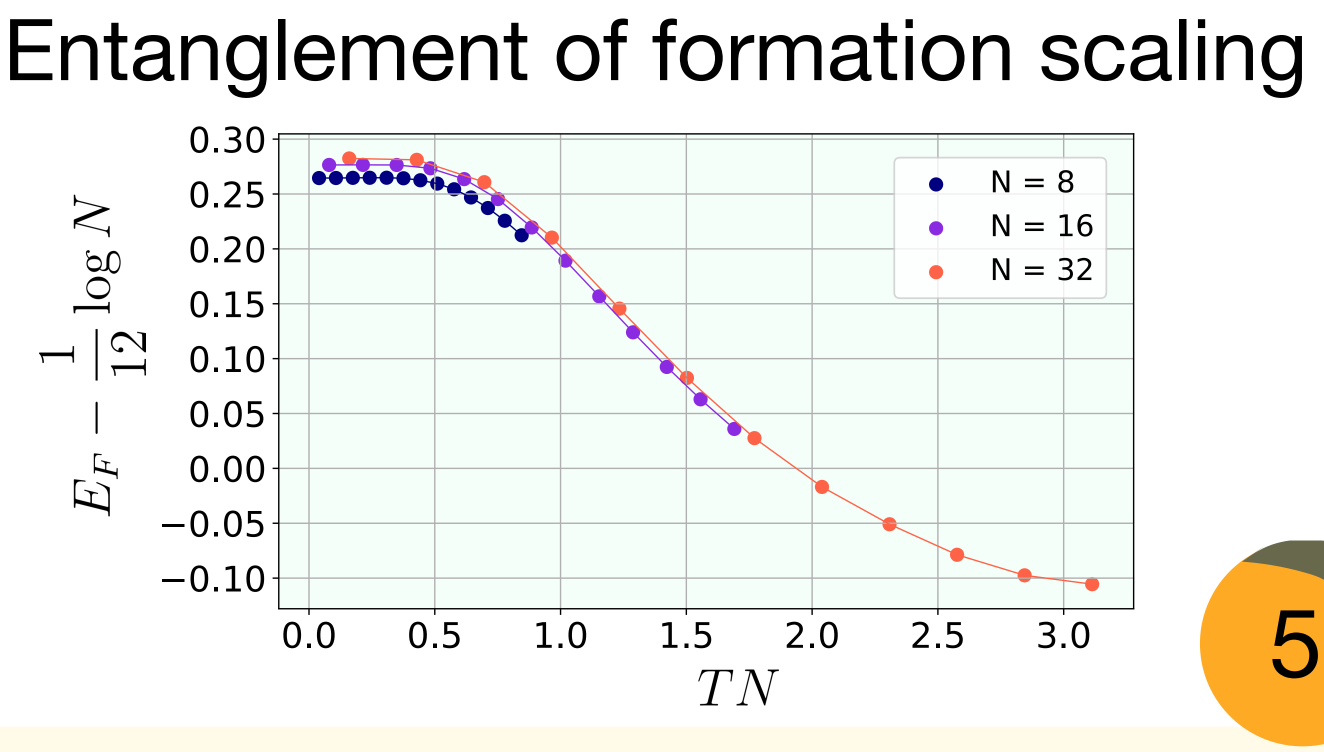
$$J = 1$$




**Entanglement:**

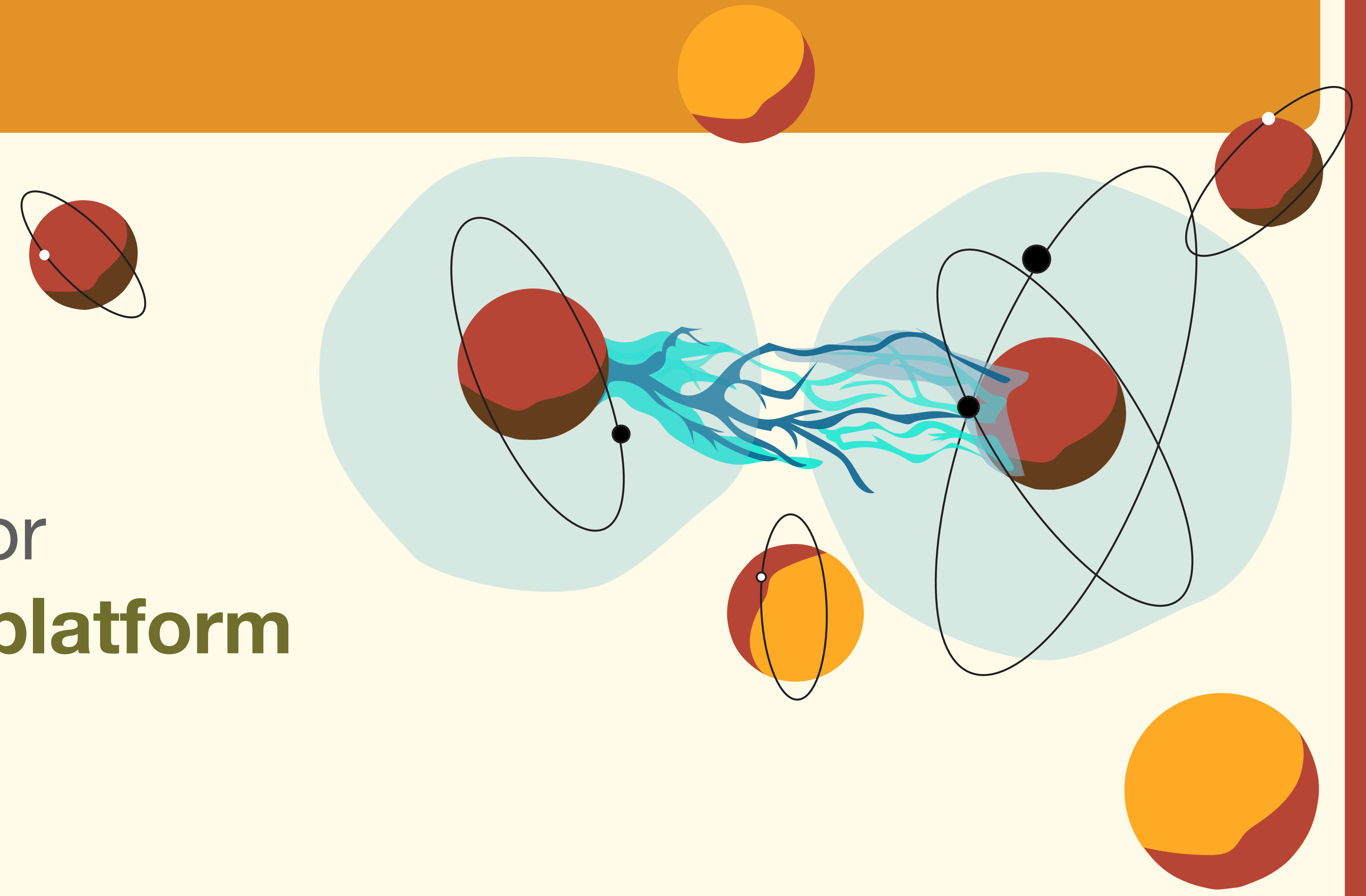


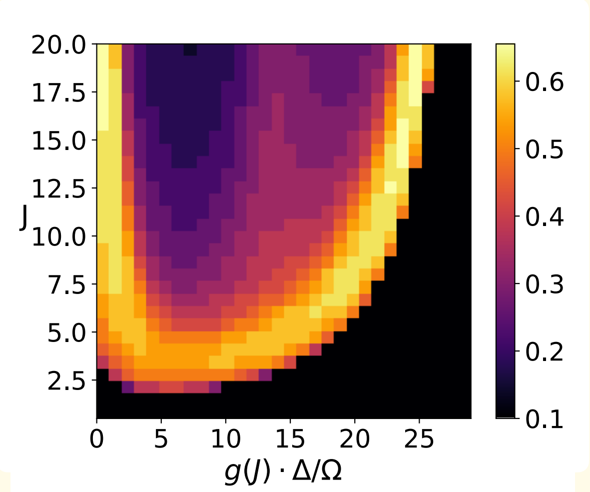



# Outlook and current research

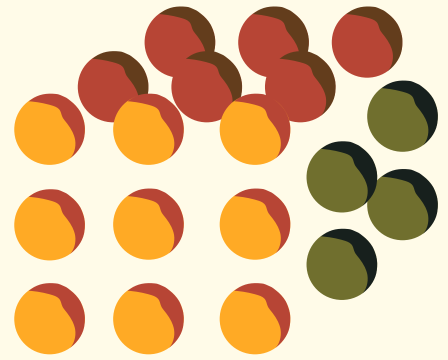
**PhD student** at the Quantum Information and Matter research group at University of Padova, Italy
**Supervisor**: prof. Simone Montangero

**Within the master thesis, we developed a method for computing the finite-T entanglement properties of quantum many-body systems**

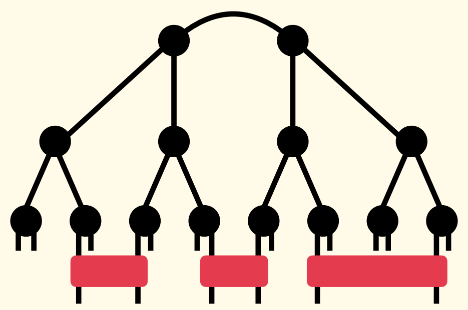





- We apply this method to obtain finite-T phase diagrams for **neutral Rydberg atom systems** - **quantum computing platform**



- Applying tensor network methods to the systems of higher dimensionality **(2D, 3D)**



- Exploring **different tensor network forms** for improving the existing algorithms

*The research leading to these results has received funding from European Union's Horizon 2020 research and innovation programme under the Marie-Sklodowska Curie grant agreement no 101034319 and from the European Union - NextGenerationEU.

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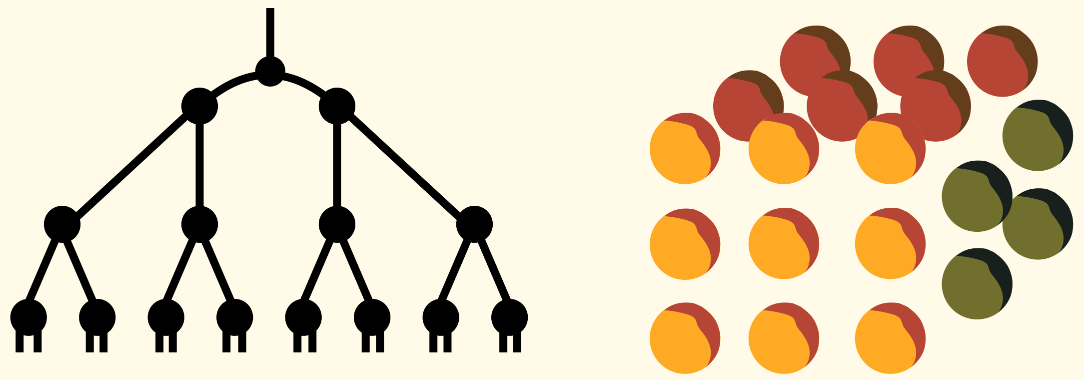
# Thank you for the attention!



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