Random Matrix Theory for fidelity decay and decoherence in quantum information systems

Master Thesis Project

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Random Matrix Theory

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Mehta, Madan Lal. Random matrices. Elsevier, 2004

Random Matrix Theory

$$P(\lambda_1, \lambda_2, ...\lambda_N) \propto \exp\left[-\frac{1}{2}\sum_n \lambda_i^2\right] \times \prod_{j<k} |\lambda_j - \lambda_k|^\beta$$

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The average spectral density

$$\langle \rho_N(E) \rangle = \left\langle \frac{1}{N}\sum_{i=1}^{N} \delta(E - E_i) \right\rangle$$

The spectral form factor

$$\langle K_2(t) \rangle = \langle \mathcal{F}[\rho(E_1)\rho(E_2)] \rangle = 1 + \delta(t) - b_2(t)$$

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- **Gaussian Orthogonal Ensamble ($\beta = 1$)**
  Rotational symmetry and time-reversal invariance
  Hamiltonian matrices *real and symmetric* $H = H^T$
- **Gaussian Unitary Ensamble ($\beta = 2$)**
  Rotational symmetry, but no time reversal invariance
  Hamiltonian matrices are *Hermitean* $H = H^\dagger$
- **Gaussian Symplectic Ensamble ($\beta = 4$)**
  No Rotational symmetry and no time reversal invariance
  Hamiltonian matrices are *quaternion real* matrices $H = H^\dagger$

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Model and echo operator

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$$H_\varepsilon = H_0 + \varepsilon V \qquad \text{where} \qquad H_0 = H_1 + H_e$$

Gorin, T., Prosen T., Seligman T., New Journal of Physics 6.1 (2004): 20.

Model and echo operator

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The fidelity can be written as

$$f(t) = \langle \psi_0(t) | \psi_\varepsilon(t) \rangle = \langle \psi(0) | \, U_0^\dagger(t) U_\varepsilon(t) \, | \psi(0) \rangle$$

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The echo operator:

$$M_\varepsilon(t) = U_0^\dagger(t) U_\varepsilon(t) \; :$$

Using Born Expansion and the linear response approximation:

$$M_\varepsilon(t) = \mathbb{1} - i\varepsilon \int_0^t d\tau \tilde{V}_\tau - \varepsilon^2 \int_0^y \int_0^\tau d\tau' \tilde{V}_\tau \tilde{V}_{\tau'} + O(\varepsilon^3)$$

Gorin, T., Prosen T., Seligman T., New Journal of Physics 6.1 (2004): 20.

Methods

We consider the coupling matrices as Random Matrices and we average over the ensamble:

$$\langle V_{ij} \rangle = 0 \qquad \langle V_{i,j} V_{k,l} \rangle = \delta_{i,l}\delta_{j,k} + \chi_{GOE}\delta_{ik}\delta_{jl}$$

Pineda, C., T. Gorin, and T. H. Seligman. New Journal of Physics 9.4 (2007): 106.

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The second-order term is related to the spectral density of the ensamble

$$\langle \tilde{V}_\tau \tilde{V}_{\tau'} \rangle_{V,|\psi\rangle} = \frac{1}{N} \sum_{\alpha,\gamma} \exp\left[i(E_\gamma - E_\alpha)(\tau - \tau')\right] = K_2(\tau - \tau'),$$

Pineda, C., T. Gorin, and T. H. Seligman. New Journal of Physics 9.4 (2007): 106.

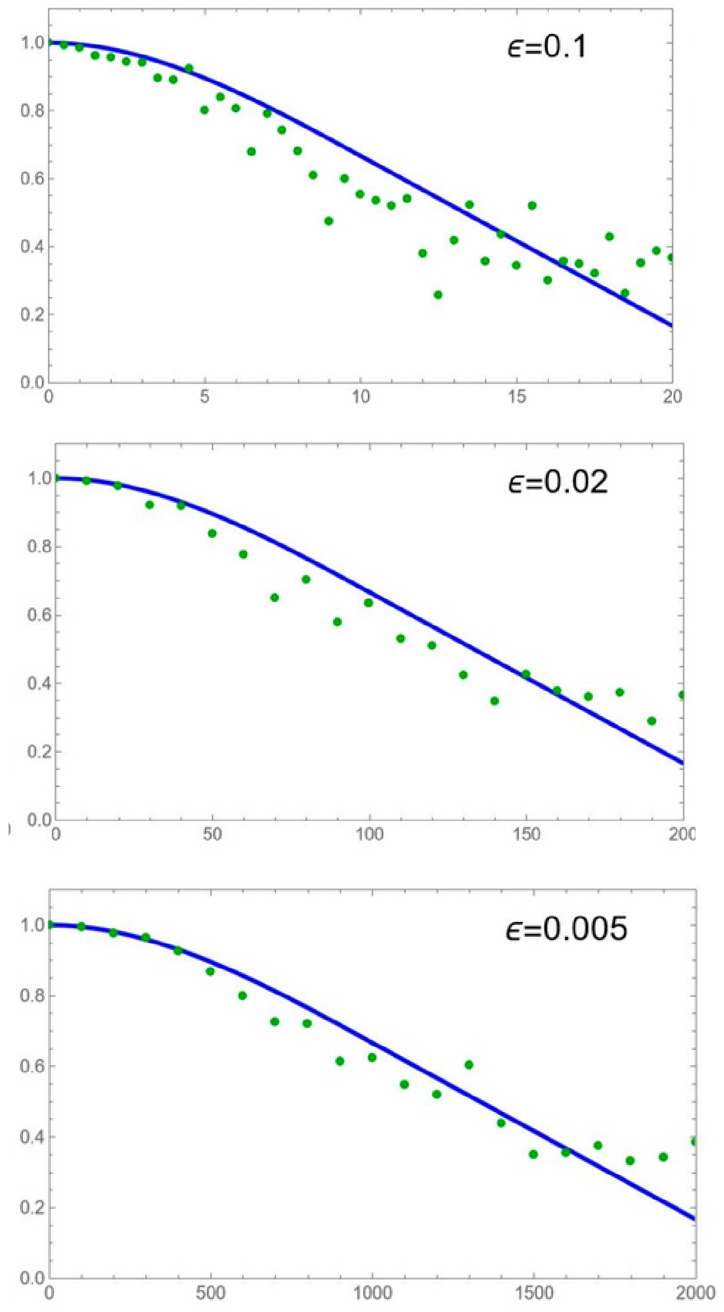
Results

Using $\langle K_2(t)\rangle = 1+\delta(t)-b_2(t)$, for the GUE case, we get

$$\langle f(t)\rangle = 1-\varepsilon^2\left[\frac{t\tau_H}{2}+\frac{t^2}{2}-f_{\tau_H}(t)\right]$$

$$f_{\tau_H}(t)=\int_0^t d\tau\int_0^\tau d\tau' b_2\left(\frac{\tau-\tau'}{\tau_H}\right)=\begin{cases}\dfrac{t\tau_H}{2}+\dfrac{t^3}{6\tau_H} & \text{if} \quad 0\le t<\tau_H\\ \dfrac{t^2}{2}+\dfrac{\tau_H^2}{6} & \text{if} \quad t\ge\tau_H\end{cases}$$

- Purity
- Von Neumann Entropy

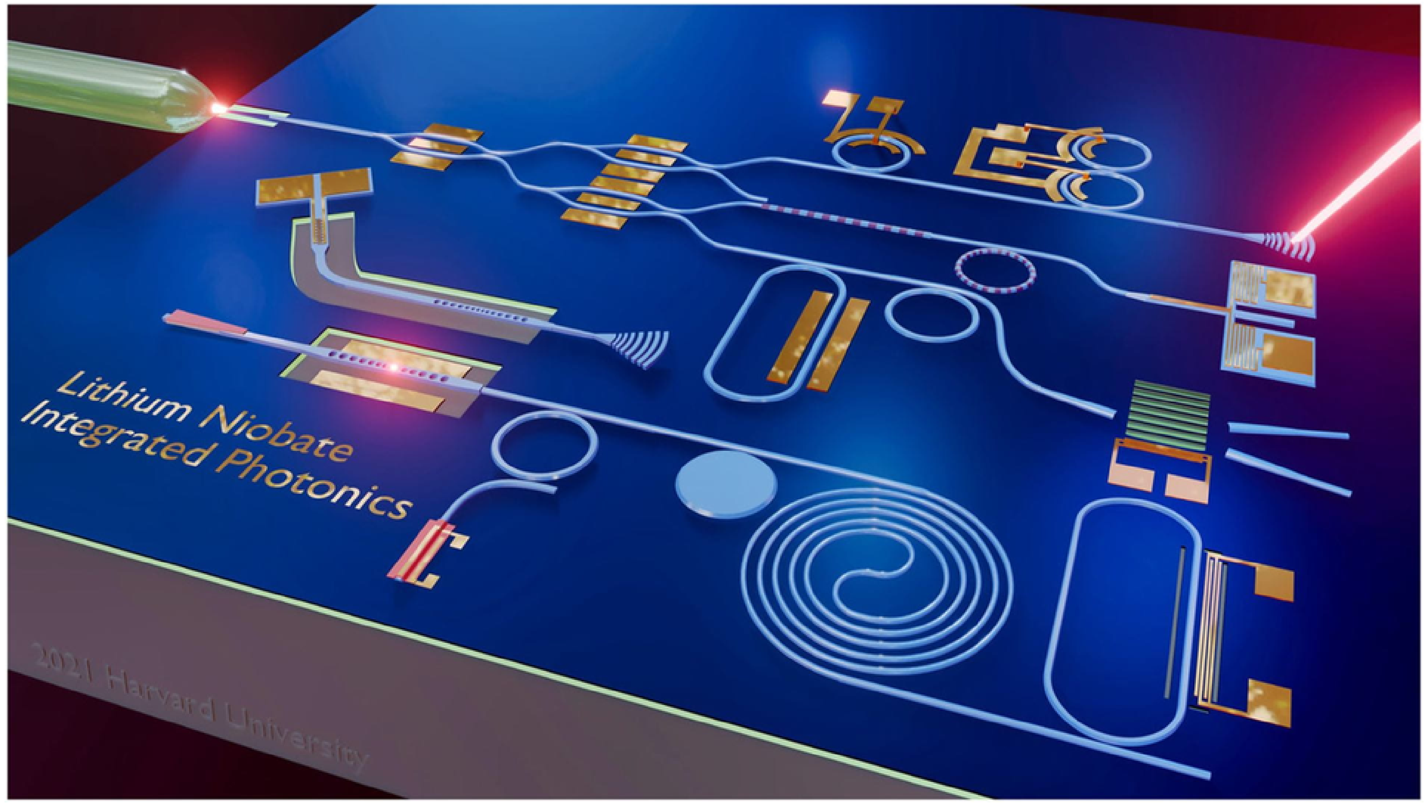


My current research activity

Theoretical design of integrated photonics platforms, to generate non-classical states of light.

Second-order nonlinear processes (spontaneous parametric down-conversion) in resonant and non-resonant structures.

Application to Quantum Technologies: Quantum Key Distribution, Quantum Computation and Quantum Sensing and Metrology



Thank you for your attention!