

Thank you for the invitation.

HOLOGRAPHIC ENTROPY

AND

INTERMEDIATE MASS BLACK HOLES

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## OUTLINE

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4. Most Likely Value of Entropy.
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7. Cosmological Entropy Considerations.
8. Observation of DMBHs a.k.a. IMBHs

## SUMMARY.

References:

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*Upper and Lower Bounds on Gravitational Entropy.*

JCAP 06:008 (2008)

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(2) P.H.F.

*High Longevity Microlensing Events and Dark Matter Black Holes*

arXiv:0806.1717 [gr-qc]

## 1. The Entropy of the Universe.

As interest grows in pursuing alternatives to the Big Bang, including cyclic cosmologies, it becomes more pertinent to address the difficult question of what is the present entropy of the universe?

Entropy is particularly relevant to cyclicity because it does not naturally cycle but has the propensity only to increase monotonically. In one recent proposal, the entropy is jettisoned at turnaround. In any case, for cyclicity to be possible there must be a gigantic reduction in entropy (presumably without violation of the second law of thermodynamics) of the visible universe at some time during each cycle.

Standard treatises on cosmology address the question of the entropy of the universe and arrive at a generic formula for a thermalized gas of the form

$$S = \frac{2\pi^2}{45} g_* V_U T^3 \quad (1)$$

where  $g_*$  is the number of degrees of freedom,  $T$  is the Kelvin temperature and  $V_U$  is the volume of the visible universe. From Eq.(1) with  $T_\gamma = 2.7^0\text{K}$  and  $T_\nu = T_\gamma(4/11)^{1/3} = 1.9^0\text{K}$  we find the entropy in CMB photons and neutrinos are roughly equal today

$$S_\gamma(t_0) \sim S_\nu(t_0) \sim 10^{88}. \quad (2)$$

Our topic here is the gravitational entropy,  $S_{grav}(t_0)$ . Following the same path as in Eqs. (1,2) we obtain for a thermal gas of gravitons  $T_{grav} = 0.91^0\text{K}$  and then

$$S_{grav}^{(thermal)}(t_0) \sim 10^{86} \quad (3)$$

This graviton gas entropy is a couple of orders of magnitude below that for photons and neutrinos.

On the other hand, while radiation thermalizes at  $T \sim 0.1eV$  for which the measurement of the black body spectrum provides good evidence and there is every reason, though no direct evidence, to expect that the relic neutrinos were thermalized at  $T \sim 1MeV$ , the thermal equilibration of the present gravitons is less definite. If gravitons did thermalize, it was at or above the Planck scale,  $T \sim 10^{19}GeV$ , when everything is uncertain because of quantum gravity effects. If the gravitons are in a non-thermalized gas their entropy will be lower than in Eq.(3), for the same number density.

But there are larger contributions to gravitational entropy from elsewhere!!!

## 2. Upper Limit on the Gravitational Entropy.

We shall assume that dark energy has zero entropy and we therefore concentrate on the gravitational entropy associated with dark matter. The dark matter is clumped into halos with typical mass  $M(\text{halo}) \simeq 10^{11}M_{\odot}$  where  $M_{\odot} \simeq 10^{57}\text{GeV} \simeq 10^{30}\text{kg}$  is the solar mass and radius  $R(\text{halo}) = 10^5\text{pc} \simeq 3 \times 10^{18}\text{km} \simeq 10^{18}r_S(M_{\odot})$ . There are, say,  $10^{12}$  halos in the visible universe whose total mass is  $\simeq 10^{23}M_{\odot}$  and corresponding Schwarzschild radius is  $r_S(10^{23}M_{\odot}) \simeq 3 \times 10^{23}\text{km} \simeq 10\text{Gpc}$ . This happens to be the radius of the visible universe corresponding to the critical density. This has led to an upper limit for the gravitational entropy is for one black hole with mass  $M_U = 10^{23}M_{\odot}$ .



Using  $S_{BH}(\eta M_{\odot}) \simeq 10^{77} \eta^2$  corresponds to the holographic principle for the upper limit on the gravitational entropy of the visible universe:

$$S_{grav}(t_0) \leq S_{grav}^{(HOLO)}(t_0) \simeq 10^{123} \quad (4)$$

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 which is 37 orders of magnitude greater than for the thermalized graviton gas in Eq.(3) and leads us to suspect (correctly) that Eq.(3) is a gross underestimate. Nevertheless, Eq.(4) does provide a credible upper limit, an overestimate yet to be refined downwards below, on the quantity of interest,  $S_{grav}(t_0)$ .

The reason why a thermalized gas of gravitons grossly underestimates the gravitational entropy is because of the 'clumping' effect on entropy. Because gravity is universally attractive its entropy is increased by clumping. This is somewhat counter-intuitive since the opposite is true for the familiar 'ideal gas'. It is best illustrated by the fact that a black hole always has 'maximal' entropy by virtue of the holographic principle. Although it is difficult to estimate gravitational entropy we will attempt to be semi-quantitative in implementing the idea.

Let us consider one halo with mass  $M(halo) = 10^{11}M_{\odot}$  and radius  $R_{halo} = 10^{18}r_S(M_{\odot}) \simeq 10^5 pc$ . Applying the holographic principle with regard to the clumping effect would give an overestimate for the halo entropy  $S_{halo}^{(HOLO)}(t_0)$  which we may correct by a purely phenomenological clumping factor

$$S_{halo}(t_0) = S_{halo}^{(HOLO)}(t_0) \left( \frac{r_S(halo)}{R(halo)} \right)^p \quad (5)$$

where  $p$  is a real parameter. Since  $r_S(halo) \leq R(halo)$ , Eq.(5) ensures that  $S_{halo} \leq S_{halo}^{HOLO}(t_0)$  provided that  $p \geq 0$ . Actually the holographic principle requires that  $S_{halo} \leq S_{BH}(M_{halo})$  and since  $S_{BH} \propto r_S^2$ , this requires that  $p \geq 2$  in Eq.(5).

The value  $p = 2$  provides a much better upper limit on the present gravitational entropy of the universe  $S_{grav}(t_0)$  than from Eq.(4). Using our average values for  $M_{halo}$  and  $R_{halo}$  and a number  $10^{12}$  of halos this gives

$$S_{grav}(t_0) < 10^{111} \quad (6)$$

which is many orders of magnitude below the holographic limit of Eq.(4). The physical reason is that the clumping to one black hole is very incomplete as there are a trillion disjoint halos. If all the halos coalesced to one black hole, and there is no reason to expect this given the present expansion rate of the universe, the entropy would reach the maximum value in Eq.(4) of  $10^{123}$  but at present the upper limit is given by Eq. (6).

### 3. Lower Limit on Gravitational Entropy

It is widely believed that most, if not all, galaxies contain at their core a supermassive black hole with mass in the range  $10^5 M_\odot$  to  $10^9 M_\odot$  with an average mass about  $10^7 m_\odot$ . Each of these carries an entropy  $S_{BH}(\text{supermassive}) \simeq 10^{91}$ . Since there are  $10^{12}$  halos this provides the lower limit on the gravitational entropy of

$$S_{grav}(t_0) \geq 10^{103} \quad (7)$$

which together with Eq.(6) provides an eight order of magnitude window for  $S_{grav}(t_0)$ .

The lower limit in Eq.(7) from the galactic supermassive black holes may be largest contributor to the entropy of the present universe but this seems to us highly unlikely because they are so very small. Each supermassive black hole is about the size of our solar system or smaller and it is intuitively unlikely that essentially all of the entropy is so concentrated.

Gravitational entropy is associated with the clumping of matter because of the long range unscreened nature of the gravitational force. This is why we propose that the majority of the entropy is associated with the largest clumps of matter: the dark matter halos associated with galaxies and cluster.

#### 4. Most Likely Value of Entropy.

In the phenomenological formula for clumping, Eq.(5), the parameter  $p$  must satisfy  $2 \leq p < \infty$  because for  $p = 2$  the halo entropy is as high as it can be, being equal to that of the largest single black hole into which it could collapse, while for  $p \rightarrow \infty$ , the halo has no gravitational entropy beyond that of the supermassive black hole at its core.

Thus, our upper and lower limits are

$$10^{111} \geq S_{grav}(t_0) \geq 10^{103} \quad (8)$$

correspond to  $p = 2$  and  $p \rightarrow \infty$  in Eq.(5) respectively. We may include the supermassive black holes in Eq.(5) by noticing that  $S_{grav}(t_0) = 10^{(125-7p)}$  and therefore, from Eq.(8),  $2 \leq p \leq 22/7$ .

Actually, the power  $p$  in Eq.(5) must depend on the halo radius  $R_{halo}$  such that  $p(R_{halo}) \rightarrow 2$  as  $R_{halo} \rightarrow r_S$ , the Schwarzschild radius, when the halo collapses to a black hole. For the present non-collapsed status of the halos,  $p > 2$  is necessary since the black hole represents the maximum possible entropy. One would also expect  $p$  to be density and therefore radial dependent, but we assume this dependence is mild enough to allow us to obtain order of magnitude estimates by setting  $p = const.$



The truth must therefore lie somewhere in between, in the range  $2 < p \leq 22/7$ . In the absence of a quantitative calculation of gravitational entropy, the integer value  $p = 3$  in Eq.(5) is one possibility. The value  $p = 3$  gives  $S_{halo} \sim 10^{92}$  and hence an estimate for  $S_{grav}(t_0)$  of  $10^{12}$  halos of

$$S_{grav}(t_0) \sim 10^{104} \quad (9)$$

which is somewhat nearer the lower than the upper limit in Eq.(8) though still 19 orders of magnitude below the holographic bound in Eq.(4).

For actual halos,  $R_{halo} \sim 10^5$  pc while for mass  $M_{halo} \sim 10^{11} M_{\odot}$  the Schwarzschild radius is  $r_S \sim 3 \times 10^{11} km \simeq 0.01$  pc which means that  $R_{halo}/r_S \gg 1$ , and we are indeed approaching the asymptotic regime  $R_{halo}/r_S \rightarrow \infty$ , for which we seek the asymptotic value of  $p_a$  defined by  $p(R_{halo}) \rightarrow p_a$  as  $R_{halo} \rightarrow \infty$ . Here, we have assumed that  $p_a = 3$  as it is the only integer satisfying  $2 < p \leq 22/7$ .

It would be more compelling to possess a derivation of  $p_a = 3$  based on general principles, for example, within the context of quantum information theory. We can rewrite Eq.(5) for  $p_a = 3$  in the more suggestive manner

$$S_{grav} \xrightarrow{R \rightarrow \infty} S_{BH} \left( \frac{\rho}{\rho_{BH}} \right) \left( \frac{R}{r_S} \right)^2 \quad (10)$$

which is similar to the quantum gravity holographic bound with the insertion of the rescaling for density for a spherical mass distribution with  $R \gg r_S$ , the Schwarzschild radius. As written in Eq.(10), this estimate of gravitational entropy can hold for all  $R$ .

We can relate this to the arguments about quantum foam by generalizing the uncertainty in a length measurement from  $(\delta l)^3 = l_P^2 l$  where  $l$  is the length in question and  $l_P$  is the Planck length to the modified generalization

$$(\delta l')^3 = l_P^2 l \left( \frac{l}{r_S} \right) = l_P^2 l \left( \frac{\rho}{\rho_{BH}} \right)^{1/3} \quad (11)$$

where  $\rho = m/l^3$  is the extant density and  $\rho_{BH} = M/l_P^3$  is the black hole density.

In this case, ignoring prefactors which are  $O(1)$  the number of  $(\delta l')^3$  cells in a volume  $l^3$  suggests, paralleling discussions of a gravitational entropy

$$S = \left( \frac{l^3}{\delta l'^3} \right) = \left( \frac{l^2}{l_P^2} \right) \left( \frac{r_S}{l} \right) = \left( \frac{l^2}{l_P^2} \right) \left( \frac{\rho_{BH}}{\rho} \right)^{1/3} \quad (12)$$

which agrees with Eq.(5) for  $p = 3$ . So the uncertainties in measurement of a length  $l$  now depend not only on the length  $l$  itself but on the extant density relative to a 'maximal' black-hole density. We know that in this framework  $\delta l$  represents a 'minimal' uncertainty so necessarily  $\delta l' \geq \delta l$  and so must increase as the extant density decreases. While this is not a rigorous derivation, as one can hardly expect because it is quantum gravity whose underlying theory is unknown, we regard it as suggestive and plausible.

At first sight, we may be concerned that Eq.(11), or

$$(\delta l') = (\delta l) \left( \frac{l}{r_S} \right)^{1/3} \quad (13)$$

might imply a  $(\delta l')$  which can grow beyond  $(\delta l)$  to an obviously unacceptable value. That this is *not* the case can be confirmed by considering specific astrophysical objects with masses ranging from the entire universe down to the equality mass  $M_e = 10^{21}$  kg in Eq.(16). Lesser masses have negligible gravitational entropy.

The point is that the final factor  $(l/r_s)$  in Eq.(13) is never extremely large, always  $\leq 10^6$ . For the universe it is  $\sim 1$  and  $(\delta l') \sim (\delta l) \sim 1$  fm. For smaller clumps of matter,  $(\delta l)$  decreases but the correction factor does not raise  $(\delta l')$  above  $\sim 1$  fm while for smaller objects such as the Sun it is less. For example, for the galactic halo parameters we have used,  $(\delta l) \sim 3 \times 10^{-3}$  fm and  $(\delta l') \sim 1$  fm. For the Sun,  $(\delta l) \sim 10^{-6}$  fm and  $(\delta l') \sim 10^{-4}$  fm. These are typical values,

We believe the pursuit of better understanding of gravitational entropy in clumps of matter with mass above  $M_e = 10^{21}$  kg. (see Eq. (16) below) may provide a very fruitful approach towards a satisfactory theory of quantum gravity. We remind the reader of our conventions  $\hbar = c = k = 1$ : restoration of units reveals the  $\hbar$  in Eq.(12), in  $l_P \propto \hbar^{1/2}$ , and the gravitational entropy we are discussing is, if it exists, a quantum mechanical phenomenon.

We can apply the same considerations based on Eq.(5) to gravitation within a single star like the Sun. The Sun has  $(R_\odot/r_S) \sim 10^5$  and with  $p_a = 3$  we find  $S_{\odot}^{(grav)} \sim 10^{72}$ , far above the standard  $S_\odot \sim 10^{57}$ , suggesting a contribution from stars to the gravitational entropy of about  $\sim 10^{95}$ .



As the gravitating object we consider becomes smaller the relative importance of gravitational entropy to non-gravitational entropy changes. Let us obtain a rough estimate of the mass  $M_e$  at which the two contribution are comparable.

Suppose  $M_e = \eta M_\odot \simeq 10^{30} \eta$  kg. and so we wish to determine  $\eta$ . We can estimate  $\eta$  by the fact that the gravitational entropy in Eq.(5) is not linear in  $\eta$  but has a quite different dependence. Let us take the typical density of the putative object to be  $\rho = 5\rho_{H_2O} = 5 \times 10^{12} \text{kg}/(\text{km})^3$ . The radius of a sphere with mass  $M_e$  is then  $R \simeq 4 \times 10^5 \eta^{1/3}$  km. Thus the gravitational entropy from Eq.(5) is

$$S_{grav} = (10^{77} \eta^2) \left( \frac{3\eta}{4 \times 10^5 \eta^{1/3}} \right) \simeq 10^{72} \eta^{8/3} \quad (14)$$

The non-gravitational entropy may be estimated by counting baryons to give the usual form linear in  $\eta$

$$S_{non-grav} \simeq 10^{57} \eta. \quad (15)$$

The two contributions,  $S_{grav}$  of Eq.( 14 ) and  $S_{non-grav}$  of Eq. ( 15 ) become comparable when  $\eta^{-5/3} \sim 10^{15}$  or  $\eta \sim 10^{-9}$ . This 'equality' mass  $M_e$  is about

$$M_e \simeq 0.1\% M_{\oplus} \simeq 10^{21} \text{kg}. \quad (16)$$

If we consider much smaller masses such as a baseball ( $\sim 1$  kg) or a primordial black hole with lifetime comparable to the age of the universe ( $\sim 10^{12}$  kg), the gravitational entropy becomes negligible.

According to our phenomenological clumping ansatz, Eq.(5), the entropy of solar system objects can be larger than conventionally assumed, the Sun by  $10^{15}$ , the Earth by  $10^5$ . We have no derivation of this new gravitational entropy component and publish this idea only to prompt more mathematically rigorous arguments to estimate the contribution of gravitational clumping to entropy.

An intuitive reason to suspect a large gravitational entropy outside of black holes comes from considering the gravitational collapse of an object of mass, say,  $M = 10M_{\odot}$  which contains  $\sim 10^{58}$  nucleons and hence non-gravitational entropy  $S \sim 10^{58}$ . Under gravitational collapse, it is conventionally believed that the entropy gradually increases, though *not* by orders of magnitude, as the radius decreases to a few times the Schwarzschild radius.

When the trapped surface of a black hole appears the entropy becomes  $\sim 10^{79}$ , an increase of some twenty orders of magnitude! While not excluded, this is intuitively implausible. On the other hand, with the clumping factor of Eq.(5) and the starting density we have employed of  $\rho = 5\rho_{H_2O}$ , the starting entropy from Eq.(14) is already  $\sim 10^{72+8/3} \sim 5 \times 10^{74}$ , and less dramatic entropy increase is needed.

There is a second consideration which provides circumstantial evidence for new gravitational entropy. If, as in Eq.(7), the cosmological entropy is dominated by the supermassive black holes, it implies that almost all the entropy is confined to a trillion objects each of radius  $\sim 10^{-6}$  pc occupying  $\sim 10^{-33}$  of the halo volume. Altogether they compose only  $\sim 10^{-36}$  of the total volume of the visible universe. Although not excluded by any deep principle, this just seems intuitively unlikely.

Let us attempt to make a somewhat more quantitative argument out of idea of how entropy grows with gravitational clumping. At last scattering density perturbations in the dark matter were small,  $\frac{\delta\rho}{\rho} \sim 10^{-5}$ , but today there are regions where  $\frac{\delta\rho}{\rho} \sim 1$  where we expect the gravitational entropy has increased enormously even though the entropy in photons has remained constant.

The non-clumped component of the universe expands adiabatically. How do we get the entropy of a clump? Assume the dark matter is in the form of very light particles. For a clump of size  $L_{gal} = 10^5$  pc, the lightest particles that can clump are of mass  $m \sim 10^{-26}$  eV. Otherwise their wavelength is larger than  $L_{gal}$ .

Recall the galactic mass is  $M_{gal} \sim 10^{12} M_{solar} \sim 10^{69}$  GeV. If this is all in dark matter (ignore baryons, etc.), then there can be at most  $N \sim \frac{M_{gal}}{m} \sim 10^{104}$  dark matter particles in a halo, or about  $N_U \sim 10^{115}$  dark matter particles in the universe that are now clumped.

If the dark matter particles start off at rest (similarly to nonthermal axions) but then start to fall into clumps, we can argue that their degrees of freedom get excited, i.e., as the particles fall into the potential well they gain kinetic energy. So these gravitational d.o.f.s give approximately zero contribution to the total entropy before density perturbations start to grow, but they now contribute  $\sim 10^{115}$ . If the masses of the dark matter are larger, the contribution to the entropy will be proportionally smaller. The mass  $m \sim 10^{-26}$  eV provides an approximate upper bound on the gravitational entropy. The lower bound for the entropy in this particulate approach is very small if the dark matter particles are far heavier such as WIMPs at the TeV scale.

## 5. Intermediate comments

Entropy is always a subtle concept, nowhere more so than for gravity. This is why we are bold enough to make such approximate estimates of the present gravitational entropy of the visible universe. Our results are concerned only with orders of magnitude and we hope our upper and lower limits  $10^{111}$  and  $10^{103}$  are credible.

These already show that the universe's entropy is dominated by gravity, being at least 13 orders of magnitude above the known entropies, each  $\simeq 10^{88}$ , for photons and relic neutrinos.



Using the clumping idea and an heuristic clumping factor dependent on a parameter  $p_a$  suggests that the gravitational entropy is dominated not by the well known galactic supermassive black holes which contribute  $\simeq 10^{103}$  but by a larger, possibly much larger, contribution from the dark matter halos which can provide (for  $p_a = 3$ ) about  $10^{104}$ , though not more than (for  $p_a \rightarrow 2$ ) about  $10^{111}$  which is still many orders of magnitude below the holographic bound  $\simeq 10^{123}$ .

It is reasonable to expect the gravitational entropy to be non-classical and an effect of quantum gravity like the holographic bound and the black hole entropy. Since string theory has had some success in those two cases, it may help in deciding whether our speculations are idle. More optimistically, the study of gravitational entropy will lead to a better theory of quantum gravity, hopefully the correct one.

## THREE APPROACHES TO QUANTUM GRAVITY

1. String theory
2. Quantum loop gravity
3. The correct theory

If our speculations are correct: the contribution of radiation to the entropy is less than 1 part in  $10^{16}$  of the total; supermassive black holes at galactic cores contribute less, possibly much less, than ten per cent; the gravitational entropy contained only in stars is already greater than the entropy of electromagnetic radiation; and the gravitational entropy contained in dark matter halos is the biggest contributor to the entropy of the universe.

## 6. Dark Matter Black Holes

If we consider normal baryonic matter, other than black holes, contributions to the entropy are far smaller. The background radiation and relic neutrinos each provide  $\sim 10^{88}$ . We have learned in the last decade about the dark side of the universe. WMAP suggests that the pie slices for the overall energy are 4% baryonic matter, 24% dark matter and 72% dark energy. Dark energy has no known microstructure, and especially if it is characterized only by a cosmological constant, may be assumed to have zero entropy. As already mentioned, the baryonic matter other than the SMBHs contributes far less than  $(S_U)^{min}$ .

This leaves the dark matter which is concentrated in halos of galaxies and clusters.

It is counter to the second law of thermodynamics when higher entropy states are available that essentially all the entropy of the universe is concentrated in SMBHs. The Schwarzschild radius for a  $10^7 M_\odot$  SMBH is  $\sim 3 \times 10^7$  km and so  $10^{12}$  of them occupy only  $\sim 10^{-36}$  of the volume of the visible universe.

Several years ago important work by Xu and Ostriker showed by numerical simulations that DMBHs with masses above  $10^6 M_\odot$  would have the property of disrupting the dynamics of a galactic halo leading to runaway spiral into the center. This provides an upper limit  $(M_{DMBH})^{max} \sim 10^6 M_\odot$ .

Gravitational lensing observations are amongst the most useful for determining the mass distributions of dark matter. Weak lensing by, for example, the HST shows the strong distortion of radiation from more distant galaxies by the mass of the dark matter and leads to astonishing three-dimensional maps of the dark matter trapped within clusters. At the scales we consider  $\sim 3 \times 10^7$  km, however, weak lensing has no realistic possibility of detecting DMBHs in the foreseeable future.

Gravitational microlensing presents a much more optimistic possibility. This technique which exploits the amplification of a distant source was first emphasized in modern times (Einstein considered it in 1912 unpublished work) by Paczynski. Subsequent observations found many examples of MACHOs, yet insufficient to account for all of the halo by an order of magnitude. These MACHO searches looked for masses in the range  $10^{-6}M_{\odot} \leq M \leq 10^2M_{\odot}$ .

The time  $t_0$  of a microlensing event is given by

$$t_0 \equiv \frac{r_E}{v} \quad (17)$$

where  $r_E$  is the Einstein radius and  $v$  is the lens velocity usually taken as  $v = 200$  km/s. The radius  $r_E$  is proportional to the square root of the lens mass and numerically one finds

$$t_0 \simeq 0.2y \left( \frac{M}{M_\odot} \right)^{1/2} \quad (18)$$

so that, for the MACHO masses considered,  $2h \leq t_0 \leq 2y$ .



Although some of the already observed MA-CHOs may be DMBHs, they do not saturate the possible mass or entropy for dark matter so let us set as definition  $(M_{DMBH})^{min} \sim 10^2 M_{\odot}$ . This provides the range for DMBH mass

$$2 \leq \log_{10} \eta = \log_{10}(M_{DMBH}/M_{\odot}) \leq 6 \quad (19)$$

which, after Eq.(20), provides a second window of interest. It corresponds to  $2y \leq t_0 \leq 200y$ .

## 7. Cosmological Entropy Considerations.

The cosmological entropy range

$$102 \leq \log_{10} S_U \leq 112 \quad (20)$$

is the first of two interesting windows which are the subject. Conventional wisdom is  $S_U \sim (S_U)^{min} = 10^{102}$ .

As mentioned already, the key guide will be the holographic principle which informs us that the cosmological entropy is in the window (20). It cannot be at the absolute maximum value because that is possible only if every halo has already completely collapsed into a single black hole.

Also, the absolute minimum although not excluded seems intuitively implausible because all the entropy is compressed into  $10^{-36}V_U$ .

The natural suggestion is that there exist DMBHs in the mass region (19). The number is limited by the total halo mass  $10^{12}M_{\odot}$ . The total entropy is higher for higher DMBH mass because  $S \propto M^2$ . Let  $n$  be the number of DMBHs per halo,  $\eta$  be the ratio ( $M_{DMBH}/M_{\odot}$ ),  $S_U$  be the total entropy for  $10^{12}$  halos and  $t_0$  be the microlensing longevity. The table below shows five possibilities

## Dark Matter Black Holes and Microlensing Longevity

$\log_{10} n_{max}$	$\log_{10} \eta$	$\log_{10} S_{halo}$	$\log_{10} S_U$	$t_0$ (years)
8	2	88	100	2
7	3	89	101	6
6	4	90	102	20
5	5	91	103	60
4	6	92	104	200

(Assumes  $\rho_{DMBH} \sim 1\% \rho_{DM}$ )

## 8. Observation of DMBHs

Since microlensing observations already impinge on the lower end of the range (19) and the Table, it is likely that observations which look at longer time periods, have higher statistics or sensitivity to the period of maximum amplification can detect heavier mass DMBHs in the halo. If this can be achieved, and it seems a worthwhile enterprise, then the known entropy of the universe could be increased by more than two orders of magnitude.

There exists interesting other analyses pertinent to existence of massive halo objects:

J. Yoo, J. Chanamé and A. Gould, *Astrophys. J.* **601**, 311 (2004). [astro-ph/0307437](#).

C. Murali, P. Arras and I. Wasserman. [astro-ph/9902028](#).

B. Moore, *Astrophys. J.* **415**, L93 (1993). [astro-ph/9306004](#).

I shall return to Yoo et al.'s article.

The previous analyses have assigned upper limits on the fraction ( $f$ ) of the halo mass that can be constituted by DMBHs.

We have no reason to suggest that all of the dark matter halo mass is from DMBHs so the fraction  $f$  could indeed be very small. Yet DMBHs can still provide a very large fraction of the entropy of the universe. For example, taking  $f = 0.01$  and  $10^6 M_\odot$  as mass allows up to  $\sim 10^4$  DMBHs per halo, a total of  $\sim 10^{16}$  Mega- $M_\odot$  black holes in the universe and the fraction of the total entropy of the universe provided by  $\sim 1\%$  of dark matter can be  $\sim 99\%!!$

According to G. Bertone (private communication, 2009) the best upper limits (from Disk Stability and Wide Binaries) appear in Fig. 7 on page 317 of

J. Yoo, J. Chanamé and A. Gould, *Astrophys. J.* **601**, 311 (2004). [astro-ph/0307437](#).

which permits 10 percent of dark matter for the range of IMBH from  $20M_{\odot}$  to  $10^6M_{\odot}$ .

\*\*\* [astro-ph/0307437](#)

It is this entropy argument based on holography and the second law of thermodynamics which is the most compelling supportive argument for DMBHs. If each galaxy halo asymptotes to a black hole the final entropy of the universe will be  $\sim 10^{112}$  as in Eq.(20) and the universe will contain just  $\sim 10^{12}$  supergigantic black holes. Conventional wisdom is that the present entropy due entirely to SMBHs is only  $\sim 10^{-10}$  of this asymptotic value. DMBHs can increase the fraction up to  $\sim 10^{-8}$ , closer to asymptopia and therefore more probable according to the second law of thermodynamics.

There are several previous arguments about the existence of DMBHs and they have put upper limits on their fraction of the halo mass. The entropy arguments are new and provide additional motivation to tighten these upper bounds or discover the halo black holes. One observational method is high longevity microlensing events. It is up to the ingenuity of observers to identify other, possibly more fruitful, methods some of which have already been explored in a preliminary way.



## SUMMARY

The best summary is to repeat this table and discuss.

### Dark Matter Black Holes (DMBHs) and Microlensing Longevity

Maximum no. DMBH/halo	Mass of <i>DMBH</i>	Entropy of Universe	Microlensing longevity
$10^8$	$100M_{\odot}$	$10^{100}$	2y
$10^7$	$1,000M_{\odot}$	$10^{101}$	6y
$10^6$	$10^4M_{\odot}$	$10^{102}$	20y
$10^5$	$10^5M_{\odot}$	$10^{103}$	60y
$10^4$	$10^6M_{\odot}$	$10^{104}$	200y

(Assumes  $\rho_{DMBH} \sim 1\% \rho_{DM}$ )

Thank you for your attention