Atom Interferometric Tests of Gravity

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Gravitational wave Detection with Atom Interferometry
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Main theme

Gravity can only be explored through the motion of test particles

Test particles

- Orbits and clocks
- Massive particles and light

What is gravity depends on the structure of the equation of motion

- Existence of inertial systems
- Order of differential equation
- Dependence on particle parameters

Applies to test particles exploring gravitational waves
Gravitational waves may influence physics of test particles

Outline

1. Questioning Newton's laws
   - Newton's first law: Inertial systems
   - Newton's second law: The law of inertia
   - Newton's third law: Law of reciprocal action

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   - Violation of UFF from space–time fluctuations
   - Universality of Free Fall for charged particles
   - Universality of Free Fall for spinning particles

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Finsler geometry

Motivation

- generic generalization of GR
- leads to deformed light cones and mass shells
- has been discussed within Quantum Gravity (Jacobson, Liberati & Mattingly)
- Very Special Relativity (Cohen & Glashow)

Finsler space

Finsler length function

$$ds^2 = F(x, dx)\,, \qquad F(x, \lambda dx) = \lambda^2 F(x, dx)$$

Finsler metric tensor $f_{\mu\nu}(x, dx)$ is defined as

$$ds^2 = g_{\mu\nu}(x, dx) dx^\mu dx^\nu\,, \quad \text{where} \quad g_{\mu\nu}(x, y) = \frac{1}{2} \frac{\partial^2 F^2(x^k, y^m)}{\partial y^\mu \partial y^\nu}$$

Finsler geometry

Geodesics

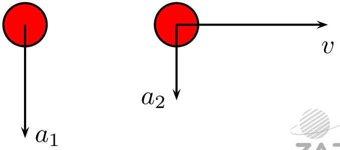
$$\delta \int ds = 0 \qquad \Rightarrow \qquad \boxed{0 = \frac{d^2 x^\mu}{ds^2} + \left\{ {}^{\mu}_{\rho\sigma} \right\}(x, \dot{x}) \frac{dx^\rho}{ds} \frac{dx^\sigma}{ds}}$$

with $\left\{ {}^{\mu}_{\rho\sigma} \right\}(x, \dot{x}) = g^{\mu\nu}(x, \dot{x}) \left(\partial_\rho g_{\sigma\nu}(x, \dot{x}) + \partial_\sigma g_{\rho\nu}(x, \dot{x}) - \partial_\nu g_{\rho\sigma}(x, \dot{x}) \right)$

Main characteristics of geodesic motion

- Geodesic equation fulfills Universality of Free Fall
- $\left\{ {}^{\mu}_{\rho\sigma} \right\}(x, \dot{x})$ cannot be transformed to zero $\forall \dot{x} \Rightarrow$ gravity cannot be transformed away locally $\Leftrightarrow$ Einstein's elevator does not hold $\Leftrightarrow$ no inertial system

- Condition to be able to transform away gravity is stronger then pure UFF.
- Acceleration toward the Earth depends on horizontal velocity.
- Speculation: violation of UGR, $G = G(T)$



Parametrizing deviations from Riemann/Minkowski

Special case: "power law" metrics

$$ds^2 = (g_{\mu_1\mu_2\ldots\mu_{2n}}(x)dx^{\mu_1}dx^{\mu_2}\cdots dx^{\mu_{2n}})^{\frac{1}{r}}$$

Deviation from Riemann/Minkowksi

$$ds^{2r} = \left(g_{\mu_1\mu_2}\cdots g_{\mu_{2r-1}\mu_{2r}} + \phi_{\mu_1\ldots\mu_{2r}}\right) dx^{\mu_1}\cdots dx^{\mu_{2r}}$$

This gives

$$ds^2 = (g_{\mu\nu} + \phi_{\mu\nu\rho_3\ldots\rho_{2r}} n^{\rho_3}\cdots n^{\rho_{2r}})\, dx^\mu dx^\nu\,, \qquad n^\mu = \frac{dx^\mu}{\sqrt{g_{\rho\sigma}dx^\rho dx^\sigma}}$$

Additional assumption: $\phi_{\mu_1\ldots\mu_{2r}}$ possesses spatial indices only (from light propagation)

Quantum mechanics in Finsler space

Finslerian Hamilton operator

$$H = H(p) \qquad \text{with} \qquad H(\lambda p) = \lambda^2 H(p)$$

"Power–law" ansatz (non–local operator)

$$H = \frac{1}{2m} \left(g^{i_1 \ldots i_{2r}} \partial_{i_1} \cdots \partial_{i_{2r}} \right)^{\frac{1}{r}}$$

Simplest case: quartic metric

$$H = \frac{1}{2m} \left(g^{ijkl} \partial_i \partial_j \partial_k \partial_l \right)^{\frac{1}{2}}$$

Deviation from standard case

$$\begin{aligned} H &= -\frac{1}{2m} \left(\Delta^2 + \phi^{ijkl} \partial_i \partial_j \partial_k \partial_l \right)^{\frac{1}{2}} \\ &= -\frac{1}{2m} \Delta \sqrt{1 + \frac{\phi^{ijkl} \partial_i \partial_j \partial_k \partial_l}{\Delta^2}} \end{aligned}$$

Quantum mechanics in Finsler space

$$H = -\frac{1}{2m}\Delta\left(1 + \frac{1}{2}\frac{\phi^{ijkl}\partial_i\partial_j\partial_k\partial_l}{\Delta^2}\right)$$

- Hughes–Drever: $H_{\mathrm{tot}} = H + \boldsymbol{\sigma}\cdot\boldsymbol{B}$
- Atomic interferometry, atom–photon interaction

$$\delta\phi \sim H(p+k) - H(p) = \frac{k^2}{2m} + \frac{1}{m}\left(\delta^{il} + \frac{\phi^{ijkl}p_j p_k}{p^2}\right)p_i k_l$$

  modified Doppler term: gives different Doppler term while rotating the whole apparatus (even in Finsler light still propagates on straight lines, anisotropy – deformed mass shell)

- cf. C.L., Lorek & Dittus 2008 for the photon sector (deformed light cone)

Outline

Order of equation of motion?

Usual framework

$$L = L(t, \boldsymbol{x}, \dot{\boldsymbol{x}}) \qquad \Rightarrow \qquad \frac{d}{dt}(m\dot{\boldsymbol{x}}) = \boldsymbol{F}(t, \boldsymbol{x}, \dot{\boldsymbol{x}})$$

Most important equation in physics!

More general equations?

- $\boldsymbol{p} = m\dot{\boldsymbol{x}}$ is a constitutive law. Can be more general (as is many cases)

  $$\boldsymbol{p} = \boldsymbol{f}(\dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}, \dddot{\boldsymbol{x}}, \ldots)$$

  Then equations of motion of higher order
- Influence of external fluctuations (e.g. space–time fluctuations, gravitational wave background): generalized Langevin equation with extra force term

  $$\int_0^t C(t - t')\dot{x}(t')dt'$$

Order of equation of motion?

Generalized framework

$$L = L(t, \boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}) \qquad \Rightarrow \qquad \frac{d^2}{dt^2}(\epsilon \ddot{\boldsymbol{x}}) = \boldsymbol{F}(t, \boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}, \dddot{\boldsymbol{x}})$$

Our specific model

$$L(t, \boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}) = L_0(t, \boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}) \quad \underbrace{-q_0 A_a \dot{x}^a}_{\text{1st order gauge fields}} \quad + \quad \underbrace{q_1 A_{ab} \dot{x}^a \dot{x}^b}_{\text{2nd order gauge fields}}$$

with (Pais–Uhlenbeck oscillator)

$$L_0(t, \boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}) = -\frac{\epsilon}{2}\ddot{\boldsymbol{x}}^2 + \frac{m}{2}\dot{\boldsymbol{x}}^2$$

C.L. & Rademaker 2009

Equation of motion

simplest case: constant electric field

$$\epsilon \ddddot{\boldsymbol{x}} + m\ddot{\boldsymbol{x}} = q\boldsymbol{E}_0$$

solution in 1D with initial conditions $x(0) = 0$, $\dot{x}(0) = 0$, $\ddot{x}(0) = 0$, and $\dddot{x}(0) = 0$

$$x(t) = \frac{q}{m}E_0\left(\frac{1}{2}t^2 + \frac{\epsilon}{m}\left(\cos(\omega t) - 1\right)\right) \qquad \text{small deviation}$$

$$\dot{x}(t) = \frac{q}{m}E_0\left(t - \sqrt{\frac{\epsilon}{m}}\sin(\omega t)\right) \qquad \text{small deviation}$$

$$\ddot{x}(t) = \frac{q}{m}E_0\left(1 - \cos(\omega t)\right) \qquad \mathcal{O}(1) \text{ deviation}$$

$$\dddot{x}(t) = \frac{q}{m}E_0\sqrt{\frac{m}{\epsilon}}\sin(\omega t) \qquad \omega = \sqrt{\frac{m}{\epsilon}} \qquad \text{large deviation}$$

- *zitterbewegung*
- Limit $\epsilon \to 0$ does not exist

Acceleration variance

(Hadamard) variance

$$\Delta\ddot{x} = \begin{cases} \dfrac{1}{\sqrt{2}}\dfrac{q}{m}E_0 & \text{for } \epsilon > 0 \\ 0 & \text{for } \epsilon = 0 \end{cases}$$

Phase shift for this *zitterbewegung* in ion interferometry ....

Other possibilities

- time of flight measurements
- electronic noise

Applies to mirrors in gw interferometers?

Linearity of law of inertia

Why is the relation between acceleration and force linear?

It is a definition

$\dot{\boldsymbol{p}} = \boldsymbol{F}$: exploration of forces through observation of orbits

Meaningful question 1: Test linearity

Taking elements of the field equation into account:

- If $\boldsymbol{F} = -\boldsymbol{\nabla} U$ with $U = \frac{M}{r}$, then one can ask

$$M \quad \rightarrow \quad \alpha M \qquad \overset{?}{\Longrightarrow} \qquad \boldsymbol{F} \quad \rightarrow \quad \alpha \boldsymbol{F}$$

- Test of field equation/dynamics in the weak field/small acceleration domain
- Applies to small relative acceleration of test masses in gw detectors?
- Same for gravity and electromagnetism?
- Experiments
  - Abramovici & Vager, PRD 1986 ($F = q\Delta\phi/L$, down to $10^{-9}\ \mathrm{m/s^2}$)
  - Gundlach et al, PRL 2007 (down to $10^{-14}\ \mathrm{m/s^2}$)
  - atom interferometry?

Linearity of law of inertia

These questions are motivated by MOND

Meaningful question 2: Free fall experiment

- MOND – dark matter: requires a certain frame of reference (galactic frame)
- MOND–situation possible on Earth once a year for 0.1 s within 1 l volume (Ignatiev, PRL 2007)
- Until now there is no (laboratory) test of MOND — MOND needs space

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Active and passive charges: Dynamics

Bondi RMP 1956; C.L., Macias, Müller, PRA 2007

Dynamics of two electrically bound particles ($\boldsymbol{E}$ = external electric field)

$$\begin{aligned} m_{1\mathrm{i}}\ddot{\boldsymbol{x}}_1 &= q_{1\mathrm{p}}q_{2\mathrm{a}}\frac{\boldsymbol{x}_2-\boldsymbol{x}_1}{|\boldsymbol{x}_2-\boldsymbol{x}_1|^3}+q_{1\mathrm{p}}\boldsymbol{E}(\boldsymbol{x}_1)\,, \\ m_{2\mathrm{i}}\ddot{\boldsymbol{x}}_2 &= q_{2\mathrm{p}}q_{1\mathrm{a}}\frac{\boldsymbol{x}_1-\boldsymbol{x}_2}{|\boldsymbol{x}_1-\boldsymbol{x}_2|^3}+q_{2\mathrm{p}}\boldsymbol{E}(\boldsymbol{x}_2)\,, \end{aligned}$$

center–of–mass and relative coordinate

$$\boldsymbol{X} := \frac{m_{1\mathrm{i}}}{M_\mathrm{i}}\boldsymbol{x}_1+\frac{m_{2\mathrm{i}}}{M_\mathrm{i}}\boldsymbol{x}_2\,, \qquad \boldsymbol{x} := \boldsymbol{x}_2-\boldsymbol{x}_1\,,$$

$M_\mathrm{i} = m_{1\mathrm{i}} + m_{2\mathrm{i}}$ = total inertial mass. Then

$$\ddot{\boldsymbol{X}} = \frac{q_{1\mathrm{p}}q_{2\mathrm{p}}}{M_\mathrm{i}}C_{21}\frac{\boldsymbol{x}}{|\boldsymbol{x}|^3}+\frac{1}{M_\mathrm{i}}\left(q_{1\mathrm{p}}+q_{2\mathrm{p}}\right)\boldsymbol{E} \qquad C_{21} := \frac{q_{2\mathrm{a}}}{q_{2\mathrm{p}}}-\frac{q_{1\mathrm{a}}}{q_{1\mathrm{p}}}\,.$$

$C_{21} = 0$: ratio between the active and passive charge is the same for both particles
$C_{21} \neq 0 \Rightarrow$ self–acceleration of center of mass along $\boldsymbol{x}$

Active and passive charges: Dynamics

Dynamics of relative coordinate

$$\ddot{\boldsymbol{x}} = -\frac{1}{m_{\rm red}} q_{1\rm p} q_{2\rm p} D_{21} \frac{\boldsymbol{x}}{|\boldsymbol{x}|^3} \,, \tag{1}$$

where

$$D_{21} = \frac{m_{1\rm i}}{M_{\rm i}} \frac{q_{1\rm a}}{q_{1\rm p}} + \frac{m_{2\rm i}}{M_{\rm i}} \frac{q_{2\rm a}}{q_{2\rm p}} = \frac{q_{1\rm a}}{q_{1\rm p}} + \frac{m_{2\rm i}}{M_{\rm i}} C_{21}$$

In the standard framework, $D_{21} = 1$.
Solutions of equation of motion (1) are ellipses, circles.
The center of mass oscillates at a frequency ω, which is related to the energy of the system.
The acceleration of the center of mass vanishes on average, $\langle \ddot{\boldsymbol{X}} \rangle = 0$. Thus, not observable for atoms.
Extends to many particle systems, e.g., to atoms having many electrons.

Interpretation

$\ddot{\boldsymbol{X}} \neq 0 \quad \Leftrightarrow \quad C_{12} \neq 0 \quad \Leftrightarrow$ violation of *actio* $=$ *reactio* for electromagnetism

Strategy to measure C_{12}

Strategy

- Electromagnetic timescales too short: self–acceleratioon canot be observed.
- Electric charges can have different signs. Therefore, we can define

  active neutrality $q_{1\mathrm{a}} + q_{2\mathrm{a}} = 0$
  passive neutrality $q_{1\mathrm{p}} + q_{2\mathrm{p}} = 0$

  $\rightarrow$ alternative tests of the equality of active and passive charges: An actively neutral system may not be passively neutral and vice versa.
  active and passive neutrality $\quad \Leftrightarrow \quad C_{21} = 0$

Experiments

tests of neutrality of atoms and molecules = tests of the equality of active and passive charge

- a passively neutral system may still generate an electric field according to

$$\phi(\boldsymbol{x}) = \frac{q_{1\mathrm{a}}}{|\boldsymbol{x} - \boldsymbol{x}_1|} + \frac{q_{2\mathrm{a}}}{|\boldsymbol{x} - \boldsymbol{x}_2|} = \frac{q_{1\mathrm{a}} + q_{2\mathrm{a}}}{|\boldsymbol{x}|} + \ldots \approx C_{21} \frac{q_{2\mathrm{p}}}{|\boldsymbol{x}|}$$

- an actively neutral atom in an external electric field may feel a force

$$M_{\mathrm{i}} \ddot{\boldsymbol{X}} = (q_{\mathrm{p}1} + q_{\mathrm{p}2}) \boldsymbol{E} = \frac{q_{2\mathrm{p}}}{q_{2\mathrm{a}}} q_{1\mathrm{a}} C_{12} \boldsymbol{E}$$

  vanishes if ratios of active and passive charges are the same for all bodies

we can distinguish two types of tests of neutrality:

- Tests of active neutrality, which measure the electric monopole field created by a passively neutral system, and
- tests of passive neutrality, which measure the force imposed by an external field onto an actively neutral system.

Experiments: Neutrality of atoms

Table: Various tests of the neutrality of atoms. If no particle is specified, q_{p} refers to the passive charge of the atoms or molecules used in the experiment, divided by the charge number of that particle (analogous for q_{a}). See Unnikrishnan & Gillies 2004 for a review.

Method	Limit $/(10^{-20}e)$
Gas efflux (350 g CO_2) [Piccard & Kessler 1925]	$q_{p,\mathrm{a}}/q_{e,\mathrm{a}} = 0.1(5)$
Gas efflux (Ar/N) [Hillas & Cranshaw 1960]	$q_{\mathrm{H,a}} = 1(3)$; $q_{n,\mathrm{a}} = -1(3)$
Gas efflux [King 1960]	$q_{\mathrm{He,a}} = -4(2)$
Superfluid He [Classen et al 1998]	$q_{\mathrm{He,a}} = -0.22(15)$
Levitator [Marinelli & Morpurgo 1982]	$\|q_{\mathrm{p}}\| \lesssim 1000$
Acoustic resonator (SF_6) [Dylla & King 1973]	$\|q_{\mathrm{p}}\| \leq 0.13$
Cs beam [Hughes 1957]	$q_{\mathrm{p}} = 90(20)$
Neutron beam [Baumann et al 1988]	$q_{n,\mathrm{p}} = -0.4(1.1)$

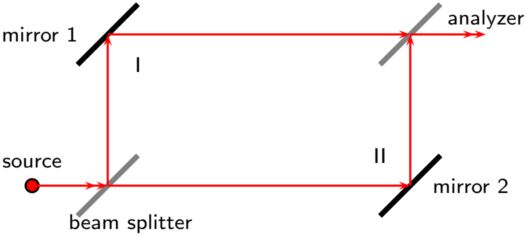
limits go down to $10^{-21}\ e$ for active and passive charge of various combinations of electrons, protons, and neutrons.

$\Rightarrow |C_{21}| \leq 10^{-21}$.

Atom interferometry

Arvanitaki et al 2008

Place atoms in different voltages – requires large beam separation in configuration space



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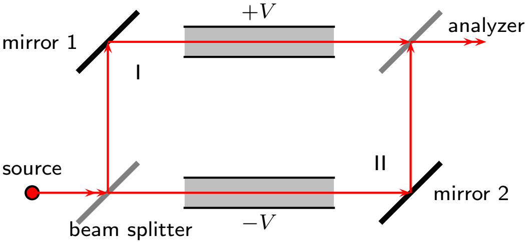

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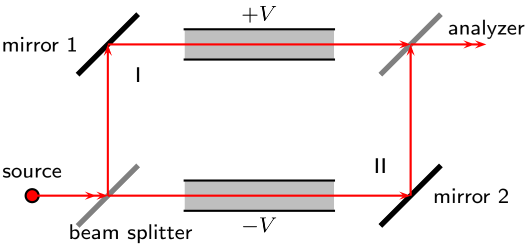

Phase shift

$$\delta\phi = 2\,\text{charge} \int V\,dt$$

For Kasevich–setup in small tower: charge $\leq 10^{-30}\,e$
Improvement by 8 orders of magnitude

Alternative experiment: fine structure constant

- Center–of–mass motion of the two–particle system cannot be quantized
- Relative motion quantizable

Hamiltonian

$$H = \frac{\boldsymbol{p}^2}{2m_{\text{red}}} + D_{21}\frac{q_{1\text{p}}q_{2\text{p}}}{|\boldsymbol{x}|}\,.$$

Energy levels are proportional to modified fine structure constant

$$\alpha_{12} = \frac{q_{1\text{p}}q_{2\text{p}}D_{12}}{\hbar c} = \frac{1}{\hbar c}q_{1\text{p}}q_{2\text{p}}\left(\frac{q_{1\text{a}}}{q_{1\text{p}}} + \frac{m_{2\text{i}}}{M_{\text{i}}}C_{21}\right)\,.$$

Spacing between energy levels depends nonlinearly on active charges $\Rightarrow$ comparison of energy levels in different atoms yields test of C_{21}.
E.g., Hydrogen (one proton $q_1 = q_p$ and one electron $q_2 = q_e$) and ionized Helium He^+ ($q_1 = 2q_p$ and $q_2 = q_e$). Then

$$\alpha_{12}(\text{He}^+) - 2\alpha_{12}(\text{H}) \approx -\frac{q_{pp}q_{ep}}{\hbar c}\frac{m_{ei}}{m_{pi}}C_{21}$$

can deduce limit

$$|C_{21}| \leq \frac{\delta\alpha}{\alpha}\frac{m_{ei}}{m_{pi}} \approx 7\times 10^{-13}\frac{m_{ei}}{m_{pi}} \approx 4\times 10^{-16}$$

Summary Newton's axioms

- Fundamental postulates have to be tested as good as possible
- Finsler: Hints from quantum gravity
- Order of equation of motion: influence from space–time fluctuations
- Small accelerations: Hints from unexplained observations (dark matter)
- No model until now for active $\neq$ passive charge;
  it is a symmetry of physics which unfortunately is not yet well analyzed
- Systematic experimental study of Newton's axioms seems worthwhile
- Atom interferometry may be of help

Outline

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The Universality of Free Fall with quantum matter

Overview

Violations of UFF from

- Scalar tensor theories (Damour, Polyakov, Piazza, Veneziano).
  Also influences Universality of Gravitational Redshift,
  PPN parameters γ, β, ...
  estimate from low energy string theory $\eta \leq 10^{-13}$, $|\gamma - 1| \leq 10^{-5}$, ...
- Scalar tensor theory coupled to quintessence (Wetterich)
  Prediction $\eta \approx 10^{-14}$.
- String theory: scattering of particles at branes = gravity. Estimate $\eta \leq 10^{-18}$
- Varying e model (Bekenstein)
  Estimate $\eta \leq 10^{-13}$
- Particle moving through space–time fluctuations (Göklü & C.L.)
  According to model and strength $\eta \leq 10^{-9}$,

Space–time fluctuations

The model

- General belief for Quantum Gravity: space–time fluctuates
- Simplest model of space–time fluctuations $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
  Amelino–Camelia PRD 2000, Schiller et al 2004
- Simplest matter system: Klein–Gordon equation in this fluctuating space-time metric $g^{\mu\nu} D_\mu D_\nu \psi - m^2 \psi = 0$

- Relativistic approximation (of metric and of quantum field) + time–dependent transformation $\psi \to \psi' = A\psi$: hermitian Hamiltonian

$$\begin{aligned} H'\psi' &= -({}^{(3)}g)^{1/4} \frac{\hbar^2}{2m} \Delta_{\rm cov} \left(({}^{(3)}g)^{-1/4} \psi' \right) + \frac{m}{2} \left(\tilde{h}^{00}_{(0)} - h^{00}_{(0)} \right) \psi' \\ & \quad -\frac{1}{2} \left\{ i\hbar\partial_i, h^{i0}_{(1)} - \tilde{h}^{i0}_{(1)} \right\} \psi' \end{aligned}$$

- Spatial average over Compton length of particle under consideration

$$i\hbar\partial_t \psi = -\frac{\hbar^2}{2m} \left(\delta^{ij} + \alpha^{ij}(t) \right) \partial_i \partial_j \psi - mU\psi$$

Space–time fluctuations

$$\alpha^{ij}(t) = \widetilde{\alpha}^{ij} + \gamma^{ij}(t), \qquad \langle \gamma^{ij}(t) \rangle_t = 0$$

- $\widetilde{\alpha}^{ij} \leftrightarrow$ spectral noise density of fluctuations
- Particular model: $m_{\mathrm{i}} = \dfrac{m}{1+\widetilde{\alpha}}, \quad \widetilde{\alpha} \sim \left(\dfrac{l_{\mathrm{Planck}}}{l_{\mathrm{Compton}}} \right)^{\beta}$

Result

$\Rightarrow$ anomalous inertial mass $\rightarrow$ apparent violation of UFF

- Alternative route for violation of UFF and LLI.
- $\beta = \frac{1}{2}$ holographic noise (Ng 2001, Hogan 2008, GEO600)
- applies also to stochastic gravitational waves

Example

For Cesium and Hydrogen: $\eta_{\beta=1} = 10^{-17}$, $\eta_{\beta=2/3} = 10^{-12}$, $\eta_{\beta=1/2} = 10^{-9}$
$\beta = \frac{1}{2}$ already ruled out (?)

Göklü & C.L. CQG 2008

Space–time fluctuations and Dirac equation

Quantum particle with spin: Dirac equation

Fluctuating metric couples to Dirac equation

$$i\gamma^\mu D_\mu \psi - m\psi = 0 \,, \qquad \gamma^{(\mu}\gamma^{\nu)} = g^{\mu\nu}$$

Nonrelativistic limit $\Rightarrow$ expected terms

$$i\hbar\partial_t\psi = \frac{1}{2m}\left(\delta^{ij} + \alpha^{ij} + \beta^{ij}{}_k\sigma^k\right)\partial_i\partial_j\psi + m(1+\gamma_i\sigma^i)U\psi + \delta_i\sigma^i\psi$$

Question: Why to keep Clifford algebra? $\leftrightarrow$ fluctuations may be adapted to the structure of the field equation under consideration (fluctuation of coefficients – space–time is what particles explore).

Klein–Gordon equation:	fluctuating $g^{\mu\nu}$	
Dirac equation:	fluctuating γ^μ:	$\rightarrow \gamma^{(\mu}\gamma^{\nu)} = g^{\mu\nu} + \delta g^{\mu\nu} + X^{\mu\nu}$
Maxwell equation:	fluctuating $g^{\mu[\rho}g^{\sigma]\nu}$	$\rightarrow k^{\mu\nu\rho\sigma} = g^{\mu[\rho}g^{\sigma]\nu} + \kappa^{\mu\nu\rho\sigma}$

Outline

UFF and charge

Standard theory

- In standard theory from ordinary coupling (deWitt & Brehme 1968) $a^\mu = \alpha \lambda_C c R^\mu{}_\nu v^\nu \Rightarrow$ violation of UFF $\sim 10^{-35}\ \mathrm{m/s}^2$

Anomalous coupling

- Anomalous coupling (Dittus, C.L., Selig, GRG 2004)

$$H = \frac{\boldsymbol{p}^2}{2m} + mU(\boldsymbol{x}) + \kappa e U(\boldsymbol{x}) = \frac{\boldsymbol{p}^2}{2m} + m\left(1 + \kappa \frac{e}{m}\right) U(\boldsymbol{x}) \, .$$

- Charge dependent anomalous gravitational mass tensor
- Also charge dependent anomalous inertial mass tensor (e.g. Rohrlich 2000)

$\Rightarrow$ Charge dependent Eötvös factor

- It is possible to choose κ's such that for neutral composite matter UFF is fulfilled while for isolated charges UFF is violated $\Rightarrow$ no constraints on κ from present UFF experiments

No underlying fundamental theory known

Outline

UFF and spin

Standard theory

- In standard theory from ordinary coupling: $a^\mu = \lambda_C R^\mu{}_{\nu\rho\sigma} v^\nu S^{\rho\sigma} \;\Rightarrow$ violation of UFF at the order $10^{-20}\ \mathrm{m/s^2}$, beyond experiment

Anomalous coupling

- Speculations: violation P, C, and T symmetry in gravitational fields (Leitner & Okubo 1964, Moody & Wilczek 1974) suggest

$$V(r) = U(r)\left[1 + A_1(\boldsymbol{\sigma}_1 \pm \boldsymbol{\sigma}_2)\cdot\widehat{\boldsymbol{r}} + A_2(\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2)\cdot\widehat{\boldsymbol{r}}\right] ,$$

- One body (e.g., the Earth) is unpolarized $\rightarrow$

$$V(r) = U(r)\left(1 + A\boldsymbol{\sigma}\cdot\widehat{\boldsymbol{r}}\right) .$$

  Hyperfine splittings of H ground state: $A_p \leq 10^{-11}$, $A_e \leq 10^{-7}$

- Hari Dass 1976, 1977, includes velocity of the particles

$$V(r) = U_0(r)\left[1 + A_1\boldsymbol{\sigma}\cdot\widehat{\boldsymbol{r}} + A_2\boldsymbol{\sigma}\cdot\frac{\boldsymbol{v}}{c} + A_3\widehat{\boldsymbol{r}}\cdot\left(\boldsymbol{\sigma}\times\frac{\boldsymbol{v}}{c}\right)\right]$$

Outline

Decoherence

- Fluctuations

$$\alpha^{ij}(t) = \underbrace{\widetilde{\alpha}^{ij}}_{\text{renormalization of } m_{\mathrm{i}}} + \underbrace{\gamma^{ij}(t)}_{\text{decoherence}}, \qquad \langle \gamma^{ij}(t) \rangle_t = 0$$

- Quantum master equation
- White noise $\gamma^{ij} = \sigma\delta^{ij}\xi(t)$, $\tau_c = \sigma^2$: Markovian master equation
  - preserves trace and positivity of the density matrix
  - generates complete positive dynamical map
  - quantum dynamical semigroup
- Increase of entropy
- Exponential decay of coherences in energy representation
- Coherence time

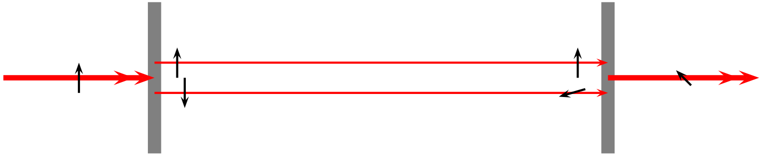
$$\tau_D = 2\left(\frac{\hbar}{\tau_c\,\Delta E}\right)\tau_c\,, \qquad \tau_c = T_{\mathrm{Pl}}$$

applies also to stochastic gravitational waves
(Breuer, Göklü & C.L. 2008 – also Wang, Bingham & Mendoca 2006, 2008)

Outline

Interference with spin

Interference in spin space



phase shift (Audretsch, Bleyer & CL, PRD 1993; CL, CQG 1998):

$$\begin{aligned} \delta\phi &= \left(H(\boldsymbol{x},\boldsymbol{p},\boldsymbol{S}) - H(\boldsymbol{x},\boldsymbol{p},-\boldsymbol{S})\right)\delta t \\ &= \left(\frac{\delta m^{ij}_{\mathrm{i}}{}_k}{m} p_i \delta_{jk} l^k - 2\left(\lambda^i_j + m\Lambda^i_j\right)\delta_{ik} l^k + 2mB_j\delta t + 2C_j mU\delta t + T_j\delta t\right) S^j \end{aligned}$$

gives estimates on anomalous Lorentz–violating spin–coupling terms

$$\left|\frac{\delta m^{ij}_k}{m}\right| \leq 5\cdot 10^{-15}, \quad \left|\Lambda^i_j\right| \leq 8\cdot 10^{-24}, \quad \left|\lambda^i_j\right| \leq 5\cdot 10^{-6}\mathrm{m}^{-1}, \quad |B_k| \leq 3\cdot 10^{-30}$$

$$|T_k| \leq 2\cdot 10^{-9}\mathrm{m}^{-1}, \quad |C_k| \leq 3\cdot 10^{-23}$$

Outline

Space–time fluctuations and spin

Classical particle with spin I

Equation of motion for pole–dipole particle (v is auxilary quantity)

$$\begin{aligned} v &= \frac{dx}{ds} \\ D_v p &= R(\cdot, v, S) \\ D_v S &= v \wedge p \\ p(S) &= 0 \end{aligned}$$

Initial conditions $x(t_0)$, $p(t_0)$, $S(t_0)$

Classical article with spin II

From quasiclassical limit of Dirac

$$\begin{aligned} D_v v &= \lambda_{\mathrm{C}} R(\cdot, v, S) \\ D_v S &= 0 \\ p(S) &= 0 \qquad \text{automatically} \end{aligned}$$

Since for Dirac $p \sim v$ no auxilary velocity needed

- Spin particle sees curvature directly!
- For fluctuations: $D_v v = R(\cdot, v, S) + \delta R(\cdot, v, S)$
- fluctuations will give additional acceleration term which may be arger than the standard term $\Rightarrow$ anomalous spin coupling $\Rightarrow$ violation of UFF for spin
- $\delta R(\cdot, v, S) \sim (\partial h)^2 v S$ – enhancement for short wavelength fluctuations

Outline

Superposition principle

Genereal non–linear Schödinger equation

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2m}\Delta\psi + F(\psi^*\psi)\psi$$

Separability of quantum sytems: non–linear Schrödinger equation of Bialnicky–Birula PRL 1977; Shimony, PRA 1978

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2m}\Delta\psi + a\left[\ln\left(b\psi^*\psi\right)\right]\psi$$

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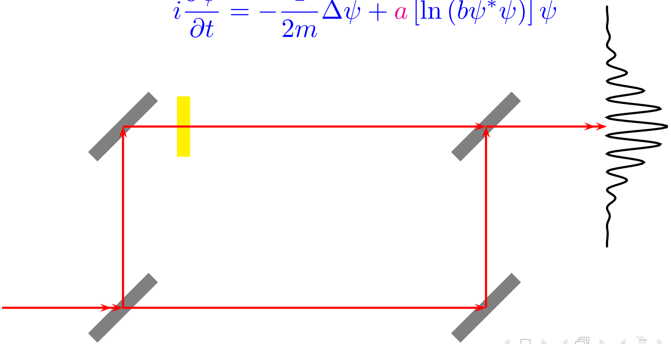

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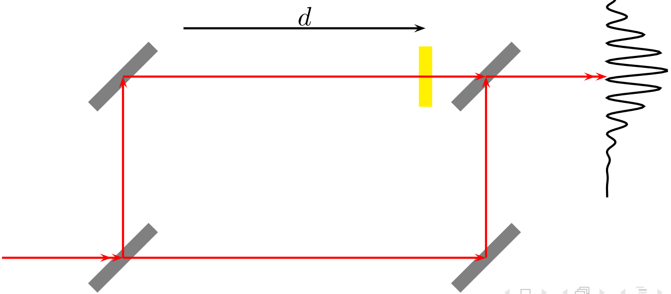


Superposition principle

phase shift:

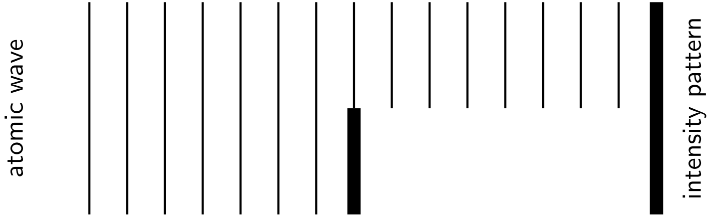
$$\delta\phi = \frac{d}{\hbar}\sqrt{\frac{m}{2E}}\left(F(|\varphi|^2) - F(\alpha^2|\varphi|^2)\right)$$

$\alpha =$ intensity attenuation.
Test with neutron interferometry (Shull et al, PRL 1980)

- result: $a \leq 3.4 \cdot 10^{-13}\ \mathrm{eV}$
- atomic interferometry should lead to orders of magnitude improvement
- from energy levels: $a \leq 4 \cdot 10^{-10}\ \mathrm{eV}$

Superposition principle

Alternative measurement: Scattering at edges



- yields best estimates for neutrons: $\alpha \leq 3 \cdot 10^{-15}\ \mathrm{eV}$
- depends on velocity of particles $\rightarrow$ should be better by many orders of magnitude for atoms
- Van der Waals, Casimir forces etc. should be included in calculation, and parameters determined by independent experiments, or by scattering at edges made of different materials

Outline

Anomalous dispersion – non–local field equations

The model

Ansatz in 3+1–form

$$i\partial_t\psi = -i\alpha^i\partial_i\psi + \alpha^{ij}\partial_i\partial_j\psi + \beta m\psi$$

$\alpha^{ij} \neq 0 \quad \Longrightarrow \quad$ violation of LLI (in a certain sense)

Non–relativistic limit

$$i\partial_t\psi = -\frac{1}{2m}\Delta\psi + \frac{1}{m^2}\left(A^{ijk} + B_l^{ijk}\sigma^l\right)\partial_i\partial_j\partial_k\psi$$

- interference experiments: Spin–flip–experiment:

$$\Delta\phi = \frac{1}{\hbar}\left(H(S) - H(-S)\right)\Delta t = \frac{2}{\hbar c}B_m^{jki}S^m m v_i v_j l_k$$

spin–$\frac{1}{2}$–field, $m = 2\times10^{-23}$ g, $l = 10$ m, $v = 1000\,$m/sec $\rightarrow$

$$B_m^{(ijk)} \leq \frac{\hbar c}{2}\frac{1}{Smv^2 l} \approx 10^{-10}$$

Anomalous dispersion

Is a generic effect of all QG approaches (string theory, LQG, NCG)
Non–locality $\leftrightarrow$ higher order derivatives

$$i\hbar\frac{\partial}{\partial t}\psi = H(p)\psi = \sum_{i=0}^{n}\alpha_i p^i$$

may also come from field equations with higher order time derivative (C.L. & Bordé 2001)
Gives dispersion relation

$$E = H(p)$$

Atom–photon interaction yields phase shift

$$\delta\varphi \sim H(p+\hbar k) - H(p)$$

Under consideration with G. Amelino–Camelia, G. Tino, ...

Outline

Summary

- Worth to do better tests of Newton's axioms
  - Existence of inertial systems
  - Order of equation of motion
  - Active vs. passive charge/mass/spin
- Many influences of space–time fluctuations (stochastic gravitational waves)
  - Violation of UFF and LLI
  - Decoherence
  - Order of equations of motion

Summary

- Worth to do better tests of Newton's axioms
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Thank you!

Thanks to

- DLR
- DFG
- Center of Excellence QUEST

Proceedings

- Publication as special issue in the journal *General Relativity and Gravitation*
- Deadline May 29, 2009
- Papers will be refereed
- All formats are welcome (LaTeX, Word)

Space–time fluctuations and spin

- Geodesic equation for static spherically symmetric metric

$$\left(\frac{dr}{ds}\right)^2 = \frac{1}{g_{tt} g_{rr}} \left(E^2 - g_{tt}\left(\epsilon + m\frac{L^2}{r^2}\right)\right)$$

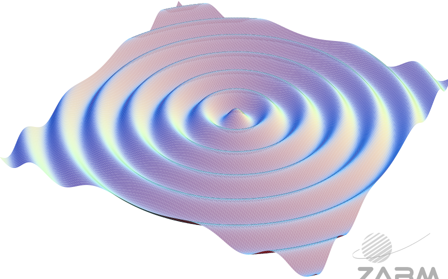
- Assuming $g_{rr} = 1/g_{tt}$ and $g_{tt} = 1 + h\cos(kr)$ (everywhere C^∞)
  Then for radial motion ($L = 0$)

$$\left(\frac{dr}{ds}\right)^2 = E^2 - 1 - h\cos(kr)$$

  Can be solved by elliptic function
- for $h \ll 1$

$$r = \sqrt{E^2 - 1}s + h\frac{\sin(\sqrt{E^2-1}ks)}{2(E^2-1)k}$$

  like zitterbewegung



Space–time fluctuations and spin

Kretschmann scalar (indicator of space–time singularities)

$$K = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \sim \frac{g''_{tt}}{g_{tt}} \sim hk^2 \frac{\cos(kr)}{1 + h\cos(kr)}$$

$\Rightarrow$ force term $R^{\mu}{}_{\nu\rho\sigma} u^{\nu} S^{\rho\sigma}$ may become large $\Rightarrow$ violation of UFF

Consequence

- For $k \to \infty$: solution of geodesic equation approaches straight line
- Kretschmann scalar $\to \infty$

$\Rightarrow$ point particles do not see fluctuating curvature
particles with spin should be sensitive to curvature

- $\longrightarrow$ may be of importance in atomic interferometry, spectroscopy, ... if fluctuations are regarded as being due to quantum gravity
- $\longrightarrow$ estimates from experiments

- needs to be compared with analysis of Dirac equation in fluctuating space-time metric

Göklü & C.L. in preparation