Calibrating D-brane networks on flux vacua

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Based on:

J. Evslin & L. M. hep-th/0703129

L. M. & P. Smyth hep-th/0507099

Outline

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Motivations

Calibrations and supersymmetric D-branes

D-brane networks

$\mathcal{N} = 1$ flux vacua as calibrated backgrounds

Networks of strings and domain walls

Possible future directions

Motivations

The relevance of supersymmetric flux vacua:

- Minimal supersymmetry
- Moduli stabilization
- Gauge/string theory correspondence
- ...

D-branes are central in many models:

- non-abelian gauge theories
- non-perturbative effects
- domain walls, cosmic strings, vacuum-changing bubbles
- flavors in gauge/string theory correspondence
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Our main interest: Type II theories on $X = \mathbb{R}^{1,3} \times M$.

Background geometry:

- $\mathcal{N} = 2$ and no fluxes $\longrightarrow$ $M = CY_3$
- $\mathcal{N} = 1$ from fluxes on M $\longrightarrow$ M is generalized complex!

[Grana, Minasian, Petrini & Tomasiello, '05]

D-brane geometry?

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Ordinary calibrations

A calibration is a p-form ω such that, for any p-submanifold Σ [Harvey & Lawson, '82]

$$d\omega = 0 \quad , \quad \omega|_\Sigma \leq \sqrt{g|_\Sigma}\, d^p\sigma$$

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Main property: a calibrated cycle Σ is volume minimizing:

$$\mathrm{Vol}(\Sigma) \leq \mathrm{Vol}(\Sigma')$$

for any Σ' in the same homology class of Σ ($\mathcal{B}$ exists such that $\partial\mathcal{B} = \Sigma - \Sigma'$).

Indeed

$$\begin{aligned} \mathrm{Vol}(\Sigma) &= \int_\Sigma \sqrt{g|_\Sigma}\, \mathrm{d}^p\sigma = \int_\Sigma \omega = \int_\mathcal{B} \mathrm{d}\omega + \int_{\Sigma'} \omega \\ &= \int_{\Sigma'} \omega \leq \int_{\Sigma'} \sqrt{g|_{\Sigma'}}\, \mathrm{d}^p\sigma = \mathrm{Vol}(\Sigma') \end{aligned}$$

On fluxless vacua $E_{\mathrm{D}p}(\Sigma) = \mathrm{Vol}(\Sigma)$
$\to$ supersymmetric branes wrap calibrated cycles [Becker, Becker & Strominger, '95]

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Inclusions of background and world-volume fluxes

Natural to demand that generalized calibrations are *energy minimizing* [Gutowski, Papadopoulos & Townsend, '99].

D-branes wrap generalized submanifolds $(\Sigma, \mathcal{F})$, with $\mathrm{d}\mathcal{F} = H|_\Sigma$

The D-brane energy density is

$$\mathcal{E}(\Sigma, \mathcal{F}) = e^{-\Phi}\sqrt{\det(g|_\Sigma + \mathcal{F})}\mathrm{d}^p\sigma - C|_\Sigma \wedge e^{\mathcal{F}}|_{\mathrm{top}}$$

Nontrivial role of $\mathcal{F}$ and H: How to include it?

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Generalized calibrations for generalized cycles

[P. Koerber; P. Smyth & L. M. '05]

A *generalized calibration* is a polyform $\omega = \sum_k \omega_{(k)}$ of definite parity such that

- Algebraic condition:
  $[\omega|_\Sigma \wedge e^{\mathcal{F}}]_{\text{top}} \leq \mathcal{E}(\Sigma, \mathcal{F})$, for any generalized submanifold $(\Sigma, \mathcal{F})$
- Differential condition: $\mathrm{d}_H \omega \equiv (\mathrm{d} + H\wedge)\omega = 0$

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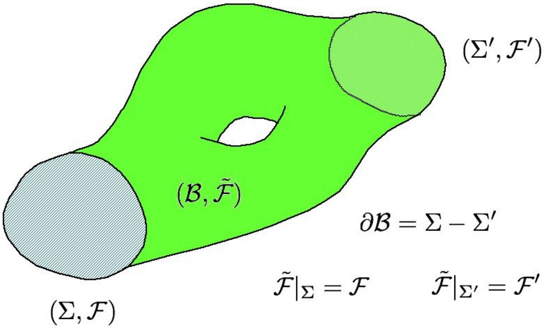
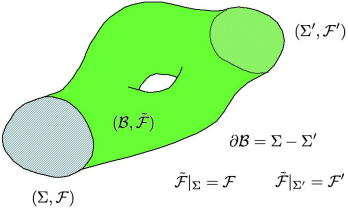
If $(\Sigma, \mathcal{F})$ is calibrated generalized cycle, then it is energy minimizing

$$E(\Sigma, \mathcal{F}) \leq E(\Sigma', \mathcal{F}')$$

for any $(\Sigma', \mathcal{F}')$ such that there is an interpolating generalized submanifold $(\mathcal{B}, \tilde{\mathcal{F}})$





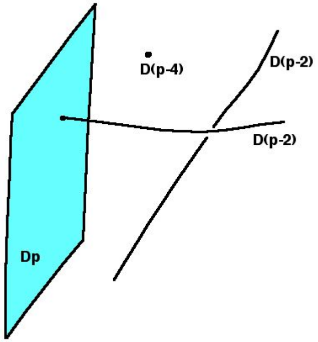
Indeed

$$\begin{aligned} E(\Sigma,\mathcal{F}) \quad &= \quad \int_\Sigma \mathcal{E}(\Sigma,\mathcal{F}) = \int_\Sigma \omega|_\Sigma \wedge e^{\mathcal{F}} = \int_{\mathcal{B}} \mathrm{d}_H \omega|_\Sigma \wedge e^{\tilde{\mathcal{F}}} + \\ &\quad + \int_{\Sigma'} \omega|_\Sigma \wedge e^{\mathcal{F}'} \leq \int \mathcal{E}(\Sigma',\mathcal{F}') = E(\Sigma',\mathcal{F}')\,. \end{aligned}$$

$\Rightarrow$ One expects that susy backgrounds have generalized calibrations such that supersymmetric D-branes are characterized as calibrated!

Outline

Generalized chains



- $\mathcal{F} \neq 0$ induces lower-dimensional charges
- Mutually supersymmetric D-branes of different dimensions and bound states
- Generalized calibrations:

$$\omega = \ldots + \omega_{(p+2)} + \omega_{(p)} + \omega_{(p-2)} + \ldots$$

Natural to consider chains of generalized submanifolds of different dimensions

$$(\Sigma_1^{(p)}, \mathcal{F}_1) + (\Sigma_2^{(p-2)}, \mathcal{F}_2) + (\Sigma_3^{(p-4)}, \mathcal{F}_3) + \ldots$$

Generalized submanifolds with magnetic sources

- Σ is a p-submanifold
- $\mathcal{C} \subset \Sigma$ is a $(p-3)$-dim magnetic source

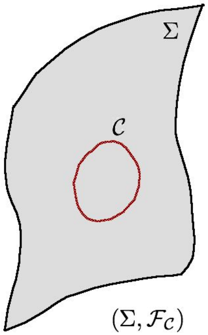
Generalized submanifolds $(\Sigma, \mathcal{F}_{\mathcal{C}})$ with

$$\mathrm{d}\mathcal{F}_{\mathcal{C}} = H|_{\Sigma} + \delta^3_{\Sigma}(\mathcal{C})$$

$$\Downarrow$$

If Σ is a compact cycle

$$\mathrm{PD}_{\Sigma}[\mathcal{C}] + [H|_{\Sigma}] = 0 \in H^3(\Sigma, \mathbb{Z})$$



Generalized boundary operator

$\hat{\partial}$ boundary operator:

$$\hat{\partial}(\Sigma, \mathcal{F}_{\mathcal{C}}) = (\partial\Sigma, \mathcal{F}_{\mathcal{C}}|_{\partial\Sigma}) - (\mathcal{C}, \mathcal{F}_{\mathcal{C}}|_{\mathcal{C}})$$

$\hat{\partial}$ is dual to the d_H-operator acting on the currents $j_{(\Sigma,\mathcal{F}_{\mathcal{C}})} \simeq \delta(\Sigma) \wedge e^{\mathcal{F}_{\mathcal{C}}}$

$$\mathrm{d}_H j_{(\Sigma,\mathcal{F}_{\mathcal{C}})} = j_{\hat{\partial}(\Sigma,\mathcal{F}_{\mathcal{C}})}$$

Extend by linearity to chains

$$(\mathfrak{S}, \mathfrak{F}) = (\Sigma_1, \mathcal{F}^{(1)}_{\mathcal{C}_1}) + (\Sigma_2, \mathcal{F}^{(2)}_{\mathcal{C}_2}) + (\Sigma_3, \mathcal{F}^{(3)}_{\mathcal{C}_3}) + \ldots$$

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RR gauge invariance and generalized cycles

Transformation of D-brane CS term under RR-gauge transformations $\delta C = \mathrm{d}_H \lambda$

$$\delta S_{\mathrm{CS}} = \int_X \langle \delta C, j_{(\mathfrak{S},\mathfrak{F})} \rangle = \int_X \langle \mathrm{d}_H \lambda, j_{(\mathfrak{S},\mathfrak{F})} \rangle = \int_X \langle \lambda, \mathrm{d}_H j_{(\mathfrak{S},\mathfrak{F})} \rangle = \int_X \langle \lambda, j_{\hat{\partial}(\mathfrak{S},\mathfrak{F})} \rangle$$

Gauge invariance demands $\hat{\partial}(\mathfrak{S},\mathfrak{F}) = 0$

$\Rightarrow$ Consistent D-brane networks wrap generalized cycles

For example $(\Sigma, \mathcal{F}_{\partial\Sigma'}) + (\Sigma', \mathcal{F}')$ is a generalized cycle: D$(p-2)$-brane wrapping $(\Sigma', \mathcal{F}')$ ends on Dp-brane wrapping $(\Sigma, \mathcal{F}_{\partial\Sigma'})$ [Strominger '95].

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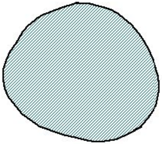
Deformations inside generalized homology classes

- Ordinary deformations

$$(\Sigma, \mathcal{F}) \quad \longrightarrow \quad (\Sigma', \mathcal{F}')$$



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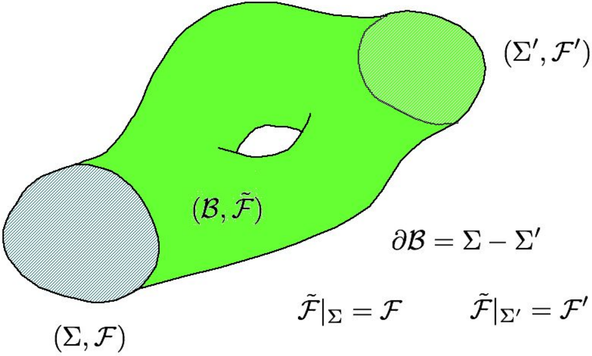

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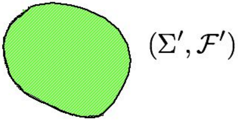

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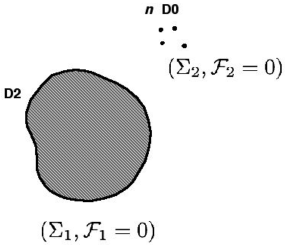

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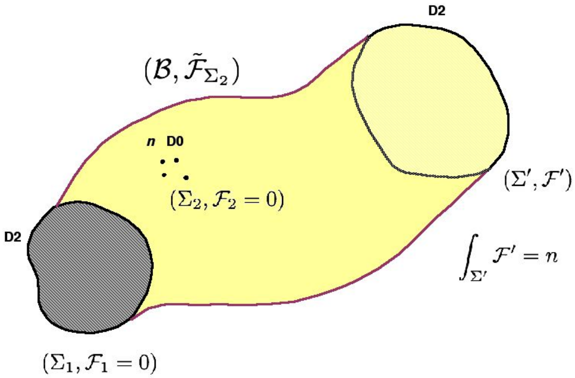

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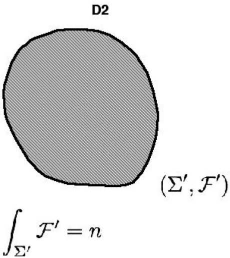

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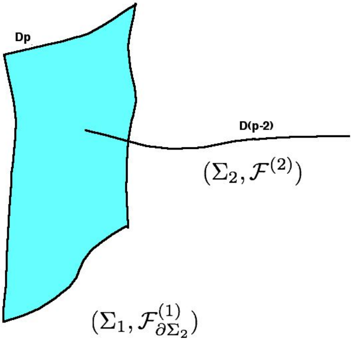

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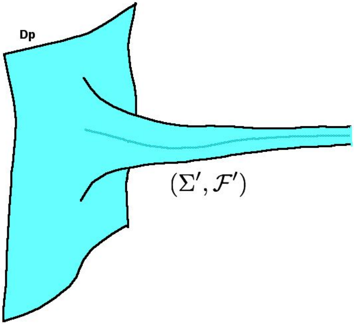
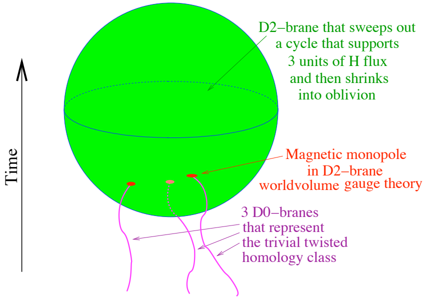


- MMS-instantons [Maldacena, Moore & Seiberg '01]

If $H \neq 0$ then we can have transitions

$$(\Sigma, \mathcal{F}) \quad \longrightarrow \quad \text{nothing}$$



if we can choose cycle $(\mathcal{B}, \tilde{\mathcal{F}}_\Sigma)$, where $\mathrm{PD}_{\mathcal{B}}[\Sigma] = [H|_{\mathcal{B}}]$

D-brane networks and calibrations

We can apply the generalized calibrations

$$\omega = \sum_{k \text{ even,odd}} \omega_{(k)}$$

to generalized cycles

$$(\mathfrak{S}, \mathfrak{F}) = (\Sigma_1, \mathcal{F}^{(1)}_{\mathcal{C}_1}) + (\Sigma_2, \mathcal{F}^{(2)}_{\mathcal{C}_2}) + (\Sigma_3, \mathcal{F}^{(3)}_{\mathcal{C}_3}) + \dots$$

Again, a calibrated generalized submanifolds minimizes its energy inside its $\hat{\partial}$-homology class:

$$E(\mathfrak{S}, \mathfrak{F}) \leq E(\mathfrak{S}', \mathfrak{F}')$$

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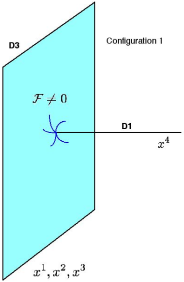
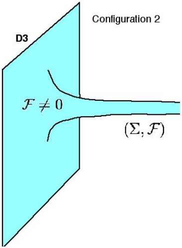
A simple example: The BIon [Callan & Maldacena; Gauntlett, Gomis & Townsend '97]





- D3+D1 system: associated generalized calibration

$$\omega = \omega_{\mathrm{D3}} + \omega_{\mathrm{D1}} = \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3 + \mathrm{d}x^4$$

- Integrating ω over configuration 1: $E_{\mathrm{min}} = \mathrm{Vol}_{D3} + \mathrm{L}_{D1}$
- D3 embedding: $x^a = \sigma^a$, $a = 1, 2, 3$, and $x^4 = X(\sigma)$

  $[\omega|_\Sigma \wedge e^{\mathcal{F}}]_{\mathrm{top}} = \sqrt{\eta|_\Sigma + \mathcal{F}}\mathrm{d}^3\sigma \;\Leftrightarrow\; \mathrm{d}X = \star\mathcal{F} \;\Rightarrow\; X = \frac{1}{4\pi|\sigma|}$ (configuration 2)

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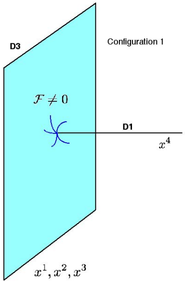

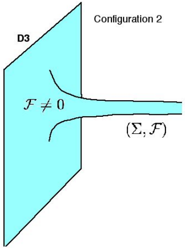

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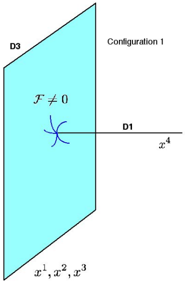

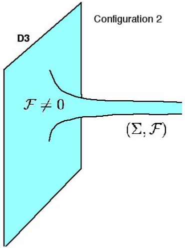

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$\mathcal{N} = 1$ flux vacua

General $\mathcal{N} = 1$ Type II vacua on $X = \mathbb{R}^{1,3} \times M$:

metric: $$ds^2 = e^{2A(y)} dx^\mu dx_\mu + g_{mn}(y) dy^m dy^n \ ,$$

RR-fluxes: $$F_{(n)} = \hat{F}_{(n)} + \mathrm{Vol}_{(4)} \wedge \tilde{F}_{(n-4)} \ ,$$

Killing spinors: $$\varepsilon_1(y) = \zeta_+ \otimes \eta_+^{(1)}(y) + \text{ c. c.}$$

$$\varepsilon_2(y) = \zeta_+ \otimes \eta_\mp^{(2)}(y) + \text{ c. c.}$$

Pure spinors Ψ^+ (even) and Ψ^- (odd)

$$\eta_+^{(1)} \otimes \eta_\pm^{(2)\dagger} \sim \sum_{k=even/odd} \frac{1}{k!} \Psi^\pm_{m_1\dots m_k} \gamma^{m_1\dots m_k} \quad \leftrightarrow \quad \Psi^\pm = \sum_{n=even,odd} \Psi^\pm_{(n)}$$

We can set

$$\Psi_1 = \begin{cases} \Psi^- & \text{in IIA} \\ \Psi^+ & \text{in IIB} \end{cases} \qquad \Psi_2 = \begin{cases} \Psi^+ & \text{in IIA} \\ \Psi^- & \text{in IIB} \end{cases}$$

Background susy conditions $\Leftrightarrow$ equations for Ψ_1 and Ψ_2

$$\mathrm{d}_H(e^{4A-\Phi}\mathrm{Re}\Psi_1) = e^{4A}\tilde{F} \quad , \quad \mathrm{d}_H(e^{2A-\Phi}\mathrm{Im}\Psi_1) = 0 \quad , \quad \mathrm{d}_H(e^{3A-\Phi}\Psi_2) = 0$$

[Graña, Minasian, Petrini & Tomasiello, hep-th/0505212]

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[Graña, Minasian, Petrini & Tomasiello, hep-th/0505212]

Since $X=\mathbb{R}^{1,3}\times M$, we have three possible calibrations on M:

$$\begin{array}{rcll}\omega^{(4d)} & = & e^{4A}\left(e^{-\Phi}\mathrm{Re}\Psi_1-\tilde{C}\right) & \text{space-time filling branes}\\ \omega^{(\mathrm{string})} & = & e^{2A-\Phi}\mathrm{Im}\Psi_1 & \text{strings}\\ \omega^{(\mathrm{DW})} & = & e^{3A-\Phi}\mathrm{Re}(e^{i\theta}\Psi_2) & \text{domain walls}\end{array}$$

They must satisfy the two conditions

- *Algebraic condition* $\longrightarrow$ checked
- *Differential condition* $\mathrm{d}_H\omega=0$ $\Leftrightarrow$ background supersymmetry conditions!

κ-symmetry $\Rightarrow$ *Supersymmetric D-branes wrap calibrated generalized cycles*

$\mathcal{N} = 1$ background supersymmetry and calibrations

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Relation with Hitchin's and Gualtieri's generalized complex geometry

[Graña, Minasian, Petrini & Tomasiello, hep-th/0505212]

- $d_H \Psi_2 \simeq 0 \quad \Rightarrow \quad$ integrable *generalized complex structure*
- $d_H \Psi_1 \quad \simeq \quad F \quad \Rightarrow \quad$ non-integrable *generalized (almost) complex structure*

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Generalized complex geometry and D-branes

Space-filling admit effective $\mathcal{N} = 1$ 4d description

- Superpotential depending on Ψ_2 $\quad\longrightarrow\quad$ relation with DW's
- D-terms depending on Ψ_1 $\quad\longrightarrow\quad$ relation with strings

[L. M. '06]

Generalized complex geometry crucial: massless spectrum in terms of a cohomology group $H^1(\Sigma, \mathcal{F})$

[P. Koerber & L. M. '06]

Outline

Motivations

Calibrations and supersymmetric D-branes

D-brane networks

$\mathcal{N} = 1$ flux vacua as calibrated backgrounds

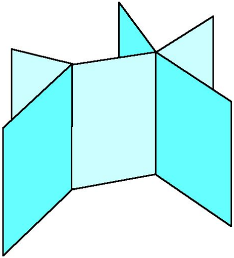
Networks of strings and domain walls

Possible future directions

The total calibration

One can also consider mutually BPS domain walls and strings that are glued together

[Gauntlett, Gibbons, Hull & Townsend '00]



The total calibration on $\mathbb{R}^3 \times M$ for these configurations is given by

$$
\begin{aligned}
\omega \quad = \quad & dx^1 \wedge dx^2 \wedge dx^3 \wedge e^{4A}\left(e^{-\Phi}\mathrm{Re}\Psi_1 - \tilde{C}\right) + dx^1 \wedge e^{2A-\Phi}\mathrm{Im}\Psi_1 + \\
& + dx^1 \wedge dx^2 \wedge e^{3A-\Phi}\mathrm{Re}\Psi_2 + dx^3 \wedge dx^1 \wedge e^{3A-\Phi}\mathrm{Im}\Psi_2
\end{aligned}
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As a subcase, M can be a warped CY

[Graña & Polchinski '00; Giddings, Kachru & Polchinski '01]

The total calibration is

$$\begin{aligned}\omega_{\mathrm{WCY}} \quad = \quad & \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3 \wedge (e^{4A} - J \wedge J - e^{4A}\tilde{C}) + \mathrm{d}x^1 \wedge (J - e^{-4A} J \wedge J \wedge J) + \\ & + \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{Re}\Omega + \mathrm{d}x^3 \wedge \mathrm{d}x^1 \wedge \mathrm{Im}\Omega\end{aligned}$$

One can consider for example networks of space-filling D3's, D5 domain walls and D7 strings.

Effects of $H \neq 0$ on M

- If M is compact, a D7-string alone is inconsistent. We need D5-domain walls ending on it
- A D5 domain wall wrapping an internal three-cycle Γ with $\int_\Gamma H \neq 0$ needs space-filling D3's ending on it

Networks on IIB warped CY compactifications

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[Graña & Polchinski '00; Giddings, Kachru & Polchinski '01]

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$$\begin{aligned}\omega_{\rm WCY} \;=\;& dx^1\wedge dx^2\wedge dx^3\wedge(e^{4A}-J\wedge J-e^{4A}\tilde{C})+dx^1\wedge(J-e^{-4A}J\wedge J\wedge J)+\\ &+dx^1\wedge dx^2\wedge \mathrm{Re}\Omega+dx^3\wedge dx^1\wedge \mathrm{Im}\Omega\end{aligned}$$

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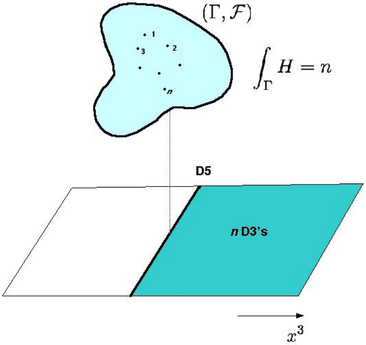
D5-D3 domain walls

The D5-wrap an internal generalized three-cycle $(\Gamma, \mathcal{F})$ with $\int_\Gamma H = n$, and n space-filling D3's ending on it, at points $p_i \in \Gamma$.



If $x^3 = X(\sigma)$, calibration condition $[\omega|_\Gamma \wedge e^{\mathcal{F}}]_{\text{top}} = \mathcal{E}$ implies

$$\mathrm{Re}\Omega|_\Gamma = \sqrt{\det(g|_\Gamma)}\, \mathrm{d}^3\sigma \quad , \quad \mathrm{d}X = \star_3 \mathcal{F}$$

Thus, Γ is a SLag cycle and $\Delta X = \star_3 [H|_\Gamma + \sum_i \delta^3_\Gamma(p_i)]$

Explicit examples on $T^6/\mathbb{Z}_2$ flux compactifications [Kachru, Schulz & Trivedi '02] and Klebanov-Strassler solution (see also [Kachru, Pearson & Verlinde '01])

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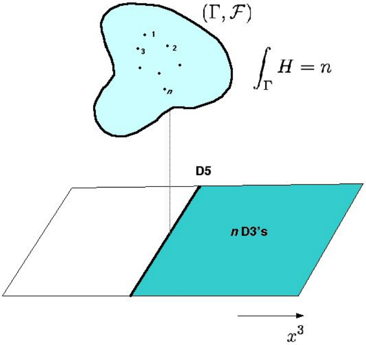

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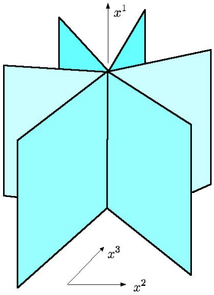
Gluing domain walls

Tension of a BPS domain wall:
$T_{\rm DW} = \frac{1}{g_s} |\int_\Gamma \Omega|$

The DW is at an angle $\alpha(\Gamma)$ with the x^2 axis in the (x^2, x^3)-plane

$$\alpha(\Gamma) = \arg\left(\int_\Gamma \Omega\right)$$

We can glue D5 domain walls wrapping different Γ_i at different angles.



If $\mathrm{PD}_M[\sum_i \Gamma_i] = [H]$ a D7 must fill the string

$H \wedge \Omega = 0 \quad \Rightarrow \quad \sum_i \int_{\Gamma_i} \Omega = 0$ equilibrium condition:

$$\sum_i T_{\rm DW}(\Gamma_i) \cos\alpha(\Gamma_i) = 0 \quad , \quad \sum_i T_{\rm DW}(\Gamma_i) \sin\alpha(\Gamma_i) = 0$$

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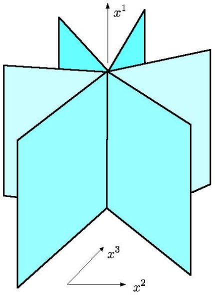

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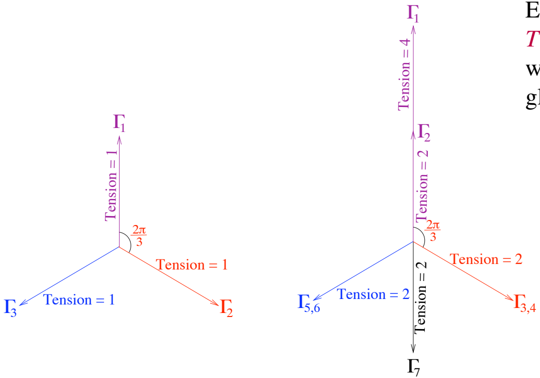
Domain wall networks on $T^6/\mathbb{Z}_2$



Explicit examples on a specific $T^6/\mathbb{Z}_2$ flux compactification with and without D7 filling the gluing string

Outline

Future directions

- Explicit examples on truly generalized complex models
- Relation with non-geometric backgrounds
- Quantization of fluxes
- Inclusion of non-abelian effects
- ...

Simplest case: D-branes on Calabi-Yau 3-folds

- Pure spinors: $\Psi_1 = e^{iJ}$ and $\Psi_2 = \Omega^{(3,0)}$
- $\mathcal{E}(\Sigma, \mathcal{F}) = \sqrt{g|_\Sigma + \mathcal{F}}\, \mathrm{d}^n\sigma$, with $\mathrm{d}\mathcal{F} = 0$.
- The generalized calibrations are

$$\omega^{(\text{even})} = \mathrm{Re}\big(e^{i\theta} e^{iJ}\big) \quad , \quad \omega^{(\text{odd})} = \mathrm{Re}\big(e^{i\theta}\Omega\big) \quad \Rightarrow \quad \mathrm{d}\omega^{(\text{even}/\text{odd})} = 0\,.$$

- The calibration condition $[\omega|_\Sigma \wedge e^{\mathcal{F}}]_{\text{top}} = \sqrt{P[g] + \mathcal{F}}\, \mathrm{d}^n\sigma$ is equivalent to the conditions found by [Mariño, Minasian, Moore & Strominger, '99]