Tadpole Resummations and One-loop Scalar Masses in String Models without Supersymmetry

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1. Motivation and Introduction

Possibility of low-scale string models without SUSY
(using D-branes at orbifold singularities
with brane SUSY breaking mechanism)

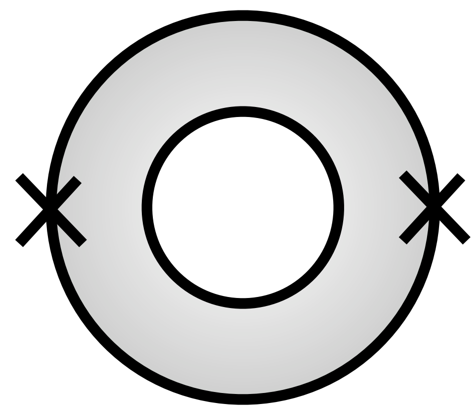
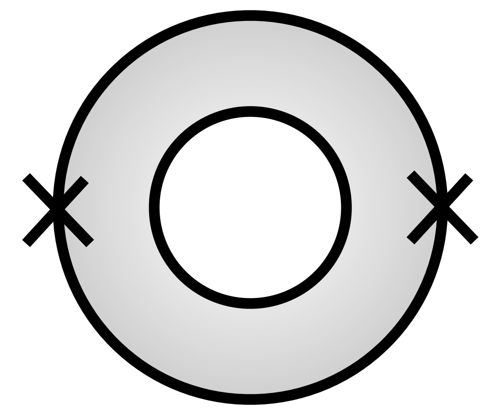
- At tree-level there are many massless scalar fields with non-trivial gauge charges.
- Non-zero one-loop corrections to the masses of those scalar fields are expected because of lack of SUSY.

Some extra scalar fields would become massive and decouple, and some others would obtain negative mass squared and would become Higgs doublet fields in the Standard Model for electroweak symmetry breaking.

Some concrete calculations have shown the possibility of radiative symmetry breaking.

NS

R

$m^2 \sim$  $+$ 

boson loop (positive)

fermion loop (negative)

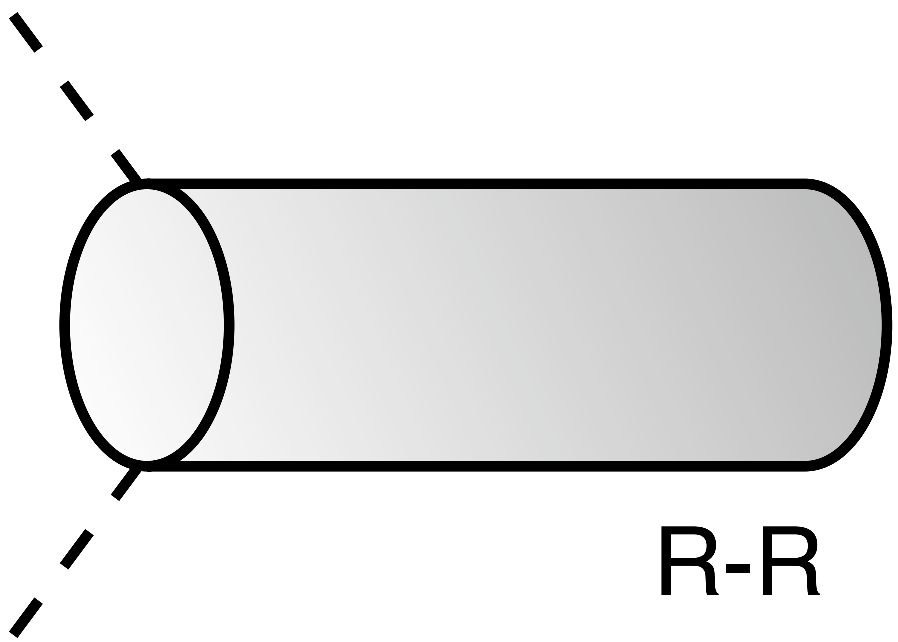
possible to have:

$$m^2 \simeq -\frac{g^2}{16\pi^2}\frac{1}{\alpha'}f(R/\sqrt{\alpha'}) < 0$$

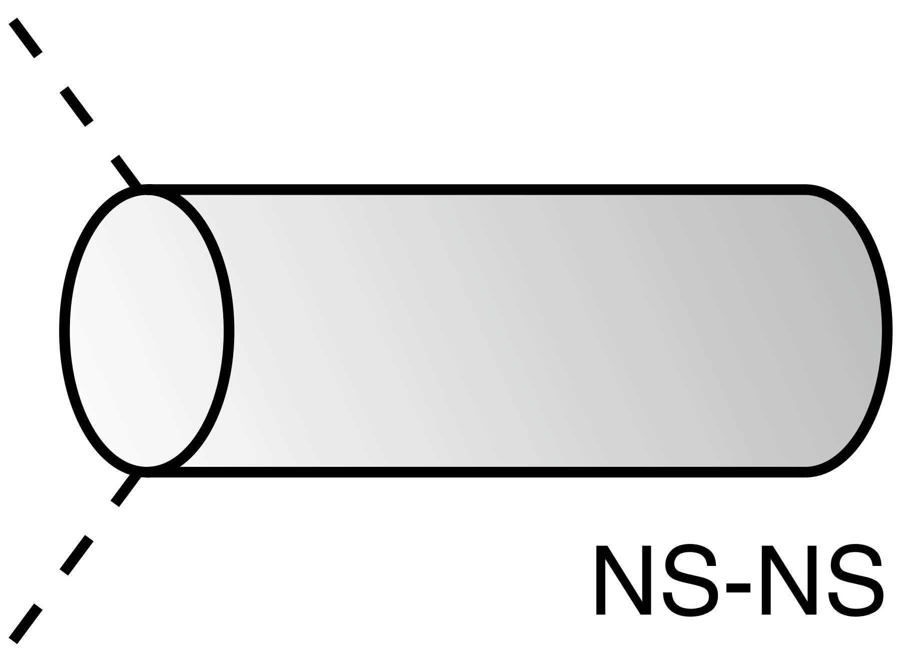
Antoniadis-Benakli-Quiros (2000), N.K. (2006)

“NS-NS tadpole problem”

Two kinds of closed-string tadpole contributions assuming open-closed string duality



massless R-R tadpoles should be canceled for the consistency of the model.



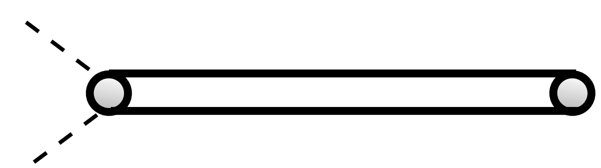
massless NS-NS tadpoles are not required to cancel for consistencies

In general string models without SUSY have
NS-NS tadpoles of massless dilaton and graviton.

- The conceptual difficulty (or understanding):

The background geometry and fields configurations are not the "solution" of String Theory.
-> This should be cured by Fishler-Susskind mechanism which is very difficult to do unfortunately.

- The actual difficulty:



$$\lim_{p \to 0} \frac{1}{p^2}$$

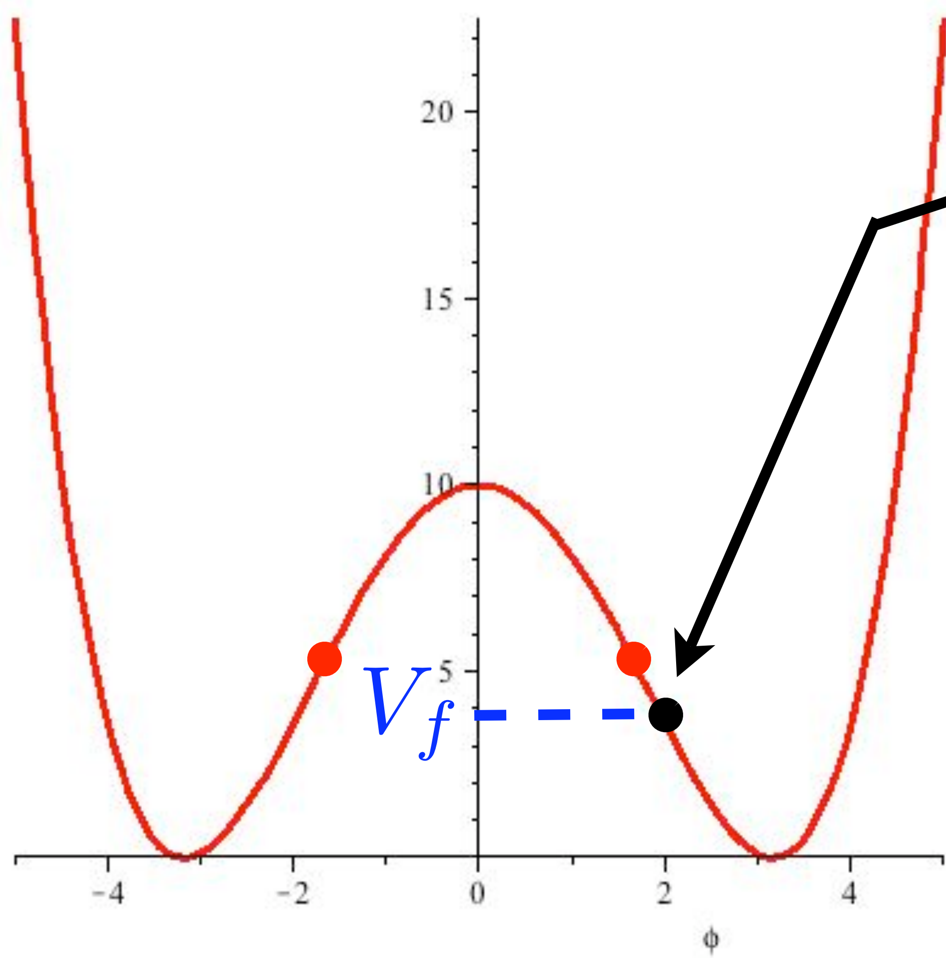
Open-string one-loop calculations get infrared divergences.
Some reasonable technique to evade these divergences?
-> "tadpole resummations"

Tadpole resummations in field theory

Dudas-Nicolosi-Pradisi-Sagnotti (2005)

Possibility to obtain true values of physical quantities in “wrong vacua”

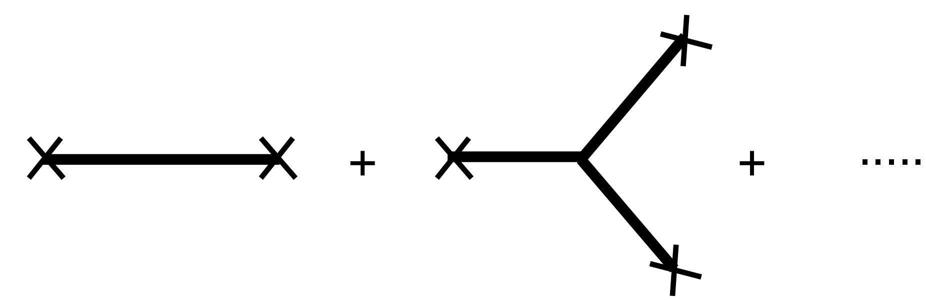
ex.



“wrong vacua”

$$V = -4a(v^2 - v_f^2)v_f\phi + \cdots$$

tadpole

$V_{\text{res}} =$ 

$$V_f + V_{\text{res}} = 0$$

Is the same is possible in String Theory?

Similar resummations are possible in String Theory using boundary state formalism

The technique gives one positive result:

The vacuum energy of a “Dp-brane” in Bosonic String Theory is canceled by the tree-level contribution from tadpole resummations,

$$\Lambda_p^{\rm cl} + \Lambda_p^{\rm res} = 0 \qquad (\Lambda_p^{\rm cl} = T_p)$$

N.K. (2008)

which is consistent with Sen’s conjecture of “Dp-brane” decay in Bosonic String Theory: “tachyon condensation”.

Some non-trivial checks and applications to actual calculations of scalar masses will be given.

2. Some techniques

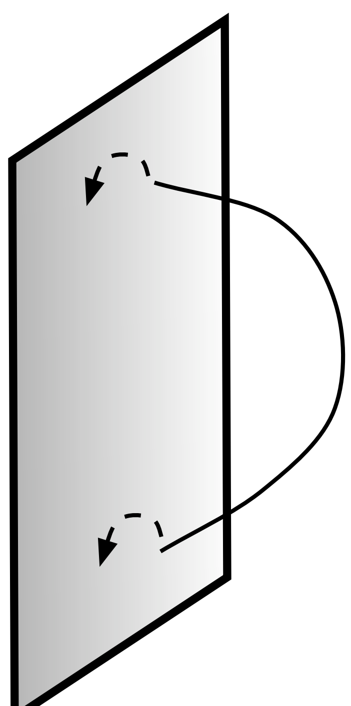
In this talk we mainly consider Bosonic String Theory, although the same is applicable to Superstring Theory.

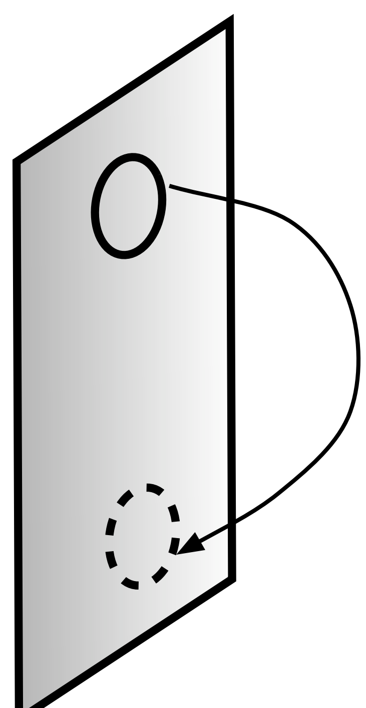
<u>Boundary State Formalism</u>

the cylinder amplitude

open string $\longleftrightarrow$ closed string

one loop — tree





Dp-brane boundary state

$$\frac{1}{2!}\langle B_p|D|B_p\rangle$$

second order of tadpole insertion

propagator operator

The Cylinder Amplitude
-- one-loop correction to the vacuum energy of Dp-brane --

$$A_p = \frac{1}{2!}\langle B_p|D|B_p\rangle = \frac{1}{2!}V_{p+1}N_p^2\Delta_p$$

$$N_p \equiv T_p/2: \text{ normalization factor of } |B_p\rangle$$

$$\begin{aligned}\Delta_p \quad &\equiv \quad \frac{\pi\alpha'}{2}\int_0^\infty ds \int \frac{d^{d_\perp}p}{(2\pi)^{d_\perp}} e^{-\frac{\pi\alpha'}{2}p_\perp^2 s}\frac{1}{(\eta(is))^{24}} \\ &= \quad \frac{\pi\alpha'}{2}\int_0^\infty ds \frac{1}{(2\pi^2\alpha' s)^{d_\perp/2}}\frac{1}{(\eta(is))^{24}}\end{aligned}$$

$$d_\perp \equiv d-(p+1) \text{ with } d=26$$

Infrared divergence due to massless NS-NS tadpoles of dilaton, graviton and tachyon.

Tadpole couplings in boundary states

$$A^{\mu\nu} \equiv \langle 0; k | a_1^{\mu} \tilde{a}_1^{\nu} | B_p \rangle = -\frac{T_p}{2} V_{p+1} S^{\mu\nu} \qquad S^{\mu\nu} \equiv (\eta^{\alpha\beta}, -\delta^{ij})$$

$$\begin{cases} A_{\rm grav} = A^{\mu\nu}\epsilon^{(h)}_{\mu\nu} = -V_{p+1} T_p \eta^{\alpha\beta}\epsilon^{(h)}_{\alpha\beta}, \\ A_{\rm dil} = A^{\mu\nu}\epsilon^{(\phi)}_{\mu\nu} = V_{p+1} T_p a. \end{cases} \qquad \left(a \equiv \frac{d-2p-4}{2\sqrt{d-2}} \right)$$

D-brane effective action in Einstein frame

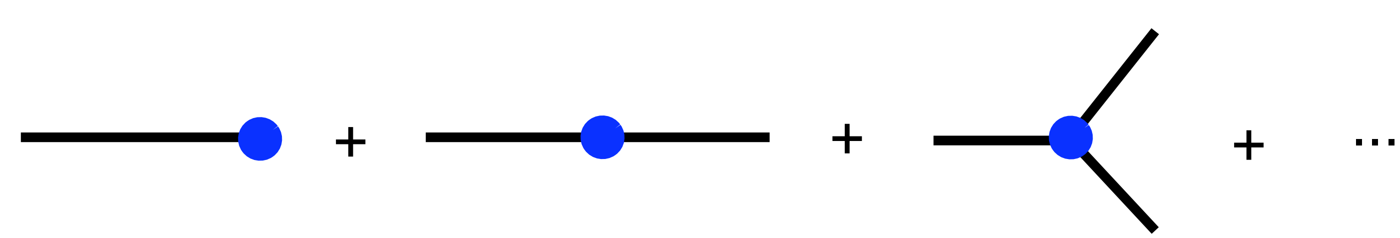
$$S_{Dp} = -T_p \int d^{p+1}\xi e^{-a\phi} \sqrt{-\det g_{\alpha\beta}}$$

ignoring B-field and gauge field

$$\mathcal{L}_{\rm dil} = -T_p e^{-a\phi} = -T_p + T_p a \phi - \frac{1}{2!} T^p a^2 \phi^2 + \frac{1}{3!} T^p a^3 \phi^3 - \cdots$$

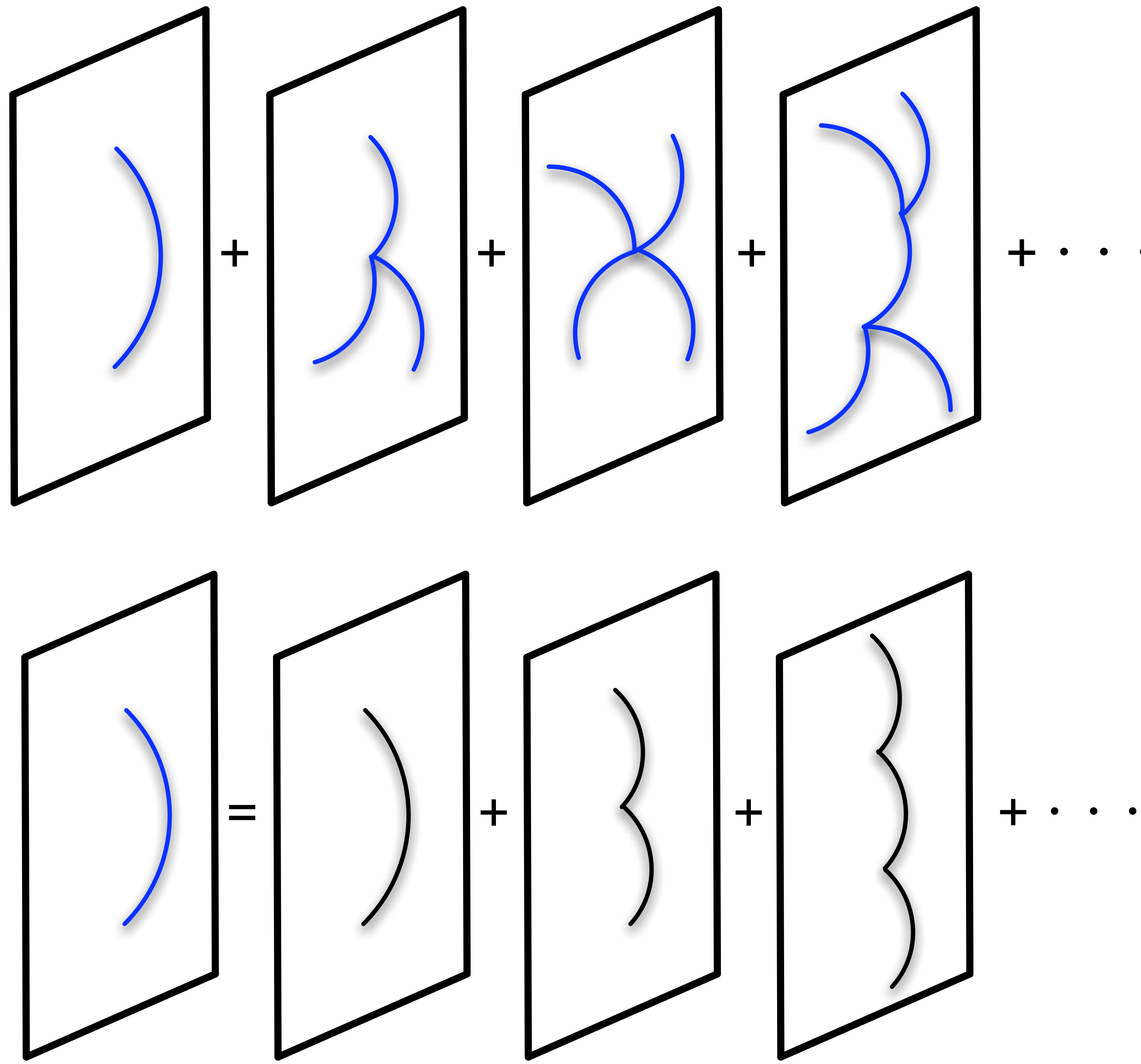
$$\Lambda_p^{\rm cl} = T_p$$

vacuum energy



Strategy for tadpole resummations

$$\Lambda_p = \Lambda_p^{\mathrm{cl}} +$$



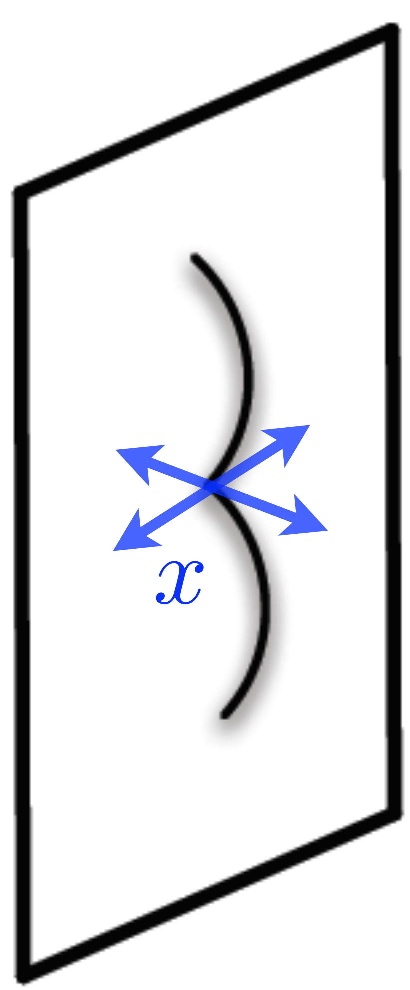
“Closed strings bouncing on a Dp-brane”

Multi-point vertices in boundary state formalism

$-T_p a^2$

$$\hat{M} \equiv \int d^d x \delta^{d_\perp}(x) |\tilde{B}_p(x)\rangle (-T_p) \langle \tilde{B}_p(x)|$$

integrating over Dp-brane world volume



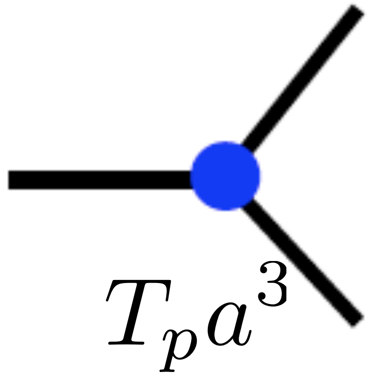
$$|\tilde{B}_p(x)\rangle \equiv \frac{1}{T_p} \delta^{p+1}(\hat{x} - x) |B_p\rangle$$

normalization
for correct coupling

specify one point
on Dp-brane

$$\left(T_p a \times \frac{1}{T_p}\right)^2 \times (-T_p) \times d^d x \delta^{d_\perp}(x) \to -T_p a^2 \times V_{p+1}$$

Similar for three-point vertex and higher



$$\hat{M}^{(3)} \equiv \frac{1}{3!} T_p \left(\frac{N_p}{T_p} \right)^3 \int d^d x \delta^{d_\perp}(x)$$
$$\times |\tilde{B}_p(x)\rangle |\tilde{B}_p(x)\rangle |\tilde{B}_p(x)\rangle$$

Closer look at "propagator"

$$\begin{aligned} \Delta_p &= \frac{\pi\alpha'}{2} \int_0^\infty ds \frac{1}{(2\pi^2\alpha' s)^{d_\perp/2}} \frac{1}{(\eta(is))^{24}} \\ &\to \frac{\pi\alpha'}{2} \int^\infty ds \frac{1}{(2\pi^2\alpha' s)^{d_\perp/2}} \left\{ e^{2\pi s} + 24 + \mathcal{O}(e^{-2\pi s}) \right\} \end{aligned}$$

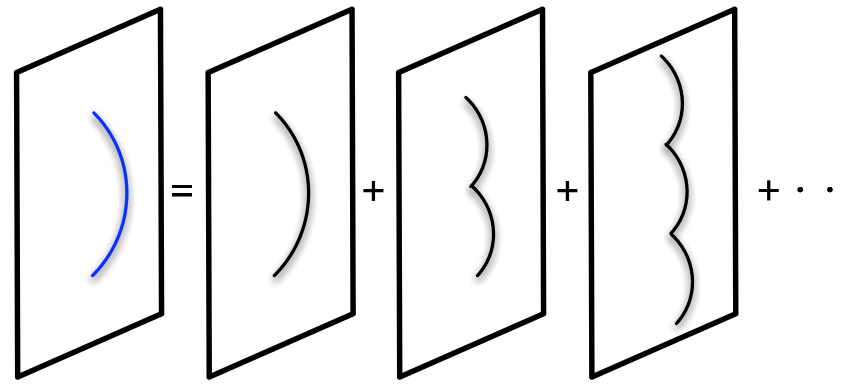
tachyon tadpoles

dilaton/graviton tadpoles

Regularize via an "ultraviolet" cutoff on s

3. Actual calculations on vacuum energies

Full two-point function



"one bounce"

$$\frac{1}{2!}\langle B_p|D\hat{M}D|B_p\rangle = \frac{1}{2!}\int d^dx\,\delta^{d_\perp}(x)\langle B_p|D|\tilde{B}_p(x)\rangle(-T_p)\langle\tilde{B}_p(x)|D|B_p\rangle$$

$$= \frac{1}{2!}V_{p+1}N_p^2\left(\frac{N_p}{T_p}\right)^2(-T_p)(\Delta_p)^2$$

similar to the cylinder

Similar for "two bounces", and more

$$A_p^{(2)} = \frac{1}{2!}\left\{\langle B_p|D|B_p\rangle + \langle B_p|D\hat{M}D|B_p\rangle + \langle B_p|D\hat{M}D\hat{M}D|B_p\rangle + \cdots\right\}$$

geometric series

$$\equiv \frac{1}{2!}\langle B_p|D_M|B_p\rangle$$

$$= \frac{1}{2!}V_{p+1}N_p^2\frac{\Delta_p}{1+T_p(N_p/T_p)^2\Delta_p} \to \frac{1}{2!}V_{p+1}T_p$$

$$\boxed{\Lambda_p^{(2)} = -\frac{1}{2!}T_p}$$

Full three-point function

vertex

\# of contractions

$$\frac{1}{3!} = \frac{1}{3!} \times \frac{1}{3!} \times 3!$$

third order of tadpole insertions

$$A_p^{(3)} = \frac{1}{3!} T_p \int d^d x \delta^{d_\perp}(x) \left(\langle B_p | D_M | \tilde{B}_p(x) \rangle \right)^3$$

$$= \frac{1}{3!} V_{p+1} T_p \left(\frac{(N_p^2/T_p)\Delta_p}{1 + T_p (N_p/T_p)^2 \Delta_p} \right)^3 \to \frac{1}{3!} V_{p+1} T_p$$

$$\boxed{\Lambda_p^{(3)} = -\frac{1}{3!} T_p}$$

Full Vacuum Energy of Dp-brane

$$\Lambda_p^{\mathrm{res}} \equiv -\left(A_p^{(2)} + A_p^{(3)} + \cdots \right) / V_{p+1}$$

$$= -T_p \sum_{n=1}^{\infty} \frac{1}{n(n+1)} \boxed{= -T_p}$$

exactly cancels

$$\boxed{\Lambda_p^{\mathrm{cl}} = T_p}$$

Consistent with Sen's conjecture

4. Some Consistency Checks

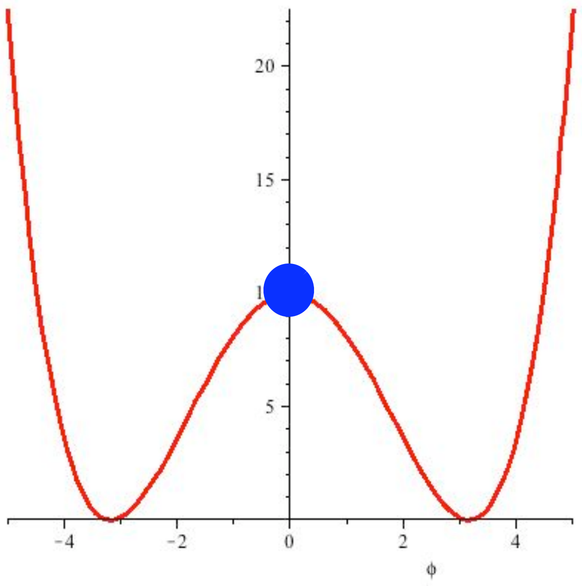
<u>“tadpole resummations” on a D25-brane in SO(8192) theory</u>

8192 D25-branes in unoriented Bosonic String Theory

Douglas-Grinstein (1987), Marcus-Sagnotti (1987), Weinberg (1987)

-> (unstable) “solution” of String Theory

- no tadpoles of dilaton and graviton
- tachyon (with tadpole)



“tadpole resumations” should not give corrections to the vacuum energy, even though tachyon exists.

two kinds of boundary states and three kinds of amplitudes

$$|B_{25}\rangle \quad \text{and} \quad |C_{25}\rangle$$

for orientifold
fixed plane: O25

$$\begin{aligned}
\mathcal{A} &= \frac{1}{2!}\langle B_{25}|D|B_{25}\rangle \\
\mathcal{M} &= \frac{1}{2!}\left(\langle B_{25}|D|C_{25}\rangle + \langle C_{25}|D|B_{25}\rangle\right) \\
\mathcal{K} &= \frac{1}{2!}\langle C_{25}|D|C_{25}\rangle
\end{aligned}$$

Full cylinder amplitude with bouncing on D25 and O25

$$\begin{aligned}
\mathcal{A}^{(2)} &= \frac{1}{2!}V_{26}N_{25}^2 \frac{\Delta_{25}}{1+T_{25}(N_{25}/T_{25})^2\Delta_{25}} \times \frac{1}{1-T_{25}(N_{25}/T_{25})^2\Delta_{25}} \\
&\to 0
\end{aligned}$$

(the same for the other amplitudes)

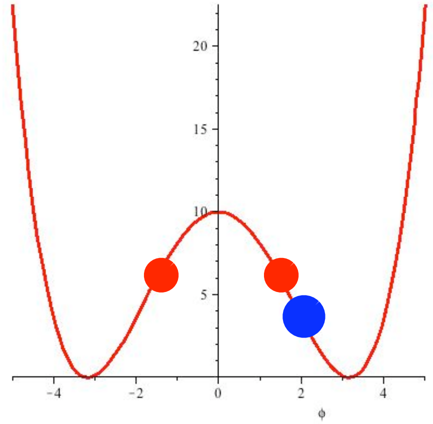
No Correction to Vacuum Energy

(Ignoring tachyon divergence -> $\mathcal{A}^{(2)} + \mathcal{M}^{(2)} + \mathcal{K}^{(2)} = 0$, and the same for multi-point functions. No correction to the vacuum energy.)

“tadpole resummations” on a D9-brane in Sugimoto model

32 D9-branes in a specially unoriented Superstring Theory
-> not a “solution” of String Theory, but stable
(D9-brane is not expected to decay.)

- NS-NS tadpoles (dilaton and graviton)
- no tachyon



“tadpole resummations” should not give the correction to the D9-brane vacuum energy which cancels the classical vacuum energy, even though the system is not a “solution”.

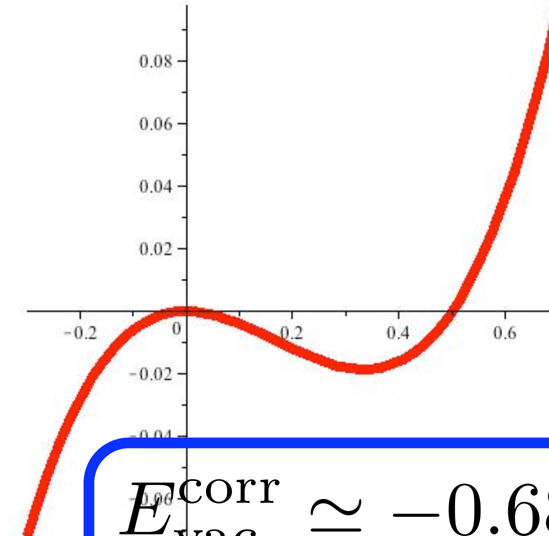
$$
\begin{aligned}
\mathcal{A}^{(2)} &= \frac{1}{2!} V_{10} N_9^2 \frac{\Delta_{\mathrm{NS}}}{1 + T_9 (N_9/T_9)^2 \Delta_{\mathrm{NS}}} \times \frac{1}{1 + T_9 (N_9/T_9)^2 \Delta_{\mathrm{NS}}} \\
&\to 0
\end{aligned}
$$

no correction to D9-brane vacuum energy

5. Problems and Limitations of this Procedure

On the relation with open-string field theory analysis on "tachyon condensation"

From Taylor-Zwiebach (TASI 2001):



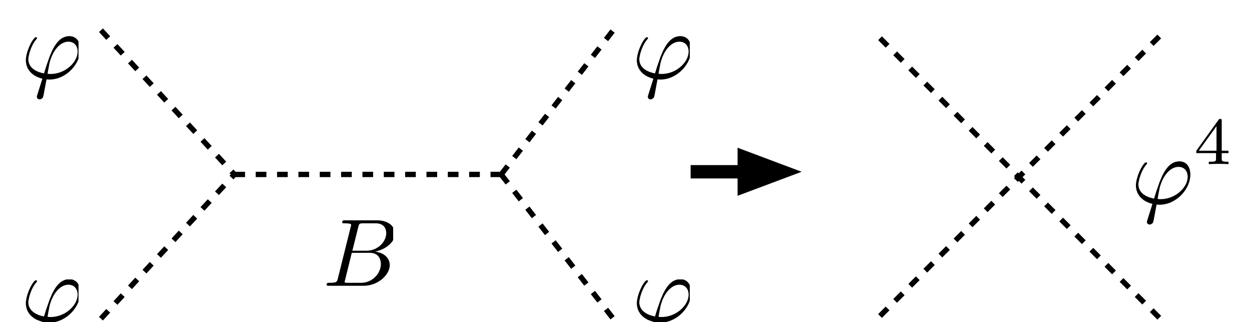
tachyon potential (tachyon mode only) $V(\varphi) = -\frac{1}{2}\varphi^2 + \mu\varphi^3$

$$\boxed{E_{\rm vac}^{\rm corr} \simeq -0.68 T_{25}}$$

a level truncation: $\Psi_s = \varphi|0\rangle + B(\alpha_{-1}\cdot\alpha_{-1})|0\rangle + \beta(b_{-1}c_{-1})|0\rangle + \cdots$

$$\begin{aligned} V &= -\frac{1}{2}\varphi^2 + 26B^2 - \frac{1}{2}\beta^2 \\ &+ \mu\left[\varphi^3 - \frac{130}{9}\varphi^2 B - \frac{11}{9}\varphi^2\beta + \cdots\right] \end{aligned}$$

$$\boxed{E_{\rm vac}^{\rm corr} \simeq -0.95938 T_{25}}$$



heavy modes are integrated out

Open string massive modes are important in the analysis of “tachyon condensation” using open-string field theory.

Their role is not evident in the procedure of “tadpole resummations”.

One thing we can say:

Open-closed string duality, which is essential in the procedure of “tadpole resummations”, requires infinite number of open string modes. The effects of open string massive modes are implicitly included.

Gravitational back reactions

The procedure of "tadpole resummations" includes "propagations" of closed string in the direction perpendicular to D-branes, assuming flat space-time.

The existence of D-branes should change the background geometry (and background fields).

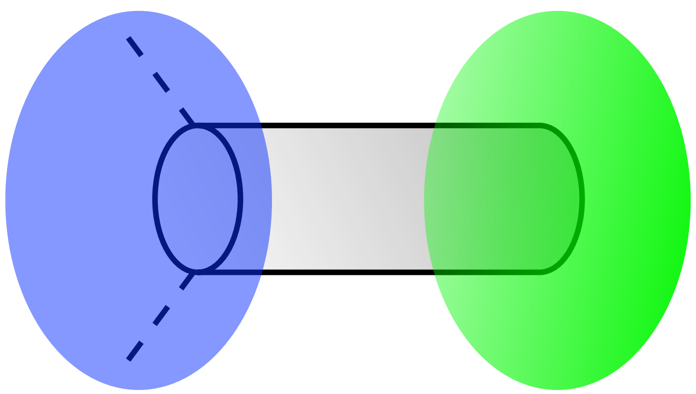
ex. spontaneous compactifications: Dudas-Mourad (2000)

↓

It may affect the results of "tadpole resummations", although it is not included in the analysis based on string field theory.

6. One-loop scalar masses

Boundary states with constant background scalar fields are required.



They can be obtained extending the formalism by Callan et al. (1988).

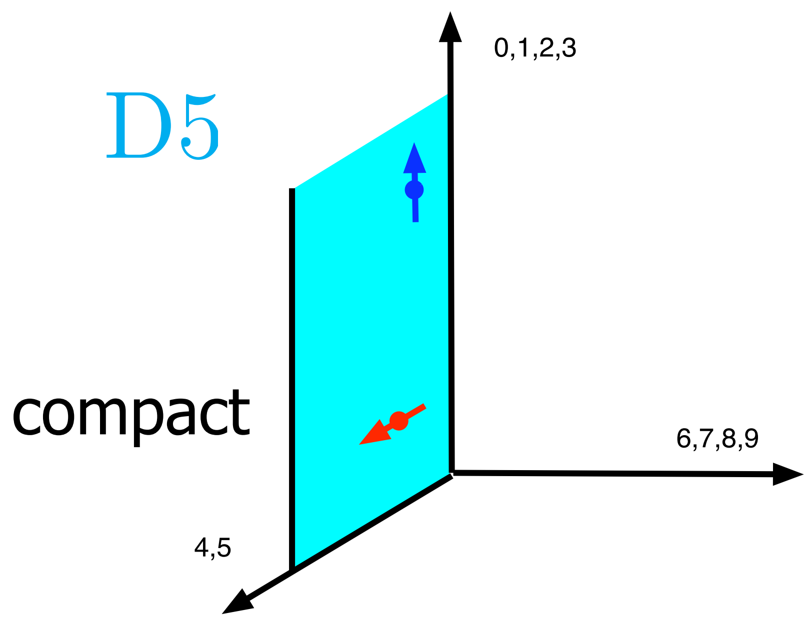
$$|B_p;\phi\rangle = N_p \int \mathcal{D}X\mathcal{D}\Theta\, e^{iS_A(\phi)}\, |x,\bar{x},x_D,\bar{x}_D\rangle|\theta,\bar{\theta},\theta_D,\bar{\theta}_D\rangle|B_{\rm gh}\rangle|B_{\rm sgh}\rangle$$

x, x_D, θ, θ_D : “boundary coordinates”

S_A : “boundary action”

Two categories of massless scalar fields

1) "gauge boson" polarized parallel to the D-brane

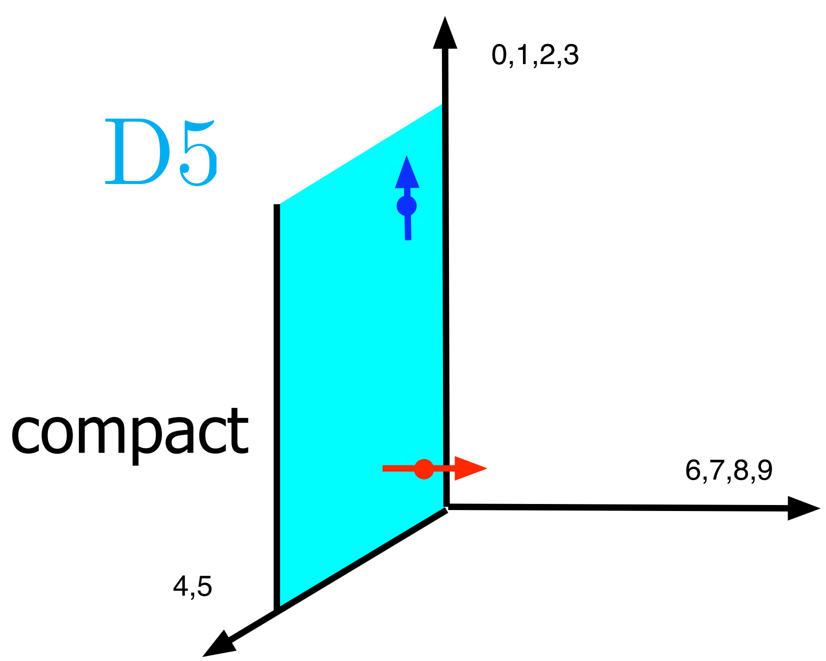


$$S_A = S_A^{\parallel} = -A_i \int_0^{2\pi} d\sigma \left[\partial_\sigma X^i\right]_{\tau=0}$$

"Wilson line"

Callan et al. (1988)

2) "gauge boson" polarized perpendicular to the D-brane



$$S_A = S_A^{\perp} = -A_I \int_0^{2\pi} d\sigma \left[\partial_\tau X^I\right]_{\tau=0}$$

"D-brane moduli"

Polchinski (1994)

Callan-Klebanov (1996)

One non-trivial example in the second category.

(the first category -> Antoniadis-Benakli-Quiros (2000))

Four D3-branes and Three anti-D7-branes

on a SUSY $\mathbf{C}^3/\mathbf{Z}_3$ orbifold singularity

("brane SUSY breaking": Antoniadis-Dudas-Sagnotti (1999), ...)

$\mathbf{Z}_3$ operation matrices on each D-brane

$$\gamma_3 = \mathrm{diag}(\mathbf{1}_2, \alpha \mathbf{1}_1, \alpha^2 \mathbf{1}_1) \qquad \mathrm{U}(2) \times \mathrm{U}(1)_1 \times \mathrm{U}(1)_2$$

$$\gamma_{\bar{7}_3} = \mathbf{1}_3 \qquad \mathrm{U}(3)$$

$$(\alpha \equiv e^{i2\pi/3})$$

non-anomalous $\mathrm{U}(1)$

No massless R-R tadpoles (anomalies) by

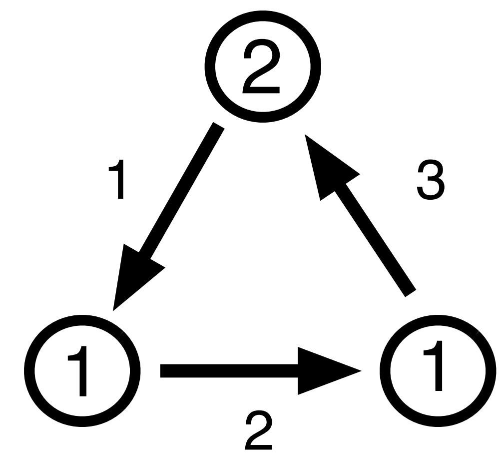
$$3\mathrm{Tr}(\gamma_3) - \mathrm{Tr}(\gamma_{\bar{7}_3}) = 0$$

Massless modes under $\mathrm{U}(3) \times \mathrm{U}(2) \times \mathrm{U}(1)$

D3-D3 open string (SUSY)

one-loop mass of $\phi \equiv \Phi_2^3$

$$
\begin{aligned}
(\Phi_1^a, \Psi_1^a) &: \qquad (2^*, 1)_{+1}, \\
(\Phi_2^a, \Psi_2^a) &: \qquad (1, 1)_0, \\
(\Phi_3^a, \Psi_3^a) &: \qquad (2, 1)_{-1}.
\end{aligned}
$$



$$a = 1, 2, 3$$

D3-anti-D7 open string (non-SUSY)

$$
\begin{aligned}
\phi_1 &: \qquad (2, 3^*)_0, \\
\phi_2 &: \qquad (2^*, 3)_0, \\
\psi_1 &: \qquad (1, 3)_{-1}, \\
\psi_2 &: \qquad (1, 3^*)_{+1},
\end{aligned}
$$

Using the boundary state with constant scalar background field (without technical details):

$$m_\phi^2 = 9g^2(2\gamma + 3\ln 3)\alpha' \left[3\langle B_{\bar{7}_3}|D|B_3\rangle + \langle B_3|D|B_3\rangle\right]/V_4,$$

two cylinder vacuum amplitudes

No R-R tadpole divergence

$$m_\phi^2|_{\mathrm{R-R}} = 9g^2(2\gamma + 3\ln 3)\alpha' \cdot (N_3^T)^2 \cdot 2\pi^2 \frac{\alpha'}{4\pi} \cdot \left(-8 \cdot \frac{3}{2\pi} + \cdots\right)$$

NS-NS tadpoles -> tadpole divergence in NS-NS contribution

$$\left[3\langle B_{\bar{7}_3}|D|B_3\rangle + \langle B_3|D|B_3\rangle\right]_{\mathrm{NS-NS}} = (N_3^T)^2 \cdot 2\pi^2 \frac{\alpha'}{4\pi} V_4 \cdot 2\int_0^\infty ds + \text{finite}$$

Only the twisted closed string contributes.

$$S_A = S_A^{\perp} = -A_I \int_0^{2\pi} d\sigma \left[\partial_\tau X^I\right]_{\tau=0} = 0$$

for untwisted sector: $X^I(\tau, \sigma + 2\pi) = X^I(\tau, \sigma)$

- The tadpole coupling of the twisted NS-NS closed string has been investigated.
  Merlatti-Sabella (2000) and Bertolini et al. (2002)
- Contact two-point interaction has been suggested.
  Marotta (2002)
- Higher order contact couplings are yet to be explored.

We can not do the complete "tadpole resummations" due to the lack of the information on contact couplings, but it is possible to make the NS-NS contribution finite by doing partial "tadpole resummations".

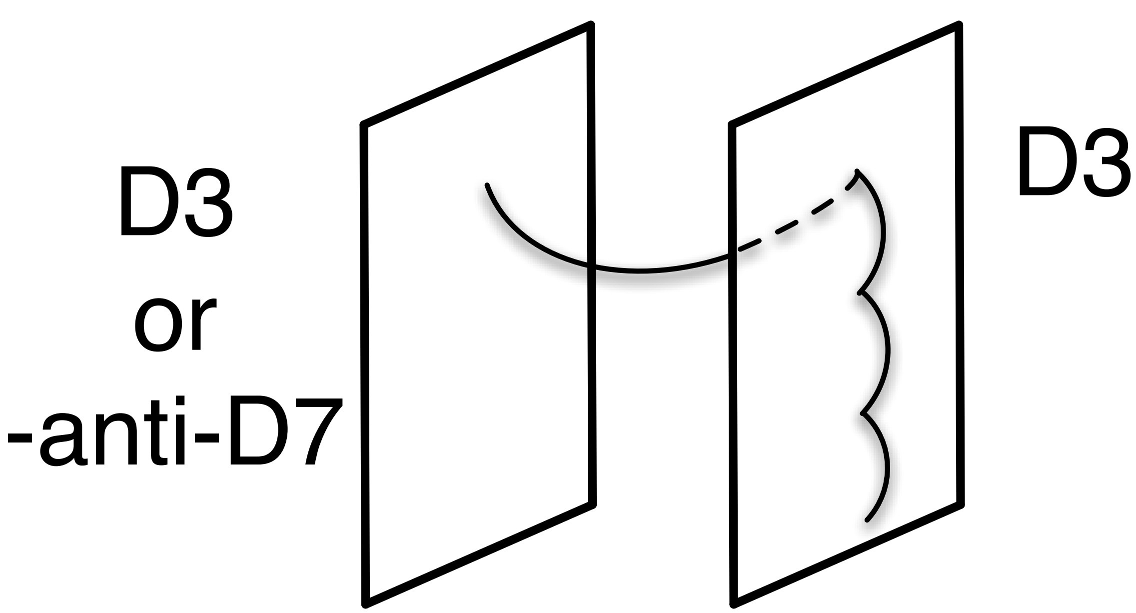
“partial tadpole resummations”

$$\begin{aligned}\langle B_3|D|B_3\rangle \quad &\longrightarrow \quad \langle B_3|D|B_3\rangle \\ &\qquad +\langle B_3|D\hat{M}^{\mathrm{T}}D|B_3\rangle + \langle B_3|D\hat{M}^{\mathrm{T}}D\hat{M}^{\mathrm{T}}D|B_3\rangle + \cdots\,, \\ \langle B_{\bar{7}_3}|D|B_3\rangle \quad &\longrightarrow \quad \langle B_{\bar{7}_3}|D|B_3\rangle \\ &\qquad +\langle B_{\bar{7}_3}|D\hat{M}^{\mathrm{T}}D|B_3\rangle + \langle B_{\bar{7}_3}|D\hat{M}^{\mathrm{T}}D\hat{M}^{\mathrm{T}}D|B_3\rangle + \cdots\,.\end{aligned}$$

$$\hat{M}^{\mathrm{T}} \equiv \int d^{10}x\delta^6(x)|\tilde{B}_3\rangle(-T_3^{\mathrm{T}})\langle\tilde{B}_3|$$

describing one bouncing on D3-brane

$T_3^{\mathrm{T}} \equiv 2N_3^{\mathrm{T}}/\kappa_{4\mathrm{D}}$: an assumption



“bouncing” only on D3-brane

NS-NS contribution

$$m_\phi^2|_{\mathrm{NS-NS}} = 9g^2(2\gamma + 3\ln 3)\alpha' \cdot 2T_3^T$$

In total

$$m_\phi^2 = 9g^2(2\gamma + 3\ln 3)\alpha' \left[\frac{4N_3^T}{\kappa_{4\mathrm{D}}} - (N_3^T)^2 \cdot 2\pi^2 \frac{\alpha'}{4\pi} \cdot \frac{8 \cdot 3}{2\pi} \right] + \cdots$$

NS-NS R-R

We need further information on the twisted sector to judge the sign of mass squared.

9. Summary

1) The importance of NS-NS tadpole problem in string models without supersymmetry is emphasized.

2) The procedure of “tadpole resummations” is formulated in String Theory.

3) “Tadpole resummations” give a consistent results with the “tachyon condensation” in Bosonic String Theory.

4) Some non-trivial checks are presented and the problems and limitations of this procedure are discussed.

5) The application to one-loop masses of scalar modes is briefly introduced.