Massive spin-2 propagation in curved space-time backgrounds

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Perspectives in String Theory
Galileo Galilei Institute
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April 2009

Massive Spin-2

1. The Problem
2. Suggestions for the solution
3. Torsion Theories
4. Massive Spin-2 in Torsion Theories in Curved Backgrounds
5. Conclusion

In Collaboration with

- V. Parameswaran Nair and
- Valery Rubakov

arXiv:0811.3781

- Work in progress with V. Rubakov

Massive Spin-2

- Any higher dimensional theory of gravity such as String Theory has infinite number of them.

  What we want is a theory in 4D with a finite number of interacting massive spin-2 particles.

- If they are massive gravitons they imply infrared modifications of gravity with important cosmological implications.

- Non metrical massive spin-2 theories offer dark matter candidates

- We need consistent interacting theories.

- Even classically we do not have a fully satisfactory model of consistent massive spin-2 particles (massive gravitons or any other).
- First we shall briefly review known models of massive gravitons.

Fierz-Pauli (1939)

$$L_{EH}^{(2)}(h_{\mu\nu}) + \frac{\alpha}{4} h_{\mu\nu} h^{\mu\nu} + \frac{\beta}{4} (h_{\mu}^{\mu})^2$$

Consistent only if

$$\beta = -\alpha = m_G^2$$

Number Of Adjustments

- Actually since the only symmetries are global Poincare' even the coefficients in the Einstein Hilbert part are arbitrary.
- Absence of Ghosts and Tachyons in the Linear Theory fix them uniquely to the FP form
- P. van Nieuwenhuizen NPB 60 (1973) 478

5 massive d.o.f propagate

- No ghosts or tachyons.

- Zero mass limit does not produce GR in leading order approximation. The light bending is predicted to be ¾ of that of GR

- Non linear terms must be taken into account.

A. I. Vainstein, Phys.Lett.39B (1972) 393

- It can then be shown that in the range

$$r_M \ll r \ll r_m \qquad r_M = 2G_N M$$

one can still use GR, where $$r_m = \frac{(m_g r_M)^{1/5}}{m_g}$$

$$3 \cdot 10^5\,\mathrm{cm} \ll r \ll 10^{21}\,\mathrm{cm}$$

Curved Space (BD Ghost)

- No Coordinate Invariant non linear completion in D=4 is known.

$$S_m = M_{\rm Pl}^2 \int d^4x \sqrt{|g|} \left(R + \frac{m_g^2}{4} \left[h_{\mu\nu}^2 - (h_\mu^\mu)^2 \right] \right) ,$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Buolware-Deser PRD6(1972)3368

Alternatives in D=4
Increase the number of fields

f-g theory :

A second symmetric second rank tensor is introduced

C.J. Isham, A. Salam and J. Strathdee
PRD3 (1971)867,

More Recently

- N.Arkani-Hamed, H. Georgi and M.D. Schwartz, arXiv:0209227

- E. Babvichev, C.Deffayet and R, Zour arXiv:0901.0393

Alternative in higher than 4D DGP

G. R. Dvali, G. Gabadadze, M. Porrati
arXiv:0005016

- C. Deffayet (A short Review with Cosmological Applications)

Int. J. Mod. Phys. D 16 (2007) 2023

DGP

$$S = M_*^3 \int d^4x \, dy \, \sqrt{|G|} \, \mathcal{R} + M_{\rm Pl}^2 \int d^4x \, \sqrt{|g|} \, R(x)$$

$$m_c \equiv \frac{1}{r_c} \equiv \frac{2 \, M_*^3}{M_{\rm Pl}^2}$$

AdS/CFT Approach

- E. Kiritsis and V.Niarchos

    arXiv:0808.3410

Non Metric Massive spin-2

- A Lorentz tensor t_{ijk} symmetric w.r.t the interchange of the i and j can also produce a spin-2 particle.

O(1,3) content

Impose two extra conditions:

$$\eta^{ij} t_{ijk} = 0$$

$$t_{ijk} + t_{jki} + t_{kij} = 0$$

Unconstrained Action

- Subject to these conditions t_{ijk} has 16 independent components,

- It can be contained in the components of a spin connection or torsion.

- We want to include gravity, thus, we end up with gravity theories including torsion.

These models do not make graviton massive!

- Cosmological applications:

- arXiv:gr-qc/0608121

  Yi Mao , Max Tegmark , Alan Guth , Serkan Cabi

- arXiv:0808.2063

  Yi Mao

Fundamental Fields

- Vierbein $$e^i_\mu$$
- Connection $$A_{ij\mu} = -A_{ji\mu}$$
- Curvature $$F_{ijmn} =$$

Invariants

$$F_{jl} = \eta^{ik} F_{ijkl},$$

$$F = \eta^{jk} F_{jk},$$

$$\varepsilon \cdot F = \varepsilon_{ijkl} F^{ijkl}$$

Torsion

- Define

$$C_{ijk} = e_j^{\mu} e_i^{\nu} (\partial_\mu e_{i\nu} - \partial_\nu e_{i\mu})$$

- Torsion

$$T_{ijk} = A_{ijk} - A_{ikj} - C_{ijk}$$

$$= -T_{ikj}$$

O(1,3) pieces

$$\boxed{T_{ijk} = \frac{2}{3}(t_{ijk} - t_{ikj}) + \frac{1}{3}(\eta_{ij} v_k - \eta_{ik} v_j) + \varepsilon_{ijkl} a^l}$$

$$S = \int d^4x \; e \; L, \qquad\qquad e = \det e^i_\mu$$

$$L = L_F + L_T,$$

$$L_T = \alpha t_{ijk} t^{ijk} + \beta v_i v^i + \gamma a_i a^i$$

Gravitational Action

$$L_F = c_1 F + c_2 + c_3 F_{ij} F^{ij}$$

$$+ c_4 F_{ij} F^{ji} + c_5 F^2$$

$$+ c_6 (\varepsilon_{ijkl} F^{ijkl})^2 + b F_{ijkl} F^{ijkl},$$

Ghost and Tachyon Free Classes

- Tachyon and ghost cancellations in flat backgrounds require restrictions on the parameters.

- In flat space only the $c_2 = 0$ has been studied

K. Hayashi and T. Shirafuji

Prog. Theor. Phys. **64**, 866 (1980)

64, 1435 (1980)

64, 2222 (1980).

Phys. Rev. D **21**, 3269 (1980).

There are 10 parameters

- We shall work with a 6-parameter family by choosing:

$$b = c_3 + c_4 + 3c_5 = 0$$

$$\alpha = -\beta = \frac{4\gamma}{9}$$

Reduced Lagrangian

$$L_T = \alpha(t_{ijk}t^{ijk} - v_i v^i + \frac{9}{4}a_i a^i)\ ,$$

$$L_F = c_1 F + c_2 +$$

$$c_3 F_{ij} F^{ij} + c_4 F_{ij} F^{ji}$$

Field equations

- Gravitational

$$\frac{\delta S}{\delta e^{i}_{m}} = 0$$

- Torsion

$$\frac{\delta S}{\delta A^{i}_{j\mu}} = 0$$

Gravitational Equations

$$c_1 F_{ji} + c_3 (F^m{}_i F_{mj} - F_j{}^{mn}{}_i F_{mn}) +$$

$$(D^k + v^k) F_{ijk} + H_{ij} - \frac{1}{2} \eta_{ij} L = 0$$

where

$$F_{ijk} = \alpha\left[(t_{ijk} - t_{ikj}) - (\eta_{ij}v_k - \eta_{ik}v_j) - \frac{3}{4}\varepsilon_{ijkl}a^l\right]$$

$$H_{ij} = T_{mni}F^{mn}{}_j - \frac{1}{2}T_{jmn}F_i{}^{mn}$$

- We shall not need H

Trace of Grav.Equ.

- If the torsion vanishes

$$F = -\frac{2c_2}{c_1}$$

- First (in torsion) order terms

$$c_1 F_{(1)} = -3\alpha\nabla_i v^i$$

Antisymmetric Grav.Equ.

$$c_1 F_{[ji]} + \frac{c_4}{2}(F^m{}_i F_{jm} - F^m{}_j F_{im}) -$$

$$\frac{1}{2}(F_j{}^{mn}{}_i - F_i{}^{mn}{}_j)(c_3 F_{mn} + c_4 F_{nm})$$

$$+2c_5 F_{[ji]} F +$$

Symmetric Grav.Equ.

Likewise the symmetric part of Gravitational equations can be derived.

1. Curl of the torsion equations

- ε -trace

$$12c_6 D_l(\varepsilon \cdot F) - \frac{2}{3}\varepsilon_{ijkl} t_n{}^{ik}(c_3 F^{jn} + c_4 F^{nj}) -$$

$$8c_6 v_l(\varepsilon \cdot F) \; - \frac{2}{3}(c_3 - c_4)\varepsilon_{ijkl} v^i F^{jk}$$

$$- 2(c_3 F_{jl} + c_4 F_{lj}) a^j + \frac{2}{9}\tilde{\alpha} a_l = 0$$

2- Trace of the torsion Equation

$$-3c_5\left(D_jF^{(ij)}-\frac{1}{2}D^iF\right)+(c_3-c_4)D_jF^{[ij]}$$

$$-2c_5\left(V_jF^{(ij)}-\frac{1}{2}V^iF\right)+(c_3-c_4)D_jF^{[ij]}$$

$$+\frac{2}{3}(c_3-c_4)V_iF^{[ij]}-3c_5t^{i(jn)}F_{(jn)}$$

$$+\frac{1}{3}(c_3-c_4)t^{i[jn]}F_{[jn]}$$

$$-\frac{1}{2}(c_3-c_4)\varepsilon^{ijnl}a_lF_{[jn]}+6c_6a^i(\varepsilon\cdot F)+\frac{3}{2}\tilde{\alpha}v^i=0$$

Vanishing Torsion Backgrounds

- Grav.Equ. + Torsion Equ.

$$c_1 R_{ij} = \frac{\lambda}{2}\eta_{ij} - 3c_5 W_{iklj} R^{kl}$$

$$\nabla^i W_{ijkl} = 0$$

Einstein Manifolds

$$R_{ijkl} = \Lambda(\eta_{ik}\eta_{jl} - \eta_{il}\eta_{jk}) + W_{ijkl},$$

$$R_{ij} = 3\Lambda\eta_{ij}, \qquad R = 12\Lambda,$$

$$\Lambda = -\frac{c_2}{6c_1}$$

Linearised Equ. in Einstein Background

- Antisymmetric Einstein

$$(c_1 - 4\Lambda c_3) F_{(1)[ji]} - \nabla^k F_{(1)[ji]k} -$$

$$(c_3 - c_4) W_j{}^{[mn]}{}_i F_{(1)[mn]} = 0$$

In terms of torsion components

$$
\begin{aligned}
F_{(1)[ji]} &= -\frac{2}{3\alpha}\nabla^k F_{[ji]k} \\
&= \frac{2}{3}(\nabla^k t_{k[ji]} - \nabla_{[j} v_{i]} + \frac{3}{4}\varepsilon_{jikl}\nabla^k a^l)
\end{aligned}
$$

$$
\nabla^k t_{k[ij]} - \nabla_{[i} v_{j]} = -\frac{3}{4}\varepsilon_{ijkl}\nabla^k a^l
$$

Important result

$$F_{(1)[ij]} = 0 = \nabla^k F_{[ij]k}$$

Axial vector a_l

$$8c_6\nabla_l(\nabla\cdot a)-(2\Lambda c_5+\frac{\tilde{\alpha}}{2})a_l=0$$

$$\boxed{\sigma = \nabla^i a_i}$$

$$\left(\nabla^2 - \frac{2\Lambda c_5 + \frac{\tilde{\alpha}}{2}}{8c_6}\right)\sigma = 0$$

Constraints

$$\nabla^k t_{k[mn]} = \nabla_{[m} v_{n]}$$

V is determined algebraically in terms of t

$$v_i = \frac{4c_5}{3\tilde{\alpha}} W_{ijkl} t^{j[kl]}$$

Constraint

Spin-2 Field

$$\chi_{ij} = \nabla^k t_{k(ij)}$$

- This is traceless and transverse if v=0

Equation for t_{ijk}

$$\nabla_i F_{(1)jk} - \nabla_j F_{(1)ik} + \frac{1}{6}(\eta_{ik}\nabla_j F_{(1)} - \eta_{jk}\nabla_i F_{(1)})$$
$$-\frac{1}{3}(2\Lambda + \frac{\tilde{\alpha}}{2c_5})\{(\eta_{ik}v_j - \eta_{jk}v_i) + 4t_{k[ij]}\} = 0$$

$$(\nabla^2 - 4\Lambda)F_{(1)jk} - W_{ijkl}F^{il}_{(1)} - \frac{1}{3}\left[\nabla_j\nabla_k + \frac{1}{2}\eta_{jk}(\nabla^2 - 6\Lambda)\right]F_{(1)}$$
$$-\frac{1}{3}(2\Lambda + \frac{\tilde{\alpha}}{2c_5})\{2(\nabla_k v_j + \nabla_j v_k) - \eta_{jk}\nabla\cdot v + 6\nabla^i t_{i(jk)}\} = 0$$

Both the trace and the divergence of this equation is zero.
There are thus five independent equations.

Symmetric Einstein

$$c_1(F_{(1)ij} - \frac{1}{2}\eta_{ij}F_{(1)}) + \nabla^k F_{(1)ijk} + 3c_5 W_{jmni}F^{mn}_{(1)} = 0$$

$$F_{(1)ij} \quad = \quad R_{(1)ij} - 2\nabla^k t_{k(ij)} + \frac{1}{3}(\nabla_i v_j + \nabla_j v_i) + \frac{1}{3}\eta_{ij}\nabla \cdot v$$

$$\nabla^k F_{(1)ijk} \quad = \quad -\alpha(3\nabla^k t_{k(ij)} - \frac{1}{2}(\nabla_i v_j + \nabla_j v_i) + \eta_{ij}\nabla \cdot v)$$

$$c_1 R_{(1)ij} = -\frac{\tilde{\alpha}}{2}\eta_{ij}\nabla \cdot v + 3\tilde{\alpha}\nabla^k t_{k(ij)} - \frac{\tilde{\alpha}}{2}(\nabla_i v_j + \nabla_j v_i) - 3c_5 W_{imnj} F^{mn}_{(1)}$$

Symmetric Spaces

$$W_{ijkl} = 0$$

$$v_i = 0$$

$$F_{(1)ij} = R_{(1)ij} - 2\nabla^k t_{k(ij)}$$

Einstein

$$F_{(1)ij} = \frac{3\alpha}{c_1}\chi_{ij}$$

$$c_1 R_{(1)ij} = 3\tilde{\alpha}\chi_{ij}$$

Massive Spin-2

$$\chi_{ij} = \nabla^k t_{k(ij)}$$

$$\eta^{ij}\chi_{ij} = 0, \qquad \nabla^i \chi_{ij} = 0$$

KG Equation

$$(\nabla^2 - M^2)\chi_{ij} = 0,$$

$$M^2 = 4\Lambda\left(1 + \frac{c}{3\alpha}\right) + \frac{\tilde{\alpha}c_1}{3\alpha c_5}$$

All components of t_{ijk}

$$t_{k[ij]} = \frac{9\alpha}{4c_1(2\Lambda + \frac{\tilde{\alpha}}{2})}(\nabla_i \chi_{jk} - \nabla_j \chi_{ik})$$

Cosmological Constant and Masses

$$\Lambda = -\frac{1}{6}\frac{c_2}{M_p{}^2}$$

$$\boxed{M^2{}_0 = \frac{1}{16c_6}(4\Lambda c_5 + \tilde{\alpha})}$$

$$3\tilde{\alpha} = 3\alpha + 2M_p{}^2$$

$$\boxed{M^2{}_2 = 4\Lambda(1 + \frac{M_p{}^2}{3\alpha}) + \frac{\tilde{\alpha}}{3\alpha c_5} M^2{}_p}$$

Unitarity Bound(1)

- Combining the above we can rewrite the mass of the spin-2 particle as,

$$M_2{}^2 = 4\Lambda + \frac{16c_6}{3\alpha c_5} M_0{}^2 M^2{}_p$$

Absence of Ghosts and Tachyons in flat backgrounds

$$c_5 < 0, c_6 > 0, \alpha < 0,$$

$$\tilde{\alpha} = \alpha + \frac{2}{3} c_1 > 0$$

Unitarity Bound (2)

- This shows that if the spin zero particle is non tachyonic the mass of the spin 2 particle is bounded from below by

$$M_2^{\ 2} \succ 4\Lambda$$

- This bound is identical to the one discovered in

- A. Higuchi, NPB 325(1989)745

Relation to Fierz Pauli

- Note that our spin-2 field is not a metric tensor.
- On the other hand FP in flat space is a unique consistent massive spin-2 theory.
- Is there a FP equation in our scheme?

Yes

- Our KG equation for the

$$\chi_{ij} = \nabla^k t_{k(ij)}$$

can be derived from a FP equation written in the curved background, but not for the metric!

- It reduces to the standard FP in flat background.

Conclusions(1)

- We have obtained a propagating spin-2 field with no pathologies (so far!) in curved backgrounds.

- The propagating d.o.f are a massless graviton, a massive spin-2 and a massive spin 0 particle.

- The inclusion of non linearities follow from the starting action. They will be coordinate invariant automatically.

Conclusions(2)

- More general models with no tuning must be studied.
- Solve the equations for t in arbitrary Einstein backgrounds and also other backgrounds.
- Non linear effects should be examined.