Holographic Models of Cosmological Singularities

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Vrije Universiteit Brussel & The International Solvay Institutes

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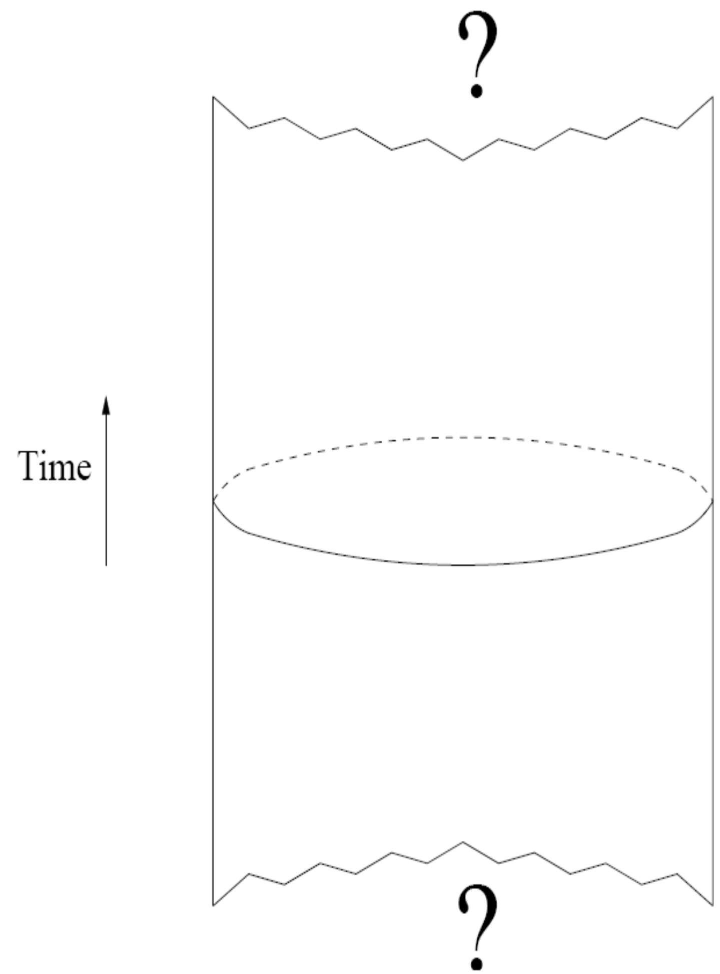
Plan

- AdS cosmology: review of basic idea
- ABJM theory and an unstable triple trace deformation
- Beyond the singularity? Self-adjoint extensions
- Summary and outlook

AdS cosmology: basic idea

Starting point: supergravity solutions in which smooth, asymptotically AdS initial data evolve to a big crunch singularity in the future.

Can a dual gauge theory be used to study this process in quantum gravity?



AdS cosmologies: basic idea

• AdS: boundary conditions required

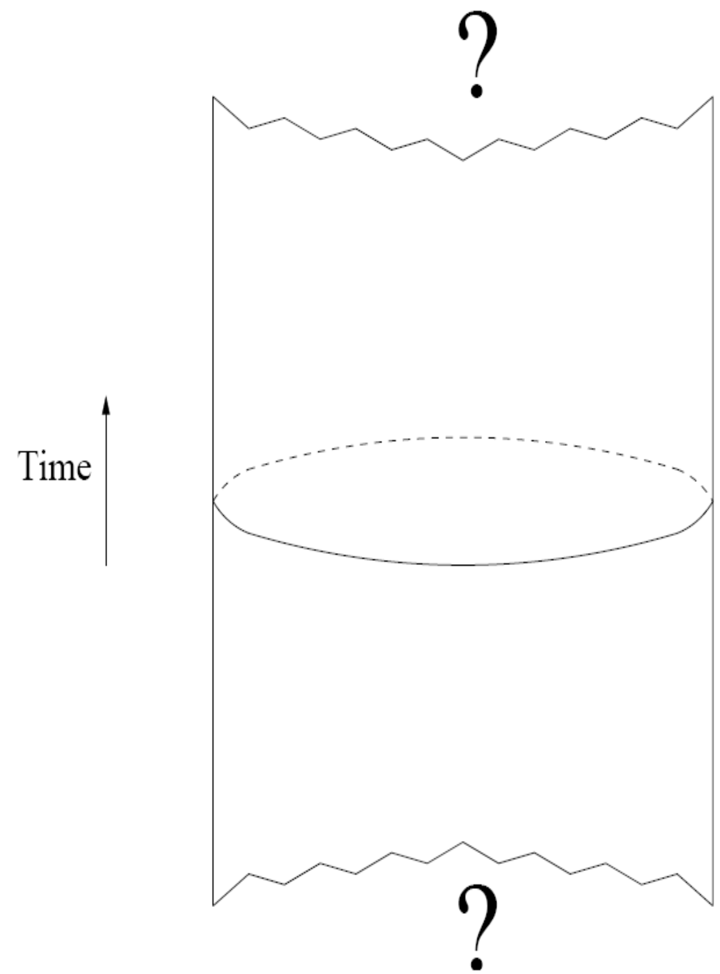
• Usual supersymmetric boundary conditions: stable

• Modified boundary conditions: smooth initial data that evolve into big crunch (which extends to the boundary of AdS in finite time)

• AdS/CFT relates quantum gravity in AdS to field theory on its conformal boundary

• Modified boundary conditions → potential unbounded below in boundary field theory; scalar field reaches infinity in finite time

• Goal: learn something about cosmological singularities (in the bulk theory) by studying unbounded potentials (in the boundary theory)



AdS cosmology: the bulk theory

Compactify 11d sugra on S^7 and truncate (consistently) to

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}(\nabla\varphi)^2 + \frac{1}{R_{AdS}^2\left(2+\cosh(\sqrt{2}\,\varphi)\right)}\right]$$

de Wit, Nicolai
Duff, Liu

This describes a scalar whose mass squared is negative but above the BF bound.

In all solutions asymptotic to the AdS_4 metric

$$ds^2 = R_{AdS}^2\left(-(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega_2\right)$$

the scalar field decays at large radius as

$$\varphi(r) \sim \frac{\alpha(t,\Omega)}{r} + \frac{\beta(t,\Omega)}{r^2}$$

Consider AdS invariant boundary conditions

$$\beta = -h\alpha^2$$

Hertog, Maeda

AdS cosmology: bulk solution

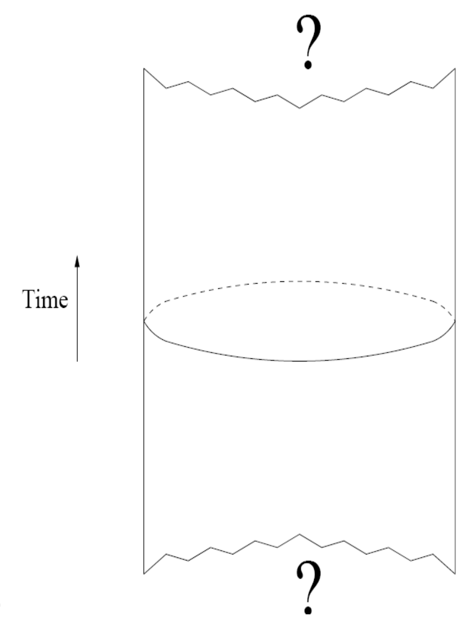
$$\varphi(r) \sim \frac{\alpha}{r} + \frac{\beta}{r^2} \qquad\qquad \beta = -h\alpha^2$$

Standard supersymmetric boundary conditions: $h = 0$

For $h \neq 0$, there exist smooth asymptotically AdS initial data that evolve to a singularity that reaches the boundary of AdS in finite global time.

Example: analytic continuation of Euclidean instanton

$$ds^2 = \frac{d\rho^2}{b^2(\rho)} + \rho^2 d\Omega_3 \qquad \text{with} \qquad \varphi(\rho) \sim \frac{\alpha}{\rho} + \frac{\beta}{\rho^2}$$



leads to Lorentzian cosmology:

- inside the lightcone (corresponding to the origin of the Euclidean instanton): open FRW universe with scale factor that vanishes at some finite time $t = \pi/2$.

- outside the lightcone: asymptotic behavior

$$\varphi \sim \frac{\alpha(t)}{r} - \frac{h\alpha^2(t)}{r^2} \qquad \text{with} \qquad \alpha(t) = \frac{\alpha(0)}{\cos t}$$

Hertog, Horowitz

AdS cosmology: dual field theory

$$\varphi(r) \sim \frac{\alpha}{r} + \frac{\beta}{r^2} \qquad \beta = -h\alpha^2$$

M-theory on AdS_4 x S^7 $\xleftrightarrow{\text{dual}}$ M2-brane CFT on $R \times S^2$

Maldacena

• With usual boundary conditions $\beta = 0$, the scalar field ϕ is dual to a dimension 1 operator

$$\mathcal{O} = \frac{1}{N} \mathrm{Tr}\, T_{ij} \phi^i \phi^j$$

Aharony, Oz, Yin

The expectation value of $\mathcal{O}$ is determined by the asymptotic behavior of ϕ: $\alpha \leftrightarrow \langle \mathcal{O} \rangle$

• Boundary conditions with $h \neq 0$ correspond to deforming the CFT by a triple trace operator:

$$S \to S + \frac{h}{3} \int \mathcal{O}^3$$

Witten; Berkooz, Sever, Shomer; Hertog, Maeda

This corresponds to a potential that is unbounded from below, and $\langle \mathcal{O} \rangle$ becomes infinite in finite time:

$$\langle \mathcal{O} \rangle = \alpha(t) = \frac{\alpha(0)}{\cos t}$$

Hertog, Horowitz

AdS cosmology: toy model for the boundary theory

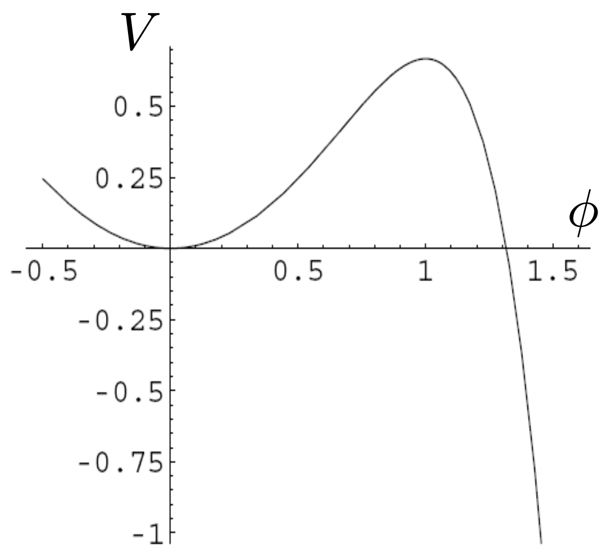
Ignore the non-abelian structure in $\mathcal{O} = \frac{1}{N} \mathrm{Tr}\, T_{ij} \phi^i \phi^j$ and replace $\mathcal{O}$ by the square of a single scalar field:

$$\mathcal{O} \to \phi^2$$

We find a scalar field theory with standard kinetic term and potential

$$V = \frac{1}{8}\, \phi^2 - \frac{h}{3}\, \phi^6$$

The quadratic term corresponds to the conformal coupling to the curvature of the S^2.



AdS cosmology: what happens when the field reaches infinity?

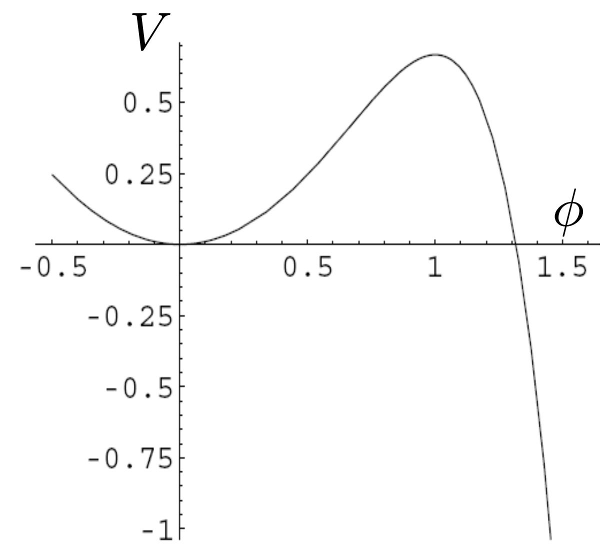
- Classical solution: $\phi = \dfrac{(3/8h)^{1/4}}{\cos^{1/2} t}$

  Field reaches infinity at finite time $t = \pi/2$

- Semiclassically: field tunnels out of metastable minimum and reaches infinity at finite time.

- Quantum mechanics of the homogeneous mode: theory of quantum mechanics with unbounded potentials.

  Self-adjoint extensions of Hamiltonian: field bounces back from infinity.



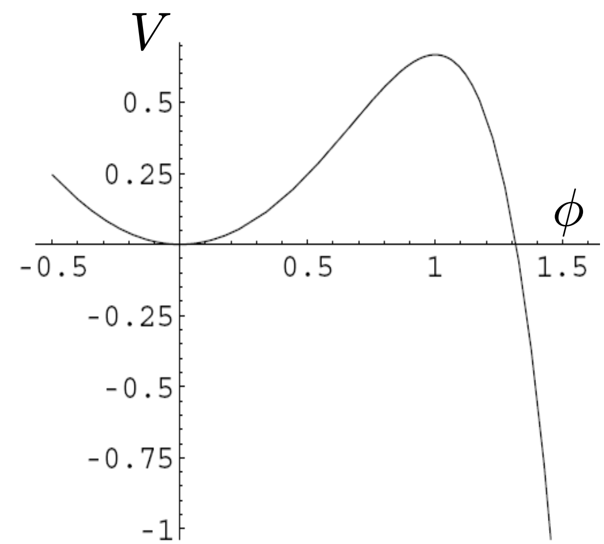
$$V = \frac{1}{8}\,\phi^2 - \frac{h}{3}\,\phi^6$$

- Quantum field theory with unbounded potentials: not much known. Particle creation may be important.

- Regularization by adding irrelevant operator $\dfrac{\phi^8}{M}$ to potential: big crunch replaced by large black hole. Thermalization?

Hertog, Horowitz; Elitzur, Giveon, Porrati, Rabinovici; Banks, Fischler

AdS cosmology: questions



- Can we perform computations in M2-brane theory?
- How can we interpret the unstable potential?
  - → Brane nucleation — Bernamonti, BC
- Do self-adjoint extensions make sense in field theory?
- If so, how does a wavepacket evolve after it reaches infinity?
- If so, what is the bulk interpretation?

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ABJM theory: action

$\mathcal{N}=6$ superconformal U(N) x U(N) Chern-Simons-matter theory with levels k and -k

- Gauge fields A_μ and $\hat{A}_\mu$
- Scalar fields $Y^A,\ A=1,\ldots 4$ in fundamental of $SU(4)_R$ and in $(N,\bar{N})$ of gauge group

$$S=\int d^3x\left[\frac{k}{4\pi}\epsilon^{\mu\nu\lambda}\mathrm{Tr}(A_\mu\partial_\nu A_\lambda+\frac{2i}{3}A_\mu A_\nu A_\lambda-\hat{A}_\mu\partial_\nu\hat{A}_\lambda-\frac{2i}{3}\hat{A}_\mu\hat{A}_\nu\hat{A}_\lambda)\right.$$

$$\left.\mathrm{Tr}(D_\mu Y^A)^\dagger D^\mu Y^A+V_{\rm bos}+\text{ terms with fermions}\right]$$

$$V_{\rm bos}=-\frac{4\pi^2}{3k^2}\mathrm{Tr}\left[Y^AY^\dagger_AY^BY^\dagger_BY^CY^\dagger_C+Y^\dagger_AY^AY^\dagger_BY^BY^\dagger_CY^C\right.$$

$$\left.+4Y^AY^\dagger_BY^CY^\dagger_AY^BY^\dagger_C-6Y^AY^\dagger_BY^BY^\dagger_AY^CY^\dagger_C\right]$$

Aharony, Bergman, Jafferis, Maldacena

ABJM theory: brane interpretation and gravity dual

ABJM theory is worldvolume action of N coincident M2-branes on $\mathbb{Z}_k$ orbifold of $\mathbb{C}^4$

$$\mathbb{Z}_k: \quad y^A \to \exp(2\pi i/k) y^A$$

Coupling constant of ABJM theory is 1/k → "'t Hooft" limit: large N with N/k fixed.

Gravity dual: $\mathbb{Z}_k$ orbifold of $AdS_4 \times S^7$:

$$ds^2 = \frac{R^2}{4} ds^2_{AdS_4} + R^2 ds^2_{S^7}$$

$$F_4 \sim N' \epsilon_4 \qquad (N' = kN)$$

$$\frac{R}{l_p} = (32\pi^2 N')^{1/6}$$

Can write $ds^2_{S^7} = (d\chi + \omega)^2 + ds^2_{\mathbb{C}P^3}$

Orbifold identification makes χ periodicity $\frac{2\pi}{k}$. In 't Hooft limit: weakly coupled IIA string theory.

Aharony, Bergman, Jafferis, Maldacena

A triple trace deformation of ABJM theory

Scalar field φ of consistent truncation of sugra survives $\mathbb{Z}_k$ quotient

→ Bulk analysis extends to k>1. Will study 't Hooft limit (large N with N/k fixed).

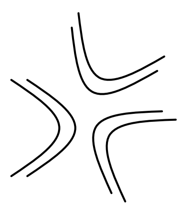
Dimension 1 chiral primary operator with same symmetry properties as φ under $SU(4)_R$:

$$\mathcal{O} = \frac{1}{N^2}\mathrm{Tr}(Y^1 Y_1^\dagger - Y^2 Y_2^\dagger)$$

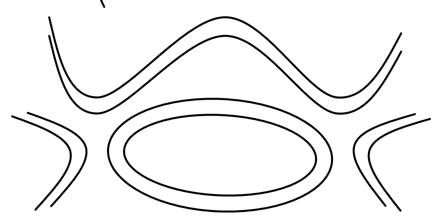
Triple trace deformation: $$V = -\frac{f}{N^4}\left[\mathrm{Tr}(Y^1 Y_1^\dagger - Y^2 Y_2^\dagger)\right]^3$$

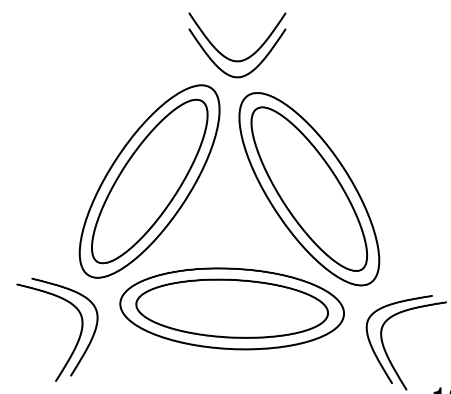
Quantum corrections: is effective potential truly unbounded below? Elitzur, Giveon, Porrati, Rabinovici
→ Sensitive to UV behavior! (Does one need to turn on irrelevant operators?)

Vertex in double line notation:



Will find beta function at order $1/N^2$

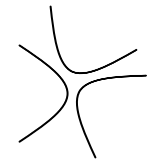




BC, Hertog, Turok

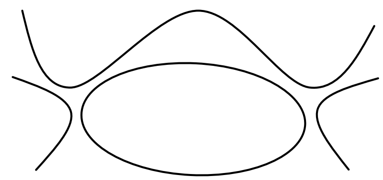
Warm-up: O(N) vector model

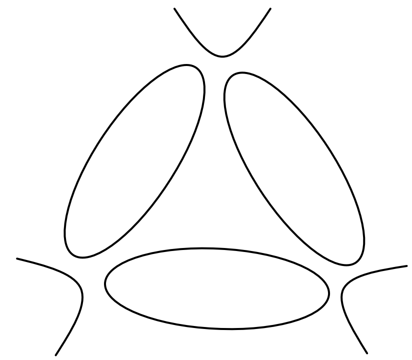
$$S = \int d^3x \left(-\partial_\mu \vec{\phi} \cdot \partial^\mu \vec{\phi} - \frac{\lambda}{N^2} \left(\vec{\phi} \cdot \vec{\phi} \right)^3 \right)$$



Perturbative beta function up to order 1/N:

$$\beta(\lambda) = \frac{9\lambda^2}{\pi^2 N} - \frac{9\lambda^3}{32\pi^2 N}$$





Stephen, McCauley; Stephen; Lewis, Adams; Pisarski

Positive coupling $(\lambda > 0)$:

- Perturbative UV fixed point: $\lambda^* = 32$
- Non-perturbatively: UV fixed point at $\lambda_c = \frac{8\pi^2}{3} < \lambda^*$ (for $N = \infty$)

Bardeen, Moshe, Bander

"instability" for $\lambda > \lambda_c$ (masses of order the cutoff)

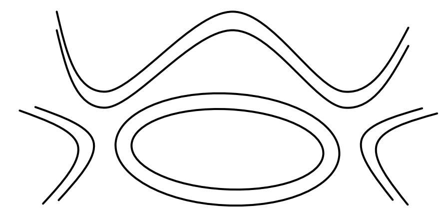
Negative coupling $(\lambda < 0)$:

- UV fixed point at $\lambda = 0$ → asymptotic freedom, effective potential truly unbounded below

Coleman, Gross

Renormalization of triple trace deformation of ABJM theory: simplified

Consider simplified potential $V = -\frac{f}{N^4}\left[\mathrm{Tr}(YY^\dagger)\right]^3$ with $f > 0$



Beta function $\beta(-f) = \frac{9f^2}{4\pi^2 N^2} + \ldots$

Callan-Symanzik: $\mu \frac{df}{d\mu} = -\frac{9f^2}{4\pi^2 N^2}$

Solution: $f_\mu = \frac{8\pi^2 N^2}{9\ln(\mu^2/M^2)}$

Coleman-Weinberg potential: $V(Y) = -\frac{8\pi^2}{9N^2 \ln[\mathrm{Tr}(YY^\dagger)/M^2]}\left[\mathrm{Tr}(YY^\dagger)\right]^3$

→ Reliable for large $\mathrm{Tr}(YY^\dagger)$

Question: is this also true for $V = -\frac{f}{N^4}\left[\mathrm{Tr}(Y^1 Y_1^\dagger - Y^2 Y_2^\dagger)\right]^3$?

Warm-up: O(N) x O(N) vector model

$$S = \int d^3x \left[-\partial_\mu \vec{\phi}_1 \cdot \partial^\mu \vec{\phi}_1 - \partial_\mu \vec{\phi}_2 \cdot \partial^\mu \vec{\phi}_2 - \frac{\lambda_{111}}{N^2} \left(\vec{\phi}_1 \cdot \vec{\phi}_1 \right)^3 - \frac{\lambda_{222}}{N^2} \left(\vec{\phi}_2 \cdot \vec{\phi}_2 \right)^3 \right.$$
$$\left. - \frac{\lambda_{112}}{N^2} \left(\vec{\phi}_1 \cdot \vec{\phi}_1 \right)^2 \left(\vec{\phi}_2 \cdot \vec{\phi}_2 \right) - \frac{\lambda_{122}}{N^2} \left(\vec{\phi}_1 \cdot \vec{\phi}_1 \right) \left(\vec{\phi}_2 \cdot \vec{\phi}_2 \right)^2 \right]$$

Rabinovici, Saering, Bardeen

Warm-up: O(N) x O(N) vector model

$$S = \int d^3x \left[-\partial_\mu \vec{\phi}_1 \cdot \partial^\mu \vec{\phi}_1 - \partial_\mu \vec{\phi}_2 \cdot \partial^\mu \vec{\phi}_2 - \frac{\lambda_{111}}{N^2} \left(\vec{\phi}_1 \cdot \vec{\phi}_1 \right)^3 - \frac{\lambda_{222}}{N^2} \left(\vec{\phi}_2 \cdot \vec{\phi}_2 \right)^3 \right.$$
$$\left. - \frac{\lambda_{112}}{N^2} \left(\vec{\phi}_1 \cdot \vec{\phi}_1 \right)^2 \left(\vec{\phi}_2 \cdot \vec{\phi}_2 \right) - \frac{\lambda_{122}}{N^2} \left(\vec{\phi}_1 \cdot \vec{\phi}_1 \right) \left(\vec{\phi}_2 \cdot \vec{\phi}_2 \right)^2 \right]$$

Rabinovici, Saering, Bardeen

$$\begin{aligned}
\beta_{111} &= \frac{9}{\pi^2 N} \left(\lambda_{111}^2 + \frac{1}{9} \lambda_{112}^2 \right) - \frac{9}{32\pi^2 N} \left(\lambda_{111}^3 + \frac{1}{3} \lambda_{111} \lambda_{112}^2 + \frac{1}{9} \lambda_{112}^2 \lambda_{122} + \frac{1}{27} \lambda_{122}^3 \right); \\
\beta_{112} &= \frac{9}{\pi^2 N} \left(\frac{1}{9} \lambda_{112}^2 + \frac{1}{9} \lambda_{122}^2 + \frac{2}{3} \lambda_{111} \lambda_{112} + \frac{2}{9} \lambda_{112} \lambda_{122} \right) \\
&\quad - \frac{9}{32\pi^2 N} \left(\lambda_{111}^2 \lambda_{112} + \frac{2}{3} \lambda_{111} \lambda_{112} \lambda_{122} + \frac{1}{9} \lambda_{112}^3 + \frac{1}{3} \lambda_{112}^2 \lambda_{222} + \frac{2}{9} \lambda_{112} \lambda_{122}^2 + \frac{1}{3} \lambda_{122}^2 \lambda_{222} \right); \\
\beta_{122} &= \frac{9}{\pi^2 N} \left(\frac{1}{9} \lambda_{122}^2 + \frac{1}{9} \lambda_{112}^2 + \frac{2}{3} \lambda_{222} \lambda_{122} + \frac{2}{9} \lambda_{122} \lambda_{112} \right) \\
&\quad - \frac{9}{32\pi^2 N} \left(\lambda_{222}^2 \lambda_{122} + \frac{2}{3} \lambda_{222} \lambda_{122} \lambda_{112} + \frac{1}{9} \lambda_{122}^3 + \frac{1}{3} \lambda_{122}^2 \lambda_{111} + \frac{2}{9} \lambda_{122} \lambda_{112}^2 + \frac{1}{3} \lambda_{112}^2 \lambda_{111} \right); \\
\beta_{222} &= \frac{9}{\pi^2 N} \left(\lambda_{222}^2 + \frac{1}{9} \lambda_{122}^2 \right) - \frac{9}{32\pi^2 N} \left(\lambda_{222}^3 + \frac{1}{3} \lambda_{222} \lambda_{122}^2 + \frac{1}{9} \lambda_{122}^2 \lambda_{112} + \frac{1}{27} \lambda_{112}^3 \right).
\end{aligned}$$

BC, Hertog, Turok

Warm-up: O(N) x O(N) vector model

$$S = \int d^3x \left[-\partial_\mu \vec{\phi}_1 \cdot \partial^\mu \vec{\phi}_1 - \partial_\mu \vec{\phi}_2 \cdot \partial^\mu \vec{\phi}_2 - \frac{\lambda_{111}}{N^2} \left(\vec{\phi}_1 \cdot \vec{\phi}_1 \right)^3 - \frac{\lambda_{222}}{N^2} \left(\vec{\phi}_2 \cdot \vec{\phi}_2 \right)^3 \right.$$
$$\left. - \frac{\lambda_{112}}{N^2} \left(\vec{\phi}_1 \cdot \vec{\phi}_1 \right)^2 \left(\vec{\phi}_2 \cdot \vec{\phi}_2 \right) - \frac{\lambda_{122}}{N^2} \left(\vec{\phi}_1 \cdot \vec{\phi}_1 \right) \left(\vec{\phi}_2 \cdot \vec{\phi}_2 \right)^2 \right]$$

Perturbative fixed points:

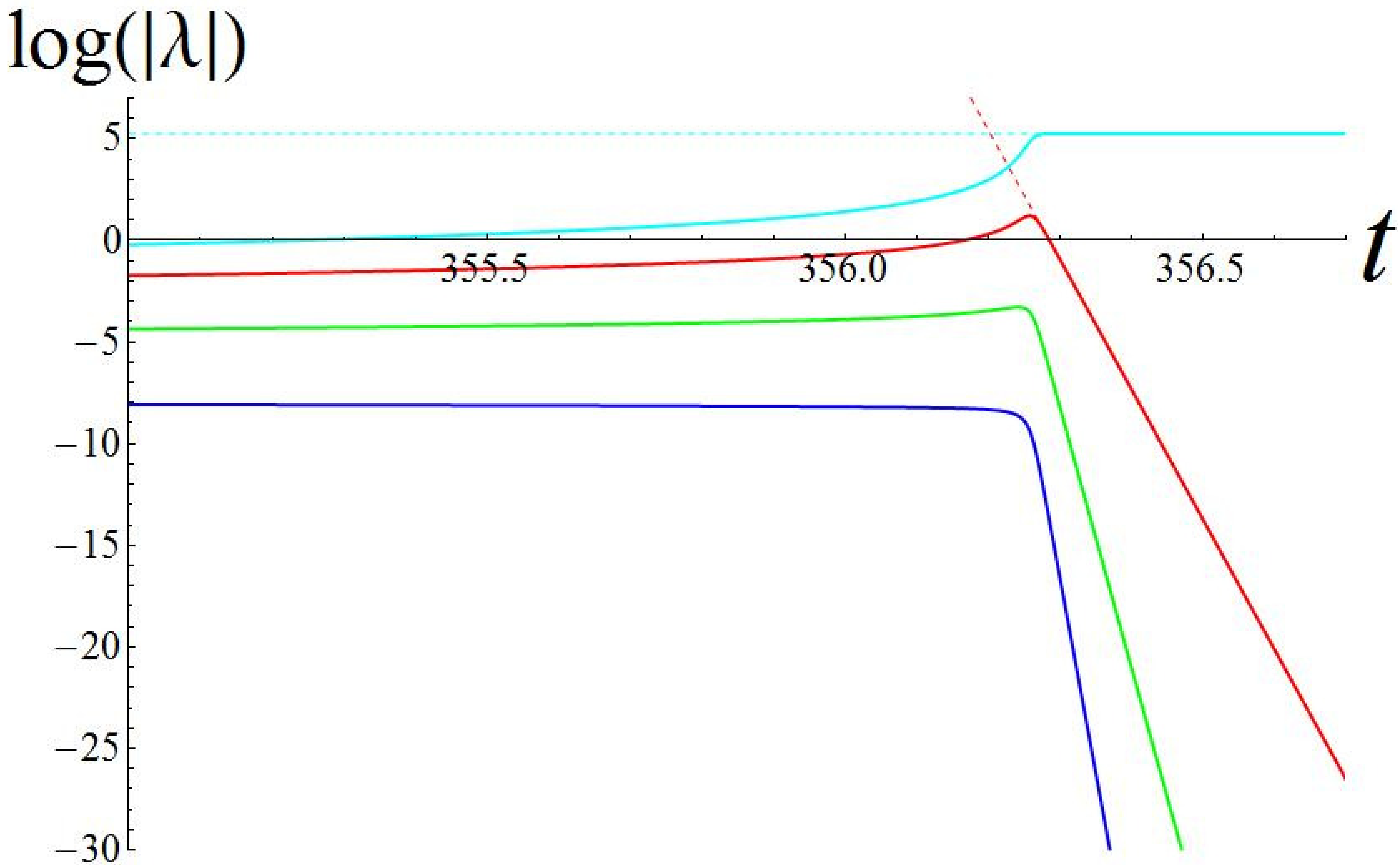
- $\lambda_{112} = \lambda_{122} = 3\lambda_{111} = 3\lambda_{222} = 3\lambda^*$
- $\lambda_{222} = \lambda^*, \quad \lambda_{112} = \lambda_{122} = \lambda_{111} = 0$

Starting from $V = \frac{\lambda}{N^2} \left(\vec{\phi}_1 \cdot \vec{\phi}_1 - \vec{\phi}_2 \cdot \vec{\phi}_2 \right)^3$ at some scale M_0 and running towards the UV, do we end up at one of these fixed points?

→ Use beta functions to compute couplings as function of $t \equiv \ln(M/M_0)$

BC, Hertog, Turok

Perturbative UV fixed point in O(N) x O(N) vector model



Starting from $V = \frac{\lambda}{N^2}\left(\vec{\phi}_1 \cdot \vec{\phi}_1 - \vec{\phi}_2 \cdot \vec{\phi}_2\right)^3$, in the UV one reaches the fixed point

$\lambda_{222} = \lambda^*, \quad \lambda_{112} = \lambda_{122} = \lambda_{111} = 0$

BC, Hertog, Turok

Non-perturbative effects in O(N) x O(N) vector model and deformed ABJM

- Perturbative analysis suggests that theory can be defined without UV cutoff

  → no cutoff-suppressed irrrelevant operators that could stabilize the potential

- Perturbative analysis for deformed ABJM theory: similar but not completely carried out

- Non-perturbatively: regions of stability/instability identified for O(N) x O(N) vector model at $N = \infty$ Rabinovici, Saering, Bardeen

- Non-perturbative analysis not yet carried out for deformed ABJM theory; probably not very important for our purposes (in progress…) BC, Hertog, Turok

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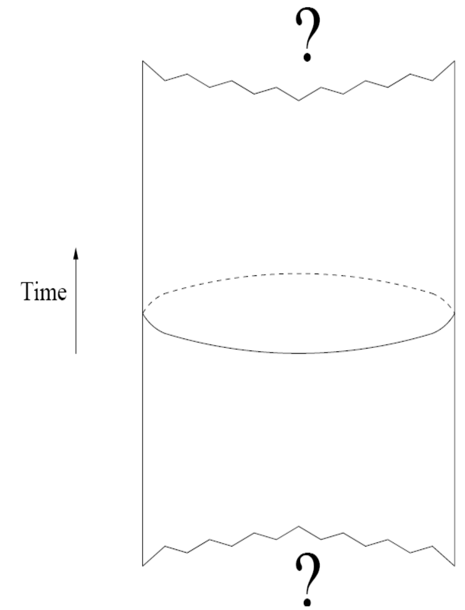
The homogeneous mode is a quantum mechanical variable

Boundary field theory lives on $R \times S^2$

time

finite volume space



Decompose $\phi(t,\mathbf{x}) = \bar{\phi}(t) + \delta\phi(t,\mathbf{x})$

First ignore inhomogeneous modes $\delta\phi(t,\mathbf{x})$,
which start out in ground state.

Kinetic term for homogeneous mode: $V_3 \int dt\, \frac{1}{2}\dot{\bar{\phi}}^2$

finite "mass"

Wave function will undergo quantum spreading. This will give rise to UV cutoff on creation of inhomogeneous modes.

BC, Hertog, Turok

Quantum mechanics with unbounded potentials

$$\hat{H} = -\frac{d^2}{dx^2} + V(x) \quad \text{with} \quad V(x) = -x^4 \quad \text{for} \quad x > 0 \;\text{and}\; \quad V(x) = 0 \quad \text{for} \quad x < 0 \;.$$

For such potentials, classical trajectories can reach infinity in finite time. So do quantum mechanical wavepackets, which would seem to lead to loss of probability/unitarity.

Unitarity can be restored by restricting the domain of allowed wavefunctions such that the Hamiltonian is self-adjoint (“self-adjoint extension”):

$$(\hat{H}\phi_1, \phi_2) = (\phi_1, \hat{H}\phi_2) \qquad \leftrightarrow \qquad \left[\frac{d\phi_1^*}{dx}\phi_2 - \phi_1^*\frac{d\phi_2}{dx}\right]_{x=\infty} = 0$$

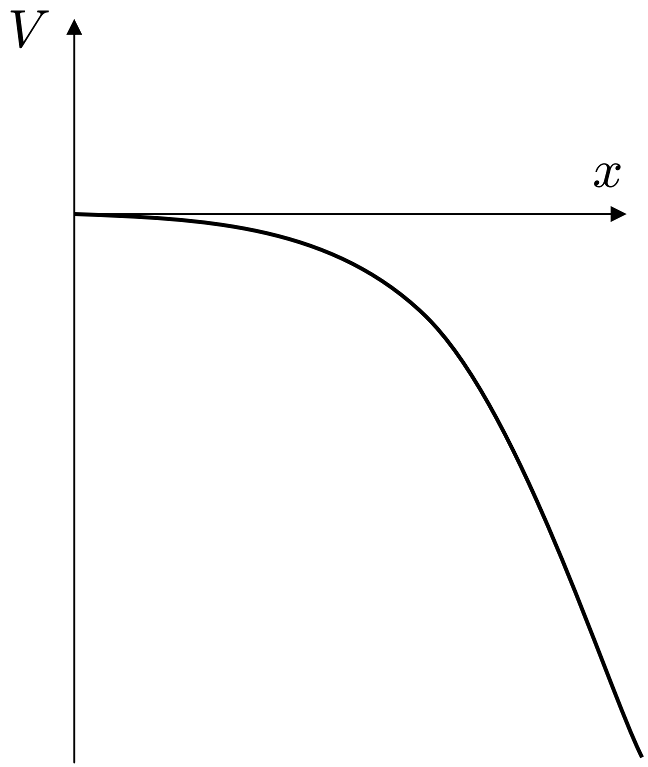
The WKB energy eigenfunctions $\quad [2(E+x^4)]^{-1/4} \exp\left(\pm i \int_0^x \sqrt{2(E+y^4)}dy\right)$

are an increasingly good approximation for large x. Unitarity can be achieved by only allowing the linear combination that for large x behaves as

$$\psi_E^{\alpha}(x) \sim \frac{1}{x}\cos\left(\frac{\sqrt{2}x^3}{3} + \underset{\substack{\uparrow \\ \text{arbitrary phase}}}{\alpha}\right)$$

Reed, Simon

Interpretation of the self-adjoint extensions

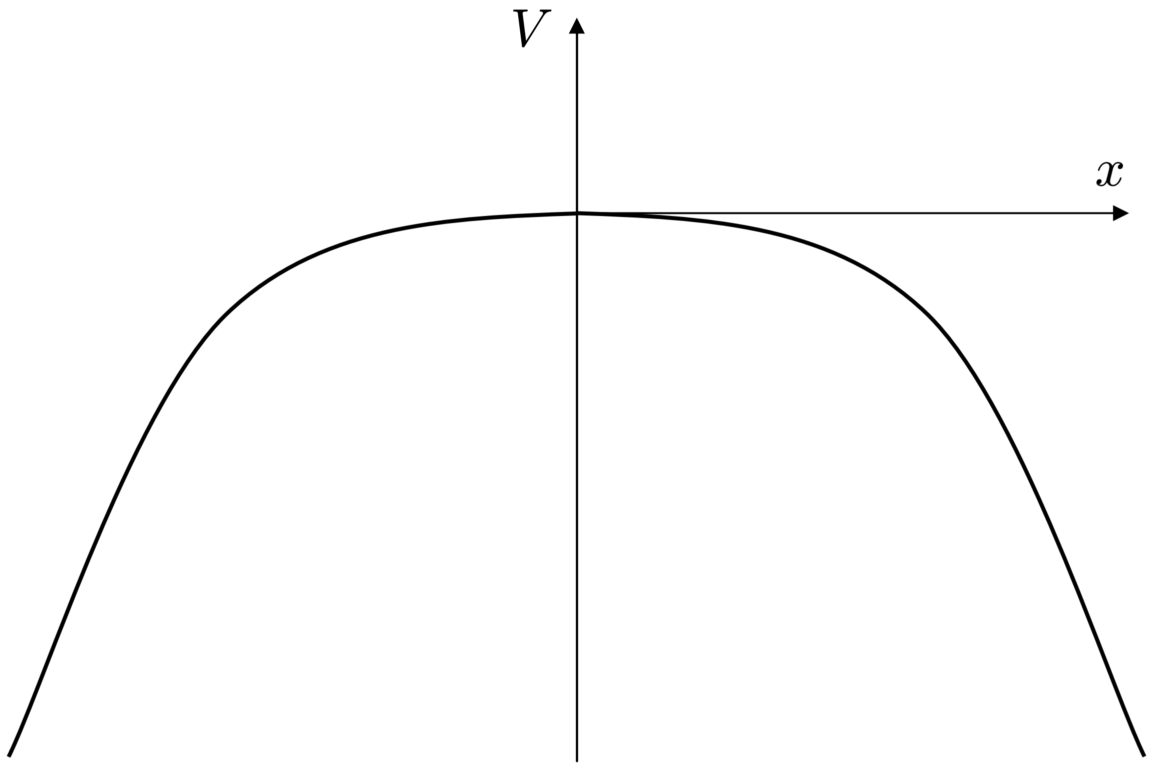


Rightmoving wavepacket disappearing at infinity is always accompanied by leftmoving wavepacket appearing at infinity (think of brick wall at infinity)

Carreau, Farhi, Gutmann, Mende

Energy spectrum consists of bound states (energy levels depend on phase α and are unbounded from below) as well as scattering states (if potential is bounded from above)

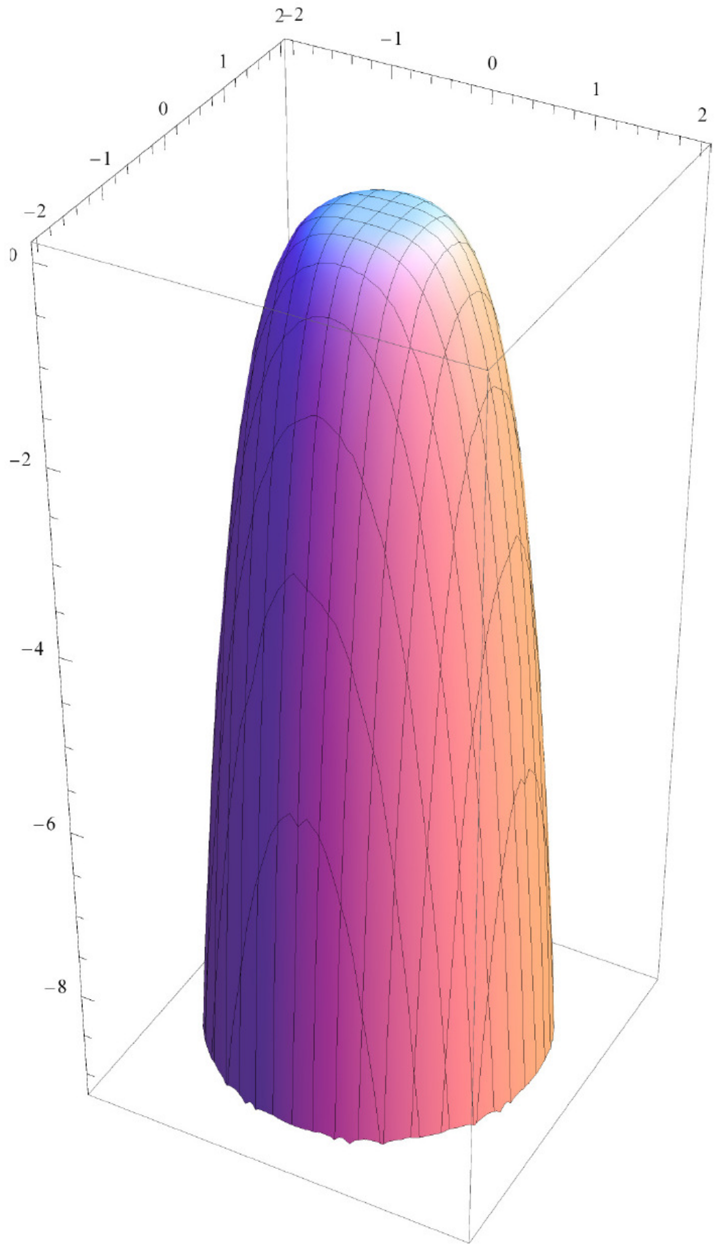
Self-adjoint extensions for potential unbounded on two sides



Self-adjoint extension has 4 parameters

Reed, Simon

Self-adjoint extensions in 2d quantum mechanics



$$H = -\frac{d^2}{dx^2} - \frac{d^2}{dy^2} - (x^2 + y^2)^2$$

- SA extensions labeled by arbitrary function $g(\theta, \theta')$ subject to $g(\theta, \theta') = g^*(\theta', \theta)$

  (infinite number of parameters)

- If rotational invariance imposed: $g(\theta - \theta')$

- If local probability conservation is imposed:
  $\alpha\delta(\theta - \theta')$

  one parameter

Carreau, Farhi, Gutmann, Mende

Self-adjoint extensions in quantum field theory

$$V = -\frac{\lambda}{4}\,\phi^4$$

Equation of motion: $$\partial^2\phi = -\lambda\phi^3 + \cancel{\frac{1}{6}R(S^3)\phi}$$

Ricci scalar; ignore for large ϕ

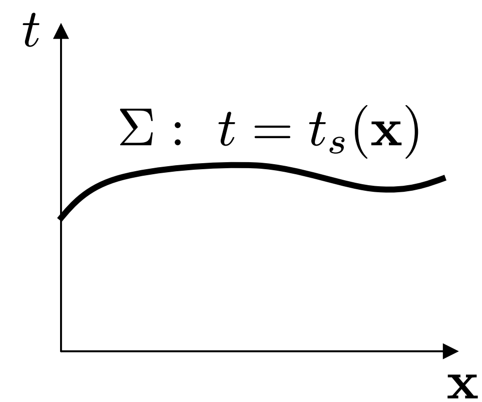
Homogeneous background solution: $\phi = \sqrt{(2/\lambda)}\,t^{-1}$. Define $\chi = (2/\lambda)^{1/2}\phi^{-1}$.

Can construct generic, spatially inhomogeneous solution to e.o.m. in expansion around space-like singular surface $\Sigma:\ t = t_s(\mathbf{x})$ where ϕ is infinite:

time delay

$$\chi(t,\mathbf{x}) = -t + t_s(\mathbf{x}) + \frac{1}{6}t^2\nabla^2 t_s - \frac{1}{24}t^4(\nabla^4 t_s) + \ldots$$
$$-\frac{\lambda\rho(\mathbf{x})}{10}\,t^5 + \cdots + \quad \text{(non-linear in } t_s, \rho\text{)}$$

energy perturbation

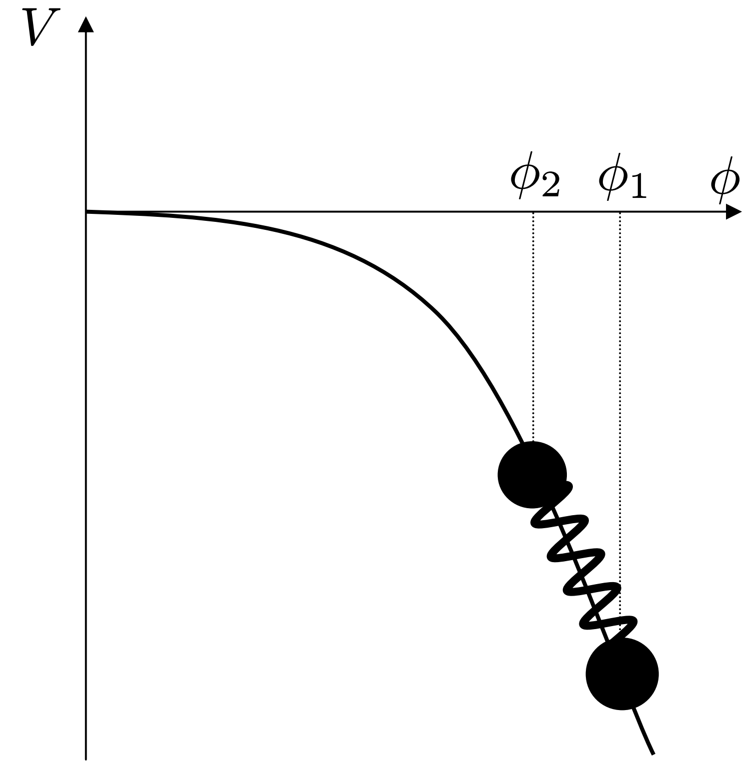


Main observation: spatial gradients become unimportant near the singularity

→ evolution becomes ultralocal

Different spatial points decouple, and we can try to define a self-adjoint extension point by point

Simplified model: two lattice points



$$H = \frac{1}{2}(\pi_1^2 + \pi_2^2) - \frac{\lambda}{4}(\phi_1^4 + \phi_2^4) + k^2(\phi_1 - \phi_2)^2$$

cf. two particles connected by spring in $-x^4$ potential

Suppose ϕ_1 hits infinity first:

$$\phi_1 \approx \sqrt{\frac{2}{\lambda}}\frac{1}{|t|} \quad (t \uparrow 0)$$

Then $\ddot{\phi}_2 \approx \frac{2k^2}{|t|}$ (because of $-2k^2\phi_1\phi_2$ coupling), leading to divergent acceleration

and velocity as $t \uparrow 0$, but finite displacement: $\phi_1 \approx \sqrt{\frac{2}{\lambda}}\frac{1}{|t|}, \quad \phi_2 \approx \text{ const} \quad (t \uparrow 0)$

→ effect of gradient interaction is small → ultralocality

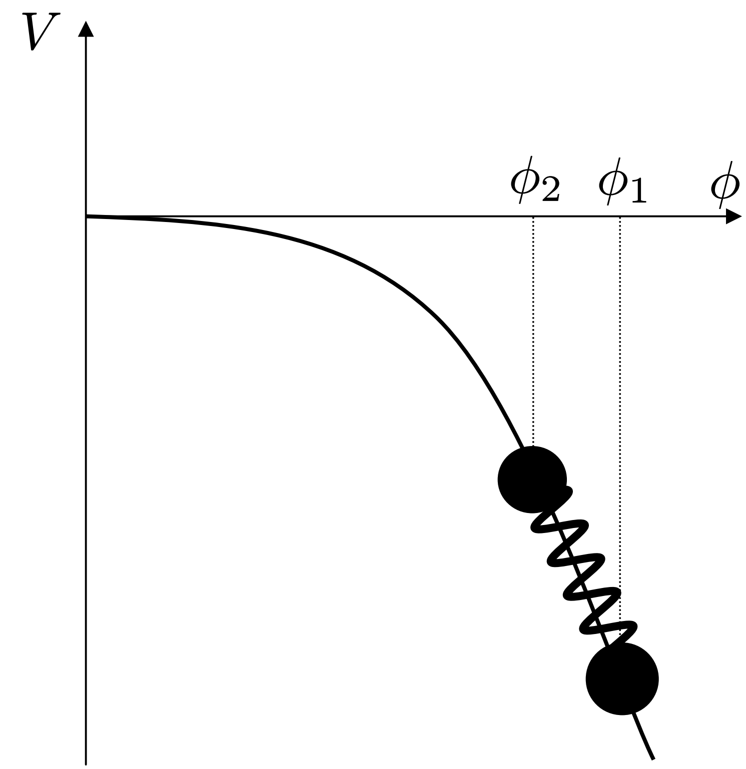
However: complications start just after ϕ_1 has hit infinity…

BC, Hertog, Turok

Simplified model: two lattice points

$$H = \frac{1}{2}(\pi_1^2 + \pi_2^2) - \frac{\lambda}{4}(\phi_1^4 + \phi_2^4) + k^2(\phi_1 - \phi_2)^2$$

$$\phi_1 \approx \sqrt{\frac{2}{\lambda}}\frac{1}{|t|} \quad (t \uparrow 0)$$



At $t = 0$, particle 2 has infinite velocity

→ immediately hits infinity → model degenerates

Cure: Do not put brick wall. Rather, let ϕ_1 reappear at $-\infty$ after disappearing at $+\infty$

BC, Hertog, Turok

Plan

- AdS cosmology: review of basic idea
- ABJM theory and an unstable triple trace deformation
- Beyond the singularity? Self-adjoint extensions
- Summary and outlook

Summary

- AdS cosmology: study cosmological singularity by studying field theories with potentials unbounded below

- Specific case: ABJM theory with unstable triple trace deformation. Studied quantum effective potential and found perturbative UV fixed point in closely related model

- Self-adjoint extensions: prescriptions to continue time evolution beyond the singularity. Subtle in QFT, but concrete proposal is being developed

Outlook: big crunch/big bang cosmology?

Program (in principle):

- Take a state in the bulk theory (with modified boundary conditions)
- Translate to state in dual boundary theory (with unbounded potential)
- Evolve state through singularity using self-adjoint extension
- Translate evolved state back to state in bulk theory and see what it looks like

- If only homogeneous mode in boundary theory: final state would roughly resemble initial state.
- Inhomogeneous modes → particle creation: potentially attractive for cosmology, but need to make sure backreaction is small enough
- Deformed ABJM theory: number of created particles suppressed by inverse power of N
  → preliminary result: large probability to bounce

  (Unlike related $\mathcal{N}=4$ SYM model)

BC, Hertog, Turok