Holography for non-relativistic CFTs

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Outline

- Review finite temperature holography for relativistic CFTs
- Holography for non-relativistic CFTs
- Heating up the non-relativistic theory
- Conformal non-relativistic hydro
- Future directions & conclusions

Holographic description of CFTs

- Identify $SO(d,2)$ conformal symmetry with isometries of AdS_{d+1}. In Poincare coordinates,

$$ds^2 = r^2 \eta_{\mu\nu} dx^\mu dx^\nu + \frac{dr^2}{r^2},$$

  Dilatation is $D : x^\mu \to \lambda x^\mu, r \to \lambda^{-1} r$.
- Normalisable perturbations $\leftrightarrow$ states; $h_{\mu\nu} \leftrightarrow \langle T_{\mu\nu} \rangle$
- $\mathcal{N} = 4$ $SU(N)$ SYM $\leftrightarrow$ IIB on $\mathrm{AdS}_5 \times S^5$
  Can obtain many more explicit examples by replacing S^5 by a Sasaki-Einstein space.
  - Classical gravity valid at strong 't Hooft coupling.
- Focus on universal subsector: stress tensor correlation functions are graviton correlation functions in bulk.

Finite temperature

▷ Bulk black hole dual to thermal ensemble.

$$ds^2 = r^2 \left[-\left(1 - \frac{r_+^4}{r^4}\right) dt^2 + d\mathbf{x}^2 \right] + \left(1 - \frac{r_+^4}{r^4}\right)^{-1} \frac{dr^2}{r^2}$$

Temperature $T = \frac{r_+}{\pi}$.
Entropy $S = \frac{1}{4G_5} r_+^3 V = \frac{\pi^3}{4G_5} V T^3$
Free energy $F \approx TI = -\frac{\pi^3}{16 G_5} V T^4$.
Can extract bdy stress tensor from metric: Henningson Skenderis Balasubramanian Kraus

$$ds^2 = \frac{dz^2}{z^2} + \frac{1}{z^2}[g_{(0)} + z^4 g_{(4)}],$$

$$\langle T_{\mu\nu} \rangle \propto g_{(4)\mu\nu} \propto r_+^4 \mathrm{diag}(3, 1, 1, 1).$$

Hydrodynamics

- Effective description of long-wavelength perturbations

$$T_{\mu\nu} \sim \pi^4 T^4 (\eta_{\mu\nu} + 4u_\mu u_\nu) - 2\eta\sigma_{\mu\nu} + \ldots$$

- Transport coefficients calculated in linearised theory on black hole background: $\frac{\eta}{s} = \frac{1}{4\pi}$.

  Policastro
  Son
  Starinets

  - Universal property for theories with a gravity dual.
  - Conjectured general lower bound.

Bhattacharyya
Hubeny
Minwalla
Rangamani

- More direct approach:
  - Consider black hole solution with $T(t, \mathbf{x})$, $u^\mu(t, \mathbf{x})$.
  - Correct order by order in derivative expansion.
  - Hydrodynamic parameters determined by bulk dynamics:

$$16\pi G_5 T_{\mu\nu} = \pi^4 T^4 (\eta_{\mu\nu} + 4u_\mu u_\nu) - 2\pi^3 T^3 \sigma_{\mu\nu} + \ldots$$

$\star$ Applications to quark-gluon plasma and condensed matter

Non-relativistic CFTs

Strongly-coupled non-relativistic systems with conformal symmetry are also interesting

▷ Example: fermions at unitarity:

$$H = \int d^d x \partial_i \psi_\alpha^\dagger \partial_i \psi_\alpha + \int d^d x \int d^d y \psi_\alpha^\dagger(x) \psi_\beta^\dagger(y) V(|x-y|) \psi_\beta(y) \psi_\alpha(x).$$

If $V(|x-y|)$ is tuned to have infinite scattering length, long-distance physics has a scaling symmetry

$$D : t \to \lambda^2 t, \quad x \to \lambda x$$

with $\Delta_\psi = \frac{d}{2}$.

Experimentally realised in cold atoms.

▷ Non-rel CFTs have primary operators, state-op corr etc — Nishida Son

⋆ Can we also describe these by a gravitational dual?

Non-relativistic conformal symmetry

Galilean symmetry: rotations M_{ij}, translations P_i, boosts K_i, Hamiltonian H, particle number N. $i = 1, \dots, d$.
Extended by the dilatation D,

$$[D, P_i] = iP_i, [D, H] = ziH, [D, K_i] = (1 - z)iK_i, [D, N] = i(2 - z)N.$$

Dynamical exponent z determines scaling of H under dilatations.

For $z = 2$, N is central, and there is a special conformal generator C: $[D, C] = -2iC, [H, C] = -iD$.
Schrödinger algebra: Symmetry of free Schrödinger equation.

$\star$ Isometries of a gravitational dual?

Embedding in SO(d+2,2)

Embed Galilean symmetry in $ISO(d+1,1)$ by light-cone quant: choose light-cone coordinates $x^{\pm} = \frac{1}{\sqrt{2}}(x^0 \pm x^{d+1})$, identify

$$M_{ij} = \tilde{M}_{ij}, \quad P_i = \tilde{P}_i, \quad K_i = \tilde{M}_{-i}, \quad H = \tilde{P}_+, \quad N = \tilde{P}_-.$$

x^+ is Galilean time coordinate; x^- momentum is particle number.

▷ Particle number should be discrete: need to identify x^-—DLCQ.

Extend to embed $Sch(d)$ in $SO(d+2,2)$ by

$$D = \tilde{D} + (z-1)\tilde{M}_{+-}$$

For $z=2$, $C = \frac{\tilde{K}_-}{2}$.

⋆ $Sch(d)$ is a subgroup of $SO(d+2,2)$

Geometrical dual

Son Balasubramanian McGreevy

Deform AdS_{d+3} to

$$ds^2 = -r^4(dx^+)^2 + r^2(-2dx^+dx^- + d\mathbf{x}^2) + \frac{dr^2}{r^2}.$$

Homogeneous space, Killing vectors

$$M_{ij} = -i(x^i\partial_j - x^j\partial_i), \quad P_i = -i\partial_i, \quad H = -i\partial_+,$$

$$D = -i(x^i\partial_i + 2x^+\partial_+ - r\partial_r),$$

$$K_i = -i(-x^+\partial_i + x^i\partial_-), \quad N = -i\partial_-.$$

- Most general geometry with these symmetries.
- Solution of a theory with a massive vector, $A^- = 1$.
- Non-rel causal structure: $I^+(x_0^+, x_0^-, \mathbf{x}_0) = \{x^\mu : x^+ > x_0^+\}$.

Dual in string theory

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Adams
Balasubramanian
McGreevy

Maldacena
Martelli
Tachikawa

Take $\mathrm{AdS}_5 \times S^5$,

$$ds^2 = r^2(-2dx^+dx^- + d\mathbf{x}^2) + \frac{dr^2}{r^2} + (d\psi + P)^2 + d\Sigma_4^2,$$

and apply a TsT transformation:

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- T-dualize the Hopf fiber coordinate ψ to $\tilde{\psi}$,
- Shift $x^- \to \tilde{x}^- = x^- + \tilde{\psi}$,
- T-dualise $\tilde{\psi}$ to ψ at fixed $\tilde{x}^-$.

Resulting solution is

$$ds^2 = -r^4(dx^+)^2 + r^2(-2dx^+dx^- + d\mathbf{x}^2) + \frac{dr^2}{r^2} + (d\psi + P)^2 + d\Sigma_4^2,$$

$$B = r^2 dx^+ \wedge (d\psi + P).$$

Heating up the non-relativistic theory

Apply the TsT transformation to the Schwarzschild-AdS solution,

$$ds^2 = r^2(-f(r)dt^2 + dy^2 + d\mathbf{x}^2) + \frac{dr^2}{r^2 f(r)} + ds^2_{S^5},$$

where $f(r) = 1 - r_+^4/r^4$.
Resulting solution in 5d is

$$\begin{aligned} ds^2 &= r^2 k(r)^{-\frac{2}{3}} \left(\left[\frac{r_+^4}{4\beta^2 r^4} - r^2 f(r) \right] (dx^+)^2 \right. \\ &\quad \left. + \frac{\beta^2 r_+^4}{r^4} (dx^-)^2 - [1 + f(r)] dx^+ dx^- \right) + k(r)^{\frac{1}{3}} \left(r^2 d\mathbf{x}^2 + \frac{dr^2}{r^2 f(r)} \right). \end{aligned}$$

$$A = \frac{r^2}{k(r)} \left(\frac{1 + f(r)}{2} dx^+ - \frac{\beta^2 r_+^4}{r^4} dx^- \right),$$

$$e^\phi = \frac{1}{\sqrt{k(r)}}; \quad k(r) = 1 + \frac{\beta^2 r_+^4}{r^2}.$$

▷ β param. choice of x^- coord in TsT; define $\gamma^2 \equiv \beta^2 r_+^4$.

Thermodynamics of bulk solution

▷ Horizon at $r = r_+$, null Killing vector $\xi = \frac{\partial}{\partial t} = \frac{\partial}{\partial x^+} + \frac{1}{2\beta^2}\frac{\partial}{\partial x^-}$.
Entropy $S = \frac{1}{4G_5} r_+^3 \beta \Delta x^- V$, Temperature $T = \frac{r_+}{\pi\beta}$.
Chemical potential $\mu = \frac{1}{2\beta^2}$ — chemical potential for particle number.
Black hole corresponds to a grand canonical ensemble.

▷ Using a saddle-point approximation to the partition function,

$$\langle N \rangle = \frac{\gamma^2}{8\pi^2 G_5}(\Delta x^-)^2 V, \quad \langle E \rangle = \frac{r_+^4}{16\pi G_5}\Delta x^- V, \quad \langle P \rangle = \frac{r_+^4}{16\pi G_5}\Delta x^-.$$

Equation of state $P = \varepsilon$. Consequence of non-relativistic conformal symmetry; $dP = 2\varepsilon$.

▷ Interesting limit: $r_+ \to 0$ at fixed γ.
Zero temperature, finite particle number density.

Hydrodynamics

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Thompson

- General bulk solution has four parameters: β, r_+, v^i. Promote to functions of t, x^i?
  ▷ Obtain by TsT from relativistic hydro solution.
  - Four parameters: r_+, unit-normalized u^μ.
  - Assume independent of x^-: functions of x^+, x^i.
- Dual hydro stress tensor?
  We constructed an action using covariant counterterms, but required stringent boundary conditions.
  $\Rightarrow$ Applying holographic renormalisation does not give a good stress tensor.
  Re-interpret AdS stress tensor in non-relativistic theory.

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Hydrodynamics

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- Dimensional reduction of relativistic stress tensor $T^{\mu\nu}$ along x^- gives non-relativistic stress tensor complex:

$$T^{++} = \rho, T^{+i} = \rho v^i, T^{ij} = \Pi^{ij}, T^{+-} = \varepsilon + \frac{1}{2}\rho v^2, T^{-i} = j^i_\varepsilon.$$

- In non-relativistic conformal theory, two transport coefficients at first order: shear viscosity η, heat conductance κ.

$$\frac{\eta}{s} = \frac{1}{4\pi}, \quad \kappa = 2\eta\frac{\varepsilon + P}{\rho T}$$

.

- Can justify this approach to non-relativistic stress tensor by arguing planar sector of D3-brane field theory is unchanged by TsT transformation.

Lifshitz theory

Kachru
Liu
Mulligan

Drop boosts, retaining anisotropic scaling symmetry.

- Theory has a d+2 dimensional gravitational dual:

$$ds^2 = -r^{2z} dt^2 + r^2 d\mathbf{x}^2 + \frac{dr^2}{r^2}.$$

- Killing vectors

$$M_{ij} = -i(x^i \partial_j - x^j \partial_i), \quad P_i = -i\partial_i, \quad H = -i\partial_t,$$

$$D = -i(x^i \partial_i + zt\partial_t - r\partial_r).$$

- Solution of a theory with p-form fields with a Chern-Simons coupling.
- Finite temperature solutions recently obtained. Danielsson Thorlacius

Discussion

- NRCFT is an interesting and challenging extension of AdS/CFT.
- Easy to embed solutions in string theory; solution-generating transformation.
- Finite temperature solutions obtained.
- Hydrodynamics related to hydrodynamics of $\mathcal{N} = 4$ SYM by TsT transformation.

Discussion

Issues:

- Role of extra x^- direction?
- Compactification of x^-.
- Large N limit?
- Asymptotics unusual; black hole solutions have slow falloff. Difficult to define boundary stress tensor directly.
- Lifshitz case avoids many of these issues, but not yet related to string theory.