ASPECTS OF FREE FERMIONIC HETEROTIC-STRING MODELS

Alon E. Faraggi

- AEF, 1990's (top quark mass, CKM, MSSM w Cleaver and Nanopoulos)
- AEF, C. Kounnas, J. Rizos, 2003-2009
- AEF, PLB2002, work in progress with Angelantonj, Tsulaia

Galileo Galilei Institute, Firenze, 26 May 2009

DATA $\longrightarrow$ STANDARD MODEL

$SU(3) \times SU(2) \times U(1)_Y \longrightarrow SU(5) \longrightarrow SO(10)$

$$\left[\begin{pmatrix} \nu \\ e \end{pmatrix} + D^c_L \right] + \left[U^c_L + \begin{pmatrix} u \\ d \end{pmatrix} + E^c_L \right] + N^c_L$$

$$\bar{5} \quad + \quad 10 \quad\quad 1 \quad\quad \underline{\quad} \; 16$$

STANDARD MODEL -> UNIFICATION

ADDITIONAL EVIDENCE:

Logarithmic running, proton longevity, neutrino masses

PRIMARY GUIDES:

3 generations

SO(10) embedding

Realistic free fermionic models

'Phenomenology of the Standard Model and string unification'

- Top quark mass $\sim$ 175–180GeV — PLB 274 (1992) 47
- Generation mass hierarchy — NPB 407 (1993) 57
- CKM mixing — NPB 416 (1994) 63 (with Halyo)
- Stringy seesaw mechanism — PLB 307 (1993) 311 (with Halyo)
- Gauge coupling unification — NPB 457 (1993) 409 (with Dienes)
- Proton stability — NPB 428 (1994) 111
- Squark degeneracy — NPB 526 (1998) 21 (with Pati)
- Minimal Superstring Standard Model — PLB 455 (1999) 135 (with Cleaver & Nanopoulos)
- Moduli fixing — NPB 728 (2005) 83

Free Fermionic Construction

Left-Movers: $\psi^{\mu}_{1,2}$, χ_i, y_i, ω_i $(i = 1, \cdots, 6)$

Right-Movers

$$\bar{\phi}_{A=1,\cdots,44} = \begin{cases} \bar{y}_i \; , \; \bar{\omega}_i & i = 1, \cdots, 6 \\ & \\ \bar{\eta}_i & i = 1, 2, 3 \\ \bar{\psi}_{1,\cdots,5} & \\ \bar{\phi}_{1,\cdots,8} & \end{cases}$$

$$f \longrightarrow -e^{i\pi\alpha(f)} f$$

$$V \longrightarrow V \qquad\qquad Z = \sum_{\substack{all\ spin\\ structures}} c\binom{\vec{\alpha}}{\vec{\beta}} Z\binom{\vec{\alpha}}{\vec{\beta}}$$

Models $\longleftrightarrow$ Basis vectors $+$ one–loop phases

Model building – Construction of the physical states

$$b_j \quad j = 1, \cdots, N \qquad \rightarrow \qquad \Xi \;=\; \sum_j n_j b_j$$

For $\vec{\alpha} \;=\; (\vec{\alpha}_L; \vec{\alpha}_R) \;\in\; \Xi \quad \Rightarrow \quad \mathsf{H}_{\vec{\alpha}}$

$$\alpha(f) = 1 \quad \Rightarrow \quad |\pm\rangle \qquad ; \qquad \alpha(f) \neq 1 \quad \Rightarrow \quad f, f^* \quad , \quad \nu_{f,f^*} = \frac{1 \mp \alpha(f)}{2}$$

$$M_L^2 = -\frac{1}{2} + \frac{\vec{\alpha}_L \cdot \vec{\alpha}_L}{8} + N_L = -1 + \frac{\vec{\alpha}_R \cdot \vec{\alpha}_R}{8} + N_R = M_R^2 \;\; (\;\equiv\; 0 \;)$$

GSO projections $\qquad e^{i\pi(\vec{b}_i \cdot \vec{F}_\alpha)} |s\rangle_{\vec{\alpha}} = \delta_\alpha c^* \begin{pmatrix} \vec{\alpha} \\ \vec{b}_i \end{pmatrix} |s\rangle_{\vec{\alpha}}$

$$Q(f) = \frac{1}{2}\alpha(f) + F(f) \quad \rightarrow \quad \text{U(1) charges}$$

Calculation of Mass Terms

nonvanishing correlators

$$\langle V_1^f V_2^f V_3^b \cdot \cdot \cdot \cdot V_N^b \rangle$$

gauge & string invariant

"anomalous" $U(1)_A$

$$\text{Tr}Q_A \neq 0 \Rightarrow \quad D_A = 0 = A + \sum Q_k^A |\langle \phi_k \rangle|^2$$

$$D_j \;\; = 0 = \qquad \sum Q_k^j |\langle \phi_k \rangle|^2$$

$$\langle W \rangle = \langle \frac{\partial W_N}{\partial \eta_i} \rangle = 0 \quad N = 3 \cdot \cdot \cdot \cdot$$

Supersymmetric vacuum $\langle F \rangle = \langle D \rangle = 0.$

nonrenormalizable terms $\longrightarrow$ effective renormalizable operators

$$V_1^f V_2^f V_3^b \cdot \cdot \cdot \cdot V_N^b \;\; \rightarrow \;\; V_1^f V_2^f V_3^b \frac{\langle V_4^b \cdot \cdot \cdot V_N^b \rangle}{M^{N-3}}$$

The NAHE set:

$$1 = \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$S = \{\psi^\mu, \chi^{1,\dots,6}\}, \qquad N=4 \text{ Vacua}$$

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \qquad N=4 \rightarrow N=2$$

$$b_2 = \{\chi^{12}, \chi^{56}, y^{12}, \omega^{56} | \bar{y}^{12}, \bar{\omega}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \qquad N=2 \rightarrow N=1$$

$$b_3 = \{\chi^{12}, \chi^{34}, \omega^{12}, \omega^{34} | \bar{\omega}^{12}, \bar{\omega}^{34}, \bar{\eta}^3, \bar{\psi}^{1,\dots,5}\}, \qquad N=2 \rightarrow N=1$$

$Z_2 \times Z_2$ orbifold compactification

$\Longrightarrow$ Gauge group $SO(10) \times SO(6)^{1,2,3} \times E_8$

beyond the NAHE set   Add $\{\alpha, \beta, \gamma\}$

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,\ldots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,\ldots,8}$
α	0	0	0	0	1	0	0	1	0	0	1	1	0	0	1	1	1 1 1 0 0	1	0	0	1 1 0 0 0
β	0	0	0	0	0	0	1	1	1	0	0	1	0	1	0	1	1 1 1 0 0	0	1	0	0 0 1 1 0
γ	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0 0 0 $\frac{1}{2}$ $\frac{1}{2}$

number of generations is reduced to three

$$SO(10) \quad \longrightarrow \quad SU(3) \times SU(2) \times U(1)_{T_{3_R}} \times U(1)_{B-L}$$

$$U(1)_Y \; = \; \tfrac{1}{2}(B-L) \; + \; T_{3_R} \; \in \; SO(10) \; !$$

$$SO(6)^{1,2,3} \quad \longrightarrow \quad U(1)^{1,2,3} \; \times \; U(1)^{1,2,3}$$

The massless spectrum

Three twisted generations $b_1, \quad b_2, \quad b_3$

Untwisted Higgs doublets

$$h_{1_{1,0,0}} \qquad \bar{h}_{1_{-1,0,0}}$$
$$h_{2_{0,1,0}} \qquad \bar{h}_{2_{0,-1,0}}$$
$$h_{3_{0,0,1}} \qquad \bar{h}_{3_{0,0,-1}}$$

Sector $b_1 + b_2 + \alpha + \beta$ $\quad h_{\alpha\beta_{-\frac{1}{2},-\frac{1}{2},0,0,0,0}} \qquad \bar{h}_{\alpha\beta_{\frac{1}{2},\frac{1}{2},0,0,0,0}}$

$\oplus$ $SO(10)$ singlets

Sectors $b_j + 2\gamma \quad j = 1, 2, 3 \quad \longrightarrow$ hidden matter multiplets

"standard" $SO(10)$ representations

NAHE + $\{ \alpha , \beta , \gamma \} \rightarrow$ exotic vector–like matter $\rightarrow$ superheavy

$\oplus$ Quasi-realistic phenomenology

Fermion mass hierarchy

Fermion mass terms

$$c g f_i f_j h \left(\frac{\langle\phi\rangle}{M}\right)^{N-3}$$

c - calculable coefficients g - gauge coupling

$$f_i,\, f_j \ \in \ b_j \quad j \ = \ 1, 2, 3$$

$h \rightarrow$ light Higgs multiplets

$$M \ \sim \ 10^{18}\, GeV$$

$\langle\phi\rangle$ generalized VEVs, several sources

Top quark mass prediction

only $\lambda_t = \langle t^c Q_t \bar{h}_1 \rangle = \sqrt{2} g \neq 0$ at $N = 3$

$$W_4 \longrightarrow b^c Q_b h_{\alpha\beta} \Phi_1 + \tau^c L_\tau h_{\alpha\beta} \Phi_1$$

$$\Longrightarrow \qquad \lambda_b = \left(c_b \frac{\langle \phi \rangle}{M} \right) \qquad \lambda_\tau = \left(c_\tau \frac{\langle \phi \rangle}{M} \right)$$

$$\longrightarrow \qquad \lambda_b = \lambda_\tau = 0.35 g^3 \sim \frac{1}{8} \lambda_t$$

Evolve λ_t , λ_b to low energies

$$m_t = \lambda_t v_1 = \lambda_t \frac{v_0}{\sqrt{2}} \sin\beta \qquad m_b = \lambda_b v_2 = \lambda_b \frac{v_0}{\sqrt{2}} \cos\beta$$

where $v_0 = \dfrac{2 m_W}{g_2(M_Z)} = 246 \mathrm{GeV}$ and $(v_1^2 + v_2^2) = \dfrac{v_0^2}{2}$

$$m_t = \lambda_t(m_t) \frac{v_0}{\sqrt{2}} \frac{\tan\beta}{(1 + \tan^2\beta)^{\frac{1}{2}}} \Longrightarrow \qquad m_t \sim 175 \mathrm{GeV} \quad \mathrm{PLB274(1992)47}$$

Cabibbo mixing PLB 307 (1993) 305

Find anomaly free solution

$$M_d \sim \begin{pmatrix} \epsilon & \frac{V_2\bar{V}_3\Phi_{\alpha\beta}}{M^3} & 0 \\ \frac{V_2\bar{V}_3\Phi_{\alpha\beta}\xi_1}{M^4} & \frac{\bar{\Phi}_2^-\xi_1}{M^2} & 0 \\ 0 & 0 & \frac{\Phi_1^+\xi_2}{M^2} \end{pmatrix} v_2,$$

$$\frac{V_2\bar{V}_3\Phi_{\alpha\beta}}{M^3} = \frac{\sqrt{5}\, g^6}{64\, \pi^3} \approx 2 - 3 \times 10^{-4}.$$

$\epsilon < 10^{-8}$. Fix ξ_1, ξ_2, $\bar{\Phi}_2^-$ to fit m_b, m_s

$$\Rightarrow |V| \sim \begin{pmatrix} 0.98 & 0.2 & 0 \\ 0.2 & 0.98 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Three generation mixing $\longrightarrow$ NPB 416 (1994) 63

$$|J| \sim 10^{-6}$$

Correspondence with $Z_2 \times Z_2$ orbifold   PLB 326 (1994) 62

NAHE $\oplus$ $(\xi_2 = \{\bar{\psi}_{1,\cdots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\} = 1)$ $\rightarrow$ $\{1, S, \xi_1, \xi_2, b_1, b_2\}$

Gauge group: $SO(4)^3 \times E_6 \times U(1)^2 \times E_8$ and 24 generations.

toroidal compactification $(6_L + 6_R)$ g_{ij}, b_{ij}

$$g_{ij} = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & -1 \\ 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & -1 & 0 & 2 \end{pmatrix} \qquad b_{ij} = \begin{cases} g_{ij} & \text{i ¡ j} \\ 0 & \text{i = j} \\ -g_{ij} & \text{i ¿ j} \end{cases}$$

R_i $\rightarrow$ the free fermionic point $\rightarrow$G.G. $SO(12) \times E_8 \times E_8$

mod out by a $Z_2 \times Z_2$ with standard embedding

$\Rightarrow SO(4)^3 \times E_6 \times U(1)^2 \times E_8$ with 24 generations

Exact correspondence

In the realistic free fermionic models

replace $X = \{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\} = 1$

with $\quad 2\gamma \; = \{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3, \bar{\phi}^{1,\cdots,4}\} = 1$

Then $\{\vec{1}, \vec{S}, \vec{\xi}_1 = \vec{1} + \vec{b}_1 + \vec{b}_2 + \vec{b}_3, 2\gamma\} \;\rightarrow\;$ N=4 SUSY and

$$SO(12) \times SO(16) \times SO(16)$$

apply $b_1 \times b_2 \;\rightarrow\; Z_2 \times Z_2 \;\rightarrow\;$ N=1 SUSY and

$$SO(4)^3 \times SO(10) \times U(1)^3 \times SO(16)$$

b_1, b_2, b_3 $\quad \Rightarrow \; (3 \times 8) \cdot 16$ of $SO(10)_O$

$b_1 + 2\gamma$, $b_2 + 2\gamma$, $b_3 + 2\gamma$ $\quad \Rightarrow \; (3 \times 8) \cdot 16$ of $SO(16)_H$

Alternatively, $c\binom{\xi_1}{\xi_2} \; = \; +1 \;\rightarrow\; -1$

$Z_2 \times Z_2$ orbifolds

A torus One complex parameter $Z = Z + n\, e_1 + m\, e_2$

$T^2 \times T^2 \times T^2 \longrightarrow$ Three complex coordinates z_1 , z_2 and z_3

Z_2 orbifold : $Z = - Z + \sum_i m_i e_i \longrightarrow$ 4 fixed points

$$Z = \{ \; 0 \; , \; 1/2\, e_1 \; , \; 1/2\, e_2 \; , \; 1/2\, (e_1 + e_2) \; \}$$

$$\frac{T^2 \times T^2 \times T^2}{Z_2 \times Z_2}$$

$\alpha : (z1 , z2 , z3) \; -> \; (-z1 , -z2 , +z3) \; -> \; 16$

$\beta : (z1 , z2 , z3) \; -> \; (+z1 , -z2 , -z3) \; -> \; 16$

$\alpha\beta : (z1 , z2 , z3) \; -> \; (-z1 , +z2 , -z3) \; -> \; 16$

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↓

$\gamma : (z1, z2, z3) \; -> \; (z1+1/2, z2+1/2, z3+1/2) \longrightarrow$ 24

Classification of fermionic $Z_2 \times Z_2$ orbifolds (FKNR, FKR)

Basis vectors: consistent modular blocks 4,8 periodic fermions

$$1 = \{\psi^\mu, \ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$S = \{\psi^\mu, \chi^{1,\dots,6}\},$$

$$z_1 = \{\bar{\phi}^{1,\dots,4}\},$$

$$z_2 = \{\bar{\phi}^{5,\dots,8}\},$$

$$e_i = \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\}, \ i = 1, \dots, 6,$$ $N = 4$ Vacua

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\},$$ $N = 4 \to N = 2$

$$b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\},$$ $N = 2 \to N = 1$

Vector bosons: NS, $z_{1,2}$, $z_1 + z_2$, $x = 1 + s + \sum e_i + z_1 + z_2$

impose: Gauge group $SO(10) \times U(1)^3 \times \text{hidden}$

Independent phases $c\left[\begin{smallmatrix} v_i \\ v_j \end{smallmatrix}\right] = \exp[i\pi(v_i|v_j)]$: upper block

	1	S	e_1	e_2	e_3	e_4	e_5	e_6	z_1	z_2	b_1	b_2
1	-1	-1	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
S			-1	-1	-1	-1	-1	-1	-1	-1	1	1
e_1				$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_2					$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_3						$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_4							$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_5								$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_6									$\pm$	$\pm$	$\pm$	$\pm$
z_1										$\pm$	$\pm$	$\pm$
z_2											$\pm$	$\pm$
b_1												$\pm$
b_2												

Apriori 55 independent coefficients $\rightarrow$ 2^{55} distinct vacua

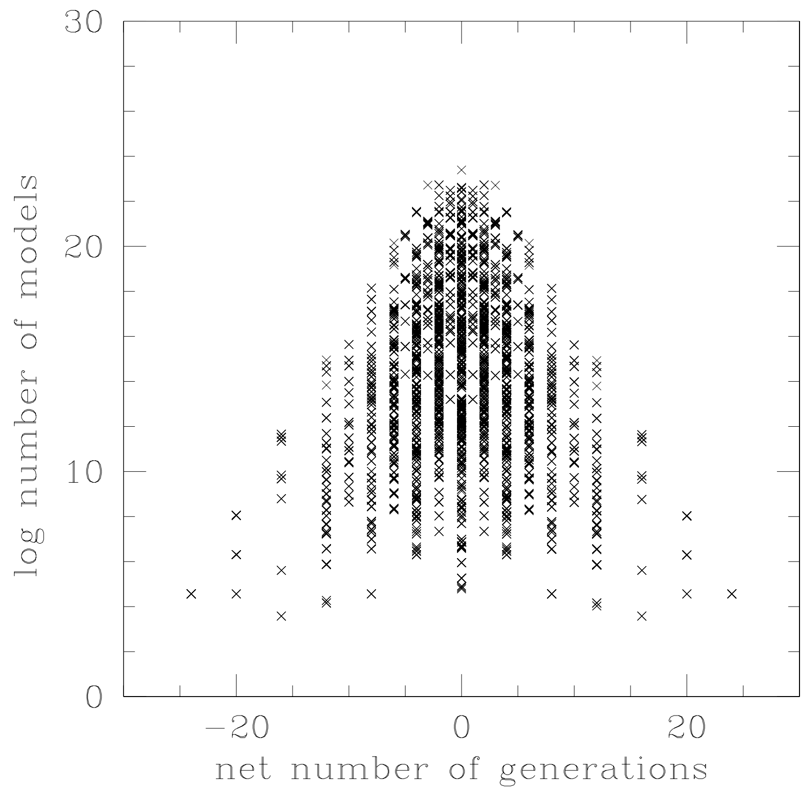
Impose: Gauge group $SO(10) \times U(1)^3 \times SO(8)^2$

$\longrightarrow$ 40 independent coefficients

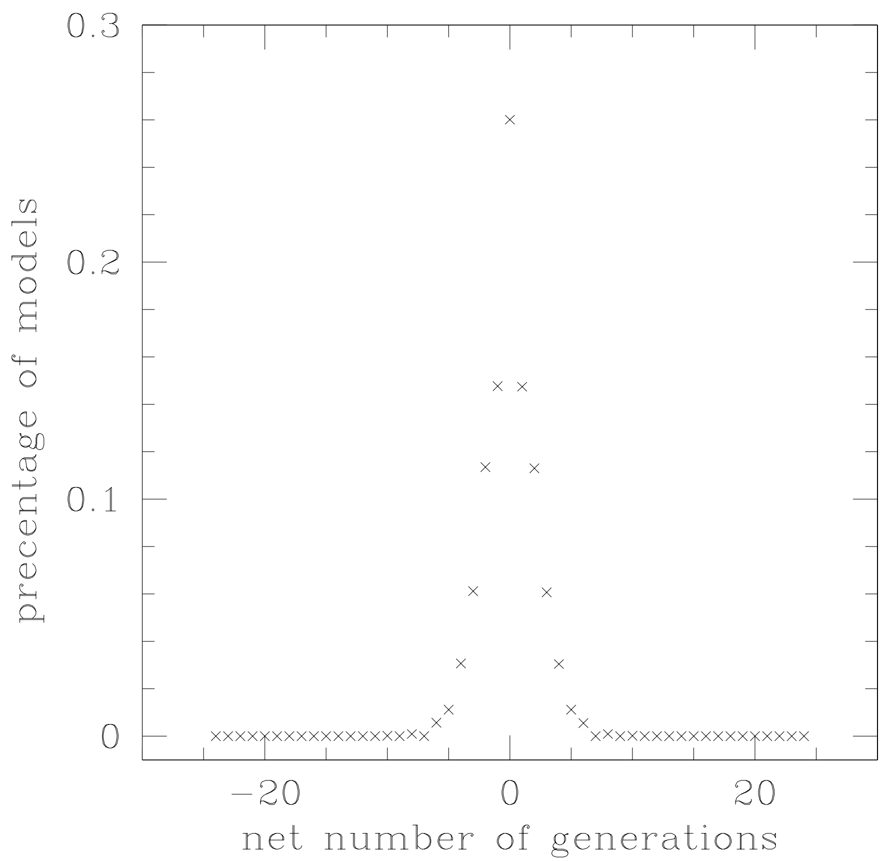
RESULTS:

FKR I: Random sampling of phases. $SO(10) \times U(1)^3 \times \text{hidden}$

FKR II: Complete classification. $SO(10) \times U(1)^3 \times SO(8)^2$



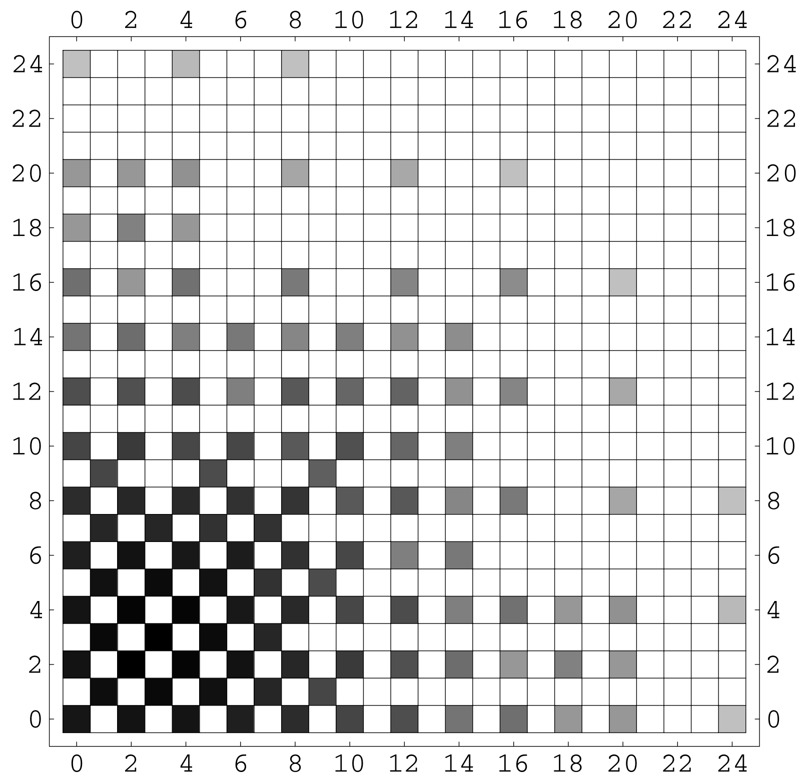
RESULTS ARE SIMILAR



7×10^9 models $\sim$ 15% with 3 gen FKRI

Spinor–vector duality:

Invariance under exchange of $\#(16+\overline{16}) \;<-> \;\; \#(10)$



Symmetric under exchange of rows and columns

Self-dual: $\#(16+\overline{16}) \;=\; \#(10)$ without E_6 symmetry

Future:

Understand in geometrical terms:

Progress:

Tristan Catelin-Jullien, AEF, C. Kounnas, J. Rizos, NPB2009 $\longrightarrow$

Operational understanding in terms of partition function free phases

NAHE-based partition functions:

Question:

$$\frac{T^6}{Z_2 \times Z_2} \;\rightarrow\; 48 \;\text{ fixed points} \qquad\qquad \frac{SO(12)}{Z_2 \times Z_2} \;\rightarrow\; 24 \;\text{ fixed points}$$

$$Z_2 \;\text{ shift } \;: 48 \;\longleftrightarrow\; 24$$

Is this the same vacuum? In general, no.

shift that reproduces the $SO(12)$ lattice at the free fermionic point?

Possible shifts:

$$
\begin{aligned}
A_1 &: \quad X_{\rm L,R} \to X_{\rm L,R} + \tfrac{1}{2}\pi R\,,\\
A_2 &: \quad X_{\rm L,R} \to X_{\rm L,R} + \tfrac{1}{2}\left(\pi R \pm \frac{\pi\alpha'}{R}\right)\,,\\
A_3 &: \quad X_{\rm L,R} \to X_{\rm L,R} \pm \tfrac{1}{2}\frac{\pi\alpha'}{R}\,.
\end{aligned}
$$

Using the level-one $\mathrm{SO}(2n)$ characters

$$
\begin{aligned}
O_{2n} &= \tfrac{1}{2}\left(\frac{\vartheta_3^n}{\eta^n} + \frac{\vartheta_4^n}{\eta^n}\right)\,, \qquad & V_{2n} &= \tfrac{1}{2}\left(\frac{\vartheta_3^n}{\eta^n} - \frac{\vartheta_4^n}{\eta^n}\right)\,,\\
S_{2n} &= \tfrac{1}{2}\left(\frac{\vartheta_2^n}{\eta^n} + i^{-n}\frac{\vartheta_1^n}{\eta^n}\right)\,, \qquad & C_{2n} &= \tfrac{1}{2}\left(\frac{\vartheta_2^n}{\eta^n} - i^{-n}\frac{\vartheta_1^n}{\eta^n}\right)\,.
\end{aligned}
$$

The partition function of the heterotic string on $SO(12)$ lattice:

$$Z_+ = (V_8 - S_8)\left[|O_{12}|^2 + |V_{12}|^2 + |S_{12}|^2 + |C_{12}|^2\right]\left(\bar{O}_{16} + \bar{S}_{16}\right)\left(\bar{O}_{16} + \bar{S}_{16}\right) ,$$

and

$$\begin{aligned} Z_- \;=\; (V_8 - S_8) \Big[&\left(|O_{12}|^2 + |V_{12}|^2\right)\left(\bar{O}_{16}\bar{O}_{16} + \bar{C}_{16}\bar{C}_{16}\right) \\ &+ \left(|S_{12}|^2 + |C_{12}|^2\right)\left(\bar{S}_{16}\bar{S}_{16} + \bar{V}_{16}\bar{V}_{16}\right) \\ &+ \left(O_{12}\bar{V}_{12} + V_{12}\bar{O}_{12}\right)\left(\bar{S}_{16}\bar{V}_{16} + \bar{V}_{16}\bar{S}_{16}\right) \\ &+ \left(S_{12}\bar{C}_{12} + C_{12}\bar{S}_{12}\right)\left(\bar{O}_{16}\bar{C}_{16} + \bar{C}_{16}\bar{O}_{16}\right)\Big] . \end{aligned}$$

where $\pm$ refers to

$$c\binom{\xi_1}{\xi_2} \;=\; \pm 1$$

connected by : $$Z_- = Z_+ / a \otimes b ,$$

$$a = (-1)^{F_{\rm L}^{\rm int} + F_\xi^1} , \qquad b = (-1)^{F_{\rm L}^{\rm int} + F_\xi^2} .$$

Starting from:

$$\mathrm{Z}_+ = (V_8 - S_8) \left(\sum_{m,n} \Lambda_{m,n} \right)^{\otimes 6} (\bar{O}_{16} + \bar{S}_{16}) \, (\bar{O}_{16} + \bar{S}_{16}) \ ,$$

where as usual, for each circle,

$$p^i_{\mathrm{L,R}} = \frac{m_i}{R_i} \pm \frac{n_i R_i}{\alpha'} \,,$$

and

$$\Lambda_{m,n} = \frac{q^{\frac{\alpha'}{4} p^2_{\mathrm{L}}} \, \bar{q}^{\frac{\alpha'}{4} p^2_{\mathrm{R}}}}{|\eta|^2} \,.$$

Add shifts : (A_1, A_1, A_1) , (A_3, A_3, A_3)

($48 \to 24$ yes)

($SO(12)$? no)

Uniquely:

$$g \ : \ (A_2, A_2, 0)\,,$$
$$h \ : \ (0, A_2, A_2)\,,$$

where each A_2 acts on a complex coordinate

($48 \rightarrow 24$ yes)

($SO(12)$? yes)

A STRINGY DOUBLET-TRIPLET SPLITTING MECHANIS

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}$
α	0	0	0	0	1	0	0	1	0	0	1	1	0	0	1	1	1 1 1 0 0	1	0	0	1 1 0
β	0	0	0	0	0	0	1	1	1	0	0	1	0	1	0	1	1 1 1 0 0	0	1	0	0 0 1
γ	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0 0 0

NAHE $\quad \rightarrow \quad \chi_j \bar{\psi}^{1,\cdots,5} \bar{\eta}_j + c.c. \rightarrow (5+\bar{5})_j = 10_j$ of $SO(10)$

$$\alpha\ ,\ \beta \quad \rightarrow \quad SO(10) \quad \rightarrow \quad SO(6)\times SO(4)$$

$$\Delta_j = |\alpha_L(T_2^j) - \alpha_R(T_2^j)|$$

$\Delta_j = 0 \rightarrow D_j, \bar{D}_j$ $\qquad$ $\Delta_j = 1 \rightarrow h_j, \bar{h}_j$

$h_1,\ \bar{h}_1,\ D_1,\ \bar{D}_1\ ,\ \ h_2,\ \bar{h}_2,\ D_2,\ \bar{D}_2$ are projected out

$h_3,\ \bar{h}_3$ remain in the spectrum

Conclusions

Phenomenological string models produce interesting lessons

Spinor–vector duality

relevance of non–standard geometries

Free Fermionic Models $\longrightarrow$ $Z_2 \times Z_2$ orbifold near the self–dual point

Duality & Self–Duality $\Leftrightarrow$ String Vacuum Selection