# Multi-particle production in the glasma at NLO and plasma instabilities

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# Talk based on:

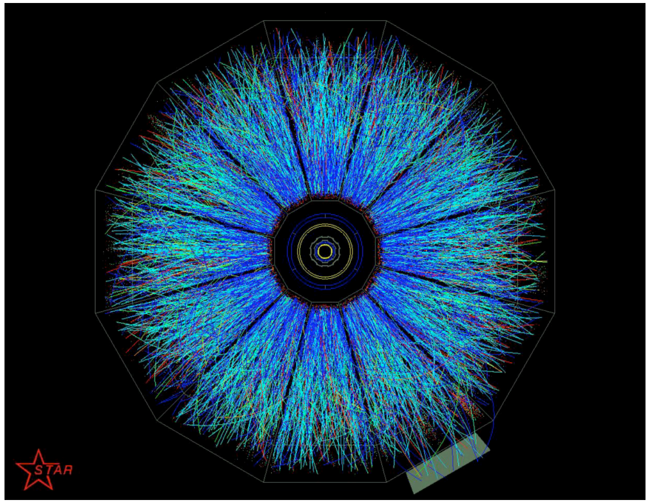
- **I)** ***Multiparticle production to NLO:*** F. Gelis & R. Venugopalan, Nucl. Phys. A776, 135 (2006); Nucl. Phys. A779, 177 (2006).

  **II)** ***Plasma Instabilities:*** P. Romatschke & R. Venugopalan, PRL 96: 062302 (2006); PRD 74:045011 (2006).

- ***Recent lectures:*** F. Gelis and R. Venugopalan, **hep-ph/0611157**

**Theme: deep connection between I) & II) in relation to thermalization of CGC -> QGP**

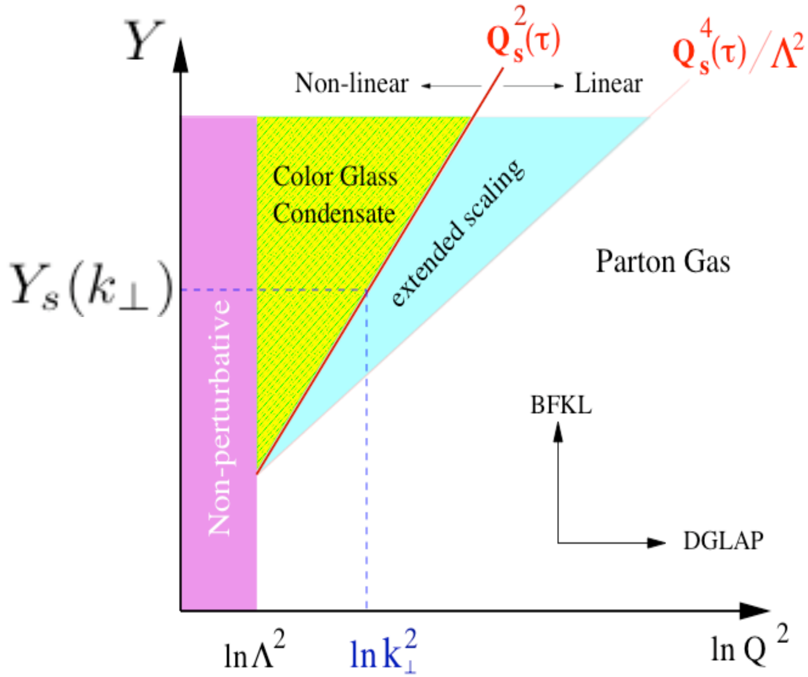
**Can we compute multiparticle production *ab initio* in heavy ion collisions ?**

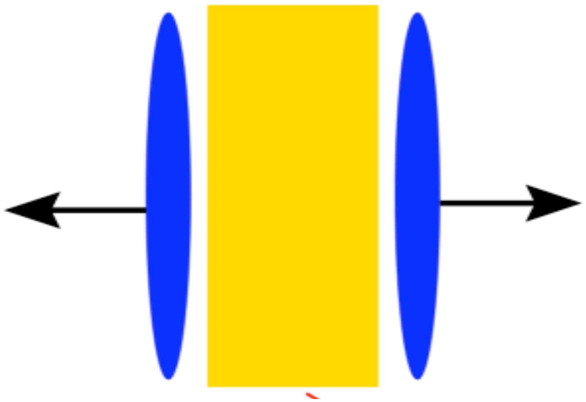


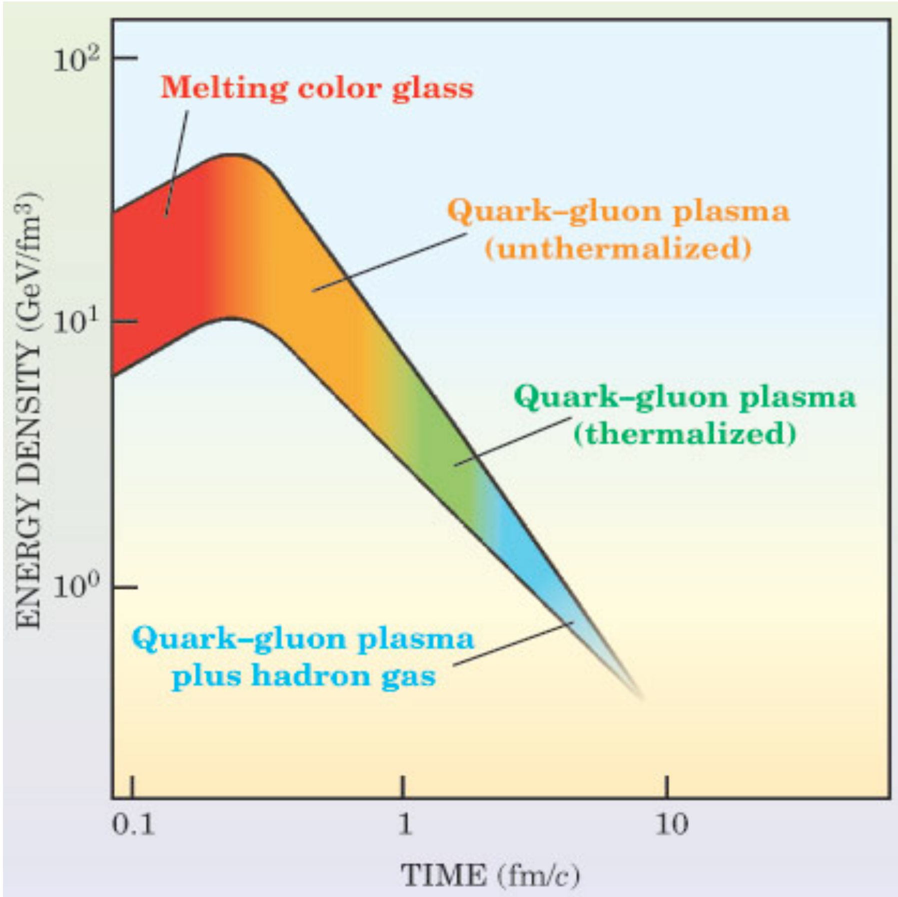

**Framework:**
**CGC- classical fields**
**+ strong sources**

$$\alpha_S(Q_s) << 1$$

$$\rho \sim \frac{1}{g}\left(\equiv \frac{1}{\sqrt{\alpha_S}}\right) \gg 1$$



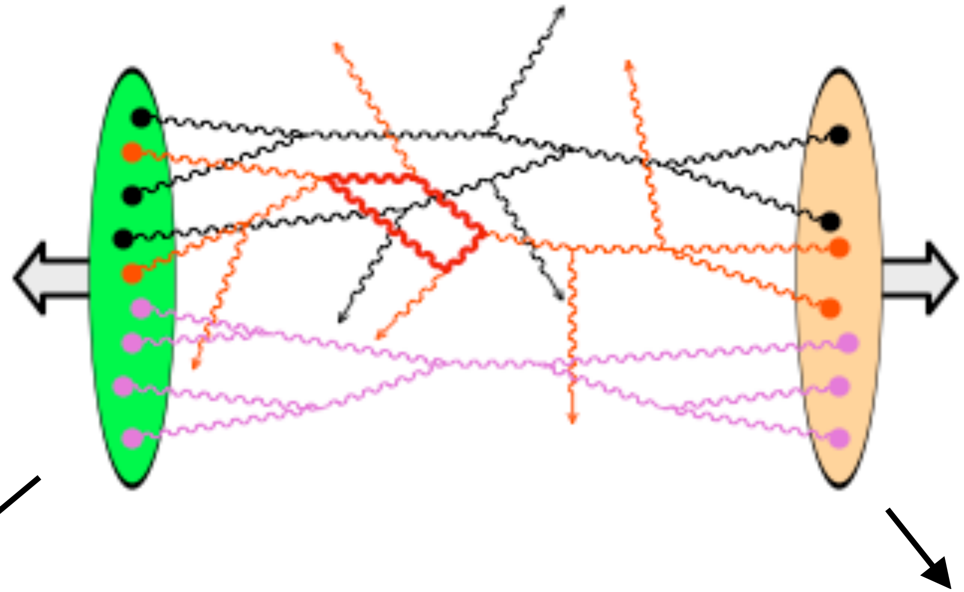





**Glasma** **(**\Glahs-maa**):**
*Noun:* **non-equilibrium phase between CGC & QGP**

**T.Lappi & L. McLerran; Kharzeev, Krasnitz, RV**

**Probability to produce n >> 1 particles in HI collisions:**



$$\rho_A \sim \frac{1}{g} >> 1$$

$$\rho_B \sim \frac{1}{g} >> 1$$

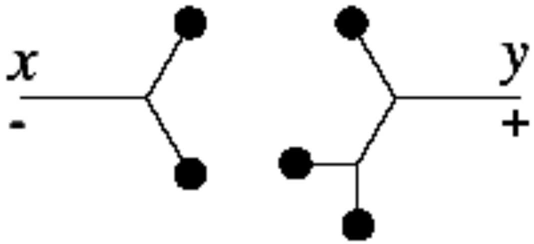
**P_n obtained from cut vacuum graphs in field theories with strong sources.**

Gelis, RV

## Systematic power counting for the average multiplicity

**I) Leading order:** $\mathrm{O}\left(\frac{1}{g^2}\right)$ but $(g\rho)^\infty$

$\langle n \rangle_{\mathrm{LO}}$ :



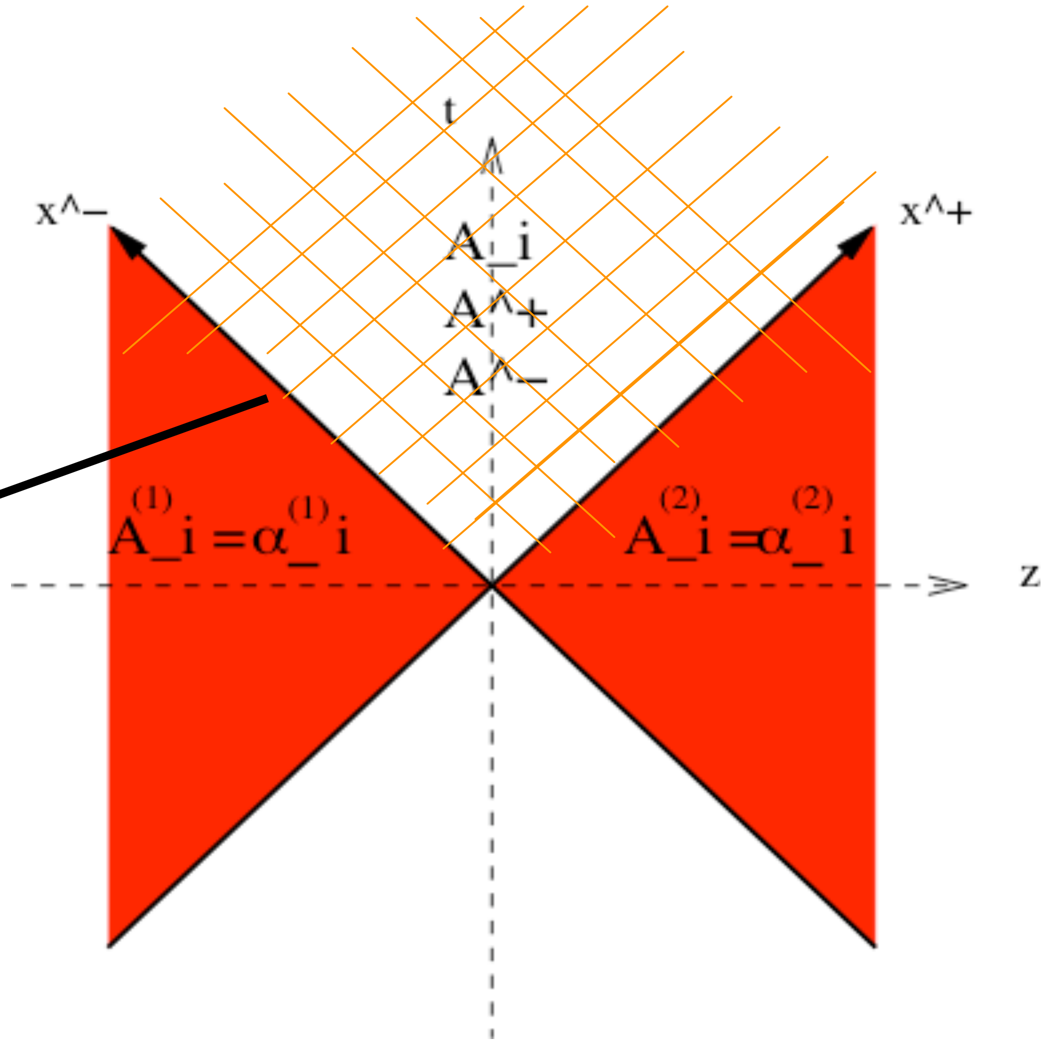

**From Cutkosky's rules, sum of all Feynman tree diagrams**

**$\Rightarrow$ solution of classical equations of motion with** ***retarded b.c.***

# Yang-Mills Equations for two nuclei

$$D_\mu F^{\mu\nu,a} = \delta^{\nu +}\rho_1^a(x_\perp)\delta(x^-) + \delta^{\nu -}\rho_2^a(x_\perp)\delta(x^+)$$

Kovner,McLerran,Weigert




**Initial conditions from matching eqns. of motion on light cone**

$$\tau = \sqrt{2x^+x^-}\,;\ \eta = \frac{1}{2}\ln\left(\frac{x^+}{x^-}\right)$$

# Lattice Formulation

Krasnitz, RV

- Hamiltonian in $A^\tau = 0$ gauge; per unit rapidity,

$$H = \frac{\tau}{2}\int d^2 r_\perp \left[ p^\eta p^\eta + \frac{1}{\tau^2} E_r E_r + \frac{1}{\tau^2}(D_r\Phi)(D_r\Phi) + F_{xy}F_{xy} \right]$$

For ``perfect'' pancake nuclei, boost invariant configurations

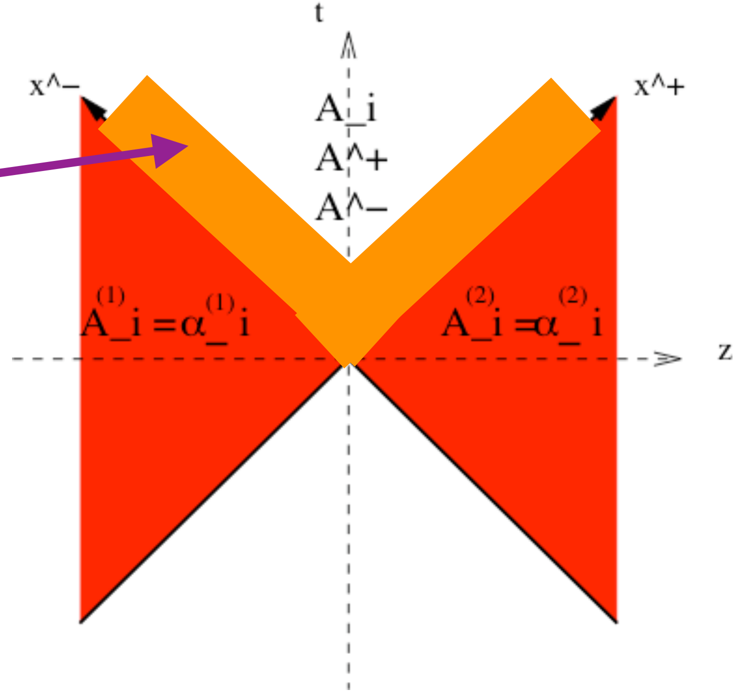
$$A_r(\tau,\eta,r_\perp) = A_r(\tau,r_\perp)\ ;\ A_\eta(\tau,\eta,r_\perp) = \Phi(\tau,r_\perp)$$

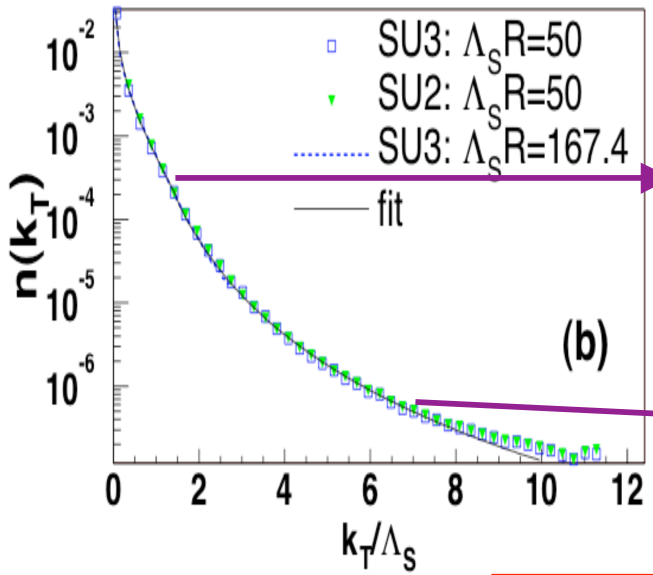
- Solve 2+1- D Hamilton's equations in real time for space-time evolution of glue in Heavy Ion collisions

Krasnitz,Nara,RV
Lappi

$$E_p \frac{d\langle n\rangle_{LO}}{d^3p} = \frac{1}{16\pi^3} \lim_{x^0,y^0\to\infty} \int d^3x\, d^3y e^{ip\cdot(x-y)} (\partial_{x^0} - iE_p)(\partial_{y^0} + iE_p)$$

$$\times \sum_{\text{phys.}\Lambda} \varepsilon_\mu^\lambda(p)\varepsilon^{\star\lambda}_\nu(p) A_a^\mu(x) A_c^\nu(y)$$




2D Bose-Einstein (generic for glassy non-eq. classical systems ?)

pert. tail

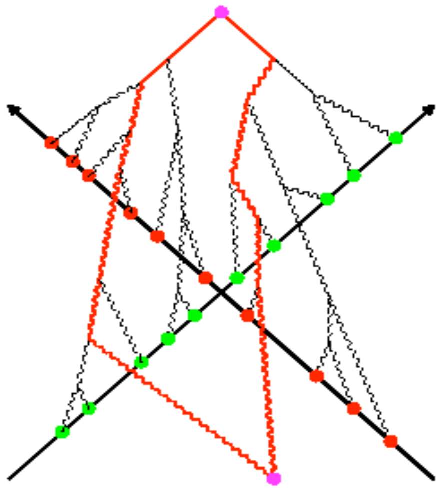
$$\boxed{\varepsilon = \frac{0.26}{g^2} \frac{\Lambda_s^4}{(\Lambda_s \tau)}}$$

Energy density
20 ~ 40 GeV/fm^3
at 0.3 fm at RHIC

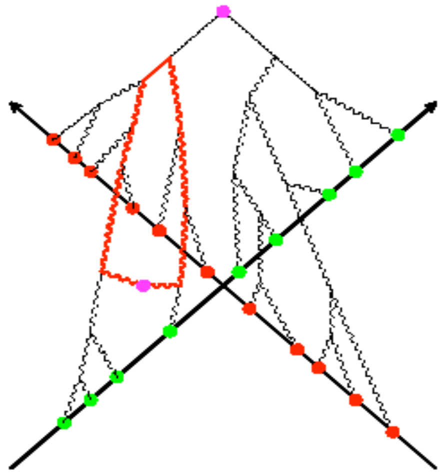
$$(\Lambda_s \approx 1.6 - 2\,\mathrm{GeV})$$

# II) Multiplicity at <u>next-to-leading order:</u> O ( g^0 )

$$\langle n \rangle_{\mathrm{NLO}} =$$



$$+$$



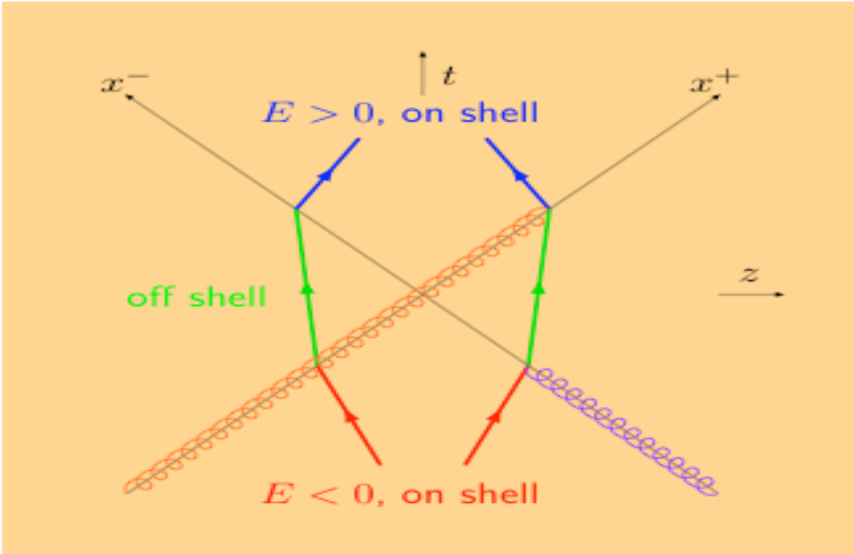
Gluon pair production contribution

One loop corrections to classical field contribute at same order

Remarkably, both terms can be computed by solving eq. of motion for the *small fluctuations* about the classical background field with retarded b.c.
- initial value problem

Gelis+RV

**Results same order in coupling as quark pair production contribution**



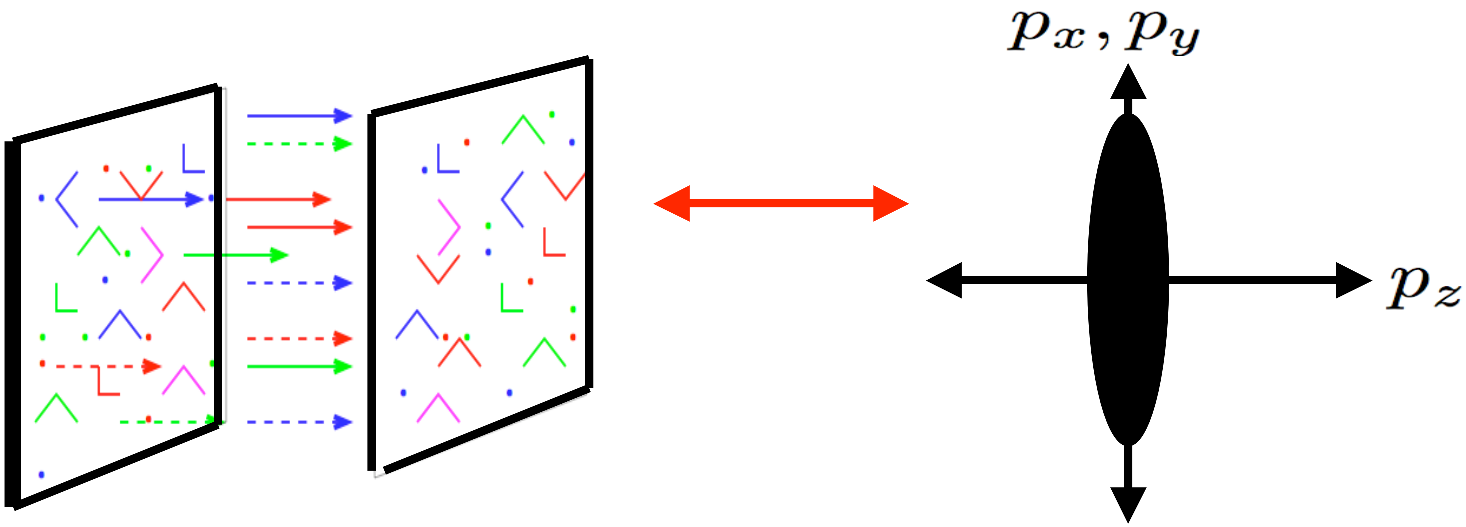

Gelis,Kajantie,Lappi

**NLO contributions may be essential to understand thermalization in heavy ion collisions**

**Also discussed in framework of Schwinger mechanism**

Kharzeev, Levin, Tuchin

In the glasma, the classical, boost invariant
E & B fields are purely longitudinal




Such anisotropic momentum distributions are very unstable-Weibel instability of E.M. plasmas

See Bödeker & Strickland talks

Small (quantum/NLO) *"rapidity dependent"* fluctuations can grow exponentially and generate longitudinal pressure - may hold key to thermalization

## Construct model of initial conditions with fluctuations:

i) $E_i(x_\perp,\eta) = 0 + \delta E_i(x_\perp,\eta) \qquad A^i = \alpha_1^i + \alpha_2^i$

$E_\eta(x_\perp,\eta) = ig[\alpha_1^i,\alpha_2^i] + \delta E_\eta(x_\perp,\eta) \qquad A_\eta = 0$

ii) **Method:**

**Generate random transverse configurations:** $\langle \delta\bar{E}_i(x_\perp)\delta\bar{E}_j(y_\perp)\rangle = \delta_{ij}\delta(x_\perp - y_\perp)$

**Generate Gaussian random function in \eta** $\langle F(\eta)F(\eta')\rangle = \Delta^2\delta(\eta-\eta')$

$$\boxed{\delta E_i(x_\perp,\eta) = \partial_\eta F(\eta)\,\delta\bar{E}_i(x_\perp)\ ;\ E_\eta(x_\perp,\eta) = -F(\eta)\,D_i\delta\bar{E}_i(x_\perp)}$$

**This construction explicitly satisfies Gauss' Law**

## Compute components of the Energy-Momentum Tensor

$$P_\perp = T^{xx} + T^{yy} = 2\,\mathrm{Tr}\left[F_{xy}^2 + E_\eta^2\right]$$

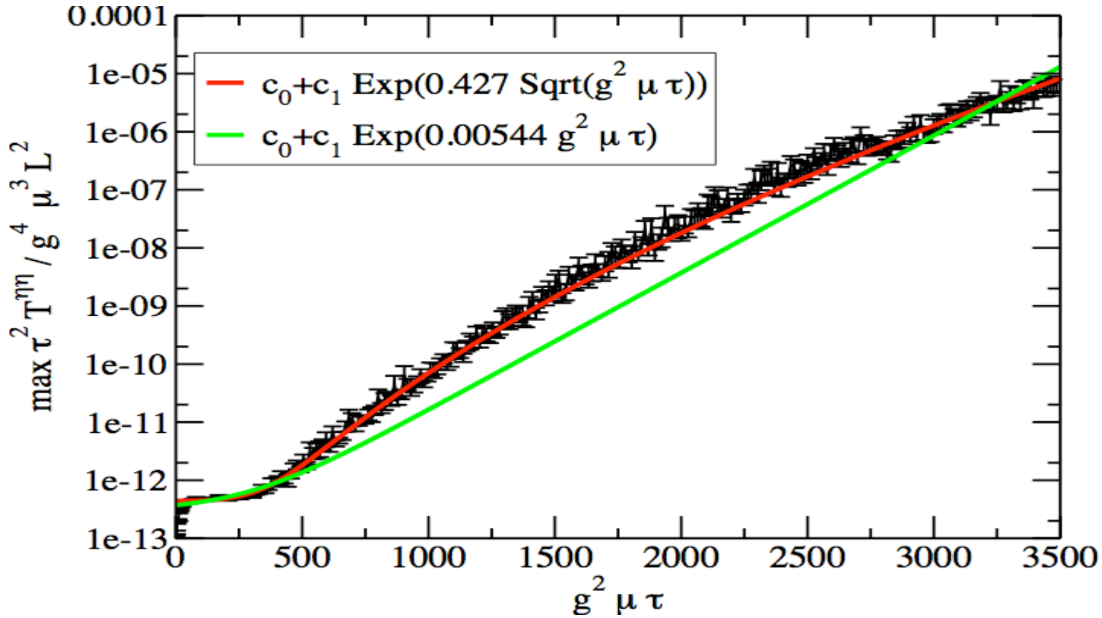
$$P_L = \tau^2 T^{\eta\eta} = \tau^{-2}\mathrm{Tr}\left[F_{\eta i}^2 + E_i^2\right] - \mathrm{Tr}\left[F_{xy}^2 + E_\eta^2\right]$$

$$\mathcal{H} = \tau T^{\tau\tau} \equiv \tau\left(P_\perp + P_L\right)$$

**Hard Loop prediction:** Arnold,Lenaghan,Moore

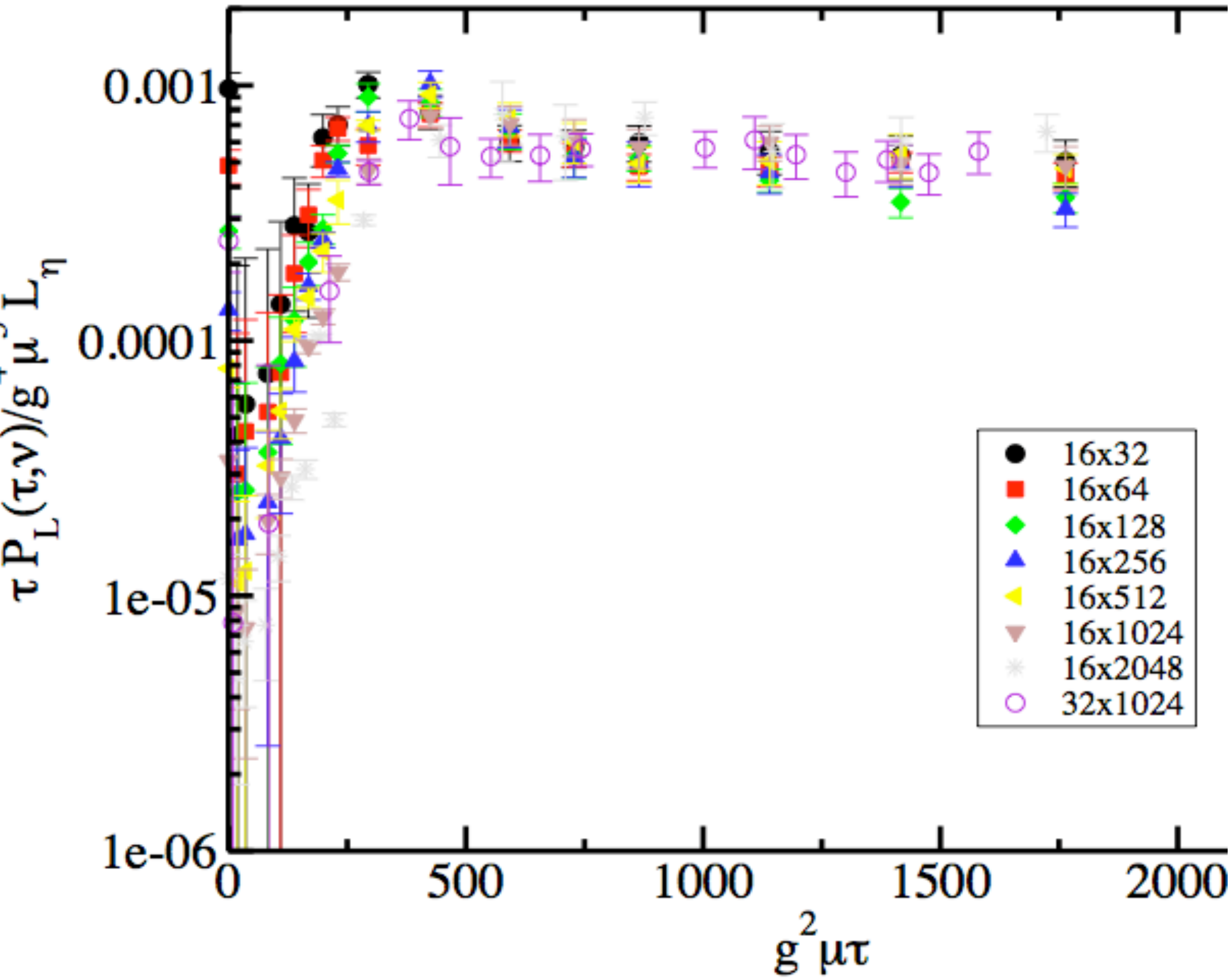
$$|A(\tau)|^2 \propto \exp\left(\int_0^\tau d\tau'\gamma(\tau')\right) \rightarrow \exp\left(C\sqrt{g^2\mu\tau}\right)$$

**Results from 3+1-D numerical simulations of Glasma exploding into the vacuum:**




Romatschke, RV

**Non-Abelian Weibel instability seen for very small rapidity dependent fluctuations**




**Instability saturates at late times-possible non-Abelian saturation of modes ?**

Fluctuations become of order of the background field when

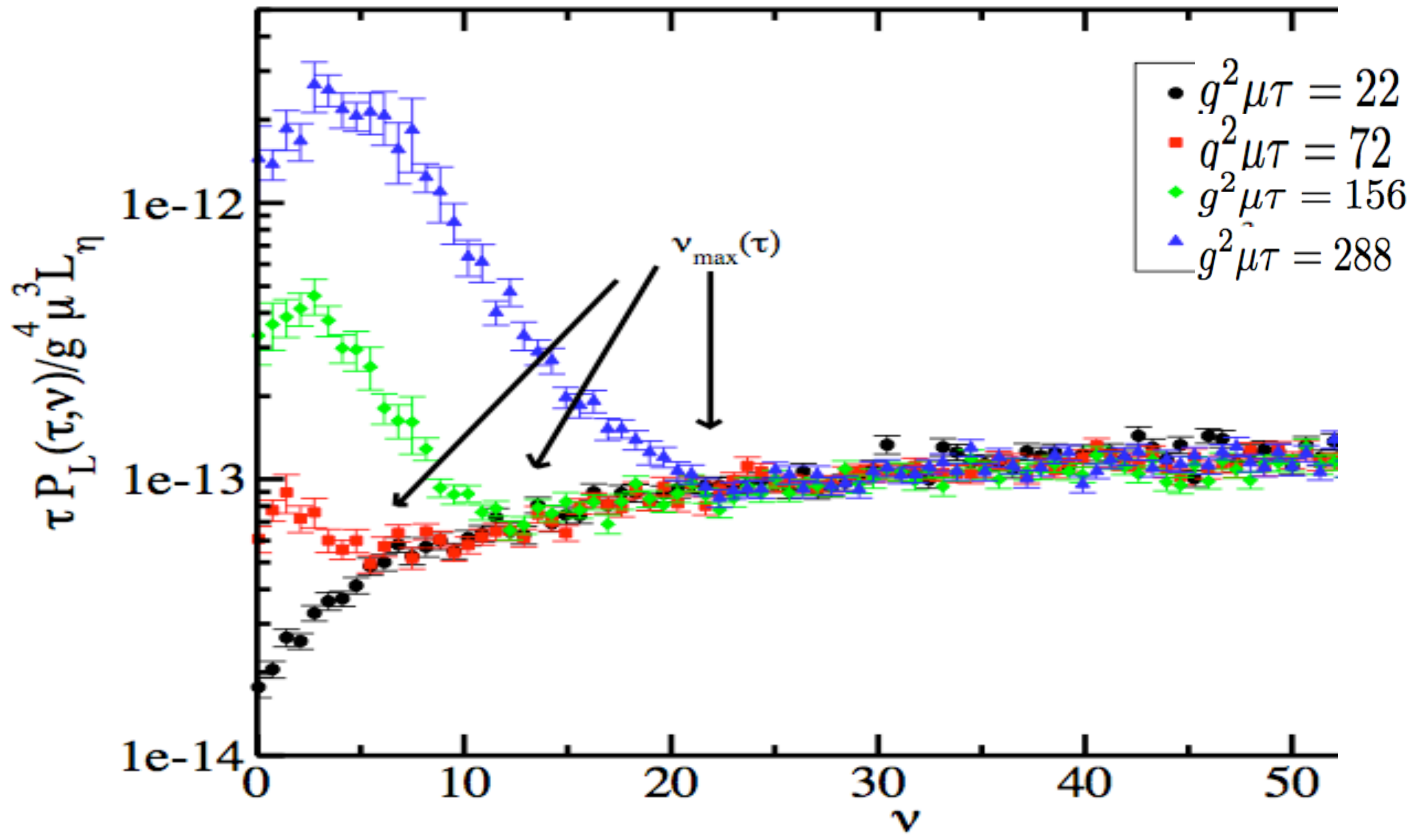
$$\tau \sim \frac{1}{C^2(g^2\mu)} \ln^2\left(\frac{1}{C_1 g^2}\right)$$

Expect $C_1 \sim O(1)$

Our numerical simulations allowed much smaller values:

$$C_1 \sim O(10^{-8} - 10^{-20})$$

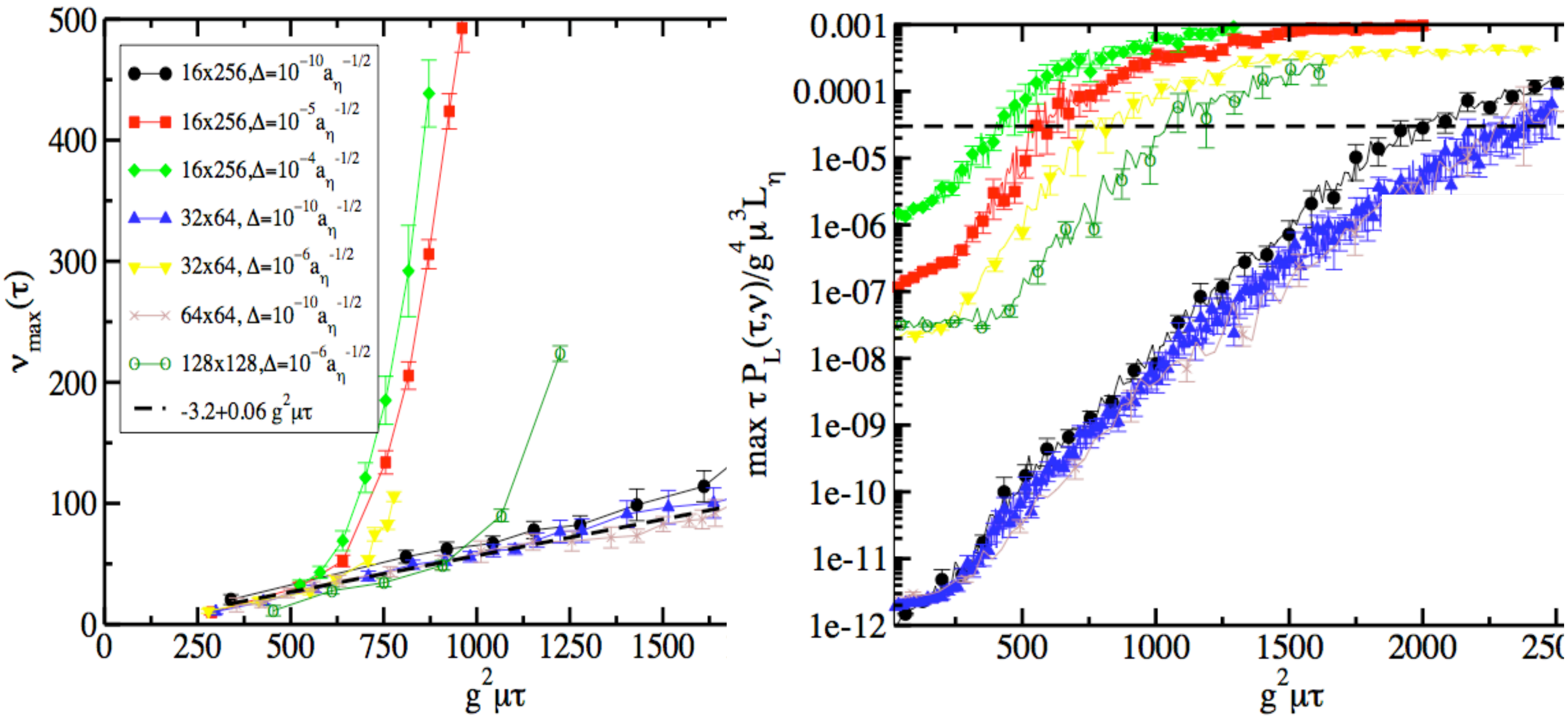
Hence the large times…




**Distribution of unstable modes also similar to kinetic theory**

Arnold, Lenaghan, Moore, Yaffe
Romatschke, Strickland, Rebhan

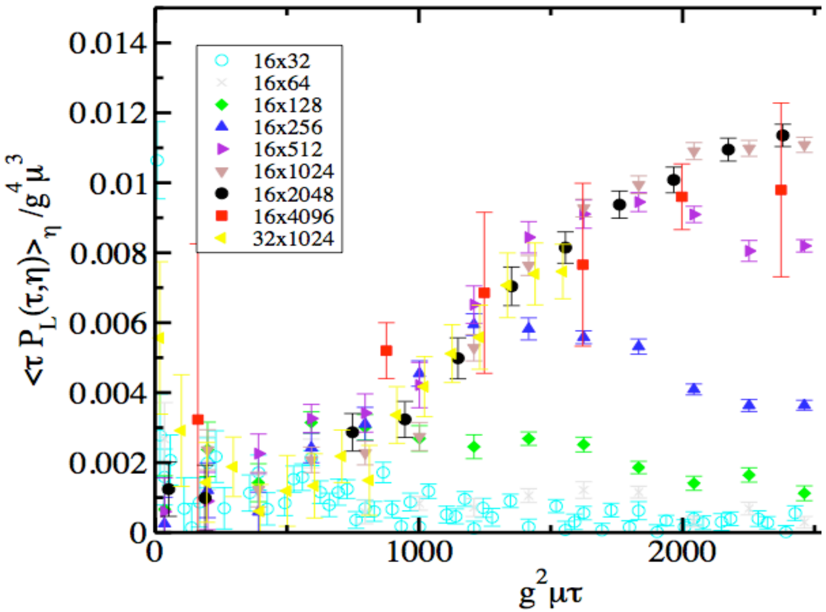
**Romatschke, RV**



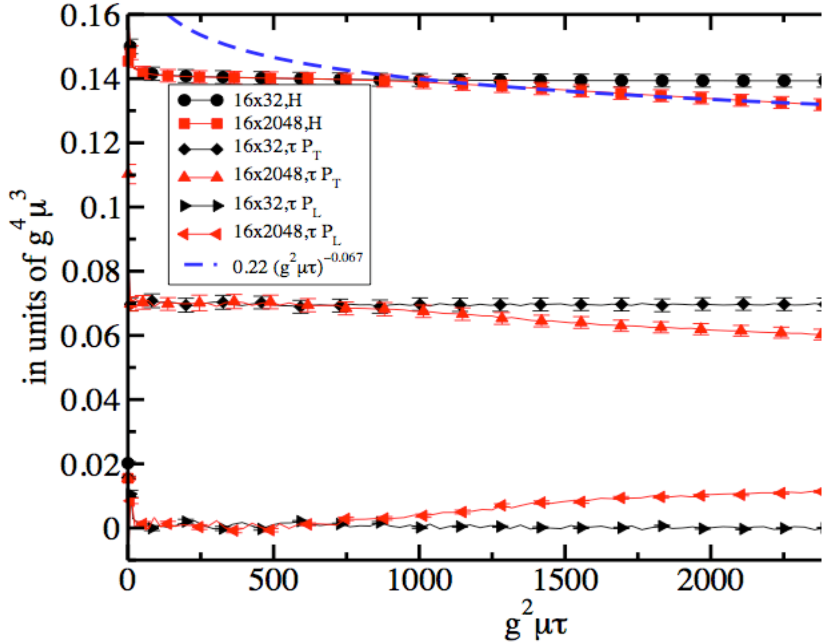

**Very rapid growth in max. frequency when modes of transverse magnetic field become large -**

**Lorentz force effect on hard transverse mom. modes ?**

Growth in longitudinal pressure…




Decrease in transverse pressure…



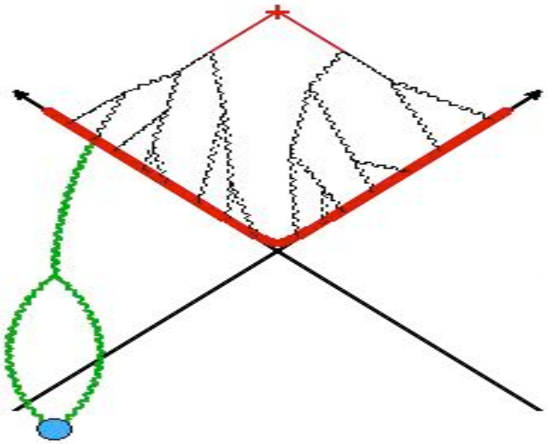

$$\varepsilon \sim \frac{1}{\tau^{1.067}}$$

**> free streaming but**
**< ideal hydro**

**Same conclusion from Hard Loop study**
**Romatschke-Rebhan**

## Comments:

a) Results very sensitive to spectrum of initial fluctuations- numerical results are for a first guess.



Fukushima, Gelis, McLerran

Recent WKB analysis of small fluctuations differs significantly…

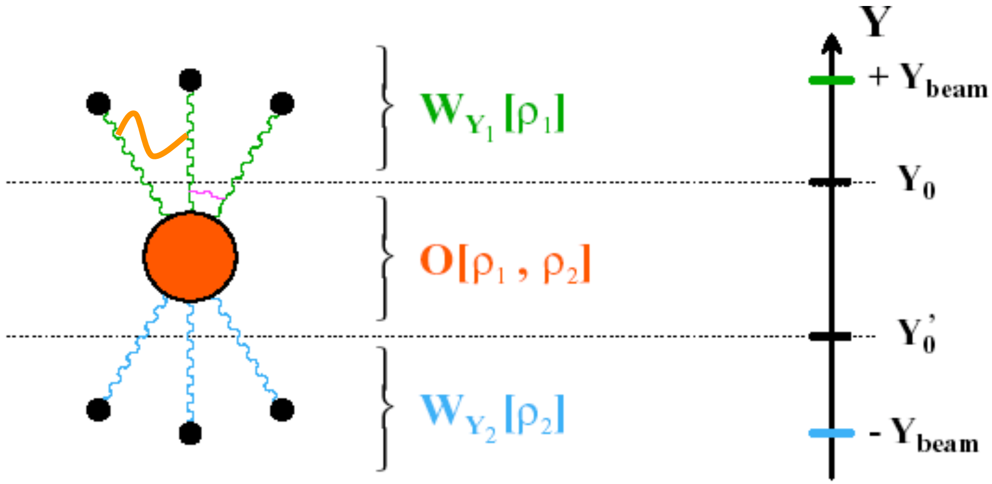
$$\langle a_i(\eta,x_\perp)a_j(\eta',x'_\perp)\rangle = \frac{1}{\tau\sqrt{-(\partial_\eta/\tau)^2-\partial_\perp^2}}\left(\delta_{ij}+\frac{\partial_i\partial_j}{(\partial_\eta/\tau)^2}\right)\delta(\eta-\eta')\delta^{(2)}(x_\perp-x'_\perp)$$

$$\langle e_i(\eta,x_\perp)e_j(\eta',x'_\perp)\rangle = \tau\sqrt{-(\partial_\eta/\tau)^2-\partial_\perp^2}\left(\delta_{ij}-\frac{\partial_i\partial_j}{(\partial_\eta/\tau)^2+\partial_\perp^2}\right)\delta(\eta-\eta')\delta^{(2)}(x_\perp-x'_\perp)$$

## Comments:

b) Understanding **high energy factorization** (analogous to proofs of collinear factorization) will be important for full NLO estimate

Gelis,Lappi,RV



# Summary and Outlook

Outlined an algorithm
to *systematically* compute particle production
in AA collisions to NLO

Pieces of this algorithm already exist:

- Pair production computation of Gelis, Lappi and Kajantie very similar

- Likewise, the 3+1-D computation of Romatschke and RV + 3+1-D computations of Lappi

- Result should include

  - All leading log small x evolution effects

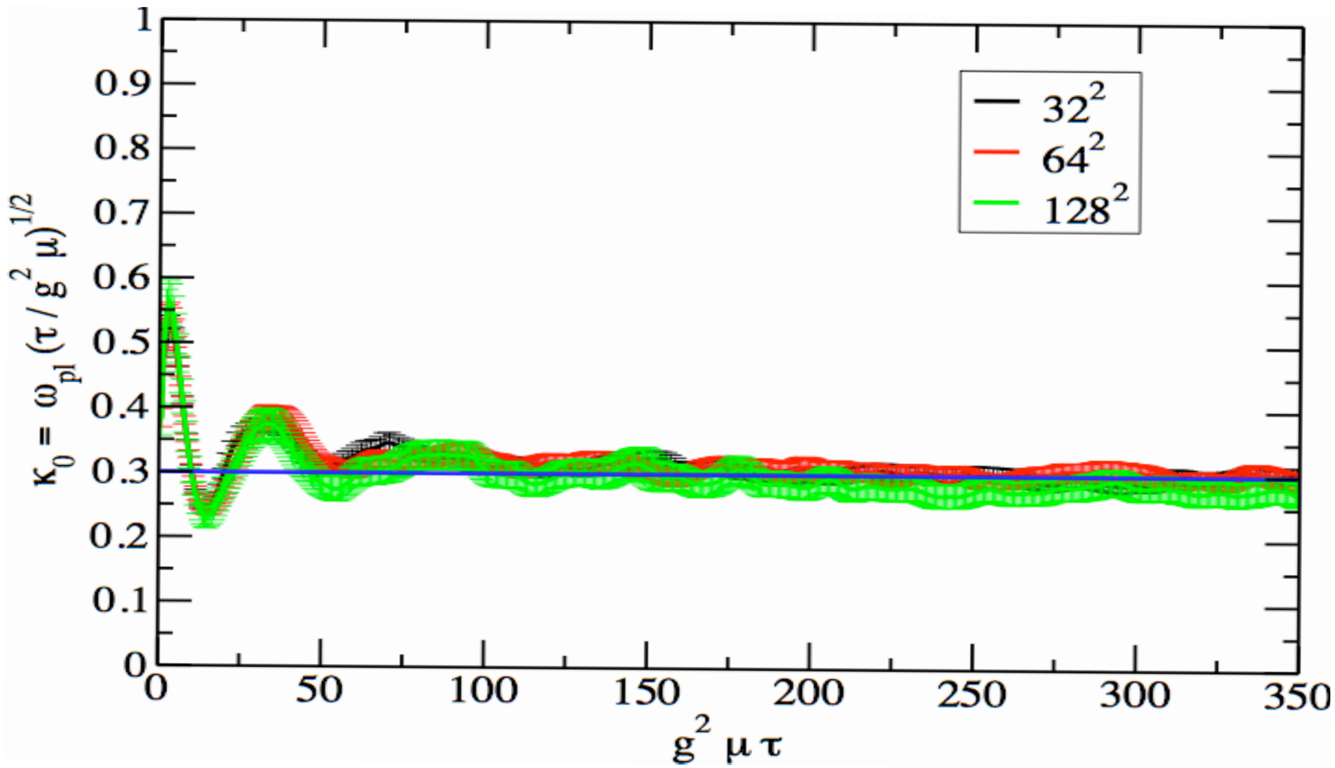
  - NLO contributions to particle production

- Very relevant for studies of energy loss, thermalization, topological charge, at early times

- Relation to kinetic theory formulation at late times - in progress (Gelis, Jeon, RV, in preparation)

# EXTRA SLIDES

## Growth rate proportional to plasmon mass…




$\frac{C}{\kappa_0}$ ~ 2 * prediction from HTL kinetic theory