Mass and Spin Measurement with $M_{T2}$ and MAOS Momentum

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Outline

1. Motivation: New Physics Events with Missing Transverse Momentum

2. $M_{T2}$-Kink Method for Mass Measurement

3. $M_{T2}$-Assisted On-Shell (MAOS) Momentum and its Applications:
   - Sparticle Spin and Higgs Mass Determination

4. Summary
Motivations for new physics at the TeV scale:

- **Hierarchy Problem**

\[ \delta m_H^2 \sim \frac{g^2}{8\pi^2} \Lambda_{\text{SM}}^2 \sim M_Z^2 \quad \Rightarrow \quad \Lambda_{\text{SM}} \sim 1 \text{ TeV} \]

- **Dark Matter**

Thermal WIMP with \( \Omega_{\text{DM}} h^2 \sim \frac{0.1}{g^4} \left( \frac{m_{\text{DM}}}{1 \text{ TeV}} \right)^2 \sim 0.1 \)

\[ \Rightarrow \quad m_{\text{DM}} \sim 1 \text{ TeV} \]

Many new physics models solving the hierarchy problem while providing a DM candidate involve a \( Z_2 \)-parity symmetry under which the new particles are \( Z_2 \)-odd, while the SM particles are \( Z_2 \)-even:

- SUSY with \( R \)-parity,
- Little Higgs with \( T \)-parity,
- UED with \( KK \)-parity, ...

* At colliders, new particles are produced always in pairs.
* Lightest new particle is stable, so a good candidate for WIMP DM.
LHC Signal

Pair-produced new particles \((Y + \bar{Y})\) decaying into visible SM particles \((V)\) plus invisible WIMPs \((\chi)\):

\[
pp \rightarrow U + Y + \bar{Y} \rightarrow U + \sum V(p_i) + \chi(k) + \sum V(q_j) + \chi(l) \\
\text{(multi-jets + leptons + } p_T) 
\]

\(U \equiv\) Upstream momenta carried by the visible SM particles not from the decay of \(Y + \bar{Y}\) \((\bar{Y} \text{ is not necessarily the antiparticle of } Y.)\)
Mass measurement of these new particles is quite challenging:
* Initial parton momenta in the beam-direction are unknown.
* Each event involves two invisible WIMPs.

Kinematic methods of mass measurement:
 i) Endpoint Method
 ii) Mass Relation Method
 iii) $M_{T2}$-Kink Method

Spin measurement appears to be even more challenging:
* It often requires a more refined event reconstruction and/or polarized mother particle state.

MAOS momentum provides a systematic approximation to the invisible WIMP momentum, and thus can be useful for spin and mass measurements.
**Kinematic Methods of Mass Measurement**

i) **Endpoint Method** Hinchliffe et. al.; Allanach et. al.; Gjelsten et. al.;…

Endpoint value of the invariant mass distribution of visible (SM) decay products depend on the new particle masses.

* 3-step squark cascade decays when $m_{\tilde{q}} > m_{\chi_2} > m_{\tilde{\ell}} > m_{\chi_1}$

\[
m_{\ell\ell}^{\text{max}} = m_{\chi_2} \sqrt{(1 - m_{\tilde{\ell}}^2 / m_{\chi_2}^2)(1 - m_{\tilde{\chi}_1}^2 / m_{\tilde{\ell}}^2)}
\]
\[
m_{q\ell\ell}^{\text{max}} = m_{\tilde{q}} \sqrt{(1 - m_{\chi_2}^2 / m_{\tilde{q}}^2)(1 - m_{\tilde{\chi}_1}^2 / m_{\chi_2}^2)}
\]
\[
m_{q\ell}^{\text{max (high)}} = m_{\tilde{q}} \sqrt{(1 - m_{\chi_2}^2 / m_{\tilde{q}}^2)(1 - m_{\tilde{\chi}_1}^2 / m_{\tilde{\ell}}^2)}
\]
\[
m_{q\ell}^{\text{max (low)}} = m_{\tilde{q}} \sqrt{(1 - m_{\chi_2}^2 / m_{\tilde{q}}^2)(1 - m_{\tilde{\ell}}^2 / m_{\chi_2}^2)}
\]
Result for SUSY SPS1a Point

Weiglein et. al. hep-ph/0410364

Input masses: \((m_{\tilde{q}}, m_{\chi_2}, m_{\tilde{\ell}}, m_{\chi_1}) = (540, 177, 143, 96)\) GeV

Fitted masses: \((543 \pm 13, 180 \pm 9, 146 \pm 11, 98 \pm 9)\) \((\int \mathcal{L} = 100 \text{ fb}^{-1})\)
Reconstruct the missing momenta using all available constraints.

* A pair of symmetric 3-step cascade decays of squark pair

- 16 unknowns: $k^\mu$, $l^\mu$, $k'^\mu$, $l'^\mu$
- 12 mass-shell constraints:
  \[ k^2 = l^2 = k'^2 = l'^2, \]
  \[ (k + p_3)^2 = (l + q_3)^2 = (k' + p'_3)^2 = (l' + q'_3)^2, \]
  \[ (k + p_2 + p_3)^2 = (l + q_2 + q_3)^2 = (k' + p'_2 + p'_3)^2 = (l' + q'_2 + q'_3)^2, \]
  \[ (k + p_1 + p_2 + p_3)^2 = (l + q_1 + q_2 + q_3)^2 = (k' + p'_1 + p'_2 + p'_3)^2 = (l' + q'_1 + q'_2 + q'_3)^2, \]
- 4 $p_T$-constraints: $k_T + l_T = p_T$, $k'_T + l'_T = p'_T$
* 8 (complex) solutions for each event-pair, some of which are real.
* Many wrong solutions from wrong combinatorics.

For given set of event-pairs, number of real solutions shows a peak at the correct masses: Cheng et. al.

Input masses: \( (m_{\tilde{q}}, m_{\chi_2}, m_{\tilde{\ell}}, m_{\chi_1}) = (568, 180, 143, 97) \) GeV

Fitted masses: \( (562 \pm 4, 179 \pm 3, 139 \pm 3, 94 \pm 3) \) \((\int \mathcal{L} = 300 \text{ fb}^{-1})\)
Remarks

- Mass relation method and endpoint method require a long decay chain, at least 3-step chain, to determine the involved new particle masses.

- However, there are many cases (including a large fraction of popular scenarios) that such a long decay chain is not available:
  A simple example: mSUGRA with \( m_0^2 > 0.6 M_{1/2}^2 \Rightarrow m_{\tilde{\ell}} > m_{\chi_2} \)

- SUSY with \( m_{\text{sfermion}} \gg m_{\text{gaugino}} \):
  (Focus point scenario, Loop-split SUSY, Some string moduli-mediation, ...)

* Mass relation method simply can not be applied.
* Endpoint method determines only the gaugino mass differences.
* \( M_{T2} \)-kink method can determine the full gaugino mass spectrum.
iii) $M_{T2}$-Kink Method

Cho, Choi, Kim, Park;  Barr, Gripaios, Lester

$M_{T2}$ is a generalization of the transverse mass to an event producing two invisible particles with the same mass.

**Transverse mass of** $Y \rightarrow V(p) + \chi(k)$:

$$M_{T}^2 = m_{V}^2 + m_{\chi}^2 + 2\sqrt{m_{V}^2 + |p_T|^2} \sqrt{m_{\chi}^2 + |k_T|^2} - 2p_T \cdot k_T$$

* Invariant Mass:  

$$M^2 = m_{V}^2 + m_{\chi}^2 + 2\sqrt{m_{V}^2 + |p|^2} \sqrt{m_{\chi}^2 + |k|^2} - 2p \cdot k$$

$$= m_{V}^2 + m_{\chi}^2 + 2\sqrt{m_{V}^2 + |p_T|^2} \sqrt{m_{\chi}^2 + |k_T|^2} \cosh(\eta_V - \eta_{\chi}) - 2p_T \cdot k_T \geq M_{T}^2$$

One can use an arbitrary trial WIMP mass $m_{\chi}$ to define $M_{T}$.  
(True WIMP mass $= m_{\chi}^{true}$).

* For each event, $M_{T}$ is an increasing function of $m_{\chi}$.

* For all events, $M_{T}(m_{\chi} = m_{\chi}^{true}) \leq m_{Y}^{true}$ in the zero width limit.
\[ M_{T2}(\text{event}; m_\chi) = \left\{ \text{event} \right\} = \left\{ m_{V_1}, p_T, m_{V_2}, q_T, p_T' \right\} \]

\[ = \min_{k_T+l_T=p_T} \left[ \max \left( M_T(p_T, m_{V_1}, k_T, m_\chi), M_T(q_T, m_{V_2}, l_T, m_\chi) \right) \right] \]
\[ M_{T2}(\text{event}; m_\chi) \quad (\{\text{event}\} = \{m_{V1}, p_T, m_{V2}, q_T, p_T^\prime\}) \]

\[ = \min_{k_T + l_T = p_T} \left[ \max \left( M_T(p_T, m_{V1}, k_T, m_\chi), M_T(q_T, m_{V2}, l_T, m_\chi) \right) \right] \]

\[ (p_T^\prime = -p_T - q_T - u_T) \]

* For each event, \( M_{T2} \) is an increasing function of \( m_\chi \).
* For all events, \( M_{T2}(m_\chi = m_{\chi}^{\text{true}}) \leq m_Y^{\text{true}} \) in the zero width limit.
**$M_{T2}$-Kink**

If the event set has a certain variety, which is in fact quite generic,

$$M_{T2}^{\text{max}}(m_\chi) = \max_{\{\text{all events}\}} \left[ M_{T2}(\text{event}; m_\chi) \right]$$

has a kink-structure at $m_\chi = m_\chi^{\text{true}}$ with $M_{T2}^{\text{max}}(m_\chi = m_\chi^{\text{true}}) = m_\chi^{\text{true}}$. 
Kink (due to different slopes) appears if

- The visible decay products of $Y \to V + \chi$ have a sizable range of invariant mass $m_V$, which would be the case if $V$ is a multi-particle state. Cho, Choi, Kim, Park

- There are events with a sizable range of upstream transverse momentum $u_T$, which would the case if $Y$ is produced with a sizable ISR or produced through the decay of heavier particle. Gripaios

* Kink is a fixed point (or a point of enhanced symmetry) at $m_\chi = m_\chi^{\text{true}}$ under the variation of $m_V$ and $u_T$.

* For cascade decays, $M_{T2}$-kink method can be applied to various sub-events:
Gluino $M_{T2}$-Kink in heavy sfermion scenario

Cho, Choi, Kim, Park

$M_{T2}$ of hard 4-jets (no $b$, no $\ell$) which are mostly generated by the gluino-pair 3-body decay: $\tilde{g}\tilde{g} \rightarrow q\bar{q}\chi\bar{q}\chi$, where $m_{\tilde{g}} \lesssim 1$ TeV and $m_{\tilde{q}} \sim$ few TeV.

Input masses: $(m_{\tilde{g}}, m_{\chi_1}) = (780 \text{ GeV}, 98 \text{ GeV})$ (Wino-like $\chi_1$)

Fitted masses: $(776 \pm \text{ few}, 97 \pm \text{ few})$ \ $(\int \mathcal{L} = 300 \text{ fb}^{-1})$
Kink itself is quite generic, but often it might not be sharp enough to be visible in the real analysis.

Related methods which might be useful:

- Number of real solutions for $M_{T2}$-assisted on-shell (MAOS) momenta, which is expressed as a function of $m_\chi$, might show a sharper kink at $m_\chi = m_{\chi}^{\text{true}}$. Cheng and Han; arXiv:0810.5178

- $M_{T2}$-kink is a fixed point under $\partial/\partial m_V$, $\partial/\partial u_T$:

$$\implies \left( \frac{\partial}{\partial m_V}, \frac{\partial}{\partial u_T} \right) \left\{ M_{T2}^{\text{max}}(m_\chi^{\text{trial}} = 0), M_{CT}^{\text{max}}, \ldots \right\}$$

can provide information which would allow mass determination in the absence of long decay chain. Torvey: arXiv:0802.2879


More general and systematic method to find a variable ( = singularity coordinate ) most sensitive to the singularity structure providing information on the unknown masses in missing energy events.
MAOS momentum is a collider event variable designed to approximate systematically the invisible particle momentum for an event set producing two invisible particles with the same mass.
Construction of the MAOS WIMP momenta $k_{\mu}^{maos}$ and $l_{\mu}^{maos}$

i) Choose appropriate trial WIMP and mother particle masses: $m_\chi$, $m_Y$.

ii) Determine the transverse MAOS momenta with $M_{T2}$:

$$M_{T2} = M_T(p^2, p_T, m_\chi, k_T^{maos}) \geq M_T(q^2, q_T, m_\chi, l_T^{maos})$$

$$\left( p_T = k_T^{maos} + l_T^{maos} \right)$$

* $M_{T2}$ selects unique $k_T^{maos}$ and $l_T^{maos}$:

![Diagram showing construction of $k_T^{maos}$ and $l_T^{maos}$ for balanced and unbalanced cases.]
iii) Two possible schemes for the longitudinal and energy components:

\[
Y(p + k) \bar{Y}(q + l) \rightarrow V_1(p) + \chi(k) + V_2(q) + \chi(l)
\]

**Scheme 1:**

\[
k_{maos}^2 = \ell_{maos}^2 = m_\chi^2, \quad (k_{maos} + p)^2 = (\ell_{maos} + q)^2 = m_Y^2
\]

**Scheme 2:**

\[
k_{maos}^2 = \ell_{maos}^2 = m_\chi^2, \quad \frac{k_{maos}}{k_0} = \frac{p_z}{p_0}, \quad \frac{\ell_{maos}}{\ell_0} = \frac{q_z}{q_0}
\]

(Scheme 2 can work even when \( Y + \bar{Y} \) are in off-shell.)

The MAOS constructions are designed to have \( k_{maos}^\mu = k_{true}^\mu \) for the \( M_{T2} \) endpoint events when \( m_\chi = m_\chi^{true} \) and \( m_Y = m_Y^{true} \).

\[\rightarrow \] One can systematically reduce \( \Delta k/k \equiv (k_{maos}^\mu - k_{true}^\mu)/k_{true}^\mu \) with an \( M_{T2} \)-cut selecting the near endpoint events.
For each event, MAOS momenta obtained in the scheme 1 are real iff
\[ m_Y \geq M_{T2}(\text{event}; m_\chi). \]

\[ \implies \text{MAOS momenta are real for all events if} \]
\[ m_Y \geq M_{T2}^{\max}(m_\chi) \equiv \max_{\{\text{events}\}} \left[ M_{T2}(\text{event}; m_\chi) \right] \left( m_Y^{\text{true}} = M_{T2}^{\max}(m_\chi^{\text{true}}) \right) \]

* If \( m_\chi^{\text{true}} \) and \( m_Y^{\text{true}} \) are known, use \( m_\chi = m_\chi^{\text{true}} \) and \( m_Y = m_Y^{\text{true}} \).

* Unless, one can use \( m_\chi = 0 \) and \( m_Y = M_{T2}^{\max}(0) \).

Precise knowledge of \( m_\chi^{\text{true}} \) and \( m_Y^{\text{true}} \) might not be essential if
\[ \left( m_\chi^{\text{true}} / m_Y^{\text{true}} \right)^2 \ll 1: \]
\[
\left( \frac{\Delta k}{k} \right)_{m_\chi^{\text{true}}, m_Y^{\text{true}}} - \left( \frac{\Delta k}{k} \right)_{m_Y = M_{T2}^{\max}(0)} = \mathcal{O} \left( \left( \frac{m_\chi^{\text{true}}}{m_Y^{\text{true}}} \right)^2 \right),
\]
\[ \frac{\Delta k_T}{k_T} = \frac{\tilde{k}_T - k_T^{\text{true}}}{k_T^{\text{true}}} \] distribution for \( \bar{q}\bar{q}^* \rightarrow q\bar{q}\bar{q}\bar{q} \):

\[ \tilde{k}_T = \frac{1}{2} p_T \quad (\tilde{k}_T + \tilde{\ell}_T = p_T) \]
\[ \tilde{k}_T = k_T^{\text{maos}} \text{ for full events} \]
\[ \tilde{k}_T = k_T^{\text{maos}} \text{ for the top 10\% of near endpoint events} \]
Example 1: Gluino/KK-gluon 3-body decay for SPS2 point and its UED equivalent:

\[ s = \left( p_q + p_{\bar{q}} \right)^2, \quad t_{\text{true}} = \left( p_{q(\bar{q})} + k_{\text{true}} \right)^2, \quad t_{\text{maos}} = \left( p_{q(\bar{q})} + k_{\text{maos}}^\pm \right)^2 \]

Without \( k_{\text{maos}}^\mu \), one may consider the \textbf{s-distribution} to distinguish \textbf{gluino} from \textbf{KK-gluon}: Csaki, Heinonen, Perelstein
With $k_{\text{maos}}^\mu$ (scheme 1), one can use the $s-t_{\text{maos}}$ distribution clearly distinguishing the gluino from the KK-gluon: arXiv:0810.4853[hep-ph]
Example 2: Drell-Yan pair production of \textbf{slepton or KK-lepton} for SUSY SPS1a point and its UED equivalent:\textsuperscript{Barr}

\[ \frac{d\Gamma}{d\cos \theta_Y} \quad \text{and} \quad \frac{d\Gamma}{d\cos \theta_\ell} \quad \text{of} \quad q\bar{q} \rightarrow Z^0/\gamma \rightarrow Y\bar{Y} \rightarrow \ell\chi\bar{\ell}\chi \]

\( Y = \text{slepton or KK-lepton, } \chi = \text{LSP or KK-photon, } \cos \theta_Y = \hat{p}_Y \cdot \hat{p}_{\text{beam}} \text{ in the CM frame of } Y\bar{Y}, \)

\( \cos \theta_\ell = \hat{p}_\ell \cdot \hat{p}_{\text{beam}} \text{ in the CR(rapidity) frame of } \ell\bar{\ell} \)
Without MAOS, one may look at the lepton angle \((\cos \theta_\ell)\) distribution to distinguish the slepton pair production from the KK-lepton pair production: Barr

With MAOS momentum (scheme 1), the mother particle production angle \((\cos \theta_Y)\) can be reconstructed: Cho, Choi, Kim, Park

\[
Y(p + k_{\text{maos}}^{\pm})\bar{Y}(q + l_{\text{maos}}^{\pm}) \rightarrow \ell(p)\chi(k_{\text{maos}}^{\pm})\bar{\ell}(q)\chi(l_{\text{maos}}^{\pm})
\]

\[
\frac{d\Gamma}{d \cos \theta_Y^{\text{maos}}} \equiv \sum_{\alpha=\pm} \sum_{\beta=\pm} \frac{d\Gamma}{d \cos \theta_{\alpha\beta}}
\]

\[
(\cos \theta_{\pm\pm} = \hat{p}_Y \cdot \hat{p}_{\text{beam}} \text{ for } k_{\text{maos}}^{\pm} \text{ and } l_{\text{maos}}^{\pm})
\]
\[
\frac{d\Gamma}{d \cos \theta_\ell} \quad \text{vs} \quad \frac{d\Gamma}{d \cos \theta_Y^{\text{maos}}}
\]

with appropriate event cut (\(\exists\) the \(M_{T2}\)-cut selecting the top 30 \%) while including the detector smearing effect for SUSY SPS1a and its UED equivalent: (Knowledge of the mass is not essential.)
$H \rightarrow WW \rightarrow \ell(p) \nu(k) \ell(q) \nu(l)$

Use the scheme 2 which approximates well the neutrino momenta even when $W$-bosons are in off-shell.

$$m_{H}^{\text{maos}} = (p + q + k^{\text{maos}} + l^{\text{maos}})^2$$

![Graph](image1.png)

**full event** 
($m_{H} = 140$ GeV)

![Graph](image2.png)

**top 30%**
Correlation between $\Delta \Phi_{ll} = \frac{p_T \cdot q_T}{|p_T||q_T|}$ and $M_{T2}$:

In the limit of vanishing ISR, $M_{T2}^2 = 2|p_T||q_T|(1 + \cos \Delta \Phi_{ll})$

Even with ISR, such correlation persists:

Using $\Delta \Phi_{ll}$ and $M_{T2}$ for the event selection, both the signal to background ratio and the efficiency of the MAOS approximation can be enhanced together.
- Event generation with PYTHIA6.4 with $\int L dt = 10 \text{ fb}^{-1}$
- Detector simulation with PGS4
- Include $q\bar{q}, gg \rightarrow WW$ and $t\bar{t}$ backgrounds
- Event selection including the optimal cut of $M_{T2}$ and $\Delta\Phi_{ll}$

$m_H = 140, \ M_{T2} > 51$

$m_H = 150, \ M_{T2} > 57$

$m_H = 180, \ M_{T2} > 68$

$m_H = 190, \ M_{T2} > 70$
1-σ error of $m_H$ from the likelihood fit to the $m_H^{\text{maos}}$ distribution
Summary

- $M_{T2}$-kink method (or related methods) might be able to determine new particle masses with missing energy events, even when a long decay chain is not available.

- MAOS momenta provide a systematic approximation to the invisible particle momenta in missing energy events, which can be useful for a spin measurement of new particle.

- MAOS momenta can be useful also for some SM processes with two missing neutrinos, particularly for probing the properties of the Higgs boson and top quark with
  
  * $H \rightarrow W^+W^- \rightarrow \ell^+\nu\ell\bar{\nu}$,
  * $t\bar{t} \rightarrow bW^+\bar{b}W^- \rightarrow b\ell^+\nu\bar{b}\ell\bar{\nu}$. 