Radiative and rare semileptonic B decays
(news 2009/2010)

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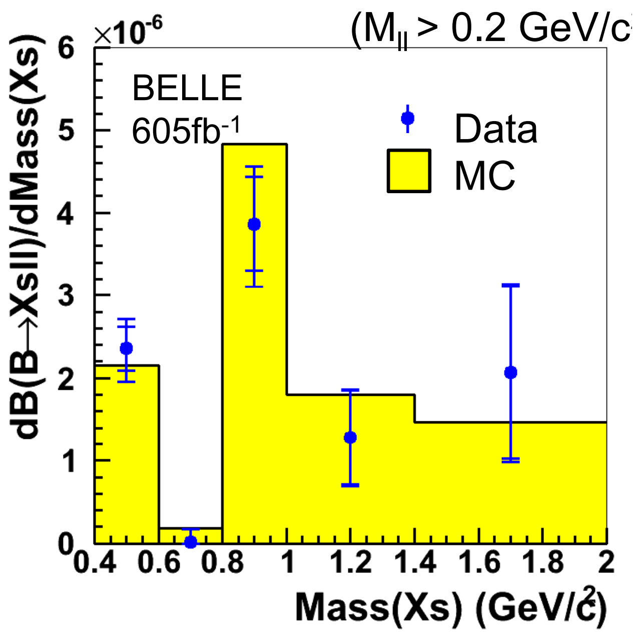
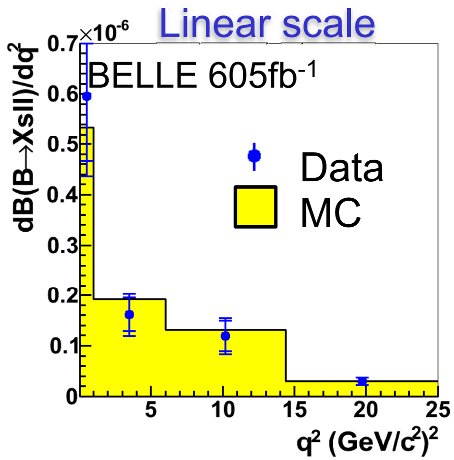
1. New, more precise determination of $\mathcal{B}(B \to X_s l^+ l^-)$ by Belle.

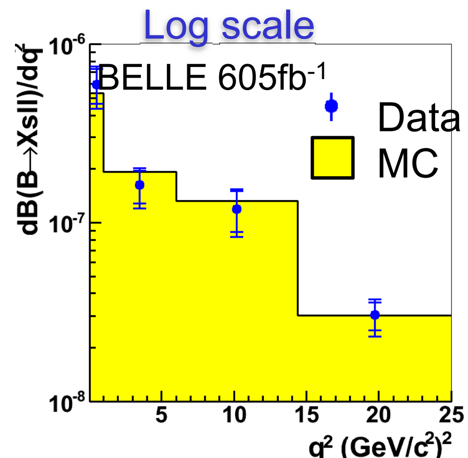
Slide from T. Ijima at Lepton-Photon 2009:

Detailed property of $B \to X_s ll$

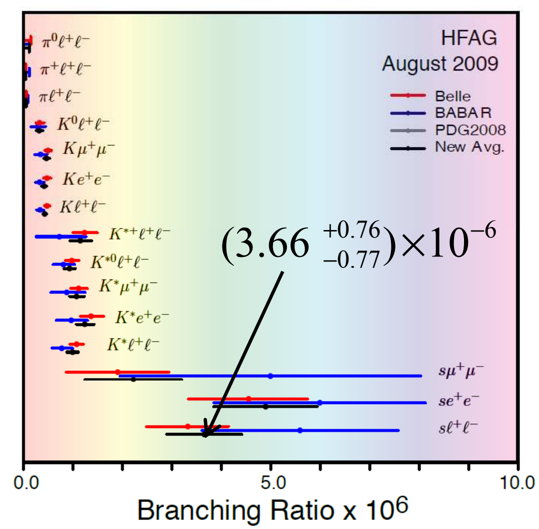
Δ(stat.) much larger than Δ(syst.)







$\mathcal{B}(B \to X_s \ell^+ \ell^-)$



For entire MXs region

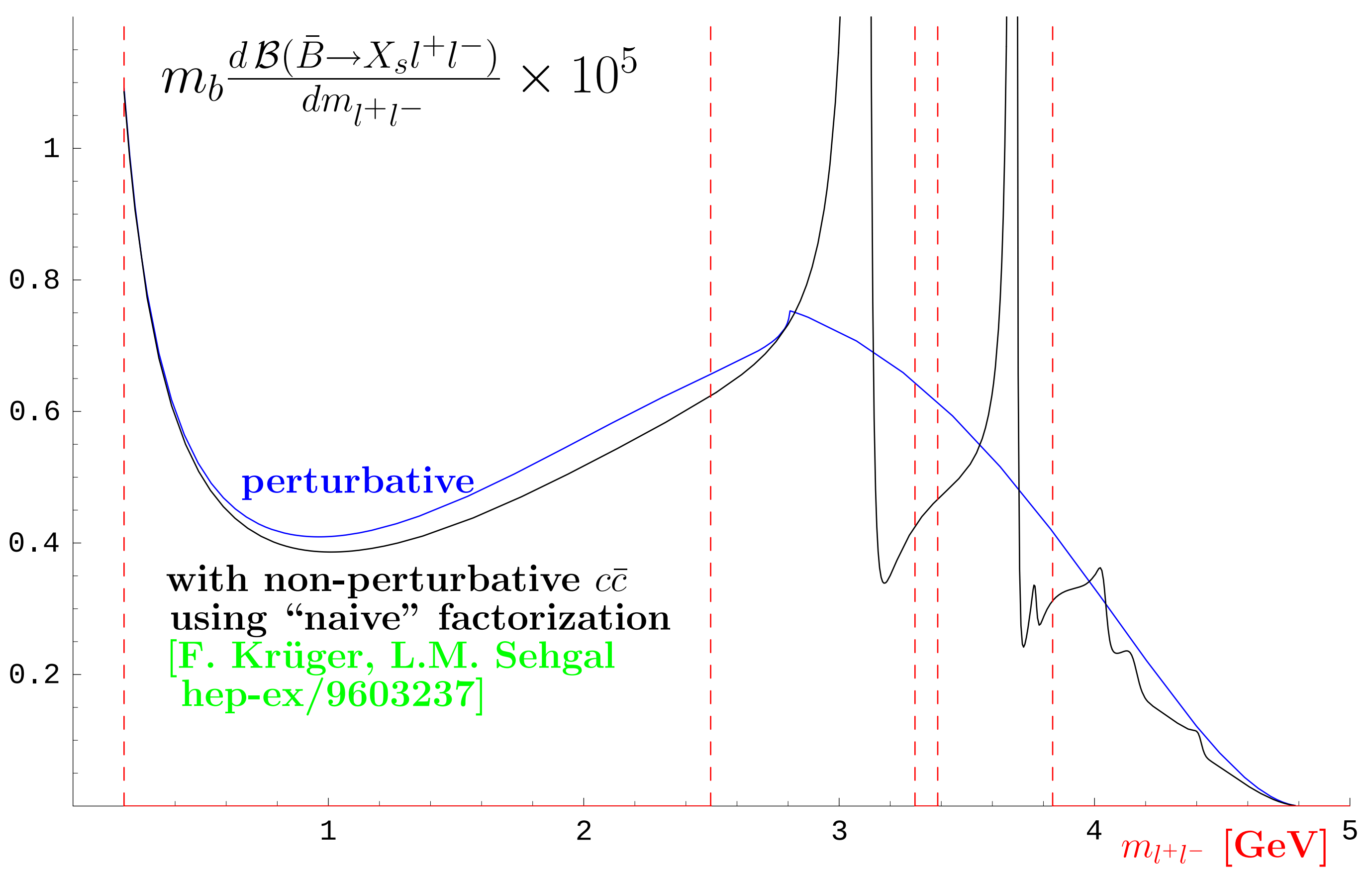
$$Br(B \to X_s ee) = (4.56 \pm 1.15^{+0.33}_{-0.40}) \times 10^{-6}$$

$$Br(B \to X_s \mu\mu) = (1.91 \pm 1.02^{+0.16}_{-0.18}) \times 10^{-6}$$

$$Br(B \to X_s \ell\ell) = (3.33 \pm 0.80^{+0.19}_{-0.24}) \times 10^{-6}$$

Note: Measured Br (M(XS):0.2-2.0 GeV/c²) x [1.10±0.002] --- based on signal MC

Dilepton mass spectrum in $\bar{B} \to X_s l^+ l^-$.



New HFAG average (2009): $\mathcal{B}(X_s \to l^+l^-) = \left(3.66^{+0.76}_{-0.77}\right) \times 10^{-6}$

$\Rightarrow$ **Non-SM sign of C_7 is excluded at more than 4σ**

(as compared to 3σ that we've had so far)

[P. Gambino, U. Haisch, MM, PRL 94 (2005) 061803] using $(4.5 \pm 1.0) \times 10^{-6}$.

provided $C_{9,10}$ remain unchanged.

$$\mathcal{L} \;=\; \underset{(q\,=\,u,d,s,c,b,\;\; l\,=\,e,\mu)}{\mathcal{L}_{QCD\times QED}(q,l)} \;+\; \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) O_i$$

$$O_i = \begin{cases} (\bar{s}\Gamma_i c)(\bar{c}\Gamma'_i b), & i = 1,2, & |C_i(m_b)| \sim 1 \\ (\bar{s}\Gamma_i b)\Sigma_q(\bar{q}\Gamma'_i q), & i = 3,4,5,6, & |C_i(m_b)| < 0.07 \\ \frac{e m_b}{16\pi^2}\bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, & i = 7, & C_7(m_b) \sim -0.3 \\ \frac{g m_b}{16\pi^2}\bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}, & i = 8, & C_8(m_b) \sim -0.15 \\ \frac{e^2}{16\pi^2}(\bar{s}_L \gamma_\mu b_L)(\bar{l}\gamma^\mu \gamma_5 l), & i = 9, \mathbf{10} & |C_i(m_b)| \sim 4 \end{cases}$$

Inclusive decay rates and the sign of C_7

$$\left(\hat{s} = \frac{q^2_{l^+l^-}}{m_b^2}\right)$$

$$\frac{d\Gamma(\bar{B} \to X_s l^+ l^-)}{d\hat{\mathbf{s}}} = \frac{G_F^2 m_{b,\text{pole}}^5 \left|V_{ts}^* V_{tb}\right|^2}{48\pi^3} \left(\frac{\alpha_{\text{em}}}{4\pi}\right)^2 (1-\hat{\mathbf{s}})^2 \times$$

$$\times \left\{(1+2\hat{\mathbf{s}}) \left(\left|\mathbf{C_9^{\text{eff}}}(\hat{\mathbf{s}})\right|^2 + \left|\mathbf{C_{10}^{\text{eff}}}(\hat{\mathbf{s}})\right|^2\right) + \left(4 + \frac{8}{\hat{\mathbf{s}}}\right) \left|\mathbf{C_7^{\text{eff}}}(\hat{\mathbf{s}})\right|^2 + 12\,\text{Re}\left(\mathbf{C_7^{\text{eff}}}(\hat{\mathbf{s}}) \mathbf{C_9^{\text{eff}*}}(\hat{\mathbf{s}})\right)\right\} + \mathbf{R_1},$$

$$\Gamma(\bar{B} \to X_s \gamma)_{E_\gamma > E_0} = \frac{G_F^2 m_{b,\text{pole}}^5 \left|V_{ts}^* V_{tb}\right|^2}{8\pi^3} \frac{\alpha_{\text{em}}}{4\pi} \left|\mathbf{C_7^{\text{eff}}}(\hat{\mathbf{s}} = \mathbf{0})\right|^2 + \mathbf{R_2}$$

are conveniently expressed in terms of the so-called effective coefficients

$$C_i^{\text{eff}}(\hat{\mathbf{s}}) = C_i(\mu_{\mathbf{b}}) + \textbf{(loop corrections)}(\hat{\mathbf{s}}).$$

The quantities R_i stand for small bremsstrahlung contributions and for the non-perturbative corrections.

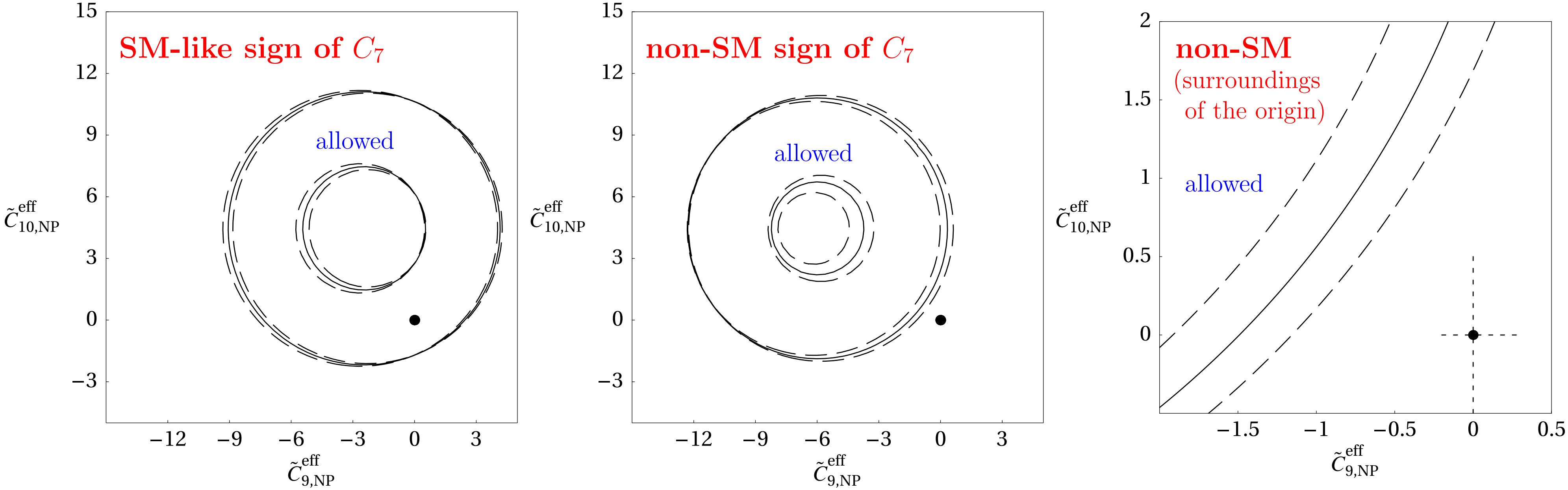
$$\text{sgn}\, \mathbf{C_7}(\mu_b) = \textbf{(“sign of the } b \to s\gamma \textbf{ amplitude”).}$$

This sign matters for the $\bar{B} \to X_s l^+ l^-$ rate and (even more) for the forward-backward asymmetry:

$$A_{\text{FB}} = \int_{-1}^{1} dy \, \frac{d^2\Gamma(\bar{B} \to X_s l^+ l^-)}{d\hat{\mathbf{s}}\, dy} \text{sgn}\, y \sim (1-\hat{\mathbf{s}})^2\, \text{Re}\left[\mathbf{C_{10}^{\text{eff}*}}(\hat{\mathbf{s}}) \left(\hat{\mathbf{s}} \mathbf{C_9^{\text{eff}}}(\hat{\mathbf{s}}) + 2\mathbf{C_7^{\text{eff}}}(\hat{\mathbf{s}})\right)\right] + \mathbf{R_3},$$

where $y = \cos\theta_l$ and θ_l is the angle between the momenta of $\bar{B}$ and l^+ in the dilepton rest frame. Forward-backward asymmetries for the exclusive $\bar{B} \to K^{(\star)} l^+ l^-$ modes are defined analogously.

AD 2005 model-independent constraints on additive new physics contributions to $C_{9,10}$ at 90% C.L.



The three lines correspond to three different values of $\mathcal{B}(\bar{B} \to X_s\gamma) \times 10^4$: the experimental central value and borders of the 90% C.L. domain for this branching ratio.

The dot at the origin indicates the SM case for $C_{9,10}$.

The SM values have been assumed for $C_1, ..., C_6$ and for C_8. New physics in $\mathbf{C_8}$ would have little effect provided one accepts the bound $\mathcal{B}(b \to \text{charmless})_{NP} =$ 3.7% @ 95% C.L. [DELPHI, PLB 426 (1998) 193].

In the rightmost plot, the maximal MFV MSSM ranges for $C_{9,\rm NP}$ and $C_{10,\rm NP}$ are indicated by the dashed cross. They were obtained in hep-ph/0112300 by A. Ali, E. Lunghi, C. Greub and G. Hiller who scanned over the following parameter ranges:

$$2.3 < \tan\beta < 50, \qquad 0 < M_2 < 1\ \text{TeV}, \qquad -1\ \text{TeV} < \mu < 1\ \text{TeV},$$

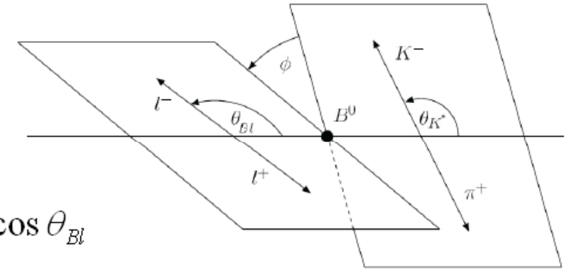
$$78.6\ \text{GeV} < M_{H^\pm} < 1\ \text{TeV}, \qquad 90\ \text{GeV} < M_{\tilde{t}_{1,2}} < 1\ \text{TeV},$$

$$-\tfrac{\pi}{2} < \theta_{\tilde{t}} < \tfrac{\pi}{2}, \qquad M_{\tilde{\nu}} \geq 50\ \text{GeV}.$$

2. Updated forward-backward asymmetries in $\mathcal{B}(B \to K^* l^+ l^-)$.

Slide from T. Ijima at Lepton-Photon 2009:

$B \to K^*ll$: FB Asymmetry

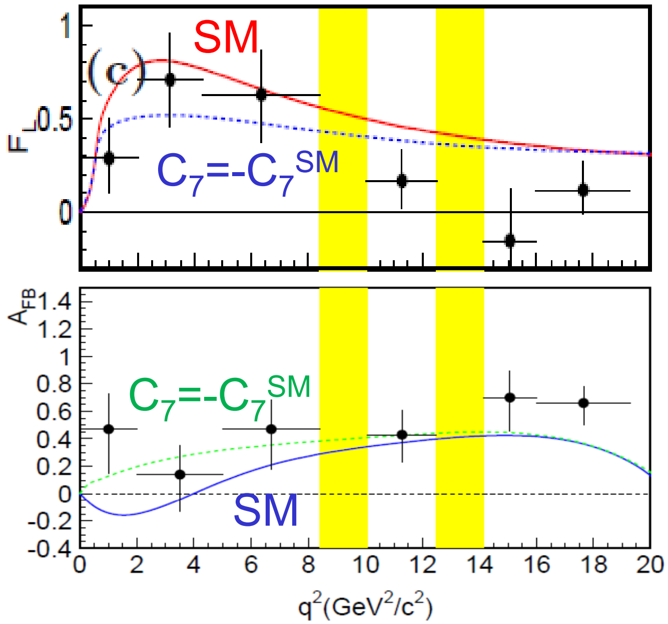


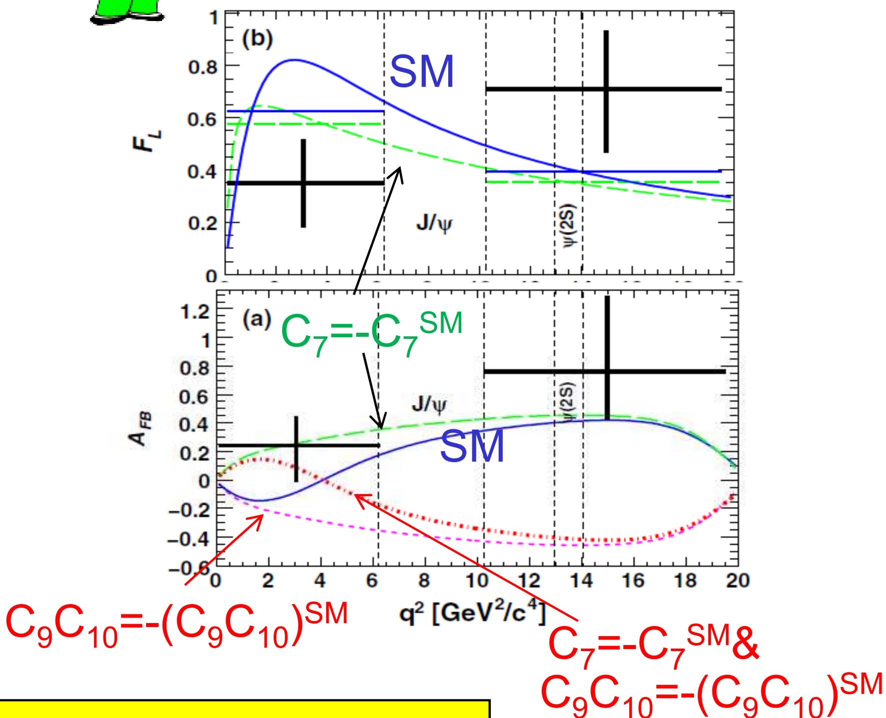
A_{FB} extracted from fits to

$$\frac{3}{4}F_L(1-\cos^2\theta_{Bl}) + \frac{3}{8}(1-F_L)(1+\cos^2\theta_{Bl}) + A_{FB}\cos\theta_{Bl}$$

657 M BB,
submitted to PRL, arXiv: 0904.0770

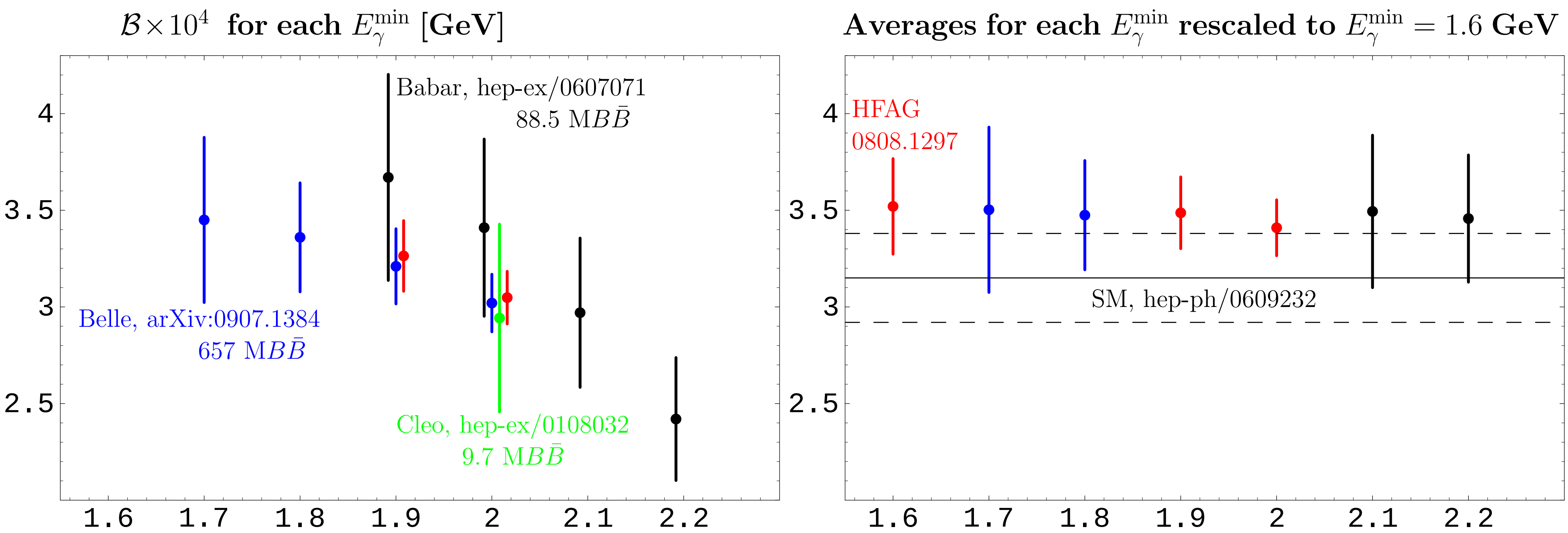
384M BB,
PRD79, 031102(R) (2009)





A_{FB} exceeds SM ?

3. Updated $\mathcal{B}(B \to X_s\gamma)$ measurement by Belle.
A. Limosani *et al*, arXiv:0907.1384, PRL 103 (2009) 241801.



The displayed measurements are only the fully-inclusive, no-hadronic-tag ones.
Other methods (included in the HFAG average):

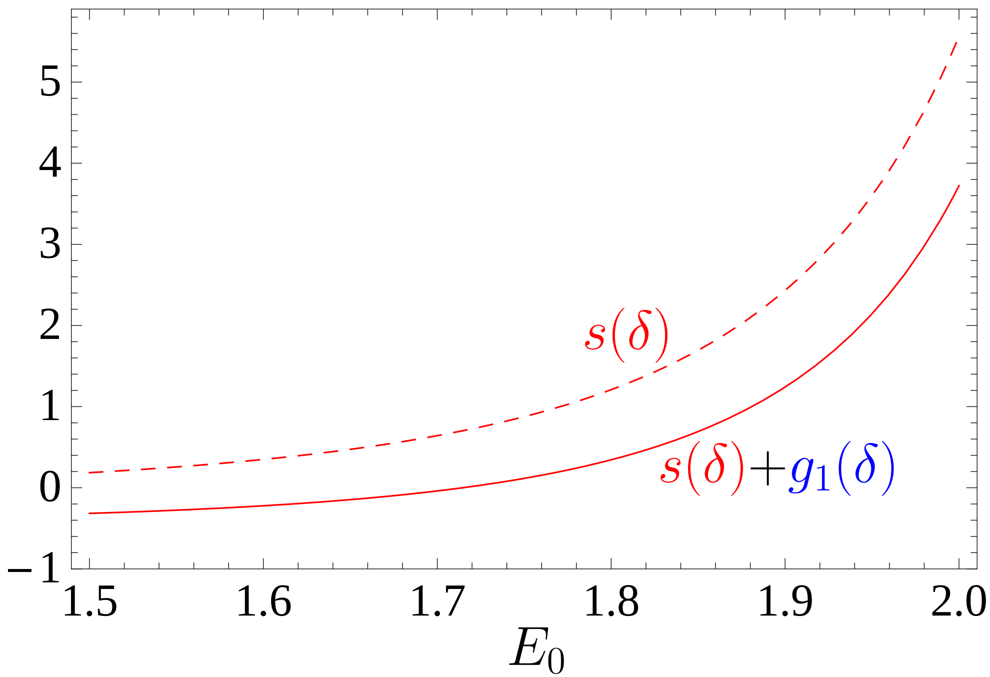
- Semi-inclusive (systematics-limited),
- With hadronic tags of the recoiling B meson (not necessarily fully reconstructed). Low systematic errors, but statistics-limited at present.

4. Evaluation of $\mathcal{O}(\alpha_s \Lambda^2/m_b^2)$ corrections to $\Gamma_{77}(\bar{B} \to X_s \gamma)$ and moments of the photon spectrum.

[T. Ewerth, P. Gambino and S. Nandi, arXiv:0911.2175, NPB 830 (2010) 278].

$$\Gamma_{77}|_{E_\gamma > E_0} = \Gamma_{77}^{(0)} \Big[1 \;+\; \frac{\lambda_1 - 9\lambda_2(\mu)}{2m_b^2} \;+\; \frac{\alpha_s(\mu)}{\pi} f_{\rm pert.NLO}(\delta) \;+\; \frac{\alpha_s^2(\mu)}{\pi^2} f_{\rm pert.NNLO}(\delta)$$
$$+\; \frac{\lambda_1 \alpha_s(\mu)}{3 m_b^2 \pi} \left(-\frac{3 + 4\ln\delta}{6\delta^2} + g_1(\delta) \right) \;+\; \frac{\lambda_2(\mu)\alpha_s(\mu)}{m_b^2 \pi} g_2(\delta) \;+\; \ldots \Big]$$

M. Neubert, 2005



$$\delta = 1 - \frac{2E_0}{m_b}$$

$g_{1,2}$ **contain** $\frac{\ln\delta}{\delta}$, $\frac{1}{\delta}$, $\ln^2\delta$, $\ln\delta$, **and non-singular terms.**

5. Clarification of quark-hadron duality issues in $\bar{B} \to X_s l^+ l^-$

[M. Beneke, G. Buchalla, M. Neubert and C. Sachrajda, arXiv:0902.4446, EJPC 61 (2009) 439].

If the intermediate J/ψ and ψ' resonances are included, $\Gamma(\bar{B} \to X_s l^+ l^-)$ exceeds the perturbative $\Gamma(b \to X_s l^+ l^-)$ by around two orders of magnitude.

Is the quark-hadron duality violated here?

G.B. 2000: No, because we need to resum Coulomb-like interactions in the $c\bar{c}$ state.

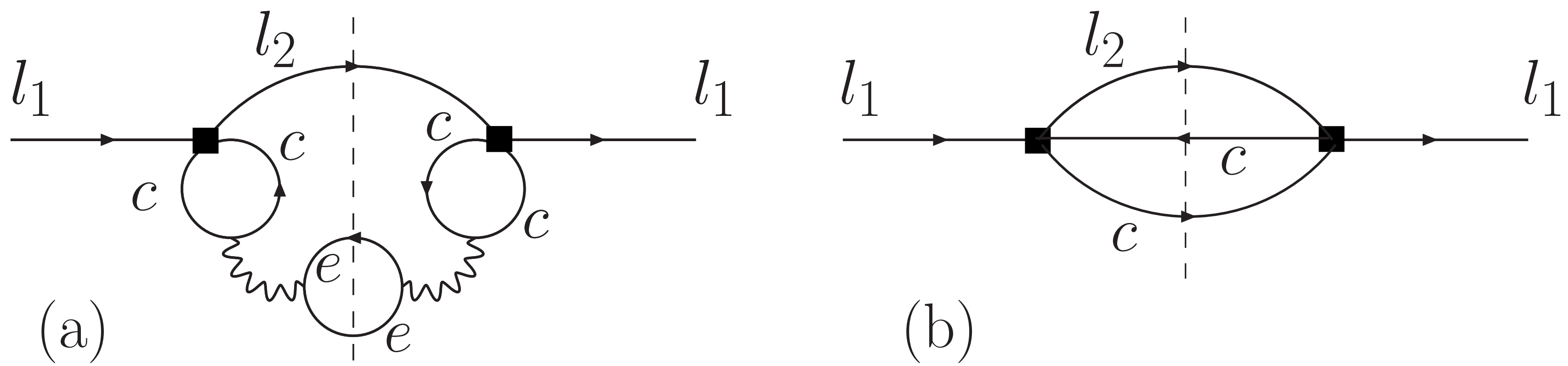
BBNS 2009: Yes, because we need to resum Coulomb-like interactions in the $c\bar{c}$ state.

Both answers are satisfactory, because they differ only linguistically, while the physics remains the same.

Technically: Coulomb resummation effects get washed out after smearing over q^2 in the correlator (as in $b \to sc\bar{c}$), but not in the squared correlator (as in $b \to se^+e^-$).

Pedagogical toy model: consider ficticious leptons (heavy l_1 instead of b, and massless l_2 instead of s) to single out bound-state effects in the $c\bar{c}$ system only.

The decays $l_1 \to l_2 c\bar{c}$ and $l_1 \to l_2 e^+e^-$ are described by:



In the case (b), we integrate imaginary part of the correlator $\Pi(q^2)$ of two $c\bar{c}$ currents. In the case (a), we get $|\Pi(q^2)|^2$.

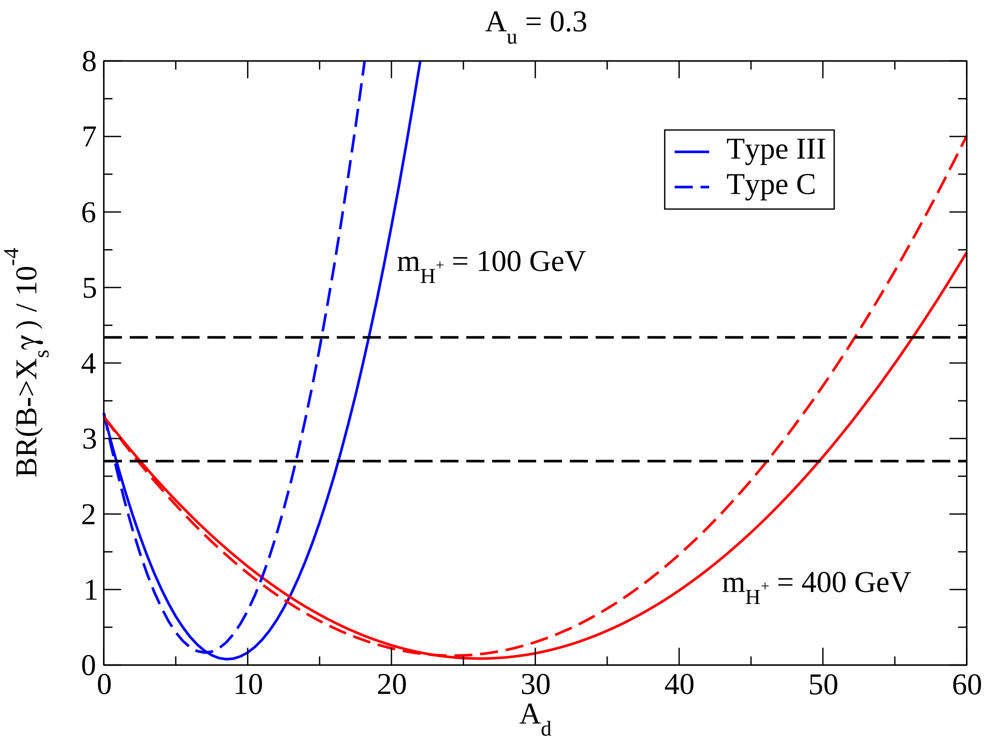
In the acknowledgments, thanks to Tobias Hurth for *persistent encouragement*.

6. Many BSM studies... Let's have a look at the past 2 weeks.

6a. G. Degrassi and P. Slavich, arXiv:1002:1071 (Feb 4th)

Evaluation of the NLO QCD corrections to R_b and $b \to s\gamma$ in generic MVF two-Higgs-doublet models.

$$\mathcal{L}_{H^+} = -\frac{g}{\sqrt{2}\, m_W} \, \Sigma_{i,j=1}^{3} \; \bar{u}_i \, T_R^{(a)} \left(A_u^i \, m_{u_i} \, \frac{1-\gamma_5}{2} - A_d^i \, m_{d_j} \, \frac{1+\gamma_5}{2} \right) V_{ij} \, d_j \, H_{(a)}^+ + \text{h.c.}$$



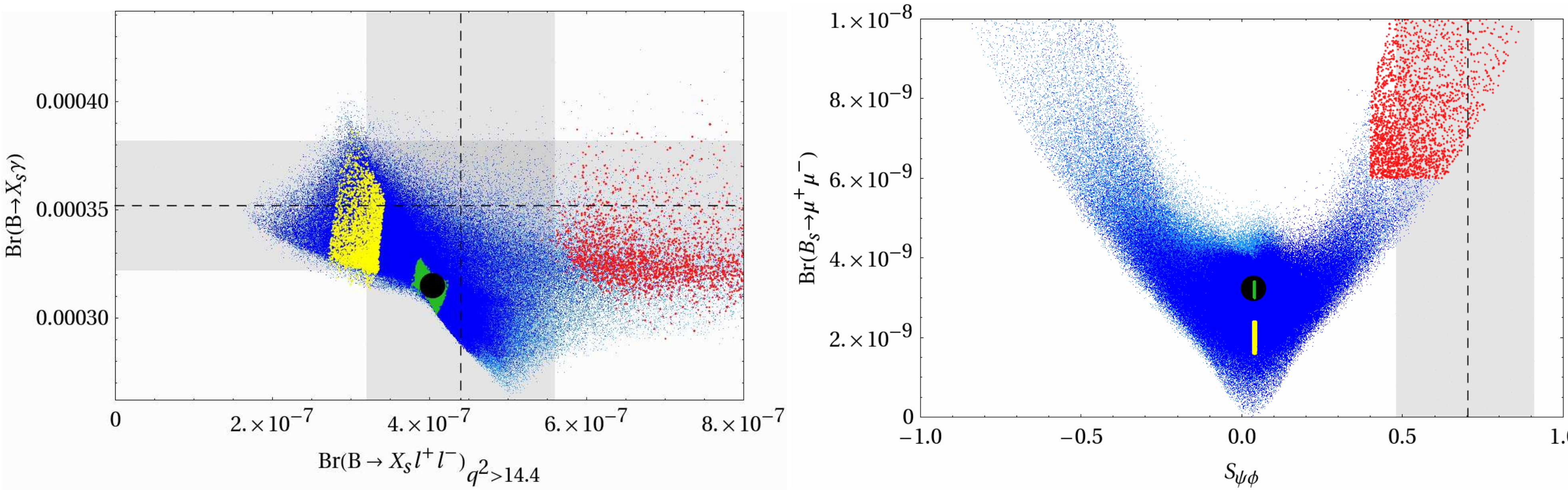
Question: Do the two-loop $b \to s\gamma$ matching results agree analytically with those from hep-ph/9904413 (C. Bobeth, J. Urban, MM)?

6b. Fourth generation (congratulations to George Hou!)

6b1. arXiv:1002.0595 (Feb 3rd), A. Soni *et al.*, 46pp.

6b2. arXiv:1002.2216 (Feb 10th), A. J. Buras *et al.*, 87pp.

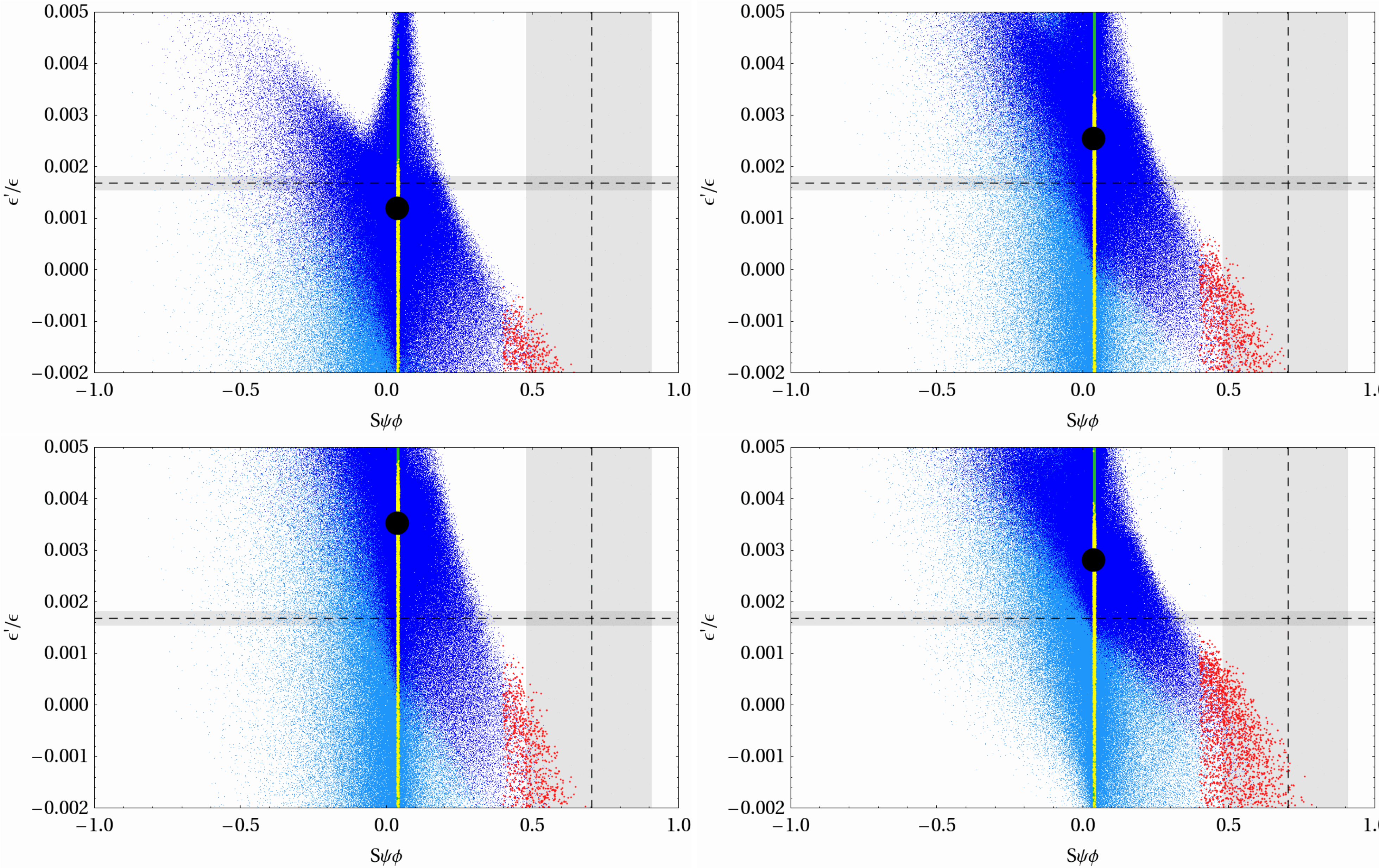
Scans over the SM4 parameter space (Fig. 16 from the latter paper):



LO $b \to s\gamma$ matching for 4th gen.

	BS1 (yellow)	BS2 (green)	BS3 (red)
$S_{\psi\phi}$	0.04 ± 0.01	0.04 ± 0.01	≥ 0.4
$\mathrm{Br}(B_s \to \mu^+\mu^-)$	$(2 \pm 0.2) \cdot 10^{-9}$	$(3.2 \pm 0.2) \cdot 10^{-9}$	$\geq 6 \cdot 10^{-9}$

Would the left plot remain qualitatively the same for $q^2 \in [1, 6]\ \mathrm{GeV}^2$ and/or with the updated HFAG result for the full q^2 range?



0.005
0.004
0.003
0.002
0.001
0.000
−0.001
−0.002
ϵ′/ϵ
−1.0
−0.5
0.0
0.5
1.0
Sψϕ

To conclude, the following topics have been missed in my list of 2009/2010 news:

- **Isospin asymmetries in** $B \to K^*\gamma$ **and** $B \to K^{(*)}l^+l^-$,
- **CP asymmetries in those decays,**
- **Theory upgrades in the full angular analyses of** $B \to K^*l^+l^-$,
- **Many other new BSM studies, some of them even more recent.**

  (see e.g. arXiv:1002.2758 (Feb 14th), Q. Chang, X.-Q. Li, Y.-D. Yang,

  "$B \to K^*l^+l^-$**,** Kl^+l^- **decays in a family non-universal** Z' **model.")**
- **....**

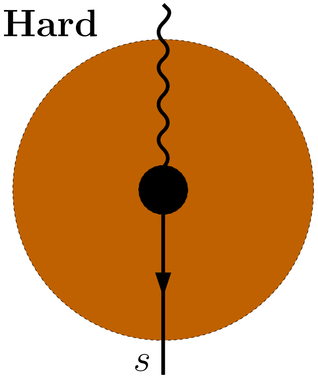
BACKUP SLIDES

Energetic photon production in charmless decays of the $\bar{B}$-meson

$(E_\gamma \gtrsim \frac{m_b}{3} \simeq 1.6\,\mathrm{GeV})$ [see MM, arXiv:0911.1651]

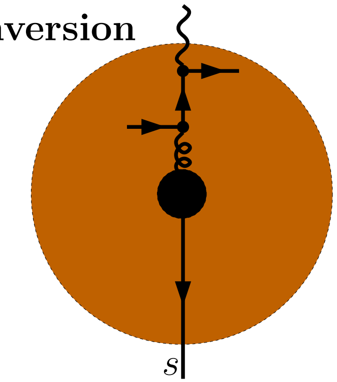
A. Without long-distance charm loops:

1. **Hard**



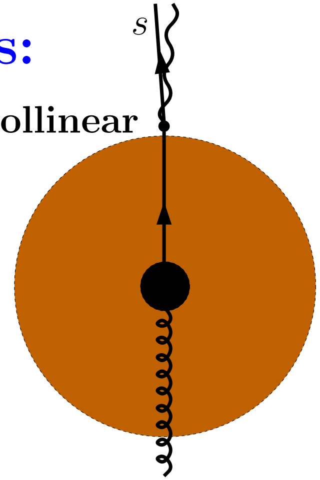
Dominant, well-controlled.

2. **Conversion**



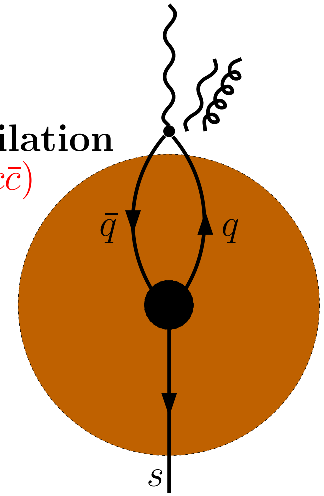
$\mathcal{O}(\alpha_s \Lambda/m_b)$, $(-1.5 \pm 1.5)\%$.
[Lee, Neubert, Paz, 2006]

3. **Collinear**



Pert. $< 1\%$, nonp. $\sim -0.2\%$.
[Kapustin,Ligeti,Politzer, 1995]

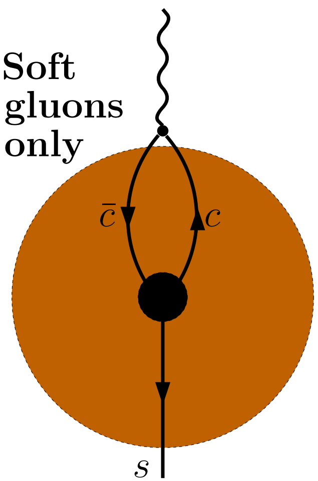
4. **Annihilation** $(q\bar{q} \neq c\bar{c})$



Exp. π^0, η, η', ω subtracted.
Perturbatively $\sim 0.1\%$.

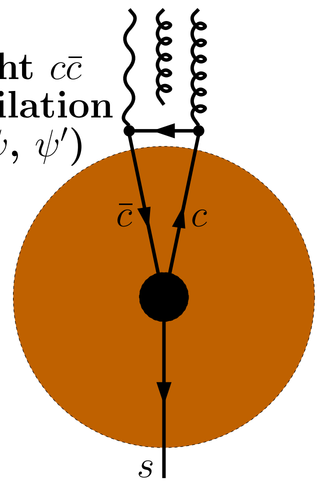
B. With long-distance charm loops:

5. **Soft gluons only**



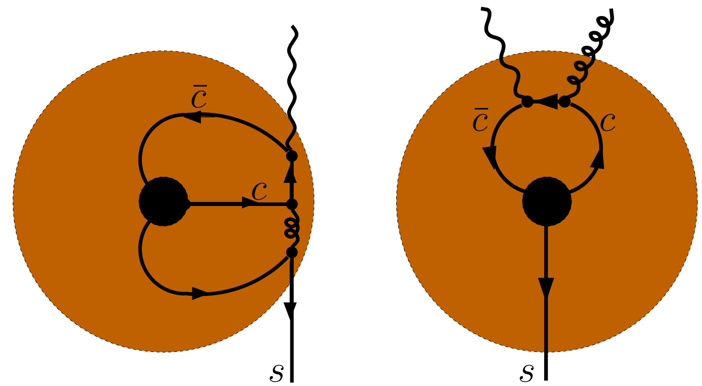
$\mathcal{O}(\Lambda^2/m_c^2)$, $\sim +3.1\%$.
[Voloshin, 1996], [...],
[Buchalla, Isidori, Rey, 1997]

6. **Boosted light $c\bar{c}$ state annihilation (e.g. η_c, J/ψ, ψ')**



Exp. J/ψ subtracted ($< 1\%$).
Perturbatively (including hard): $\sim +3.6\%$.
$\phi_{ij}^{(1)}(\delta)$, $\phi_{ij}^{(2)\beta_0}(\delta)$, $i,j = 1,2$

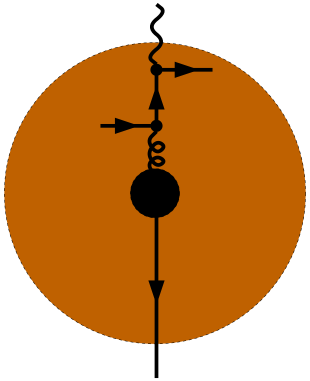
7. **Annihilation of $c\bar{c}$ in a heavy $(\bar{c}s)(\bar{q}c)$ state**



$\mathcal{O}(\alpha_s(\Lambda/M)^2)$ $\mathcal{O}(\alpha_s \Lambda/M)$
$M \sim 2m_c, 2E_\gamma, m_b$.

e.g. $\mathcal{B}[B^- \to D_{sJ}(2457)^- \; D^*(2007)^0 \;] \simeq 1.2\%$,
$\mathcal{B}[B^0 \to D^*(2010)^+ \bar{D}^*(2007)^0 K^-] \simeq 1.2\%$.

Gluon-to-photon conversion in the QCD medium



This is hard gluon scattering on the valence quark or a "sea" quark that produces an energetic photon. The quark that undergoes this Compton-like scattering is assumed to remain soft in the $\bar{B}$-meson rest frame to ensure effective interference with the leading "hard" amplitude. Without interference the contribution would be negligible ($\mathcal{O}(\alpha_s^2\Lambda^2/m_b^2)$).

Suppression by Λ can be understood as originating from dilution of the target (size of the $\bar{B}$-meson $\sim \Lambda^{-1}$).

A rough estimate using vacuum insertion approximation gives

$$\Delta\Gamma/\Gamma \in [-3\%, -0.3\%] \qquad (\mathcal{O}(\alpha_s\Lambda/m_b)).$$

[Lee, Neubert, Paz, hep-ph/0609224]

However:

1. Contribution to the interference from scattering on the "sea" quarks vanishes in the $SU(3)_{\text{flavour}}$ limit because $Q_u + Q_d + Q_s = 0$.

2. If the valence quark dominates, then the isospin-averaged $\Delta\Gamma/\Gamma$ is given by:

$$\frac{\Delta\Gamma}{\Gamma} \simeq \frac{Q_d+Q_u}{Q_d-Q_u}\,\Delta_{0-} = -\frac{1}{3}\Delta_{0-} = \left(+0.2 \pm 1.9_{\text{stat}} \pm 0.3_{\text{sys}} \pm 0.8_{\text{ident}}\right)\%,$$

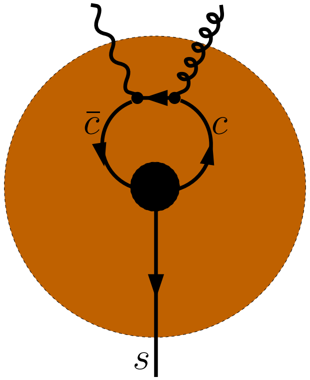
using the BABAR measurement (hep-ex/0508004) of the isospin asymmetry

$$\Delta_{0-} = [\Gamma(\bar{B}^0 \to X_s\gamma) - \Gamma(B^- \to X_s\gamma)]/[\Gamma(\bar{B}^0 \to X_s\gamma) + \Gamma(B^- \to X_s\gamma)],$$

for $E_\gamma > 1.9$ **GeV**.

Quark-to-photon conversion gives a soft s-quark and poorly interferes with the "hard" $b \to s\gamma g$ amplitude.

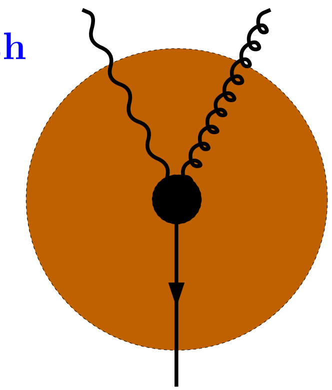
Annihilation of $c\bar{c}$ in a heavy $(\bar{c}s)(\bar{q}c)$ state



Heavy $\Leftrightarrow$ **Above the $D\bar{D}$ production threshold**

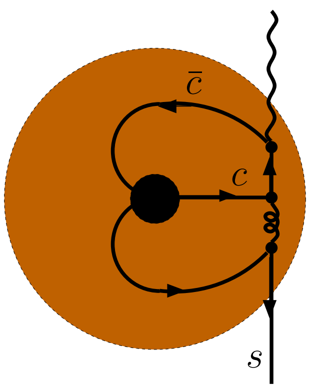
Long-distance $\Rightarrow$ **Annihilation amplitude is suppressed with respect to the open-charm decay due to the order Λ^{-1} distance between c and $\bar{c}$. By analogy to the B-meson decay constant $f_B \sim \Lambda(\Lambda/m_b)^{1/2}$, we may expect that the suppression factor scales like $(\Lambda/M)^{3/2}$, where $M \sim 2m_c, 2E_\gamma, m_b$.**

Hard gluon $\Leftrightarrow$ **Suppression by α_s of the interference with**
(non-soft)



Altogether: $\mathcal{O}\left(\alpha_s(\Lambda/M)^{3/2}\right)$.

To stay on the safe side, assume $\mathcal{O}\left(\alpha_s\Lambda/m_b\right)$ for numerical error estimates.



This type of amplitude interferes with the leading term but receives an additional Λ/M suppression (at least) due to participation of the s-quark in the hard annihilation.

The inclusive branching ratio in the SM:

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\rm NNLO}_{E_\gamma>1.6\ {\rm GeV}} = \begin{cases} (3.15 \pm 0.23) \times 10^{-4}, & \text{hep-ph/0609232, using the 1S scheme,} \\ (3.26 \pm 0.24) \times 10^{-4}, & \text{following the kin scheme analysis of arXiv:0805.0271, but } \overline{m}_c(\overline{m}_c)^{\rm 2loop} \text{ rather than } \overline{m}_c(\overline{m}_c)^{\rm 1loop}. \end{cases}$$

Contributions to the total uncertainty:

5% non-perturbative, mainly $\mathcal{O}\left(\alpha_s \frac{\Lambda}{m_b}\right)$ $\longrightarrow$ Improved measurements of Δ_{0-} should help.

3% parametric $\left(\alpha_s(M_Z),\ \mathcal{B}^{\rm exp}_{\rm semileptonic},\ m_c\ \&\ C,\ \ldots\right)$

2.0% 1.6% 1.1% (1S)
2.5% (kin)

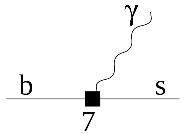
3% m_c-interpolation ambiguity $\longrightarrow$ The calculation of G_{17} and G_{27} for $m_c = 0$ should help a lot.

3% higher order $\mathcal{O}(\alpha_s^3)$ $\longrightarrow$ This uncertainty will stay with us.

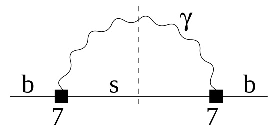
Missing ingredients in the perturbative NNLO matrix elements

$$\Gamma(b \to X_s^{\rm parton}\gamma)\Big|_{E_\gamma > E_0} = \frac{G_F^2 m_b^5 \alpha_{\rm em}}{32\pi^4} |V_{ts}^* V_{tb}|^2 \sum_{i,j=1}^{8} C_i(\mu_b) C_j(\mu_b) G_{ij}(E_0, \mu_b)$$

LO: $G_{ij} = \delta_{i7}\delta_{j7}$



$\Leftrightarrow$



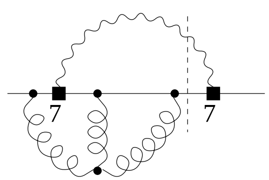
$|C_{1,2}(\mu_b)| \sim 1,\ \ |C_{3,4,5,6}(\mu_b)| < 0.07,$
$C_7(\mu_b) \sim -0.3,\ \ C_8(\mu_b) \sim -0.15.$

NLO: **The most important** G_{ij} $(i,j = 1,2,7,8)$ **are known since 1996.** [Greub, Hurth, Wyler, 1996] [Ali, Greub, 1991-1995]

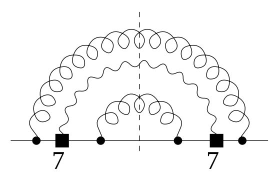
The remaining G_{ij} **are known since 2002.** [Buras, Czarnecki, MM, Urban, 2002] [Pott, 1995]

NNLO: **Only** $i,j = 1,2,7,8$ **have been considered so far.**

Only G_{77} **is fully known:**



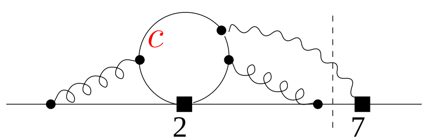
$+$



$+ \ldots$

[Blokland et al., 2005]
[Melnikov, Mitov, 2005]
[Asatrian et al., 2006-2007]

G_{27}**:**
(and analogous G_{17})



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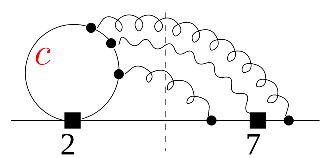


$+ \ldots$

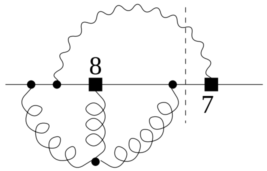
Two-particle cuts:
~ 160 **four-loop**
master integrals $(m_c = 0)$
recently completed
by T. Schutzmeier.

Three- and four-particle cuts:
R. Boughezal,
M. Czakon,
T. Schutzmeier,
in progress...

Previous status reports: arXiv:0712.1676, arXiv:0807.0915.

Diagrams with quark loops on gluon lines for $m_c \neq 0$: arXiv:0707.3090.

G_{78}:



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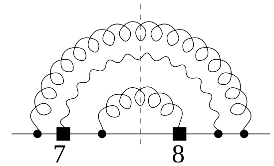


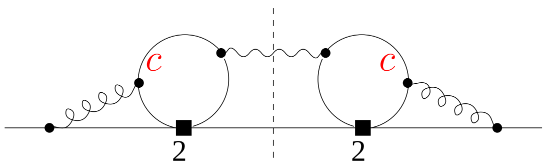
$+ \ldots$

Two-particle cuts:
finished in 2007
(unpublished)

Three- and four-particle cuts:
in progress...

H.M. Asatrian, T. Ewerth, A. Ferroglia, C. Greub, G. Ossola.

G_{22}:
(and analogous G_{11} & G_{12})



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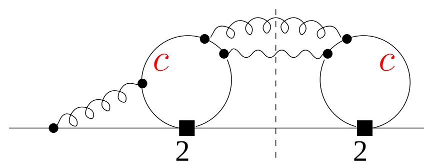

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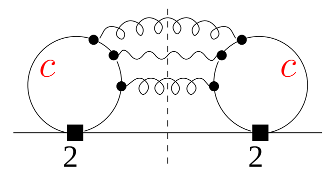


$+ \ldots$

Two-particle cuts
are known (just $|\mathrm{NLO}|^2$).

Three- and four-particle cuts
vanish at the endpoint $E_\gamma = m_b/2$.
Analogous NLO corrections are not big $(+3.6\%)$.

The current phenomenological analysis at the NNLO relies on using the BLM approximation together with the large-m_c asymptotics of the non-BLM correction. The latter correction is interpolated in m_c under the assumption that it vanishes at $m_c = 0$.

Large-m_c asymptotics of G_{ij}^{NNLO} $(m_c \gg m_b/2)$:

1	2	7	8	
+	+	+	+	1
	+	+	+	2
		+	−	7
			−	8

[MM, Steinhauser, 2006]

The BLM approximation for G_{ij}^{NNLO} (arbitrary m_c):

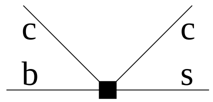
1	2	7	8	
+	+	+	−	1
	+	+	−	2
		+	+	7
			+	8

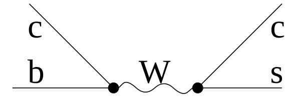
The BLM corrections to G_{78}, G_{88} are small.

G_{18} and G_{28} are small at the NLO.

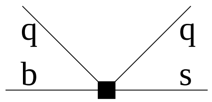
[Bieri, Greub, Steinhauser, 2003]
[Ligeti, Luke, Manohar, Wise, 1999]
[Ferroglia, Haisch, 2007]

The operators Q_i that matter for $b \to s\gamma$ read:

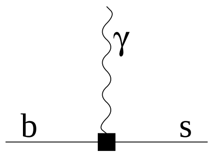
$O_{1,2} =$ 

$= (\bar{s}\Gamma_i c)(\bar{c}\Gamma'_i b)$, **from** 

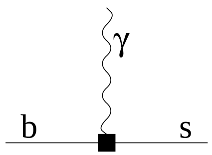
, $|C_i(m_b)| \sim 1$

$O_{3,4,5,6} =$ 

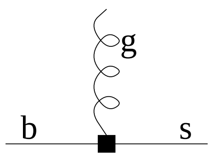
$= (\bar{s}\Gamma_i b)\Sigma_q(\bar{q}\Gamma'_i q)$, $|C_i(m_b)| < 0.07$

$O_7 =$ 

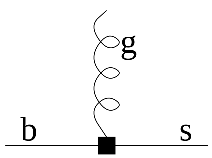
$= \frac{em_b}{16\pi^2}\,\bar{s}_L\sigma^{\mu\nu}b_R F_{\mu\nu}$, $C_7^{\rm SM}(m_b) \simeq -0.3$

$O'_7 =$ 

$= \frac{em_b}{16\pi^2}\,\bar{s}_R\sigma^{\mu\nu}b_L F_{\mu\nu}$, $C_7^{\prime\rm SM} = \frac{m_s}{m_b}C_7^{\rm SM}$

$O_8 =$ 

$= \frac{gm_b}{16\pi^2}\,\bar{s}_L\sigma^{\mu\nu}T^a b_R G^a_{\mu\nu}$, $C_8^{\rm SM}(m_b) \simeq -0.15$

$O'_8 =$ 

$= \frac{gm_b}{16\pi^2}\,\bar{s}_R\sigma^{\mu\nu}T^a b_L G^a_{\mu\nu}$, $C_8^{\prime\rm SM} = \frac{m_s}{m_b}C_8^{\rm SM}$

Their SM Wilson coefficients are known up to $\mathcal{O}(\alpha_s^2)$ (NNLO).

Assumption: no relevant NP effects in the 4-quark operators.

$$\Gamma(\bar{B}^0 \to K^{*0}\gamma)_{\rm exp} = (4.01 \pm 0.20) \times 10^{-5}$$ **[HFAG],**

$$\Gamma(\bar{B}_s \to \phi\gamma)_{\rm exp} = \left(5.7^{+1.8}_{-1.5}({\rm stat})^{+1.2}_{-1.1}({\rm syst})\right) \times 10^{-5}$$ **[BELLE, PRL 100 (2008) 121801].**

The decay rates $\Gamma(\bar{B} \to \bar{K}^*\gamma)$ and $\Gamma(\bar{B}_s \to \phi\gamma)$ are proportional to (practically) the same combinations of the Wilson coefficients as the inclusive rate $\Gamma(\bar{B} \to X_s\gamma)$.

Errors in the inclusive rate are $\mathcal{O}(7\%)$, both EXP and TH.

Theory uncertainties in the exclusive rates are $\mathcal{O}(30\%)$ due to non-perturbative form-factors.

A promising exclusive observable for constraining the Wilson coefficients:

The mixing-induced CP asymmetry

$$A_{\rm CP}(t) = \frac{\Gamma\big[\bar{B}^0(t) \to \bar{K}^{*0}\gamma\big] - \Gamma\big[B^0(t) \to K^{*0}\gamma\big]}{\Gamma\big[\bar{B}^0(t) \to \bar{K}^{*0}\gamma\big] + \Gamma\big[B^0(t) \to K^{*0}\gamma\big]} = C_{K^*\gamma}\cos(\Delta m_B t) + S_{K^*\gamma}\sin(\Delta m_B t).$$

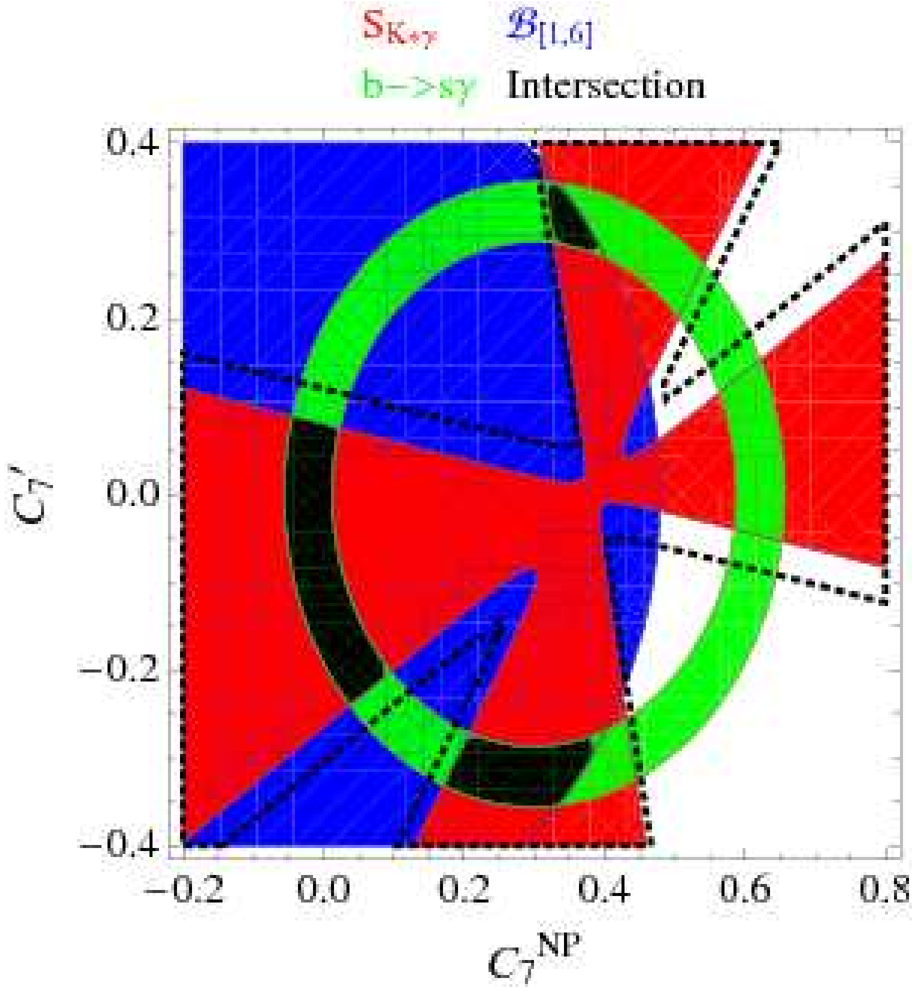
$$S^{\rm th}_{K^*\gamma} = -\frac{2|z|}{1+|z|^2}\sin\left[2\beta - \arg\left(C_7 C_7'\right)\right] + \ldots \overset{\rm SM}{\simeq} -0.03, \qquad z = \frac{C_7'}{C_7} \overset{\rm SM}{\simeq} \frac{m_s}{m_b}.$$

$$S^{\rm exp}_{K^*\gamma} = -0.19 \pm 0.23$$ **[BaBar,Belle $\to$ HFAG].**

Constraints in the $(C_7^{\rm NP} \equiv C_7 - C_7^{\rm SM}, C_7')$ plane from

C. Bobeth, G. Hiller and G. Piranishvili, **arXiv:0805.2525**

Fig. 2a



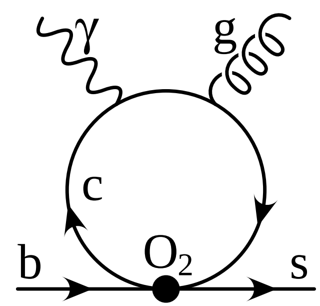
Green: $\bar{B} \to X_s \gamma$,

Blue: $\bar{B} \to X_s l^+ l^-$

$q^2_{\rm dilept} \in [1,6]$ GeV^2,

Red: $S_{K^*\gamma}$

Black dotted lines: Effect of enlarging the uncertainty in the SM prediction for $S_{K^*\gamma}$ due to the $\mathcal{O}(\Lambda/m_b)$ fraction of right-handed photons originating from:



B. Grinstein, Y. Grossman, Z. Ligeti and D. Pirjol, Phys. Rev. D 71 (2005) 011504.

Assumptions for the above plot:

(i) $C_7^{\rm NP}$ and C_7' are real.

(ii) All the other Wilson coefficients are fixed at their SM values.

The operators Q_i that matter for $\bar{B} \to \bar{K}^* \mu^+ \mu^-$ and $\bar{B}_s \to \phi \mu^+ \mu^-$ are the same as those for $\bar{B} \to \bar{K}^* \gamma$ and $\bar{B}_s \to \phi \gamma$, plus:

$$O_9 = \frac{\alpha_{\rm em}}{4\pi} \left(\bar{s}_L \gamma^\nu b_L\right) \left(\bar{\mu} \gamma_\nu \mu\right), \qquad O_9' = \frac{\alpha_{\rm em}}{4\pi} \left(\bar{s}_R \gamma^\nu b_R\right) \left(\bar{\mu} \gamma_\nu \mu\right),$$

$$O_{10} = \frac{\alpha_{\rm em}}{4\pi} \left(\bar{s}_L \gamma^\nu b_L\right) \left(\bar{\mu} \gamma_\nu \gamma_5 \mu\right), \qquad O_{10}' = \frac{\alpha_{\rm em}}{4\pi} \left(\bar{s}_R \gamma^\nu b_R\right) \left(\bar{\mu} \gamma_\nu \gamma_5 \mu\right),$$

and, in principle, also the four chirality-violating operators that do not contribute to $\bar{B}_s \to \mu^+ \mu^-$:

$$O_S' = \frac{\alpha_{\rm em}}{4\pi} \left(\bar{s} b\right) \left(\bar{\mu} \mu\right), \qquad O_P' = \frac{\alpha_{\rm em}}{4\pi} \left(\bar{s} b\right) \left(\bar{\mu} \gamma_5 \mu\right),$$

$$O_T = \frac{\alpha_{\rm em}}{4\pi} \left(\bar{s} \sigma^{\nu\lambda} b\right) \left(\bar{\mu} \sigma_{\nu\lambda} \mu\right), \qquad O_T' = \frac{\alpha_{\rm em}}{4\pi} \left(\bar{s} \sigma^{\nu\lambda} b\right) \left(\bar{\mu} \sigma_{\nu\lambda} \gamma_5 \mu\right).$$

The full angular distribution of $\bar{B} \to \bar{K}^*(\to \bar{K}\pi)\mu^+\mu^-$:

[e.g.: C. Bobeth, G. Hiller and G. Piranishvili, arXiv:0805.2525]

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_l\, d\cos\theta_{K^*}\, d\phi} = \frac{3}{8\pi} J(q^2, \theta_l, \theta_{K^*}, \phi),$$

$$\begin{aligned} J(q^2, \theta_l, \theta_{K^*}, \phi) \;=\;& J_1^s \sin^2\theta_{K^*} + J_1^c \cos^2\theta_{K^*} + (J_2^s \sin^2\theta_{K^*} + J_2^c \cos^2\theta_{K^*}) \cos 2\theta_l \\ &+ J_3 \sin^2\theta_{K^*} \sin^2\theta_l \cos 2\phi + J_4 \sin 2\theta_{K^*} \sin 2\theta_l \cos\phi \\ &+ J_5 \sin 2\theta_{K^*} \sin\theta_l \cos\phi + J_6 \sin^2\theta_{K^*} \cos\theta_l + J_7 \sin 2\theta_{K^*} \sin\theta_l \sin\phi \\ &+ J_8 \sin 2\theta_{K^*} \sin 2\theta_l \sin\phi + J_9 \sin^2\theta_{K^*} \sin^2\theta_l \sin 2\phi. \end{aligned}$$

$q^2 =$ **dilepton invariant mass squared,**

$\theta_l =$ **angle between the μ^- and $\bar{B}$ momenta in the dilepton c.m.s.,**

$\theta_{K^*} =$ **angle between the $\bar{K}$ and $\bar{B}$ momenta in the $\bar{K}\pi$ c.m.s.,**

$\phi =$ **angle between the normals to the $\bar{K}\pi$ and $\mu^+\mu^-$ planes in the $\bar{B}$-meson rest frame.**

The forward-backward asymmetry:

$$A_{FB}(q^2) = \left(\frac{d\Gamma}{dq^2}\right)^{-1} \left[I_0^1 - I_{-1}^0\right] d\cos\theta_l \frac{d^2\Gamma}{dq^2\, d\cos\theta_l} = \left(\frac{d\Gamma}{dq^2}\right)^{-1} J_6(q^2)$$

Quantities similar to $A_{FB}(q^2)$ can be obtained by integrating the full distribution with various angular weighting functions. Such quantities are functions of ratios of the Wilson coefficients C_i/C_j and ratios of q^2-dependent form-factors.

In general: 7 independent form-factors
[see e.g. F. Krüger, J. Matias, Phys. Rev. D71 (2005) 094009].

In the large E_{K^*} limit $(m_{K^*}/E_{K^*} \sim \Lambda/m_b \ll 1)$: only $\xi_\perp(q^2)$ and $\xi_\parallel(q^2)$, up to $\mathcal{O}(\alpha_s, \Lambda/m_b)$.
[see e.g. M. Beneke and T. Feldmann, Nucl. Phys. B 612 (2001) 3].

Two strategies:

1. **Determine $\xi_\perp/\xi_\parallel$ together with C_i/C_j from experiment.**
2. **Search for quantities in which the form-factors cancel out.**

Example: see next slide