V_{cb} and V_{ub}

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GGI, Feb. 2010

Some Transparencies borrowed from
A. Khodjamirian, F. Tackman, E. Gardi, R. van der Water

Contents

Inclusive Vcb

- Structure of the expansion (@ tree):

$$\begin{aligned} d\Gamma &= d\Gamma_0 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^2 d\Gamma_2 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 d\Gamma_3 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^4 d\Gamma_4 \\ &\quad + d\Gamma_5 \left(a_0 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^5 + a_2 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^2 \right) \\ &\quad + \ldots + d\Gamma_7 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^4 \end{aligned}$$

- Power counting $m_c^2 \sim \Lambda_{\rm QCD} m_b$

Present state of the $b \to c$ semileptonic Calculations

- Tree level terms up to and including $1/m_b^5$ known
- $\mathcal{O}(\alpha_s)$ and full $\mathcal{O}(\alpha_s^2)$ for the partonic rate known
- $\mathcal{O}(\alpha_s)$ for the μ_π^2/m_b^2 is known
- In the pipeline:
  - Complete α_s/m_b^2, including the μ_G terms
- The upshot:

$$V_{cb} = (41.54 \pm 0.44 \pm 0.58_{HQE}) \times 10^{-3}$$

(PDG 2010)

Relative uncertainty of 1.7% !!

Higher Orders in the $1/m$ Expansion

- Beyond μ_π^2, μ_G^2, ρ_D and ρ_{LS}: At $\mathcal{O}(1/m^4)$
  Dimension 7 matrix elements = four derivatives

Spin-independent

$$
\begin{aligned}
2M_B m_1 &= \langle \left((\vec{p})^2\right)^2 \rangle \\
2M_B m_2 &= g^2 \langle \vec{E}^2 \rangle \\
2M_B m_3 &= g^2 \langle \vec{B}^2 \rangle \\
2M_B m_4 &= g \langle \vec{p} \cdot \mathrm{rot}\, \vec{B} \rangle
\end{aligned}
$$

Spin-dependent

$$2M_B m_5 = g^2 \langle \vec{S} \cdot (\vec{E} \times \vec{E}) \rangle$$

$$2M_B m_6 = g^2 \langle \vec{S} \cdot (\vec{B} \times \vec{B}) \rangle$$

$$2M_B m_7 = g \langle (\vec{S} \cdot \vec{p})(\vec{p} \cdot \vec{B}) \rangle$$

$$2M_B m_8 = g \langle (\vec{S} \cdot \vec{B})(\vec{p})^2 \rangle$$

$$2M_B m_9 = g \langle \Delta(\vec{\sigma} \cdot \vec{B}) \rangle$$

- In the published paper three of the matrix elements were ommitted!
- An erratum and a more sophisticated estimate of the matrix elements will be published soon

Quantitative Results

- Estimate by "Ground State Saturation": (Spatial Components only)

$$\langle B(v)|\bar{b}(iD_{\mu_1})(iD_{\mu_2})(iD_{\mu_3})(iD_{\mu_4})b|B(v)\rangle \approx$$
$$\frac{1}{2M_B}\langle B(v)|\bar{b}(iD_{\mu_1})(iD_{\mu_2})b|B(v)\rangle\,\langle B(v)|\bar{b}(iD_{\mu_3})(iD_{\mu_4})b|B(v)\rangle$$
$$+\frac{1}{2M_B}\sum_{\mathrm{Pol}}\langle B(v)|\bar{b}(iD_{\mu_1})(iD_{\mu_2})b|B^*(v)\rangle\,\langle B^*(v)|\bar{b}(iD_{\mu_3})(iD_{\mu_4})b|B(v)\rangle$$

- Calculate via "Trace Formulae"
  $\longrightarrow$ reduce them to μ_π and μ_G (Bigi, Zwicky, Uraltsev)

- With two time derivatives:

$$\langle B(v)|\bar{b}(iD_{\mu_1})(iD_0)(iD_0)(iD_{\mu_4})b|B(v)\rangle$$
$$\approx \bar{\epsilon}^2\langle B(v)|\bar{b}(iD_{\mu_1})(iD_{\mu_4})b|B(v)\rangle$$

- $\bar{\epsilon}$: Excitation energy to the first excited state
- Numerical values
  ($\mu_\pi^2 = 0.45\,\mathrm{GeV}^2, \mu_G^2 = 0.35\,\mathrm{GeV}^2, \bar{\epsilon} = 400\,\mathrm{MeV}$)

m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9
0.11	-0.07	-0.08	0.39	-0.06	-0.16	0.42	1.26	0.40

(all values in GeV^4)

- Effect has been studied in detail on the moments
- $\rightarrow$ small effects of expected size!
- Effect on the total rate:
  $(\delta\Gamma|_{1/m_b^i} = (\Gamma|_{1/m_i} - \Gamma|_{1/m_{i-1}})/\Gamma_{\text{parton}})$

$$\delta\Gamma\Big|_{1/m_b^4} \approx +0.29\% \quad \delta\Gamma\Big|_{1/m_b^3} \approx -2.84\% \quad \delta\Gamma\Big|_{1/m_b^2} \approx -4.29\%$$

- Impact on V_{cb}: Slight improvement of the uncertainly related to the application of the HQE
  Total improvement small, $\mathcal{O}(0.25\%)$

$\mathcal{O}(\alpha_s \mu_\pi^2 / m_b^2)$ corrections

- One-Loop α_s corrections known since a long time
- Corrections to the leading (partonic) rate
- Make use of Reparametrization invariance:

$$v \to v' = v + \frac{k}{m_b}$$

- Relates different orders of the $1/m_b$ expansion
- Valid to all orders in α_s
- $\to$ Compute $\mathcal{O}(\alpha_s)$-Correction with $p_b = m_b v + k$ and expand in k

$$k_\mu k_\nu \to (g_{\mu\nu} - v_\mu v_\nu) \frac{\mu_\pi^2}{3}$$

- For the complete α_s/m_b^2 also the $\mathcal{O}(\alpha_s\mu_G^2/m_b^2)$ Corrections need to be computed
- Significantly more complicated
- $\to$ Needs the one gluon matrix elements at one loop
- Doable, is in the pipeline
- The knowledge of the partonic α_s^2 corrections also give us the $\alpha_s^2\mu_\pi^2/m_b^2$ by RPI
- .... and the $\alpha_s m_1/m_b^4$, and the $\alpha_s^2 m_1/m_b^4$

$\mathcal{O}(\alpha_s^2)$ corrections Czarnecki, Pak; Melnikov

- Technically challenging
- Partially numerical calculation
- Analytic Results for limiting cases
- $\rightarrow$ allows for an interpolation
- Recenty: Complete differential distributions available

- Contributions to the Moments ($d\Gamma_0$: Partonic rate)

$$L_n(E_{\text{cut}}) = \frac{\langle (E_l/m_b)^n \, \theta(E_l - E_{\text{cut}}) d\Gamma \rangle}{\langle d\Gamma_0 \rangle}$$

$$H_n(E_{\text{cut}}) = \frac{\langle (E_h/m_b)^n \, \theta(E_l - E_{\text{cut}}) d\Gamma \rangle}{\langle d\Gamma_0 \rangle}$$

- Expansion:

$$L_n(E_{\text{cut}}) = L_n^{(0)} + \frac{\alpha_s}{\pi} L_n^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \left[\beta_0 L_n^{2,\text{BLM}} + L_n^{(2)}\right]$$

$$H_n(E_{\text{cut}}) = H_n^{(0)} + \frac{\alpha_s}{\pi} H_n^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \left[\beta_0 H_n^{2,\text{BLM}} + H_n^{(2)}\right]$$

- $\beta_0 = 11 - 2N_f/3$ and $\alpha_s = \alpha_s^{\overline{\text{MS}},\text{N}_\text{f}=5}(m_b)$

n	$E_{\rm cut}$, GeV	$L_n^{(0)}$	$L_n^{(1)}$	$L_n^{(2,\rm BLM)}$	$L_n^{(2)}$
0	0	1	-1.77759	-1.9170	3.40
1	0	0.307202	-0.55126	-0.6179	1.11
2	0	0.10299	-0.1877	-0.2175	0.394
0	1	0.81483	-1.4394	-1.5999	2.63
1	1	0.27763	-0.49755	-0.5667	1.00
2	1	0.09793	-0.17846	-0.20875	0.382

TABLE I: Lepton energy moments.

Tables from Melnikov

n	$E_{\rm cut}$, GeV	$H_n^{(0)}$	$H_n^{(1)}$	$H_n^{(2,\rm BLM)}$	$H_n^{(2)}$
1	1	0.334	-0.57728	-0.6118	1.02
2	1	0.14111	-0.23456	-0.2343	0.362

TABLE II: Hadronic energy moments.

Even higher orders: $\mathcal{O}(1/m_b^n)$, $n > 4$ Corrections

- $1/m_b^5$ has been studied in the context of "intrinsic charm"
  Numerical estimates of the $1/m_b^5$ are available
- General Structure of the higher order terms have been studied
- Proliferation of new parameters

Estimates of $1/m_b^5$

- In total 18 parameters (Singlet and Triplet)
- Estimates of the parametes by "Ground State Saturation"
- Full expressions for doubly differential rates are available
- Numerical estimates

$$\frac{\Gamma|_{\text{complete}}^{1/m_b^5}}{\Gamma_0} \approx 0.36\% \qquad \frac{\Gamma|_{1/m_c^2}^{1/m_b^5}}{\Gamma_0} \approx 0.46\%$$

Beyond $1/m_b^5$

- Proliferation of parameters in high orders $1/m_b$:

	Dim 5	Dim 6	Dim 7	Dim 8	Dim 9	Dim 10	Dim 11
1	1	1	4	7	24	60	216
σ	1	1	5	11	48	150	624
tot	2	2	9	18	72	210	840

- At high orders: (n = Dim - 3)

$$N_1(n) \approx \frac{1}{2} \sum_{n_g=1}^{[\frac{n}{2}]} (2n_g - 1)!! \binom{n-2}{n-2n_g}$$

$$N_\sigma(n) \approx \frac{1}{2} \sum_{n_g=1}^{[\frac{n}{2}]-1} (2n_g - 1)!! \binom{n-2}{n-2n_g-2} \binom{2+2n_g}{2}$$

Exclusive Vcb

- Kinematic variable for a heavy quark: Four Velovity v
- Differential Rates

$$\frac{d\Gamma}{d\omega}(B \to D^* \ell \bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3} |V_{cb}|^2 m_{D^*}^3 (\omega^2 - 1)^{1/2} P(\omega) (\mathcal{F}(\omega))^2$$
$$\frac{d\Gamma}{d\omega}(B \to D \ell \bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3} |V_{cb}|^2 (m_B + m_D)^2 m_D^3 (\omega^2 - 1)^{3/2} (\mathcal{G}(\omega))^2$$

- with $\omega = vv'$ and
- $P(\omega)$: Calculable Phase space factor
- $\mathcal{F}$ and $\mathcal{G}$: Form Factors

Heavy Quark Symmetries

- Normalization of the Form Factors is known at $vv' = 1$: (both initial and final meson at rest)
- Corrections can be calculated / estimated

$$\begin{aligned} \mathcal{F}(\omega) &= \eta_{\mathrm{QED}}\eta_A \left[1 + \delta_{1/\mu^2} + \cdots\right] (\omega - 1)\rho^2 + \mathcal{O}((\omega - 1)^2) \\ \mathcal{G}(1) &= \eta_{\mathrm{QED}}\eta_V \left[1 + \mathcal{O}\left(\frac{m_B - m_D}{m_B + m_D}\right)\right] \end{aligned}$$

- Parameter of HQS breaking: $\dfrac{1}{\mu} = \dfrac{1}{m_c} - \dfrac{1}{m_b}$
- $\eta_A = 0.960 \pm 0.007$, $\eta_V = 1.022 \pm 0.004$, $\delta_{1/\mu^2} = -(8 \pm 4)\%$, $\eta_{\mathrm{QED}} = 1.007$

$B \to D^{(*)}$ Form Factors from the Lattice

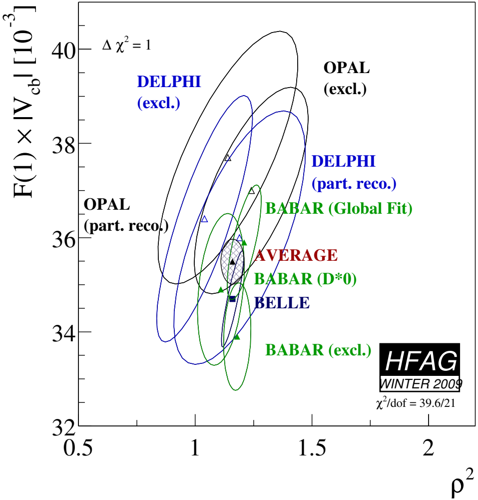
- Unquenched Calculations become available!
- Heavy Mass Limit is not used
- Lattice Calculations of the deviation from unity

$$\boxed{\mathcal{F}(1) = 0.927 \pm 0.024}$$

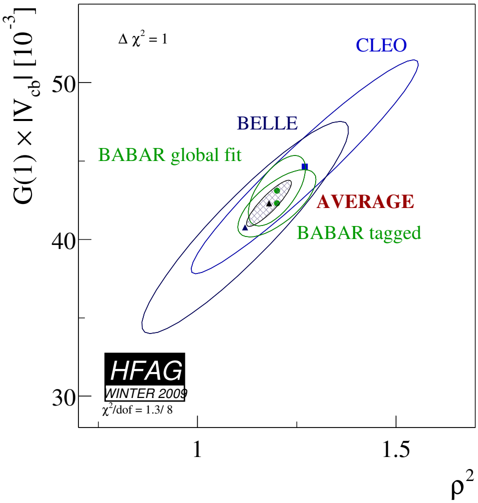
$$\boxed{\mathcal{G}(1) = 1.074 \pm 0.018 \pm 0.016}$$

F(1): Milc/Fermilab 2009, G(1): A. Kronfeld et al. 2005

$B \to D^* \ell \bar{\nu}_\ell$



$B \to D\ell\bar{\nu}_\ell$



$$\boxed{V_{cb,excl} = (38.7 \pm 1.1) \times 10^{-3}}$$

- from Zero Recoil Sum Rules: Hints that $\mathcal{F}(1) \leq 0.9$

  Uraltsev, Gambino, TM: Work to be completed at GGI

- Likewise, old estimate by Kolya for $\mathcal{G}(1)$

$$\mathcal{G}(1) = 1.04 \pm 0.02$$

Ist there a problem $V_{cb}^{\rm incl}$ vs. $V_{cb}^{\rm excl}$?

- Take the V_{cb} value from the (very mature!) inclusive determination
  ... and compute $\mathcal{F}(1)$ and $\mathcal{G}(1)$!

$$\mathcal{F}(1) = 0.86 \pm 0.03$$
$$\mathcal{G}(1) = 1.02 \pm 0.04$$

- Both lattice results are on the high side.

Exclusive Vub: $B \longrightarrow \pi\ell\bar{\nu}_\ell$

- Problem: Calculation of the form factor:

$$\frac{d\Gamma(\bar{B}^0 \rightarrow \pi^+ l^- \nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} p_\pi^3 \, |f_{B\pi}^+(q^2)|^2 + O(m_l^2)$$

- Lattice QCD
- QCD (Light Cone) Sum Rules

Tools: Form Factor Parametrizations

- Becirevic Kaidalov Parametrization

$$f_+(q^2) = \frac{f_+(0)}{(1 - q^2/m_{B^*}^2)(1 - \alpha q^2/m_{B^*}^2)}$$

- z parametrization

$$P(t)\phi(t, t_0)f_+(t) = \sum_{k=0}^{\infty} a_k(t_0) z^k(t, t_0)$$

with

$$z(t, t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}, \quad t_+ = (m_B + m_\pi)^2$$

$$
\begin{aligned}
|V_{ub}| \times 10^3 &= 3.38 \pm 0.36 \\
a_0 &= 0.0218 \pm 0.0021 \\
a_1 &= -0.0301 \pm 0.0063 \\
a_2 &= -0.059 \pm 0.032 \\
a_3 &= 0.079 \pm 0.068
\end{aligned}
$$

R. van der Water, Vxb @ SLAC 2009, Fermilab Milc

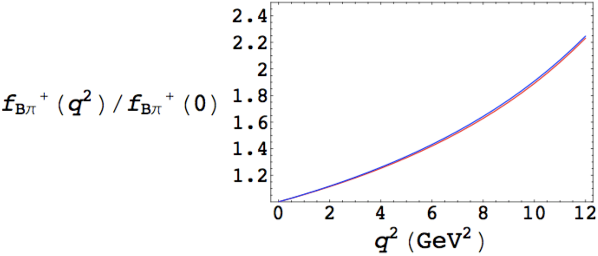
Status of LCSR calculation

- last update: $f^+_{B\pi}(q^2)$ at $0 \leq q^2 \leq 12$ GeV^2 ,

  [G.Duplancic, A.K., B.Melic, Th.Mannel, N.Offen JHEP, 0804, 014 (2008)]

  - with f_B from two-point QCD sum rules (exp. f_B still with a larger error)
  - fitting the shape to the *BABAR* data to constrain LCSR input (twist-2 DA)
  - in this region fitted to BK parameterization with $\alpha_{BK} = 0.53 \pm 0.06$

  (equally well to Boyd-Grinstein-Lebed series-parameterization)

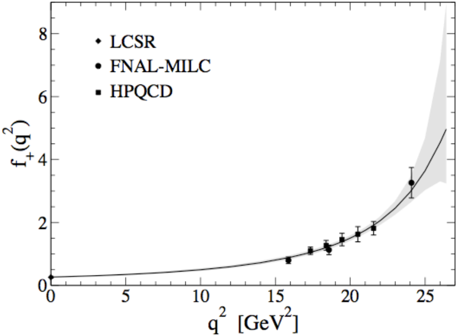


- the result: $\boxed{f^+_{B\pi}(0) = 0.26^{+0.04}_{-0.03}}$

Combining LCSR⊕BaBar shape ⊕ lattice QCD

[Bourrely, Caprini, Lellouch, 0807.222 hep-ph]

- Modified series-parameterization used :



$q^2 = 0$: LCSR *[DKMMO]* , $q^2 > 15$ GeV^2: lattice QCD *[FNAL-MILC, HPQCD]*

$|V_{ub}|$ from $B \to \pi l \nu_l$

a sample of most recent results

[Ref.]	$f^+_{B\pi}(q^2)$ calculation	$f^+_{B\pi}(q^2)$ input	$\lvert V_{ub}\rvert \times 10^3$
FNAL-MILC '08	lattice	-	3.38±0.35
HPQCD '07	lattice	-	3.55±0.25±0.50
Ball, Zwicky '04	LCSR	-	$3.5 \pm 0.4 \pm 0.1$
Flynn, Nieves '07	-	lattice $\oplus$ LCSR	$3.47 \pm 0.29 \pm 0.03$
DKMMO '07	LCSR	-	$3.5 \pm 0.4 \pm 0.2 \pm 0.1$
Bourrely, Caprini, Lellouch '08	-	lattice$\oplus$ LCSR	3.54 ± 0.24

- Exclusive V_{ub}: PDG Average 2010

$$V_{ub} = (3.38 \pm 0.36) \times 10^{-4}$$

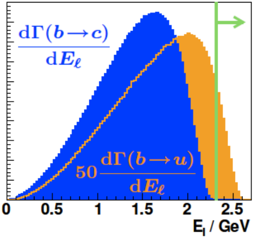
(PDG 2010)

$|V_{ub}|$ from Inclusive $B \to X_u \ell \nu$

Removing huge charm background requires stringent phase space cuts

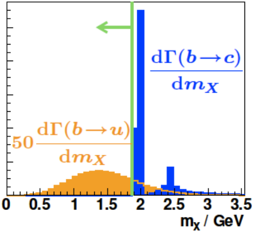
$$\mathcal{B}(B \to X_c \ell \nu) / \mathcal{B}(B \to X_u \ell \nu) \simeq 50$$

- Cuts can drastically enhance perturbative and nonperturbative corrections



Rates become sensitive to b-quark PDFs in B meson

- Determine shape of spectra
- Leading order: Universal shape function (SF) [Neubert (1993); Bigi et al. (1993)]
- $\mathcal{O}(\Lambda_{QCD}/m_b)$: Several more subleading shape functions [Bauer, Luke, Mannel (2001)]
- Need to be extracted from data (like any PDF)



Regions of Phase Space

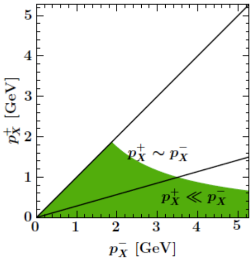
Kinematic variables: $p_X^\pm = E_X \mp |\vec{p}_X|$

Shape function region (SCET region): $p_X^+ \ll p_X^-$

- Leading order in $1/m_b$ requires nonperturbative shape function $S(\omega)$
  [Korchemsky, Sterman (1994); Bauer et al. (2001)]

$$\mathrm{d}\Gamma = H(E_\ell, p_X^\pm)\int \mathrm{d}\omega\, J[p_X^-(p_X^+ - \omega)]S(\omega)$$

- $\mathcal{O}(\alpha_s^2)$ corrections recently completed
  [Becher, Neubert (2005, 2006); Bonciani, Ferroglia; Asatrian et al.; Beneke et al.; Bell (2008)]



Local OPE region: $p_X^+ \sim p_X^-$ (q^2 spectrum, small E_ℓ)

- Leading order in $1/m_b$ given by quark decay (as in $B \to X_c \ell\nu$) known to $\mathcal{O}(\alpha_s, \alpha_s^2\beta_0)$ [De Fazio, Neubert (1999); Gardi, Ridolfi, Gambino (2006)]

Cut on $m_X < m_D$ does not imply $p_X^+ \ll p_X^-$ $\Rightarrow$ depends on both regions

Non-Experimental Uncertainties

Theoretical uncertainties

- Unknown higher orders in α_s, $1/m_b$ expansions
- Weak annihilation (open question $\Rightarrow$ separate data into B^+ and B^0)

Dominant uncertainties on $|V_{ub}|$ come from input parameters

- m_b: Total rate $\sim |V_{ub}|^2 m_b^5$, partial rates with cuts $\sim |V_{ub}|^2 m_b^{\mathcal{O}(10)}$
  - Need as precise as possible m_b to get precise $|V_{ub}|$
- Shape function(s): Sensitivity depends on phase space region
  - (a) SCET region: small p_X^+, very large E_ℓ
    $\Rightarrow$ Need the full shape (i.e. all moments)
  - (b) Local OPE region: total rate, q^2 spectrum, small E_ℓ
    $\Rightarrow$ Only need 1st moments (i.e. m_b, μ_π^2)
  - (c) something in between: m_X, moderately large E_ℓ
- m_b and SF uncertainties are separate but correlated

The OPE hard–cutoff approach

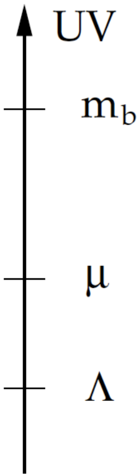
- Gambino, Giordano, Ossola & Uraltsev propose to write each structure function as a convolution

$$W_i = \int dk^+ F_i(k^+, q^2; \mu) W_i^{\rm pert}(\mu) \,,$$

  A hard cutoff $\mu = 1$ GeV is implemented in the 'kinetic scheme'. $F_i(k^+, q^2; \mu)$ are non-perturbative functions, parametrized subject to constrains on the moments of W_i computed by OPE.

- Advantages: simple and prudent! Perturbation theory is used in a safe regime above 1 GeV; the infrared is parametrized.

- Limitations:
  - Extensive parametrization: the unknown functions $F_i(k^+, q^2; \mu)$ depends on *two* kinematic variables!
  - Known structure of infrared singularities not used!

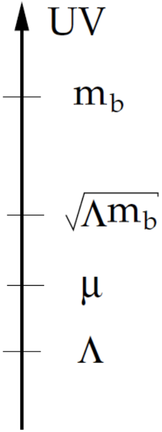


The elaborate shape function approach

- For jet kinematics $P^+ \ll P^- \simeq m_b$ one has

$$\frac{d\Gamma}{dP^- dP^+ dE_l} = HJ \otimes S(k^+, \mu) + \frac{\sum H_n J_n \otimes S_n(k^+, \mu)}{m_b} + \cdots \quad (1)$$

- The shape function approach by Bosch, Lange, Neubert & Paz combines (1), valid for jet kinematics, with the local OPE.
- Advantages: elaborate use of theoretical tools. Sudakov resummation of jet logs.
- Limitations:
  - starting at $\mathcal{O}(1/m_b)$ more unknowns than observables
  - Even the first $S(k^+, \mu)$ cannot be computed non-perturbatively. It is parametrized based on known center (m_b) and width (μ_π^2) alone.



Dressed Gluon Exponentiation (DGE)

- Resummed perturbation theory (on-shell heavy quark) yields:

$$\frac{1}{\Gamma_{\rm tot}}\frac{d\Gamma}{dP^+dP^-dE_l} = \int_{-i\infty}^{i\infty}\frac{dN}{2\pi i}\left(1-\frac{P^+-\bar\Lambda}{P^--\bar\Lambda}\right)^{-N} H(N,P^-,E_l)\,\overline{\rm Sud}(P^-,N)$$

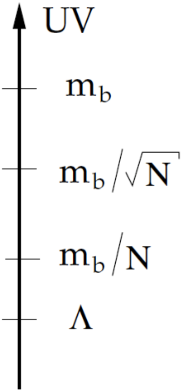
soft and collinear radiation is summed into a Sudakov factor

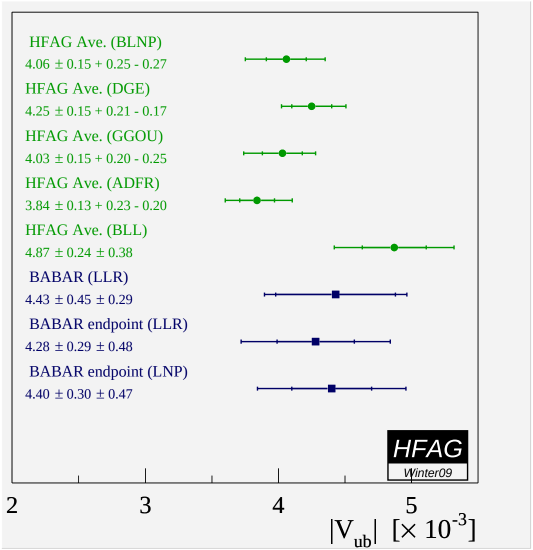
$$\overline{\rm Sud}(p^-,N) = \exp\left\{\frac{C_F}{\beta_0}\int_0^\infty \frac{du}{u}\,T(u)\left(\frac{\Lambda}{p^-}\right)^{2u}\left[\underbrace{B_{\mathcal{J}}(u)\Gamma(-u)\left(1-N^u\right)}_{\rm Jet} - \underbrace{B_{\mathcal{S}}(u)\Gamma(-2u)\left(1-N^{2u}\right)}_{\rm Quark\ Distribution}\right]\right\}$$

- Renormalon resummation indicates the presence of specific power corrections $(N\Lambda/p^-)^k$ in the exponent!
    - $u=1/2$ ambiguity cancels with the pole mass renormalon.
    - $u=1$ renormalon is missing ($B_{\mathcal{S}}(1)=0$).
    - $u\geq 3/2$ ambiguities are present in the on-shell spectrum.

Dressed Gluon Exponentiation (DGE)

- Resummed on-shell calculation in moment space, with no cutoff!
  resummation includes:
  - Sudakov logs of both jet and quark–distribution — both currently at NNLL accuracy!
  - Renormalon resummation in the exponent.
- Parametrization of power corrections in moment space
- Advantages: Ultimate use of resummed perturbation theory; minimal parametrization.
- Limitations: difficult to relate the magnitude of power corrections to conventional cutoff based definitions.




HFAG Ave. (BLNP)
4.06 ± 0.15 + 0.25 - 0.27
HFAG Ave. (DGE)
4.25 ± 0.15 + 0.21 - 0.17
HFAG Ave. (GGOU)
4.03 ± 0.15 + 0.20 - 0.25
HFAG Ave. (ADFR)
3.84 ± 0.13 + 0.23 - 0.20
HFAG Ave. (BLL)
4.87 ± 0.24 ± 0.38
BABAR (LLR)
4.43 ± 0.45 ± 0.29
BABAR endpoint (LLR)
4.28 ± 0.29 ± 0.48
BABAR endpoint (LNP)
4.40 ± 0.30 ± 0.47
HFAG
Winter09
2
3
4
5
$|V_{ub}|$ $[\times 10^{-3}]$

- Inclusive V_{ub}: PDG Average 2010

$$V_{ub} = (4.27 \pm 0.24) \times 10^{-4}$$

(PDG 2010)

- Recent BELLE Analysis of inclusive V_{ub}:

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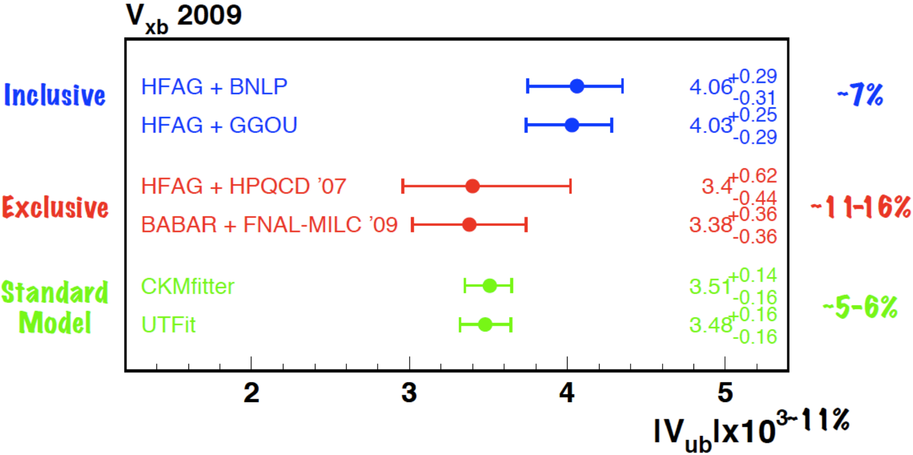
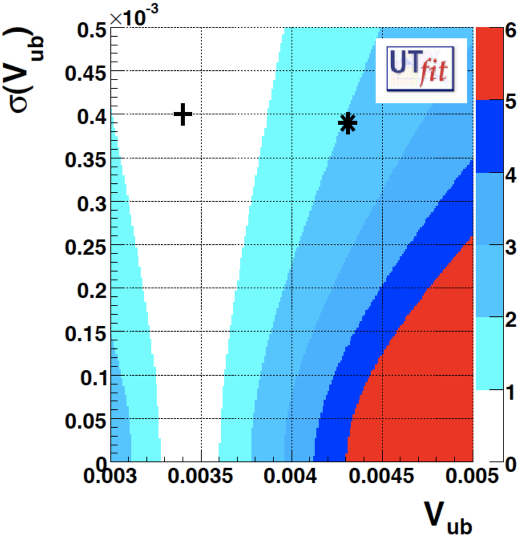
TABLE II. Values for $|V_{ub}|$ with relative errors (in %).

Theory	$\lvert V_{ub}\rvert \times 10^3$	Stat	Syst	m_b	Th.
BLNP [5]	4.37	4.3	4.0	$^{+3.1}_{-2.7}$	$^{+4.3}_{-4.0}$
DGE [6]	4.46	4.3	4.0	$^{+3.2}_{-3.3}$	$^{+1.0}_{-1.5}$
GGOU [7]	4.41	4.3	4.0	1.9	$^{+2.1}_{-4.5}$

Instead of conclusions ...

- Exclusive V_{cb} vs Inclusive V_{cb}:
  My point of view: The Problem lies in the Latice calculation of the Form Factors
- Exclusive V_{cb} vs Inclusive V_{cb}: More severe ...


V$_{xb}$ 2009
Inclusive
Exclusive
Standard Model
HFAG + BNLP
HFAG + GGOU
HFAG + HPQCD '07
BABAR + FNAL-MILC '09
CKMfitter
UTFit
$4.06^{+0.29}_{-0.31}$
$4.03^{+0.25}_{-0.29}$
$3.4^{+0.62}_{-0.44}$
$3.38^{+0.36}_{-0.36}$
$3.51^{+0.14}_{-0.16}$
$3.48^{+0.16}_{-0.16}$
~7%
~11-16%
~5-6%
2
3
4
5
$|V_{ub}| x 10^{3}$
~11%


$\times 10^{-3}$
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
$\sigma(V_{ub})$
0.003
0.0035
0.004
0.0045
0.005
V_{ub}
6
5
4
3
2
1
0
UTfit