Lattice QCD and Flavour

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Indirect Searches for New Physics at the time of the LHC

GGI, Florence, March 23rd 2010

1. Introduction

- There has been a huge improvement in the precision of lattice calculations in the last 3 years or so.
- There are a number of groups focussing on different aspects on flavour physics.
- I will talk about progress in kaon physics, particularly from the RBC-UKQCD collaboration using Domain Wall Fermions (set in context).
  - RBC=RIKEN, Brookhaven National Laboratory, Columbia University.
  - UKQCD in this project = Edinburgh and Southampton Universities.
  - We coordinate the generation of (expensive) ensembles and work in subgroups on a wide variety of physics topics.
  - The 2008 paper describing our old ensembles had 33 authors and we are preparing the analogous paper for our new ensembles.
- A set of references is found at the end of the talk.
- I also exploit preliminary results of the Flavianet Lattice Averaging Group (FLAG): G. Colangelo, S. Dürr, A. Jüttner, L. Lellouch, H. Leutwyler, V. Lubicz, S. Necco, C. Sachrajda, S. Simula, A. Vladikas, U. Wenger, H. Wittig.

Plan of the Talk

1. Introduction
2. Determination of V_{us}
   - 2.i f_K/f_π .
   - 2.ii $K_{\ell 3}$ decays.
3. B_K
4. η and η' mesons and mixing.
5. $K \to \pi\pi$ Decays
6. Conclusions and Prospects

- We use two datasets of DWF with the Iwasaki Gauge Action with a lattice spacing of about 0.114 fm:
  - $24^3 \times 64 \times 16 \quad (L \simeq 2.74\,\text{fm})$
  - $(16^3 \times 32 \times 16 \quad (L \simeq 1.83\,\text{fm})\,)$
- On the 24^3 lattice measurements have been made with 4 values of the light-quark mass:

$$ma = 0.03\ (m_\pi \simeq 670\,\text{MeV}); \qquad ma = 0.02\ \ (m_\pi \simeq 555\,\text{MeV});$$
$$ma = 0.01\ (m_\pi \simeq 415\,\text{MeV}); \qquad ma = 0.005\ (m_\pi \simeq 330\,\text{MeV})\,.$$

  - (Using partial quenching the lightest pion in our analysis has a mass of about 240 MeV .)

  On the 16^3 lattice results were obtained with $ma = 0.03,\, 0.02$ and 0.01.
- For the (sea) strange quark we take $m_s a = 0.04$, although a posteriori we see that this is a little too large.
- We are completing the analysis of an ensemble on a $32^3 \times 64 \times 16$ lattice with $a \simeq 0.081$ fm ($L \simeq 2.6$ fm) with three dynamical masses ($m_\pi \simeq 310,\ 365$ and 420 MeV).

  This will enable us to reduce the discretization errors significantly.

  Some preliminary results were presented at Lattice 2008, 2009 and elsewhere.

- Imagine an idealized situation where simulations are possible at all quark masses for a variety of βs ($\beta = \beta_i$, $i = 1, 2, \cdots, N$). We can choose to fix $m_{ud}(\beta_i)$, $m_s(\beta_i)$ and $a(\beta_i)$ by requiring that 3 physical quantities take their physical values. This defines a *Scaling Trajectory*.
  - We use m_π, m_K and m_Ω.
- We can then calculate other physical quantities ($f_\pi(\beta_i)$, $B_K(\beta_i)$, $\cdots$). These will have lattice artefacts of $O(a_i^2\Lambda_{\mathrm{QCD}}^2)$ and we imagine extrapolating the results to the continuum limit.
- At present however, we have to extrapolate to the physical values of m_{ud} (and interpolate to m_s). We have invested considerable effort in defining and performing global fits in which we keep physical Low Energy Constants at all (both) β_i and yet treat the artefacts consistently. **ALMOST DONE.**

$$O(m_\pi^2/\Lambda_\chi^2),\ O(a^2\Lambda_{\mathrm{QCD}}^2)\ \surd, \quad O((m_\pi/\Lambda_\chi)^4),\ O(a^2m_\pi^2),\ O((a\Lambda_{\mathrm{QCD}})^4)\cdots\times .$$

  - We use other ansatz also.

- \- Topology Changing.
  - Although the algorithms used in the generation of field ensembles are formally ergodic, in a finite simulation it may be that the space of field configurations has not been fully sampled.
  - Procedures for calculating autocorrelations exist, but can not be 100% reliable.
  - It has recently been stressed that for fine lattices ($a \lesssim 0.04$ fm), the topological charge does not change (for the actions generally used).

    Zeuthen and CERN groups, $\cdots$.
  - There is a large amount of algorithmic work being devoted to overcome this problem.
- Step Scaling — Alpha Collaboration.
  - Although the idea of step-scaling and the *femto universe* have been advocated for a long time by the Alpha collaborations, up to recently they have only been used by a small number of groups.
  - Improved precision in the calculation of physical quantities $\Rightarrow$ this is becoming a more widely used technique (B-physics, Non-perturbative renormalization etc.)
  - Match lattices at different β until we end up with a very fine, but small, lattice where connection with continuum QCD can be made reliably.

- *- Reweighting*
  - Although we can simulate at $m_s^{\rm phys}$, we only know its value a posteriori.
  - We therefore have to estimate what m_s is before performing the simulations.
  - Imagine that we wish to compute (Dirac operator $D_q = D[U, m_q]$)

$$\langle O\rangle_2 = \frac{\int d[U]e^{-S_g}\sqrt{\det(D_2^\dagger D_2)}\,O(U)}{\int d[U]e^{-S_g}\sqrt{\det(D_2^\dagger D_2)}}$$

  - Imagine also that we performed the simulation with mass m_1. Now

$$\langle O\rangle_2 = \frac{\int d[U]e^{-S_g}\sqrt{\det(D_1^\dagger D_1)}\,O(U)\,w(U)}{\int d[U]e^{-S_g}\sqrt{\det(D_1^\dagger D_1)}\,w(U)}$$

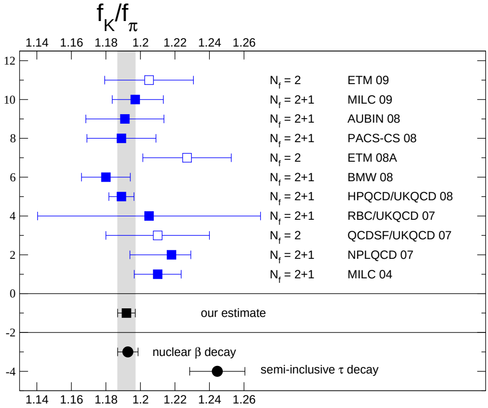
    where

$$w[U] = \det\left(\frac{D_2^\dagger[U]D_2[U]}{D_1^\dagger[U]D_1[U]}\right)^{1/2} \equiv \det{}^{-1/2}(\Omega) = \left(\frac{\int D\xi\, e^{-\xi^\dagger\sqrt{\Omega[U]}\xi}}{\int D\xi\, e^{-\xi^\dagger\xi}}\right).$$

  - Jointly sampling U and ξ fields $\Rightarrow \langle O\rangle_2$.
    - One (small) systematic error removed.

2. V_{us} – f_K/f_π FLAG Compendium – Preliminary

- All groups calculate f_K/f_π.



- Flag Compendium – Preliminary:
  - $f_K/f_\pi = 1.190(2)(10)$ – Direct $N_f = 2+1$;
  - $f_K/f_\pi = 1.210(6)(17)$ – Direct $N_f = 2$.

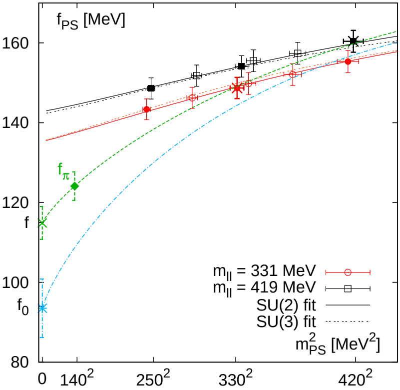
- The calculation requires a reliable chiral extrapolation.

$$\Rightarrow \quad \text{SU(2) ChPT}\,.$$

RBC/UKQCD, arXiv:0804:0473

- Is the chiral extrapolation as well under control for all quantities as we think?
- Very soon, as the simulated masses $\to m_\pi^{\text{phys}}$ the chiral extrapolation will be a smaller concern.

Comparison of Results obtained using SU(2) and SU(3) ChPT



RBC/UKQCD, arXiv:0804:0473

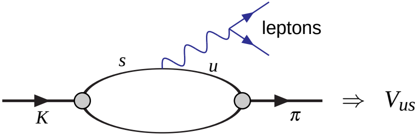
- Study is performed at NLO in the chiral expansion.
- black points - partially quenched results with $am_l = 0.01$ ($m_\pi^{\text{unitary}} \simeq 420$ MeV).
- red points - partially quenched results with $am_l = 0.005$ ($m_\pi^{\text{unitary}} \simeq 330$ MeV).
- We find:

$$f_\pi/f \simeq 1.08\,, \quad f/f_0 = 1.23(6)\,.$$

- The corresponding results from the MILC collaboration, who do an NNLO analysis (partly in staggered chiral perturbation theory), with NNNLO analytic terms:

$$f_\pi/f = 1.052(2)\begin{pmatrix}+6\\-3\end{pmatrix}\,, \quad f/f_0{}_{\,\mathrm{MILC}} = 1.15(5)\begin{pmatrix}+13\\-3\end{pmatrix}\,,$$

- The large value of f_π/f_0 (and even larger values of f_{PS}/f_0 of ~ 1.6 where we have data) lead RBC/UKQCD (and ETMC) to present results based on SU(2)$\times$ SU(2) ChPT.

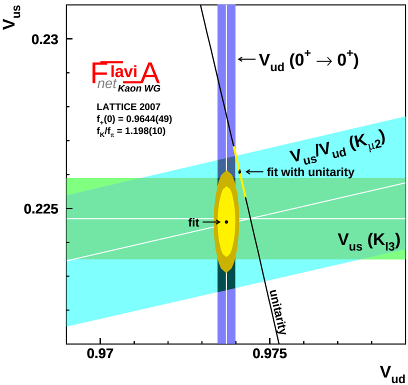


$$\langle\,\pi(p_\pi)\,|\bar{s}\gamma_\mu u\,|\,K(p_K)\,\rangle = f_0(q^2)\,\frac{M_K^2-M_\pi^2}{q^2}q_\mu + f_+(q^2)\,\left[(p_\pi+p_K)_\mu - \frac{M_K^2-M_\pi^2}{q^2}q_\mu\right]$$

where $q \equiv p_K - p_\pi$.
To be useful in extracting V_{us} we require $f_0(0) = f_+(0)$ to better than about 1% precision.

$$\chi\text{PT} \Rightarrow f_+(0) = 1 + f_2 + f_4 + \cdots \quad \text{where} \quad f_n = O(M^n_{K,\pi,\eta}).$$

Reference value $f_+(0) = 0.961 \pm 0.008$ where $f_2 = -0.023$ is relatively well known from χPT and $f_4, f_6, \cdots$ are obtained from models. Leutwyler & Roos (1984)



$$
\begin{aligned}
f_+^{K\pi}(0) &= 0.9644(33)(34) \\
&\Rightarrow |V_{us}| = 0.2247(12)
\end{aligned}
$$

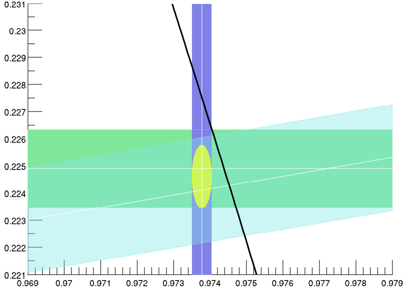
$$
\begin{aligned}
\frac{f_K}{f_\pi} &= 1.198(10) \\
&\Rightarrow |V_{us}| = 0.2241(24)
\end{aligned}
$$

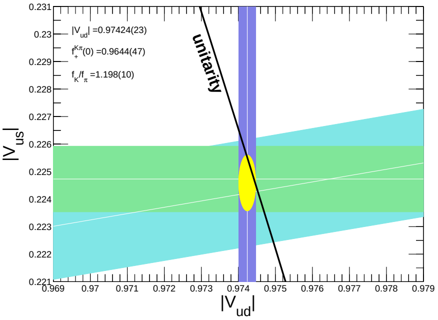
A.Jüttner, Lattice 2007

Our final result from the $K_{\ell 3}$ project is

$$f_+^{K\pi}(0) = 0.964(5)\,.$$

P.A.Boyle et al. [RBC&UKQCD Collaborations – arXiv:0710.5136 [hep-lat]]





$$V_{ud} = 0.97372(10)(15)(19)$$

W.Marciano, Kaon2007

$$V_{ud} = 0.97424(23)$$

I.Towner and J.Hardy, CKM(2008)

Courtesy of Flavianet Kaon WG and A.Jüttner

- The uncertainties on $|V_{ud}|^2$ and $|V_{us}|^2$ are comparable!

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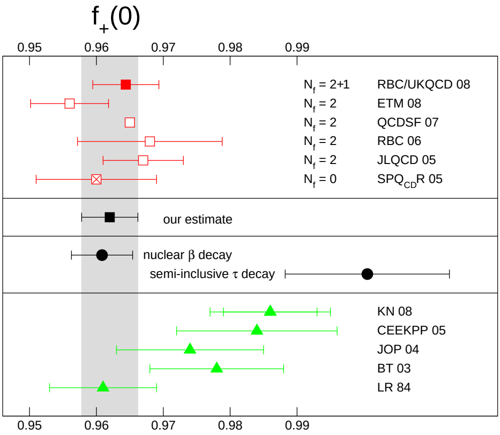
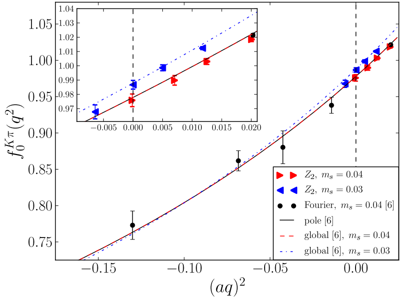


- RBC-UKQCD and ETM to lighter masses.

Improving the Precision – q^2 Extrapolation

P.A.Boyle *et al.* March 2010

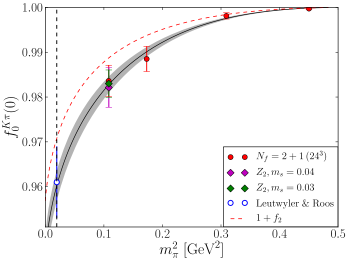


- We are now able to calculate the form-factor directly at $q^2 = 0$ (using twisted boundary conditions).
- For example for the 330 MeV pion:

$$f^{K\pi}(0)_{\rm pole} = 0.9774(35);\quad f^{K\pi}(0)_{\rm polynomial} = 0.9749(59);\quad f^{K\pi}(0)_{\rm TBC} = 0.9757(44).$$

  - An important source of systematic error has been eliminated.

Improving the Precision – Chiral Extrapolation

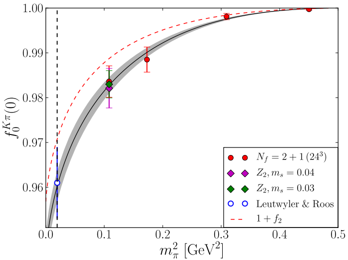


- Where we have data the results are robust.
- The principal uncertainty is in the chiral extrapolation.
- For example, what value should we take for f in

$$f_2 = \frac{3}{2}H_{\pi K} + \frac{3}{2}H_{\eta K}; \quad H_{PQ} = -\frac{1}{64\pi^2 f^2}\left[M_P^2 + M_Q^2 + \frac{2M_P^2 M_Q^2}{M_P^2 - M_Q^2}\log\frac{M_Q^2}{M_P^2}\right]?$$

- Examples (all of which fit the lattice data well):

$$f = 100,\ 115,\ 131.5\,\text{MeV} \Rightarrow f_+^{K\pi}(0) = 0.9556,\ 0.9599,\ 0.9631\,\text{respectively}\,.$$



- The emphasis must now be to reduce the error due to the chiral extrapolation.
- Lattice simulations are being performed at lighter masses.
- Need theoretical guidance in optimizing the chiral extrapolation.
- *Hard Pion SU(2) Chiral Perturbation Theory*: J.Flynn & CTS, arXiv:0809.1229

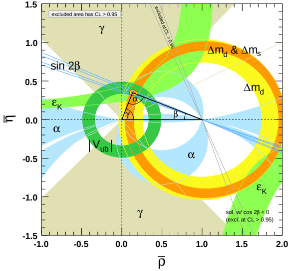
$$f_0(0) = f_+(0) = F_+\left(1 - \frac{3}{4}\frac{m_\pi^2}{16\pi^2 f^2}\log\left(\frac{m_\pi^2}{\mu^2}\right) + c_+ m_\pi^2\right)$$

$$f_-(0) = F_-\left(1 - \frac{3}{4}\frac{m_\pi^2}{16\pi^2 f^2}\log\left(\frac{m_\pi^2}{\mu^2}\right) + c_- m_\pi^2\right).$$

- It would be useful to know the result at NNLO.

3. B_K

University of Southampton School of Physics and Astronomy



- Flavour and Chiral symmetry properties of DWF well suited to this calculation.
- $\Delta S = 2$ operator renormalizes multiplicatively and is renormalized nonperturbatively.

- Our published results are arXiv:hep-ph/0702042, 0804.0473

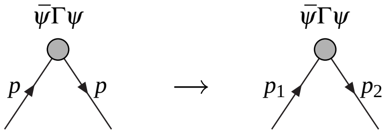
$$B_K^{\overline{\rm MS}}(2\,{\rm GeV}) = 0.524(10)(28) \qquad (\hat{B}_K = 0.720(13)(37))\,.$$

- The largest component of the uncertainty is due to the single lattice spacing.
- Analysis with a second a and continuum extrapolation almost ready (v18 of draft).
- Aubin, Laiho, Van de Water, $\hat{B}_K = 0.724(8)(28)$, (DWF/Staggered Mixed Action) arXiv:0905.3947
- Other groups have preliminary results.

B_K – Current Work

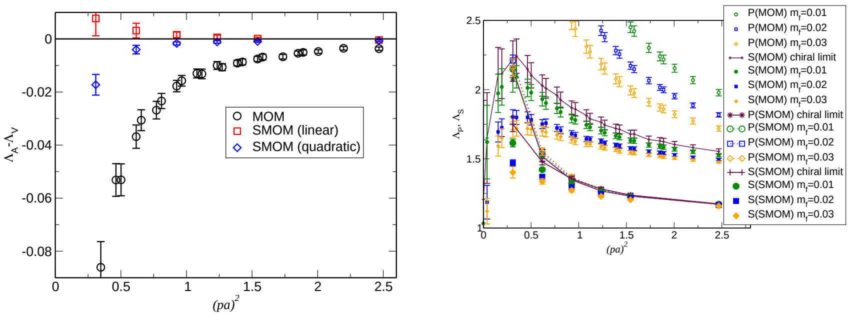
- We are almost completed the full analysis of B_K on our finer lattice and hence to be able to compute the continuum extrapolation.
- We are currently repeating the procedure for all the possible dimension 6 $\Delta S = 2$ operators which contribute in extensions of the standard model.
- We have been generalizing the Rome-Southampton Non-Perturbative Renormalization method (RI-MOM) to non-exceptional momenta.

RBC-UKQCD - arXiv:0712.1061, arXiv:0901.2599



$$p_1^2 = p_2^2 = (p_1 - p_2)^2$$

Evidence for small chiral symmetry breaking



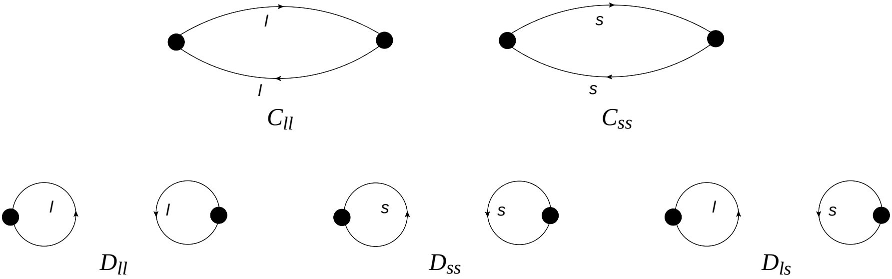
- $\Lambda_A - \Lambda_V$.
- Λ_S and Λ_P.

Y.Aoki arXiv:0901.2595 [hep-lat]

- We have also renormalized $O^{\Delta S=2}$ using non-exceptional momentum configurations.

RBC-UKQCD – arXiV:1002.2999

- To study η and η' we need to evaluate *disconnected* diagrams.



- Here l represents the u or d quark ($m_u = m_d$) and s the strange quark.
- For disconnected diagrams the needed exponential decrease in t comes from increasingly large statistical cancelations implying a rapidly vanishing signal-to-noise ratio.

RBC-UKQCD – arXiV:1002.2999

- Let

$$O_l = \frac{\bar{u}\gamma_5 u + \bar{d}\gamma_5 d}{\sqrt{2}} \quad \text{and} \quad O_s = \bar{s}\gamma_5 s\,.$$

- We calculate the correlation functions

$$X_{\alpha\beta}(t) = \frac{1}{32}\sum_{t'=0}^{3} 1\,\langle\, O_\alpha(t+t')\,O_\beta(t')\,\rangle \quad \text{where} \quad \alpha,\beta = l,s\,.$$

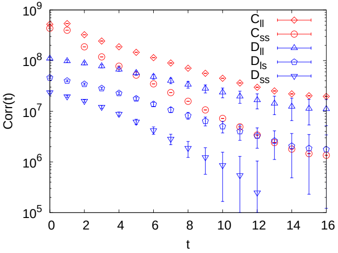
  - Sources are generated for each time slice (T=32).
  - $X_{ls} \neq 0$ because of the $D_{ls} = D_{sl}$ diagrams.

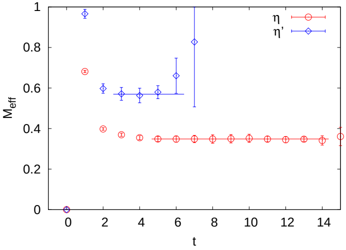
- The four correlation functions correspond to the diagrams as follows:

$$\begin{pmatrix} X_{ll} & X_{ls} \\ X_{sl} & X_{ss} \end{pmatrix} = \begin{pmatrix} C_{ll} - 2D_{ll} & -\sqrt{2}D_{ls} \\ -\sqrt{2}D_{sl} & C_{ss} - D_{ss} \end{pmatrix}.$$

- The usual expectation that disconnected diagrams and the resulting mixing are small does not apply here.

RBC-UKQCD – arXiV:1002.2999





- We diagonalize $X(t)$ at each t:

$$X(t) = A^{\mathrm{T}} \begin{pmatrix} e^{-m_\eta t} & 0 \\ 0 & e^{-m_{\eta'} t} \end{pmatrix} A\,, \quad \text{where} \quad A = \begin{pmatrix} \langle \eta \,|\, O_l \,|0\rangle & \langle \eta \,|\, O_s \,|0\rangle \\ \langle \eta' \,|\, O_l \,|0\rangle & \langle \eta' \,|\, O_s \,|0\rangle \end{pmatrix}$$

- To be more precise we diagonalize $X(t_0)^{-1} X(t)$. Lüscher and Wolff (1990)

- In the standard phenomenological treatment of $\eta - \eta'$ mixing

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |8\rangle_{\rm sym} \\ |1\rangle_{\rm sym} \end{pmatrix}$$

- In the O_8 and O_1 basis

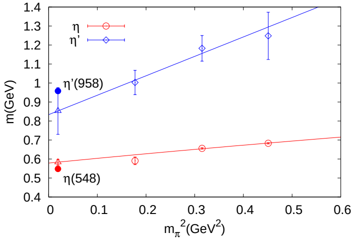
$$A = \begin{pmatrix} \sqrt{Z_8}\cos\theta & -\sqrt{Z_1}\sin\theta \\ \sqrt{Z_8}\sin\theta & \sqrt{Z_1}\cos\theta \end{pmatrix} \quad \text{where} \quad {}_{\rm sym}\langle a|O_b|0\rangle = \sqrt{Z_a}\delta_{ab}\,.$$

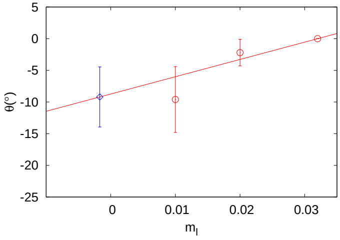
- If this model is correct then the columns of A are orthogonal. We find for the dot product - $-0.009(49)$ for $m_l = 0.01$ and $0.008(24)$ for $m_l = 0.02$.
- The mixing angle can be determined from

$$\frac{A_{\eta 1}A_{\eta' 8}}{A_{\eta 8}A_{\eta' 1}} = -\tan^2\theta\,.$$

$\eta - \eta'$ mixing

RBC-UKQCD – arXiV:1002.2999





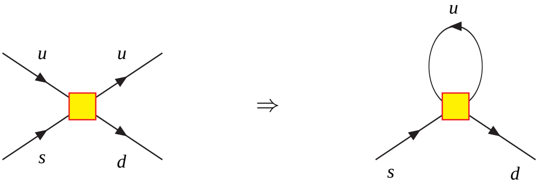
- We find $m_\eta = 583(15)$ MeV and $m_{\eta'} = 853(123)$ MeV and $\theta = -9.2(4.7)^\circ$. (Statistical errors only.)
- To our accuracy, our calculation demonstrates that QCD can explain the relatively large mass of the ninth pseudoscalar meson and its small mixing with the SU(3) octet state.
- There is plenty more to do!

5. $K \to \pi\pi$ decay amplitudes from $K \to \pi$ Matrix Elements

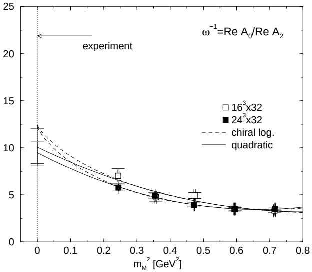
- At lowest order in the SU(3) chiral expansion one can obtain the $K \to \pi\pi$ decay amplitude by calculating $K \to \pi$ and $K \to$ vacuum matrix elements.
- In 2001, two collaborations published some very interesting (quenched) results on non-leptonic kaon decays in general and on the $\Delta I = 1/2$ rule and ε'/ε in particular:

Collaboration(s)	Re A_0/Re A_2	ε'/ε
RBC	25.3 ± 1.8	$-(4.0 \pm 2.3) \times 10^{-4}$
CP-PACS	$9 \div 12$	$(-7 \div -2) \times 10^{-4}$
Experiments	22.2	$(17.2 \pm 1.8) \times 10^{-4}$

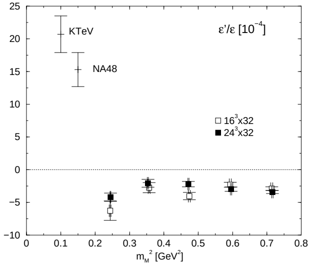
- This required the control of the *ultraviolet problem*, the subtraction of power divergences and renormalization of the operators – highly non-trivial.
    - Four-quark operators mix, for example, with two quark operators $\Rightarrow$ power divergences:



- Re A_0/Re A_2 as a function of the meson mass.



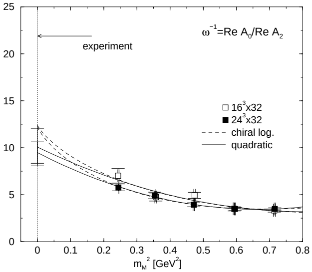
- ε'/ε as a function of the meson mass.



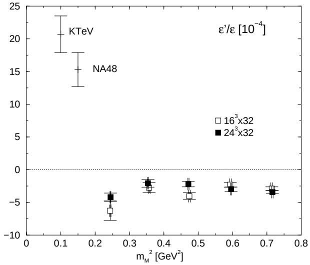
- The RBC and CP-PACS simulations were quenched, and relied on the validity of lowest order χPT in the region of approximately 400-800 MeV.
- Given the cancellations between different matrix elements (particularly O_6 and O_8) the negative value of ε'/ε is not such an embarrassment but

Must do better!

- Re A_0/Re A_2 as a function of the meson mass.



- ε'/ε as a function of the meson mass.



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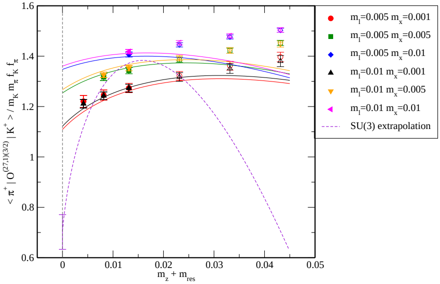


N.Christ arXiV:0912.2917

$$O^{3/2}_{(27,1)} = (\bar{s}d)_L\left\{(\bar{u}u)_L - (\bar{d}d)_L\right\} + (\bar{s}u)_L\,(\bar{u}d)_L$$

- RBC/(UKQCD) have repeated the calculation with the 24^3 DWF ensembles in the pion-mass range 240-415 MeV.
- For illustration consider the determination of α_{27}, the LO LEC for the (27,1) operator. Satisfactory fits were obtained, but again the corrections were found to be huge, casting serious doubt on the approach.
- Soft pion theorems are not sufficiently reliable $\Rightarrow$ need to compute $K \to \pi\pi$ matrix elements.
- To arrive at this important conclusion required a major effort.

- To make progress we need to be able to calculate $K \to \pi\pi$ matrix elements directly and the RBC/UKQCD Collaboration is undertaking a major study.
  T.Blum, P.Boyle, D. Broemmel, J. Flynn, E. Goode, T. Izubuchi, C. Kim, M. Lightman, Qi Liu, R. Mawhinney, N. Christ, C. Sachrajda, A. Soni.
- The main theoretical ingredients of the *infrared* problem with two-pions in the s-wave are now understood.
- Two-pion quantization condition in a finite-volume

$$\delta(q^*) + \phi^P(q^*) = n\pi\,,$$

where $E^2 = 4(m_\pi^2 + q^{*\,2})$, δ is the s-wave $\pi\pi$ phase shift and ϕ^P is a kinematic function.
M.Lüscher, 1986, 1991, $\cdots$.

- The relation between the physical $K \to \pi\pi$ amplitude A and the finite-volume matrix element M

$$|A|^2 = 8\pi V^2\,\frac{m_K E^2}{q^{*\,2}}\,\left\{\delta'(q^*) + \phi^{P\,\prime}(q^*)\right\}\,|M|^2\,,$$

where $'$ denotes differentiation w.r.t. q^* .
L.Lellouch and M.Lüscher, hep-lat/0003023; C.h.Kim, CTS and S.Sharpe, hep-lat/0507006; N.H.Christ, C.h.Kim and T.Yamazaki hep-lat/0507009

$K \to (\pi\pi)_{I=2}$ - Evaluating the LL Factor

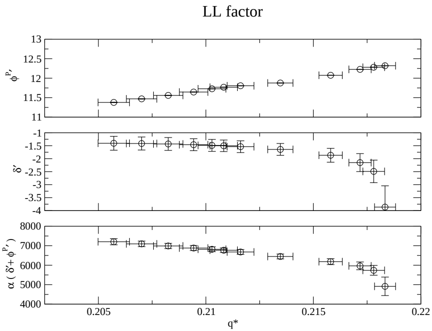
C.h. Kim and CTS, arXiv:1003.3191

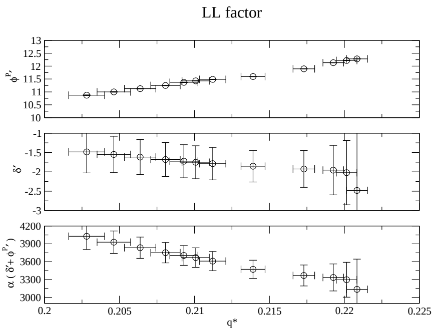
- Use the Wigner-Eckart Theorem to relate the physical $K \to \pi^+\pi^0$ matrix element to that for $K \to \pi^+\pi^+$

$$ {}_{I=2}\langle \pi^+(p_1)\pi^0(p_2)\,|O^{3/2}|K^+\rangle = \frac{3}{2}\langle \pi^+(p_1)\pi^+(p_2)\,|O'^{3/2}|K^+\rangle\,, $$

- Calculate the $K \to \pi^+\pi^+$ matrix element with the u-quark with twisted boundary conditions with twisting angle θ.
- Perform a Fourier transform of one of the pion interpolating operators with additional momentum $-2\pi/L$.
  The ground state now corresponds to one pion with momentum θ/L and the other with momentum $(\theta-2\pi)/L$.
- The corresponding $\pi\pi$ s-wave phase-shift can then be obtained by the Lüscher formula as a function of θ $\Rightarrow$ this allows for the derivative of the phase-shift to be evaluated directly at the masses being simulated.
- We have carried this procedure out in an exploratory calculation. Fig
- Unfortunately this technique does not work for $K \to (\pi\pi)_{I=0}$ decays.

C.h.Kim and CTS, arXiv:1003.3191





C.h. Kim and CTS, arXiv:1003.3191

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- We are starting a major project to calculate the $\Delta I = 3/2$ $K \to \pi\pi$ Decay Amplitudes. There are no significant obstacles to completing this.
  - An exploratory quenched study with improved Wilson fermions was completed in 2004 but at the time we did not understand the Finite-Volume corrections at non-zero total momentum.

    P. Boucaud, V. Gimenez, C. J. D. Lin, V. Lubicz, G. Martinelli, M. Papinutto and C. T. Sachrajda, Nucl. Phys. B **721** (2005) 175
  - The first results of an exploratory quenched study with Domain Wall Fermions were presented at Lattice 2009.

    M.Lightman and E.J.Goode, arXiv:0912.1667

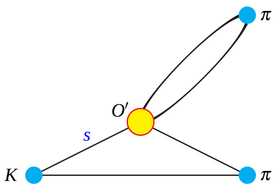
    Novel features included:
    - using the Wigner-Eckart Theorem:

      $$_{I=2}\langle \pi^+(p_1)\pi^0(p_2)\,|O^{3/2}|K^+\rangle = \frac{3}{2}\langle \pi^+(p_1)\pi^+(p_2)\,|O'^{3/2}|K^+\rangle\,,$$

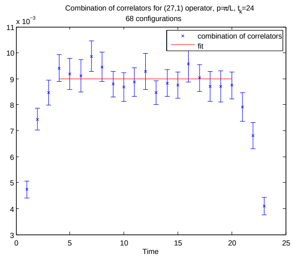
      where $O'^{3/2}$ has the flavour structure $(\bar{s}d)\,(\bar{u}d)$.
    - using antiperiodic boundary conditions so that the final state is $\langle \pi^+(\pi/L)\pi^+(-\pi/L)\,|$.

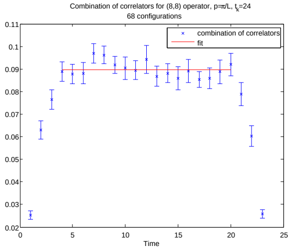
      C-h Kim, Ph.D. Thesis

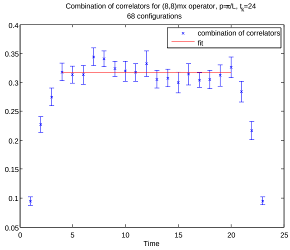
Preliminary $\Delta I = 3/2$ Matrix Elements



- We have been using an exploratory quenched study to learn about suitable parameters for the main simulation.
- The plots show the matrix elements as a function of the t for the insertion of the operator. $t_{\pi\pi} = 0$, $t_K = 24$.

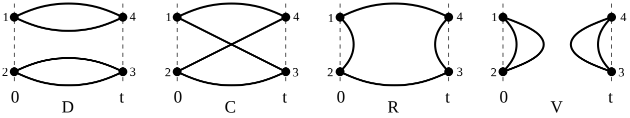






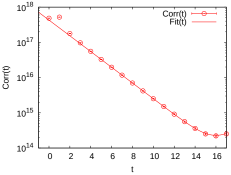
$$O'^{3/2}_{(27,1)} = (\bar{s}d)_L\,(\bar{u}d)_L \qquad O'^{3/2}_{7} = (\bar{s}d)_L\,(\bar{u}d)_R \qquad O'^{3/2}_{8} = (\bar{s}^i d^j)_L\,(\bar{u}^j d^i)_R$$

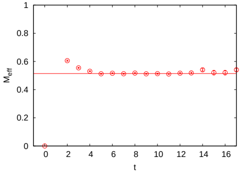
Quenched RBC-UKQCD Study, Courtesy of E.Goode

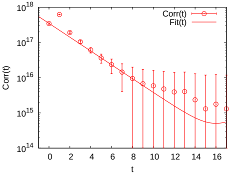


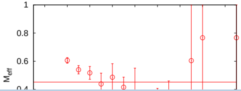
- For I=2 $\pi\pi$ states the correlation function is proportional to D-C.
- We are also exploring whether it will be feasible to compute the $\Delta I = 1/2\ K \to \pi\pi$ Decay Amplitudes.
- For I=0 $\pi\pi$ states the correlation function is proportional to 2D+C-6R+3V.

The major practical difficulty is to subtract the vacuum contribution with sufficient precision.









- RBC/UKQCD, Preliminary, Qi Liu et al. arXiv:0910.2658
- $I = 2$ (Correlator and Effective Mass)
- $I = 0$ (Correlator and Effective Mass)

6. Conclusions and Prospects

- Huge recent improvement in reliability and precision of lattice computations of quantities relevant for flavour physics.
- As $m_\pi \to m_\pi^{\rm phys}$ the chiral extrapolation becomes less of a problem.
  - LECs of Chiral Pert. Th. being computed with unprecedented precision.
  - (I am not convinced that the current representation of lattice data by NNLO/models is fully under control yet!)
- Future:
  - Improve precision still further.
  - Extend the physics reach of the computations.
    Discussions with wider flavour community needed here.
- Other speakers would have focussed on different important topics, e.g.:
  - Alpha Collaboration: HQET at $O(1/m)$ using NPR and step-scaling.
  - HPQCD: Large range of B-physics with NRQCD and charm physics using *highly improved* actions.
  - FNAL, CP-PACS, RBC-UKQCD - Symanzik-improvement based approach.
  - However, I am very much of the opinion that power divergences must be subtracted non-perturbatively. Maiani, Martinelli, CTS (1992)
  - We still don't know how to study $B \to M_1 M_2$ decays, even in principle.

- The precision of lattice calculations is now reaching the point where we need good interactions with the N^nLO QCD perturbation theory community.
- The traditional way of dividing responsibilities is:

$$\text{Physics} \quad = \quad \underset{\substack{\uparrow \\ \text{Perturbative} \\ \text{QCD}}}{C} \quad \times \quad \underset{\substack{\uparrow \\ \text{Lattice} \\ \text{QCD}}}{\langle f\,|\,O\,|\,i\rangle}$$

- The two factors have to be calculated in the same scheme.
- Can we meet half way?

$$\begin{array}{c} \text{bare} \\ \text{lattice} \\ \text{operators} \end{array} \longrightarrow \ \textbf{?} \ \longleftarrow \begin{array}{c} \text{operators} \\ \text{renormalized} \\ \text{in } \overline{\text{MS}} \text{ scheme} \end{array}$$

- What is the best scheme for **?** (RI-SMOM, Schrödinger Functional, $\cdots$)?

1. *Physical Results from 2+1 Flavor Domain Wall QCD and SU(2) Chiral Perturbation Theory,*
C. Allton *et al.*, (32 Authors, 133 pages)
Phys.Rev. **D78** (2008) 114509; [arXiv:0804.0473 [hep-lat]]

2. $K_{\ell 3}$ *semileptonic form factor from 2+1 flavour lattice QCD,*
P.A. Boyle, A. Jüttner, R.D. Kenway, C.T. Sachrajda, S. Sasaki, A. Soni, R.J. Tweedie and J.M. Zanotti,
Phys. Rev. Lett. **100** (2008) 141601; [arXiv:0710.5136 [hep-lat]].

3. *Hadronic form factors in lattice QCD at small and vanishing momentum transfer,*
P. A. Boyle, J. M. Flynn, A. Juttner, C. T. Sachrajda and J. M. Zanotti,
JHEP **0705** (2007) 016 [arXiv:hep-lat/0703005].

4. *The pion's electromagnetic form factor at small momentum transfer in full lattice QCD,*
P.A. Boyle, J.M. Flynn, A. Jüttner, C. Kelly, H. Pedroso de Lima, C.M. Maynard, C.T. Sachrajda and J.M. Zanotti,
JHEP 0807:112,2008; [arXiv:0804.3971 [hep-lat]].

5. *Neutral kaon mixing from 2+1 flavor domain wall QCD,*
D. J. Antonio *et al.*, (19 Authors)
Phys. Rev. Lett. **100** (2008) 032001 [arXiv:hep-ph/0702042].

6. *Non-perturbative renormalization of quark bilinear operators and* B_K *using domain wall fermions,*
Y. Aoki *et al.*, (14 Authors, 81 pages)
Phys. Rev. D78 (2008) 054510 [arXiv:0712.1061 [hep-lat]].

7. *Renormalization of quark bilinear operators in a MOM-scheme with a non-exceptional subtraction point,*
C. Sturm, Y. Aoki, N. H. Christ, T. Izubuchi, C. T. C. Sachrajda and A. Soni,
arXiv:0901.2599 [hep-ph].

8. *SU(2) Chiral Perturbation Theory for* $K_{\ell 3}$ *Decay Amplitudes,*
J. Flynn and C.T. Sachrajda,
Nucl. Phys. B812 (2009) 64 [arXiv:0809.1229 [hep-ph]].

9. *The* η *and* η' *mesons from Lattice QCD,*
N.H. Christ *et al.* (9 Authors, 4 pages) [arXiv:1002.2999 [hep-lat]].

10. $K \to (\pi\pi)_{I=2}$ *decays and twisted boundary conditions,*
C.h. Kim and C.T. Sachrajda [arXiv:1003.3191 [hep-lat]].

FLAG – Preliminary

- We have already seen the two precise results:

$$\left|\frac{V_{us} f_K}{V_{ud} f_\pi}\right| = 0.27599(59) \quad \text{and} \quad |V_{us} f_+(0)| = 0.21661(47)$$

Flavianet – arXiv:0801.1817

- We can view these as two equation for the four unknowns f_K/f_π, $f_+(0)$, V_{us} and V_{ud} .
- Within the Standard Model we also have the unitarity constraint:

$$|V_{ud}|^2 + |V_{us}|^2 \cancel{+|V_{ub}|^2} = 1$$

- Thus we now have 3 equations for four unknowns.
- There has been considerable work recently in updating the determination of V_{ud} based on 20 different superallowed transitions. Hardy and Towner, arXiV:0812.1202

$$|V_{ud}| = 0.97425(22)\,.$$

- If we accept this value then we are able to determine the remaining 3 unknowns:

$$|V_{us}| = 0.22544(95), \quad f_+(0) = 0.9608(46), \quad \frac{f_K}{f_\pi} = 1.1927(59)\,.$$