GGI-Florence, 23 March '10

Status of Neutrino Masses and Mixings

Guido Altarelli
Universita' di Roma Tre/CERN

Where we stand. What we have learnt. Open problems

Evidence for solar and atmosph. ν oscillatn's confirmed on earth by K2K, KamLAND, MINOS...

Δm^2 values:

$\Delta m^2_{atm} \sim 2.5\ 10^{-3}\ eV^2$,

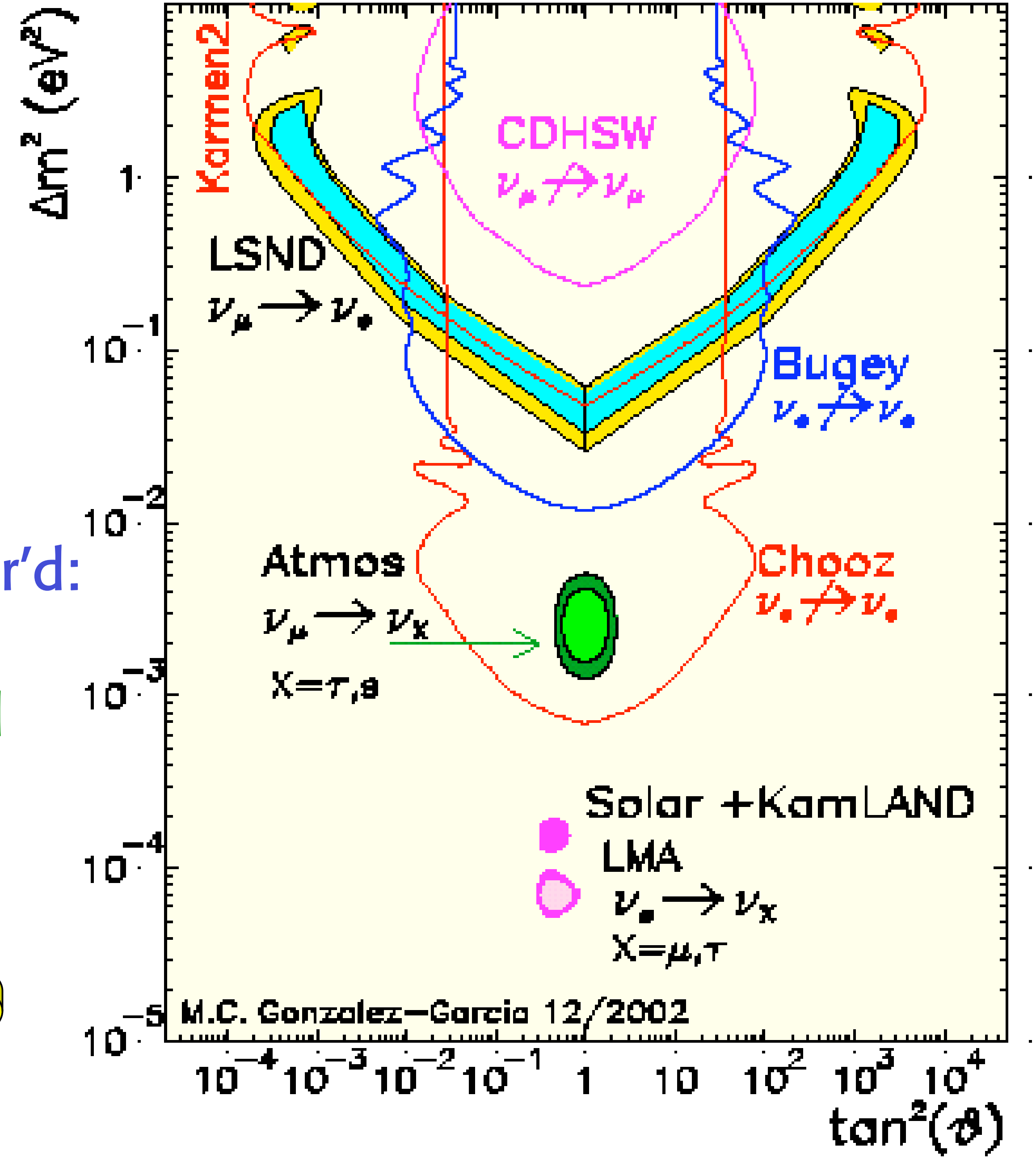
$\Delta m^2_{sol} \sim 8\ 10^{-5}\ eV^2$

and mixing angles measur'd:

θ_{12} (solar) large

θ_{23} (atm) large,~ maximal

θ_{13} (CHOOZ) small



Miniboone has not confirmed LSND

3 ν's are enough!

We do not need to add new neutrinos:
e.g. sterile neutrinos

The 3 known species are enough
Also, we can assume CPT invariance

Additional ν's or CPT violations are not completely excluded but for economy we can assume that they do not exist

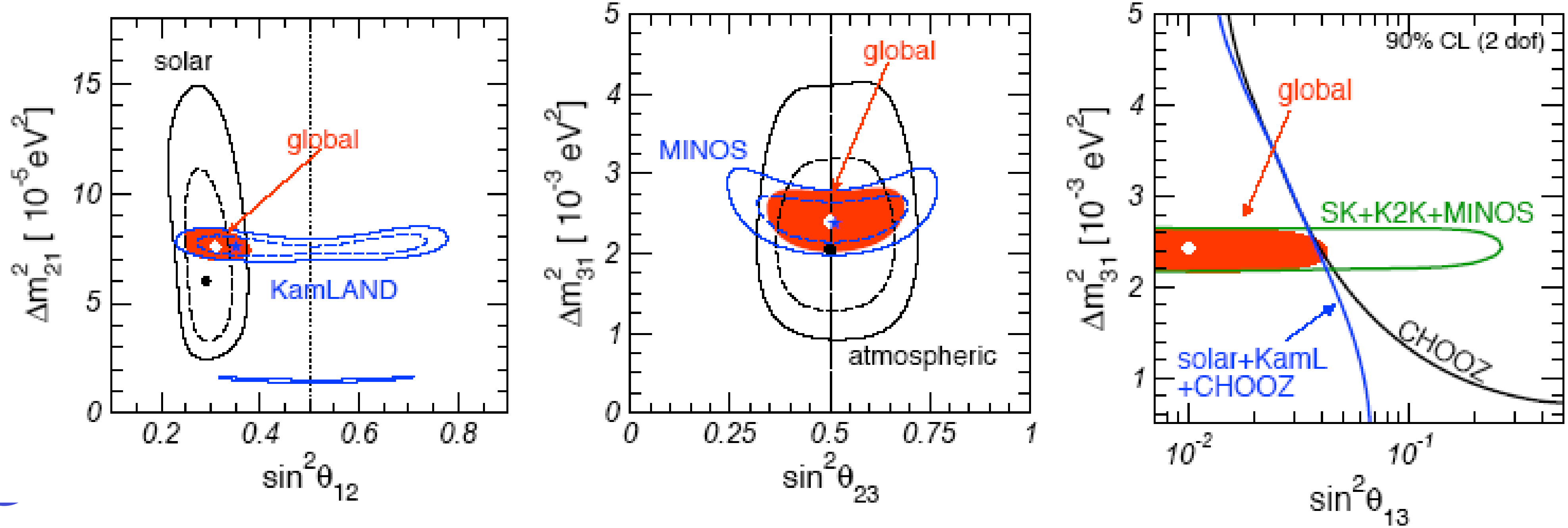
Neutrino oscillation parameters

- 2 distinct frequencies
- 2 large angles, 1 small

parameter	best fit	2σ	3σ
$\Delta m^2_{21}\ [10^{-5}\mathrm{eV}^2]$	$7.65^{+0.23}_{-0.20}$	7.25–8.11	7.05–8.34
$\lvert\Delta m^2_{31}\rvert\ [10^{-3}\mathrm{eV}^2]$	$2.40^{+0.12}_{-0.11}$	2.18–2.64	2.07–2.75
$\sin^2\theta_{12}$	$0.304^{+0.022}_{-0.016}$	0.27–0.35	0.25–0.37
$\sin^2\theta_{23}$	$0.50^{+0.07}_{-0.06}$	0.39–0.63	0.36–0.67
$\sin^2\theta_{13}$	$0.01^{+0.016}_{-0.011}$	≤ 0.040	≤ 0.056

Schwetz et al '08

Best measured angle

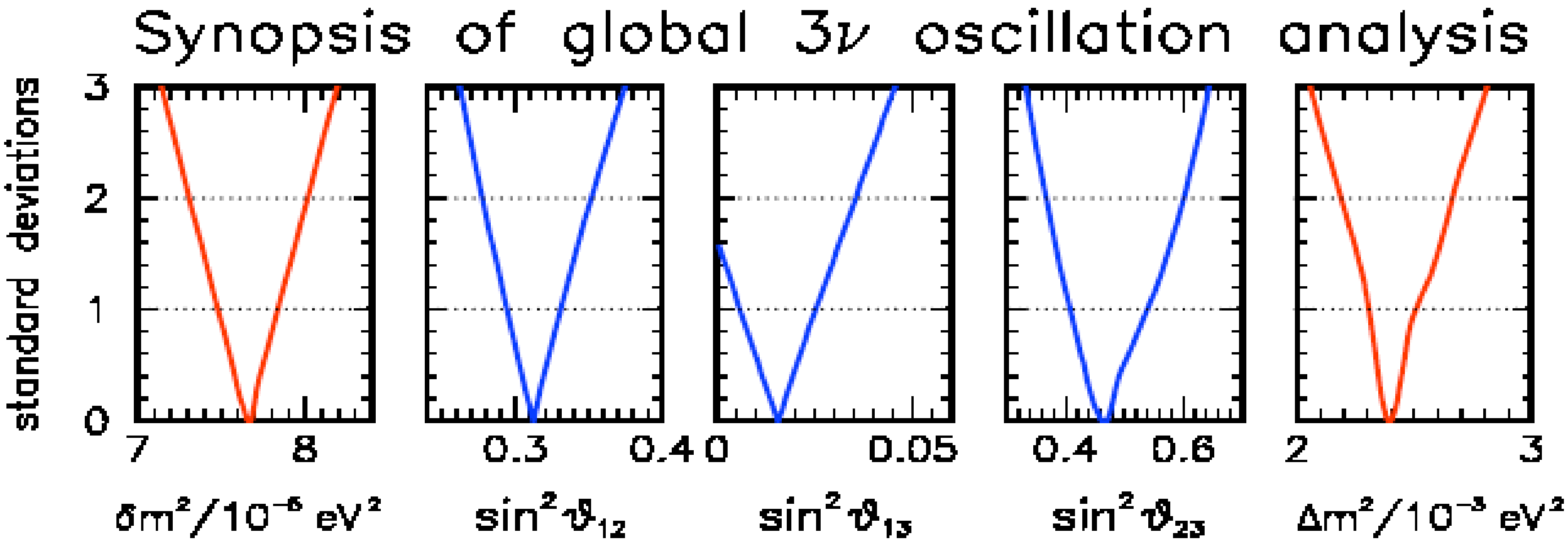


Different fits of the data agree

Fogli et al '08

Table 1: Global 3ν oscillation analysis (2008): best-fit values and allowed n_σ ranges, from Ref. [4].

Parameter	$\delta m^2/10^{-5}$ eV^2	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	$\Delta m^2/10^{-3}$ eV^2
Best fit	7.67	0.312	0.016	0.466	2.39
1σ range	7.48 – 7.83	0.294 – 0.331	0.006 – 0.026	0.408 – 0.539	2.31 – 2.50
2σ range	7.31 – 8.01	0.278 – 0.352	< 0.036	0.366 – 0.602	2.19 – 2.66
3σ range	7.14 – 8.19	0.263 – 0.375	< 0.046	0.331 – 0.644	2.06 – 2.81

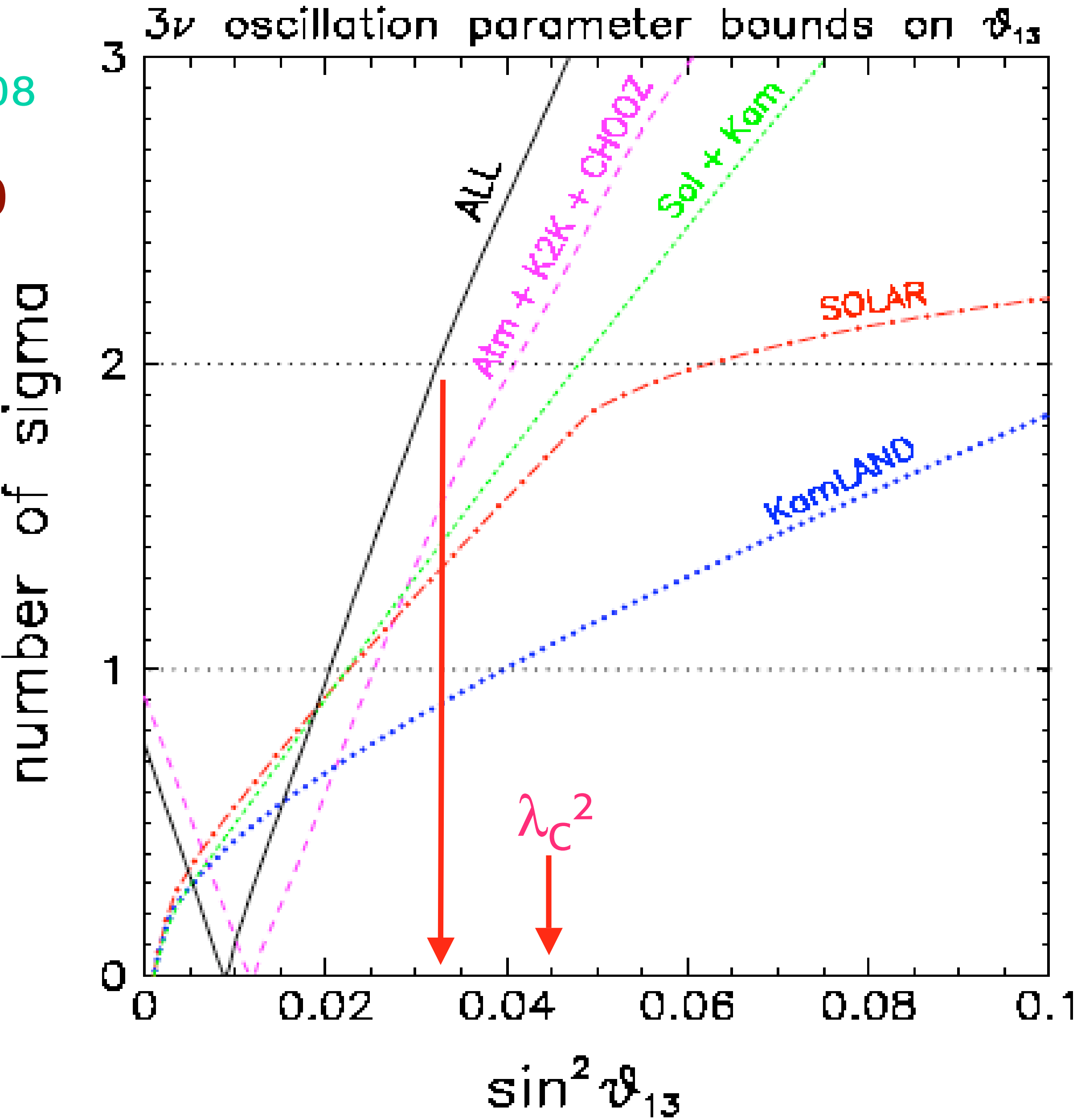


θ_{13} bounds

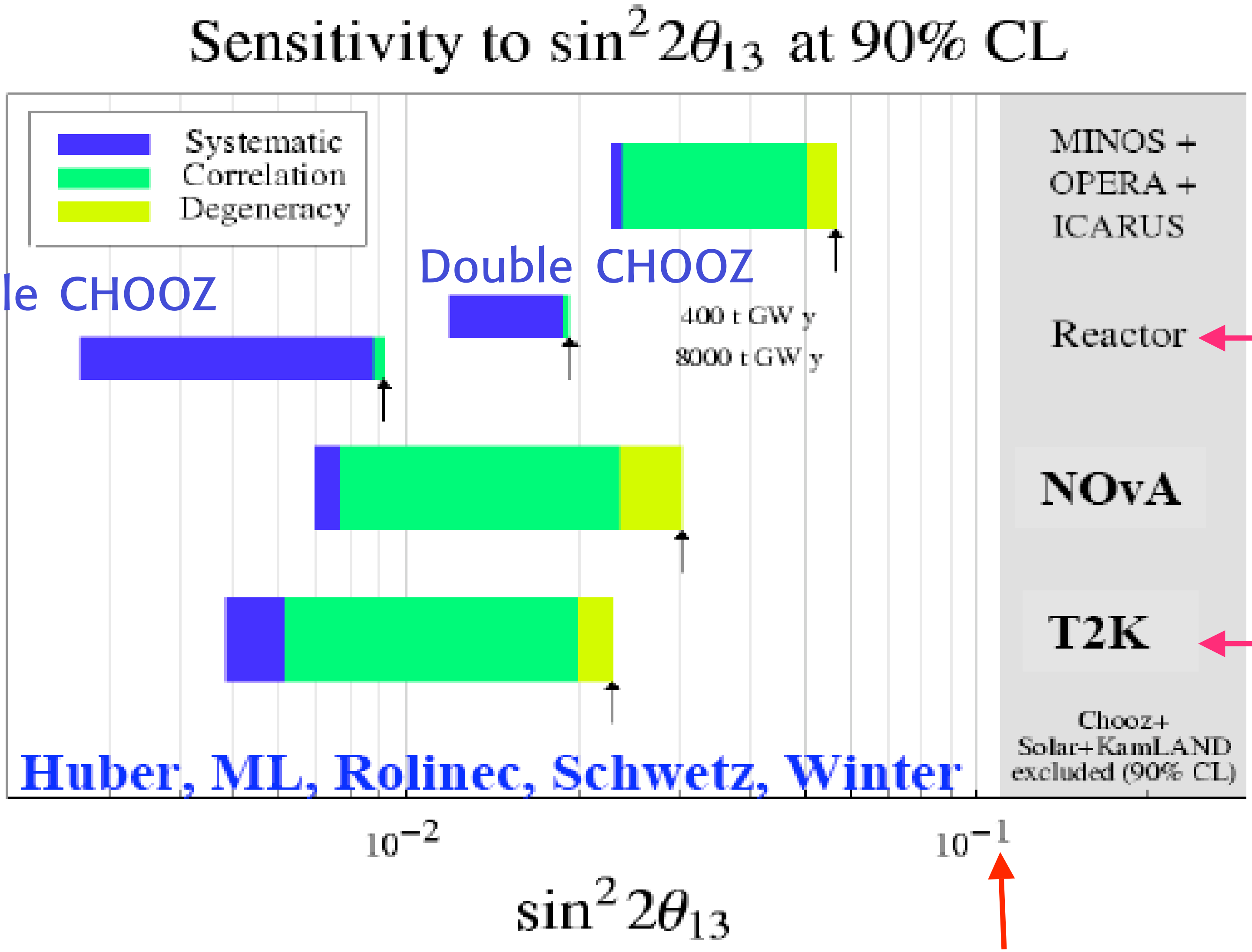
Fogli et al '08

$\sin^2\theta_{13}=0.016\pm0.010$

The 95% upper bound on $\sin\theta_{13}$ is close to $\lambda_C = \sin\theta_C$



Measuring θ_{13} is crucial for future ν-oscill's physics (eg CP violation)



Triple CHOOZ

Also
Daya Bay
RENO

1st ν event detected on Feb. 24 '10

~Present limit

ν oscillations measure Δm^2. What is m^2?

$\Delta m^2_{atm} \sim 2.5\ 10^{-3}\ eV^2 = (0.05\ eV)^2$; $\Delta m^2_{sun} \sim 8\ 10^{-5}\ eV^2 = (0.009\ eV)^2$

- Direct limits

$m_{"\nu e"} < 2.2$ eV

$m_{"\nu\mu"} < 170$ KeV

$m_{"\nu\tau"} < 18.2$ MeV

End-point tritium β decay (Mainz, Troitsk)
Future: Katrin
0.2 eV sensitivity
(Karsruhe)

$m_{ee} = |\Sigma\, U_{ei}^2\, m_i|$

- $0\nu\beta\beta$ $m_{ee} < 0.2 - 0.7 - ?$ eV (nucl. matrix elmnts)

Evidence of signal? Klapdor-Kleingrothaus

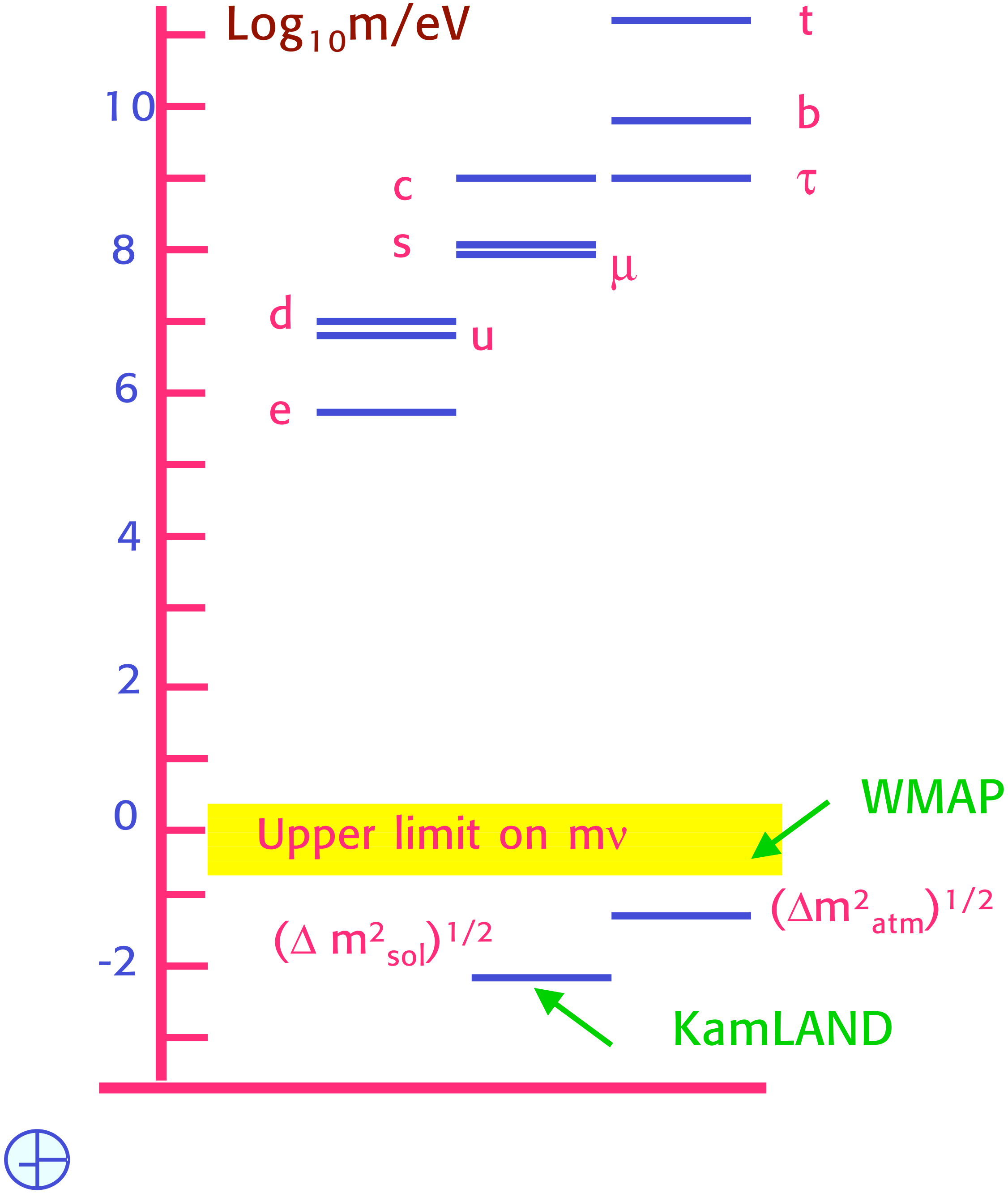
- Cosmology $\Omega_\nu\, h^2 \sim \Sigma_i m_i\ /94eV$ $(h^2 \sim 1/2)$

$\Sigma_i m_i < 0.2\text{-}0.7$ eV (dep. on data&priors)

WMAP, SDSS, 2dFGRS, Ly-α

→ Any ν mass $< 0.06 - 0.23 - \sim 1$ eV

depending on your weight on cosmology



Neutrino masses are really special!

$m_t/(\Delta m^2_{atm})^{1/2} \sim 10^{12}$

Massless ν's?

- no ν_R
- L conserved

Small ν masses?

- ν_R very heavy
- L not conserved

A very natural and appealing explanation:

ν's are nearly massless because they are Majorana particles and get masses through L non conserving interactions suppressed by a large scale M (the scale of ν_{RH} Majorana mass)

$$m_\nu \sim \frac{m^2}{M}$$

m: $\leq m_t \sim v \sim 200$ GeV
M: scale of L non cons.

Note:

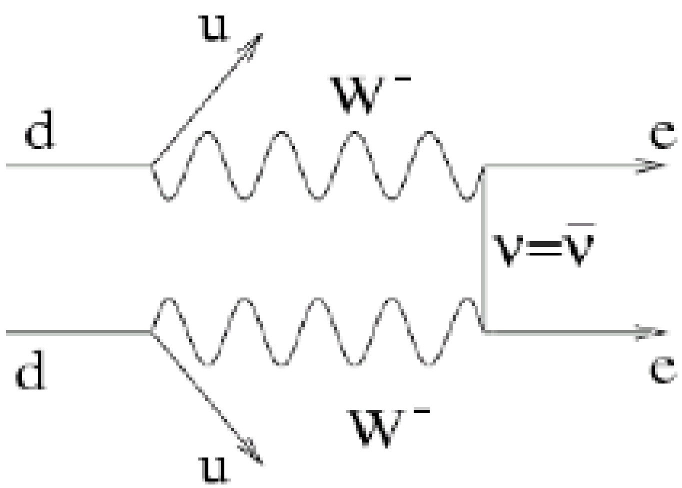
$m_\nu \sim (\Delta m^2_{atm})^{1/2} \sim 0.05$ eV
$m \sim v \sim 200$ GeV

$M \sim 10^{15}$ GeV

Neutrino masses are a probe of physics at M_{GUT} !

All we know from experiment on ν masses strongly indicates that ν's are Majorana particles and that L is not conserved (but a direct proof still does not exist).

Detection of 0νββ would be a proof of L non conservation. Thus a big effort is devoted to improving present limits and possibly to find a signal.



$0\nu\beta\beta = dd \to uue^-e^-$

Heidelberg-Moscow
IGEX
Cuoricino-Cuore
Nemo
Sokotvina
DAMA
Lucifero
•••••

Baryogenesis by decay of heavy Majorana ν's

BG via Leptogenesis near the GUT scale

$T \sim 10^{12\pm3}$ GeV (after inflation)

Buchmuller,Yanagida, Plumacher, Ellis, Lola, Giudice et al, Fujii et al ….

Only survives if Δ(B-L) is not zero
(otherwise is washed out at T_{ew} by instantons)

Main candidate: decay of lightest ν_R ($M \sim 10^{12}$ GeV)

L non conserv. in ν_R out-of-equilibrium decay:
B-L excess survives at T_{ew} and gives the obs. B asymmetry.

Quantitative studies confirm that the range of m_i from ν oscill's is compatible with BG via (thermal) LG

In particular the bound was derived for hierarchy

$m_i < 10^{-1}$ eV

Buchmuller, Di Bari, Plumacher; Giudice et al; Pilaftsis et al; Hambye et al
Hagedorn et al

Can be relaxed for degenerate neutrinos
So fully compatible with oscill'n data!!

We cannot exclude that ν's are Dirac particles

We cannot exclude that ν masses arise at the EW scale

But if we believe in some form of GUT's and that L conservation is violated near the GUT scale:

then it is very economical and natural to assume that ν's are Majorana particles and their mass is inversely related to the large scale of L non conservation.

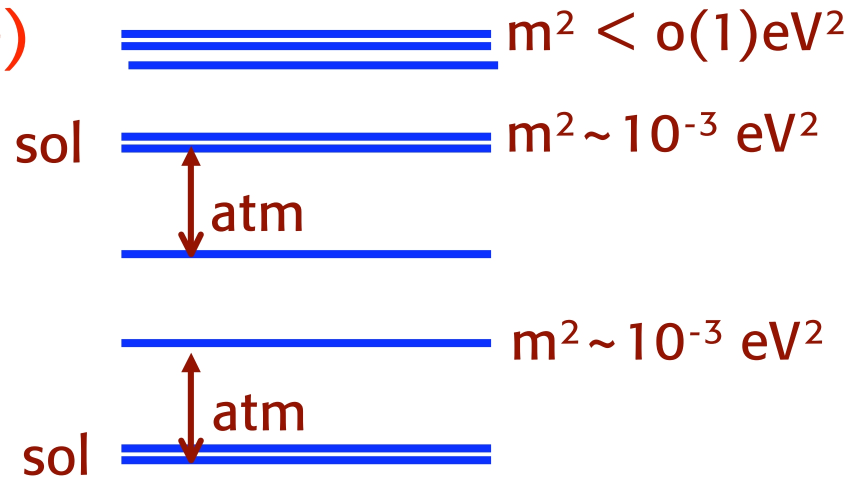
In turn ν's support GUT's

The current experimental situation on ν masses and mixings has much improved but is still incomplete

- what is the absolute scale of ν masses?
- value of θ_{13}......
- pattern of spectrum (sign of Δm^2_{atm})
- no detection of $0\nu\beta\beta$ (i.e. no proof that ν's are Majorana)
  see-saw?

3 light ν's are OK (MiniBoone)

- Degenerate ($m^2 >> \Delta m^2$)
- Inverse hierarchy
- Normal hierarchy



→ Different classes of models are still possible

$0\nu\beta\beta$ would prove that L is not conserved and ν's are Majorana
Also can tell degenerate, inverted or normal hierarchy

$$|m_{ee}|=c_{13}^2\ [m_1 c_{12}^2+e^{i\alpha}m_2 s_{12}^2]+m_3 e^{i\beta}s_{13}^2$$

Degenerate: $\sim|m||c_{12}^2+e^{i\alpha}s_{12}^2|\sim|m|(0.3\text{-}1)$

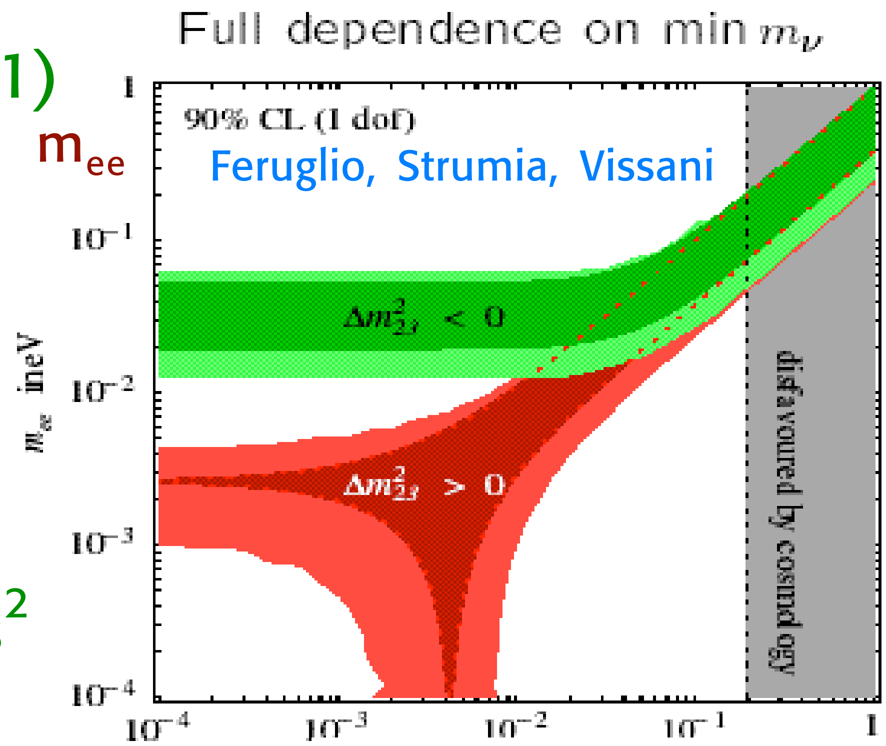
$|m_{ee}|\sim\ |m|\ (0.3\ \text{-}1)\leq\ 0.23\text{-}1$ eV

IH: $\sim(\Delta m^2_{atm})^{1/2}|c_{12}^2+e^{i\alpha}s_{12}^2|$

$|m_{ee}|\sim\ (1.6\text{-}5)\ 10^{-2}$ eV

NH: $\sim(\Delta m^2_{sol})^{1/2}s_{12}^2+(\Delta m^2_{atm})^{1/2}e^{i\beta}s_{13}^2$

$|m_{ee}|\sim$ (few) 10^{-3} eV



Present exp. limit: $m_{ee}< 0.3\text{-}0.5$ eV
(and a hint of signal????? Klapdor Kleingrothaus)

General remarks

• After KamLAND, SNO and WMAP.... not too much hierarchy is found in ν masses:

$$r \sim \Delta m^2_{sol}/\Delta m^2_{atm} \sim 1/30$$

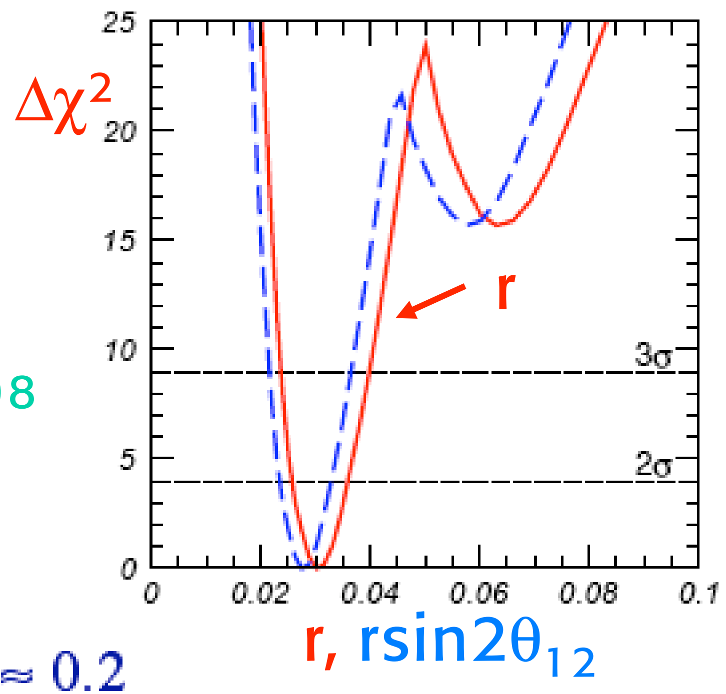
Only a few years ago could be as small as 10^{-8}!

Precisely at 3σ: $0.025 < r < 0.039$

Schwetz et al '08

or

$m_{heaviest} < 0.2 - 0.7$ eV
$m_{next} > \sim 8\ 10^{-3}$ eV



For a hierarchical spectrum: $\frac{m_2}{m_3} \approx \sqrt{r} \approx 0.2$

Comparable to $\lambda_C = \sin\theta_C$: $\lambda_C \approx 0.22$ or $\sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$

Suggests the same "hierarchy" parameters for q, l, ν
(small powers of λ_C) ⟶ e.g. θ_{13} not too small!

- θ_{13} not necessarily too small
  probably accessible to exp.

  Very small θ_{13} theoretically hard [typically $\theta_{13} > 0.01$]

- Still large space for non maximal 23 mixing

  2-σ interval $0.37 < \sin^2\theta_{23} < 0.60$ Fogli et al '08

  Maximal θ_{23} theoretically hard

- θ_{12} is at present the best measured angle
  $\Delta\sin^2\theta_{12}/\sin^2\theta_{12} \sim 6\%$

For constructing models we need the data but also to decide which feature of the data is really relevant

Examples:

Is Tri-Bimaximal (TB) mixing really a significant feature or just an accident?

Is lepton-quark complementarity (LQC) a significant feature or just an accident?

Here we already see 3 different classes of models that can fit the data:
TB & LQC are accidents or TB is relevant or LQC is relevant

Accidents: a wide spectrum of (mostly old) models
Anarchy, Anarchy in 2-3 sector, Lopsided models,
$U(1)_{FN}$, ... GUT versions exist [SU(5), SO(10)]

Typically there are free parameters fitted to the angles

TB

$$U=\begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

A coincidence or a hint?

TB mixing agrees with data at ~ 1σ

At 1σ:

G.L.Fogli et al '08

$\sin^2\theta_{12} = 1/3$: 0.29-0.33

$\sin^2\theta_{23} = 1/2$: 0.41-0.54

$\sin^2\theta_{13} = 0$: $< \sim 0.02$

Called:
Tri-Bimaximal mixing

Harrison, Perkins, Scott '02

$$\nu_3 = \frac{1}{\sqrt{2}}(-\nu_\mu + \nu_\tau)$$

$$\nu_2 = \frac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)$$

LQC: Lepton Quark Complementarity

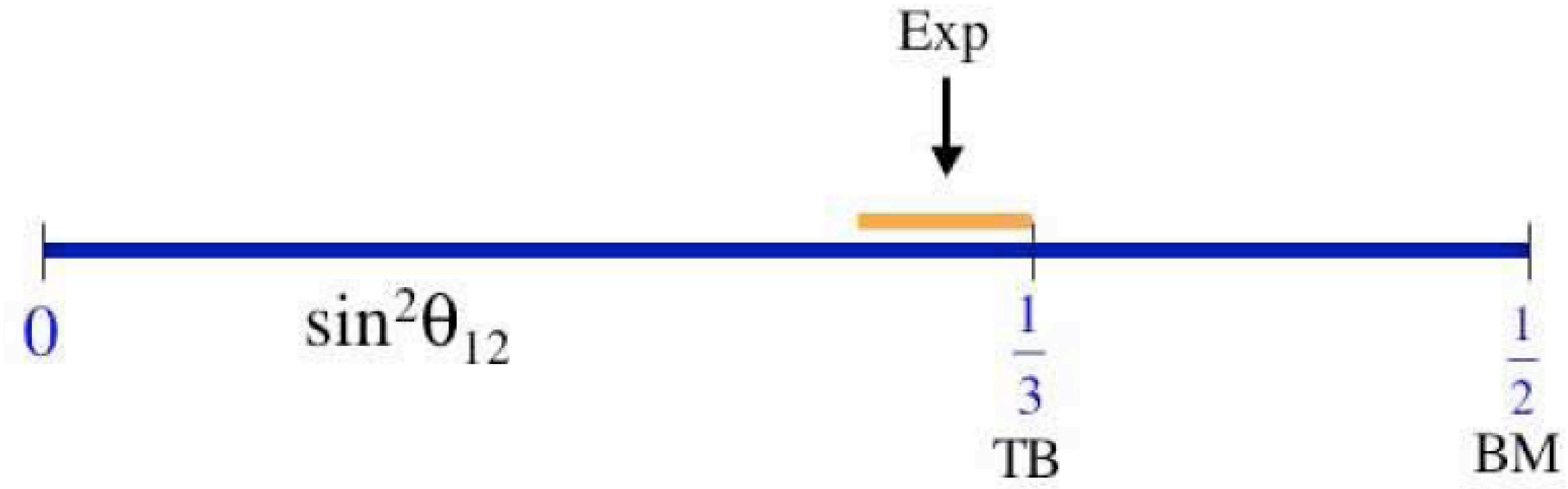
$$\theta_{12} + \theta_C = (47.0 \pm 1.2)^o \sim \pi/4$$

Raidal'04

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Suggests Bimaximal mixing corrected by diagonalisation of charged leptons

A coincidence or a hint?



Suggests that deviations from BM mixing arise from charged lepton diagonalisation

For the corrections from the charged lepton sector, typically $|\sin\theta_{13}| \sim (1-\tan^2\theta_{12})/4\cos\delta \sim 0.15$

GA, Feruglio, Masina
Frampton et al
King
Antusch et al........

Needs $|\sin\theta_{13}|$ near the present bound!

$$\bar{U}_{12} = -\frac{e^{-i(\alpha_1+\alpha_2)}}{\sqrt{2}} + \frac{s^e_{12}e^{-i\alpha_2} + s^e_{13}e^{i\delta_e}}{2}$$

$$\bar{U}_{13} = \frac{s^e_{12}e^{-i\alpha_2} - s^e_{13}e^{i\delta_e}}{\sqrt{2}}$$

$$\bar{U}_{23} = -e^{-i\alpha_2}\frac{1 + s^e_{23}e^{i\alpha_2}}{\sqrt{2}}$$

$\theta_{12} + \theta_C \sim \pi/4$

difficult to get. Rather:

$\theta_{12} + o(\theta_C) \sim \pi/4$

"weak" LQC

Corr.'s from s^e_{12}, s^e_{13} to U_{12} and U_{13} are of first order (2nd order to U_{23})

One can construct a model, based on S4, where BM mixing holds in 1st approximation and is then corrected by terms $o(\lambda_C)$

G.A., Feruglio, Merlo '09

In our model BM mixing is exact at LO

For the special flavon content chosen, only θ_{12} and θ_{13} are corrected from the charged lepton sector by terms of $o(\lambda_C)$ (large correction!) while θ_{23} gets smaller corrections (great!) [for a generic flavon content also $\delta\theta_{23} \sim o(\lambda_C)$]

An experimental indication for this model would be that θ_{13} is found near its present bound at T2K

$\mu-\tau$ symmetry

Consider models with $\theta_{13}= 0$ and θ_{23} maximal and θ_{12} generic [includes both BM and TB]

The most general mass matrix is given by (after ch. lepton diagonalization!!!) and it is 2-3 or $\mu-\tau$ symmetric

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix}$$

Inspired models based on $\mu-\tau$ symmetry

Grimus, Lavoura..., Ma,....
Mohapatra, Nasri, Hai-Bo Yu ....

Neglecting Majorana phases it depends on 4 real parameters (3 mass eigenvalues and 1 mixing angle: θ_{12})

But actually θ_{12} is the best measured angle (after KamLAND, SNO....). And it is directly compatible with TB mixing.

TB mixing

By adding $\sin^2\theta_{12} \sim 1/3$ to $\theta_{13} \sim 0$, $\theta_{23} \sim \pi/4$:

Tribimaximal Mixing

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix} \longrightarrow m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

$m_{11} + m_{12} = m_{22} + m_{23}$

$$\sin^2 2\theta_{12} = \frac{8y^2}{(x-w-z)^2 + 8y^2}$$

$= 8/9$ for TB

$m_1 = x - y$

$m_2 = x + 2y$

$m_3 = x - y + 2v$

The 3 remaining parameters are the mass eigenvalues

TB mixing

Harrison, Perkins, Scott

A simple mixing matrix compatible with all present data

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

In the basis of diagonal ch. leptons:

$m_\nu = U\mathrm{diag}(m_1, m_2, m_3)U^T$

$$m_\nu = \frac{m_3}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} + \frac{m_2}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \frac{m_1}{6}\begin{bmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{bmatrix}$$

Eigenvectors: $m_3 \to \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ $\quad m_2 \to \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad m_1 \to \frac{1}{\sqrt{6}}\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

Note: mixing angles independent of mass eigenvalues

Compare with quark mixings $\lambda_C \sim (m_d/m_s)^{1/2}$

- For the TB mixing matrix all mixing angles are fixed to particularly symmetric values

Sparked interest in constructing models that can naturally produce this highly ordered structure --> discrete flavour groups

A recent review: GA, Feruglio 1002.0211

Models based on the A4 discrete symmetry (even permutations of 1234) offer a minimal solution

Ma...; GA, Feruglio, GA, Feruglio, Lin; hep-ph/0610165; GA, Feruglio, Hagedorn; Y. Lin; Csaki et al; GA, Meloni.......

Larger finite groups: T', S4, $PSL_2(7)$ have also been studied

Feruglio et al; Chen, Mahanthappa;Frampton, Kephart; Lam; Bazzocchi et al , King et al

Alternative models based on $SU(3)_F$ or $SO(3)_F$ or their finite subgroups

Verzielas, G. Ross .... King ....

Discrete symmetries coupled with Sequential Dominance or Form Dominance

King ...., Chen, King......

A4 A4 is the discrete group of even perm's of 4 objects. (the inv. group of a tetrahedron). It has $4!/2 = 12$ elements.

A4 transformations can be written in terms of S and T with: $S^2 = T^3 = (ST)^3 = 1$ as:

$1, T, S, ST, TS, T^2, TST, STS, ST^2, T^2S, T^2ST, TST^2$

An element is abcd which means 1234 --> abcd

C_1:	$1 = 1234$			
C_2:	$T = 2314$	$ST = 4132$	$TS = 3241$	$STS = 1423$
C_3:	$T^2 = 3124$	$ST^2 = 4213$	$T^2S = 2431$	$TST = 1342$
C_4:	$S = 4321$	$T^2ST = 3412$	$TST^2 = 2143$	

C_1, C_2, C_3, C_4 are equivalence classes

x, x′ in same class if $[x' \sim gxg^{-1}]$ g: group element

Irr. reprent'ns $1, 1', 1'', 3$

L lepton doublet ~ 3

$e^c, \mu^c, \tau^c \sim 1, 1'', 1'$

Why discrete groups, in particular A4, work?

TB mixing corresponds to m
in the basis where
charged leptons are diagonal

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

m is the most general matrix invariant under
$SmS = m$ and $A_{23} m A_{23} = m$ with:

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

$$A_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

2-3
symmetry

$S^2 = A_{23}^2 = 1$

Charged lepton masses:
a generic diagonal matrix,
is invariant under T
(or ηT with η a phase):

$$m_l = v_T \frac{v_d}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

a possible T is

$$m_l^+ m_l = T^+ m_l^+ m_l T$$

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

$S^2=T^3=(ST)^3=1$ define A4

$\omega^3=1 \rightarrow T^3=1$

Invariance under S and T can be made automatic in A4 while A_{23} is not in A4 (2<->3 exchange is an odd permutation)
But 2-3 symmetry happens in A4 if 1' and 1" symm. breaking flavons are absent.

S, T and A_{23} are all contained in S4

$S^4=T^3=(ST^2)^2=1$ define S4

Lam

Structure of A4 models

The model is invariant under the flavour group A4

There are flavons ϕ_T, ϕ_S , ξ... with VEV's that break A4:

ϕ_T down to G_T, the subgroup generated by 1, T, T^2, in the charged lepton sector

ϕ_S , ξ down to G_S, the subgroup generated by 1, S, in the neutrino sector

$$\langle\varphi_T\rangle = (v_T, 0, 0)$$
$$\langle\varphi_S\rangle = (v_S, v_S, v_S)$$
$$\langle\xi\rangle = u \quad , \quad \langle\tilde{\xi}\rangle = 0$$

ϕ_T, ϕ_S ~ 3

ξ ~ 1

The 2-3 symmetry occurs in A4 if 1′ and 1″ flavons are absent

The aligment occurs because is based on A4 group theory

TB mixing broken by higher dimension operators

Typically $\delta\theta \sim o(\lambda_C^2)$

Many versions of A4 models exist by now

- with dim-5 effective operators or with see-saw
- with SUSY or without SUSY
- in 4 dimensions or in extra dimensions

  e.g G.A., Feruglio'05; G.A., Feruglio, Lin '06; Csaki et al '08.....
- with different solutions to the alignment problem

  e.g Hirsch, Morisi, Valle '08
- with sequential (or form) dominance

  e.g King'07 ; Chen, King '09
- with charged lepton hierarchy also following from a special alignment (no $U(1)_{FN}$) Lin'08; GA, Meloni'09
- extension to quarks, possibly in a GUT context

In lepton sector TB (or BM) mixing point to discrete flavor groups

What about quarks?

A problem for GUT models is how to reconcile the quark with the lepton mixings

quarks: small angles, strongly hierarchical masses
abelian flavour symm. [e.g. $U(1)_{FN}$]
neutrinos: large angles, perhaps TB or BM
non abelian discrete symm. [e.g. A4]

A4: Extension to quarks

If we take all fermion doublets as 3 and all singlets as 1, 1′, 1″ (as for charged leptons): $Q_i \sim 3$; $u^c, d^c \sim 1$; $c^c, s^c \sim 1''$; $t^c, b^c \sim 1'$

Then u and d quark mass matrices, like for charged leptons, are BOTH diagonal in the T-diagonal basis

As a result V_{CKM} is unity: $V_{CKM} = U_u^+ U_d \sim 1$

So, in first approx. (broken by loops and higher dim operators), ν mixings are TB and quark mixings ~identity: NOT BAD

BUT the hierarchy of q mixing angles is not given and the above A4 transf. properties are not compatible with GUT's

- Larger discrete flavour groups for quark mixings (no GUT's)

Carr, Frampton
Feruglio et al
Frampton, Kephart

....

- GUT models with approximate TB mixing

it is indeed possible, also for A4, but not easy!
[SU(5) less difficult than SO(10)]

Ma, Sawanaka, Tanimoto; Ma; GA, Feruglio, Hagedorn 0802.0090
Morisi, Picarello, Torrente Lujan; Bazzocchi et al;
de Madeiros Verzielas, King, Ross [$\Delta(27)$];
King, Malinsky [$SU(4)_C xSU(2)_L xSU(2)_R$]; Antusch et al;
Chen, Mahanthappa [T']; Bazzocchi et al [$\Delta(27)$];
King, Luhn [$PSL_2(7)$]; Dutta, Mimura, Mohapatra [S4];

....

SUSY-SU(5) GUT with A4 and TB

GA, Feruglio, Hagedorn 0802.0090

A satisfactory ~realistic model

Key ingredients:

- SUSY

  In general SUSY is crucial for hierarchy, coupling unification and p decay
  Specifically it makes simpler to implement the required alignment

- GUT's in 5 dimensions

  In general GUT's in ED are most natural and effective
  Here also contribute to produce fermion hierarchies

- Extended flavour symmetry: $A4 \times U(1) \times Z_3 \times U(1)_R$

  $U(1)_R$ is a standard ingredient of SUSY GUT's in ED

  Hall-Nomura'01

ED effects contribute to the fermion mass hierarchies

A bulk field is related to its zero mode by: $B = \frac{1}{\sqrt{\pi R}} B^0 + \ldots$

This produces a suppression parameter for couplings with bulk fields $s \equiv \frac{1}{\sqrt{\pi R \Lambda}} < 1$

Λ: UV cutoff

- In bulk: N=2 SUSY Yang-Mills fields + H_5, H_5^{bar}+ T_1, T_2, T_1', T_2'

(doubling of bulk fermions to obtain chiral massless states at y=0)

also crucial to avoid too strict mass relations for 1,2 families:

(b-τ unification only for 3rd family)

- All other fields on brane at y=0 (in particular N, F, T_3)

$$m_u = \begin{pmatrix} s^2t^5t'' + s^2t^2t''^4 & s^2t^4 + s^2tt''^3 & stt''^2 \\ s^2t^4 + s^2tt''^3 & s^2t''^2 & st'' \\ stt''^2 & st'' & 1 \end{pmatrix} sv_u^0 \sim \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \lambda v_u^0$$

Note: all m of rank 1 in LO: only $m_{33} \sim o(1)$!

dots=0 in 1st approx

$$m_d = \begin{pmatrix} st^3 + st''^3 & \ldots & \ldots \\ st^2t'' & st & \ldots \\ stt''^2 & st'' & 1 \end{pmatrix} v_T sv_d^0 \sim \begin{pmatrix} \lambda^4 & \ldots & \ldots \\ \lambda^4 & \lambda^2 & \ldots \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} v_T \lambda v_d^0$$

$$m_e = \begin{pmatrix} st^3 + st''^3 & st^2t'' & stt''^2 \\ \ldots & st & st'' \\ \ldots & \ldots & 1 \end{pmatrix} v_T sv_d^0 \sim \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^4 \\ \ldots & \lambda^2 & \lambda^2 \\ \ldots & \ldots & 1 \end{pmatrix} v_T \lambda v_d^0$$

with

$$\overbrace{\frac{\langle\varphi_T\rangle}{\Lambda} = (v_T, 0, 0) \quad , \quad \frac{\langle\varphi_S\rangle}{\Lambda} = (v_S, v_S, v_S) \quad , \quad \frac{\langle\xi\rangle}{\Lambda} = u}^{\text{A4 breaking}} \qquad \overbrace{\frac{\langle\theta\rangle}{\Lambda} = t \quad , \quad \frac{\langle\theta''\rangle}{\Lambda} = t''}^{U(1)_{FN}\text{ breaking}}$$

$s \sim t \sim t'' \sim \lambda \sim 0.22$ $\quad v_T \sim \lambda^2 \sim m_b/m_t$ $\quad v_S, u \sim \lambda^2$

For ν's after see-saw

$$m_\nu = \frac{1}{3a(a+b)} \begin{pmatrix} 3a+b & b & b \\ b & \dfrac{2ab+b^2}{b-a} & \dfrac{b^2-ab-3a^2}{b-a} \\ b & \dfrac{b^2-ab-3a^2}{b-a} & \dfrac{2ab+b^2}{b-a} \end{pmatrix} \frac{s^2(v_u^0)^2}{\Lambda}$$

with

$$a \equiv \frac{2x_a u}{(y^D)^2} \quad , \qquad b \equiv \frac{2x_b v_S}{(y^D)^2}$$

m_ν is of the form

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix} \qquad U = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & +1/\sqrt{2} \end{pmatrix}$$

with

TB mixing is exact in LO

$$m_1 = \frac{1}{(a+b)} \quad , \qquad m_2 = \frac{1}{a} \quad , \qquad m_3 = \frac{1}{(b-a)}$$

or

$$\frac{2}{m_2} = \frac{1}{m_1} - \frac{1}{m_3}$$

Finally:

By taking $s \sim t \sim t'' \sim \lambda \sim 0.22$ $v_T \sim \lambda^2 \sim m_b/m_t$ $v_S, u \sim \lambda^2$

a good description of all quark and lepton masses is obtained.
As for all U(1) models only $o(\lambda^p)$ predictions can be given
(modulo o(1) coeff.s)

TB mixing for neutrinos is reproduced in first approximation

Quark hierarchies force corrections to TB mixing to be $o(\lambda^2)$
(in particular we predict $\theta_{13} \sim o(\lambda^2)$, accessible at T2K).

A moderate fine tuning is needed to fix λ_C and r
(nominally of $o(\lambda^2)$ and 1 respectively)

Normal or inverse hierarchy are possible, degenerate ν's
are excluded

SO(10) is even more difficult

A sketch of an SO(10) model with TB mixing

Dutta, Mimura, Mohapatra '09

$$W_Y = h\,\psi\psi H + f\,\psi\psi\bar{\Delta} + h'\,\psi\psi\,(\Sigma \text{ or } H')$$

16 10 126 10′ or 120

$$Y_u = h + r_2 f + r_3 h',$$
$$Y_d = r_1(h + f + h'),$$
$$Y_e = r_1(h - 3f + c_e h'),$$
$$Y_{\nu D} = h - 3r_2 f + c_\nu h',$$

$$\mathcal{M}_\nu = f v_L - M_D \frac{1}{f v_R} M_D^t$$

type II type I

assume type II dominant

$$\mathcal{M}_\nu = f v_L.$$

ν's are only fixed by f in LO (f is of the TB type)

f and h′ correct fermion masses (h has only 33 in LO, h>>f,h′)

f and h′ give quark mixing AND corrections to TB mixing

$$W_Y = h\,\psi\psi H + f\,\psi\psi\bar{\Delta} + h'\,\psi\psi\,(\Sigma \text{ or } H')$$

$$h \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \Bigg| \quad f \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} + \lambda \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad \Bigg| \quad h' \propto \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

rank 1

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

Nice predictions:

Normal hierarchy and r ~ $\Delta m^2_{sol}/\Delta m^2_{atm} \sim \lambda_C$

$$\sin\theta_{13} \equiv U_{e3} \sim \frac{V_{us}}{3\sqrt{2}} \simeq 0.05.$$

......

The problem is to realize the different conditions in a natural model (a crude S4 version is proposed)

Conclusion

- No need for more than 3 light neutrinos or CPT violation
- Majorana ν's, the see-saw mechanism and $M \sim M_{GUT}$ explain the data (we expect L non cons. in GUT's)
  - needs confirmation from $0\nu\beta\beta$ decay
  - ν's support GUT's
- Different models can accommodate the data on ν mixing
  - e. g. TB mixing accidental or a hint?

Anarchy
Lopsided models
$U(1)_{FN}$,
......

discrete groups

Value of θ_{13} important for deciding

- θ_{13}, sign Δm^2_{23}, CP phase δ, absolute m^2 scale.... ?????