

Extra-Dimensions & Flavor

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(CERN)

22/3/2010
GGI, Firenze

4D → 5D

Why?

4D \rightarrow 5D

Why not?

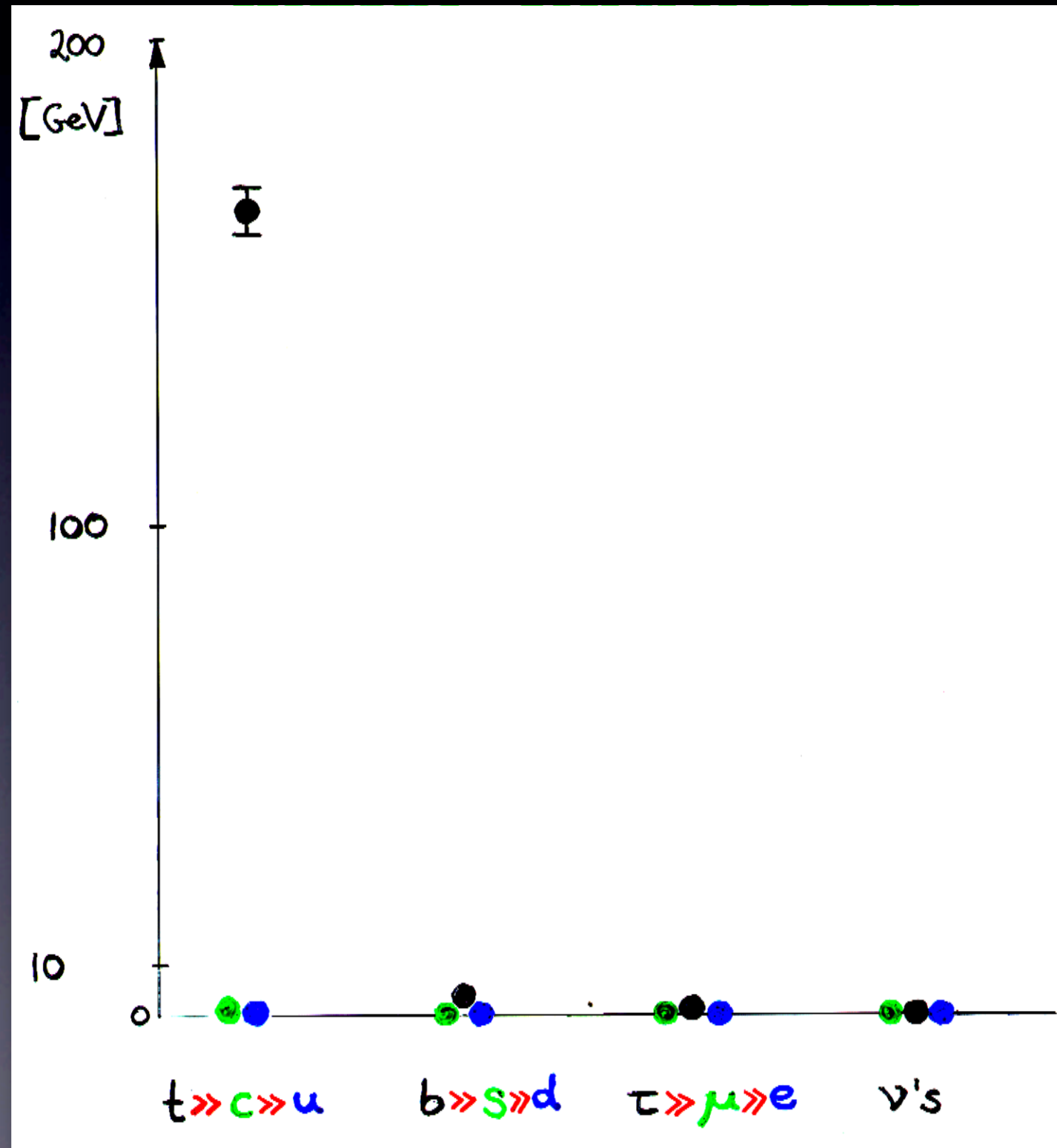
Crazier than a fermionic extra dimension?

4D → 5D

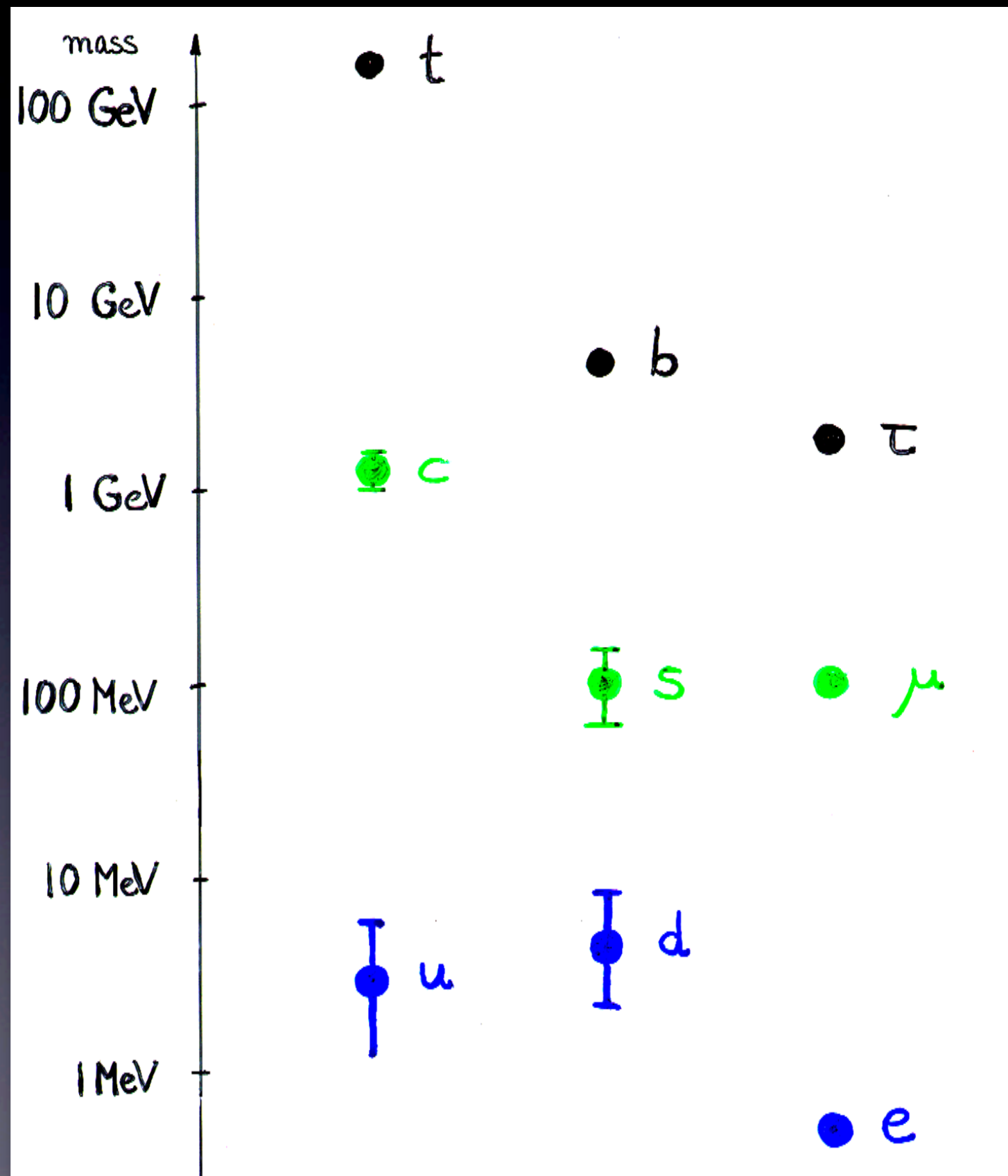
Why 4D?

The flavor puzzle

Quark and Lepton mass hierarchy



Masses on a Log-scale



The SM flavor puzzle

$$Y_D \approx \text{diag} (2 \cdot 10^{-5} \quad 0.0005 \quad 0.02)$$

$$Y_U \approx \begin{pmatrix} 6 \cdot 10^{-6} & -0.001 & 0.008 + 0.004i \\ 1 \cdot 10^{-6} & 0.004 & -0.04 + 0.001i \\ 8 \cdot 10^{-9} + 2 \cdot 10^{-8}i & 0.0002 & 0.98 \end{pmatrix}$$

Why this structure?

Other dimensionless parameters of the SM:

$$g_s \sim 1, \quad g \sim 0.6, \quad g' \sim 0.3, \quad \lambda_{\text{Higgs}} \sim 1, \quad |\theta| < 10^{-9}$$

Log(SM flavor puzzle)

$$-\log |Y_D| \approx \text{diag} (11 \quad 8 \quad 4)$$

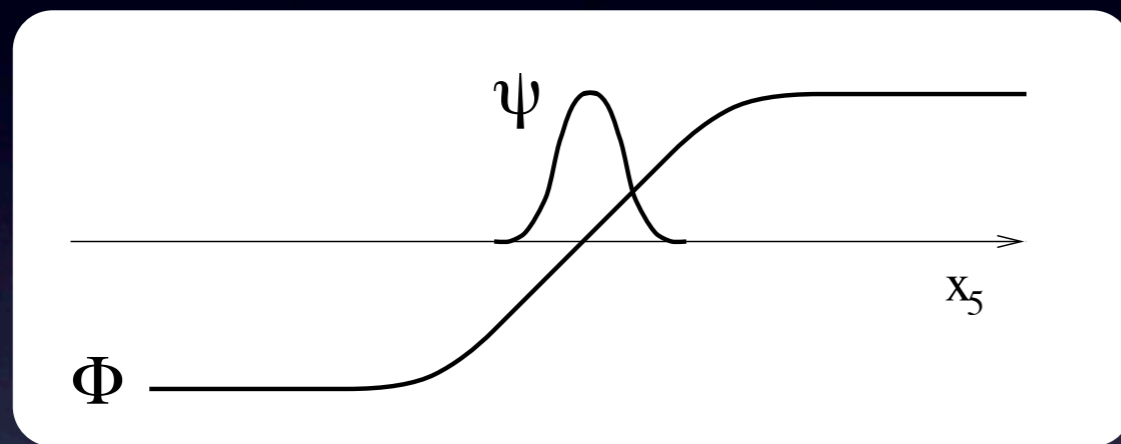
$$-\log |Y_U| \approx \begin{pmatrix} 12 & 7 & 5 \\ 14 & 6 & 3 \\ 18 & 9 & 0 \end{pmatrix}$$

If $Y = e^{-\Delta}$, then the Δ don't look crazy.

Hierarchies w/o Symmetries

Arkani-Hamed, Schmaltz

SM on thick brane & domain wall \Rightarrow chiral localization



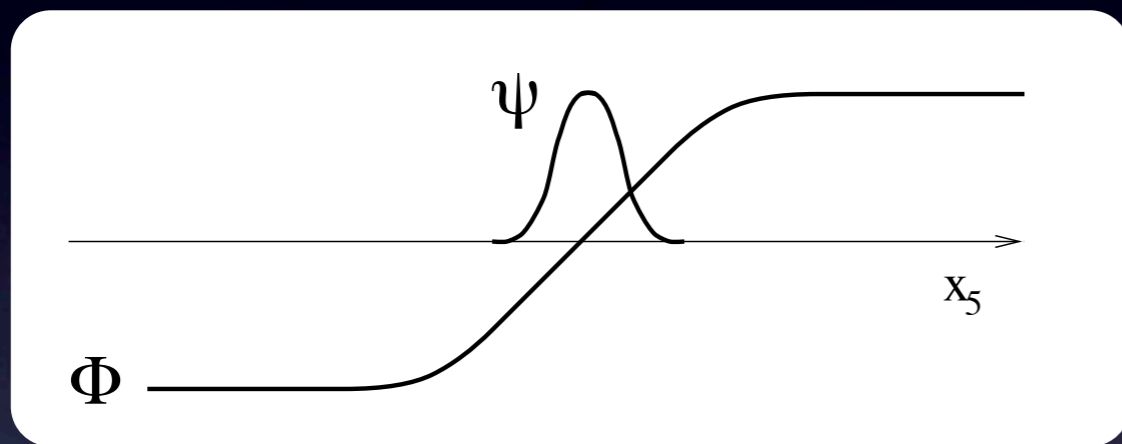
$$\mathcal{S} = \int d^5x \sum_{i,j} \bar{\Psi}_i [i \not{\partial}_5 + \lambda \Phi(x_5) - m]_{ij} \Psi_j$$

$$\Psi = \begin{pmatrix} \Psi_L \\ \Psi_R \end{pmatrix} = \begin{pmatrix} \psi_L^0 \\ 0 \end{pmatrix} + \text{KK modes}$$

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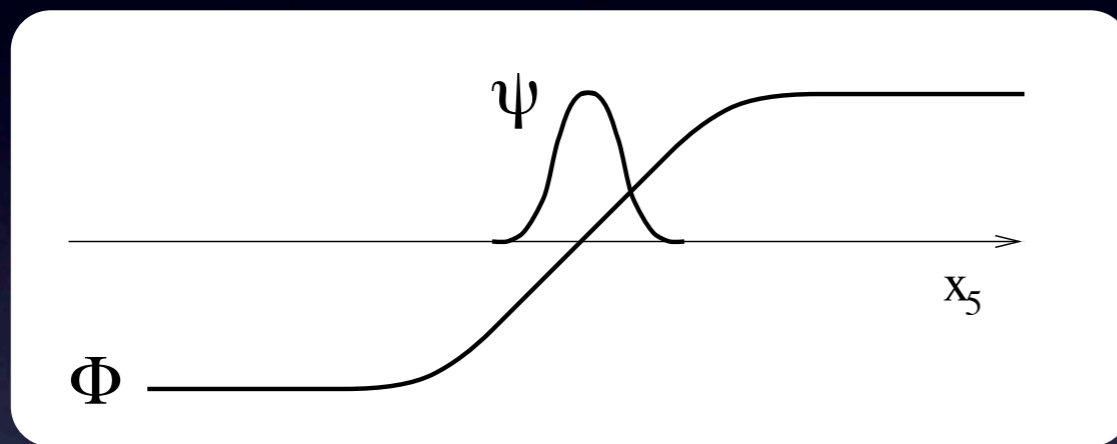
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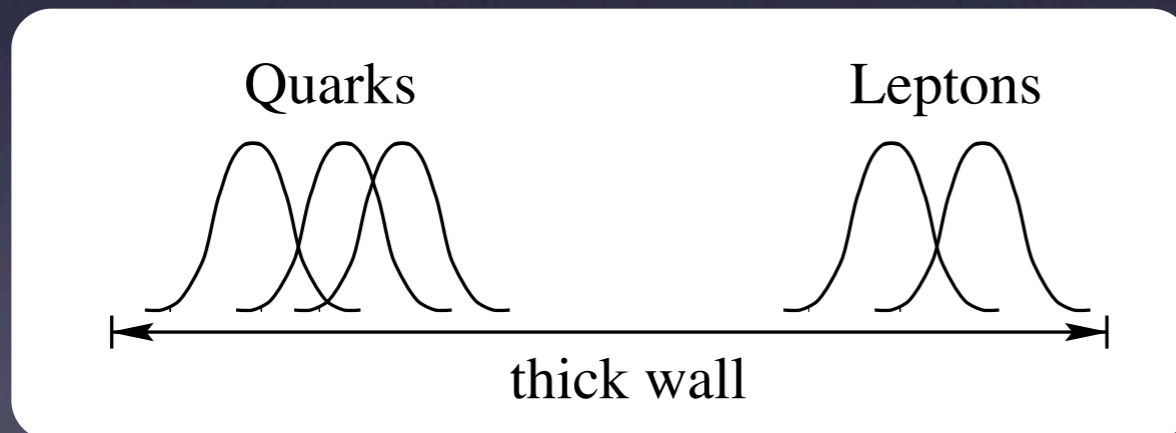
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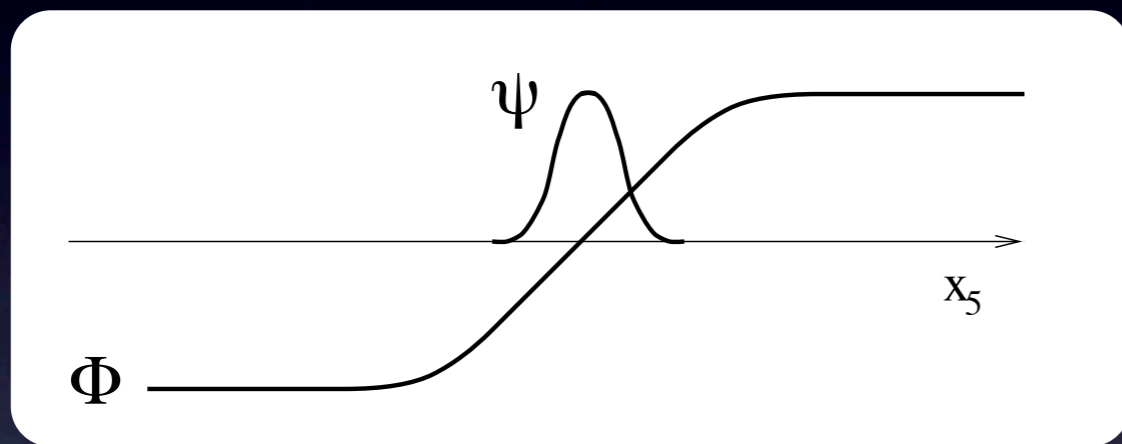
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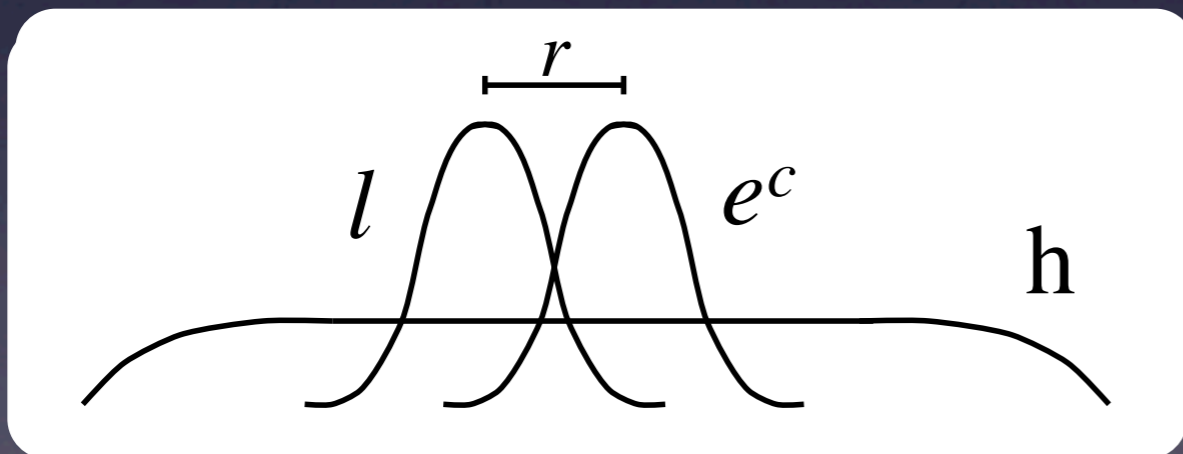
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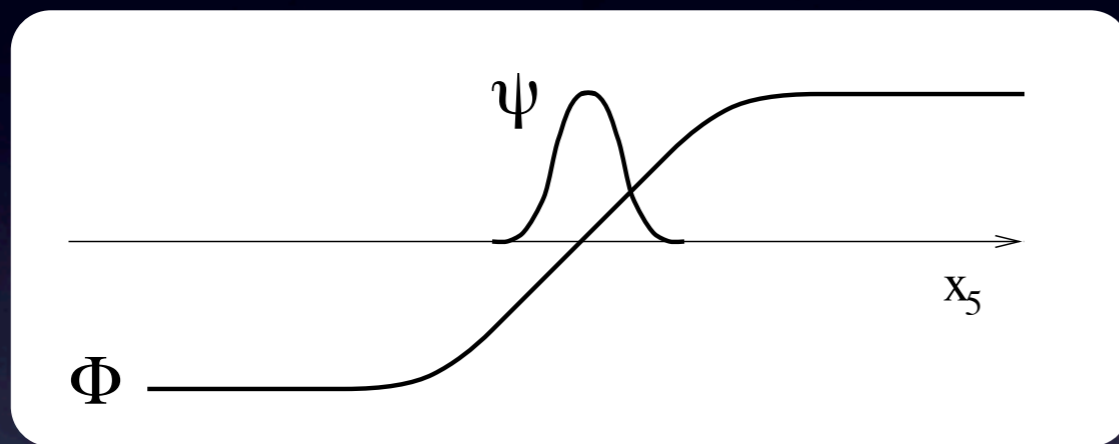
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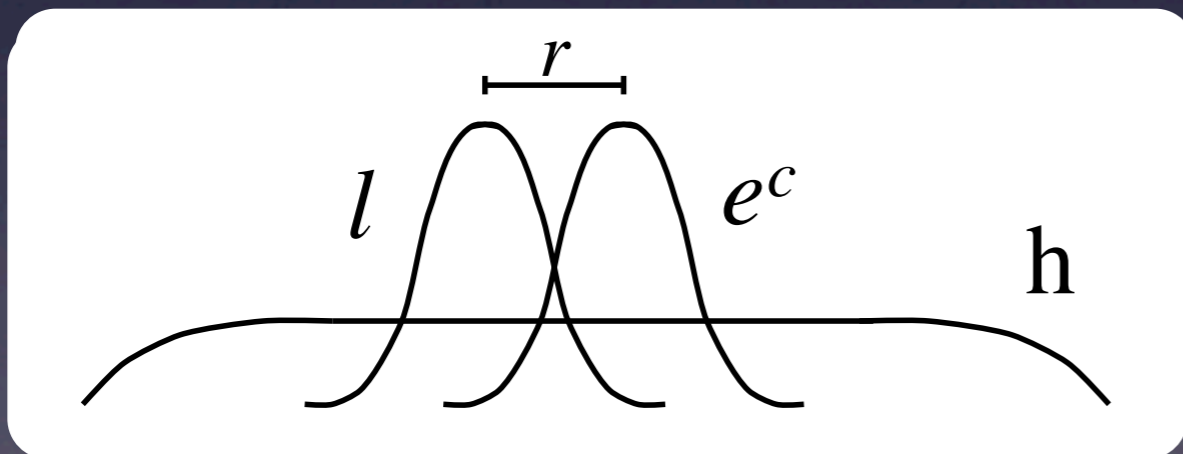
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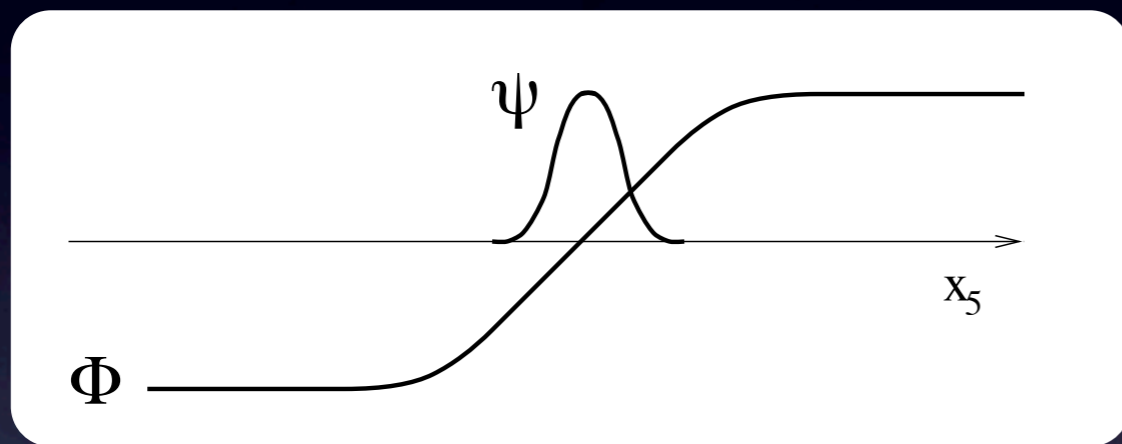


$$\int dx_5 \phi_l(x_5) \phi_{ec}(x_5) = \frac{\sqrt{2}\mu}{\sqrt{\pi}} \int dx_5 e^{-\mu^2 x_5^2} e^{-\mu^2 (x_5 - r)^2} = e^{-\mu^2 r^2 / 2}$$

Hierarchies w/o Symmetries

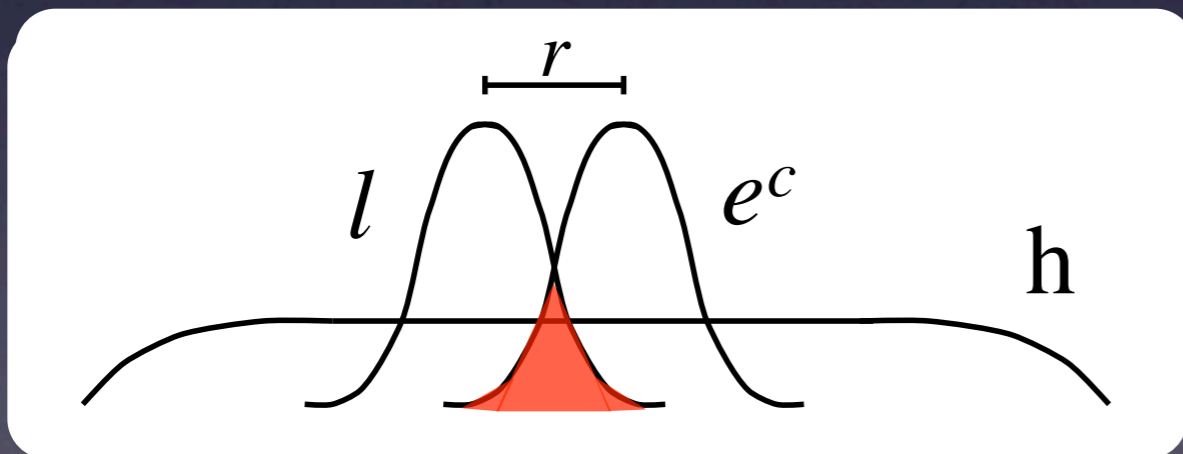
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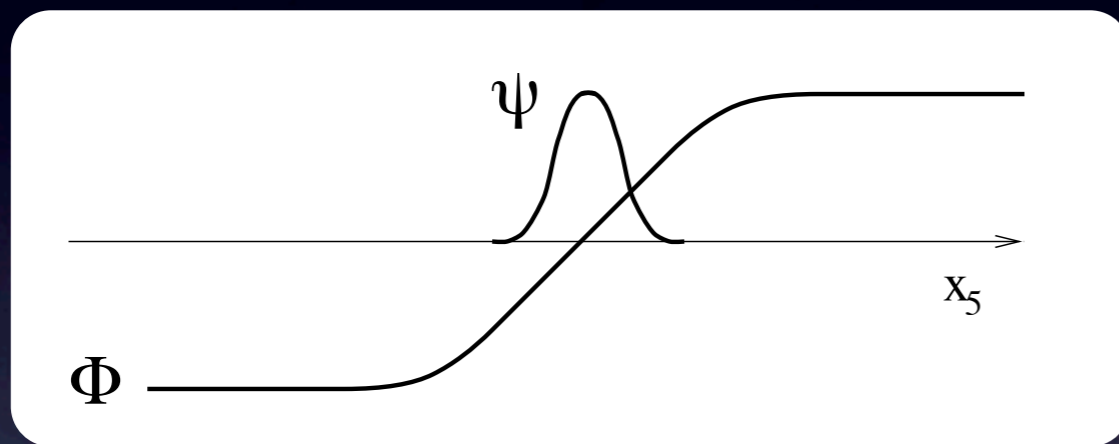


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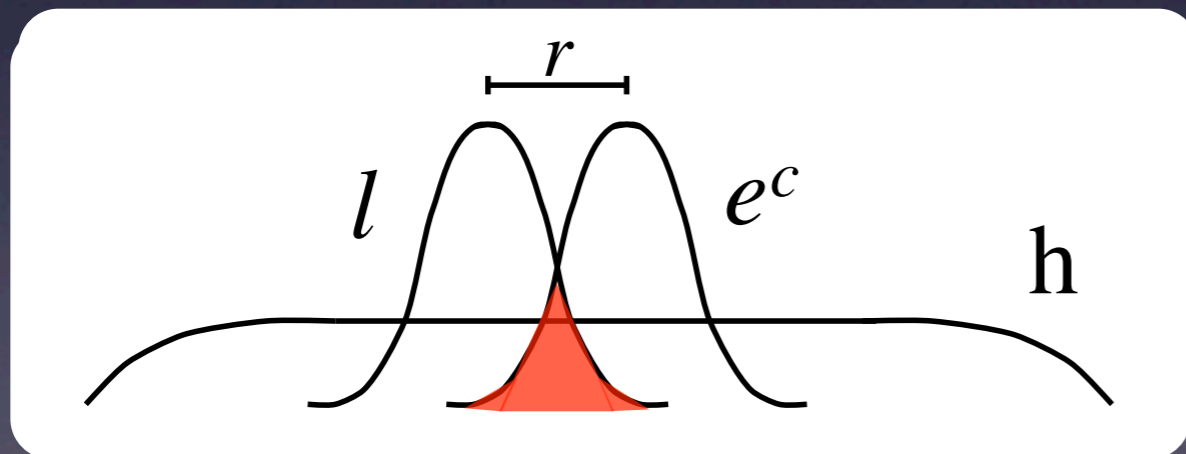
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Log(flavor hierarchy)!

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New sources for FCNCs

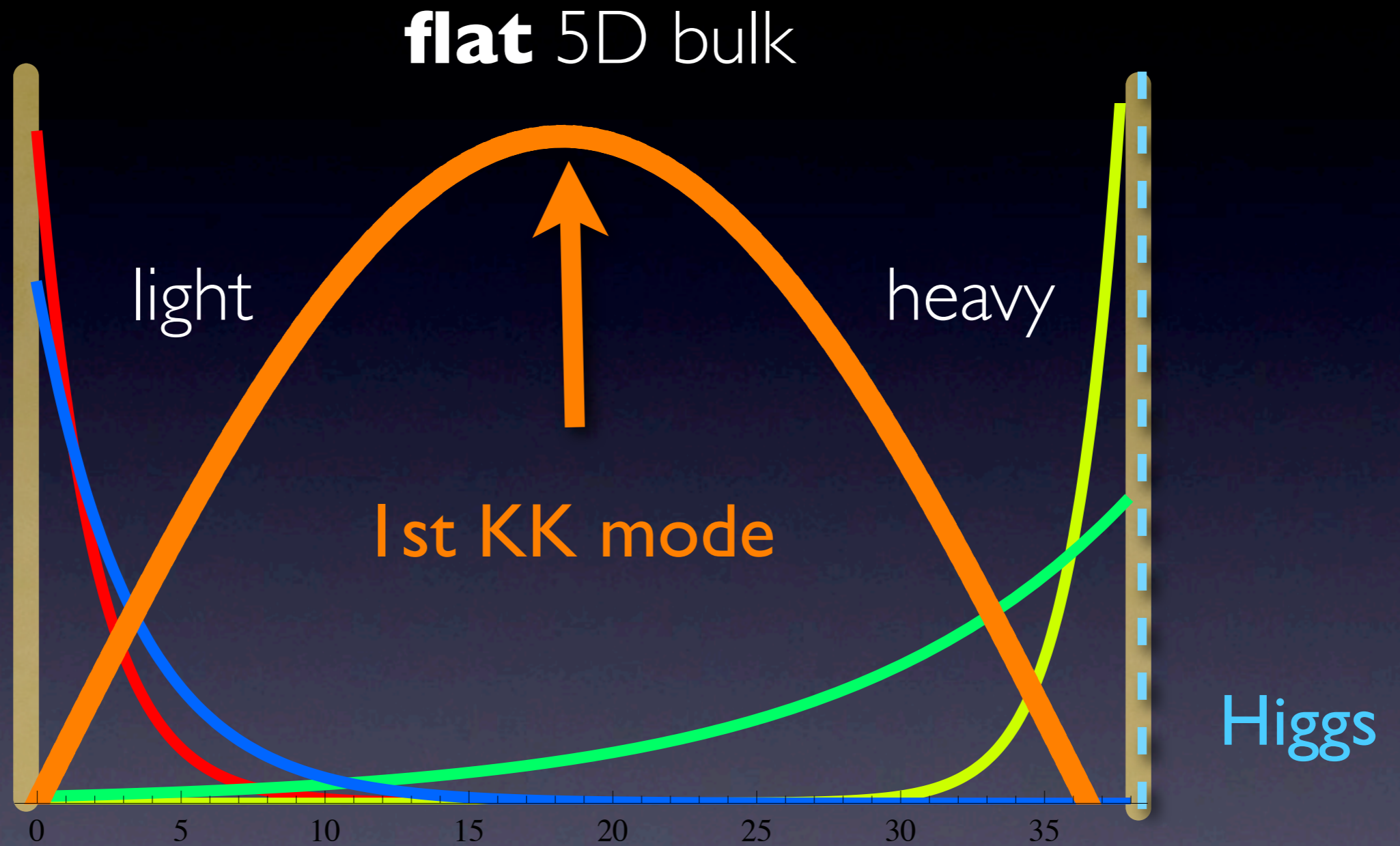
Delgado, Pomarol, Quiros '99

flat 5D bulk



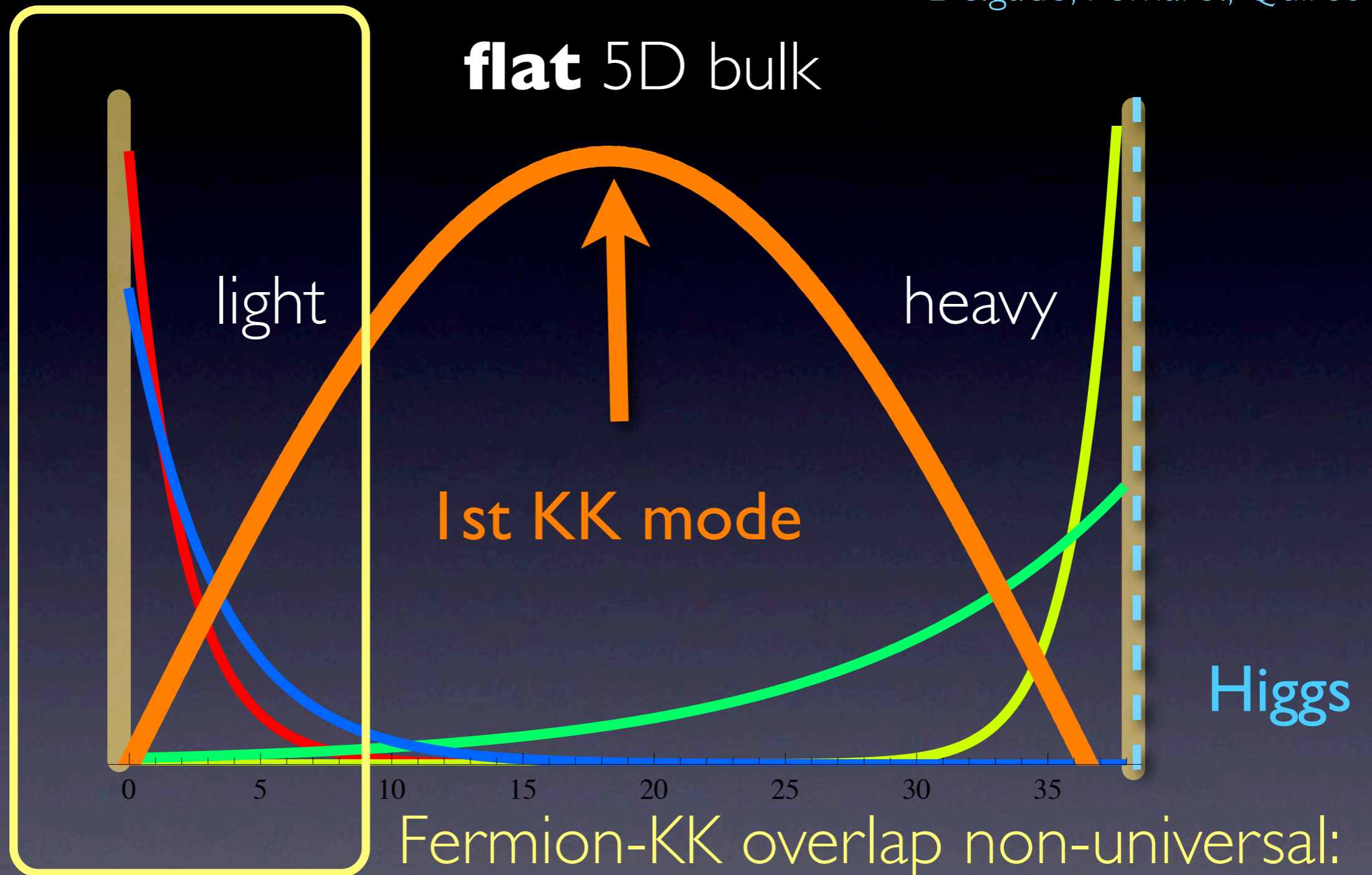
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Fermion-KK overlap non-universal:
induces FCNCs among light fermions

$$\Rightarrow M_{\text{KK}} \sim 1/R > 5000 \text{ TeV}$$

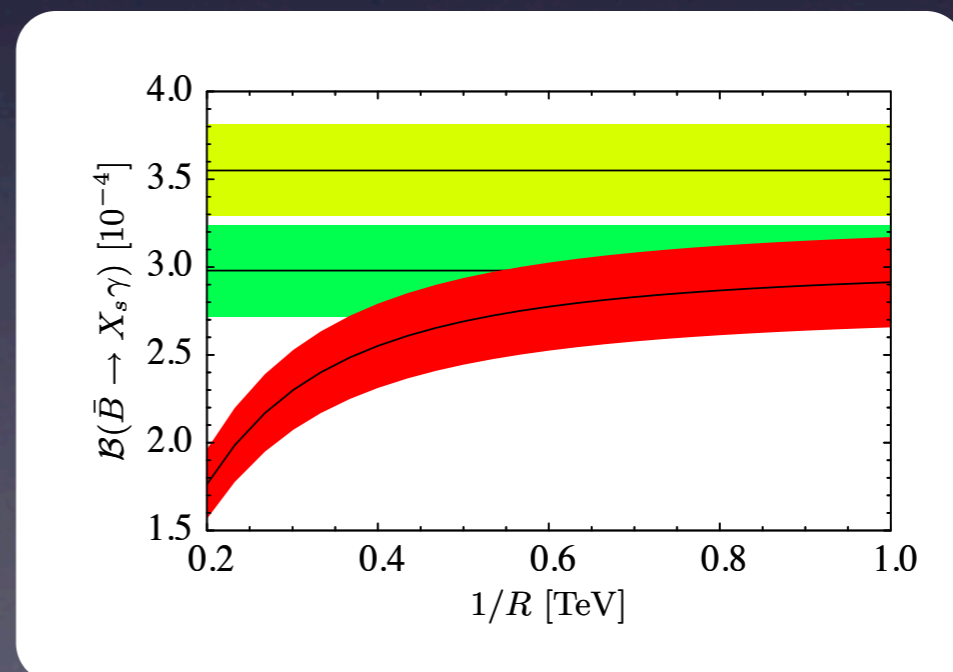
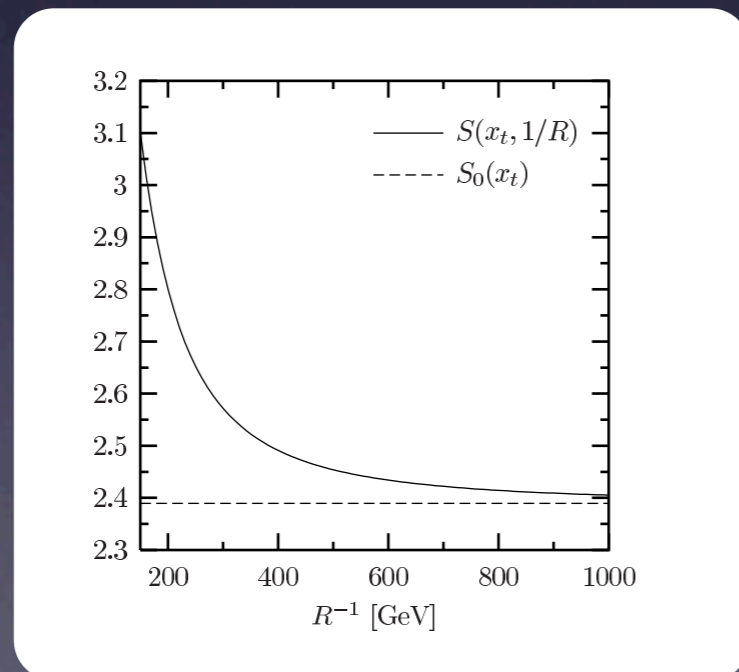
Universal extra dimensions

Cheng, Matchev, Schmaltz

Even with flat wave-functions for all fermions (and the higgs) remains a conceptual problem.

Agashe, Deshpande; Buras, Spranger, W

Calculable contributions are MFV, but...



4D \rightarrow 5D EFT

$$\int d^4x \lambda_4 \bar{\psi} h \psi \longrightarrow \int d^4x \int_0^R dy \lambda_5 \bar{\Psi} H \Psi$$

$$[\lambda_4] = 0 \longrightarrow [\lambda_5] = -1/2 \quad \lambda_5 = \sqrt{2\pi R} \lambda_4$$

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$$\int d^4x \int_0^R dy \lambda_5 \bar{\Psi} H \Psi + \dots \simeq \int d^4x \lambda_4 \bar{\psi} h \psi + \frac{1}{\Lambda^2} \bar{\psi} \psi \bar{\psi} \psi + \dots$$

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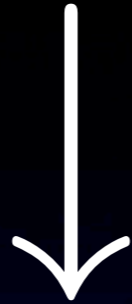
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NDA: $\Lambda \simeq \frac{8\pi}{\lambda_4^2} \frac{1}{R}$ vs. $\Lambda_{\epsilon_K} > 10^5 \text{ TeV}$

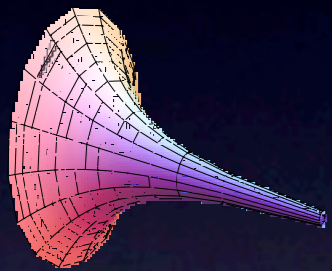
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From flat to warped ED

$$ds^2 = dx_\mu dx_\nu - dy^2$$



Randall, Sundrum

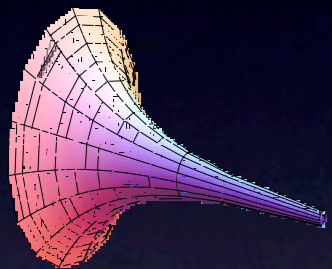


$$ds^2 = \left(\frac{R}{z}\right)^2 (dx_\mu dx_\nu - dz^2)$$

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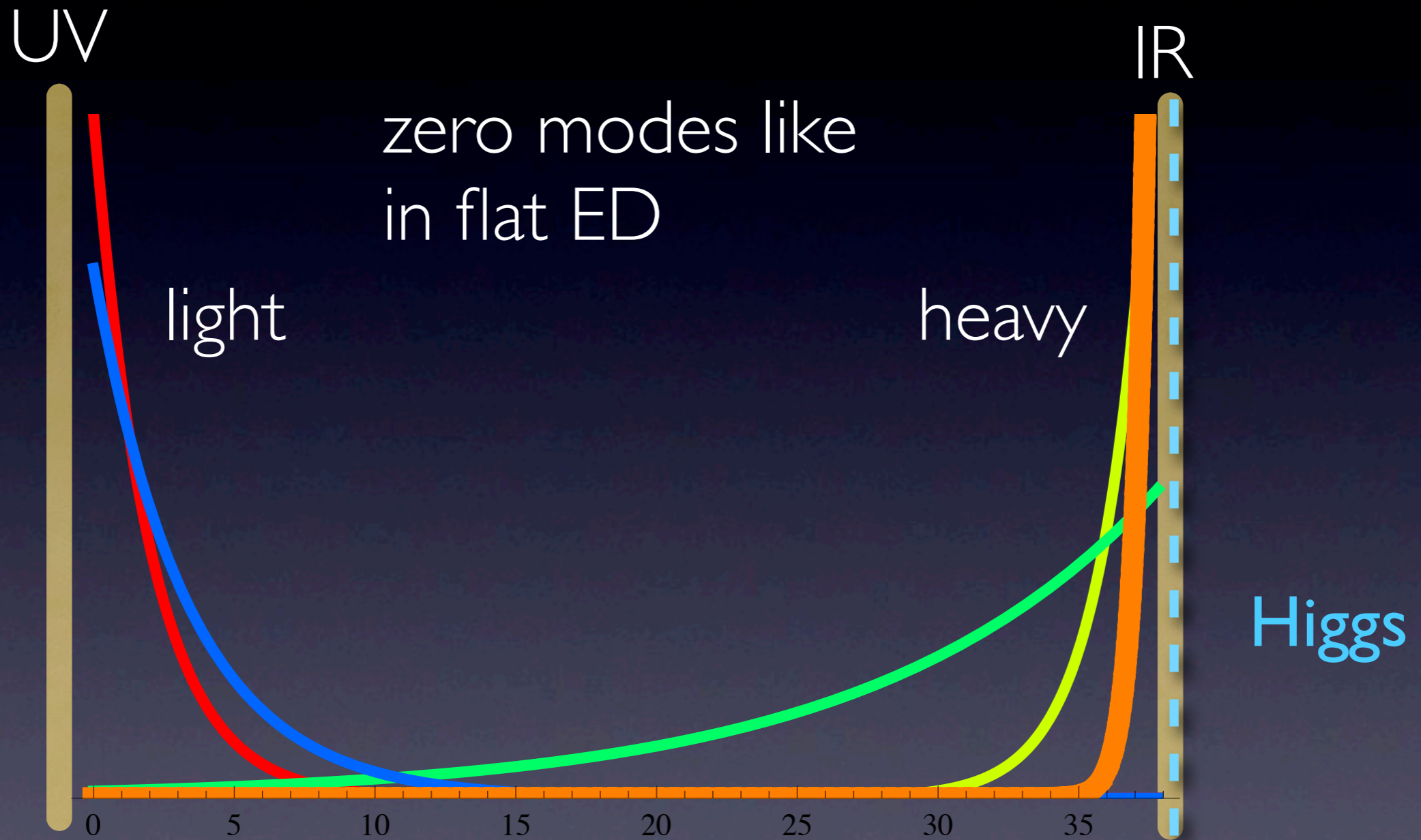


$$ds^2 = \left(\frac{R}{z}\right)^2 (dx_\mu dx_\nu - dz^2)$$

- ✓ solution to the hierarchy problem
- ✓ AdS/CFT description: reappraisal of strong EW symmetry breaking (composite Higgs, technicolor,...)
- ✓ high scale unification, log running of gauge couplings

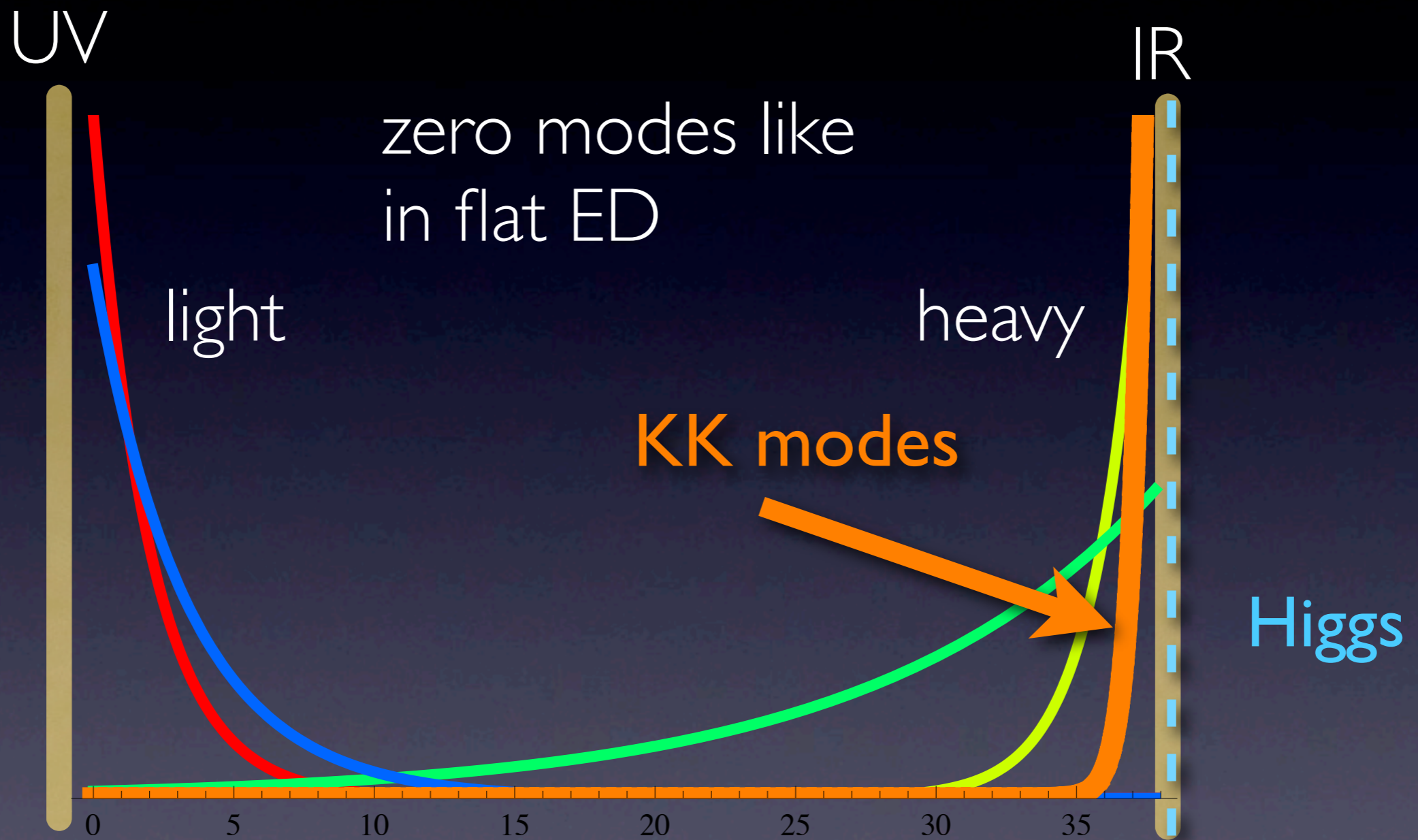
Flavor in RS

Grossman, Neubert; Gherghetta, Pomarol; Huber;



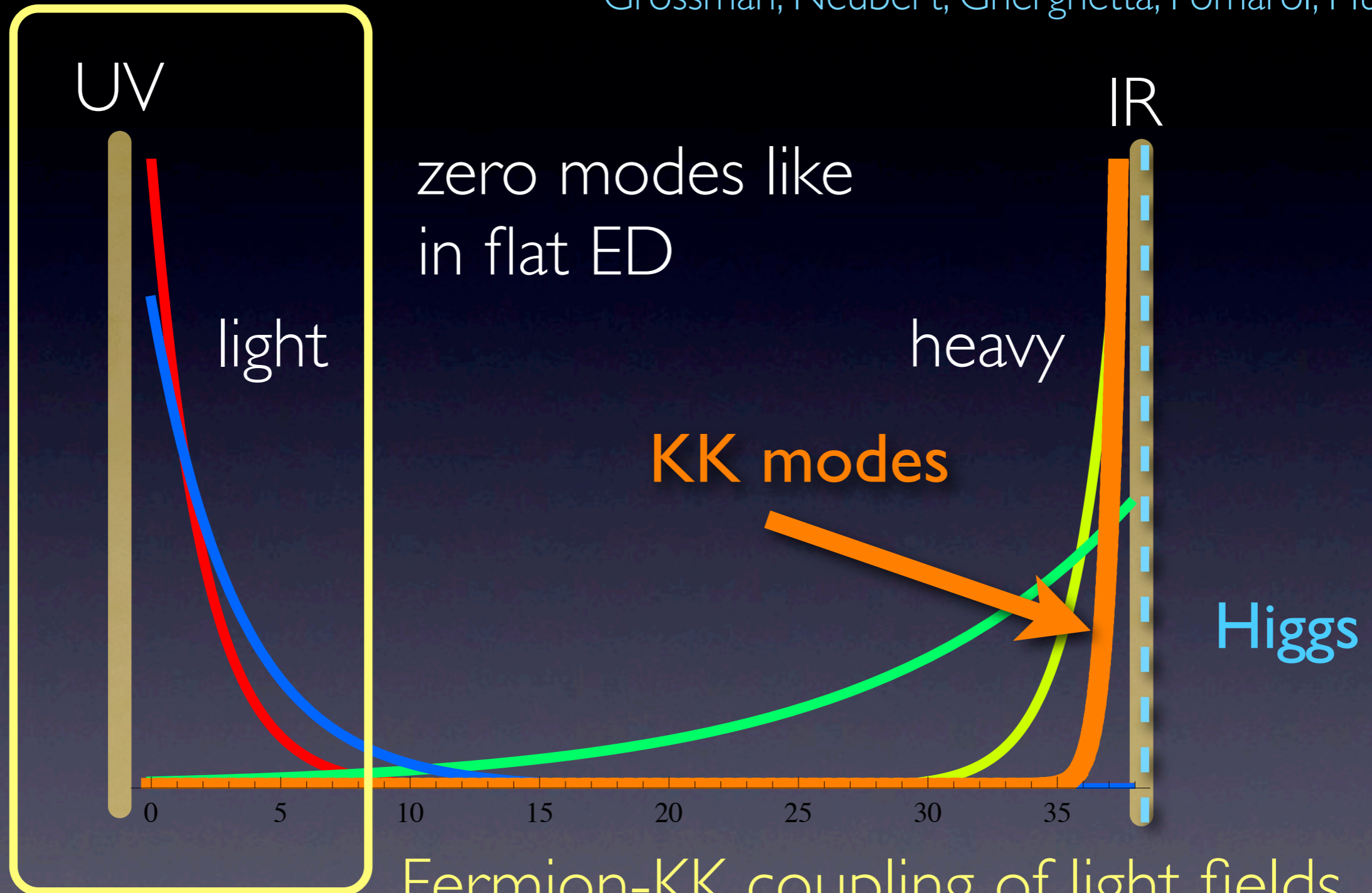
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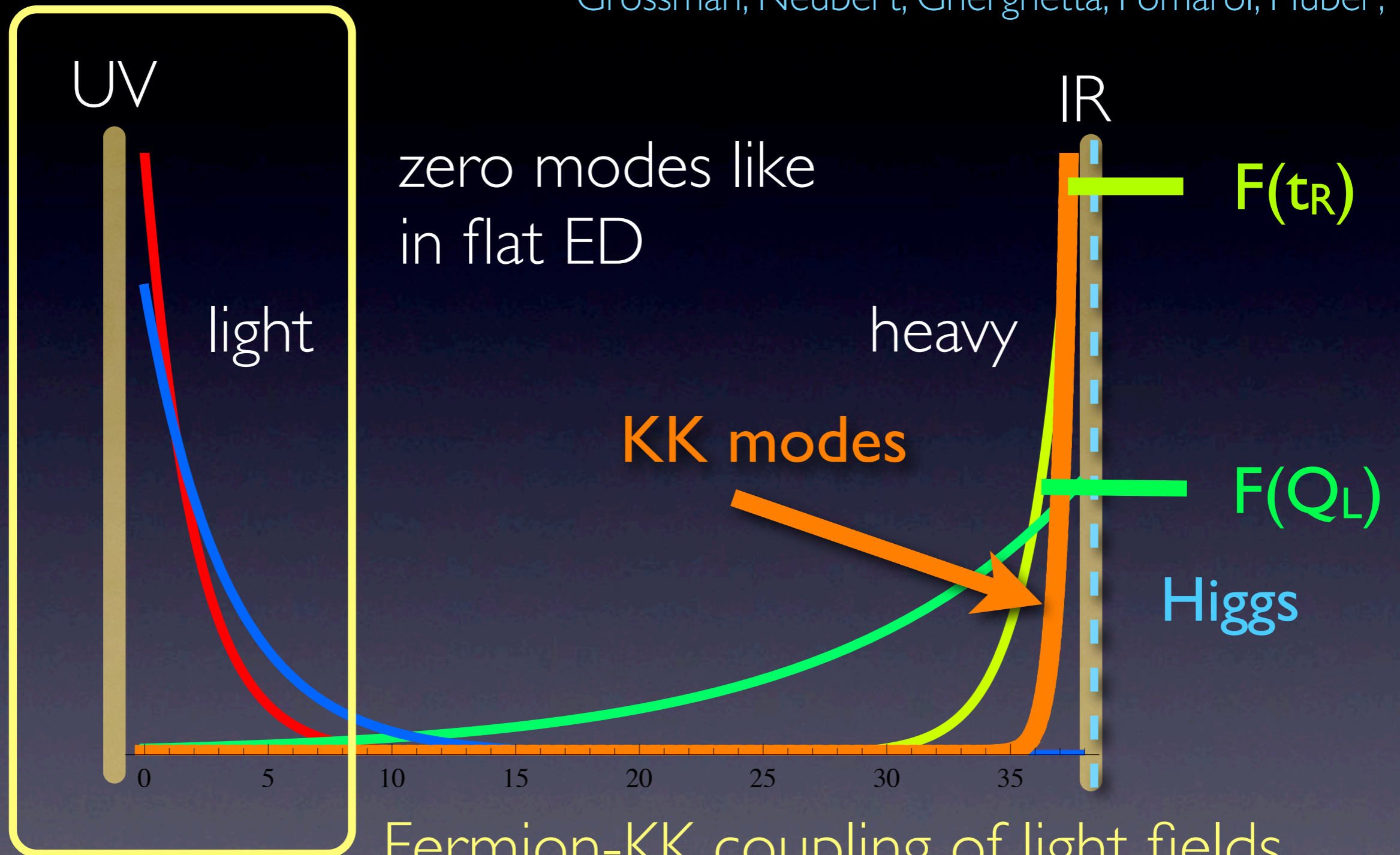
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Fermion-KK coupling of light fields almost universal!

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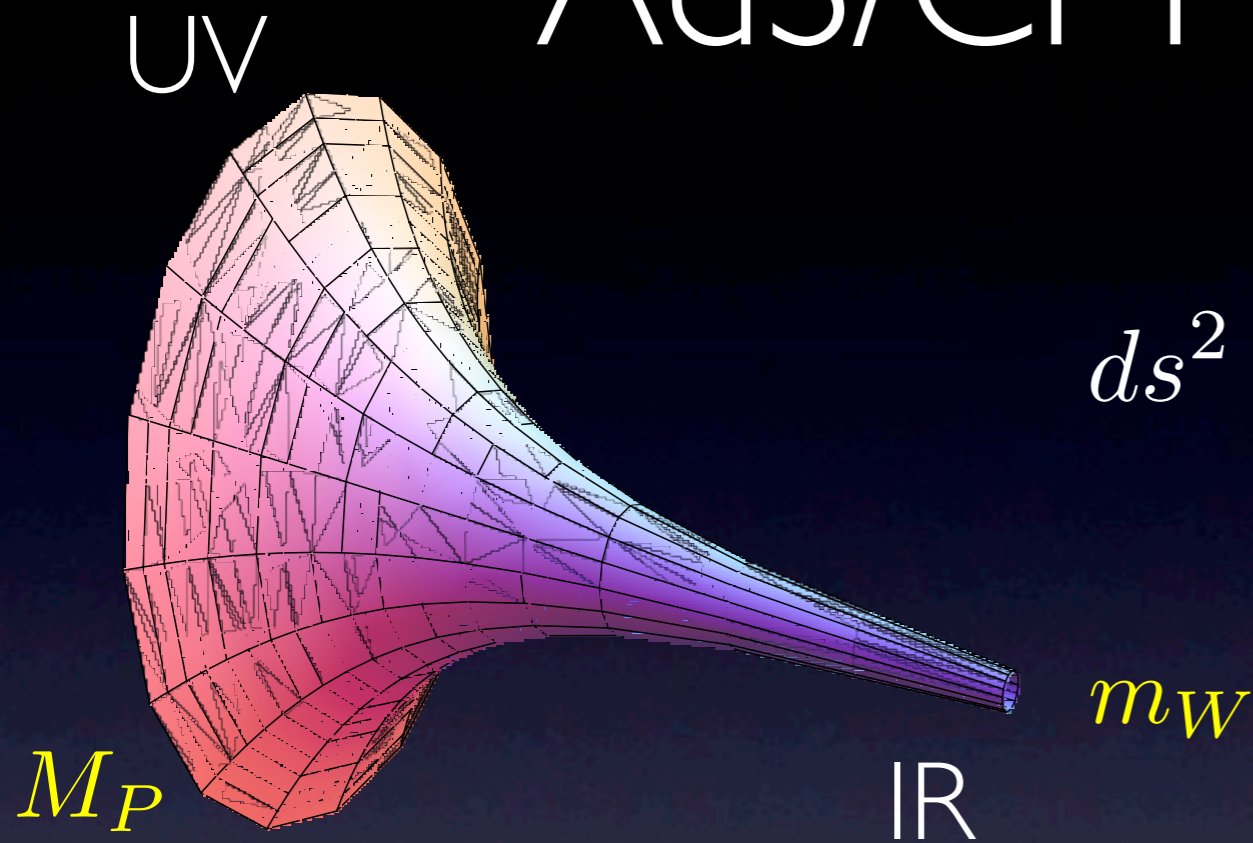
Fermion-KK coupling of light fields almost universal!

Why are FCNCs protected?

Excursion into AdS/CFT

AdS/CFT (popular science realization)

Randall, Sundrum



$$ds^2 = \left(\frac{R}{z}\right)^2 (dx_\mu dx_\nu - dz^2)$$

Anti-de-Sitter (AdS)



Conformal (CFT)

Compactification



Mass gap

Red-shifting of scales



Dimensional trans-
mutation

$$m_W = \sqrt{\frac{g(IR)}{g(UV)}} M_P \ll M_P$$

$$m_W \sim e^{-4\pi/\alpha} M_P$$

Two ways of giving mass to fermions...

Bi-linear:

$$\mathcal{L} = y f_L \mathcal{O}_H f_R, \quad \mathcal{O}_H \sim (1, 2)_{\frac{1}{2}}$$

Linear:

D.B. Kaplan '91

$$\mathcal{L} = y f_L \mathcal{O}_R + y_R f_R \mathcal{O}_L + m \mathcal{O}_L \mathcal{O}_R, \quad \mathcal{O}_R \sim (3, 2)_{\frac{1}{6}}$$

Partial compositeness

$$\mathcal{L} = \mathcal{L}_{elem}(g_{elem}) + \mathcal{L}_{comp}(g_*) + \mathcal{L}_{mix}$$

$$1 \lesssim g_* \lesssim 4\pi$$

Contino, Kramer, Son, Sundrum

$$|SM\rangle = \cos \phi |elem.\rangle + \sin \phi |comp.\rangle$$

$$|heavy\rangle = -\sin \phi |elem.\rangle + \cos \phi |comp.\rangle$$


1) Linear coupling of SM fields to composites

$$\mathcal{L}_{UV} \supset \lambda \bar{\mathcal{O}}_R \psi_L$$

Contino, Pomarol

2) Strong sector conformal over large energy range

$$\mu \frac{d\lambda}{d\mu} = \gamma \lambda \quad \gamma = \dim[\mathcal{O}_R] + 3/2 - 4$$


$$\lambda \sim \left(\frac{\text{TeV}}{M_{Pl}} \right)^\gamma$$

AdS/CFT translation:

$$\gamma = c - \frac{1}{2}$$

Partial compositeness

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Degree of compositeness:

$$\sin \phi = F(c) \sim \left(\frac{\text{TeV}}{M_{pl}} \right)^{c - \frac{1}{2}}$$

Meanwhile in the Extra-Dimension

Fermion zero mode on the IR brane

$$F(c) \sim \begin{cases} (\text{TeV/Planck})^{c-\frac{1}{2}} & c > 1/2 \\ \sqrt{1-2c} & c < 1/2 \end{cases}$$

Structure of the mass matrix

$$m_u^{SM} = \frac{v}{\sqrt{2}} F_q \mathbf{Y}_u F_u,$$

$$m_d^{SM} = \frac{v}{\sqrt{2}} F_q \mathbf{Y}_d F_d$$

$\mathbf{Y}_u, \mathbf{Y}_d \sim \mathcal{O}(1)$ & anarchic and $F_i \ll F_j$ for $i < j$.

Match SM spectrum and V_{CKM}

Hierarchical mass eigenvalues (6 conditions)

$$(m_{u,d})_{ii} \sim \frac{v}{\sqrt{2}} F_{Q_i} Y_{u,d} F_{u_i,d_i} \quad F_q = \left(\frac{F}{\Lambda} \right)^q$$

and hierarchical mixing angles (2 conditions)

$$F_{Q_1}/F_{Q_3} \sim \theta_{13} \sim \lambda^3$$

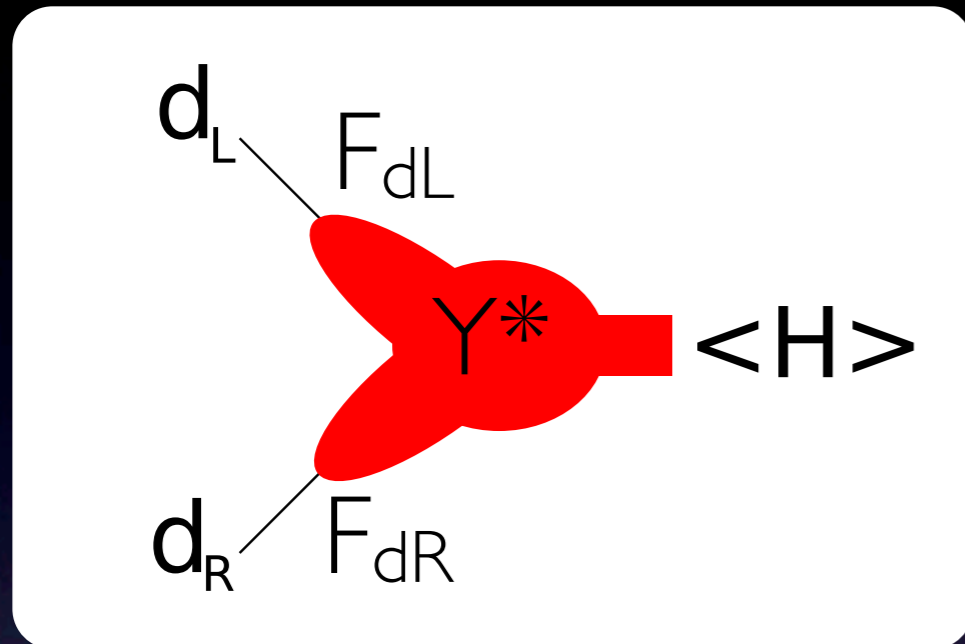
$$F_{Q_2}/F_{Q_3} \sim \theta_{23} \sim \lambda^2$$

check Cabibbo:

$$\theta_{12} \sim F_{Q_1}/F_{Q_2} \sim F_{Q_1}/F_{Q_3} \cdot F_{Q_3}/F_{Q_2} \sim \lambda \quad \checkmark$$

RS GIM - partial compositeness

Gherghetta, Pomarol; Huber; Agashe, Perez, Soni;

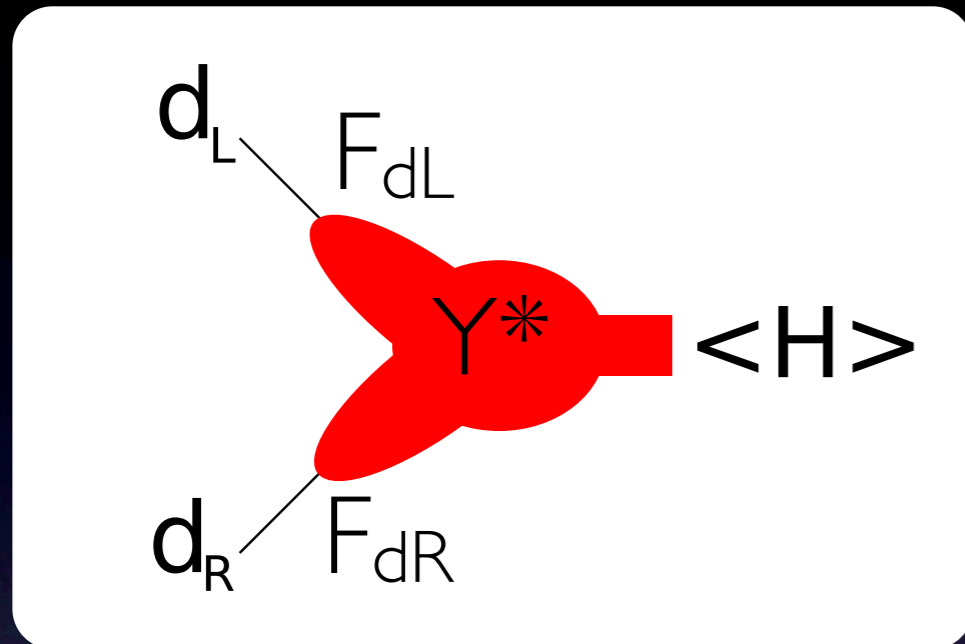


Flavor hierarchy from hierarchy in F_i

$$m_d \sim v F_{dL} Y^* F_{dR}$$

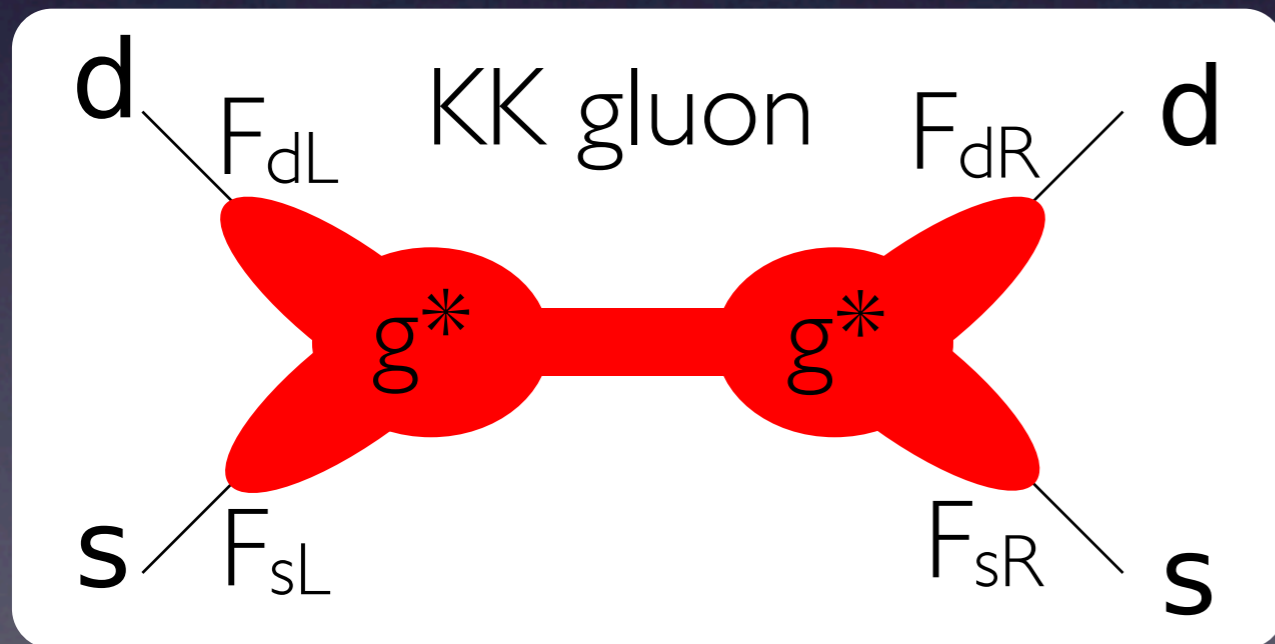
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KK gluon FCNCs proportional to the same small F_i :

$$\sim \frac{(g^*)^2}{M_{KK}^2} F_{dL} F_{dR} F_{sL} F_{sR}$$

$$\sim \frac{(g^*)^2}{M_{KK}^2} \frac{m_d m_s}{(v Y^*)^2}$$

RS vs. Composite Higgs

Georgi, Kaplan

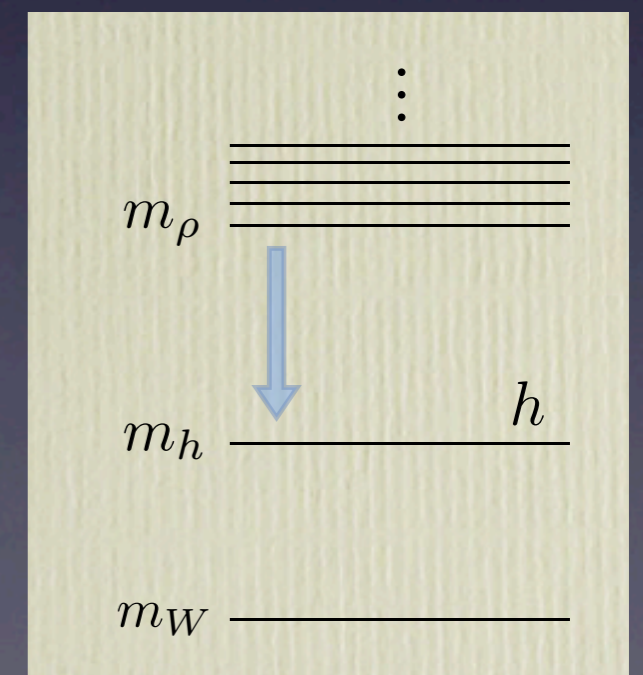
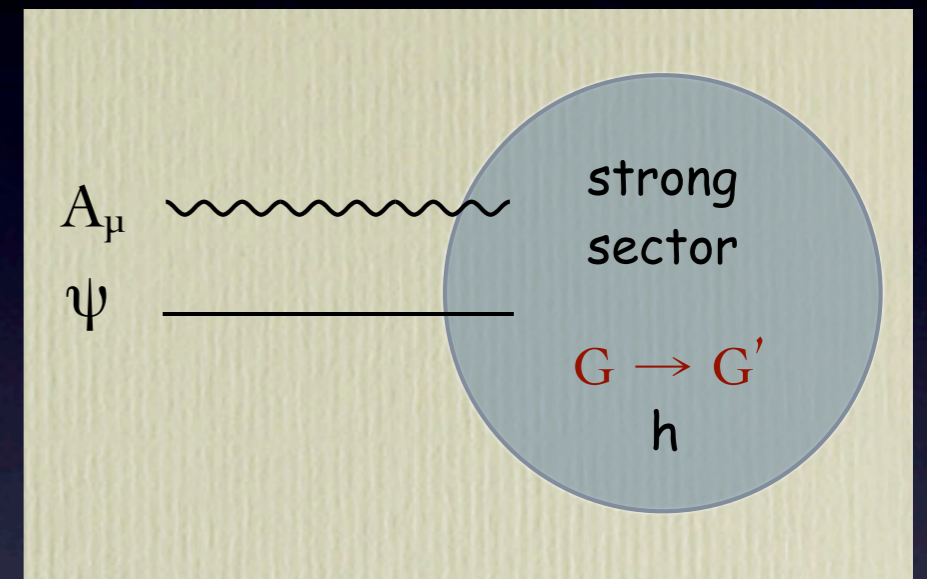
Light Higgs-like scalar arises as a bound state from a strongly-interacting EWSB sector

Motivations:

- A composite Higgs solves the hierarchy problem
- A light Higgs is preferred by the electroweak fit

A light composite Higgs can naturally arise as a (pseudo) Goldstone boson

Brane/bulk scalar in RS \equiv 4D composite Higgs.



FCNCs from composite Higgs

Agashe, Contino; Azatov, Toharia, Zhu

Further constraints from compositeness of Higgs

$$Y_d \bar{Q}_L H d_R + \frac{\tilde{Y}^d}{\Lambda^2} \bar{Q}_L H d_R (H^\dagger H) + \frac{\tilde{Z}}{\Lambda^2} \bar{Q}_L i \not{D} Q_L (H^\dagger H) + \dots$$

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$$M^d = v Y^d - \left(\tilde{Y}^d + \tilde{Z} Y^d + \dots \right) \frac{v^3}{\Lambda^2},$$

$$H = v + h(x)$$

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$$h \bar{d}_L d_R \left[Y^d - \mathbf{3} \left(\tilde{Y}^d + \tilde{Z} Y^d + \dots \right) \frac{v^2}{\Lambda^2} \right],$$

FCNCs from composite Higgs

Agashe, Contino; Azatov, Toharia, Zhu

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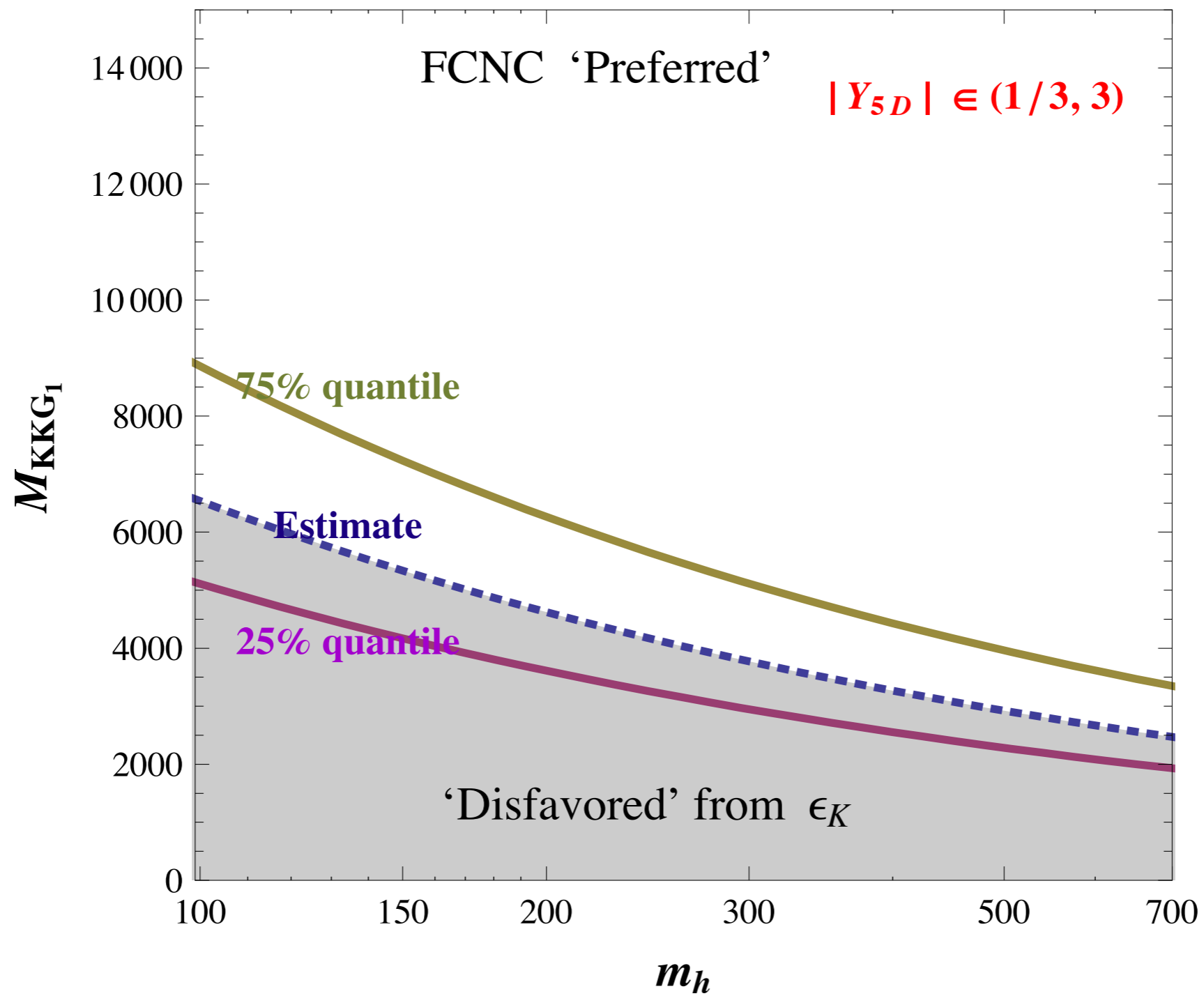
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⚡ FCNCs

$$h \bar{d}_L d_R \left[Y^d - 3 \left(\tilde{Y}^d + \tilde{Z} Y^d + \dots \right) \frac{v^2}{\Lambda^2} \right],$$



If composite Higgs is not just ordinary bound state but **pGB** associated with **G** → **H** in strong sector

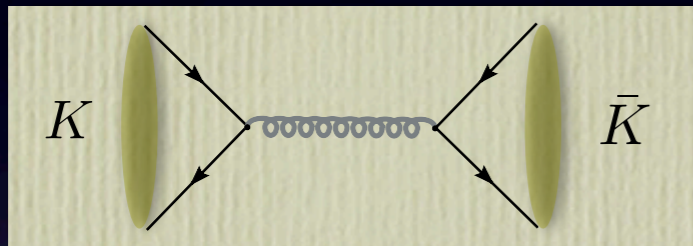
$$\bar{Q}_L H \left(Y^d + \tilde{Y}^d \frac{H^\dagger H}{\Lambda^2} + \dots \right) d_R \longrightarrow \bar{\psi}_L^i P_{ij}(\Sigma) \psi_R^j$$

Constraints are **less severe** (only from kinetic terms, suppressed by small quark masses).

FCNCs assuming anarchy

Csaki, Falkowski, W; Buras et al; Casagrande et al

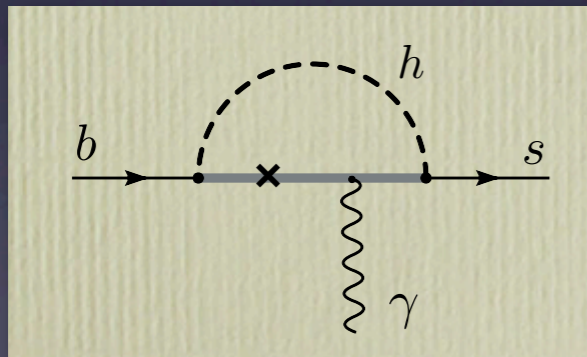
$\Delta F = 2$ (strongest constraint from ϵ_K)



$$C^4(M_*) \sim \frac{1}{M_*^2} \frac{2m_d m_s}{v^2} \left(\frac{g_*}{Y_*} \right)^2$$

$$M_* \gtrsim 10 \left(\frac{g_*}{Y_*} \right) \text{ TeV}$$

$\Delta F = 1$ (strongest constraint from ϵ'/ϵ)



$$M_* \gtrsim 1.3 Y_* \text{ TeV}$$

Combined constraints strong \Rightarrow flavor problem w/ anarchy

Spurion analysis

Without the Yukawas SM has

$$SU(3)_{Q_L} \times SU(3)_{u_R} \times SU(3)_{d_R}$$

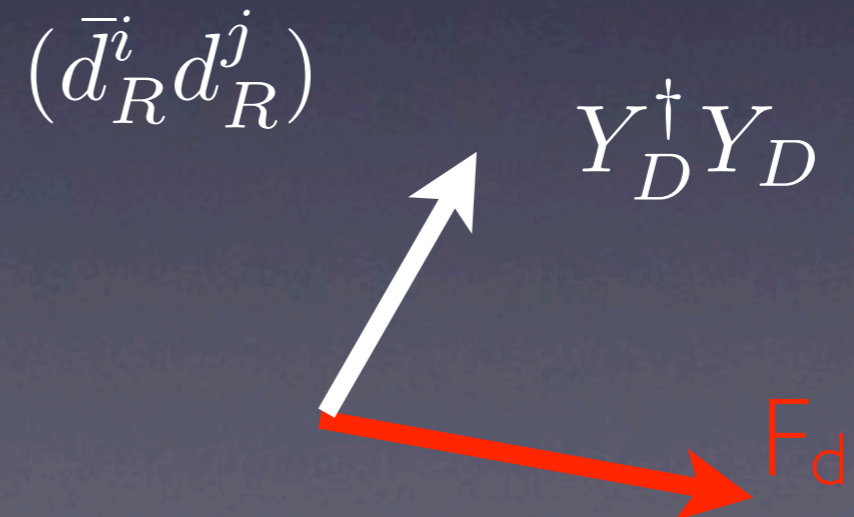
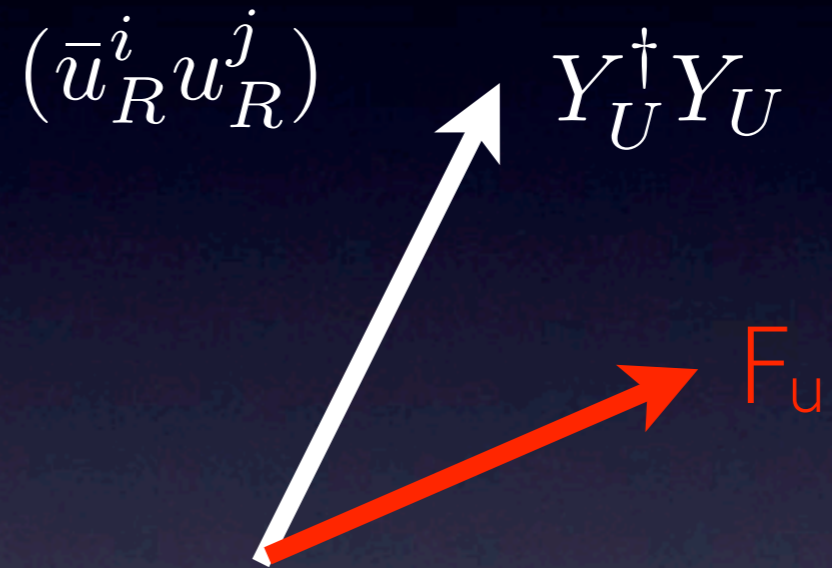
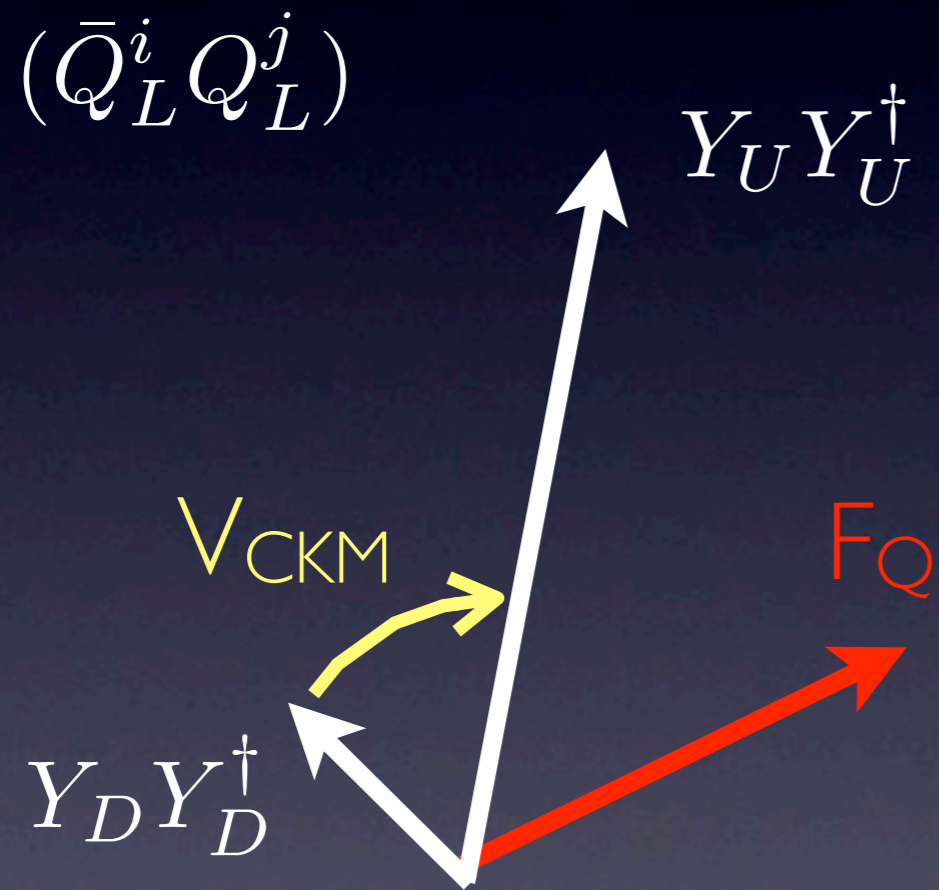
global flavor symmetry.

In RS broken by $Y_u^*, Y_d^* + F_Q, F_d, F_u$

No dangerous FCNCs in the down sector if

$Y_d^* + F_Q, F_d$ aligned (diagonal in the same basis)

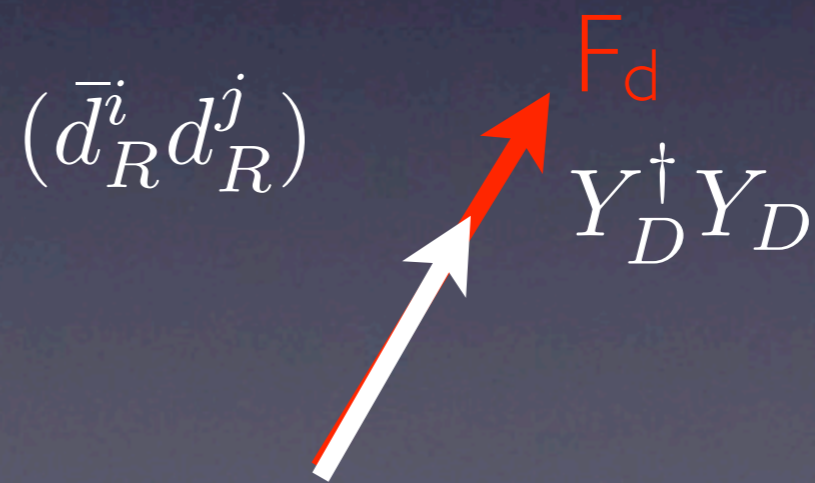
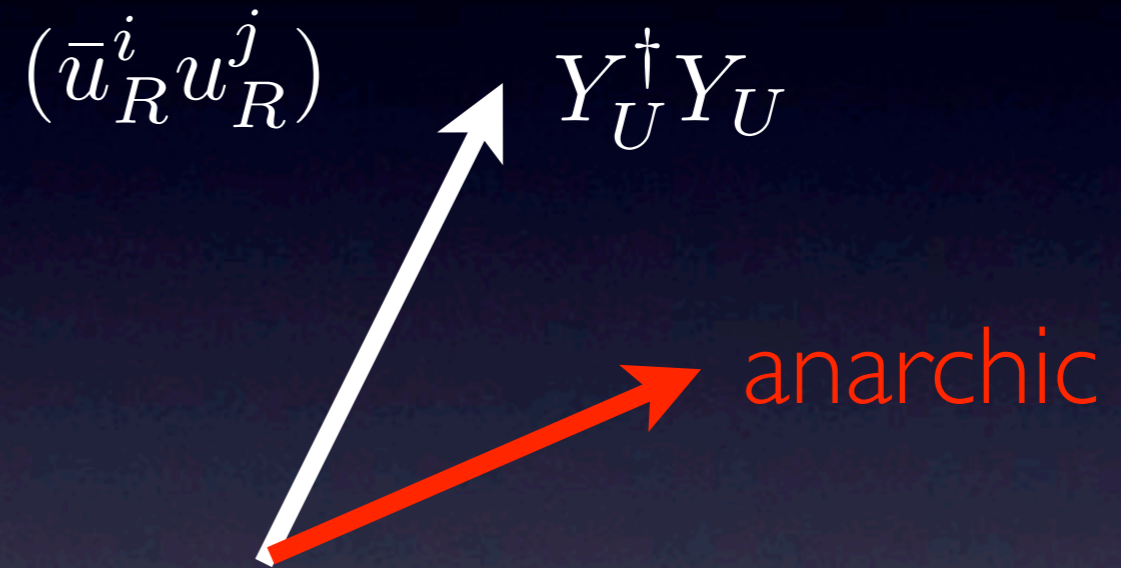
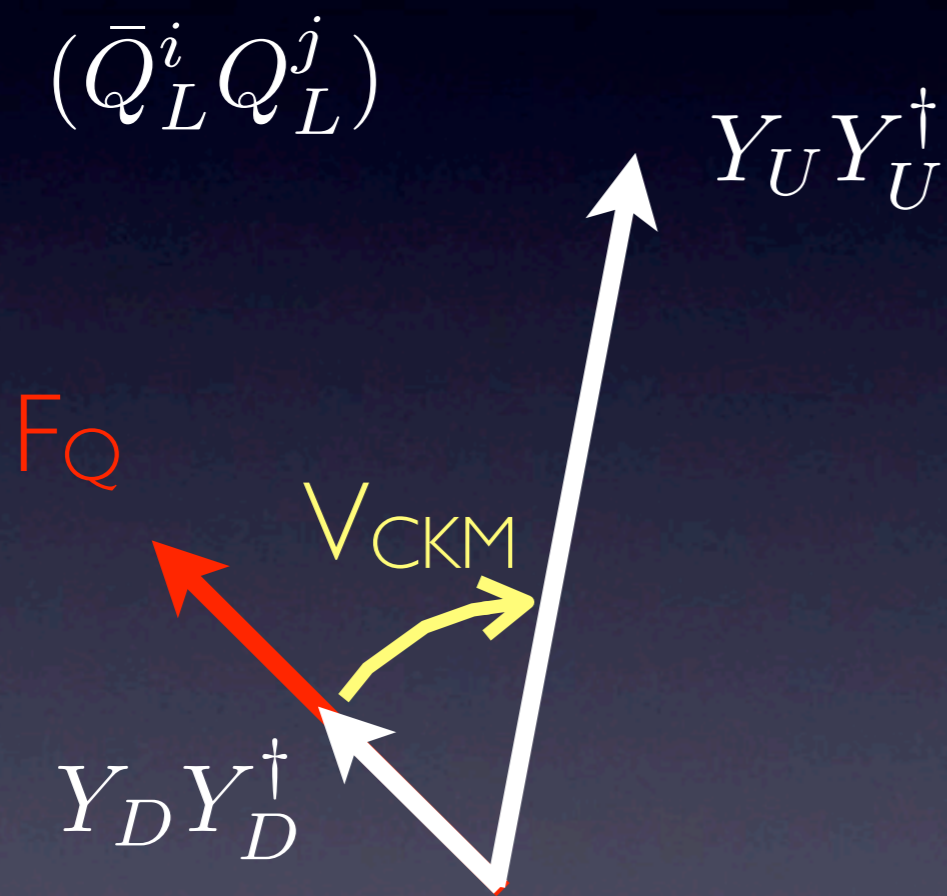
Anarchy



+ LR, RL

Align down sector

similar to Nir, Seiberg '93 for MSSM



+ LR, RL

Aligning 5D MFV

Fitzpatrick, Randall, Perez; Csaki, Perez, Surujon, A.W.,

$SU(3)^3$ flavor symmetry broken by Yukawas only

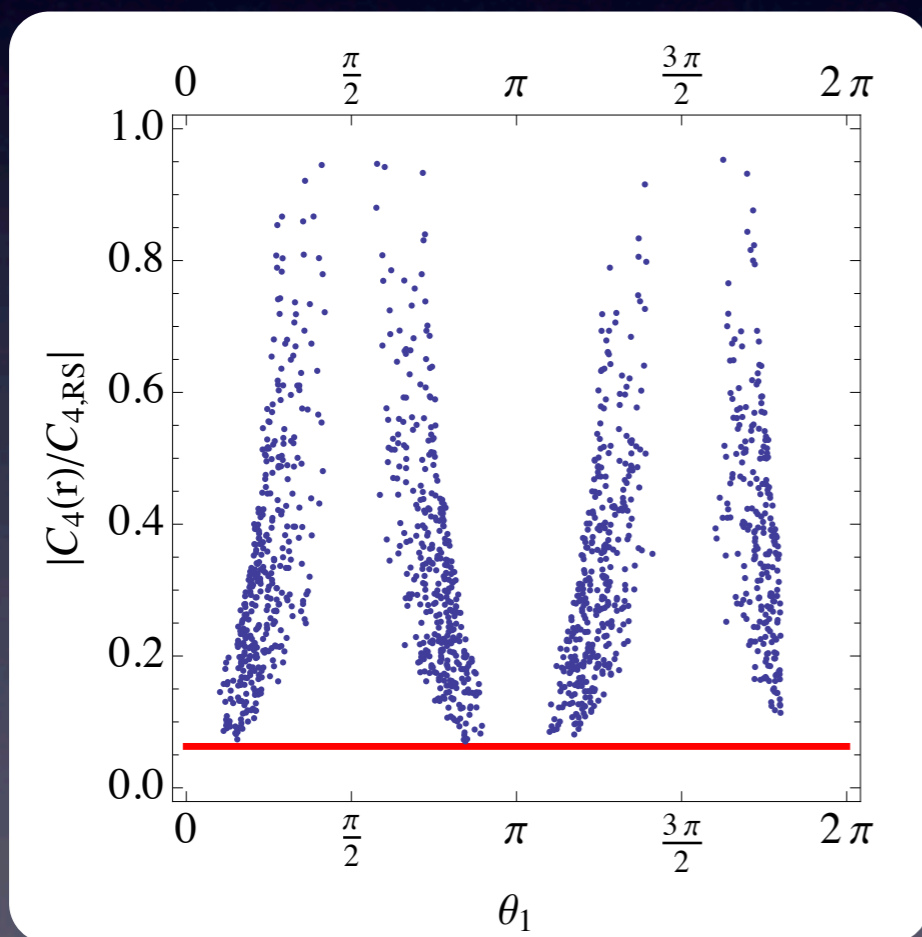
$$c_Q \sim Y_d Y_d^\dagger + \epsilon Y_u Y_u^\dagger \quad c_d \sim Y_d^\dagger Y_d \quad c_u \sim Y_u^\dagger Y_u$$

Need $\epsilon \ll 1$ to align F_Q , F_d , and Y_d

Aligning 5D MFV

Fitzpatrick, Randall, Perez; Csaki, Perez, Surujon, A.W.,

Scan 5D CKM and test suppression $C_{4\text{MFV}} / C_{4,\text{RS}}$
Keep $\epsilon = 0.2$ fixed.



Need $\epsilon \rightarrow 0$
 \Rightarrow symmetry?

Alignment due to shining

Csaki, Perez, Surujon, A.W.

In the bulk: gauged $SU(3)_Q \times SU(3)_d$ flavor

$$F(c_Q) = F(Y_{*d} Y_{*d}^\dagger), \quad F(c_d) = F(Y_{*d}^\dagger Y_{*d})$$

Flavor broken by vev of Yukawa field Y_{*d} only

UV breaking 'shines' into the bulk via **marginal operator**

Rattazzi, Zafaroni

$$\Phi_d : (\mathbf{3}, \mathbf{1}, \underline{\mathbf{3}}), \quad \langle \Phi_d \rangle = Y_{*d} (z/R)^{-\epsilon}$$

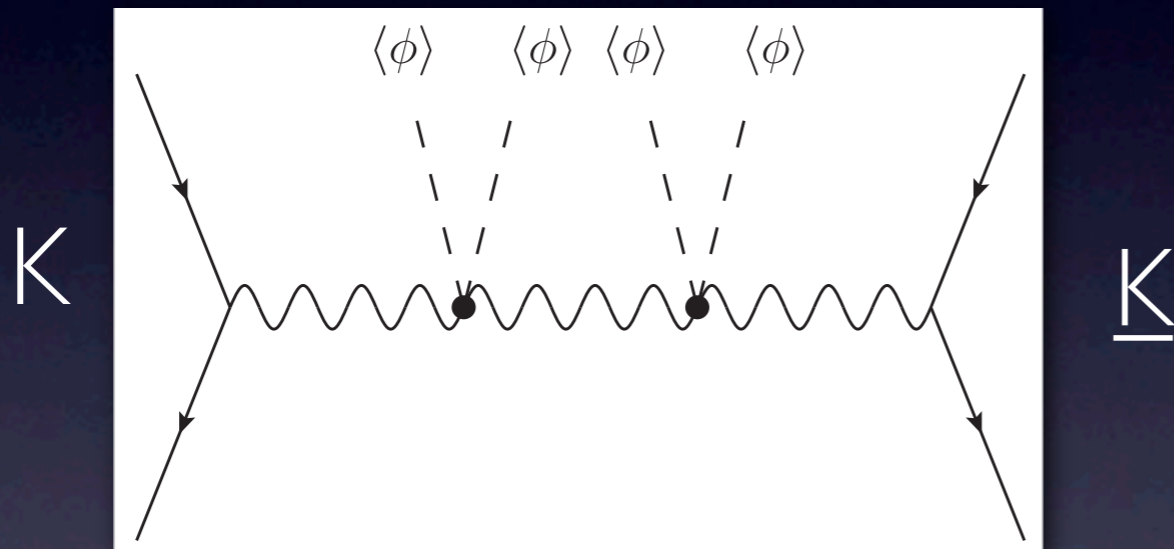
Big effects in up-FCNCs expected.

A theory of flavor at the
LHC?

Flavor gauge boson FCNCs

Csaki, Lee, Perez, AW in preparation

In Rattazzi-Zaffaroni model: **dynamical MFV**



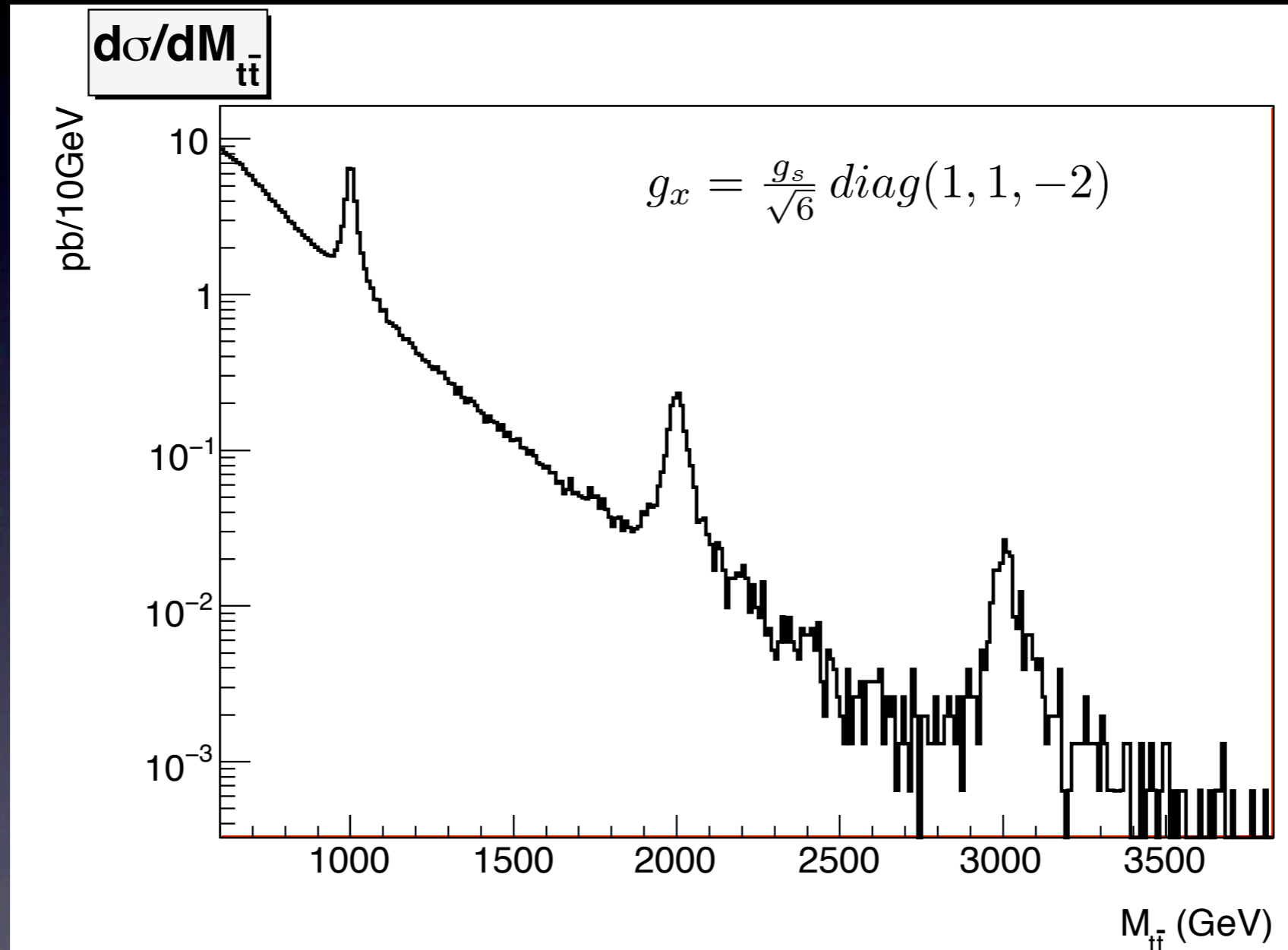
$$\mathcal{L}_{mass} = M_{KK}^2 \text{Tr}[A_Q A_Q] + \frac{4g_{Q^*}^2 R^3}{3R'^2} \text{Tr}[\phi_u A_Q A_Q \phi_u]$$

$g^* \sim 4$ allowed

$$\frac{1}{M_{KK}^2} \frac{2g_{Q^*}^6 y_t^4}{27(M_{KK} R')^4 + 42(g_{Q^*} y_t)^2 (M_{KK} R')^2 + 16(g_{Q^*} y_t)^4} ((V^\dagger Y_u^2 V)_{ij})^2 (\bar{d}_i \gamma_L^\mu d_j) (\bar{d}_i \gamma_L^\mu d_j)$$

Flavor scalars & gauge bosons

Csaki, Lee, Perez, AW in preparation



thanks to Seung Lee for the plot

Conclusions

Extra dimensions allow new approaches to the flavor puzzle.

Warped flavor is a calculable realization of partial compositeness, a linear mixing with the composite sector.

RS-GIM suppresses dangerous FCNCs, tension with CPV in Kaon sector. Can decouple problems but decouple from LHC.

Anarchy alone needs tuning to survive, additional flavor structure indicated.

Signal: large FCNCs in the **up-sector** (top FCNCs, $D-\underline{D}$), Flavor gauge bosons?

