The muon g-2 discrepancy: new physics or a relatively light Higgs?

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Firenze - March 3 2010

Work in collaboration with W.J. Marciano & A. Sirlin
PRD78 (2008) 013009 [updated in arXiv:1001.4528]

The present experimental values:

$$a_e = 1159652180.73\ (28) \times 10^{-12}$$

0.24 parts per billion !! Hanneke et al., PRL100 (2008) 120801

$$a_\mu = 116592089\ (63) \times 10^{-11}$$

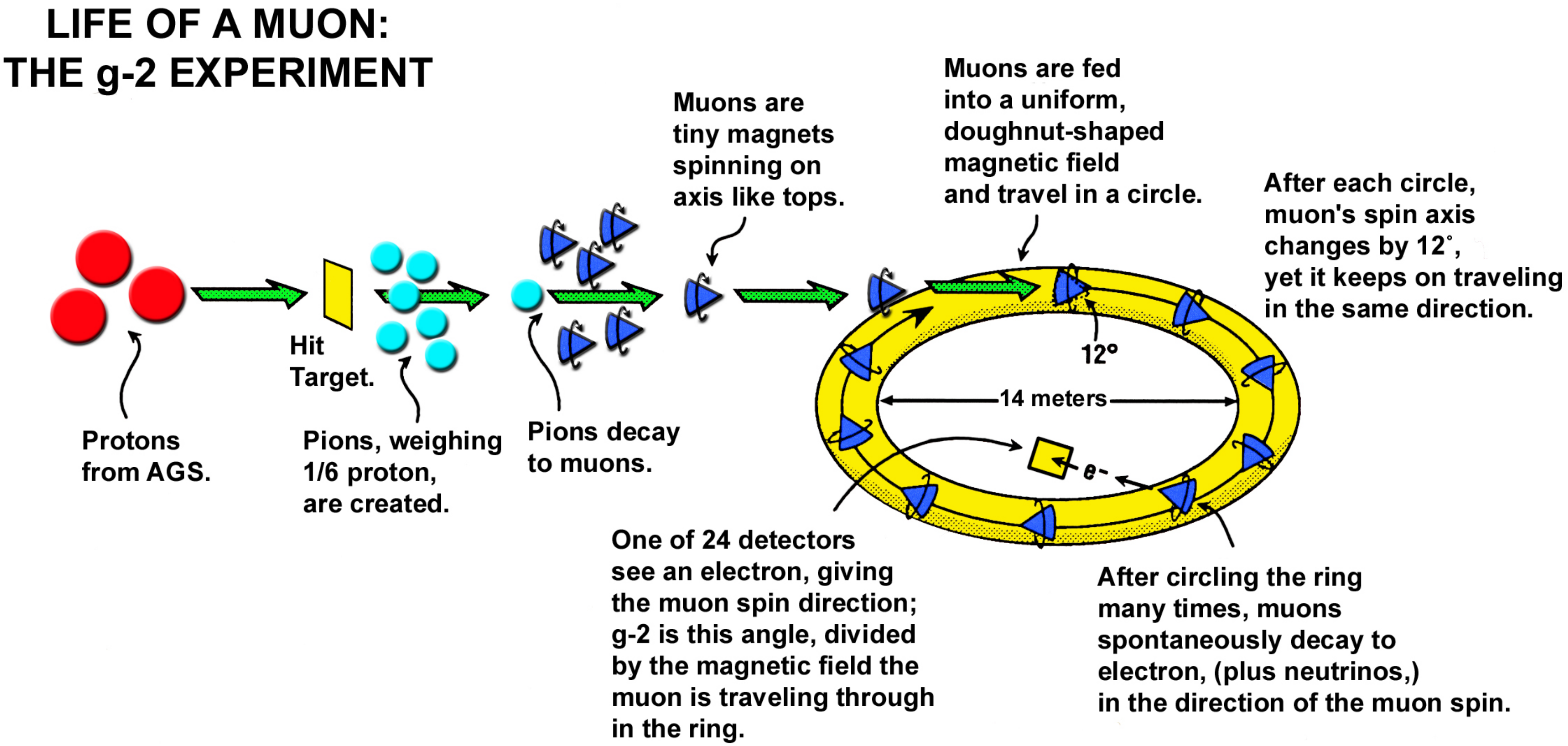
0.5 parts per million !! E821 – Final Report: PRD73 (2006) 072003

$$a_\tau = -0.018\ (17)$$

DELPHI - EPJC35 (2004) 159 [$a_\tau^{SM} = 117721(5) \times 10^{-8}$, Eidelman & MP ’07]

The muon g-2 discrepancy

The experiment E821



LIFE OF A MUON:
THE g-2 EXPERIMENT

Protons from AGS.

Hit Target.

Pions, weighing 1/6 proton, are created.

Pions decay to muons.

Muons are tiny magnets spinning on axis like tops.

Muons are fed into a uniform, doughnut-shaped magnetic field and travel in a circle.

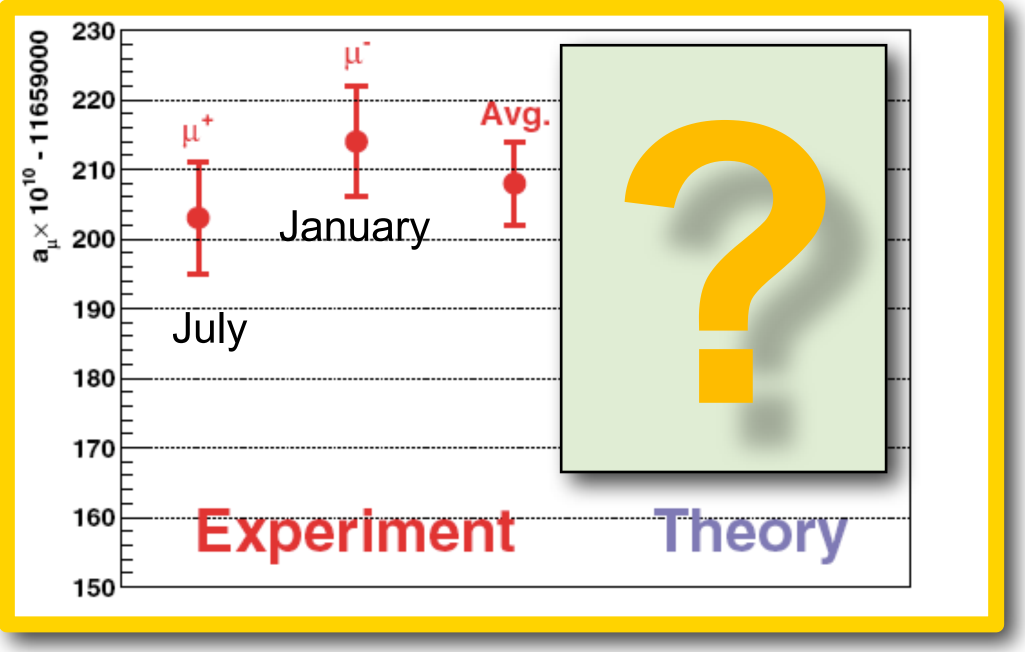
After each circle, muon's spin axis changes by 12˚, yet it keeps on traveling in the same direction.

One of 24 detectors see an electron, giving the muon spin direction; g-2 is this angle, divided by the magnetic field the muon is traveling through in the ring.

After circling the ring many times, muons spontaneously decay to electron, (plus neutrinos,) in the direction of the muon spin.

Homepage of E821

The muon g-2: the experimental result



- **Today:** $a_\mu^{EXP} = (116592089 \pm 54_{stat} \pm 33_{sys}) \times 10^{-11}$ [0.5ppm].
- **Future:** new muon g-2 experiments proposed at
  - Fermilab (E969), aims at 0.14ppm
  - J-PARC aims at 0.1 ppm
    [D.Hertzog & N.Saito, U. Paris VI meeting, Feb 26 2010]
- **Are theorists ready for this (amazing) precision?** [not yet]

The anomalous magnetic moment: the basics

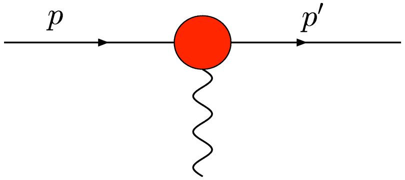
- The Dirac theory predicts for a lepton l=e,μ,τ:

$$\vec{\mu}_l = g_l \left(\frac{e}{2m_l c}\right)\vec{s} \qquad g_l = 2$$

- QFT predicts deviations from the Dirac value:

$$g_l = 2\,(1 + a_l)$$

- Study the photon–lepton vertex:



$$\bar{u}(p')\Gamma_\mu u(p) = \bar{u}(p')\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m}F_2(q^2) + \ldots\right]u(p)$$

$$\boxed{F_1(0) = 1 \qquad F_2(0) = a_l}$$

a_μ^{SM}: the QED contribution

$a_\mu^{QED} = (1/2)(\alpha/\pi)$ Schwinger 1948

$+ \ 0.765857408\ (27)\ (\alpha/\pi)^2$

Sommerfield; Petermann; Suura & Wichmann '57; Elend '66; MP '04

$+ \ 24.05050959\ (42)\ (\alpha/\pi)^3$

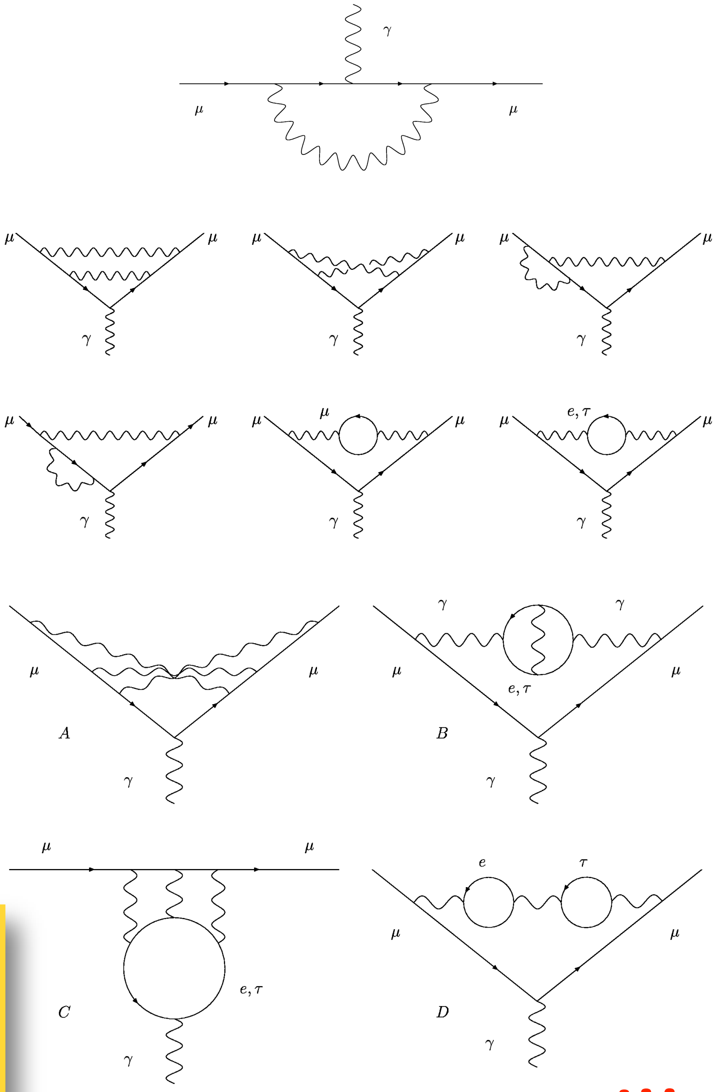
Remiddi, Laporta, Barbieri … ; Czarnecki, Skrzypek; MP '04;
Friot, Greynat & de Rafael '05, Mohr,Taylor & Newell '08

$+ \ 130.805\ (8)\ (\alpha/\pi)^4$

Kinoshita & Lindquist '81, … , Kinoshita & Nio '04, '05;
Aoyama, Hayakawa, Kinoshita & Nio, June & Dec 2007

$+ \ 663\ (20)\ (\alpha/\pi)^5$ In progress

Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta, Karshenboim,…, Kataev, Kinoshita & Nio '06, Kinoshita et al. 2010



…

Adding up, we get:

$a_\mu^{QED} = 116584718.08\ (14)(04) \times 10^{-11}$

from coeffs, mainly from 5-loop unc ↲ ↳ from new $\delta\alpha$('08)

with α=1/137.035999084(51) [0.37 ppb]

[A parenthesis on the electron g-2…

a_e^{SM} = $(1/2)(\alpha/\pi)$ - $0.328\,478\,444\,002\,89(60)\,(\alpha/\pi)^2$

Schwinger 1948 Sommerfield; Petermann; Suura & Wichmann '57; Elend '66; MP '06

$A_2^{(4)}(m_e/m_\mu) = 5.197\,386\,78\,(26) \times 10^{-7}$

$A_2^{(4)}(m_e/m_\tau) = 1.837\,62\,(60) \times 10^{-9}$

$+\ 1.181\,234\,016\,827\,(19)\,(\alpha/\pi)^3$

Kinoshita, Barbieri, Laporta, Remiddi, … , Li, Samuel; Mohr, Taylor & Newell '08, MP '06

$A_2^{(6)}(m_e/m_\mu) = -7.373\,941\,73\,(27) \times 10^{-6}$

$A_2^{(6)}(m_e/m_\tau) = -6.5819\,(19) \times 10^{-8}$

$A_3^{(6)}(m_e/m_\mu, m_e/m_\tau) = 1.909\,45\,(62) \times 10^{-13}$

$-\ 1.9144\,(35)\,(\alpha/\pi)^4$

Kinoshita & Lindquist '81, … , Kinoshita & Nio '05; Aoyama, Hayakawa, Kinoshita & Nio, 2007

$+\ 0.0\,(4.6)\,(\alpha/\pi)^5$ In progress (12672 mass ind. diagrams!)

Aoyama, Hayakawa, Kinoshita, Nio '07; Aoyama et al. 2008 & 2010.

$+\ 1.676\,(20) \times 10^{-12}$ Hadronic

Krause 1997, Jegerlehner & Nyffeler 2009

$+\ 0.02973\,(52) \times 10^{-12}$ Electroweak

Mohr, Taylor & Newell, '08; Czarnecki, Krause, Marciano '96

... and the best determination of alpha]

- The new (2008) measurement of the electron g-2 is:

  $a_e^{exp} = 1159652180.73\ (28) \times 10^{-12}$ Hanneke et al, PRL100 (2008) 120801

  vs. old (factor of 15 improvement, 1.8σ difference):

  $a_e^{exp} = 1159652188.3\ (4.2)\ \times 10^{-12}$ Van Dyck et al, PRL59 (1987) 26

- Equating $a_e^{SM}(\alpha) = a_e^{exp}$ → best determination of alpha to date:

$\alpha^{-1} = 137.035\,999\,084\ (12)(37)(2)(33)\ [0.37\text{ppb}]$ Hanneke et al, '08

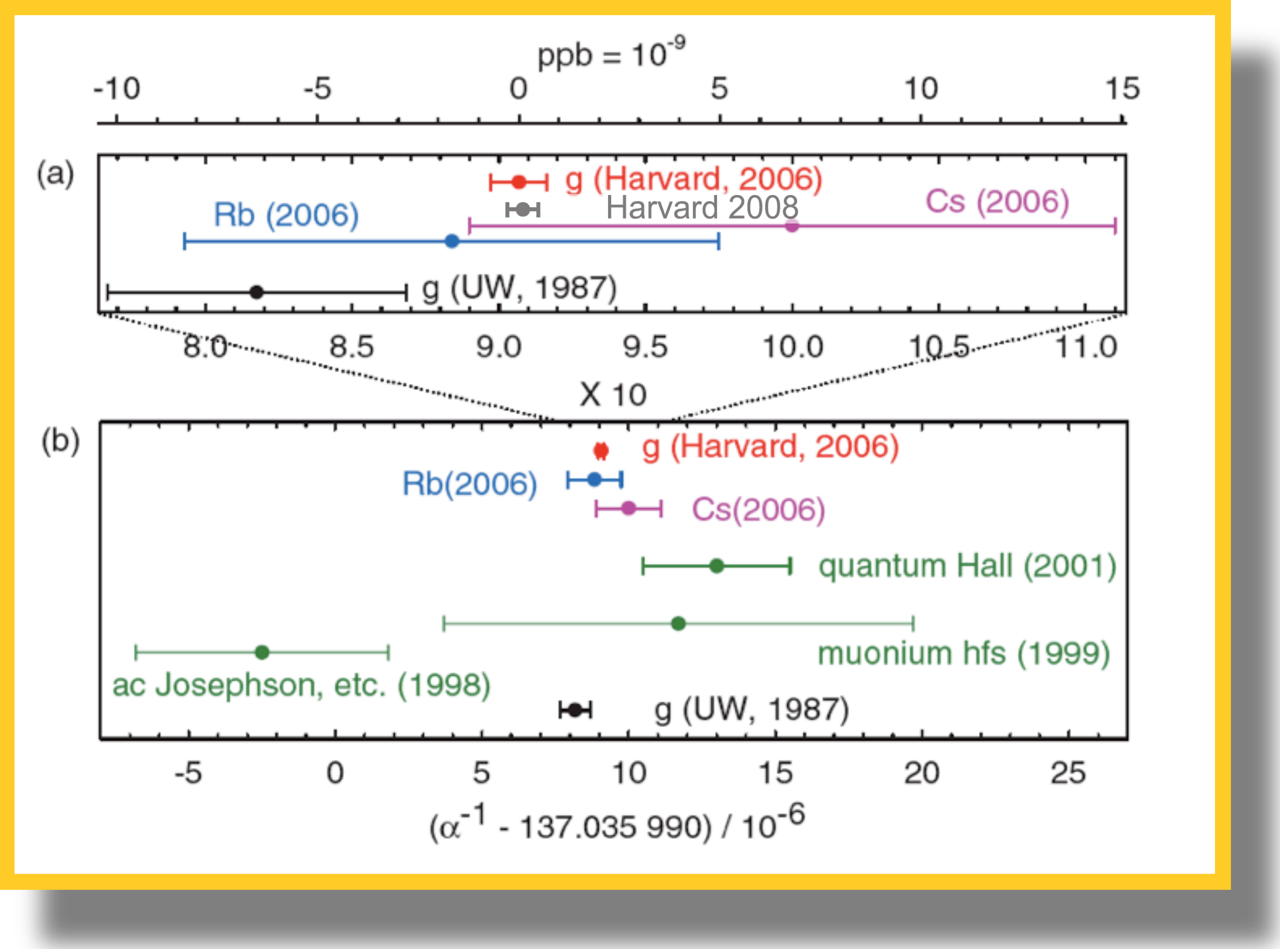
δC_4^{qed} δC_5^{qed} δa_e^{had} δa_e^{exp} (smaller than th!)

- Compare it with other determinations (independent of a_e):

$\alpha^{-1} = 137.036\,000\,00$	(110)	[7.7 ppb]	PRA73 (2006) 032504 (Cs)
$\alpha^{-1} = 137.035\,998\,78$	(91)	[6.7 ppb]	PRL96 (2006) 033001 (Rb)
$\alpha^{-1} = 137.035\,999\,45$	(62)	[4.6 ppb]	PRL101 (2008) 230801 (Rb)

Excellent agreement → beautiful test of QED at 4-loop level!

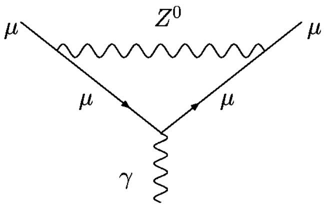
Old and new determinations of alpha

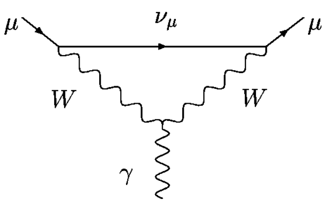


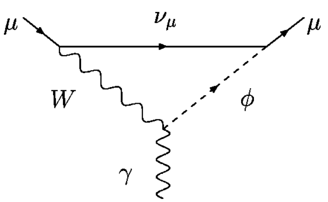
Gabrielse, Hanneke, Kinoshita, Nio & Odom, PRL99 (2007) 039902
Hanneke, Fogwell & Gabrielse, PRL100 (2008) 120801

a_μ^{SM}: the Electroweak contribution

- **One-loop term:**







$$a_\mu^{\rm EW}(\text{1-loop}) = \frac{5G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1+\frac{1}{5}\left(1-4\sin^2\theta_W\right)^2 + O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right)\right] \approx 195\times10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda;

Studenikin et al. '80s

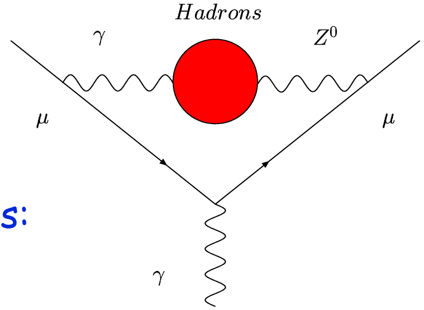
- **One-loop plus higher-order terms:**

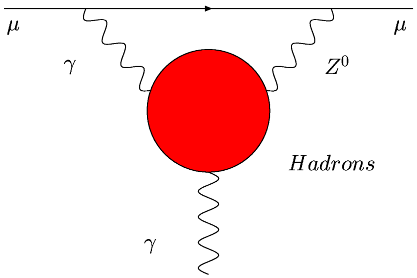
$a_\mu^{EW} = 154\ (2)\ (1) \times 10^{-11}$

Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano, Vainshtein '02; Degrassi, Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk, Czarnecki '05; Vainshtein '03.

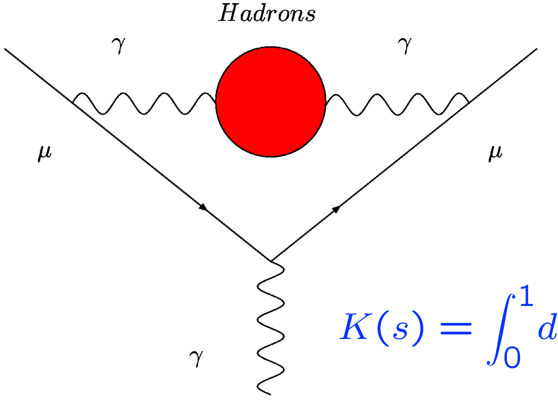
Higgs mass variation, M_{top} error, 3-loop nonleading logs

Hadronic loop uncertainties:





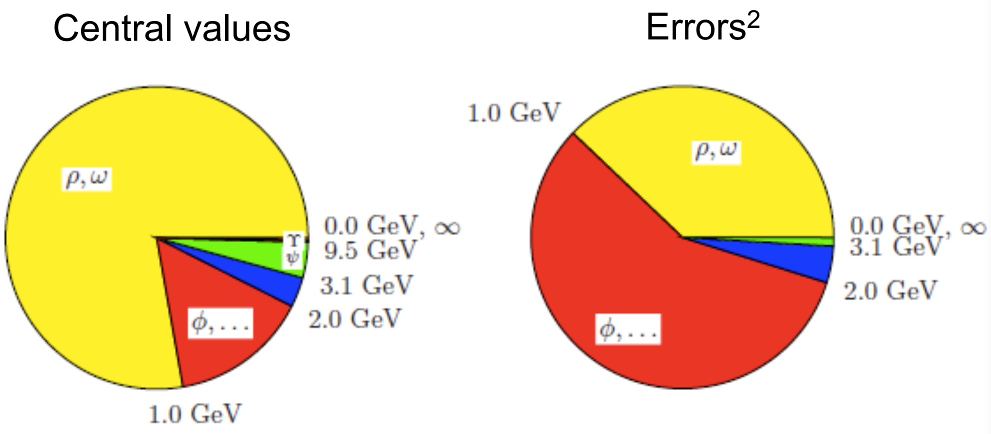
a_μ^{SM}: the hadronic leading-order (HLO) contribution



$$a_\mu^{HLO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} \frac{ds}{s} K(s)\, R(s)$$

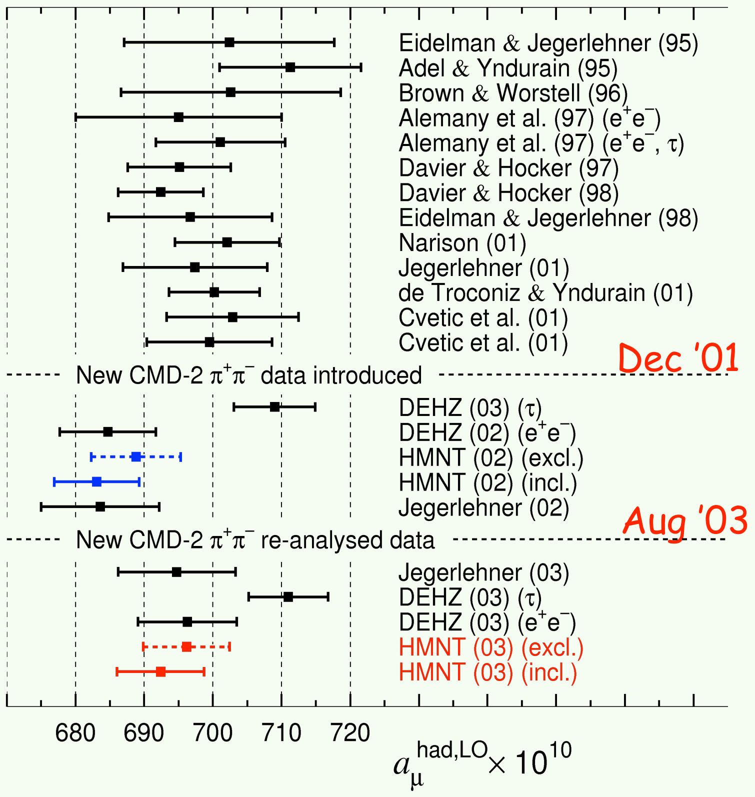
$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)s/m_\mu^2}$$

Bouchiat & Michel 1961; Gourdin & de Rafael 1969


Central values

Errors^2

F. Jegerlehner and A. Nyffeler, Phys. Rept. 477 (2009) 1



Hagiwara et al., PRD 69 (2004) 093003

a_μ^{SM}: the hadronic leading-order (HLO) contribution (II)

$a_\mu^{HLO} = 6909\,(39)_{exp}\,(19)_{rad}\,(7)_{qcd} \times 10^{-11}$ S. Eidelman, ICHEP06; M. Davier, TAU06

$= 6894\,(42)_{exp}\,(18)_{rad} \times 10^{-11}$ Hagiwara, Martin, Nomura, Teubner, PLB649(2007)173

$= 6903\,(53)_{tot} \times 10^{-11}$ F. Jegerlehner, A. Nyffeler, Phys. Rept. 477 (2009) 1

$= 6955\,(40)_{exp}\,(7)_{qcd} \times 10^{-11}$ Davier et al, arXiv:0908.4300 (includes new BaBar 2π)

$= 6894\,(36)_{exp}\,(18)_{rad} \times 10^{-11}$ HLMNT09, arXiv:1001.5401 [hep-ph]

- Radiative Corrections (**Luminosity, Initial-State Radiation, Vacuum Polarization, Final-State Radiation) are a very delicate issue! All under control? Dedicated effort by large group of theorists and experimentalists: ongoing WG on RC and MC tools for hadronic cross sections at low energy ("Radio MontecarLow")** S.Actis et al, arXiv:0912.0749.

a_μ^{SM}: the HLO contribution: e^+e^- data

- The overall agreement of all e^+e^- data looks reasonably good, but discrepancies exists.
- **CMD2:** The 1998 $\pi^+\pi^-$ data in the ρ energy range, published in 2007, agree with their earlier 1995 ones.
- **SND's** $\pi^+\pi^-$ 2006 data analysis in agreement with CMD2.
- **Initial State Radiation (ISR) Method.** Collider operates at fixed energy but s_π can vary continuously. Important independent method made possible by strong th & exp interplay.
- **KLOE**: The "Small angle" (2008) and "large angle" (2009) $\pi^+\pi^-$ independent analyses are in agreement in the overlap region. More results coming (muon normalization).
- Reasonable agreement between **KLOE** and **CMD2-SND** (especially below the ρ). The contributions to a_μ^{HLO} agree.
- **BaBar:** 2009 $\pi^+\pi^-$ results (from 0.5 to 3 GeV) now published. Discrepancies with other experiments, mainly with **KLOE**.

a_μ^{SM}: the HLO contribution: Tau-decay data

- A long-standing problem: The τ data of ALEPH and CLEO are significantly higher than the CMD2-SND-KLOE ones, particularly above the ρ.
- The 2008 $a_\mu^{\pi\pi}$ τ result of Belle agrees with Aleph-Cleo-Opal (some deviations from Aleph's spectral functions).
- June 2009: Davier et al, arXiv:0906.5443v3

$$\boxed{a_\mu^{HLO} = 7053\ (45) \times 10^{-11}}$$

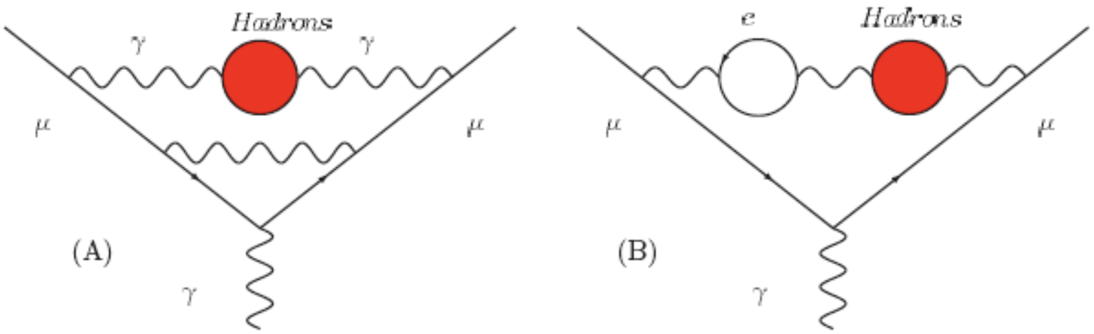
Belle's data included + IB corrections revisited & updated

(Marciano & Sirlin '88; Cirigliano, Ecker, Neufeld '01-'02, Flores-Baez et al. '06 &' 07)

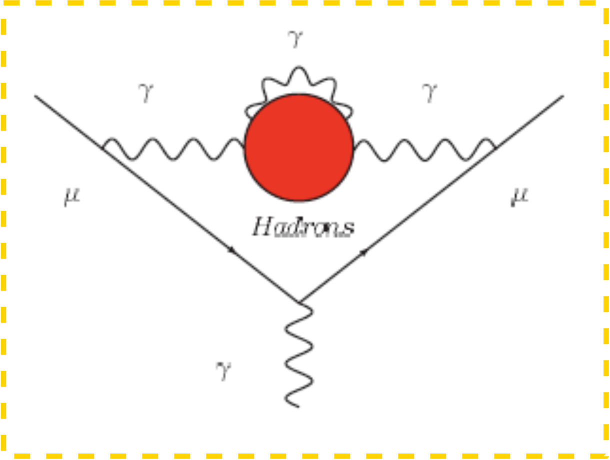
- The discrepancy with e^+e^- data is smaller, but it's still there! Inconsistencies in e^+e^- or τ data? All possible isospin-breaking (IB) effects taken into account? Recent claims that e^+e^- & τ data are consistent after IB effects & vector meson mixings considered (Benayoun et al.'07 & '09).

a_μ^{SM}: the hadronic higher-order (HHO) contributions - VP

HHO: Vacuum Polarization



Already included in a_μ^{HLO}



O(α^3) contributions of diagrams containing hadronic vacuum polarization insertions:

$$\boxed{a_\mu^{HHO}(vp) = -98\ (1) \times 10^{-11}}$$

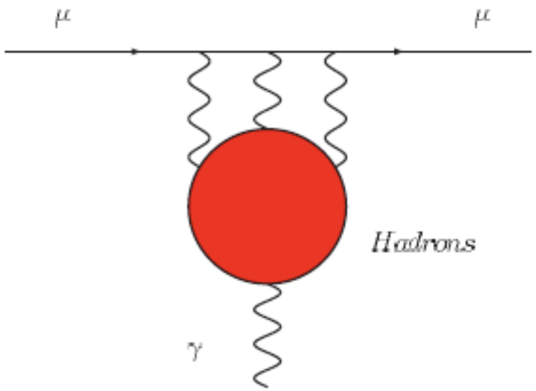
Krause '96, Alemany et al. '98, Hagiwara et al. '03, '06 & '10

Shifts by ~ -3×10^{-11} if τ data are used instead of the e^+e^- ones

Davier & Marciano '04.

a_μ^{SM}: the hadronic higher-order (HHO) contributions - LBL

HHO: Light-by-light contribution



- Unlike the HLO term, for the hadronic l-b-l term we must rely on theoretical approaches.
- This term had a troubled life! Its recent determinations vary between:

$a_\mu^{HHO}(lbl)$ = + 80 (40) × 10^{-11} Knecht & Nyffeler '02

$a_\mu^{HHO}(lbl)$ = +136 (25) × 10^{-11} Melnikov & Vainshtein '03

$a_\mu^{HHO}(lbl)$ = +105 (26) × 10^{-11} Prades, de Rafael, Vainshtein '09

$a_\mu^{HHO}(lbl)$ = +116 (39) × 10^{-11} Jegerlehner & Nyffeler '09

(results based also on Hayakawa, Kinoshita '98 & '02; Bijnens, Pallante, Prades '96 & '02)

- Upper bound: $a_\mu^{HHO}(lbl) < \sim 160 \times 10^{-11}$ Erler & Sanchez 2006, Pivovarov 2002
- Lattice? Very hard, but in progress: Rakow et al, Hayakawa et al., ...
- It's likely to become the ultimate limitation of the SM prediction.

The muon g-2: Standard Model vs. Experiment

Adding up all the above contribution we get the following SM predictions for a_μ and comparisons with the measured value:

a_μ^{EXP} = 116592089 (63) × 10^{-11}

E821 – Final Report: PRD73 (2006) 072
with latest value of $\lambda=\mu_\mu/\mu_p$ (CODATA'06)

	$a_\mu^{\rm SM} \times 10^{11}$	$(\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM}) \times 10^{11}$	σ
[1]	116 591 773 (53)	316 (82)	3.8
[2]	116 591 782 (59)	307 (86)	3.6
[3]	116 591 834 (49)	255 (80)	3.2
[4]	116 591 773 (48)	316 (79)	4.0
[5]	116 591 929 (52)	160 (82)	2.0

with a_μ^{HHO}(lbl) = 105 (26) × 10^{-11}

[1] HMNT06, PLB649 (2007) 173
[2] F. Jegerlehner and A. Nyffeler, Phys. Rept. 477 (2009) 1
[3] Davier et al, arXiv:0908.4300 August 2009 (includes BaBar)
[4] HLMNT09: Hagiwara, Liao, Martin, Nomura, Teubner, arXiv:1001.5401
[5] Davier et al, arXiv:0906.5443v3 August 2009 (τ data).

Note that the th. error is now about the same as the exp. one

The effective fine-structure constant at the scale M_Z and the bounds on the Higgs mass

The Hadronic Contribution to $\alpha(M_Z^2)$ ...

The effective fine-structure constant at the scale M_Z is given by:

$$\alpha(M_Z) = \frac{\alpha}{1-\Delta\alpha(M_Z)} \qquad \text{with} \qquad \Delta\alpha = \Delta\alpha_{\rm lep} + \Delta\alpha^{(5)}_{\rm had} + \Delta\alpha_{\rm top}$$

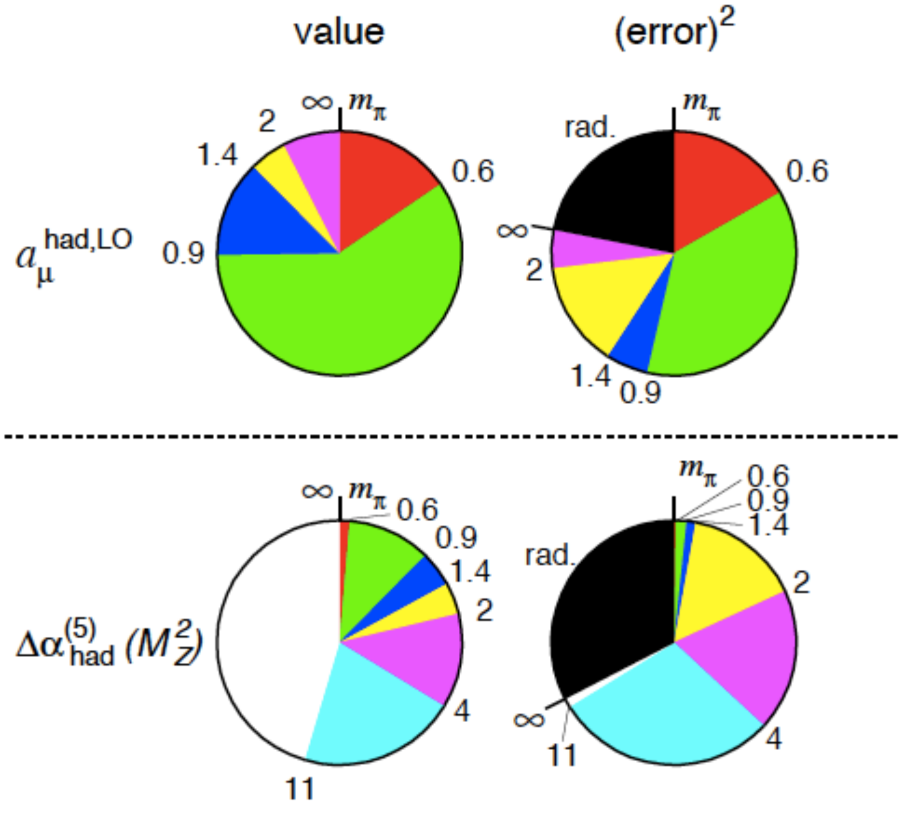
The light quarks part is given by:

$$\Delta\alpha^{(5)}_{\rm had}(M_Z) = \frac{M_Z^2}{4\alpha\pi^2}\, P\int_{4m_\pi^2}^{\infty} ds\, \frac{\sigma(s)}{M_Z^2 - s}$$

Progress due to significant data improvement (in particular BES):

$\Delta\alpha_{had}^{(5)}(M_z^2)$ =

0.02800 (70)	Eidelman, Jegerlehner'95
0.02775 (17)	Kuhn, Steinhauser 1998
0.02749 (12)	Troconiz, Yndurain 2005
0.02758 (35)	Burkhardt, Pietrzyk 2005
0.02768 (22)	HMNT 2006
0.02761 (23)	F. Jegerlehner 2008
0.02760 (15)	HLMNT 2009



Teubner at PHIPSI 09, Beijing, October 2009

… and the EW Bounds on the SM Higgs mass

- The dependence of SM predictions on the Higgs mass, via loops, provides a powerful tool to set bounds on its value.
- Comparing the theoretical predictions of M_W and $\sin^2\theta_{\rm eff}^{\rm lept}$
  [convenient formulae in terms of M_H, M_{top}, $\Delta\alpha_{had}^{(5)}(M_Z)$ and $\alpha_s(M_Z)$ by Degrassi, Gambino, MP, Sirlin '98; Degrassi, Gambino '00; Ferroglia, Ossola, MP, Sirlin '02; Awramik, Czakon, Freitas, Weiglein '04 & '06]

with $M_W = 80.399\ (23)$ GeV [LEP+Tevatron, Aug' 09]

$\sin^2\theta_{\rm eff}^{\rm lept} = 0.23153\ (16)$ [LEP+SLC]

and

$\Delta\alpha_{had}^{(5)}(M_Z) = 0.02760\ (15)$ [HLMNT09]

$M_{top} = 173.1\ (1.3)$ GeV [CDF-D0, Mar '09]

$\alpha_s(M_Z) = 0.118\ (2)$ [PDG '08]

we get

$M_H = 96^{+32}_{-25}$ GeV & $M_H < 153$ GeV 95%CL

- The value of $\Delta\alpha_{had}^{(5)}(M_Z)$ is a key input of these EW fits…

The a_μ - M_H connection

How do we explain Δa_μ?

- Δa_μ can be explained in many ways: errors in HHO-LBL, QED, EW, HHO-VP, g-2 EXP, HLO; or New Physics.
- Can Δa_μ be due to hypothetical mistakes in the hadronic $\sigma(s)$?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{had}^{(5)}(M_Z)$.
- Consider:

$$a_\mu^{HLO}: \qquad a = \int_{4m_\pi^2}^{s_u} ds\, f(s)\,\sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^3},\ s_u < M_Z^2,$$

$$\Delta\alpha_{had}^{(5)}: \qquad b = \int_{4m_\pi^2}^{s_u} ds\, g(s)\,\sigma(s), \qquad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)},$$

and the increase

$$\Delta\sigma(s) = \epsilon\sigma(s)$$

(ε>0), in the range:

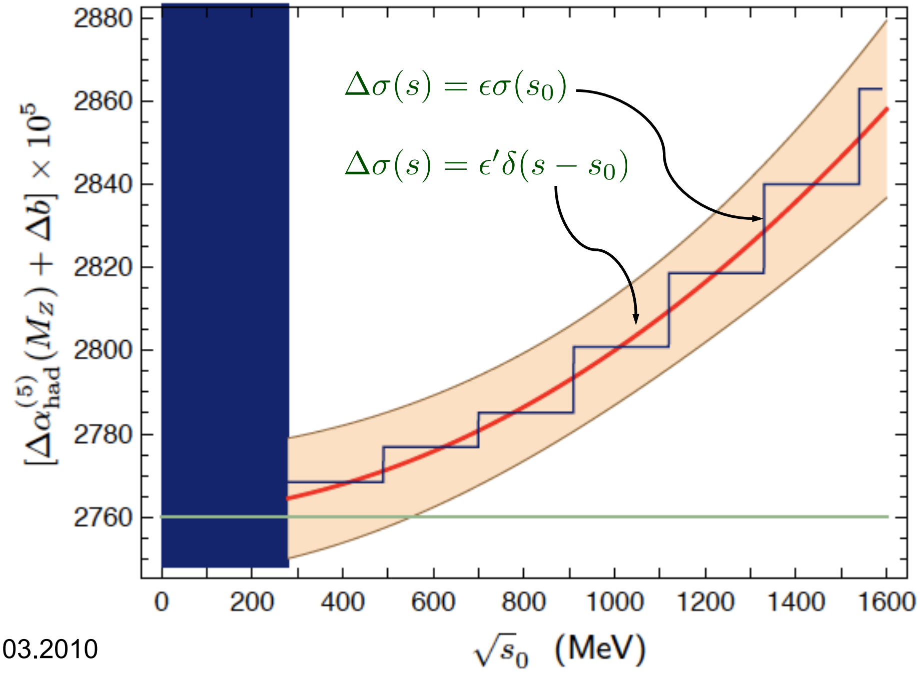
$$\sqrt{s} \in [\sqrt{s}_0 - \delta/2, \sqrt{s}_0 + \delta/2] \qquad \longrightarrow$$

Shifts of a_μ^{HLO} and $\Delta\alpha_{had}^{(5)}(M_Z)$

- If this shift $\Delta\sigma(s)$ in $[\sqrt{s_0}-\delta/2, \sqrt{s_0}+\delta/2]$ is adjusted to bridge the g-2 discrepancy, the value of $\Delta\alpha_{had}^{(5)}(M_Z)$ increases by:

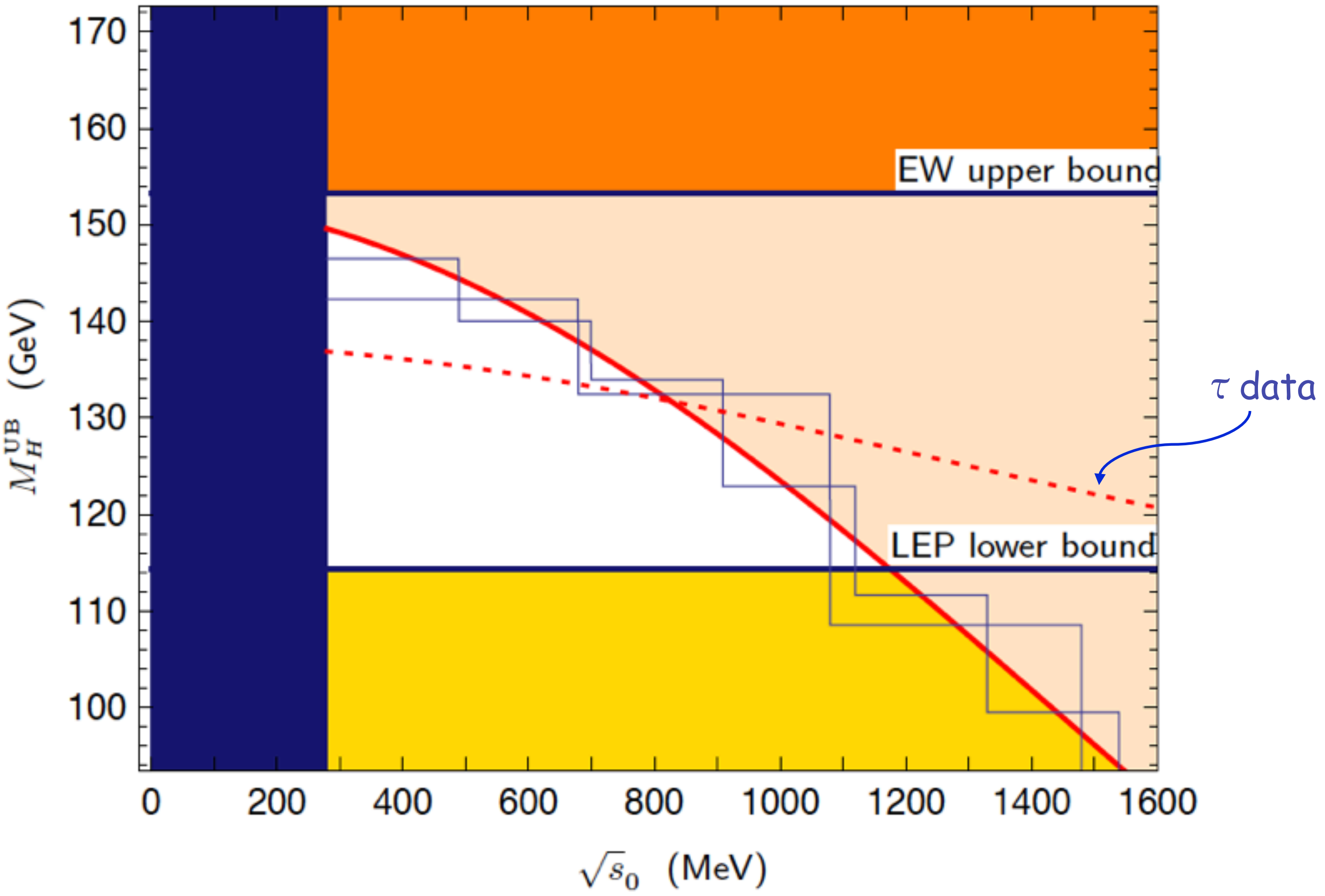
$$\Delta b(\sqrt{s_0},\delta) = \Delta a_\mu \frac{\int_{\sqrt{s_0}-\delta/2}^{\sqrt{s_0}+\delta/2} g(t^2)\,\sigma(t^2)\,t\,dt}{\int_{\sqrt{s_0}-\delta/2}^{\sqrt{s_0}+\delta/2} f(t^2)\,\sigma(t^2)\,t\,dt}$$

- Adding this shift to $\Delta\alpha_{had}^{(5)}(M_Z)$ = 0.02760(15) [HLMNT09] , with Δa_μ= 316(79) x 10^{-11} [HLMNT09], we obtain:



The muon g-2: connection with the SM Higgs mass

- **How much does the M_H upper bound change when we shift $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{had}^{(5)}(M_Z)$ by Δb] to accommodate Δa_μ ?**



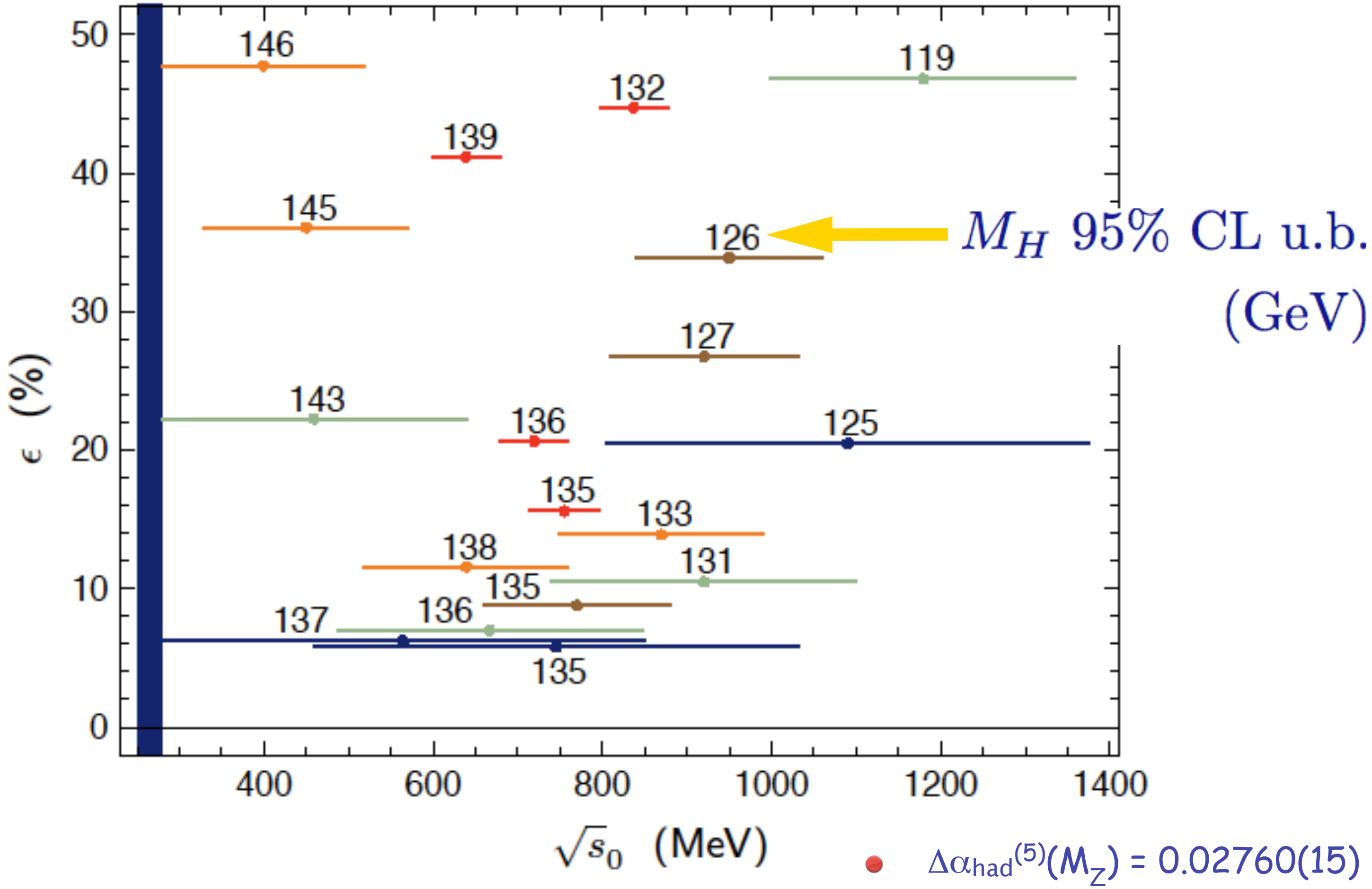
- $\Delta\alpha_{had}^{(5)}(M_Z) = 0.02760(15)$
- $\Delta a_\mu = 316(79) \times 10^{-11}$

The muon g-2: connection with the SM Higgs mass (2)

- The LEP direct-search lower bound is $M_H^{LB} = 114.4$ GeV (95%CL).
- The hypothetical shifts $\Delta\sigma = \varepsilon\sigma(s)$ that bridge the muon g-2 discrepancy conflict with the LEP lower limit when $\sqrt{s_0} >$ ~1.2GeV (for bin widths δ up to several hundreds of MeV).
- While the use of τ data in the calculation of a_μ^{HLO} reduces the muon g-2 discrepancy, it increases $\Delta\alpha_{had}^{(5)}(M_Z)$, lowering the M_H upper bound to ~138GeV: near-conflict with the M_H lower bound if the remaining Δa_μ (~2σ) is bridged by a further increase $\Delta\sigma$.
- In a scenario where τ data agree with e^+e^- ones below ~1GeV after isospin viol. effects & vector meson mixings (Benayoun et al.'07 & '09), we could assume that Δa_μ is bridged by hypothetical errors above ~1GeV. If so, M_H^{UB} falls below M_H^{LB} !!
- Scenarios where Δa_μ is accommodated without affecting M_H^{UB} are possible, but considerably more unlikely.

How realistic are these shifts Δσ(s) ?

- **How realistic are these shifts Δσ(s) when compared with the quoted exp. uncertainties? Study the ratio ε=Δσ(s)/σ(s):**



- $\Delta\alpha_{had}^{(5)}(M_Z) = 0.02760(15)$
- $\Delta a_\mu = 316(79) \times 10^{-11}$.

How realistic are these shifts Δσ(s) ? (2)

- The minimum ε is ~ +5%. It occurs if σ is multiplied by (1+ε) in the whole integration region (!), leading to M_H^{UB} ~ 75 GeV (!!)

- As the quoted exp. uncertainty of σ(s) below 1 GeV is ~ a few per cent (or less), the possibility to explain the muon g-2 with these shifts Δσ(s) appears to be unlikely.

- If, however, we allow variations of σ(s) up to ~6% (7%), M_H^{UB} is reduced to less than ~137 GeV (138 GeV). E.g., the ~6% shift in [0.6, 1.2] GeV, required to fix Δa_μ, lowers M_H^{UB} to 133 GeV. Some tension with the M_H >~120GeV "vacuum stability" bound.

- Reminder: the above M_H upper bounds, like the LEP-EWWG ones, depend on the value of $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$: usual problems. They also depend on M_t & δM_t: we made simple formulae to translate the M_H upper bounds above into new values corresponding to M_t & δM_t inputs different from those employed here.

Conclusions

- Beautiful example of interplay between theory and experiment: g_e probed at <ppt ! → α and extraordinary test of QED's validity; g_μ probed at <ppb ! tests full SM → great to unveil New Physics!

- The discrepancy Δa_μ is more than 3σ if e+e- data are used. With tau data, the deviation is ~2σ. More e^+e^- data & analyses eagerly awaited! QED & EW ready for new g-2 exp! LBL??

- Δa_μ can be due to New Physics, or to problems in a_μ^{SM} (or a_μ^{EXP}!). Can it be due to hypothetical mistakes in the hadronic σ(s)?

- An increase Δσ(s) could bridge Δa_μ, leading however to a decrease on the EW upper bound on the SM Higgs mass M_H .

- We conclude that solving Δa_μ via an increase of σ(s) is unlikely in view of current experimental error estimates. However, if this turns out to be the solution, then the M_H upper bound drops to about 135 GeV which, in conjunction with the LEP 114 GeV lower limit, leaves a narrow window for M_H.

The End