Determining all gas properties in galaxy clusters from the dark matter profile alone

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SUMMARY

1 – BACKGROUND

Observations show that most of the *baryonic* mass in galaxy clusters is in form of *gas*

$$f_g \equiv \frac{M_{\rm gas}}{M_{\rm tot}} \simeq 0.12 - 0.18 \ , \tag{1}$$

more than 1 order of magnitude larger than the stellar mass

$$f_* \equiv \frac{M_*}{M_{\rm tot}} \simeq 0.02 - 0.03 \ . \tag{2}$$

Observations also show that the gas is *hot*

$$1 \cdot 10^7 \, \mathrm{K} < T_g < 1.5 \cdot 10^8 \, \mathrm{K} \ , \tag{3}$$

and so it is *ionized*. Therefore it emits via thermal Bremsstrahlung in the X-ray band – this is just how it is detected – and it is found that its luminosity is

$$6 \cdot 10^{42} \, \mathrm{erg\,s^{-1}} < L_X < 2 \cdot 10^{45} \, \mathrm{erg\,s^{-1}} \ . \tag{4}$$

Note that

$$\langle T_g \rangle \simeq T_{\rm vir} \ . \tag{5}$$

Indeed, for $\langle T_g \rangle > T_{\rm vir}$ the gas would have evaporated while for $\langle T_g \rangle < T_{\rm vir}$ it would have collapsed towards the centre: in either case it would *not* be observed.
I consider throughout only *regular* and *relaxed* clusters. Because this occurs via *violent relaxation*, it means that

$$t_{\rm cluster} > t_{\rm cross} \ . \tag{6}$$

I also assume *spherical symmetry*. Then the gas is in *hydrostatic equilibrium* inside the region

$$r_* < r < r_{\rm vir} \ , \tag{7}$$

with r_* defined by

$$t_{\rm cool}(r_*) = t_{\rm dyn}(r_*) \ , \tag{8}$$

with $t_{\rm cool}(r) \propto r^{3/2}$ and $t_{\rm dyn}(r) \propto (G\rho)^{-1/2}$. Note that this hot gas cannot clump, and so it must be *diffuse* within the cluster potential well.
Hydrostatic equilibrium is formalized by

$$\frac{dP_g(r)}{dr} = -\frac{G\,M_{\rm tot}(r)\rho_g(r)}{r^2}\ . \qquad (9)$$

Assuming further that the gas is a *perfect gas*

$$P_g(r) = n_g(r) k_B T_g(r) \qquad (10)$$

it follows that condition for hydrostatic equilibrium becomes

$$\sigma_g^2(r)\,\left(\frac{d\ln\rho_g(r)}{d\ln r} + \frac{d\ln T_g(r)}{d\ln r}\right) + \frac{G\,M_{\rm tot}(r)}{r} = 0\ . \qquad (11)$$

with the gas 1-dim velocity dispersion defined as

$$\sigma_g^2(r) \equiv \frac{k_B T_g(r)}{\mu m_p}\ . \qquad (12)$$

2 – FROM GAS TO DARK MATTER

Observations of X-ray emission from hot diffuse gas in regular relaxed clusters allow for the determination of the DM properties.

2.1 – DM density profile

Observations yield

- X-ray temperature profile of the gas $T_g(r)$,
- X-ray emissivity profile of the gas $j_X(r)$,
- X-ray luminosity profile of the gas $L_X(r)$.

Note that the deprojection of the data is *unique* because of the supposed spherical symmetry. Assuming Bremsstrahlung emission, the gas number density profile is

$$\rho_g(r) \propto \left(\frac{j_X(r)}{L_X(T_g(r))} \right)^{1/2} \quad . \tag{13}$$

Since $\rho_g(r)$ and $T_g(r)$ are now known, Eq. (11) yields $M_{\rm tot}(r)$. Due to the fact that the *leading* mass component is dark matter (DM), in first approximation we get $M_{\rm DM}(r) = M_{\rm tot}(r)$, so that the DM profile is fixed in a unique way. Higher-order corrections taking the gas mass into account can be computed in a straightforward fashion.

2.2 – DM anisotropy profile

A plausible assumption in that in first approximation the DM distribution is characterized by *complete spherical symmetry*. In such a situation it is described by the following Jeans equation

$$\sigma_r^2(r) \left(\frac{d \ln \rho_{\rm DM}(r)}{d \ln r} + \frac{d \ln \sigma_r^2(r)}{d \ln r} + 2\beta(r) \right) + \frac{G \, M_{\rm tot}(r)}{r} = 0 \ , \tag{14}$$

with $\rho_{\rm DM}(r)$ the DM density profile. Complete spherical symmetry forces the 2 tangential components of the velocity dispersion

$\sigma_\varphi(r)$ and $\sigma_\theta(r)$ of the DM particles to be the same – they are denoted by $\sigma_t(r)$ – but they are *unrelated* to the radial component $\sigma_r(r)$. So the departure to orbital isotropy of DM particles is parameterized by

$$\beta(r) \equiv 1 - \frac{\sigma_t^2(r)}{\sigma_r^2(r)} \ . \tag{15}$$

Correspondingly, I define the mean DM 1-dim velocity dispersion as

$$\sigma_{\rm DM}^2(r) \equiv \frac{1}{3}\Big(\sigma_r^2(r) + 2\,\sigma_t^2(r)\Big) = \left(1 - \frac{2}{3}\beta(r)\right)\sigma_r^2 \ . \tag{16}$$

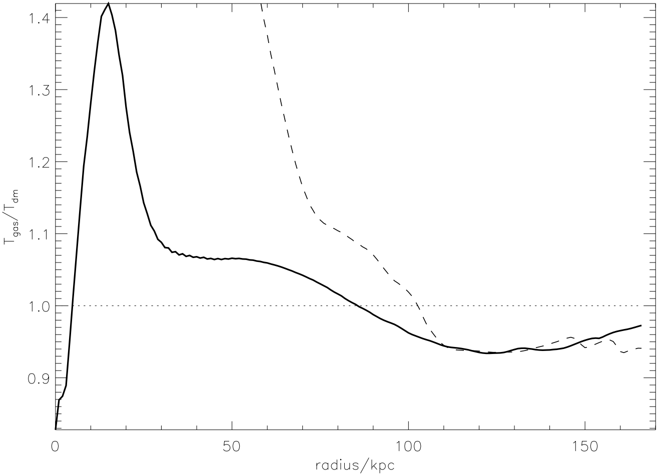
I further define

$$\kappa(r) \equiv \frac{\sigma_{\rm DM}(r)}{\sigma_g(r)} \ . \tag{17}$$

Now, because the cluster equilibrium is achieved through *violent relaxation*, equipartition of the kinetic energy *per unit mass* takes place, thereby strongly suggesting that

$$\sigma_{\rm DM}(r) = \sigma_g(r) \ . \tag{18}$$

Actually, this circumstance is further supported by extended numerical simulations which can be trusted precisely in the region where the gas is expected to be in hydrostatic equilibrium, so that I will take $\kappa(r) = 1$ from now on. Moreover, even the presence of a central AGN does *not* change the situation.



Effect of an AGN feedback.

Thanks to Eq. (18), Eqs. (11) and (14) can be combined to give

$$\sigma_r^2(r)\,\left(\frac{d\ln\rho_{\rm DM}(r)}{d\ln r}+\frac{d\ln\sigma_r^2(r)}{d\ln r}+3\right)=\psi(r)\;, \qquad (19)$$

where I have set

$$\psi(r)\equiv\sigma_g^2(r)\,\left(\frac{d\ln\rho_g(r)}{d\ln r}+\frac{d\ln T_g(r)}{d\ln r}+3\right)\;. \qquad (20)$$

By the previous argument, X-ray observations fix both $\psi(r)$ and $\rho_{\rm tot}(r)$. And recalling that I am working in the approximation $M_{\rm tot}=M_{\rm DM}$, also $\rho_{\rm DM}(r)$ is fixed. So, Eq. (19) can be solved to get

$$\sigma_r^2(r)=\frac{1}{\rho_{\rm DM}(r)\,r^3}\int_0^r\,dr'\,\rho_{\rm DM}(r')\,r'^2\;. \qquad (21)$$

3 – FROM DARK MATTER TO GAS

All observational uncertainties are avoided altogether when the argument is turned around. So, I am now assuming a *given* DM density profile and I work out the resulting gas properties under the same assumptions as above.
Central to this strategy is again the hydrostatic equilibrium condition

$$\sigma_g^2(r)\left(\frac{d\ln\rho_g(r)}{d\ln r}+\frac{d\ln T_g(r)}{d\ln r}\right)+\frac{G\,M_{\rm tot}(r)}{r}=0\ . \qquad (22)$$

Because $M_{\rm tot}(r)$ is given - recall my approximation $M_{\rm tot}(r)=M_{\rm DM}(r)$ – the job is to solve Eq. (11) for both $\rho_g(r)$ and $T_g(r)$. Of course, this can be done only if some *further* information about the state of the gas is available.

3.1 – An incorrect start

Simulations have shown that $\langle T_g \rangle \simeq T_{\rm vir}$, so that specific dependence of $\langle T_g \rangle$ on $r_{\rm vir}$ shows up.
Makino, Sasaki and Suto (MSS) assume that just the same relation holds *locally* as well! So, MSS take

$$T_g(r) = T_{\rm vir}(r) \ , \tag{23}$$

with $T_{\rm vir}(r)$ defined as $T_{\rm vir}(r_{\rm vir})$ with $r_{\rm vir} \to r$. By inserting Eq. (23) into Eq. (22), MSS proceed to evaluate $\rho_g(r)$. Further, MSS restrict themselves to isothermal and polytropic gas distributions. They find – not unexpectedly – a *disagreement* with observations.

3.2 – A correct approach

A very different attitude is taken by Frederiksen, Hansen, Host and Roncadelli.

As a preliminary step, we trivially combine Eqs. (11) and (14).

Our *first* assumption is again the relation

$$\sigma_{\rm DM}(r) = \sigma_g(r) \ . \tag{24}$$

Expressing $\sigma_{\rm DM}(r)$ in terms of $\sigma_r(r)$ and $\beta(r)$ with the help of Eq. (16), we put the resulting equation into the form

$$\gamma_g(r) = \frac{1}{1-\frac{2}{3}\beta(r)}\left(\gamma_{\rm DM}(r) + 2\beta(r) + \frac{2}{3}\beta(r)\frac{d\ln\sigma_r^2(r)}{d\ln r} + \frac{2}{3}\frac{d\beta(r)}{d\ln r}\right) \ , \tag{25}$$

where the density slopes $\gamma_{\rm DM}(r)$ of the DM and $\gamma_g(r)$ of the gas are defined as

$$\gamma_X(r) \equiv \frac{d\ln\rho_X}{d\ln r} \ . \tag{26}$$

I stress that Eq. (25) captures a crucial point of the present strategy: only the gas density slope appears on its l. h. s., whereas only quantities pertaining to the DM appear on its r. h. s. Further, the Jeans equation for DM can be rewritten as

$$r\frac{d\sigma_r^2}{dr} + \sigma_r^2\left(\gamma_{\rm DM}(r) + 2\beta(r)\right) + \frac{G\,M_{\rm tot}(r)}{r} = 0\ , \tag{27}$$

and its solution is

$$\sigma_r^2(r) = \frac{G}{B(r)}\int_r^\infty dr'\,\frac{B(r')\,M_{\rm tot}(r')}{r'^2}\ , \tag{28}$$

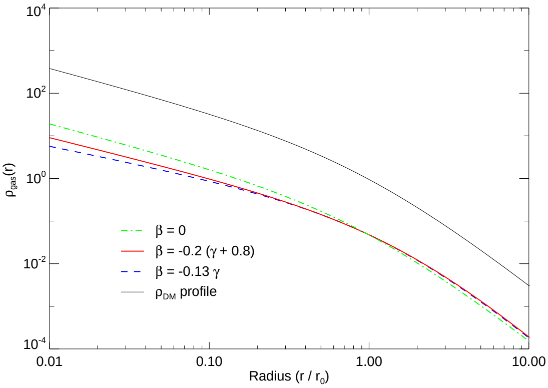
with

$$B(r) \equiv \rho_{\rm DM}(r)\exp\left\{-2\int_r^\infty dr'\,\frac{\beta(r')}{r'}\right\}. \tag{29}$$

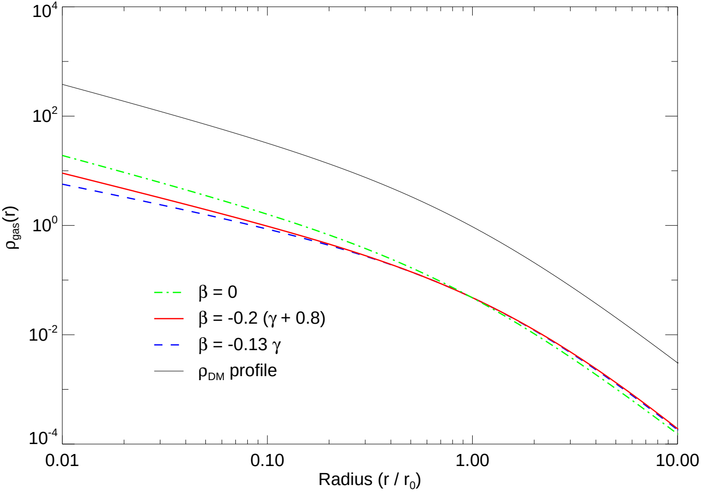
Our *second* assumption is the relation

$$\beta(r) \propto \gamma_{\rm DM}(r)\ , \tag{30}$$

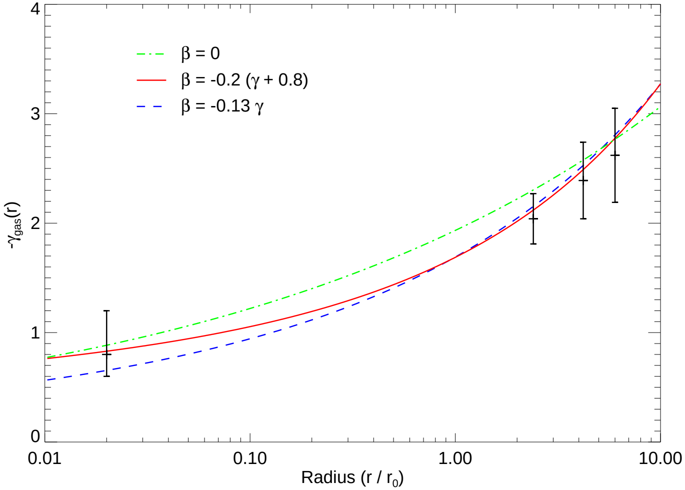
which emerges from computer simulations with a scatter of about 0.05. Using this relation we finally obtain the gas density profile.



The derived gas density profile, assuming that $\rho_g/\rho_{\rm DM} = 10\%$ at r_0, which is the scale length of the NFW profile. The upper curve (black) is the DM density, and the 3 lower lines show gas profiles modelled with extreme variations in the possible DM velocity anisotropy (green dot-dashed is isotropic ($\beta = 0$), red solid is using $\beta = -0.2\,(\gamma + 0.8)$ and blue dashed is using $\beta = -0.13\,\gamma$.



The derived slope of the gas density profile, assuming an NFW profile for the DM. Inner points are taken from Vikhlinin et al. 2006 while outer points are taken from Ettori and Balestra 2008.



The slope of the gas density profile, assuming a Sersic profile with $n = 5$ for the DM. Inner points are taken from Vikhlinin et al. 2006 while outer points are taken from Ettori and Balestra 2008.