Single cut spinor integration and tadpole coefficients

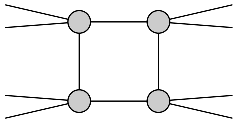
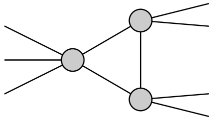
Edoardo Mirabella

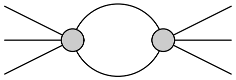
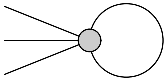
In collaboration with R. Britto

Outline

- Introduction
- Double cut & Tadpoles
- Single cut & Tadpoles
- Conclusions

Introduction

$\mathcal{A}_{\text{1-loop}} = \sum_i$  $d_i + \sum_j$  c_j

$\sum_k$  $b_k + \sum_m$  $a_m + \mathcal{R}$

Computation of the coefficients $\&$ rational part

- Passarino-Veltman reduction, [Passarino, Veltman ’79]
- singular limits of $\mathcal{A}_{\text{1-loop}}$ + on-shell formalism, [Bern, Dixon, Dunbar, Kosower ’94,’95]
  - – Generalized unitarity multiple-cuts

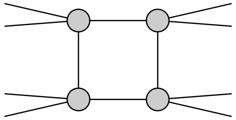
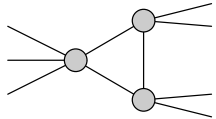
    [Bern, Dixon, Kosower ’98] [Bern *et al.* ’05][Britto, Cachazo, Feng ’05] [Mastrolia, ’07] [Forde’ 07] [Kilgore ’08] [Ellis, Giele, Kunszt ’08] [Badger ’09] [Ellis *et al.* ’08] [Berger *et al.* ’08] [Ossola, Papadopoulos, Pittau ’07]
  - – Double cut $\&$ spinor integration

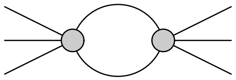
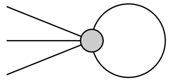
    [Britto *et al.* ’07] [Britto, Feng, Mastrolia ’06][Anastasiou *et al.* ’07,’07]

    $\rightsquigarrow$ closed formluae for d’ s c’ s and b’ s

    $\rightsquigarrow$ cannot compute a’ s

Introduction

$\mathcal{A}_{\text{1-loop}} = \sum_i$  $d_i + \sum_j$  c_j

$\sum_k$  $b_k + \sum_m$  $a_m + \mathcal{R}$

This talk: tadpole coefficients using single cut & spinor integration

- Tricky business ...
  - – n-leg $\mathcal{A}_{\text{1-loop}} \leftrightarrow (n+2)$-leg tree level amplitude,
  - – tree-level recursion relations break down, [Hall '07][Schwinn, Weinzierl '07]
  - – gauge dependence arises, [Ellis *et al.* '08]
  - – connection tadpoles & massless bubbles.
- ... But promising!
  - – single cut to fully reconstruct $\mathcal{A}_{\text{1-loop}}$. [Catani *et al.* '08] [Glover, Williams '08]

Alternative approach: find a' s from the UV & IR structure of $\mathcal{A}_{\text{1-loop}}$.

[Bern, Morgan '96] [Badger '08][Badger, Sattler, Yundin '10]

Auxiliary propagator method

Tadpole coefficients from the double cut

Auxiliary propagator method

$$\mathcal{I}_{2,2} = \frac{(2R_1 \cdot k)(2R_2 \cdot k)}{D_0 D_1}, \qquad \left[\, D_0 = k^2 - m_0^2,\; D_1 = (k - K_1)^2 - m_1^2,\; f_1 = K_1^2 + m_0^2 - m_1^2 \,\right]$$

Auxiliary propagator method

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We want $a(0)$, the coefficient of $A_0(m_0^2)$.

General idea: [Britto, Feng '09] 1) apply the OPP decomposition,

$$\mathcal{I}_{2,2} = \frac{a(0)}{D_0} + \frac{\tilde{a}(q;\,0)}{D_0} + \frac{b(0,1)}{D_0 D_1} + \frac{\tilde{b}(q;\,0,1)}{D_0 D_1} + \dots$$

Auxiliary propagator method

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K is arbitrary, ($D_K = (k-K)^2 - m_K^2$)

$$\mathcal{I}_{2,2} \frac{1}{D_K} = \frac{a(0)}{D_0 D_K} + \frac{\tilde{a}(q;\,0)}{D_0 D_K} + \frac{b(0,1)}{D_0 D_1 D_K} + \frac{\tilde{b}(q;\,0,1)}{D_0 D_1 D_K} + \ldots$$

$$\mathcal{I}_{2,2} \frac{1}{D_K} = \frac{a_K(0)}{D_0} + \frac{b_K(0,K)}{D_0 D_K} + \frac{c_K(0,1,K)}{D_0 D_1 D_K} + \ldots$$

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K is arbitrary, ($D_K = (k-K)^2 - m_K^2$)

2) apply the double cut,

$$\begin{aligned}
\Delta_2\left[\mathcal{I}_{2,2}\frac{1}{D_K}\right] &= \Delta_2\left[\frac{a(0)}{D_0 D_K}\right] + \Delta_2\left[\frac{\tilde{a}(q;\,0)}{D_0 D_K}\right] + \Delta_2\left[\frac{b(0,1)}{D_0 D_1 D_K}\right] \\
&+ \Delta_2\left[\frac{\tilde{b}(q;\,0,1)}{D_0 D_1 D_K}\right] + \dots
\end{aligned}$$

$$\Delta_2\left[\mathcal{I}_{2,2}\frac{1}{D_K}\right] = \Delta_2\left[\frac{a_K(0)}{D_0}\right] + \Delta_2\left[\frac{b_K(0,K)}{D_0 D_K}\right] + \Delta_2\left[\frac{c_K(0,1,K)}{D_0 D_1 D_K}\right] + \dots$$

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   K is arbitrary, ($D_K = (k - K)^2 - m_K^2$)
2) apply the double cut,
3) apply the decoupling conditions:
   $K \cdot K_1 = 0, \qquad K^2 + m_K^2 - m_0^2 = 0.$

$$\Delta_2 \left[\mathcal{I}_{2,2} \frac{1}{D_K} \right] = \Delta_2 \left[\frac{a(0)}{D_0 D_K} \right] + \Delta_2 \left[\frac{\tilde{a}(q;\, 0)}{D_0 D_K} \right] + \Delta_2 \left[\frac{b(0,1)}{D_0 D_1 D_K} \right] + \frac{f_1}{12} \Delta_2 \left[\frac{\tilde{b}_{00}(0,1)}{D_0 D_K} \right] + \dots$$

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$$a(0) = b_K(0,K) - \frac{f_1}{12}\,\tilde{b}_{00}(0,1)$$

$$\tilde{b}_{00}(0,1) = \frac{3}{K_1^2}\left[b(0,1)_{|\mu^2} - c_K(0,1,K)_{|\mu^2}\right]$$

$$\left\{ \begin{array}{l} b(0,1)_{|\mu^2} \\ c_K(0,1,K)_{|\mu^2} \end{array} \right. \quad \text{from} \quad \left\{ \begin{array}{l} \mathcal{I}_{2,2} \\ \mathcal{I}_{2,2}/D_K \end{array} \right. \quad \text{using double cut in D-dimensions.}$$

Auxiliary propagator method

$$\mathcal{I}_{2,2} = \frac{(2R_1 \cdot k)(2R_2 \cdot k)}{D_0 D_1}, \qquad \left[\; D_0 = k^2 - m_0^2,\;\; D_1 = (k - K_1)^2 - m_1^2,\;\; f_1 = K_1^2 + m_0^2 - m_1^2 \;\right]$$

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$$a(0) = \frac{f_1}{3(K_1^2)^2} \left[K_1^2\, (R_1 \cdot R_2) - 4 (K_1 \cdot R_1)(K_1 \cdot R_2) \right]$$

[OK with PV]

Auxiliary propagator method

$$\mathcal{I}_{2,2} = \frac{(2R_1 \cdot k)(2R_2 \cdot k)}{D_0 D_1}, \qquad \left[\ D_0 = k^2 - m_0^2,\ D_1 = (k-K_1)^2 - m_1^2,\ f_1 = K_1^2 + m_0^2 - m_1^2\ \right]$$

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- The auxiliary propagator method:
  - is valid in general,
  - requires few spurious terms,
    - $\rightsquigarrow$ traded with non-spurious ones . . .
    - $\rightsquigarrow$ . . . double cut in D-dimension sufficient
  - introduces extra D_K,
    - $\rightsquigarrow$ computation is harder
  - assumes $K_1^2 \neq 0$.

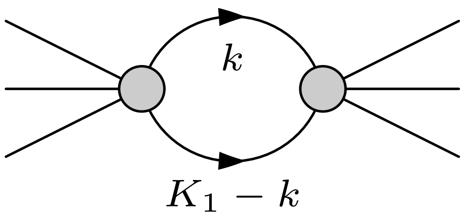
Evaluation of the double cut

How to get the double cut

Evaluation of the double cut

$$\Delta_2 [\mathcal{I}] \equiv \int d^4k \; [\mathcal{I}\, D_0\, D_1] \; \delta^+ (D_0) \; \delta^+ (D_1)$$

Definitions $D_0 = k^2 - m_0^2;\;\; D_1 = (k - K_1)^2 - m_1^2;$



Evaluation of the double cut

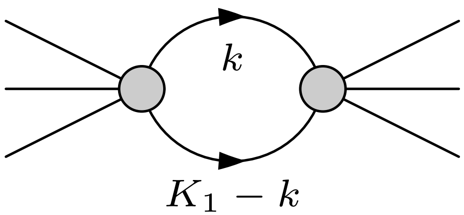
$$\Delta_2\left[\mathcal{I}\right] = \int dt \int dz d\bar{z}\, \frac{\left[\mathcal{I}\, D_0\, D_1\right]}{(1+z\bar{z})^2}$$

Definitions $D_0 = k^2 - m_0^2; \quad D_1 = (k-K_1)^2 - m_1^2;$

1) Reparametrizations

i) $k = \tilde{\ell} + \xi K_1$ [Anastasiou *et al.* '07]

ii) $\tilde{\ell}^\mu = t(p^\mu + \alpha q^\mu + z\epsilon_1^\mu + \bar{z}\epsilon_2^\mu)$ [Mastrolia '09]

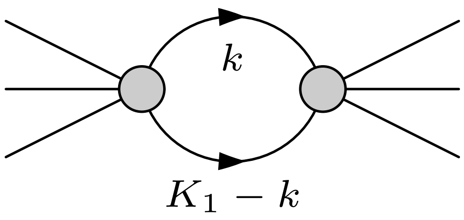


Evaluation of the double cut

$$\Delta_2[\mathcal{I}] = \int dt \left\{ \lim_{\Lambda\to\infty} \frac{1}{2} \int_0^{2\pi} d\alpha \, \Lambda e^{i\alpha} \mathcal{G}(\Lambda e^{i\alpha}, \Lambda e^{-i\alpha}) - \pi \sum_{\text{poles } z_j} \text{Res}\{\mathcal{G}(z,\bar{z}), z_j\} \right\}$$

Definitions $D_0 = k^2 - m_0^2; \quad D_1 = (k-K_1)^2 - m_1^2; \quad \mathcal{G}(z,\bar{z}) = \int d\bar{z} \, \frac{[\mathcal{I} D_0 D_1]}{(1+z\bar{z})^2}$

1) Reparametrizations



i) $k = \tilde{\ell} + \xi K_1$ [Anastasiou *et al.* '07]

ii) $\tilde{\ell}^\mu = t(p^\mu + \alpha q^\mu + z\epsilon_1^\mu + \bar{z}\epsilon_2^\mu)$ [Mastrolia '09]

2) Generalized Cauchy formula

i) $z = \Lambda e^{i\alpha} \qquad \bar{z} = \Lambda e^{-i\alpha}$

ii) the double cut given by the residues of $\mathcal{G}$

Evaluation of the single cut

What about the single cuts ?

Evaluation of the single cut

$$\Delta_1 \left[\mathcal{I} \right] \equiv \int d^4k \; \left[\mathcal{I} \, D_0 \right] \; \delta^+ \left(D_0 \right)$$

Definitions $D_0 = k^2 - m_0^2;$

Evaluation of the single cut

$$\Delta_1\,[\mathcal{I}] = \int dt \left\{ \lim_{\Lambda\to\infty} \frac{1}{2} \int_0^{2\pi} d\alpha\; \Lambda e^{i\alpha} \mathcal{G}(\Lambda e^{i\alpha}, \Lambda e^{-i\alpha}) - \pi \sum_{\text{poles } z_j} \text{Res}\{\mathcal{G}(z,\bar{z}), z_j\} \right\}$$

Definitions $D_0 = k^2 - m_0^2; \quad \mathcal{G}(z,\bar{z}) = \frac{2K^2 t}{4} \int d\bar{z}\,[\mathcal{I}\,D_0]\,;$

1) Reparametrizations

i) $k = \tilde{\ell} + \xi K$ (K arbitrary)

ii) $\tilde{\ell}^\mu = t(p^\mu + \alpha q^\mu + z\epsilon_1^\mu + \bar{z}\epsilon_2^\mu)$ [Mastrolia '09]

iii) let $K^2 \to \infty$

2) Generalized Cauchy formula

i) $z = \Lambda e^{i\alpha} \quad \bar{z} = \Lambda e^{-i\alpha}$

Evaluation of the single cut

$$\Delta_1\left[\frac{1}{D_0}\right]=\int dt\left\{\lim_{\Lambda\to\infty}\frac{1}{2}\int_0^{2\pi}d\alpha\,\frac{2K^2t}{4}\Lambda^2-\pi\sum_{\text{poles } z_j}\text{Res}\left\{\frac{2K^2t}{4}\bar{z},z_j\right\}\right\}$$

Definitions $D_0=k^2-m_0^2;\quad \mathcal{G}(z,\bar{z})=\frac{2K^2t}{4}\bar{z};$

Tadpole integral $\int d^4k\frac{1}{D_0}\longrightarrow\mathcal{I}=\frac{1}{D_0}$

Evaluation of the single cut

$$\Delta_1\left[\frac{1}{D_0}\right] = \lim_{\Lambda\to\infty} \frac{\pi K^2}{2}\Lambda^2\left(\int dt\ t\right)$$

Definitions $D_0 = k^2 - m_0^2; \quad \mathcal{G}(z,\bar{z}) = \frac{2K^2 t}{4}\bar{z};$

Tadpole integral $\int d^4k \frac{1}{D_0} \longrightarrow \mathcal{I} = \frac{1}{D_0}$

- Single cuts of a tadpole $\sim \Lambda^2$
- bubble, triangle, box, behave differently

  e.g. for a bubble $\Delta_1[1/(D_0 D_1)] \sim \log(\alpha\Lambda^2)$
- Strategy

  Find the tadpole coefficient of $\int \mathcal{I}$ *selecting the* $\Lambda^2 t-$*terms of* $\Delta_1[\mathcal{I}]$
- Caveat: spurious terms enter!

  i.e. $\int \mathcal{I} = 0 \not\Rightarrow \Delta_1[\mathcal{I}] = 0$

Single cut & Tadpoles – I

Tadpole coefficients, (next-to-) simplest example

Single cut & Tadpoles – I

$$\mathcal{I}_{2,1} = \frac{(2R \cdot k)}{D_0 D_1}, \qquad \left[\; D_0 = k^2 - m_0^2,\;\; D_1 = (k - K_1)^2 - m_1^2,\;\; f_1 = K_1^2 + m_0^2 - m_1^2 \;\right]$$

We want a(0), the coefficient of $A_0(m_0^2)$.

1) OPP decomposition ($K_1 \perp$ to n, ℓ_7, ℓ_8)

$$\mathcal{I}_{2,1} \;=\; \frac{1}{D_0} \quad a(0) + \qquad \cdots$$

Single cut & Tadpoles – I

$$\mathcal{I}_{2,1} = \frac{(2R \cdot k)}{D_0 D_1}, \qquad \left[\, D_0 = k^2 - m_0^2,\;\; D_1 = (k - K_1)^2 - m_1^2,\;\; f_1 = K_1^2 + m_0^2 - m_1^2 \,\right]$$

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2) Take the $\Lambda^2\, t$ part of the cut (recall the strategy!)

$$\Delta_1 \left[\mathcal{I}_{2,1}\right] = \;\Delta_1 \left[\frac{1}{D_0}\right]\; a(0) + \;\Delta_1 \left[\cdots\right]$$

Single cut & Tadpoles – I

$$\mathcal{I}_{2,1} = \frac{(2R \cdot k)}{D_0 D_1}, \qquad \left[\, D_0 = k^2 - m_0^2,\;\; D_1 = (k - K_1)^2 - m_1^2,\;\; f_1 = K_1^2 + m_0^2 - m_1^2 \,\right]$$

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1) OPP decomposition ($K_1 \perp$ to n, ℓ_7, ℓ_8)

2) Take the $\Lambda^2\, t$ part of the cut (recall the strategy!)

3) Compute the single cut

$$0 = \frac{q^\mu}{(q \cdot K_1)} \left[K_{1\mu} \;\; a(0) \qquad\quad + R_{1\mu} + \;\; (\text{terms} \perp \text{to } K_1^\mu) \right]$$

Single cut & Tadpoles – I

$$\mathcal{I}_{2,1} = \frac{(2R\cdot k)}{D_0 D_1}, \qquad \left[\, D_0 = k^2 - m_0^2,\;\; D_1 = (k-K_1)^2 - m_1^2,\;\; f_1 = K_1^2 + m_0^2 - m_1^2 \,\right]$$

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4) decompose $R^\mu = \alpha_1 K_1^\mu + \alpha_n n^\mu + \alpha_7 \ell_7^\mu + \alpha_8 \ell_8^\mu$

$$0 = \frac{q^\mu}{(q\cdot K_1)}\left[K_{1\mu}\,(a(0)+\alpha_1) \qquad + \;\; (\text{terms} \perp \text{to } K_1^\mu)\right]$$

Single cut & Tadpoles – I

$$\mathcal{I}_{2,1} = \frac{(2R \cdot k)}{D_0 D_1}, \qquad \left[\; D_0 = k^2 - m_0^2,\;\; D_1 = (k - K_1)^2 - m_1^2,\;\; f_1 = K_1^2 + m_0^2 - m_1^2 \;\right]$$

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$$0 = \frac{q^\mu}{(q \cdot K_1)} \left[K_{1\mu} \left(a(0) + \alpha_1 \right) \qquad + \;\; (\text{terms} \perp \text{to } K_1^\mu) \right]$$

$$a(0) = -\alpha_1 = -\frac{2K_1 \cdot R_1}{K_1^2}$$

Single cut & Tadpoles – I

$$\mathcal{I}_{2,2} = \frac{(2R_1\cdot k)(2R_2\cdot k)}{D_0 D_1}, \qquad \left[\, D_0 = k^2 - m_0^2,\;\; D_1 = (k-K_1)^2 - m_1^2,\;\; f_1 = K_1^2 + m_0^2 - m_1^2 \,\right]$$

We want a(0), the coefficient of $A_0(m_0^2)$.

1) OPP decomposition ($K_1 \perp$ to n, ℓ_7, ℓ_8)

2) Take the $\Lambda^2\, t$ part of the cut (recall the strategy!)

3) Compute the single cut

4) decompose $R_1^\mu R_2^\nu = \alpha_1 K_1^\mu K_2^\mu + \alpha_n n^\mu n^\nu + \cdots$

$$0 = \frac{q^\mu q^\nu}{(q\cdot K_1)^2}\left[\left(\alpha_1 + \frac{a(0)}{f_1} + \frac{\tilde{b}_{00}}{3}\right) K_{1\mu}K_{1\nu} - \left(\tilde{b}_{00} - \alpha_n\right) n_\mu n_\nu + \cdots\right]$$

$$\alpha_1 + \frac{a(0)}{f_1} + \frac{\tilde{b}_{00}}{3} = 0, \qquad \tilde{b}_{00} - \alpha_n = 0$$

Single cut & Tadpoles – I

$$\mathcal{I}_{2,2} = \frac{(2R_1 \cdot k)(2R_2 \cdot k)}{D_0 D_1}, \qquad [\ D_0 = k^2 - m_0^2,\ \ D_1 = (k - K_1)^2 - m_1^2,\ \ f_1 = K_1^2 + m_0^2 - m_1^2\ ]$$

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$$a(0) = -f_1 \left(\alpha_1 + \frac{\alpha_n}{3} \right) = \frac{f_1}{3(K_1^2)^2} \left[K_1^2\ (R_1 \cdot R_2) - 4 (K_1 \cdot R_1)(K_1 \cdot R_2) \right] \qquad \text{[OK!]}$$

Single cut & Tadpoles – II

More difficult: massless momentum & degenerate masses

Single cut & Tadpoles – II

$$\mathcal{I}_{2,2} = \frac{(2R_1 \cdot k)(2R_2 \cdot k)}{D_0 D_1}, \qquad \left[\, D_0 = k^2 - m^2, \;\; D_1 = (k - K_1)^2 - m^2, \;\; f_1 = 0 \,\right]$$

We want the coefficient of $A_0(m^2)$, a_{total}

- $K_1^2 = 0$ & $m = m_0 = m_1$, OPP & PV decomposition has to be changed

$$\mathcal{I}_{2,2} = a(0)\frac{1}{D_0} + a(1)\frac{1}{D_1} + b\frac{1}{D_0 D_1} + \hat{b}_{00}\frac{(2k \cdot v)^2}{D_0 D_1} + \text{spurious terms}$$

Single cut & Tadpoles – II

$$\mathcal{I}_{2,2} = \frac{(2R_1 \cdot k)(2R_2 \cdot k)}{D_0 D_1}, \qquad \left[\, D_0 = k^2 - m^2,\;\; D_1 = (k - K_1)^2 - m^2,\;\; f_1 = 0 \,\right]$$

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  - $\displaystyle\int \frac{(2k \cdot v)^2}{D_0 D_1} = \frac{(2K_1 \cdot v)^2}{3m^2} A_0(m^2) \;\rightsquigarrow\; \hat{b}_{00}$ needed
  - $\displaystyle\int \frac{1}{D_0} = \int \frac{1}{D_1} = A_0(m^2) \;\rightsquigarrow\; a(0)$ & $a(1)$ needed
  - $\displaystyle\int \frac{1}{D_0 D_1} = \frac{A_0(m^2)}{m^2} \;\rightsquigarrow\; b$ needed

Single cut & Tadpoles – II

$$\mathcal{I}_{2,2} = \frac{(2R_1 \cdot k)(2R_2 \cdot k)}{D_0 D_1}, \qquad \left[\, D_0 = k^2 - m^2, \;\; D_1 = (k - K_1)^2 - m^2, \;\; f_1 = 0 \,\right]$$

We want the coefficient of $A_0(m^2)$, $a_{\text{total}} = a(0) + a(1) + \frac{b}{m^2} + \hat{b}_{00}\frac{4(K_1 \cdot v)^2}{3m^2}$

Strategy: 1) extract $a(0) + a(1)$ & $\hat{b}_{00}$ using the $\Lambda^2 t$ terms of the cut

2) extract b selecting the logarithmic coefficients of the cut

$$\begin{aligned} a(0) + a(1) &\quad : \quad (R_1 \cdot R_2) \\ \frac{4(K_1 \cdot v)^2}{3m^2}\hat{b}_{00} &\quad : \quad \frac{4(K_1 \cdot R_1)(K_1 \cdot R_1)}{3m^2} \\ \frac{1}{m^2}b &\quad : \quad (R_1 \cdot R_2) \end{aligned}$$

Single cut & Tadpoles – II

$$\mathcal{I}_{2,2} = \frac{(2R_1 \cdot k)(2R_2 \cdot k)}{D_0 D_1}, \qquad \left[\, D_0 = k^2 - m^2,\;\; D_1 = (k - K_1)^2 - m^2,\;\; f_1 = 0 \,\right]$$

We want the coefficient of $A_0(m^2)$, $a_{\text{total}} = a(0) + a(1) + \frac{b}{m^2} + \hat{b}_{00}\frac{4(K_1 \cdot v)^2}{3m^2}$

Strategy: 1) extract $a(0) + a(1)$ & $\hat{b}_{00}$ using the $\Lambda^2 t$ terms of the cut

2) extract b selecting the logarithmic coefficients of the cut

$$\begin{aligned}
a(0) + a(1) &\quad : \quad (R_1 \cdot R_2) && + \\
\frac{4(K_1 \cdot v)^2}{3m^2}\hat{b}_{00} &\quad : \quad \frac{4(K_1 \cdot R_1)(K_1 \cdot R_1)}{3m^2} && + \\
\frac{1}{m^2} b &\quad : \quad (R_1 \cdot R_2) && = \\
a_{\text{total}} &\quad : \quad 2R_1 \cdot R_2 + \frac{4}{3m^2}(K_1 \cdot R_1)(K_1 \cdot R_2) && \text{[OK with PV]}
\end{aligned}$$

Conclusions & Outlook

Conclusions

- Auxiliary propagator method
  - exploits D-dimensional double cut
  - gives the tadpole coefficients
  - "free" from spurious terms
- Single cut
  - useful to compute the tadpole coefficients . . .
  - . . . and the bubble coefficients as well!
  - spurious terms enter
  - ok when the Gram determinant, Δ_G, vanishes

Outlook

- Study the auxiliary propagator method in the $\Delta_G = 0$ case
- Apply the single cut to get the (massless) bubble coefficients
- Apply both methods to physically relevant cases